Evolutionary equilibrium in Tullock contests: spite and overdissipation

B. Hehenkamp[a], W. Leininger[b,*] and A. Possajennikov[c]

[a] University of Dortmund, Department of Economics, 44221 Dortmund
[b] University of Dortmund, Department of Economics, 44221 Dortmund
[c] University of Nottingham, School of Economics, Nottingham NG7 2RD

Received 7 November 2002; received in revised from 28 April 2003; accepted 18 August 2003

Abstract

Tullock's analysis of rent-seeking as a contest is reconsidered from an evolutionary point of view. We show that evolutionarily stable behavior in a Tullock contest exists and differs from behavior in Nash equilibrium. Evolutionarily stable behavior in these contests is robust in a strong sense and may entail overdissipation of the contested rent.

JEL Classification Numbers: C 79, D 72

Keywords: Contests, Evolutionary stable strategy, Spite, Overdissipation

*W.Leininger@wiso.uni-dortmund.de
Tel. 0231-7553297, Fax: 0231-7553027

1. Introduction

Lockard and Tullock (2001) have recently edited a volume on "Efficient Rent-Seeking", which documents Tullock's classic analysis of rent-seeking competition as a contest and its influence on the subsequent rent-seeking literature. This literature by now also forms an important part of general contest theory as many of its modeling features – like the specification of contest success functions – are of more universal nature. The theory of rent-seeking and contest theory as a whole are based on Nash equilibrium as a solution concept. Evolutionary equilibrium – as presented in this paper – is based on the notion of an evolutionarily stable strategy (ESS) as a solution concept, which focusses on behavior (and its diffusion) itself rather than on choice of behavior. An ESS is the simplest solution concept from evolutionary game theory, which has been used to justify rationalistic Nash equilibrium. However, the popular view that an ESS represents a *refinement* of Nash equilibrium is only correct, if one applies the evolutionary argument – often implicitly, but not explicitly assumed – to an *infinite* population of players. For finite populations, a staple in the rent-seeking and contest literature, an ESS *may differ* from Nash equilibrium (Schaffer, 1988). It indeed does so in the case of Tullock contests: we prove existence of an ESS and explore the implications of these differences in contest behavior in detail.

This difference is of interest from a principal point of view: Alchian (1950) argued in his classic essay on evolutionary economics that the postulate of maximization (as in Nash equilibrium) may be false but that its use is justified by the tenets of "survival of the fittest" under (recurrent) competition. Economists usually read "competition" as "perfect competition" (or "price-taking" behavior), which implicitly requires a large ("almost" infinite) population. In more general terms, however, competition means a contest or fight between several claimants, who aim to "beat" each other. Evolutionary behavior precisely corresponds to this second meaning and is therefore highly relevant for contest theory. Yet, if only finitely many contestants compete recurrently, e.g. lobbyists whenever elections or law-making initiatives come up, then Alchian's argument is shown to lead to a maximization postulate, which is different from the one embodied in Nash equilibrium, namely, relative payoff maximization. Leininger (2003) shows that for the class of contest success functions axiomatized by Skaperdas (1996) these two maximization postulates cannot determine the same solution; i.e. evolutionary equilibrium and Nash equilibrium always differ. Moreover, he relates behavior in evolutionary equilibrium of a contest to behavior in Nash equilibrium in a transformed zero-sum game, which can be interpreted as a transfer contest game. His paper, however, is completely silent on the issue of existence of both types of

equilibria, in particular ESS, in these games. For a class of games with aggregative payoff structure Possajennikov (2003) adapts the notion of "price-taking" behavior in large player populations to finite populations just as Schaffer (1988) adapted ESS from large populations to finite populations and relates his new notion of "aggregate-taking equilibrium" to Schaffer's notion of ESS. He shows that these two solution concepts can determine the same solutions.
The present paper considers the special, but important class of contests introduced by Tullock (1980) and *determines* behavior in evolutionary equilibrium by explicitly providing a proof of existence and uniqueness of ESS for these games. It then relates the unique ESS to the unique Nash equilibrium. The difference is larger the smaller the number of contestants. Prominent and important examples of contests between just two contestants are provided by litigation lawsuits. Hirshleifer and Osborne (2001) model trials as contests with the help of a "litigation success function" of the Tullock type considered here. Baye et al. (2001) model litigation systems as (first-price) all-pay auctions, which form a limiting case of the family of Tullock contest success functions. In both of these models litigants influence the merit of their cases – and hence their winning probabilities – by hiring skillful attorneys and other inputs. Leaving aside the agency problem between litigant and attorney and supposing equal access to legal services – as these authors do – portrays typical litigation trials as recurrent games played through attorneys sufficiently often to warrant an evolutionary approach. This approach is extended to contests over public goods by Leininger (2002), who proves existence of a *local* evolutionarily stable strategy, which does coincide with Nash equilibrium if and only if competing groups (with at least two members) are of the same size.

Section 2 recollects Tullock's analysis of efficient rent-seeking[1], in which he introduced an important class of contest success functions. Section 3 motivates our new solution concept and shows that efficient rent-seeking is not evolutionarily stable. Section 4 proves existence of an evolutionarily stable strategy and discusses its properties. Most remarkably, an overdissipation result can be proven. Section 5 shows that the type of "spiteful" behavior, which drives the results obtained in the previous section, is extremely robust, which motivates a short inquiry into its general relationship to Nash equilibrium behavior in section 6. Section 7 concludes.

[1]Following Tullock (1980) and the subsequent literature we refer to Nash equilibrium behavior in a rent-seeking contest as "efficient" rent-seeking, although it is not efficient in the usual economic sense of the word.

2. Efficient rent-seeking

Recall that Tullock (1980) proposed a model, in which n rent-seekers compete for a rent of size V. If the contestants expend $x = (x_1, \ldots, x_n), x_i \geq 0$, the probability of success for player $i, i = 1, \ldots, n$ is given by

$$p_i(x_1, \ldots, x_n) = \frac{x_i^r}{\sum_{j=1}^n x_j^r}$$

and expected profit for player i is given by

$$\Pi_i(x_1, \ldots, x_n) = p_i(x_1, \ldots, x_n) \cdot V - x_i = \frac{x_i^r}{\sum_{j=1}^n x_j^r} \cdot V - x_i.$$

One can show that for $r \leq \frac{n}{n-1}$ a unique Nash equilibrium in pure strategies exists in this game, in which each player maximizes expected payoff by bidding

$$x^* = \frac{n-1}{n^2} \cdot r \cdot V.$$

Aggregate rent-dissipation then amounts to

$$n \cdot x^* = \frac{n-1}{n} \cdot r \cdot V.$$

Equilibrium expenditures never exceed V, the value of the rent, but may be strictly less than V. The "full rent dissipation"-hypothesis does not hold; yet overdissipation is incompatible with individually rational payoff maximization as it would imply, that at least one contestant has a negative payoff in equilibrium (and would therefore be better off by non-participation or bidding zero).

Important variations of this basic model include Corcoran and Karels (1985), Higgins et al. (1987), Hillman and Samet (1987), Leininger (1993), Leininger and Yang (1994) and Baye et al. (1994), in which the analysis is extended to mixed strategies and extensive form, dynamic games. Most recently, Baye and Hoppe (2002) have shown that a Tullock contest of this type is strategically equivalent to innovation tournaments in the form of patent races.

3. Evolutionarily stable strategies

An alternative to the rational-expectations approach of the previous section is to resort to a more biologically inspired view of human behavior; namely that economic behavior diffuses, stabilizes, mutates and disappears along an evolutionary path that leads to the survival of best or best adapted strategies or standards of behavior. A particularly useful concept in this respect is the notion of an evolutionarily stable strategy (ESS) as defined by Maynard Smith (Maynard Smith and Price (1973), Maynard Smith (1974, 1982)), because it allows one to say something about the stable dynamic properties of an evolutionary system without the need to commit oneself to specific dynamics. We follow Schaffer (1988), who adapted the ESS notion to finite populations of interacting agents.

A strategy is evolutionarily stable, if a whole population using that strategy cannot be invaded by a sufficiently small group of "mutants" using another strategy. Similarly, a standard of behavior in an economic contest is evolutionarily stable, if – upon being adopted by all participants in the contest – no small subgroup of individuals using a different standard of behavior can invade and "take over". Obviously, in the context of finite populations the smallest meaningful number of mutants is one. The emphasis of the evolutionary approach is not on explaining actions (as a result of particular choice or otherwise), but on the diffusion of forms of behavior in groups (as a result of learning, imitation, reproduction or otherwise).

The definition of invadability is all important:

Definition 1:

i) Let a strategy (standard of behavior) x be adapted by all players $i, i = 1, \ldots, n$. A mutant strategy $\bar{x} \neq x$ can invade x, if the payoff for a single player using $\bar{x}$ (against x of the $(n-1)$ other players) is strictly higher than the payoff of a player using x (against $(n-2)$ other players using x and the mutant using $\bar{x}$).

ii) A strategy x^{ESS} is evolutionarily stable, if it cannot be invaded by any other strategy.

Roughly speaking, an ESS is such that, if almost all members of a group adopt it, there is no other strategy that could give a higher relative payoff, if used by a group member. The dynamic justification for this notion of equilibrium is, that more successful strategies diffuse or "reproduce" faster than less successful ones and ultimately extinguish the latter. One obvious source of such dynamics behind finite population ESS is imitative behavior on behalf of contestants. In real-life rent-seeking this may occur in situations in which contestants do not perfectly know each other's payoff functions and hence resort to simply comparing expected payoffs by averaging over several contests, which they observe.

We now formalize ESS in the context of Tullock's contests and search for an effort or expenditure level x which qualifies as an ESS.

Denote by x the expenditure profile $(x_1, \dots, x_n) = (x, \dots, x); x$ can be invaded, if there exists $x_1 = \bar{x}$, say, such that

$$\Pi_1(\bar{x}, x, \dots, x) > \Pi_i(\bar{x}, x, \dots, x) \qquad \text{for} \quad i = 2, \dots, n.$$

Consequently, a strategy x^{ESS} is an ESS if and only if

$$\Pi_1(\bar{x}, x^{ESS}, \dots, x^{ESS}) < \Pi_i(\bar{x}, x^{ESS}, \dots, x^{ESS})$$
$$\text{for} \quad i = 2, \dots, n \ \text{ and for all} \quad \bar{x} \neq x^{ESS}.$$

This is equivalent (see Schaffer, 1988) to demanding that x^{ESS} must maximize the relative payoff function

$$\Pi_1(\bar{x}, x^{ESS}, \dots, x^{ESS}) - \Pi_i(\bar{x}, x^{ESS}, \dots, x^{ESS})$$

of player 1 with $i \in \{2, \dots, n\}$. Our first result states that the unique Nash equilibrium strategy x^* does *not* do so:

Theorem 1: The unique (pure strategy) Nash equilibrium of a Tullock contest for $r \leq \frac{n}{n-1}$ is not evolutionarily stable.

Proof: We claim that $x^* = \frac{n-1}{n^2} \cdot r \cdot V$ – the Nash standard of behavior – is not an ESS!

We prove this by showing that x^* does not locally maximize the relative payoff function of player 1:

$$\Pi_1(\bar{x}, x^*, \dots, x^*) - \Pi_i(\bar{x}, x^*, \dots, x^*) = \frac{\bar{x}^r - x^{*^r}}{\bar{x}^r + (n-1) \cdot x^{*^r}} \cdot V - (\bar{x} - x^*)$$

Clearly, $\frac{\partial \Pi_1}{\partial \bar{x}} = 0$ at $\bar{x} = x^*$ since x^* is a Nash equilibrium strategy for player 1; however,

$$\frac{\partial \Pi_i}{\partial \bar{x}} = \frac{-x^{*^r} \cdot r \cdot \bar{x}^{r-1}}{(\bar{x}^r + (n-1)x^{*^r})^2} \cdot V < 0 \qquad \text{for } i = 2, \dots, n$$

since a marginal increase in $\bar{x}$ *decreases* an opponent's probability of winning. Consequently, a marginal increase of $\bar{x}$ from x^* to $x^* + \triangle$ has a *second-order* negative effect on i's own payoff and a *first-order* negative effect on an opponent's payoff. Therefore i's *relative* payoff *increases*, if expenditures rise above the Nash level x^*. q.e.d.

The proof shows that more aggressive behavior than shown by a (rational) Nash-strategist does better in *relative* terms! The more aggressive mutant does – of course – not better in absolute terms, because he does not play a *best response* (that would be x^*). This "loss" in absolute terms is more than offset by a gain in *relative* terms, he now has the advantage of the highest payoff realized among all the contestants, because his higher aggressiveness lowers the opponents payoffs by more than it lowers his own! This kind of behavior has been called "spiteful" (Hamilton, 1971) in the sense that an ESS-strategist pursues not only a larger payoff for himself but also a lower payoff for his competitors. Consequently, in a stable group of all ESS-strategists and no mutants an ESS-strategist is not in general maximizing his payoff or fitness, but the difference between his own payoff and the average payoff of the other players (Schaffer, 1988).

Relative considerations are undoubtedly important in a contestant's calculus. The contest success function of the Tullock model is homogeneous of degree zero; i.e. $p_i(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) = p_i(x_1, \ldots, x_n)$ for all $i = 1, \ldots, n$, and thus only depends on the *relative* size of bids: e.g. set $\lambda = \frac{1}{x_1}$ to get $p_i = p(1, \frac{x_2}{x_1}, \ldots, \frac{x_n}{x_1})$. Axiomatic characterisations of contest success functions (e.g. Kooreman and Schoonbeek, 1997) precisely use zero-homogeneity as an axiom to account for the importance of *relative* concerns in contests. E.g. Hirshleifer and Osborne (2001), who use these functions as 'litigation success functions' in their model of trials, observe that "in lawsuits *relative* success depends on two main factors: the true degree of fault and the efforts invested on each side" (p. 131, our italics). The latter serve primarily to be able to present a *relatively* better case, they may explicitly include expenditures that serve to weaken the opponent's case.
It therefore makes sense to explore the implications of evolutionary equilibrium, which can be thought of as guiding behavior of contestants, who not only care about relative efforts but also about relative success.

4. Evolutionary rent-seeking

We now show existence of an ESS for Tullock contests and discuss its properties.

Theorem 2: There exists a unique ESS of a Tullock contest for $r \leq \frac{n}{n-1}$. It is given by

$$x^{ESS} = (\frac{r}{n} \cdot V, \frac{r}{n} \cdot V, \ldots, \frac{r}{n} \cdot V).$$

Proof: Let $\bar{x}$ denote the mutant strategy and x denote the candidate strategy for an ESS.

Consider the maximization problem

$$max_{\bar{x}} \ \frac{\bar{x}^r - x^r}{\bar{x}^r + (n-1)\cdot x^r} \cdot V - \bar{x} + x$$

whose first-order condition reduces (in a straightforward manner) to

$$\text{(FOC)} \qquad \frac{n(\bar{x}\cdot x)^r}{(\bar{x}^r + (n-1)\cdot x^r)^2 \cdot \bar{x}} \cdot r \cdot V = 1$$

Furthermore, since ESS requires identical behavior of contestants, we set $\bar{x} = x$ in search of a symmetric solution. This gives

$$n \cdot x^{2r} \cdot r \cdot V = n^2 \cdot x^{2r+1}$$

which is solved by $x = \frac{r}{n} \cdot V$. This is our only *interior* solution candidate. The second-order condition for a local maximum becomes – upon differentiating FOC –

$$\frac{[r\cdot \bar{x}^{2r}\cdot x^r + r\cdot \bar{x}^r x^{2r}\cdot n - r\cdot \bar{x}^r \cdot x^{2r} - 2r\cdot \bar{x}^{2r}\cdot x^r}{\bar{x}^2[\bar{x}^r + (n-1)x^r]^3}$$
$$+\frac{-\bar{x}^{2r}\cdot x^r - \bar{x}^r \cdot x^{2r}\cdot n + \bar{x}^r x^{2r}]\cdot r\cdot V\cdot n}{\bar{x}^2[\bar{x}^r + (n-1)x^r]^3} < 0.$$

Simplification leads to

$$\frac{\bar{x}^r \cdot x^r[(r-1)(n-1)x^r - \bar{x}^r(r+1)]\cdot r\cdot V\cdot n}{\bar{x}^2[\bar{x}^r + (n-1)x^r]^3} < 0 \ ; \qquad \text{(SOC)}$$

this holds in the symmetric solution $\bar{x} = x = \frac{r\cdot V}{n}$, if

$$(r-1)(n-1)(\frac{r\cdot V}{n})^r - (\frac{r\cdot V}{n})^r(r+1) < 0$$

or, equivalently, $r\cdot n - 2r - n < 0$. Thus, the second-order condition for local maximization holds for

$$r < \frac{n}{n-2}.$$

Also note that (SOC) implies global concavity of the relative payoff function, if $r \leq 1$ holds: The bracketed term in the numerator is always negative, the one in the denominator always positive. Hence, the local optimum is a global one for all $r \leq 1$. If $r > 1$, then (SOC) shows that the relative payoff function is locally convex for sufficiently small x: substituting

$$x = \frac{r\cdot V}{n}$$

into (SOC) yields

$$\frac{\bar{x}^r \left(\frac{r\cdot V}{n}\right)^r \left[(r-1)(n-1)\left(\frac{r\cdot V}{n}\right)^r - \bar{x}^r(r+1)\right]\cdot r\cdot V\cdot n}{\bar{x}^2\left[\bar{x}^r + \left(\frac{r\cdot V}{n}\right)^r\cdot n - \left(\frac{r\cdot V}{n}\right)^r\right]^3}.$$

This expression is positive for all $0 < \bar{x} < \hat{x}$ and negative for all $\bar{x} > \hat{x}$, where $\hat{x}$ solves the equation

$$(r-1)(n-1)\left(\frac{r\cdot V}{n}\right)^r - \bar{x}^r(r+1) = 0$$

i.e.

$$\hat{x} = \left(\frac{r-1}{r+1}\cdot(n-1)\left(\frac{r\cdot V}{n}\right)^r\right)^{\frac{1}{r}} = \left(\frac{r-1}{r+1}\right)^{\frac{1}{r}}\cdot(n-1)^{\frac{1}{r}}\cdot\frac{r\cdot V}{n}.$$

As a consequence, the relative payoff function is convex on $(0,\hat{x})$ and concave on $(\hat{x},\infty)$. Moreover, we have

$$\hat{x} < \frac{r\cdot V}{n} = x^{ESS}$$

if and only if

$$\left(\frac{r-1}{r+1}\right)^{\frac{1}{r}}\cdot(n-1)^{\frac{1}{r}} < 1.$$

The latter is equivalent to $r < \frac{n}{n-2}$. Hence, in this range the only candidates for a global maximum of the relative payoff function are 0 and $\frac{r\cdot V}{n}$.
Obviously, $\Pi_1(x,x,\ldots,x) - \Pi_i(x,x,\ldots,x) = 0$
whereas $\Pi_1(0,x,\ldots,x) - \Pi_i(0,x,\ldots,x) = \frac{n(r-1)-r}{n(n-1)}\cdot V$.
The latter is less or equal to zero, if $r \leq \frac{n}{n-1}$.
This completes the proof that x^{ESS} is indeed an ESS as $\frac{n}{n-1} < \frac{n}{n-2}$. Finally note, that the corner solution $x = (0,\ldots,0)$ is obviously not an ESS as it can be invaded by any $x \in (0,V)$.
This completes the proof of Theorem 2.

Theorem 2 says that a unique evolutionary equilibrium exists under precisely those conditions, which imply existence of a unique Nash equilibrium in pure strategies. Combined with Theorem 1 this means that the two equilibrium concepts always yield different predictions of behavior for this class of Tullock contests.

The following properties of evolutionary equilibrium are now immediate:

Theorem 3:

i) Individual expenditures and aggregate rent-dissipation in the unique evolutionary equilibrium $x = (\frac{r}{n} \cdot V, \ldots, \frac{r}{n} \cdot V)$ are always higher than in the unique efficient (Nash) equilibrium $x^* = (\frac{n-1}{n^2} \cdot r \cdot V, \ldots, \frac{n-1}{n^2} \cdot r \cdot V)$.

ii) Aggregate rent-dissipation in the unique evolutionary equilibrium is independent from the number of contestants, it is solely determined by the rent-seeking technology (contest success function) and the value of the rent:

$$n \cdot \frac{r}{n} \cdot V = r \cdot V$$

iii) For $r > 1$ there is overdissipation of the rent in the unique evolutionary equilibrium; for $r = 1$ there is full dissipation of the rent and for $r < 1$ there is underdissipation of the rent.

The occurence of overdissipation of the rent in a *pure strategy* ESS is the consequence of spiteful behavior in the presence of increasing returns to expenditures ($r > 1$). Evolutionary stability, or alternatively behavior driven by relative payoff maximization, *necessitates* spite and spite fuelled by increasing returns to expenditures leads to overdissipation of the contested prize. This – partly – vindicates Tullock's "overdissipation postulate", a target of much (justified) criticism, in the confines of his very own model (for a discussion of the overdissipation debate see Baye et al. (1999), who discuss overdissipation as a possible *ex post* feature of mixed strategy equilibria in the all-pay auction, which results from the Tullock family of contests, if $r \to \infty$).
Note that for $n = 2$ and $r = 2$, the limiting case of our existence result in Theorem 2, rent dissipation is *twice* the amount of the rent as each contestant spends up to the value of the rent! Hirshleifer and Osborne (2001, p. 174) argue that in judicial systems, in which "judges were to limit themselves to procedural issues and refrain from instructing juries on the substantive merits of the case", contest expenditures (e.g. for 'star' attorneys) may well exhibit increasing returns; i.e. trials are then contests with $r > 1$. Our findings are then in line with the often observed and lamented fact, that litigants spend too much compared to what is at stake in litigation cases.

An interesting property of Tullock contests is that allowing for several entrants at a time does *not* affect the expenditure level in an ESS. This strong stability property is shown in the next section.

5. Global stability

The definition of an ESS given above demands stability of behavior against *precisely* one mutant. A generalized ESS (Schaffer 1988) also accounts for the possibility of the occurence of several identical mutants:

Definition 2:

i) A strategy x^{ESS} is evolutionarily M-stable, if a population of n players using it cannot be invaded by m identical mutants all using any other strategy $\bar{x}$ for $1 \leq m \leq M$ and $M \leq n-1$.

ii) A strategy, that is $(n-1)$-stable, is called globally stable[2].

We now claim that an ESS of a Tullock contest, whose existence is assured by Theorem 2, is always globally stable. This gives further strength to the properties of an ESS described in Theorem 3.

Theorem 4 (Global stability):

Let $r \leq \frac{n}{n-1}$, then it holds that the unique ESS is globally stable.

Proof : We use induction over the number of mutants $m \leq n-1$. Obviously, Theorem 2 provides an induction start for our candidate $x = \frac{r}{n} \cdot V$ as it is 1-stable. Suppose then, that $x = \frac{r}{n}V$ is m-stable; i.e. if Π_M^m denotes the relative payoff function of one of m mutants, then our induction hypothesis means, that

$$\text{(IH)} \qquad \Pi_M^m = \frac{\bar{x}^r - (\frac{r}{n}V)^r}{m \cdot \bar{x}^r + (n-m)(\frac{r}{n}V)^r} \cdot V - (\bar{x} - \frac{r}{n}V) < 0 \qquad \text{for all } \bar{x} \neq \frac{r}{n}V.$$

We now claim, that $x = \frac{r}{n}V$ is in fact $(m+1)$-stable; i.e. our claim is

$$\text{(C)} \quad \Pi_M^{m+1} = \frac{\bar{x}^r - (\frac{r}{n}V)^r}{(m+1)\bar{x}^r + (n-m-1)(\frac{r}{n}V)^r} \cdot V - (\bar{x} - \frac{r}{n}V) < 0 \quad \text{for all } \bar{x} \neq \frac{r}{n}V.$$

Rewrite (C) as

[2] If a strategy x is M-stable, but not $(M+1)$-stable for $M < n-2$, then M is called the 'degree of stability' of x (Schaffer 1988).

$$\left(\frac{\bar{x}^r - (\frac{r}{n}V)^r}{(m+1)\bar{x}^r + (n-m-1)(\frac{r}{n}V)^r} \cdot V - \frac{\bar{x}^r - (\frac{r}{n}V)^r}{m\bar{x}^r + (n-m)(\frac{r}{n}V)^r} \cdot V\right)$$
$$+ \left(\frac{\bar{x}^r - (\frac{r}{n}V)^r}{m\bar{x}^r + (n-m)(\frac{r}{n}V)^r} \cdot V - (\bar{x} - \frac{r}{n}V)\right)$$

Then it follows from (IH), that the term in the second bracket is negative for all $\bar{x} \neq \frac{r}{n}V$. It hence remains to be shown that the term in the first bracket is also negative for all $x \neq \frac{r}{n}V$.
If $\bar{x} < (>)\frac{r}{n}V$, then the denominator of the first term is *smaller* (*larger*) than the one of the second term, while the common numerator is *negative* (*positive*). In both cases it follows, that the difference must be negative and hence (C) must hold. This completes the proof.

A more heuristic argument sheds further light on the somewhat unintuitive property, that in a Tullock contest 1-stability implies $(N-1)$-stability: consider again the relative payoff function of one out of m mutants, if the other contestants in the invaded population use $x^{ESS} = \frac{r}{n} \cdot V$,

$$\Pi_M^m = \frac{\bar{x}^r - (\frac{r}{n}V)^r}{[m \cdot \bar{x}^r + (n-m)(\frac{r}{n}V)^r]} \cdot V - (\bar{x} - \frac{r}{n}V) \ ;$$

and note that m only exerts influence on the probability of winning the contest and not expenditures. We now show that this influence is always a negative one. For this we treat m as a continuous variable and compute

$$\frac{\partial \Pi_M^m}{\partial m} = \frac{-(\bar{x}^r - (\frac{r}{n}V)^r)(\bar{x}^r - (\frac{r}{n}V)^r)}{[m \cdot \bar{x}^r + (n-m)(\frac{r}{n}V)^r]^2} \cdot V = -\left[\frac{\bar{x}^r - (\frac{r}{n}V)^r}{m \cdot \bar{x}^r + (n-m)(\frac{r}{n}V)^r}\right]^2 \cdot V.$$

This expression is negative whenever $\bar{x} \neq \frac{r}{n} \cdot V$ and zero otherwise. Hence it follows, that any strategy that cannot invade x^{ESS} with a single carrier (i.e. $\Pi_M^1 < 0$) cannot do so, if it replicates. The effect of an additional 'clone' of a by itself unsuccessful strategy, which *replaces* an ESS-strategist in the contest, works to the disadvantage of the mutants by unambiguously reducing the *difference* in winning probabilities between them and ESS-strategists. Stability against 'small' invasions is synonymous with stability against *any* invasion (by identical mutants).

6. Evolutionary versus Nash equilibrium

The stability result of section 5 adds to the significance of the difference between behavior in evolutionary and Nash equilibrium. The difference is a consequence of considering a finite population of contestants: according to Definition 1 any single mutant will never meet another mutant, whereas every ESS-strategist will always meet exactly one mutant. This changes the nature of the evolutionary stability argument: it now pays to reduce one's own success, if that reduces the other strategy's success even more. In contrast, such spiteful behavior cannot survive in an infinite population, since there is always a large number of matchings involving ESS-strategies exclusively. The following construction is due to Schaffer (1988) and shows – translated into our context – how spiteful behavior becomes less and less rewarding with an increasing number of finitely many contestants. The framework takes account of a full range of different contest scenarios, its polar cases correspond to the "playing-the-field" scenario of section 4 and the infinite population scenario originally proposed by Maynard Smith, respectively.

Consider a group of N potential contestants, who may recurrently engage in a contest of size $n < N$; i.e. only n out of N contestants compete in the contest (so far we have considered the case $n = N$, in which each contestant "plays the field"). The expected payoff of a single mutant among the N contestants if he is drawn into a contest and plays a strategy $\bar{x}$ against (n-1) other players using strategy x is then still given by

$$\Pi_1(\bar{x}, x, \ldots, x) = \frac{\bar{x}^r}{(n-1)x^r + \bar{x}^r} \cdot V - \bar{x} \qquad ;$$

but one of the other (n-1) players $i, i \in \{2, \ldots, n\}$, chosen for the contest expects

$$\Pi_i = \left(1 - \frac{n-1}{N-1}\right) \cdot \Pi_i(x, \ldots, x) + \frac{n-1}{N-1} \cdot \Pi_i(\bar{x}, x, \ldots, x)$$

as the probability, that a chosen player i will face the mutant player 1 from the remaining (N-1) potential players among the further chosen (n-1) players is $\frac{n-1}{N-1}$. Note, that we have assumed that players are chosen for participation randomly and with equal probability.
Consequently, an ESS strategy x^{ESS} must now solve the problem (Schaffer 1988) of relative payoff maximization

$$max_x \ \Pi_1(x, x^{ESS}, \ldots, x^{ESS}) - \left(1 - \frac{n-1}{N-1}\right) \cdot \Pi_i(x^{ESS}, \ldots, x^{ESS})$$
$$- \ \frac{n-1}{N-1} \cdot \Pi_1(x, x^{ESS}, \ldots, x^{ESS})$$

Eliminating the constant term $\left(1 - \frac{n-1}{N-1}\right) \cdot \Pi_i(x^{ESS}, \ldots, x^{ESS})$ equivalently yields

$$max_x \ \Pi_1(x, x^{ESS}, \ldots, x^{ESS}) - \frac{n-1}{N-1} \cdot \Pi_1(x, x^{ESS}, \ldots, x^{ESS})$$

Again, if x^{ESS} solves the above problem, then $x^* = (x^{ESS}, \ldots, x^{ESS})$ cannot be invaded by any strategy $x \neq x^{ESS}$. We can directly read off from the maximand, that as $N \to \infty$ we approach the Nash equilibrium problem and hence the difference in behavior among n contestants in Nash equilibrium and among n contestants (chosen out of a large population of potential contestants) in evolutionary equilibrium disappears.

In fact, it is not difficult to show, that for any n out of N contestants an ESS of a Tullock contest exists; hence we record

Theorem 5: Assume $r \leq \frac{n}{n-1}$ and let N be the number of potential contestants, who are drawn into a Tullock contest of size n, $n \leq N$. Then the unique ESS is given by

$$x^{ESS} = \frac{(n-1)N}{(N-1)n^2} \cdot r \cdot V \qquad .$$

This expression specializes to $x^{ESS} = \frac{r}{n}V$, if $n = N$, and approaches $x^* = \frac{n-1}{n^2}rV$, the Nash equilibrium effort, if N approaches infinity. Also note, that aggregate expenditures are $\frac{(n-1)N}{(N-1)n} \cdot rV$, so the overdissipation result still holds for r sufficiently close to $\frac{n}{n-1}$: if $r = \frac{n}{n-1}$, then $n \cdot x^* = \frac{N}{N-1} \cdot V > V$.
The framework of this section fits the case of litigation lawsuits fought by attorneys well. Plaintiff and defendant each choose a lawyer from a pool of authorized lawyers, who then engage in the contest.

7. Conclusion

We have examined behavior in Tullock's classic rent-seeking contest from the point of view of evolutionary stability in finite populations, our findings have bearings for contest theory in general.
Evolutionarily stable behavior in Tullock contests leads to higher efforts of contestants than Nash behavior and may entail overdissipation of the contested "rent" or stakes. Contestants behave as if they aim to maximize relative payoff, a goal which is not only furthered by increasing one's own payoff, but also by lowering the payoff of others. This additional motive justifies additional, "spiteful" investments into the contest. Moreover, spiteful behavior in an ESS is a very robust property; i.e. it is stable against any number of identical deviants. Finally, we have shown

that Nash equilibrium behavior in Tullock contests results from evolutionary equilibrium behavior of an infinite population, which engages in contests with finitely many participants.

Acknowledgements

We wish to thank an anonymous referee for helpful comments and participants in the PET 2002 conference (Parais) for useful suggestions on an earlier draft. We also thank Anna Ania for insightful discussions.

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