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Daniel J. Seidmann

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Karina Terry  
Centre for Decision Research and Experimental Economics  
School of Economics  
University of Nottingham  
University Park  
Nottingham  
NG7 2RD  
Tel: +44 (0) 115 95 15620  
Fax: +44 (0) 115 95 14159  
[karina.terry@nottingham.ac.uk](mailto:karina.terry@nottingham.ac.uk)

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# A Theory of Voting Patterns and Performance in Private and Public Committees

Daniel J. Seidmann <sup>\*†</sup>

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## Abstract

We analyze voting in private and public committees whose members care about the selected decision and the rewards which outsiders pay for representing their interests. If the agenda is binary or outsiders are symmetric then a private committee reaches decisions which better serve organizational goals than either a public committee or a randomly chosen committee member; whereas symmetric outsiders are best served by a public committee. The voting patterns of both private and public committees may fail Duverger's Law, but they both satisfy a weaker condition: Dissidents in private [resp. public] committees all vote decisions which better [resp. worse] serve organizational goals than the plurality decision; so single-peakedness implies that all dissents lie on one side of the plurality decision.

## 1. INTRODUCTION

Many committees conceal their voting patterns from outsiders: for example, the doctrine of collective responsibility requires all members of the British Cabinet to deny that they ever dissented; while the ECB's Governing Council and the Bundesbank only report their collective decisions. Issing (1999) pp. 512-3 defends the ECB's procedure by claiming that privacy insulates members from outside pressure, allowing the committee to better pursue its organizational goals and to reach consensual decisions; while Pound (1988) claims that confidential voting would prevent management from rewarding shareholders with business links to the firm for voting in favor of proposals which reduce shareholder value.<sup>1</sup>

While Issing's claim seems plausible, the argument requires some elaboration, as privacy can only completely insulate members if outsiders are naive: for a sophisticated

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<sup>\*</sup>School of Economics, Nottingham University, University Park, Nottingham NG7 2RD, England, daniel.seidmann@nottingham.ac.uk

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<sup>1</sup>Institutional investors have often proposed such a reform, which is a core policy of the Council of Institutional Investors.

outsider would infer the voting pattern from a private committee’s collective decision, and punish/reward members according to this inference. Members’ incentives then depend on the inferences which outsiders draw from unexpected decisions; and these, in turn, depend on the (endogenous) voting pattern. Issing’s argument does not address these issues and, to that extent, is unpersuasive.<sup>2</sup>

We explore Issing’s defence of private voting by analyzing complete information models of committee voting in which members each have dual loyalties: they care about the committee’s performance (how well its decisions serve organizational goals) and about the rewards they each receive from interested outsiders; while outsiders reward members for their votes. We compare the performance of committees which vote in private and in public, as well as the performance of a one-member committee (a ‘dictator’), whose voting pattern is, by definition, public.

Each extensive form is very simple: members of a committee vote simultaneously over a fixed agenda which consists of two or more feasible (collective) decisions, with the committee reaching a decision which secures most votes; and each outsider then rewards a member: after observing the decision alone in a private committee, or the decision and the voting pattern in a public committee. These are variants on conventional models of simultaneous, plurality-rule voting, which are known to possess multiple Nash equilibria. We obtain sharp results by focusing on equilibria which are both trembling-hand perfect and coalition proof: the former refinement pins down the rewards paid after the committee reaches an unexpected decision; the latter solves the coordination problem that generates the multiplicity in conventional models.

We characterize performance and voting patterns in private and public committees for two benchmark cases: when the agenda consists of two decisions; and when outsiders are symmetric in the sense that they share a common preference ordering over the members’ votes, but the agenda may contain three or more decisions. We also use examples to describe performance and voting patterns in more general settings. We show that

- The three mechanisms perform equally if members either care enough about performance or, in the benchmark cases, if they care enough about their rewards. However, private committees outperform both public committees and a randomly selected dictator in benchmark cases with an intermediate trade-off between performance and rewards. This result supports Issing’s claim, and may explain why collective decisions are often delegated to committees, even if members do not bring independent information to the table. More surprisingly, these benchmark results do not fully generalize to committees with asymmetric outsiders who vote over a nonbinary agenda: each of the three mechanisms could be uniquely optimal, even if members care enough about their own rewards;
- The voting pattern in both private and public committees can fail Duverger’s Law (with at least three decisions securing some votes), even if outsiders are symmetric. However, a private committee’s voting pattern must satisfy another condition: each

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<sup>2</sup>Issing remarks that “attempts to influence policy-makers will occur even in the absence of published votes” (p.513); but does not explain why publicity exacerbates their effect.

dissident votes a decision which better serves organizational goals than the plurality decision. Consequently, if performance is single-peaked on the agenda then every dissent on a private committee lies on one side of its plurality decision. By contrast, each dissident on a symmetric public committee votes a decision which worse serves organizational goals than the plurality decision and secures a greater reward; so every dissent again lies on one side of the plurality decision if the outsiders' utility function is also single-peaked.

Our model is closely related to Seidmann (2008), who analyzes voting in a private committee with a supramajority quota;<sup>3</sup> while Groseclose and Milyo (2006) study a public voting game with no common interest and a binary agenda. These models and our paper differ from the literature on lobbying by assuming that outsiders cannot commit to rewards. Outsiders can ensure that both public and private committees reach their ideal decision - at least in benchmark lobbying models - either by conditioning rewards on all members' votes (cf. dal Bo (2007)) or by committing to punish members if the committee reaches any other decision. By contrast, we preclude any precommitment, and use trembling hand perfection arguments to determine the rewards paid after unexpected decisions.

Our model shares important properties and addresses similar questions to a recent literature on Condorcet juries, where members care both about performance and outsiders' beliefs about their privately known ability.<sup>4</sup> We simplify this framework by supposing that preferences are commonly known: so outsiders can only be uncertain about the voting pattern, and give an exogenously fixed reward for a known vote; but we drop the prevailing assumption that the agenda is binary, which has both negative and positive advantages. We show that results on optimal mechanisms do not generalize to a larger agenda if outsiders are asymmetric; and our approach allows us to study the distribution of dissents.

Cox (1997) surveys the theoretical literature and electoral evidence on Duverger's Law, while Rietz (2005) surveys the experimental evidence.

A significant literature in social psychology has compared the performance of individuals and groups in a variety of tasks;<sup>5</sup> but it has not considered how outside influence might affect relative performance.

We present our model and some general results on voting patterns in Section 2. We analyze games with a binary agenda and with symmetric outsiders respectively in Sections 3 and 4; and then use examples to show how previous results may fail when outsiders are asymmetric in Section 5. We summarise our results in Section 6, and present some proofs in an Appendix.

## 2. VOTING GAMES

In this section, we present models of committees whose members each care about the decision which the committee reaches and about the reward they receive for representing

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<sup>3</sup>In crucial contrast to our model, pivotal members are arbitrarily most likely to be pivotal between the status quo and another decision in neighboring games.

<sup>4</sup>Recent exemplars include Levy (2007) and Visser and Swank (2007). Li et al. (2001) use a related model to compare the performance of a dictator and a committee.

<sup>5</sup>See Baron and Kerr (2003) and Blinder and Morgan (2005).

outside interests. We will analyze two games: a public voting game in which outsiders observe the voting pattern, and a private voting game in which outsiders can only observe the decision which the committee reaches. These games only differ according to the definition of outsider strategies; so it is convenient to present them together, distinguishing between strategies when appropriate. We describe the model in Section 2.1 and provide some results which will be useful throughout the paper in Section 2.2.

## 2.1. Model

We analyze play in a game with a very simple time line: members vote simultaneously, and outsiders then reward members. In this subsection, we define and motivate the game and its solution concept, presenting some subsequently useful concepts along the way.

### *Players*

We denote the set of committee members by  $j \in \{1, \dots, J\}$ . We refer to a one-member committee as a “dictator”. We suppose, for convenience and without loss of generality, that there is a bijection between the set of members (“she”) and the outsiders (“he”).

The outsider-member relationship can be interpreted very widely: if members are elected representatives then outsiders might be thought of as voters or lobbyists; or outsiders might be thought of as the members’ peer group (e.g. fellow bankers).

### *Members’ strategies in a plurality rule committee*

We suppose that the committee chooses from some fixed and finite set of decisions (the “agenda”), which we denote by  $D$ . We assume that members vote simultaneously over the agenda, so member  $j$ ’s strategy in each game is a vote:  $v_j \in D$ .<sup>6</sup> We refer to a strategy combination by members ( $v$ ) as a “voting pattern”.

If several decisions tie for most votes then the committee is assumed to reach each of these decisions with equal probability. We describe the decision which the committee reaches as the “plurality decision”, and denote it by  $\delta$ ; any other decision which secures some votes is then a “dissent”. Members who vote  $\delta$  will be called the “plurality”; all other members being “dissidents”.

A committee’s voting pattern is said to satisfy Duverger’s Law if there is no more than one dissent. We will subsequently show that symmetric private and public committees may each fail Duverger’s Law. Our analysis will generate a weaker condition, which we will describe in context.

Our solution concept will imply that each member’s choice depends on the effect of varying her vote on the plurality decision. It will prove useful to formalize this notion. Fix the votes cast by all members except  $j$ . We say that member  $j$  is “pivotal” with respect to some decisions if the probability that the committee reaches each of these decisions depends on her vote; and that a voting pattern is pivotal if some member is pivotal. A dictator must, crucially, be pivotal with respect to the whole agenda.

### *Outsiders’ strategies*

After members choose their votes, each outsider  $j$  chooses the reward ( $r_j \in \mathfrak{R}$ ) he gives member  $j$  at his information set. We distinguish below between outsider information

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<sup>6</sup>We exclude abstentions for expositional convenience. Abstentions play a crucial role in Gersbach and Hahn (2000), as expert members alone vote in private committees.

sets in the two games. A reward can be given several interpretations, which include monetary payments, media coverage, the degree of peer approval and the probability of reappointment.

Our assumption that outsiders move after the committee votes contrasts with the lobbying literature, where the principal commits to a reward schedule. In our model, outsiders best respond to the inferences they draw about voting patterns after every history, including those in which the committee reaches an unexpected decision. By contrast, a principal can deter some votes by committing to punishments off the equilibrium path.

#### *Outsider payoffs*

We write outsider  $j$ 's payoff as the sum of two terms: the first term only depends on other players' strategies, whereas the second, behavior-relevant term depends on outsider  $j$ 's strategy. It is expositionally convenient to imagine that outsider  $j$  cares directly about the committee decision, and that he seeks to reward member  $j$  for voting according to these preferences. We represent outsider  $j$ 's preferences over decisions with the "utility function"  $g_j : D \rightarrow \mathfrak{R}$ , which we normalize such that  $\min_{d \in D} g_j(d) = 0$  and  $\max_{d \in D} g_j(d) = 1$ . Outsider  $j$  uses his preferences over decisions to evaluate member  $j$ 's vote: so  $g_j(v_j)$  represents the value to outsider  $j$  of member  $j$ 's vote  $v_j$ . We assume that outsider  $j$ 's payoff depends on the distance between  $g_j(v_j)$  and the reward he pays to member  $j$ .

Formally, we write outsider  $j$ 's payoff as  $G_j - \mu[r_j - g_j(v_j)]^2$ : where  $\mu$  is a positive constant and  $G_j$  is independent of  $r_j$ . Hence, outsider  $j$ 's best response is to equate his reward ( $r_j$ ) to his expectation of  $g_j(v_j)$ . As  $G_j$  is independent of  $r_j$ , its interpretation does not affect play in a given game. However, we will use the following assumption (solely) to compare outsiders' payoffs across mechanisms:

**A1 (outsider payoffs)**  $G_j = \alpha g_j(d) + (1 - \alpha)g_j(v_j)$  for some  $\alpha \in [0, 1]$ . ■

A1 implies that outsider  $j$ 's payoff depends directly on his evaluation of the committee decision and of his member's vote, as well as on how accurately he targets his reward.

According to our model, an outsider's optimal reward only depends on his belief about member  $j$ 's vote rather than, for example, on whether he believes that member  $j$  was pivotal. This approach corresponds to Mayhew's (1974) influential model of position taking, and has also been adopted by Diermeier and Myerson (1999), Groseclose and Milyo (2006) and others.<sup>7</sup> According to this interpretation, rewards are retrospective; but they could alternatively be thought of as actions which affect members' payoffs in unmodelled subgames starting next period, such as commercial relations between management and shareholders.

In sum, outsider payoffs can be interpreted quite flexibly. Indeed, for formal purposes, we do not need to interpret  $g_j$  as a preference ordering over decisions. However, our results below rely on outsiders influencing members in equilibrium. This would be impossible if rewards were purely monetary, as each member would then anticipate that her outsider would pay as low a reward as possible, irrespective of her vote.<sup>8</sup> We exclude this possibility

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<sup>7</sup>Snyder and Ting (2005) explain why constituents might reward members of Congress solely for their position taking.

<sup>8</sup>This feature distinguishes our model from the lobbying literature, where 'outsiders' can commit to their reward schedule.

by adopting the otherwise weak assumption that  $g_j(\cdot)$  is strict: that is,  $g_j(d) \neq g_j(e)$  for any distinct decisions  $d$  and  $e$ .

We dub the decision which maximizes outsider  $j$ 's utility function: “member  $j$ 's populist decision”; we call the modal such decision across members “the populist decision”, and denote it by  $d^P$ . We simplify exposition by focusing on games in which the populist decision is unique.

Some of our results depend on the distribution of outsider utility functions on a committee. We call this distribution the committee's “composition”. In one of our benchmark cases, all outsiders share the same utility function. We then say that the committee is “symmetric”. Symmetry corresponds to (extra-model) cases in which a single outsider rewards each of the members.<sup>9</sup>

#### *Performance*

We suppose that members all share a strict, commonly known preference ordering over the agenda, which we represent by the function  $u : D \rightarrow \mathfrak{R}$ , and refer to as “performance”. Member preferences therefore have a “valence dimension”, as in Fiorina (1981) *inter alia*.<sup>10</sup> We will say that decision  $d$  is “better than” decision  $e$  if  $u(d) > u(e)$ .

It is important to remember that performance ranks decisions from the organization's rather than from a welfare point of view.

Note that the function  $u$  could be an average of outsiders' utility functions. We will say that the decision is “uncontroversial” if the best decision maximizes each outsider's utility function.

#### *Member payoffs*

We suppose that members' incentives are partially aligned with performance via (unmodelled) wage contracts and/or each member's stake in the committee's future. Accordingly, we assume that each member's payoff is a weighted sum of performance and the reward she receives.<sup>11</sup> We write member  $j$ 's payoff as  $u(\delta) + sr_j$  when the committee reaches decision  $\delta$  and outsider  $j$  gives a reward of  $r_j$ : where the trade-off parameter  $s$  is positive.<sup>12</sup> We will refer to  $s$  as the “sensitivity” of the decision, saying that the decision is sensitive [resp. insensitive] enough if  $s$  is large [resp. small] enough. If outsiders were interpreted as journalists then sensitivity would track media interest in the voting pattern, which could vary over time for a standing committee. Our main results would also hold if sensitivity varied across members.

#### *Games*

We will analyze two games, which only differ according to the definition of outsider strategies. Both games share the same set of players, member strategy sets and payoff functions.

- In the public voting game, an information set for outsider  $j$  is a voting pattern  $v$

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<sup>9</sup>Recall our supposition of a bijection between members and outsiders.

<sup>10</sup>Groseclose and Milyo's (2006) Dual Conflict assumption excludes any such common interest across members; while the literature on Condorcet juries assumes that  $u$  is not commonly known.

<sup>11</sup>In Grossman and Helpman's (1994) lobbying model, the decision-maker's payoff is a weighted sum of aggregate welfare and rewards.

<sup>12</sup>If outsiders sought to minimize rewards, as in the lobbying literature, then members would act as if  $s = 0$ .



(and a decision  $\delta$ ); so outsider  $j$  can condition his reward on member  $j$ 's vote in a public voting game.

- In the private voting game, each outsider only observes, and can therefore only condition his reward on the decision which the committee reaches.<sup>13</sup>

Note that a dictator's outsider can always infer her voting pattern, even if he can only observe the plurality decision. In other words, privacy is a characteristic of multi-member committees.

Outsider strategies in private voting games induce member preference orderings over the agenda which are “one and a half dimensional” in Groseclose's (2005) terms: members agree on the performance dimension, but may differ on another dimension because they receive heterogeneous rewards when the committee selects a given decision. Groseclose proves that a majority of such committee members prefer the median member's top-ranked decision over any alternative on the agenda. Our solution concept will imply that a private committee must select such a decision; but, for reasons to be explained below, this does not imply that private voting games have a unique outcome.

#### *Solution concept*

Pure strategy Nash equilibrium has little power in a private voting game for conventional reasons: a member is indifferent across her possible votes unless she is pivotal with respect to some decisions; so, irrespective of preferences, every decision on the agenda can be reached in a Nash equilibrium by a nonpivotal voting pattern.<sup>14</sup> Furthermore, subgame perfection does not pin down rewards when a private committee reaches an unexpected decision. Like other authors, we address these problems using a refinement which requires that the voting pattern be both coalition proof and trembling hand perfect. The former refinement addresses the coordination problem intrinsic to all voting games; the latter refinement pins down rewards off the path.

- Committee members usually talk before voting. We interpret this pre-play communication as facilitating coordinated deviations. Accordingly, we require that the voting pattern be coalition proof.<sup>15</sup>

We need some further notation to formalize this condition. For any subset of members  $K \subseteq J$ , we write a strategy combination in some game as  $\{v_K, \sigma_{-K}\}$ , where  $\sigma_{-K}$  specifies the strategies of all players who are not members in  $K$ . We say that  $v'_K$  is a “mutually profitable deviation” from  $v_K$  if every member in  $K$  is strictly better off at strategy combination  $\{v'_K, \sigma_{-K}\}$  than at strategy combination  $\{v_K, \sigma_{-K}\}$ .

We now define coalition proof strategy combinations inductively. Fix a strategy combination and some subset of members  $K \subseteq J$ , and suppose that “self-enforcing deviations” have been defined for every strict subset of  $K$ . We say that members  $K$

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<sup>13</sup>Formally, an outsider has  $\#D$  information sets, each containing the set of voting patterns such that the committee reaches decision  $\delta$ .

<sup>14</sup>de Sinopoli (2000) demonstrates that refinements based on stability have little power in voting games without outsiders.

<sup>15</sup>Grossman and Helpman (1994) use analogous arguments to justify their use of coalition proofness.

have no self-enforcing deviation from  $v_K$  if a strict subset of  $K$  have a self-enforcing deviation from  $v'_K$  whenever  $v'_K$  is a mutually profitable deviation for  $K$ . We say that a voting pattern is coalition proof if no subset of members have a self-enforcing deviation.<sup>16</sup>

- Trembling hand perfection is a technical condition which ensures that members vote as if they were pivotal, and therefore precludes any member from voting her bottom-ranked decision.<sup>17</sup>

We again need some notation to define this condition. For any fixed game  $\Gamma$ , we define a “perturbed game”  $\Gamma_\varepsilon$ , which only differs from  $\Gamma$  in terms of members’ strategy sets. In  $\Gamma_\varepsilon$ , a feasible strategy for member  $j$  is a mixture over the agenda such that  $pr(v_j = d) \geq \varepsilon_j^d$  for each decision  $d$ , where  $\varepsilon_j^d \in (0, \frac{1}{\#D})$  for all  $d \in D$ . We (conventionally) interpret any strategy for member  $j$  in  $\Gamma_\varepsilon$  which assigns probability  $\varepsilon_j^e$  to every vote  $e \neq d$  as an intention to vote  $d$  and a tremble to every other vote. Note that each member is pivotal with positive probability in any perturbed game, and her best response depends on the events in which she is pivotal.

We define  $\varepsilon$  as the maximum of  $\{\varepsilon_j^d\}$  over members  $j$  and decisions  $d$ ; and we describe  $\Gamma_\varepsilon$  as “neighboring” if  $\varepsilon$  is small enough. We say that a strategy combination is trembling hand perfect if it is both a Nash equilibrium and the limit of Nash equilibria of some sequence of perturbed games  $\{\Gamma_\varepsilon\}$  as  $\varepsilon$  approaches 0.<sup>18</sup>

We describe a pure, trembling hand perfect strategy combination whose voting pattern is coalition proof as a “CP equilibrium”. We will characterize CP equilibria by imposing trembling hand perfection and coalition proofness separately; so it is useful to call a pure, trembling hand perfect strategy combination an “equilibrium”. We could, alternatively, have required that the intended voting pattern be coalition proof in every perturbed game. This alternative is technically more elegant; but it impedes exposition without affecting our results.

Note that members alone tremble in perturbed games, and that we focus on joint deviations of members. Allowing outsiders to tremble would complicate exposition without affecting our results. By contrast, our results would fail if members and outsiders could jointly deviate: for according to our theory, private committees differ from public committees precisely because outsiders cannot determine which members deviated when the committee reaches an unexpected decision.<sup>19</sup>

To see the power of coalitional proofness, consider the limiting case of  $s = 0$  (so outsider strategies are irrelevant to members). Both the private and the public voting games possess an equilibrium in which all members vote some decision other than the worst; but only the best decision can be reached in a CP equilibrium.

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<sup>16</sup>Seidmann (2008) uses a refinement of coalition proofness. Our positive results are correspondingly tighter.

<sup>17</sup>Iterative admissibility has little bite for asymmetric committees; so trembling hand perfection is widely used in the literature on voting games. See, in particular, Austen-Smith and Feddersen (2005).

<sup>18</sup>Every Nash equilibrium of a perturbed game must, of course, be subgame-perfect.

<sup>19</sup>Recall our justification for coalition proofness in terms of pre-play conversation.

Conditional on outsiders' strategies, coalitional proofness implies that a private committee must select the median member's top-ranked decision. However, trembling hand perfection does not pin down outsider strategies; so the median member's top-ranked decision need not be uniquely defined. Consequently, even a symmetric private committee could reach several decisions, as we will subsequently show.

### *Optimal mechanisms*

We will compare the performance of private and public committees and of a dictator randomly selected from the committee members. We will say that a mechanism is optimal if the committee uniquely reaches a decision better than either of the other mechanisms reach in any CP equilibrium. This exercise implicitly supposes that the mechanism is chosen by the organization rather than by members or outsiders. We will also compare outsiders' payoffs in private and public committees under assumption A1.

## **2.2. Some general results**

In this subsection, we prove a pair of Lemmas which are both interesting in their own right and expositionally useful throughout the paper:

### **Lemma PR**

- a. *One decision ( $\delta$ ) secures most votes in any equilibrium of a private voting game;*
- b. *Every dissent in an equilibrium of a private voting game is better than  $\delta$ ;*
- c. *If the decision is insensitive enough ( $s$  is small enough) or the decision is uncontroversial then every member of a private committee votes the best decision in the unique CP equilibrium.*

### **Proof**

a. Suppose, per contra, that several decisions (say,  $\Delta$ ) secure the maximal number of votes; and define  $\underline{\delta}$  and  $\bar{\delta}$  respectively as the worst and the best decision in  $\Delta$ . If member  $j$  votes  $\underline{\delta}$  then she receives a reward of  $g_j(\underline{\delta})$ , whichever decision in  $\Delta$  is reached. She can therefore profitably deviate to voting  $\bar{\delta}$ , as the committee would then reach  $\bar{\delta}$  for sure.

b. Fix a neighboring game and a member (say,  $j$ ) who intends to vote  $d \neq \delta$  in that game. We focus wlog on the vote combinations of the other members such that  $j$  is pivotal.

The other members' strategies and the perturbation structure define the probability with which  $j$  is pivotal with respect to any given set of decisions. Part a implies that the committee reaches  $\delta$  with conditional probability close to 1 if member  $j$  votes either  $\delta$  or a decision which then secures fewer votes than  $\delta$ . Hence, member  $j$  can profitably deviate to voting  $\delta$  unless she prefers that the committee reach  $d$  than that it reach  $\delta$ : that is,

$$u(d) + sr_j(d) > u(\delta) + sr_j(\delta).$$

By definition of a Nash equilibrium, member  $j$  receives a reward close to  $g_j(d)$  if the committee reaches decision  $\delta$  in a neighboring game. Moreover, if the committee reaches decision  $d$  in a neighboring game then the posterior probability that member  $j$  voted  $d$  is

at least  $1 - \varepsilon$ , so her reward must also be close to  $g_j(d)$  if the committee reaches decision  $d$  in a neighboring game. Hence, the inequality above implies that  $u(d) > u(\delta)$ .

c. If the committee reached another decision then a coalition of members who do not vote the best decision have a self-enforcing deviation to doing so. Part b implies that all members must vote the best decision in any CP equilibrium; and existence of such a CP equilibrium is trivial. ■

Lemma PR relies on an outsider’s inability to observe which member deviated if the committee reaches an unexpected decision in a private voting game: a property that does not hold in public voting games. Consequently,

### Lemma PU

- a. *Any member who is nonpivotal in an equilibrium of a public voting game votes her populist decision;*
- b. *If the decision is insensitive enough then a public committee reaches the best decision in every CP equilibrium;*
- c. *If the case is uncontroversial then every member of a public committee votes the best decision in the unique CP equilibrium;*
- d. *If the decision is sensitive enough then every member of a public committee votes her populist decision in the unique CP equilibrium. ■*

Part a holds because a member who is nonpivotal and did not vote her populist decision could profitably deviate to so doing. If  $s$  is small enough or the case is uncontroversial then the committee can only reach the best decision, as a coalition of plurality voters otherwise have a self-enforcing deviation. Existence is easy to establish in both cases. If  $s$  is large enough then every member votes her populist decision in every equilibrium; and existence is again easy to prove.

Groseclose and Milyo (2006) Lemma 1 asserts part a for the special case of a binary public voting game.

## 3. BINARY AGENDA

In this section, we follow the bulk of the related literature by focusing on committees which vote over a binary agenda denoted  $\{h(\text{igh}), l(\text{ow})\}$ , where we suppose wlog that  $u(l) = 0$  and  $u(h) = 1$ .<sup>20</sup> Accordingly, we refer to “binary voting games”.

We allow outsiders to be asymmetric; so we write  $J$  as  $J_h \cup J_l$ , where decision  $d$  is populist for all members in  $J_d$ . We abuse notation by also writing  $J_d$  for the cardinality of  $J_d$  ( $d \in \{h, l\}$ ).

A member in  $J_h$  obviously votes  $h$  in every equilibrium of each game; so the committee must reach decision  $h$  whenever  $J_l \leq \frac{J-1}{2}$ .

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<sup>20</sup>Note that  $g_j(d) \in \{0, 1\}$  for every member  $j$  and each decision  $d$ .

On one interpretation of a binary game, the agenda consists of a proposal, which is pitted against the status quo. Management, which is identified with the outsider(s), can reward some shareholders ( $J_l$ ) with future business if they vote for a project ( $l$ ) which reduces shareholder value ( $u$ ). On this interpretation, we will consider how keeping the voting pattern private would affect management's scope to successfully propose bad projects.

We divide our analysis into three subsections: Sections 3.1 and 3.2 respectively analyze public and private voting games, while Section 3.3 characterizes optimal mechanisms by comparing a randomly selected dictator's performance with those exhibited by private and public committees.

### 3.1. Public committees

In this subsection, we characterize CP equilibria of binary public voting games. Our assumptions that  $u$  and  $g_j$  are strict functions implies that every pure strategy subgame perfect equilibrium of a generic public voting game is strict, and is therefore an equilibrium of a public voting game. Accordingly, we use Lemma PU to characterize equilibria of generic public voting games, and then eliminate those equilibria which are not coalition proof.

#### Proposition 3.1: Binary public voting games

- a. *Binary public voting games possess a unique CP equilibrium for generic sensitivity.*
- b. *The committee reaches decision  $l$  unless  $J_l \leq \frac{J-1}{2}$  or  $J$  is even and  $s < \frac{1}{2}$  or  $J$  is odd and  $s < 1$ .*

Proposition 3.1 implies that there is a critical sensitivity below which the committee reaches decision  $h$ . This value is lower for even- than odd-numbered committees because both decisions secure  $\frac{J-1}{2}$  votes when a plurality member of an odd-numbered committee is pivotal; whereas decision  $d$  secures one vote more than decision  $e$  when a plurality member of an even-numbered committee is pivotal.

#### Proof

If  $J_l \leq \frac{J-1}{2}$  then there is a unique equilibrium in which all members in  $J_h$  vote  $h$  and all other members vote  $l$ . This equilibrium is coalition proof because all members earn  $1 + s$ . Accordingly, we henceforth focus on games in which  $J_l > \frac{J-1}{2}$ .

If  $J$  is even then a member is pivotal if and only if one of the decisions secures  $\frac{J}{2}$  of the other members' votes. If  $s < \frac{1}{2}$  then  $l$  secures  $\frac{J}{2} - 1$  votes in every pivotal equilibrium, and a coalition of members in  $J_l$  has a self-enforcing deviation from any nonpivotal equilibrium; and if  $s > \frac{1}{2}$  then  $l$  secures  $J_l$  votes in every CP equilibrium.

If  $J$  is odd then a member is pivotal if and only if other members' votes are equally split between the decisions. Members in  $J_h$  must all vote  $h$ ; so consider members in  $J_l$ . Lemma PU implies that either no such members vote  $h$  (so the committee reaches  $l$ ) or that  $J_l - \frac{J-1}{2}$  members do so and  $s \leq 1$ . Any such equilibrium must be coalition proof because members who vote  $h$  are individually pivotal.

We now turn to equilibria in which the committee reaches  $l$ . If  $J_l = \frac{J+1}{2}$  then every member in  $J_l$  must vote  $l$ . All such members must then be pivotal; so such an equilibrium generically exists if and only if  $s > 1$ , and is then coalition proof. If  $J_l > \frac{J+1}{2}$  then Lemma PU implies that every member in  $J_l$  must vote  $l$ . No member is then pivotal, so such an equilibrium exists for every  $s$ . However, any coalition of at least  $J_l - \frac{J-1}{2}$  members in  $J_l$  has a self-enforcing deviation if and only if  $s < 1$ : which implies the remainder of the result. ■

Proposition 3.1 contrasts with Groseclose and Milyo (2006) Proposition 1, which states that every member of a public committee must vote her populist decision from a binary agenda. Pivotal equilibria can exist in our model because we assume that members all care about performance; Groseclose and Milyo's result depends on their Dual Conflict assumption, which excludes any common interest across all members. We delay an intuition for this part till Section 3.3.

Lemma PR states that a single decision secures most votes in a private committee. By contrast, Proposition 3.1 implies that both decisions secure half the votes in a binary public committee with an equal number of outsiders in  $J_l$  and  $J_h$  if the decision is sensitive enough.

### 3.2. Private committees

In this subsection, we characterize CP equilibria of binary private voting games. Our main result is

#### Proposition 3.2: Binary private voting games

- a. *Binary private voting games possess a unique CP equilibrium decision for generic sensitivity.*
- b. *The committee reaches decision  $l$  unless  $J_l \leq \frac{J}{2}$  or  $J$  is odd and  $s < \frac{2J_l}{2J_l - J + 1}$  or  $J$  is even and  $s < \frac{2J_l}{2J_l - J}$ .*

#### Proof

Lemma PR implies that all members must vote  $h$  in any equilibrium in which the committee reaches decision  $h$ . No member in  $J_h$  can profitably deviate; so such an equilibrium exists if and only if no member  $j$  in  $J_l$  can profitably deviate in any neighboring game: viz. when  $sr_j(l) < 1$ , where  $r_j(l)$  is the limit of the probability that member  $j$  trembled when the committee reaches decision  $l$  in a sequence of neighboring games. If  $J$  is odd [resp. even] then it is arbitrarily most likely that  $l$  secured  $\frac{J+1}{2}$  [resp.  $\frac{J}{2}$ ] votes when the committee reaches  $l$ . The incentive constraints are easiest to satisfy if all members in  $J_h$  are arbitrarily more likely to tremble than any other member, and all members in  $J_l$  are equally likely to tremble. If  $J_l \leq \frac{J}{2}$  then  $r_j(l)$  is close to 0 for every member in  $J_l$ , and no member can profitably deviate.

If  $J_l > \frac{J}{2}$  then it is arbitrarily most likely that  $J_l - \frac{J-1}{2}$  members in  $J_l$  trembled if an odd-numbered committee reached  $l$  in a neighboring game: so  $r_j(l)$  is close to  $\frac{2J_l - J + 1}{2J_l}$ , and

no member can profitably deviate if  $s < \frac{2J_l}{2J_l - J + 1}$ . A joint deviation can only change payoffs if at least  $\frac{J+1}{2}$  members participate; and the deviation can only be mutually profitable if all deviators  $j$  are in  $J_l$ , each receiving a reward close to  $r_j(l)$ : so the joint deviation is mutually unprofitable exactly when it is individually unprofitable for a member in  $J_l$  to deviate in a neighboring game. An analogous argument implies that an even-numbered committee with a majority of members in  $J_l$  reaches  $h$  in a CP equilibrium if and only if  $s < \frac{2J_l}{2J_l - J}$ .

We now consider putative equilibria in which the committee reaches decision  $l$ : equilibria which can only exist if  $J_l > \frac{J}{2}$ . Write  $J^l$  as the set of members in  $J_l$  who vote  $l$  in such an equilibrium (and its cardinality): where  $J^l \geq \frac{J+1}{2}$ , and each member in  $J^l$  earns  $s$ . Suppose that  $J$  is odd. If the committee reached  $h$  in a neighboring game then it is arbitrarily most likely that exactly  $\frac{J-1}{2}$  members in  $J^l$  did not tremble. No member outside  $J^l$  can profitably deviate; so such an equilibrium exists if and only if  $s > \frac{2J^l}{2J^l - J + 1}$ . This condition is easiest to satisfy if all members in  $J_l$  vote  $l$ : in which case, the incentive condition is  $s > \frac{2J_l}{2J_l - J + 1}$ . Any equilibrium in which the committee reaches decision  $l$  is coalition proof. An analogous argument implies that an even-numbered committee can reach  $l$  in a CP equilibrium if and only if  $s < \frac{2J_l}{2J_l - J}$ , proving the result. ■

Notice that the *performance* of a binary private committee is generically unique, whereas the *voting pattern* of a binary public committee is generically unique. We delay the intuition for Proposition 3.2 till the next subsection.

### 3.3. Comparing mechanisms

In this subsection, we compare performance under three mechanisms: a public committee, a private committee and a dictator selected at random from the committee. As there are only two feasible decisions, performance can be measured by the probability that decision  $h$  is reached. Propositions 3.1a and 3.2a imply that, for generic cases, each mechanism reaches a unique CP equilibrium decision; so performance is uniquely defined. Accordingly, we will compare mechanism performance across cases.

We will show that private committees outperform public committees for generic cases. However, we need some assumptions about the distribution of cases to compare committees with a randomly selected dictator. We adopt two benchmark assumptions to this end:

**A2 (distribution of cases)** *Composition and sensitivity are independently distributed; and sensitivity has full support on  $\mathfrak{R}_+$ .* ■

**A3 (distribution of composition)** *Every composition is equally likely.* ■

We will show that some comparisons require much weaker conditions, but that our main result might be overturned if A3 failed.

In light of A2, we write  $\Pi_s(x)$  for the probability that sensitivity is at least  $x$ .

A dictator in  $J_h$  always chooses decision  $h$ , whereas a dictator in  $J_l$  chooses the same decision as a pivotal member of an odd-numbered public committee who is in  $J_l$ . Consequently, there are two cases to consider:

- If  $s < 1$  then a dictator and an odd-numbered public committee achieve the same performance, irrespective of committee composition.
- If  $s > 1$  then an odd-numbered public committee reaches decision  $h$  if and only if  $J_l < \frac{J+1}{2}$ , whereas a randomly selected dictator chooses  $h$  with probability  $\frac{J-J_l}{J}$ ; so, for every  $s > 1$ , an odd-numbered public committee and a randomly selected dictator yield the same expected performance if all compositions are equally likely.

In sum, A2 and A3 imply that an odd-numbered public committee and a randomly selected dictator yield the same expected performance.

By contrast, A2 and A3 imply that a dictator randomly selected from an even-numbered committee chooses  $h$  with probability  $\frac{1}{2} + \frac{1}{2}[1 - \Pi_s(1)]$ ; and an even-numbered public committee reaches  $h$  with probability  $\frac{1}{2} - \frac{1}{J+1} + (\frac{1}{2} + \frac{1}{J+1})[1 - \Pi_s(\frac{1}{2})]$ : so a dictator outperforms an even-numbered public committee.

If  $J_l < \frac{J+1}{2}$  then both public and private committees reach  $h$  for every sensitivity; so suppose otherwise. Propositions 3.1a and 3.2a then imply the following generic conditions for the committees to reach each decision:

	$h$	$l$
Private committee	$s < \frac{2J_l}{2J_l - J + 1}$	$s > \frac{2J_l}{2J_l - J + 1}$
Public committee	$s < 1$	$s > 1$

Table 3.1a: Odd-numbered committees

	$h$	$l$
Private committee	$s < \frac{2J_l}{2J_l - J}$	$s > \frac{2J_l}{2J_l - J}$
Public committee	$s < \frac{1}{2}$	$s > \frac{1}{2}$

Table 3.1b: Even-numbered committees

Inspection of Table 3.1 reveals that a private committee generically weakly outperforms a public committee for every realized case.

These observations and our calculations of a dictator's performance imply

**Proposition 3.3: Optimal mechanisms (binary agenda)** *If the agenda is binary and both A2 and A3 hold then the optimal mechanism is a private committee. ■*

A dictator is, by definition, a public committee with one member. However, there is a potentially important difference between cases with  $J = 1$  and  $J > 1$ : a dictator is pivotal with respect to the whole agenda, in contrast to members of a public committee. This feature is unimportant when the agenda is binary and  $J$  is odd; so A2 and A3 imply that a dictator and an odd-numbered public committee yield the same expected performance. We will demonstrate that the difference matters when the agenda is larger.

Table 3.1 reveals that the superior performance of private committees only relies on  $s$  having a large enough support. The intuition is that members of a private committee



cannot signal that their deviation is responsible for the committee unexpectedly reaching  $l$  in a neighboring game. Consequently, members in  $J_l$  who are expected to vote  $h$  but deviate to voting  $l$  receive a lower reward when a private committee unexpectedly reaches  $l$  than when a public committee does so.

Absent A3, a dictator could be the optimal mechanism: for example, if the decision is sensitive enough and the support of  $J_l$  is contained in  $[\frac{J+1}{2}, J)$  then a private committee cannot reach  $h$ , as it can only do so unanimously (cf. Lemma PR); whereas a dictator in  $J_h$  would choose  $h$ .

Dissidents always vote  $h$  in a binary private voting game, but may vote  $l$  in a public voting game. Table 3.2 compares the voting pattern in each odd-numbered committee, with  $\{x, y\}$  denoting the number of votes which decisions  $h$  and  $l$  respectively secure and  $K \leq J - J_h$ :

	Private committee	Public committee
$J_l \leq \frac{J-1}{2}$	$\{J, 0\}$	$\{J - J_l, J_l\}$
$J_l > \frac{J-1}{2}, s < 1$	$\{J, 0\}$	$\{\frac{J+1}{2}, \frac{J-1}{2}\}$
$J_l > \frac{J-1}{2}, 1 < s < \frac{2J_l}{2J_l - J + 1}$	$\{J, 0\}$	$\{J - J_l, J_l\}$
$J_l > \frac{J-1}{2}, s > \frac{2J_l}{2J_l - J + 1}$	$\{K, J - K\}$	$\{J - J_l, J_l\}$

Table 3.2: Dissident numbers (odd-numbered committee)

Table 3.2 indicates that privacy raises the quality of voting as well as performance:<sup>21</sup> a result which is reminiscent of the claim that privacy raises the quality of debate: cf. Issing (1999).

At first sight, Proposition 3.3 conflicts with the widespread view that the voting patterns of monetary policy and related committees should be publicized.<sup>22</sup> However, this is a matter of perspective. Privacy best serves organizational interests in our model; whereas publicity allows outsiders/principals to monitor members/agents, as in the literature on transparency. If outsider payoffs also satisfy A1 then it is easy to confirm, using Table 3.2, that publicizing the voting pattern makes some outsiders better off and none worse off unless a majority of outsiders are in  $J_l$  and  $s \in (1, \frac{2J_l}{2J_l - J + 1})$ : in which case, publicizing the voting pattern makes a majority of outsiders (those in  $J_l$ ) better off, and the other outsiders worse off.

## 4. LARGE AGENDA, SYMMETRIC COMMITTEE

In this section, we extend our analysis to another benchmark case where

- The agenda may contain more than two feasible decisions; and

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<sup>21</sup>It is easy to confirm that this claim also applies to even-numbered committees.

<sup>22</sup>See, for example, Sibert (2003).

- The committee is symmetric. We write outsiders' common utility function as  $g(d)$ .

We refer to games which satisfy these conditions as “symmetric”. These games are equivalent to a model in which all members are rewarded by a single outsider.

Extending our analysis to nonbinary agendas allows Duverger's Law to fail and for a majority of members to dissent. We will show that symmetric committees can exhibit each of these features.

We will use some additional notation throughout this section. We define the function  $F^D(d; s)$  as  $u(d) + sg(d)$ : which is the payoff function for a dictator whose outsider has utility function  $g$ ; and  $d^D$  as such a dictator's choice. We will focus on generic games, in which  $d^D$  is unique. It is easy to confirm that  $d^D$  is at least as good a decision as the populist decision  $d^P$ ; and that it coincides with the populist decision if the decision is sensitive enough, and with the best decision if the decision is insensitive enough.

There are two sorts of symmetric binary committee (with agenda  $\{h, l\}$ ). If the case is uncontroversial then  $d^D = d^P = h$ ; otherwise ( $J_l = J$  in Section 3 notation)  $d^P = l$ , and  $d^D = h$  if and only if  $s < 1$ .

Voting patterns in a binary voting game are either unanimous or have one dissent: so Duverger's Law is trivially satisfied. In this section, we consider the structure of dissents.

We again divide our analysis into three subsections: Sections 4.1 and 4.2 respectively analyze public and private committees, while Section 4.3 characterizes optimal mechanisms.

#### 4.1. Public committees

Our main result in this subsection is

##### **Proposition 4.1: Symmetric public voting games**

- (Decisions) *In any CP equilibrium of a symmetric public voting game with generic sensitivity, the committee reaches a single decision, which is no worse than the populist decision and no better than a dictator's choice;*
- (Number of dissidents) *A majority of members may be dissidents;*
- (Dissents)
  - The voting pattern in a symmetric public voting game may fail Duverger's Law.*
  - Any dissent is worse than the plurality decision, and dissidents receive greater rewards than plurality members.*
  - If all decisions are scalars and if performance and utility are both single-peaked functions then all dissents are on one side of the plurality decision.*

Note that Proposition 4.1 does not assert that performance is uniquely defined. However, the bounds on performance will suffice to compare mechanisms.

**Proof**

a. Call the minimal difference between the number of votes cast for the plurality decision and for any other decision: “the voting margin”. We will prove part a by characterizing the voting margins which can occur in a CP equilibrium.

We start by demonstrating that, for generic sensitivity, the committee reaches a single decision in any equilibrium. Write  $\Delta$  for the decisions which secure most votes; and suppose, per contra, that  $\Delta$  consists of the pair  $\{d, e\}$ . Incentive conditions for members who respectively vote  $d$  and  $e$  then require

$$\begin{aligned} \frac{1}{2}[u(d) + u(e)] + sg(d) &\geq u(e) + sg(e) \text{ and} \\ \frac{1}{2}[u(d) + u(e)] + sg(e) &\geq u(d) + sg(d), \end{aligned}$$

which can only both hold if  $s = \frac{u(d)-u(e)}{2[g(e)-g(d)]}$ . For generic sensitivity, there is also no equilibrium in which  $\Delta$  consists of three or more decisions: for if there were such an equilibrium then we must have

$$\frac{1}{\#\Delta} \sum_{e \in \Delta} u(e) + sg(d) \geq \max_{e \in \Delta \setminus d} u(e) + sg(e)$$

for every  $d \in \Delta$ . Summing the  $\#\Delta$  incentive constraints yields

$$\sum_{e \in \Delta} F^D(e; s) \geq F_1^D + (\Delta - 1)F^D(d^D, s),$$

where  $F_1^D$  is the second highest value that  $F^D$  takes on the agenda. This condition fails by definition of  $d^D$ . In sum, the committee reaches a single decision ( $\delta$ ) in equilibrium; so plurality members earn  $u(\delta) + sg(\delta)$ .

If the committee reaches a nonpopulist decision in equilibrium then the populist decision must secure the second highest number of votes, else a dissident could profitably deviate; and, for analogous reasons: a coalitional deviation is only self-enforcing if every coalition member is pivotal with respect to  $d^P$  after the deviation.

The first argument implies that the committee cannot reach a decision worse than  $d^P$ . We complete the proof by demonstrating that there is no equilibrium in which the committee reaches a decision which is better than  $d^D$  (and therefore nonpopulist). Our argument turns on the voting margin.

We first argue that a nonpopulist decision ( $\delta$ ) cannot be reached with a voting margin of two. Suppose otherwise. We have to distinguish between two cases, to which purpose we define  $\bar{d}$  as  $\arg \max_{d \neq d^P} g(d)$ : viz. as the second best decision from the outsiders’ point of view. The committee cannot reach any  $\delta \neq \bar{d}$ : for  $\bar{d}$  must also secure two votes less than  $\delta$ , else each plurality member could profitably deviate to voting  $\bar{d}$ ; but any member who votes  $\bar{d}$  could profitably deviate to voting the populist decision. The committee can also not reach  $\bar{d}$  if it is better than  $d^P$ : for  $d^D$  must then be the populist decision; so Lemma PU implies that every dissident must vote  $d^D$ : which is only possible if  $J$  is even. Two plurality members then have a mutually profitable deviation to voting  $d^D$ ; and this deviation is self-enforcing by the initial supposition that the committee reaches  $\bar{d}$  in equilibrium. Lemma

PU then implies that the committee can only reach a nonpopulist decision which is better than  $d^D$  in a CP equilibrium with a voting margin of 1.

Write  $V$  for the number of votes that  $\delta$  secures, and  $\Delta_{-1}$  for the decisions which secure  $V - 1$  votes, where  $d^P \in \Delta_{-1}$ . If  $d^D \in \Delta_{-1}$  then each plurality member could profitably deviate to voting  $d^D$ ; so suppose otherwise, and let  $d^D$  secure  $W < V - 1$  votes. The incentive conditions for each plurality member imply that  $F^D(d, s) < F^D(\delta, s)$  for every decision  $d \in \Delta_{-1}$ , and that

$$\frac{1}{\#\Delta_{-1} + 1} \sum_{d \in \Delta_{-1}} [u(d) + u(\delta)] + sg(e) < F^D(\delta, s)$$

for every dissent  $e$  which is not in  $\Delta_{-1}$ . A coalition of  $V - W$  plurality members have a mutually profitable deviation to voting  $d^D$ , as they would then earn  $F^D(d^D, s)$ . No further deviation to voting  $d \in \Delta_{-1}$  can be profitable because  $F^D(d, s) < F^D(\delta, s)$ . As  $\delta$  is better than  $d^D$ , we have

$$\frac{1}{\#\Delta_{-1} + 1} \sum_{d \in \Delta_{-1}} [u(d) + u(d^D)] + sg(e) < \frac{1}{\#\Delta_{-1} + 1} \sum_{d \in \Delta_{-1}} [u(d) + u(\delta)] + sg(e) < F^D(d^D, s),$$

so a further deviation to voting  $e \notin \Delta_{-1}$  is also unprofitable. In sum, there is a self-enforcing deviation from any decision which is better than  $d^D$ .

**b and c.** We prove part b, and therefore part c.i, by example. Suppose that a seven-member committee selects a decision from the triple  $\{l, m, h\}$ , and that performance and outsider utilities satisfy

	$l$	$m$	$h$
$u(d)$	0	$u$	1
$g(d)$	$g$	1	0

Table 4.1

If  $\{g, u\} \in (0, 1)^2$  and  $s < 1 - u$  then the public voting game possesses a CP equilibrium in which three members vote  $h$ , two members vote  $l$  and the other two members vote  $m$ : for the committee must reach a decision other than  $h$  after any profitable deviation; and every deviation which changes the decision is unprofitable because the decision is insensitive enough. In sum, the voting pattern of a symmetric public committee may violate Duverger's Law, with a majority of members being dissidents.

Any dissident must receive a reward greater than  $g(\delta)$ , else she could profitably deviate to voting  $\delta$ . Write  $V$  and  $W$  for the votes respectively secured by  $\delta$  and some dissent  $d$  in a putative CP equilibrium. If  $d$  were better than  $\delta$  then a coalition of  $V - W$  plurality members could mutually profitably deviate to voting  $d$ ; and this deviation would be self-enforcing by the initial supposition that the committee reaches  $\delta$  in a CP equilibrium. In sum, all dissidents must vote decisions which are worse than  $\delta$ . This proves part c.ii.

Part c.iii follows immediately from part c.ii. ■

We now strengthen part a, demonstrating by example that a public committee can reach a decision worse than a dictator's choice in every CP equilibrium.

**Example 4.1** *A seven member committee selects a decision from the triple  $\{l, m, h\}$ . Performance and outsider utilities satisfy Table 4.1 above, with  $\{g, u\} \in (0, 1)^2$  and  $\frac{1}{2}(1 + u) < u + s < 1 < u + 2sg$ .■*

Decision  $m$  is populist in Example 4.1; and a dictator would choose  $h$ , but the committee cannot reach either  $l$  or  $h$  in equilibrium: for in each case, a plurality member could profitably deviate to voting either  $l$  or  $m$ . However, decision  $m$  can be reached by unanimous vote in a CP equilibrium. The committee cannot reach  $l$  with positive probability after any mutually profitable deviation because members who vote  $l$  would earn less than  $u + s$ . A coalition of at least  $\frac{J+1}{2}$  members can mutually profitably deviate to voting  $h$ ; but this deviation is not self-enforcing because a coalition member could profitably further deviate to voting  $l$ , earning  $\frac{1}{2}(1 + u) + sg > 1$ . The committee cannot reach  $h$  after any other self-enforcing deviation because those members who deviate to voting  $l$  would have a further profitable deviation back to voting  $m$ . Finally, if  $\frac{J-1}{2}$  members deviated to voting  $h$  and another member deviated to voting  $l$  then the former members would earn  $\frac{1}{2}(1 + u)$ , so the deviation would not be mutually profitable. In sum, a public committee can reach a decision worse than a dictator's choice in every CP equilibrium.

The intuition for this result is that a pivotal member can vote a decision which the committee does not reach if the agenda contains more than two decisions, preventing the committee from reaching a dictator's choice. This effect is impossible for either a dictator or a committee selecting from a binary agenda.<sup>23</sup>

Duverger's Law necessarily holds if the agenda is binary; so part c has no counterpart in Proposition 3.1. Lemma PU implies that Duverger's Law can easily fail in asymmetric public voting games. Its possible failure in symmetric games is more surprising, especially as the decision is insensitive enough in Example 4.1 that the committee necessarily reaches the best decision. Any nonDuvergerian CP equilibrium must satisfy the condition in our construct: some pivotal dissident does not vote the populist decision.

## 4.2. Private committees

In this subsection, we characterize CP equilibria of symmetric private voting games, and compare the voting patterns of symmetric private and public committees. In light of Proposition 4.1a, we describe performance relative to  $d^D$ : which is both a dictator's choice and the best decision that a symmetric public committee can reach.

If the decision is sensitive enough then every member of a symmetric private committee votes the populist decision in the unique CP equilibrium. To see this, suppose that the committee reached another decision in a CP equilibrium. Any member who does not vote  $d^P$  would receive a greater reward if the committee were to reach  $d^P$  in a neighboring game. Consequently, some coalition (say,  $K$ ) have a mutually profitable deviation to voting  $d^P$  whenever the decision is sensitive enough. If all members in  $K$  prefer that the committee reach another decision  $d$  (so the deviation to voting  $d^P$  is not self-enforcing) then the members in  $K$  have a self-enforcing deviation to voting  $d$ . Conversely, it is

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<sup>23</sup>Recall that a randomly selected dictator chooses an odd-numbered public committee's plurality decision if the agenda is binary.

easy to prove that there is a CP equilibrium in which all members vote the populist decision if the decision is sensitive enough. This observation and Lemma PU imply that the decisions reached by symmetric public and private committees only differ for intermediate sensitivities.

Our results depend on the identity of a particular decision. We will need some additional notation to identify this decision. We first define the function

$$F^{PR}(d; s) = \begin{cases} u(d) + \frac{1}{2}sg(d) & \text{if } J \text{ is even} \\ u(d) + \frac{J+1}{2J}sg(d) & \text{if } J \text{ is odd} \end{cases}$$

and then define  $d^{PR}$  as  $\arg \max_{d \in D} F^{PR}$ . We will focus on generic games, in which  $d^{PR}$  is unique. It is easy to confirm that  $d^{PR}$  is at least as good as  $d^D$ ; and that it coincides with the populist decision if the decision is sensitive enough, and with the best decision if the decision is insensitive enough.

If the agenda is  $\{h, l\}$  and the case is uncontroversial then  $d^{PR} = h$ ; otherwise ( $J_l = J$  in Section 3 notation)  $d^{PR} = h$  if and only if either  $J$  is odd and  $s < 2$  or  $J$  is even and  $s < \frac{2J}{J+1}$ .

**Proposition 4.2: Symmetric private voting games**

- a. (Decisions) *A symmetric private committee reaches  $d^{PR}$  in a CP equilibrium, cannot reach any other decision unanimously, but can reach other decisions which are no worse than a dictator's choice by split vote;*
- b. (Number of dissidents) *A majority of members of a symmetric private committee vote the plurality decision in any CP equilibrium;*
- c. (Dissents)
  - i. *The voting pattern in a symmetric private voting game may fail Duverger's Law.*
  - ii. *Any dissent is better than the plurality decision.*
  - iii. *If all decisions are scalars and if performance is a single-peaked function then all dissents are on one side of the plurality decision.*

**Proof**

It is convenient to prove part b first.

**b.** The claim is trivial if the equilibrium satisfies Duverger's Law. If not, then there is a dissent (say,  $d$ ) such that members who vote  $d$  earn more than any other dissidents (as  $g(\cdot)$  is strict). If dissidents form a majority then a subset of dissidents who do not vote  $d$  have a mutually profitable deviation to voting  $d$ . The deviation is self-enforcing if the deviating coalition is chosen such that its members are pivotal between  $d$  and the plurality decision after the deviation. Consequently, the plurality decision secures a majority of the votes.

**a.** We now prove the first claim in part a by construction. Suppose that all members intend to vote  $d^{PR}$ , and that they each tremble to every other decision with probability  $\varepsilon$  in every neighboring game. All members then receive a reward close to  $g(d^{PR})$  if the committee reaches  $d^{PR}$ , and a reward close to  $\frac{J-1}{2J}g(d^{PR}) + \frac{J+1}{2J}g(d)$  [resp.  $\frac{1}{2}g(d^{PR}) + \frac{1}{2}g(d)$ ] if an odd-numbered [resp. an even-numbered] committee reaches  $d$ . The definition of  $d^{PR}$  immediately implies that no member can profitably deviate when pivotal in a neighboring game: so the strategy combination forms an equilibrium. Furthermore, no coalition of members have a mutually profitable joint deviation: so the equilibrium is also coalition proof. An analogous argument implies that the committee cannot reach any other decision unanimously.

The following example, which we analyze in the Appendix, demonstrates that the committee can reach decisions other than  $d^{PR}$ :

**Example 4.2** *A five-member committee chooses from the triple of decisions  $\{l, m, h\}$ . Performance and outsider utilities satisfy*

$$\begin{array}{ccc} & l & m & h \\ u(d) & 0 & \frac{1}{5} & 1 \\ g(d) & 1 & \frac{1}{5} & 0 \end{array}$$

Now suppose, contrary to the Proposition, that the plurality decision  $\delta$  is worse than a dictator's choice ( $d^D$ ), and let  $K$  and  $L$  members respectively vote  $\delta$  and  $d^D$ . If  $K + L$  is odd [resp. even] then a coalition of  $\frac{K-L+1}{2}$  [resp.  $\frac{K-L+1}{2}$ ] plurality members would have a self-enforcing joint deviation to voting  $d^D$  if and only if

$$u(d^D) + sr_j(d^D) > u(\delta) + sg(\delta):$$

where  $r_j(d^D) \geq \min\{g(\delta), g(d^D)\}$ . This inequality necessarily holds if  $u(d^D) > u(\delta)$  by definition of  $d^D$ . Consequently, a symmetric private committee cannot reach a decision worse than  $d^D$ .

**c.** We prove c.i by example:

**Example 4.3** *A six-member committee chooses from the triple of decisions  $\{l, m, h\}$ . Performance and outsider utilities satisfy*

$$\begin{array}{ccc} & l & m & h \\ u(d) & 0 & u & 1 \\ g(d) & 1 & g & 0 \end{array}$$

and  $s > \max\{\frac{1}{g}, \frac{4u}{1-g}, 4\}$ .

Lemma PR implies that dissents are better than the plurality decision, which implies c.iii. ■

Part a generalizes Proposition 3.2 for the special case of symmetric outsiders: if the case is uncontroversial and the agenda is binary then  $d^{PR}$  is the better decision, irrespective of  $s$ ; and if the case is controversial and the agenda is binary then  $d^{PR}$  is the better decision when  $s < \frac{2J}{J+1}$  if  $J$  is odd, and when  $s < 2$  if  $J$  is even.

Part a illustrates the power of our solution concept: on the one hand, we can show that at most one decision *cannot* be reached in an equilibrium of a symmetric private voting game; on the other hand, coalition proofness alone is unrestrictive because it does not pin down rewards when the committee reaches an unexpected decision.

Example 4.2 illustrates a tension between the implications for voting patterns entailed by trembling-hand perfection and coalition proofness. On the one hand, we can show that a decision which can be reached in equilibrium with a plurality of  $K < J$  can be reached in equilibrium with a plurality of  $K + 1$ , but that the converse is false. On the other hand, decision  $l$  cannot be reached unanimously in a CP equilibrium because some members would then have a self-enforcing deviation to voting  $h$ .

Consider Example 4.3 with the additional condition that  $s < \frac{4(1-u)}{g}$ . We can show that the specified voting pattern satisfies a refinement akin to properness: members are arbitrarily more likely to tremble to less costly votes.

Part c.i recalls Myerson and Weber's (1993) example of a plurality rule voting game with a nonDuvergerian equilibrium. However, there are several important differences:

- Myerson and Weber assume that all members are equally likely to be pivotal between any given pair of decisions  $\{d, e\}$ ; and that they are equally likely to be pivotal when  $d$  and  $e$  secure the same number of votes and when  $d$  secures one vote more than  $e$ . They then construct a nonDuvergerian equilibrium in a voting game with agenda  $\{l, m, h\}$  in which  $l$  is the plurality decision. Every member believes that she is almost equally likely to be pivotal between  $l$  and  $m$  and between  $l$  and  $h$ . Myerson and Weber's assumption of common pivot probabilities is inconsistent with our model. In our example, all members believe that they are arbitrarily most likely to be pivotal when  $l$  secures one vote more than  $h$ . Member 5 cannot profitably deviate from voting  $m$  because she prefers that the committee reach  $l$  than  $h$ ; and we construct perturbed games such that she is next most likely to be pivotal with respect to  $m$ . Myerson and Weber's assumption that all pivotal voting margins are equally likely precludes this effect in their model.
- In Myerson and Weber's example, the committee reaches a Condorcet loser in a nonDuvergerian equilibrium where most members are dissidents: so the voting pattern is not coalition proof. We use a stronger solution concept than Myerson and Weber, proving by example that some nonDuvergerian equilibria may be coalition proof. In further contrast to Myerson and Weber, we show that coalition proofness precludes any voting pattern in which dissidents form a majority if outsiders are symmetric. However, we will demonstrate in the next section that private voting games with asymmetric outsiders may possess CP equilibria in which most members are dissidents.

### 4.3. Comparing mechanisms

In Section 3, we demonstrated that the performance of generic binary private and public committees is well-defined; so we used A2 and A3 to define optimal mechanisms, showing that a private committee outperforms both a public committee and a randomly selected



dictator. By contrast, symmetric committees may reach several decisions (cf. Proposition 4.2c). Nevertheless, Propositions 4.1a and 4.2a respectively bound the performance of a public and a private committee, and imply that the result for binary committees carries over to symmetric committees:

**Proposition 4.3: Optimal mechanisms (symmetric outsiders)** *If outsiders are symmetric and sensitivity has full support then the optimal mechanism is a private committee. ■*

The assumption of full support ensures that sensitivity may be intermediate: Lemmas PR and PU and the discussion at the beginning of the last subsection imply that all three mechanisms would reach the populist [resp. the best] decision if the decision is sensitive [resp. insensitive enough]. In the next section, we will show that each mechanism may be optimal when outsiders are asymmetric and the decision is sensitive enough.

A symmetric private committee weakly outperforms a symmetric public committee because outsiders fully reward any member of a public committee who unexpectedly votes the populist decision. By contrast, they only reward such an unexpected vote by a private committee member if the committee reaches an unexpected decision; and even then, outsiders partially reward all members, as they cannot identify the member(s) who have deviated. Members of a public committee therefore face greater incentives to deviate from an equilibrium in which the committee reaches a decision better than  $d^P$ ; so such decisions can only be reached in equilibrium when members are less sensitive. Private and public committees also reach different decisions because a member of the latter could profitably deviate to voting a decision which is not reached with positive probability. This is impossible in private committees because rewards only depend on the plurality decision.

Proposition 4.3 also asserts that a (private) committee may outperform an individual decision-maker with the same ability as the committee's members. In our model, a dictator is a one-member public committee. Privacy improves performance by reducing each member's reward for grandstanding.

Propositions 4.1c, 4.2c and 4.3 imply that the quality of dissents is at least as high in private as in public committees if outsiders are symmetric: a result that we also obtained for binary voting games. However, the multiplicity of CP equilibria precludes the analog of Table 3.2, which compares the distribution of dissident numbers when there are two feasible decisions.

We end this section by comparing outsiders' payoffs when the voting pattern is private and public, again assuming that their payoff function satisfies A1. In any equilibrium, member  $j$  receives a reward of  $g(v_j)$ . Any dissident in a public committee must receive a reward greater than  $g(\delta)$ , else she could profitably deviate to voting  $\delta$ ; so an outsider earns at least  $g(\delta)$  in any equilibrium of a public voting game. On the other hand, dissents are better than the plurality decision in private committees; so dissidents must receive a lower reward than plurality members, else the latter could profitably deviate to dissenting. Consequently, an outsider earns no more than  $g(\delta)$  in any equilibrium of a private voting game. Propositions 4.1a and 4.2a assert that private committees reach decisions which are at least as good as those reached by a public committee. Plurality members must therefore receive higher rewards in equilibrium in a public than in a private voting game,

else some plurality members in a public committee would have a self-enforcing deviation to voting the decision reached by a private committee. In sum, publicizing the voting pattern benefits all outsiders if they are symmetric (though this result does not generalize to all games with asymmetric outsiders).

## 5. LARGE AGENDA, ASYMMETRIC COMMITTEE

Sections 3 and 4 have established some strong results on the voting patterns and performance of binary and symmetric committees. These results carry over, of course, to committees which are not too asymmetric. In this section, we demonstrate that some of our previous results can be overturned if outsiders are asymmetric enough and the agenda contains three or more decisions.

**Proposition 5.1: Asymmetric committees** *If outsiders are asymmetric then*

- a. *Any of the three mechanisms can be uniquely optimal when the decision is sensitive enough;*
- b. *A majority of members of a private committee may be dissidents;*
- c. *Dissents may be better than a public committee's plurality decision. ■*

We prove this result in the Appendix, using Examples A.1-A.3.

If the decision is sensitive enough then all three mechanisms reach the populist decision in binary agenda and symmetric voting games. Part a asserts that this result can be overturned if outsiders are asymmetric. If the decision is sensitive enough then all members of a public committee vote their populist decisions. In Example A.1, the populist and the best decisions coincide, so a public committee is an optimal mechanism; but some members' populist decision is worse, so a randomly selected dictator might reach another decision, and the private committee cannot reach the best decision because it would have to do so unanimously. This property clearly generalizes beyond Example A.1. A public committee is always *an* optimal mechanism if a plurality of outsiders earn most utility when the committee reaches the best decision: a condition which is satisfied in uncontroversial cases.

The optimal mechanism may be a randomly selected dictator if a majority of members share a common populist decision which is worse than the minority's populist decision.

In Example A.2, the private committee can reach the best and the worst of three decisions; the public committee must reach the worst decision; and a randomly selected dictator chooses the best [resp. the worst] decision with probability  $\frac{3}{7}$  [resp.  $\frac{2}{7}$ ]. Hence, the private committee is the optimal mechanism if each CP equilibrium is equally likely to be played.

In sum, each mechanism can be uniquely optimal when the decision is sensitive enough. Lemmas PU and PR imply that the three mechanisms are equivalent if the decision is insensitive enough; but we can demonstrate that each mechanism can be uniquely optimal when sensitivity is intermediate.

We prove part b by providing an example in which all members vote their populist decisions, and outsiders are heterogeneous enough that dissidents are a majority of members. By contrast, a majority of members of binary and symmetric private committee members must vote the plurality decision.

In Section 4.3, we demonstrated that all dissents are worse than the plurality decision in symmetric public voting games. If the decision is sensitive enough then every member votes her populist decision (cf. Lemma PU). Consequently, dissidents may vote decisions which are better than the plurality decision if outsiders are asymmetric. By contrast, Lemma PR implies that dissents are better than the plurality decision in all private voting games.

## 6. SUMMARY

We have analyzed and compared performance and voting patterns in private and public voting games, where members have dual loyalties. In the two benchmark cases (a binary agenda and symmetric outsiders), a private committee outperforms both a public committee with the same composition and a dictator selected at random from the committee members; and, conversely, publicity better serves the interests of outsiders. These results formalize the notion that performance in public committees is stymied by grandstanding, and provide a possible explanation for secret ballots and open roll calls: in each case, citizens are the beneficiaries.

We have also shown that the voting pattern in public and private committees may fail Duverger's Law. However, dissents are worse than the plurality decision in binary and symmetric public committees, and are better than the plurality decision in private committees. If decisions are real numbers and performance is single-peaked then dissents on a private committee must all lie on one side of the plurality decision: a condition which is also satisfied on symmetric public committees if outsiders' utility is also single-peaked.

Anecdotal evidence (e.g. on monetary policy committees) suggests that private committees have fewer dissidents than public committees. This stylized fact is consistent with a tradition in political science that committees are more likely to reach consensual opinions if they deliberate in private (cf. Elster (1998)). However, members with divergent opinions may vote unanimously, as Meade and Stasavage (forthcoming) document for the FOMC. Our model allows us to address this difference:

If the agenda contains more than two decisions then our model does not pin down plurality size. However, our model suggests that private committees are more likely to vote unanimously *without* holding consensual opinions. Specifically, in an earlier version of this paper (which is available on request), we showed that

- The modal number of dissidents on a binary [resp. symmetric] public committee is a bare minority if all compositions are equally likely [resp. if the decision is sufficiently insensitive and if the best and populist decisions differ]; whereas
- The modal number of dissidents on a binary [resp. symmetric] private committee is zero if a majority of members is likely to top-rank the better decision [resp. if every

number of dissidents which can be reached in some CP equilibrium is realized with equal probability].

Furthermore, our model implies that only private committees can vote unanimously when outsiders are asymmetric.

We derive our results from a simple model, in which outsiders reward members for representing their interests, and members vote simultaneously. It would be interesting to know whether our results extend to more complicated reward schemes and to voting around the table.

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## APPENDIX

**Example 4.2** *A five-member committee chooses from the triple of decisions  $\{l, m, h\}$ . Performance and outsider utilities satisfy*

	$l$	$m$	$h$
$u(d)$	0	$\frac{1}{5}$	1
$g(d)$	1	$\frac{1}{5}$	0

If  $\frac{3}{2} < s < \frac{5}{3}$  then  $d^{PR} = h$  and  $d^D = l$ : so a committee which votes unanimously must reach  $h$ . It is easy to confirm that the committee cannot reach  $m$  in any CP equilibrium if  $s < 8$ .

We claim that, if  $s > \frac{3}{2}$ , then there is a CP equilibrium in which members  $j = 1, 2, 3$  vote  $l$  and members  $j = 4, 5$  vote  $m$ . To see this, consider the following perturbation structure:

	$l$	$m$	$h$
$j = 1, 2, 3$	$\varepsilon$	$\varepsilon$	$\varepsilon^2$
$j = 4, 5$	$\varepsilon$	$\varepsilon$	$\varepsilon^3$

Members  $j < 4$  then receive a reward of about  $\frac{11}{15}$  [resp.  $\frac{1}{3}$ ] if the committee reaches  $m$  [resp.  $h$ ] in a neighboring game, whereas members 4 and 5 always receive a reward of about  $\frac{1}{5}$ . The inequalities above imply that members  $j < 4$  most prefer that the committee reach  $l$ ; so they can neither profitably deviate from voting  $l$  nor be in a coalition with a mutually profitable deviation. Members 4 and 5 are arbitrarily most likely to be pivotal when a member  $j < 4$  trembles to voting  $m$ , and therefore cannot profitably deviate. In sum, we

have demonstrated that a symmetric private committee can reach a decision other than  $d^{PR}$  by split vote.

If  $s < 2$  then the committee cannot reach  $l$  with four members voting  $l$  because some plurality members would have a self-enforcing deviation to voting  $h$ , for any perturbation structure. Indeed, there can only be one dissident if  $s > 2$  because the committee cannot reach  $m$  for these sensitivities, and can only reach  $h$  unanimously (by Lemma PR).

The committee might only reach  $l$  with two dissidents because a larger proportion of plurality members must then have trembled to voting  $h$  when the committee reaches  $h$  in a neighboring game; so three plurality members prefer that the committee reach  $l$  than  $h$ , but four or five plurality members would have the opposite preference ordering. ■

**Example 4.3** *A six-member committee chooses from the triple of decisions  $\{l, m, h\}$ . Performance and outsider utilities satisfy*

$$\begin{array}{rcc} & l & m & h \\ u(d) & 0 & u & 1 \\ g(d) & 1 & g & 0 \end{array}$$

and  $s > \max\{\frac{1}{g}, \frac{4u}{1-g}, 4\}$ .

We claim that the symmetric private voting game has a CP equilibrium in which members  $j < 5$  vote  $l$ , member 5 votes  $m$ , and member 6 votes  $h$ .

To see this, consider the following perturbation structure:

$$\begin{array}{rcc} & l & m & h \\ j \leq 4 & \varepsilon & \varepsilon^6 & \varepsilon^4 \\ j = 5 & \varepsilon & \varepsilon & \varepsilon \\ j = 6 & \varepsilon & \varepsilon & \varepsilon \end{array}$$

Members  $j \leq 4$  then receive a reward of  $\frac{1}{4}(3 + g)$  if the committee reaches  $m$ , and of  $\frac{3}{4}$  if the committee reaches  $h$ ; while member 5 [resp. 6] receives a reward of 0 [resp.  $g$ ] if the committee reaches  $h$  [resp.  $m$ ]. These conditions imply that members  $j \leq 4$  can neither profitably deviate when pivotal in a neighboring game nor be in a coalition with a mutually profitable deviation.

The perturbation structure implies that member 5 is most likely to be pivotal when one member  $j \leq 4$  trembles to voting  $h$ : so  $l$  and  $h$  respectively secure three and two votes. Member 5's reward function and  $s > \frac{1}{g}$  imply that she prefers the committee to reach  $l$  than to reach  $h$ ; so she cannot profitably deviate from intending to vote  $m$  in this event. Member 5 is next most likely to be pivotal when  $l$  and  $m$  respectively secure three and two votes; so she can again not profitably deviate from intending to vote  $m$  in this event.

The perturbation structure implies that member 6 is most likely to be pivotal when member 5 and one member  $j \leq 4$  both tremble to voting  $h$ . Member 6 strictly prefers the committee to reach  $h$  than  $l$ ; so she cannot profitably deviate from intending to vote  $h$  in this event.

In sum, the private voting game possesses a nonDuvergerian equilibrium. This equilibrium is also coalition proof because any coalition with a mutually profitable deviation must include some members  $j \leq 4$ . ■

**Proposition 5.1: Asymmetric committees** *If outsiders are asymmetric then*

- a. *Any of the three mechanisms can be uniquely optimal when the decision is sensitive enough;*
- b. *A majority of members of a private committee may be dissidents;*
- c. *Dissents may be better than a public committee's plurality decision.*

**Proof**

We will exploit a number of examples, in all of which there are seven members and the agenda consists of the triple  $\{l, m, h\}$ , where

$$0 = u(l) < u(m) = u < u(h) = 1.$$

**a. Public committee**

We first demonstrate by example that the optimal mechanism may be a public committee:

**Example A.1**  $u = \frac{2}{3}$  and outsiders' utility functions satisfy

	$l$	$m$	$h$
$j = 1, 2$	1	$g$	0
$j = 3, 4$	$g$	1	0
$j = 5, 6, 7$	0	$g$	1

where  $g$  is close to but less than 1.

If  $s > \frac{1}{3}$  then dictators  $j > 4$  alone would choose  $h$ . By contrast, all members earn  $1 + s$  in equilibrium if they each vote their populist decisions: so the public voting game has a unique CP equilibrium in which the committee reaches the best decision for sure. A public committee must therefore be *an* optimal mechanism for this asymmetric game.

We now show that a private committee cannot reach  $h$  in any equilibrium. Lemma PR implies that a private committee can only reach decision  $h$  unanimously. It is easiest to satisfy the various incentive constraints in neighboring games with the following perturbation structure:

	$l$	$m$	$h$
$j = 1, 2$	$\varepsilon^3$	$\varepsilon^2$	$\varepsilon$
$j = 3, 4$	$\varepsilon^2$	$\varepsilon^3$	$\varepsilon$
$j = 5, 6, 7$	$\varepsilon$	$\varepsilon$	$\varepsilon$

The committee is then most likely to reach  $l$  if

- All  $j > 4$  and some  $j \in \{3, 4\}$  tremble to voting  $l$ ; or
- All  $j > 4$  tremble to voting  $l$  and some  $j \in \{1, 2\}$  trembles to voting  $m$ ; or

- Two members in  $\{5, 6, 7\}$  and some  $j \in \{3, 4\}$  tremble to voting  $l$  and the other member in  $\{5, 6, 7\}$  trembles to voting  $m$ ;

and analogously when the committee reaches  $m$ .

The payoffs that each member earns when the committee reaches each decision in a neighboring game are then approximated in the table below:

	$l$	$m$	$h$
$j = 1, 2$	$\frac{1}{6}sg$	$\frac{2}{3} + \frac{1}{3}sg$	1
$j = 3, 4$	$\frac{1}{3}sg$	$\frac{2}{3} + \frac{1}{6}sg$	1
$j = 5, 6, 7$	$\frac{1}{9}sg$	$\frac{2}{3} + \frac{8}{9}sg$	$1 + s$

Any member  $j < 4$  is arbitrarily most likely to be pivotal if

- All  $j > 4$  tremble to voting  $l$ ; or
- All  $j > 4$  tremble to voting  $m$ ; or
- Two members  $j > 4$  tremble to voting  $l$  and the other one trembles to voting  $m$ ; or
- Two members  $j > 4$  tremble to voting  $m$  and the other one trembles to voting  $l$ ;

each of these events occurring with equal probability. Substituting above, it is easy to confirm that members 1 and 2 can profitably deviate to voting  $m$  whenever  $s > \frac{12}{7}$ .

In sum, if  $s > \frac{12}{7}$  then a public committee reaches  $h$  in every CP equilibrium; a private committee cannot reach  $h$  in any equilibrium;<sup>24</sup> and a randomly selected dictator chooses  $h$  with probability  $\frac{3}{7}$ . Consequently, the optimal mechanism is a public committee.

#### *Dictator*

The optimal mechanism may obviously be a randomly selected dictator in other asymmetric cases. Suppose that decision  $d$  is populist for all members except  $j$ , for whom a better decision is populist. If the decision is sensitive enough then a committee would reach  $d$  in every CP equilibrium (cf. Lemmas PU and PR); but member  $j$  would reach a better decision as dictator.

#### *Private committee*

We complete the proof of part a by analyzing another example:

### **Example A.2** *Outsider utility functions satisfy*

	$l$	$m$	$h$
$j = 1, 2, 3$	1	0	$g$
$j = 4, 5$	0	1	$g$
$j = 6, 7$	0	$g$	1

---

<sup>24</sup>If  $s > \frac{4}{3}$  then it reaches  $m$  with members  $j > 4$  voting  $h$  in a CP equilibrium.



If the decision is sensitive enough then a dictator chooses her populist decision, and Lemma PU implies that a public committee reaches decision  $l$ .

A private committee cannot reach  $m$ , as a subset of members  $j \notin \{4, 5\}$  who vote  $m$  in a putative CP equilibrium would have a self-enforcing deviation to voting  $h$ . It can reach  $l$  in a CP equilibrium;<sup>25</sup> but only when every member votes her populist decision.<sup>26</sup> However, a private committee reaches decision  $h$  (unanimously) in a CP equilibrium. To see this, consider the following perturbation structure:

	$l$	$m$	$h$
$j = 1, 2, 3$	$\varepsilon^2$	$\varepsilon$	$\varepsilon$
$j = 3, 4$	$\varepsilon$	$\varepsilon^3$	$\varepsilon$
$j = 5, 6, 7$	$\varepsilon$	$\varepsilon^2$	$\varepsilon$

The payoffs that each member earns when the committee reaches any decision in a neighboring game are then approximated in the table below:

	$l$	$m$	$h$
$j = 1, 2, 3$	$sg$	$u$	$1 + sg$
$j = 4, 5$	$0$	$u + sg$	$1 + sg$
$j = 6, 7$	$0$	$u + \frac{1}{5}s(4 + g)$	$1 + s$

Decision  $h$  strictly Pareto-dominates each of the other decisions for every  $s$ ; so the voting pattern forms part of a CP equilibrium. If each CP equilibrium is equally likely to be played (for which A4 is sufficient) then a private committee's expected performance is  $\frac{1}{2}$ , which exceeds a randomly selected dictator's expected performance if  $g < \frac{3}{4}$ .

In sum, we have shown that each mechanism may be uniquely optimal when outsiders are asymmetric and the decision is sensitive enough.

**b.** We prove this part by example:

**Example A.3**  $u = \frac{1}{2}$  and outsiders' utility functions satisfy

	$g_j(l)$	$g_j(m)$	$g_j(h)$
$j = 1, 2, 3$	$1$	$\frac{1}{2}$	$0$
$j = 4, 5$	$\frac{1}{2}$	$1$	$0$
$j = 6, 7$	$0$	$\frac{1}{2}$	$1$

where  $s \in (6, \frac{21}{2})$ .

We claim that the private voting game possesses a CP equilibrium in which all members vote their populist decisions, so Duverger's Law fails. The committee reaches decision  $l$ , but a majority of members vote otherwise.

To see that this voting pattern forms part of a CP equilibrium, consider neighboring games in which every member trembles to each unintended vote with probability  $\varepsilon$ . Simple calculations reveal that members earn approximately the following payoffs when the

<sup>25</sup>Consider perturbed games in which every member trembles to every decision with probability  $\varepsilon$ .

<sup>26</sup>There are no equilibria in which a member  $j > 3$  votes  $l$ .

committee reaches each decision in a neighboring game:

$$\begin{array}{rcc}
 & l & m & h \\
 j = 1, 2, 3 & s & \frac{1}{2} + \frac{8}{9}s & 1 + \frac{7}{9}s \\
 j = 4, 5 & s & \frac{1}{2} + s & 1 + \frac{5}{6}s \\
 j = 6, 7 & s & \frac{1}{2} + \frac{11}{12}s & 1 + s
 \end{array}$$

- Conditional on being pivotal, a member  $j \leq 3$  is arbitrarily most likely to be pivotal when each decision secures two votes from the other members. The conditions on sensitivity imply that she most prefers that the committee reach decision  $l$ , and can therefore not profitably deviate from voting  $l$ .
- When none of the other members tremble, member  $j \in \{4, 5\}$  is pivotal between decisions  $l$  and  $h$ . Member  $j$  cannot profitably deviate from voting  $m$  in this event because she prefers that the committee reach  $l$  than that it reach  $h$ . Conditional on one other member trembling, member  $j$  is arbitrarily most likely to be pivotal in four (equi-probable) events. Simple calculations reveal that intending to vote  $m$  is her best response across these events.
- An analogous argument implies that members  $j \in \{6, 7\}$  have no profitable deviation from voting  $h$ .

In sum, the voting pattern forms part of an equilibrium. No coalition which has a mutually profitable deviation can contain any members  $j \leq 3$ : for, by construction, they prefer that the committee reach  $l$  than either of the other decisions. The committee must therefore either reach  $m$  or reach  $h$  after any mutually profitable deviation. However, by construction again, members 4 and 5 prefer that the committee reach  $l$  than that it reach  $h$ , while members 6 and 7 prefer that the committee reach  $l$  than that it reach  $m$ . Consequently, the voting pattern forms part of a CP equilibrium.

We can show that there is another CP equilibrium strategy combination in which members  $j \leq 5$  vote  $m$  and the other members vote  $h$ : so Duverger's Law need not fail in such games.<sup>27</sup>

**c.** In Section 4.3, we argued that all dissents are worse than the plurality decision in symmetric public voting games. If the decision is sensitive enough then every member votes her populist decision (cf. Lemma PU). Consequently, dissidents may vote decisions which are better than the plurality decision if outsiders are asymmetric. By contrast, Lemma PR implies that dissents are better than the plurality decision in all private voting games. ■

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<sup>27</sup>Construct perturbed games in which members  $j \leq 5$  are arbitrarily more likely to tremble to voting  $h$  than  $l$ , and are arbitrarily more likely than members 6 and 7 to tremble to voting  $l$ .