Lend Out IOU: A Model of Money-Creating Banks and Unconventional Central Banking

Tianxi Wang

University of Essex

March 2013

Abstract

Banks create inside money by lending out what they do not have yet, namely their liabilities, which then circulate in the economy helping bring different types of capital together into production. The paper first present a general equilibrium analysis on the quantity and efficiency of inside money creation by competitive banks. Then it extends the model to show that if banks' wealth is low, it is unable to back circulation of adequate inside money. In this case, the central bank can improve efficiency with an unconventional policy: it prints fiat money and lends it to *all* banks at zero interest rate. However, there is a limit for this way of sizing up the central bank's balance sheet to affect efficiency; it helps the real economy only if the real interest of bank lending is higher than a threshold, otherwise the issuance of fiat money stays in banks' vaults uncirculated.

1 Introduction

How did a financial crisis damage the economy? Yes, banks suffer huge losses, but unlike what happens in a war, earthquake or tsunami, a financial crisis involves very little, if not nothing, of real capital loss; physical capital, human capital, knowledge, labor, etc., are still there. But financial crises do damage the real economy, sometime harshly. Then, could the central bank help by expanding money supply via its printing machine? If it could, how and when? These are the questions the paper sets out to address.

Regarding the first question, as banks' loss in a financial crisis impairs no real capital, its damaging the real economy, then, must be by influencing the way in which different types of real capital are brought together into producing real goods and services. The most important channel through which this influence is exerted, the paper thinks, is derived from the money creation function of banks. This function, in turn, is derived from the privilege of banks that they can lend out what they *currently* do not have yet, namely their liabilities. For example, suppose a colleague economist of mine, Adam, wants to borrow $5000 to buy a car. If he asks me, I have to check my pockets, my drawers, and my bank accounts. If I cannot find $5000 altogether, then I have to say no, disappointingly. But if he turns to the HSBC, the bank does not need to search its vault and say yes; it simply asks him to open an account with it (if he has not had one yet) and then with a click of mouse, the money is deposited there. With the issuance of the loan to Adam, the HSBC's balance sheet becomes as follows, which is perfectly balanced:

Asset	Liability
Old Asset ($H) Loan to Adam ($5000)	Old Liability ($H) Deposit of Adam ($5000)

After the transaction Adam gets the car and the $5000 in his deposit account is credited to the car dealer's account with the HSBC (or, if he has none, becomes part of HSBC's borrowing from of his bank).

The HSBC can lend to Adam with a click of mouse, but I cannot, because its liability is broadly accepted as means of payment, namely money (when its soundness is not in doubt), but never so is my IOU. That is, with $5000 of the bank's IOU, the car dealer can buy flight tickets or anything he wants, but nothing with my IOU; in the terminology of Kiyotaki and Moore (2001), the bank's IOU faces no resale constraint, but mine is scarcely resalable.

This money creation function of banks, so far, receives not as much analysis as its importance deserves. For example, macroeconomics textbooks usually demonstrate how banks through cycles of loan-deposit-loan enlarge money supply, but do not offer a general equilibrium analysis on the extent and efficiency of this enlargement, which the paper sets out to explore. In particular, the paper examines four questions. (a) Do banks earn a lot with the privilege of lending by creating money, whereby they seem to possess an unlimited lending capacity? (b) Do they create too much or too little money in terms of real efficiency? (c) In case of inefficiency arising, could the central bank help by printing fiat money and lending it to banks for free? (d) Does this policy subsidize banks?

To consider these questions, let me first explain the paper's approach to why bank liability is accepted as means of payment, but mine is not. The paper assumes that due to some reasons beyond the scope of the paper, there is certain material (such as gold or a special type of paper), called hard money, that has been accepted as means of payment,[1] and banks' IOU is *believed to be* equivalent to this hard money, in the sense that it can be converted into the hard money on demand.[2]

With this approach, the paper considers an economy where entrepreneurs cannot use their own promises to hire workers, who, however, accept to be paid with hard money, which only bankers have. Therefore, in order to hire workers, entrepreneurs have to borrow from bankers hard money, or*, its equivalent*, by which I mean the papers that certify the possession of hard money (like gold certificates) and can be redeemed with it on demand. By lending out certificates of a notional value higher than the quantity of hard money they actually have, bankers lend out what they do not have yet, namely their liabilities, and create inside money.[3] The overissue by banks may naturally arise through a cycle of loan-deposit-loan. Bankers, directly, lend out only hard money to entrepreneurs, who then pass it on to the workers hired. The workers, worried about the

[1] For the literature where some fiat or commodity endogenously emerges as media of exchange, see Samulson (1958), Wallace (1980), and Kiyotaki and Wright (1989), among others; and for reviews of the literature see Ostroy and Starr (1990) and Wright (2008).

[2] In the above example, if the car dealer has any doubt about using the HSBC's liability to buy flight tickets, he can simply go to a cashier of the bank demanding to convert the deposit into banknotes, namely the hard money. He will be satisfied normally.

[3] An early example of this overissue might be found in the establishment of the Bank of England in1694. In the year, the Bank received from its proprietors not more than £720,000 but advanced to the Exchequer £1,200,000. How did it do it? By printing numbers on various papers (mainly sealed bills, which were afterwards deposited back into the Bank). See Clapham (1944), page 20.

safety of storing it under their mattresses, deposit it back to bankers and take papers that certify their deposits. Then, bankers can lend out the deposited hard money to other entrepreneurs.

The total lending by a banker, which forms her liability, is backed by her wealth (i.e. her stock of hard money) which forms the equity of her bank, and the loans she makes to the entrepreneurs. If her liability is higher than a threshold, then when the banker's loans turn sour, the loss is too big to be fully absorbed by the equity and the banker defaults, namely, unable to redeem all her papers according to their notional values. Default triggers bank run. Default means the papers are worth, *on average*, less than the notional values. But if a paper holder gets to the banker early enough to have his holding redeemed with hard money, he will obtain the notional values of his holding. Therefore, upon the arrival of the bad news on the defaulting banker's assets, all the paper holders rush to the banker for redemption. *Bank run* arises.

The paper considers two cases, one where bank run is costless, the other where it is extremely costly and disallowed, which caps the quantity of bankers' issue. The main results of the paper are as follows.

First, as for questions (a) and (b) above, the profit margin of issuance to bankers is 0 in equilibrium if bank run is costless or bankers have adequate wealth, because either condition ensures bankers have so large lending capacities that the Bertrand competition between them suffices to dissipate all the profit. This sufficient competition also ensures that bank issue is at the second best level, namely, in equilibrium entrepreneurs hire the same quantity of workers as they would if they could hire workers by issuing their own promises rather than borrowing money from bankers. But if bank run is disallowed and bankers' wealth inadequate, which eases competition by limiting their lending capacity, then bank issue earns a positive profit-margin and is insufficient to draw the second best quantity of labor to the entrepreneur sector. Namely, the economy is in credit crunch. In this case, bankers' wealth affects efficiency; a decrease in bankers' wealth may reduce the economic efficiency, thus being a public concern.

Second, bank run arises in equilibrium if it is allowed and bankers' wealth are inadequate. Note that this bank run arises not because of mis-coordination between paper holders, as is in Diamond and Dybvig (1983), but because of concerns about bankers' assets.[4] Also note that unlike in their paper, here the liquidity need (i.e. the need of immediate consumption) would not prompt a certificate holder to demand redemption of his holding, because he can use it buy what he wants.

Third, as for questions (c) and (d) above, if the economy is starved of inside money, as bank run is disallowed and bankers' wealth are inadequate, the central bank improves efficiency by printing fiat money and lending it into all the bankers at zero interest. The quantity of fiat printing, however, should not be too large, or the real interest of bank lending will be so low that bankers will not lend out all the fiat money they have received, but keep part of it in their vaults. Moreover, this monetary policy, though giving bankers free funds, does not always subsidize them, but very like squeeze their profit, because it intensifies competition between them by enlarging all their lending

[4] Bank run that is induced by information (about the asset) instead of by mis-coordination is also examined by Jacklin and Bhattacharya (1988) and Chari and Jagannathan (1988). This fundamental-based view of bank run, as reviewed by Goldstein (2010), is strongly supported by empirical research.

capacity.[5]

To summarize, the paper presents an equilibrium analysis on banks' function of creating money by lending out their IOU. Based on this analysis, moreover, the paper endogenizes a role for the central bank printing to improve efficiency.

Recently, Hart and Zingales (2011, 2012) (HZ hereafter) and Stein (2012) also consider the efficiency of private money creation in a framework of general equilibrium.[6] Both HZ and this paper model the transaction role of the privately created liability, while to this role Stein (2012) takes a shortcut, who, on the other hand, derives richer implications for monetary policy than this paper does. But those papers are not concerned with bank overissue, namely, their lending out what they do not have yet, which is the focus of this paper.[7][8] Furthermore, all those papers feature excessive money creation due to certain pecuniary externalities,[9] none of which presents itself in this paper,

[5] Even when the banking sector as a whole is worse-off by the policy, each single banker wants the central bank funds to enlarge her capacity, since she takes into no account the effects of her request upon the market conditions facing all the bankers.

[6] With matching-search frameworks based on Kiyotaki and Wright (1989), alternatively, the money-creation role of banks is studied by Cavalcanti et al (1999), Cavalcanti and Wallace (1999), Araujo and Minetti (2005), and Wallace and Zhu (2007), among others. This literature does not examine competition between banks at length.

[7] Stein (2012) does not explicitly model the transaction role of the risk free claims, engendering thus no need of overissue. In HZ, the claims used for means of payment, when being issued, are fully backed by the hard money (namely wheat) stored, which, as storage delivers a lower return, is a source of inefficiency.

[8] Freeman (1996) analyzes overissue of clearingbanks, but in his paper overissue means, differently from this paper, that part of the banks' liabilities (i.e. bank notes) is not redeemed, whereas in this paper bankers' liabilities are all redeemed.

[9] These externalities arise because the decentralized agents fail to internalize the feeding of price (the

where, therefore, the equilibrium issuance is never beyond the second best level.

Champ et al. (1996) shows that the private bank issuance in a pure exchange economy is at the socially optimal level, but they consider neither bank overissue, nor central bank printing, as the present paper does.

This paper shows that when banks create inadequate inside money, the central bank improves efficiency by turning on its printing machine and enlarging the quantity of money supply. This captures, in a stylized way, the policy of quantitative easing in particular and unconventional monetary policy in general. The paper thus contributes to the recent literature on unconventional monetary policy; see Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Zeng (forthcoming), and the special issue of the *Economic Journal*[10], among others. Two distinctive features of the present paper are that the monetary policy in it is nominal, concerned only with central bank printing, and that the policy does not always subsidize banks.

The rest of the paper is organized as follows. Section 2 examines the case where bank run is costless, Section 3 the case where it is disallowed and the bankers' wealth is low, which engenders a role for central-bank printing to improve efficiency. Section 4 discusses the role bankers' stocks of hard money play. Section 5 concludes. All proofs are relegated in Appendix.

---

price of services in HZ, or the fire sale price in Stein 2012) onto the binding liquidity constraint (in the former) or borrowing constraint (in the latter) facing all the other agents.

[10] "Unconventional Monetary Policy After the Financial Crisis", The *Economic Journal*, Volume 122, Issue 564.

2 The Basic Model

There are three dates, $t = 0$, 1, and 2, and one storable consumption good, corn, which is also used as numeraire. Production occurs at $t = 0$, yielding and *consumption* at $t = 2$. There are N bankers, N^2 entrepreneurs and N^3 workers, with N being a large number; the idea is that bankers are in perfect competition and each banker serves a large numbers of entrepreneurs and there are more workers than can be hired by entrepreneurs. All agents are risk neutral and protected by limited liability.

Workers and entrepreneurs form the "real" side of the economy, as they own production factors. Workers each have 1 unit of labor. They either produce w kilograms (kg) of corn in autarky or are hired by entrepreneurs. Entrepreneurs each have h units of human capital (or physical capital). If an entrepreneur hires L units of labor at $t = 0$, then his project returns at $t = 2$

$$y = \widetilde{A} h^{1-\alpha} L^{\alpha}$$

kg of corn, where $0 < \alpha < 1$. Without loss of generality, let $h = 1$. Productivity $\widetilde{A}$ is subject to a common shock, which is realized at $t = 1$. At $t = 0$, it is common knowledge that $\widetilde{A} = \overline{A}$ with probability q and $\widetilde{A} = \underline{A}$ with probability $1 - q$. The realization of $\widetilde{A}$ at $t = 1$ is public known. Let $A_e \equiv q\overline{A} + (1 - q)\underline{A}$ denote the mean. Assume:

$$\underline{A} < A_e \alpha. \tag{1}$$

As there are much more workers than entrepreneurs, w, the output of a worker in autarky, is the real wage at which entrepreneurs hire workers.

Bankers form the monetary side of the economy. They each have $G \times N$ kg of corn. This corn held by bankers, it is important to note, is meant to represent "hard money" of the economy: it is not a factor for producing real good, but is excepted as means of payment.[11] Henceforth, it would be convenient use N kg as a unit since each banker in equilibrium will serve N entrepreneurs and if each of them borrow x kg corn from the banker, then the overall borrowing will be $x \cdot N$ kg, namely, x units.

Frictions

In the basic model, the economy suffers two frictions on the "real side" of the economy.

Friction 1: *entrepreneurs cannot hire workers at* $t = 0$ *with their promises to repay them at* $t = 2$.[12]

But workers certainly are willing to be paid with corn, which only bankers have at $t = 0$. Entrepreneurs, in order to hire workers, need to borrow corn or *its equivalent* from bankers. By "equivalent", I mean not real corn but *corn certificates,* namely the papers that certify the issuing banker owes the papers holders certain amount of corn.

[11] Indeed, the model would work equally well, but with reduced elegance, if I assume bankers each own $G \times N$ units of gold and 1 unit of gold can be used to exchange 1 kg of corn from a foreign economy at any date.

[12] There are two ways to understand the real life relevance of this friction. Directly, it represents a borrowing constraint facing entrepreneurs, because they are unable to commit to repay workers. Alternatively, this friction captures a *resale constraint*: suppose there are many, say K, types of goods and each is necessary for subsistence and produced by N/K entrepreneurs; the workers of an entrepreneur trust his promise of repaying them with the good he will produce, that is, the entrepreneur faces no borrowing constraint; but his workers cannot use his promise for exchanging other goods, nor can they easily bring his product around for doing so. This extended version of the model will deliver the same qualitative results as the present version.

The amount printed on a paper is called its *notional value*. I assume that the notional value of a paper cannot be contingent on the realization of the productivity shock, $\widetilde{A}$. Unlike corn, which is the liability of no one, the corn certificates are the liability of the issuing banker. Thus, by lending out corn certificates instead of corn, bankers are circulating their liabilities.

To sustain the perception that corn certificates are equivalent to real corn, bankers are assumed to commit to redeem their issue with corn on demand at $t = 1$ according to the notional values, until they exhaust their corn reserve, G; in this case they suspend redemption at date 1 and reopen for redemption at date 2 after receiving repayments from the debtor entrepreneurs. The assumption of redemption on demand is made to endogenize bank run in equilibrium and is dispensable to the results unconcerned with bank run. Then, G in the paper represents both bankers' wealth and hard money reserve. I will show in section 4 that it is the wealth that G really captures.

How well a banker can redeem her certificates determines the discount at which her certificates are valued to their notional values at $t = 0$, denoted by δ, and depends on the total notional value of her issue, denoted by D, which is assumed to be publicly observed at $t = 0$. If D is small, for example, no bigger than her corn reserve, then she can always redeem 1-to-1 her certificates, which are thus valued as their notional values, namely $\delta = 1$. However, if D is too large the banker will default, unable to redeem 1-to-1 her certificates. If so, the certificates are valued at a discount $\delta < 1$ of their notional values. In the basic model, bank default is assumed costless.

The other friction of the economy is:

Friction 2: *entrepreneurs are unable to make commitments on the scale of their* *projects in terms of the number of workers they hire.*

Due to this friction, a banker-entrepreneur loan contract cannot be contingent on the scale of the entrepreneur's project and is as follows. The entrepreneur borrows from the banker certificates of E kg corn at $t = 0$ and at $t = 2$ he has duty to repay the banker $E(1 + r)$ kg corn or its equivalent, with x kg corn equivalent to the certificates of the overall notional value x issued by the banker. r is thus the interest rate charged by the banker.

The timing is as follows.

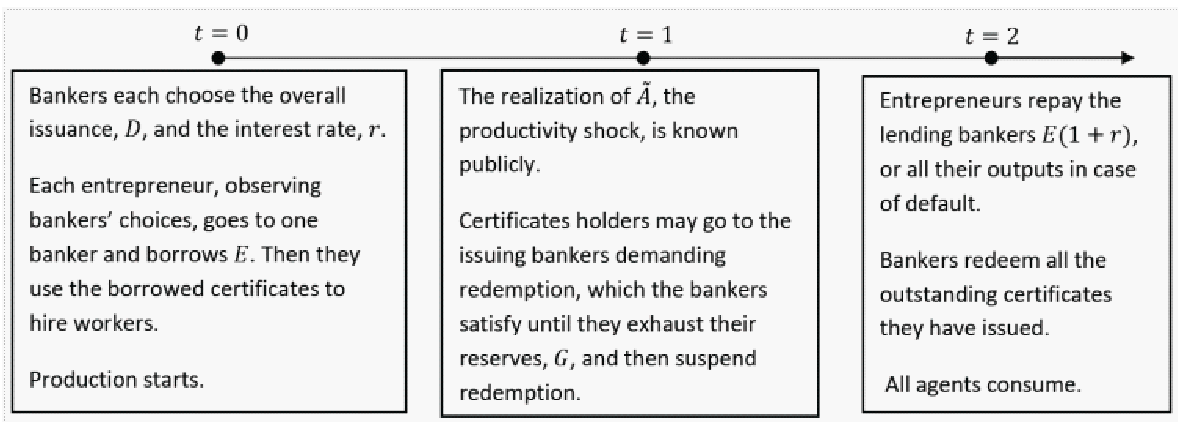

Figure 1: Timing of Events

Passing on to the equilibrium analysis of bank issuance and its efficiency, I figure out the socially efficient allocations, as benchmarks.

The First Best and Second Best Allocations

What efficiency concerns is the amount of labor allocated to entrepreneurs. Due to universal risk neutrality and the opportunity cost of labor being w, in the first best, the social planner's problem is $\max_L A_e L^\alpha - wL$. The first best amount of labor allocated to each entrepreneur, denoted by L^{FB}, is therefore:

$$L^{FB} = (\frac{A_e \alpha}{w})^{\frac{1}{1-\alpha}}. \tag{2}$$

The second best allocation is defined as the allocation in absence of Friction 1, namely, the amount of labor entrepreneurs hire in the competitive equilibrium if they can hire workers with their own promises, which makes bankers irrelevant for the organization of production. The equilibrium allocation is as follows.

Lemma 1 *The second best amount of labour drawn to the entrepreneur sector is:*

$$L^{SB} = (\frac{q\overline{A}\alpha + (1-q)\underline{A}}{w})^{\frac{1}{1-\alpha}} \tag{3}$$

Proof. See Appendix. ■

$L^{SB} > L^{FB}$, namely, the equilibrium scale of entrepreneurs' projects is too large. It arises because of Friction 2 due to which entrepreneurs can only compete by posting wages; if they could compete with posting both wage and scale, then the equilibrium allocation would conform with the first best. The analysis of this case is to be found in Wang (2010). The feature that entrepreneurs would hire too many workers in the competitive equilibrium even in the absence of Friction 1 is interesting because, as I will show, while competition between bankers overcomes Fiction 1 perfectly, it has no bite on Fiction 2, but when I add the third friction that disallows bank default, then the optimal monetary policy may be able to restore the first best allocation, that is, overcome both Fictions 1 and 2 and achieve more than the competitive market can.

3 Bank Issue in the Competitive Equilibrium

I first consider the demand side of the papers (i.e. corn certificates) market, namely entrepreneurs.

3.1 The Demand Side of the Papers Market

Consider the entrepreneurs who go to a banker that offers deal (D, r), which, as will be shown, determines δ, the discount at which the banker's certificates are valued to their notional values at $t = 0$. Each of the entrepreneur is small to the banker, who serves a large number of them, and his demand has negligible effects on the bank's deal. Therefore each entrepreneur takes (δ, r) as given when deciding on his demand of the banker's papers. If, for hiring workers, he borrows certificates of E kg corn, which are worth δE at $t = 0$, then, given the real wage to workers is w, he hires

$$L = \frac{\delta E}{w} \tag{4}$$

workers.

At $t = 2$, the entrepreneurs have duty to repay the bank $E(1 + r)$, in either real corn, or the certificates of as amount, or a mix in between. They will strictly prefer to repay with the certificates, if the certificates are worth less than the notional value at $t = 2$, which, however, occurs only if they default; otherwise, their demand in total of the certificates is $E(1 + r)$ units, but the supply of the certificates, namely the amount issued at $t = 0$, is only E, and this over-demand would push up the values of certificates to the notional values. Therefore, the entrepreneurs either outlay $E(1 + r)$ kg corn to repay the loans or default. Their problem is thus

$$\max_{E} q(\overline{A}L^{\alpha} - E(1+r)) + (1-q)\max(\underline{A}L^{\alpha} - E(1+r), 0), s.t.(4) \tag{5}$$

They indeed default in the bad state:

Lemma 2 *For any* (w, δ, r)*, at the optimum* $\underline{A}L^{\alpha} < E(1+r)$*, that is, the entrepreneurs default in the bad state.*

Proof. See Appendix. ■

Therefore, the solution to problem (5), namely, the demand of the certificates by each entrepreneur coming to the banker, is:

$$E(\delta, r) = (\frac{\overline{A}\alpha}{1+r})^{\frac{1}{1-\alpha}}(\frac{\delta}{w})^{\frac{\alpha}{1-\alpha}} \tag{6}$$

Then, the amount of labor entrepreneurs each hire and their profit are are respectively:

$$L(\delta, r) = (\frac{\overline{A}\alpha}{wR})^{\frac{1}{1-\alpha}} \tag{7}$$

$$V(\delta, r) = q(1-\alpha)(\frac{\overline{A}^{\frac{1}{\alpha}}\alpha}{wR})^{\frac{\alpha}{1-\alpha}}, \tag{8}$$

where

$$R \equiv \frac{1+r}{\delta}$$

is the *real* gross interest rate of the borrowing: the gross "nominal" interest rate $1+r$ is charged on the notional values, which are inflated by $1/\delta$ compared to the market value with which the entrepreneurs hire workers. Naturally, both L and V depend only on R and inversely. Define the real interest rate at which $L = L^{FB}$ in (7) (i.e. the first best amount of labor hired by entrepreneurs) as the first best interest rate, denoted by R^{FB}, and that at which $L = L^{SB}$ (i.e. the second best amount of labor hired) as the second best interest rate, denoted by R^{SB}. Then,

$$R^{FB} = \frac{\overline{A}}{A_e} \tag{9}$$

$$R^{SB} = \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}}. \tag{10}$$

3.2 The Supply Side of the Papers Market

In equilibrium, all the entrepreneurs get the same profit whatever bankers they go to and thus only one real interest rate, $\widehat{R}$, prevails, which then characterizes the market conditions bankers face. A representative banker takes this $\widehat{R}$ as given and decides on (D, r), namely the overall national value of her issue and the nominal interest rate. As the profit of entrepreneurs from the banker's deal is inversely related to $R = (1 + r)/\delta$, the banker can attract entrepreneurs to come only if of her deal $(1 + r)/\delta \leq \widehat{R}$. r is directly chosen by her. How δ is determined by (D, r) is examined below.

The value of δ depends on whether the banker defaults. In the good state, when the entrepreneurs on her asset side do not default, the banker has no difficulty redeeming her papers on the liability side. But in the bad state, the default of all the entrepreneurs, which occurs by Lemma 1, drags the banker down to default if her issue is too large, as the proposition below states.

Proposition 1 *If the representative banker chooses* (D, r)*, then*

(i): In the bad state, the value of her loans is

$$\underline{Y} = \frac{\underline{A}(1+r)}{\overline{A}\alpha} D, \tag{11}$$

and she does not default if and only if her issuance, D, is small enough that

$$D \cdot (1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}) \leq G; \tag{12}$$

(ii): the discount of her issue at $t = 0$ is

$$\delta(D, r) = \left\{ \begin{array}{c} 1, \text{ if } D(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}) \leq G \\ q + (1-q)(\frac{G}{D} + \frac{\underline{A}(1+r)}{\overline{A}\alpha}), \text{ otherwise} \end{array} \right\}. \tag{13}$$

Proof. See Appendix. ■

Intuitively, first, (11) arises because at $t = 1$, upon the news of $\widetilde{A} = \underline{A}$, the defaulting entrepreneurs hand over to the banker the whole of their projects, the value of which is in a proportion to the amount of her issue (i.e. D) and the proportion is positively related to the interest rate (i.e. $1 + r$) because the higher the rate, the smaller the scale of the projects, and the greater is their value as they are of decreasing returns to scale. Second, the banker's balance sheet is (in the unit of N kg corn):

Asset	**Liability**
Corn reserve (G)	Equity
Loans to the entrepreneurs ($\underline{Y}$)	Debt (D)

Table 1: The balance-sheet of a banker in the bad state

She does not default, thus, if and only if

$$D \leq G + \underline{Y}, \tag{14}$$

which, with (11), is equivalent to (12). Lastly, certainly $\delta = 1$ (i.e. the certificates are not discounted) if the banker will never default, while if she will default in the bad state, then the discount is $(G+\underline{Y})/D$ in that state and is 1 in the good state (occurring with probability q). So arises (13).

Bank default triggers bank run. A banker known to default means her papers are known to worth less than their notional values on average (i.e. $G+\underline{Y}<D$). But if a paper holder gets to the banker early enough before her gold reserve is exhausted and thus get his holding redeemed into corn, he will get the notional values of his holding. Therefore, upon the arrival of the bad news on the banker's assets, all the paper holders rush to the defaulting banker for redemption. *Bank run* arises.

At $t=0$, if the representative banker chooses (D,r), then economic her profit is

$$\Pi(D,r)=q(G+Dr)+(1-q)\max(G-(1-\frac{\underline{A}(1+r)}{\overline{A}\alpha})D,0)-G. \tag{15}$$

In the good state, as no entrepreneurs default, the banker earns earns interest rate Dr from the loans and keeps his reserve G, whereas in the bad state, she might default if $G+\underline{Y}<D$, and thus her profit is thus $\max(G+\underline{Y}-D,0)$, which, with $\underline{Y}$ given by (11), gives rise to the term multiplied by $1-q$.

Taking the market conditions characterized by $\widehat{R}$ as given, the banker chooses (D,r) to maximize this profit function, subject to the constraint that her deal can attract entrepreneurs to come, that is,

$$\frac{1+r}{\delta}\leq\widehat{R}, \tag{16}$$

where δ is determined by (D,r) through (13).

The banker's problem is thus:

$$\max_{D,r,\delta}\Pi(D,r),\ s.t.(16)\text{ and }(13) \tag{17}$$

The solution to this problem certainly depends on $\widehat{R}$, the market conditions facing all bankers, as given below (remember $R^{SB} = \frac{\overline{A}\alpha}{q\overline{A}\alpha+(1-q)\underline{A}}$ by 10):

Proposition 2 *The solution to and the value of the banker's problem is:*

(i) if $\widehat{R} > R^{SB}$*, then* $D = \infty$ *and* $\Pi = \infty$*;*

(ii) if $\widehat{R} = R^{SB}$*, then the banker is indifferent with any* D*, and* r *is settled by* D *through the binding (16), namely,*

$$\frac{1+r}{\delta} = \widehat{R}, \tag{18}$$

with δ *as a function of* (D, r) *given by (13), and* $\Pi = 0$*; and*

(iii) if $R^{SB} > \widehat{R}$*, then* $D = 0$*, namely, lending to entrepreneurs is not profitable to the banker.*

Proof. See Appendix. ■

The core messages of the proposition are as follows.

First, the profit margin of lending to entrepreneurs is positive, thus lending generating a profit, if and only if $\widehat{R} > R^{SB}$. For an intuition, note that the banker wants to increase the interest rate, r, so long as she can still attract entrepreneurs, that is, until (16) is binding, thus at the optimum the banker's real interest rate $R = \widehat{R}$; and that the profit margin of lending, $\widehat{\pi}$, is the surplus of the social value of a project invested minus the profit to the entrepreneur, that is, $\widehat{\pi} = A_e L^\alpha - wL - V$, which, with (7) and (8) and $R = \widehat{R}$, gives rise to

$$\widehat{\pi} = (\frac{\overline{A}\alpha}{w})^{\frac{\alpha}{1-\alpha}}(q\overline{A}\alpha + (1-q)\underline{A}) \cdot \widehat{R}^{\frac{-1}{1-\alpha}}(\widehat{R} - R^{SB}). \tag{19}$$

Therefore, the profit margin is positive, namely $\widehat{\pi} > 0$, if and only if $\widehat{R} > R^{SB}$.

Second, if the bankers' profit from lending is positive (namely $\widehat{R} > R^{SB}$), then it is infinitely large (i.e. $\Pi = \infty$), because the banker can lend out an infinitely large amount (i.e. $D = \infty$). This capability of the banker, in spite of her finite stock of funds (i.e. corns), is derived from the privilege of her liability (i.e. corn certificates) being accepted as a means of payment, a feature of the paper's focus, by which the banker can finance assets by lending out her liability and therefore faces no budget constraint.

3.3 The Equilibrium: the Irrelevance of Bankers' Wealth and Bank Run

The equilibrium is defined as follows, where D is in the unit of N kg, bankers are indexed with $i \in \mathbb{N} \equiv \{1, 2, ..., N\}$, and $\widehat{x}$ is used to denote the equilibrium value of variable x:

Definition 1 *A profile* $(\{\widehat{D}_i, \widehat{r}_i, \widehat{\delta}_i, \beta_i, \widehat{E}_i\}_{i \in \mathbb{N}}; \widehat{R})$ *forms an equilibrium, if:*

(i) given all other bankers offer real interest rate $\widehat{R}$*, which thus prevails,* $(\widehat{D}_i, \widehat{r}_i, \widehat{\delta}_i)$ *solves problem (17) (i.e. among banker i's optimal choices) and* $(1 + \widehat{r}_i)/\widehat{\delta}_i = \widehat{R}$*;*

(ii) given bankers offer $\{\widehat{D}_i, \widehat{r}_i, \widehat{\delta}_i\}_{i \in \mathbb{N}}$*, entrepreneurs get the same profit from any banker and each go to banker i with probability* β_i*;*

(iii) given banker i offers $(\widehat{D}_i, \widehat{r}_i, \widehat{\delta}_i)$*, the entrepreneurs coming to her demand* $\widehat{E}_i$*, that is,* $\widehat{E}_i = E(\widehat{\delta}_i, \widehat{r}_i)$ *given by (6);*

(iv) market clears: $\widehat{D}_i = N\beta_i\widehat{E}_i$ *and* $\sum_{i \in \mathbb{N}} \beta_i = 1$.

In equilibrium, bankers neither get an infinitely large profit, nor abstain from lending, which, by Proposition 2, is the case if and only if

$$\widehat{R} = R^{SB}. \tag{20}$$

At this value of $\widehat{R}$, the amount of labour hired by entrepreneurs is

$$\widehat{L} = L^{SB},$$

the second best amount which the competitive markets would implement in the absence of Friction 1. Moreover, by Proposition 2, bankers get 0 profit and are indifferent with any D. This indeterminacy leads to a continuum of equilibrium. In the symmetric equilibrium among them, where all banker offer the same deal, $(\widehat{D}, \widehat{r}, \widehat{\delta})$, and $\beta_i = 1/N$ for each $i \in \mathbb{N}$, $(\widehat{D}, \widehat{r}, \widehat{\delta}, \widehat{E})$ is characterized by (18) (the binding attracting-entrepreneurs constraint), (13) (the equation settling the discount of certificates), $\widehat{D} = \widehat{E}$ (condition iv of the definition), and (6) (condition iii).

The main feature of the equilibria described in the following proposition (in the proof of which the characterization of the symmetric equilibrium is to be found).

Proposition 3 *(i) In any equilibrium, however small bankers' wealth (G) is, the profit margin of bank issuance is 0 and the quantity of bank issue exactly suffices to draw the second best amount of labor, L^{SB}, to entrepreneurs, that is, is pinned by the real side of the economy.*

(ii) In any equilibrium, a positive mass of bankers default (and in the symmetric equilibrium all bankers default), and thus bank run occurs to them at $t = 1$ upon the arrival of bad news $\widetilde{A} = \underline{A}$, if and only if bankers' wealth is so low that

$$G < q(\overline{A}\alpha - \underline{A})(\frac{q\overline{A}\alpha + (1-q)\underline{A}}{w})^{\frac{\alpha}{1-\alpha}} \equiv G^{SB}, \tag{21}$$

where G^{SB} is the level of bankers' wealth that exactly suffices to back bank issue for the second best amount of labour drawn to entrepreneurs without inducing bank default.

Proof. See Appendix. ■

Some intuitions and comments for the proposition are as follows.

First, result (i) arises because bankers are engaged into a Betrand competition with *unlimited* capacity: If the profit margin of bank issuance is positive, namely if $\widehat{R} > R^{SB}$, a banker would undercut all the others, and thus earn a huge profit, by offering a lower real interest rate (i.e. $(1+r)/\delta$). The banker can undercut *all the others,* namely serve all the entrepreneurs, because her lending capacity is unlimited. This, in turn, is because they can lend out not only what they have (i.e. their wealth), but also what they do not have yet, namely her liabilities, as was noted in the second comment to Proposition 2. Put differently, *bankers are hurt by their very privilege of liability being accepted as means of payment.*

Second, as the aggregate value of bank issues is fixed at wL^{SB}, exactly sufficient to draw the second best amount of labour to entrepreneurs, their aggregate notional value $\widehat{D} \geq wL^{SB}$. Therefore, if bankers' wealth, G, is small enough to honor (21), then it cannot back the issuance of such a quantity of papers without resorting to default, which triggers bank run as was noted. Therefore, if $G < G^{SB}$ bank run occurs to many bankers in any equilibrium (and to all bankers in the symmetric equilibrium).

But bank run is usually very costly. The next section considers a case where it is assumed extremely costly and thus disallowed. Then, bankers' lending capacity, D, is anchored by their wealth, G, through (12), the condition ensuring no-default. When their wealth is low, bankers pump into the economy only inadequate amount of money (which, indeed, is regarded by by Kiyotaki and Moore 2001 as the blood of the economy). This shortage of inside money, as will be shown, engenders room for the central bank printing to affect the real economy by supplementing money supply.

4 Extension: The (Unconventional) Central Banking: to Print and Give

As was said, in this section I introduce the third friction:

Friction 3: *bank default is disallowed.*

This friction serves to anchor the lending capacity of bankers (D) to their wealth (G). The same effect could be served by a moral-hazard related assumption, such as the equity value of a banker (i.e. $G + Y - D$; see table 1) should never fall below αG, with $\alpha \in (0, 1)$ (otherwise the banker will abscond with α fraction of her wealth without being caught) and is indeed so served in Getler and Kiyotaki (2010).

Moreover, in this section I assume bankers' corn reserves are small enough to honor (21), that is,

$$G < G^{SB}, \tag{22}$$

Otherwise equilibrium entails no bank default, according to Proposition 3.

Below I first work out the market equilibrium without intervention by the central bank and then proceed to demonstrate how the central bank can improve social efficiency by printing fiat money.

4.1 The Wealth of Bankers Matters: the Quantity of Inside Money, Interest Rate and Efficiency

What occurs is summarized by the following proposition.

Proposition 4 *Suppose Fiction 3 is present and (22) is honored. Then there is a unique equilibrium in which:*

(i) the profit margin of bank issuance is positive and bankers lend to the most where they stay solvent. The quantity of bank issue is pinned by the monetary side (i.e. bankers), anchored by G *through*

$$\widehat{D} = \frac{G}{1 - \frac{A}{\overline{A}\alpha}\widehat{R}}, \tag{23}$$

where $\widehat{R}$*, the equilibrium real interest rate, is pinned down by* G *through*

$$\frac{1 - \frac{A}{\overline{A}\alpha}R}{R^{\frac{1}{1-\alpha}}} = \frac{w^{\frac{\alpha}{1-\alpha}}}{(\overline{A}\alpha)^{\frac{1}{1-\alpha}}}G; \tag{24}$$

(ii) $\widehat{R}(G)$ *is decreasing with* G *and* $\widehat{D}$ *increases with* G*;*

(iii) the amount of labor drawn to the entrepreneur sector increases with G*, but always fall below the second best quantity.*

Proof. See Appendix. ■

Below are intuitions and comments for the proposition.

First, Friction 3 anchors the amount of bank issue to bankers' wealth, which, assumed in (22), is inadequate. Consequently, aggregate inside money (i.e. bank issue) is in dearth, falling below the second best amount, causing the real interest rate above the second best level, namely $\widehat{R} > R^{SB}$. At this real rate, as was noted in the first comment following Proposition 2, the profit margin of bank issuance is positive. The positive margin drives each bankers to issue as much as long as she is solvent, namely, until the non-default constraint, (12), is binding, which gives rise to (23). Equalizing this quantity of supply to the quantity of demand, $\widehat{E}$ (given by 6), gives rise to (24) and determines the real interest rate ($\widehat{R}$).

Second, since the quantity of inside money (D) is anchored by bankers' wealth (G), the higher is the wealth, the more inside money is pumped into the economy, and as a result, the lower the interest rate of borrowing it ($\widehat{R}$). So arises result (ii).

Third, workers are hired with inside money. Thus the quantity of workers hired pinned down by the quantity of inside money pumped. As the latter increases with G and falls below the second best level, so does the former. That is result (iii).

Inside money falling below the second best level engenders room for the central bank printing to make a difference, as I proceed to show.

4.2 The Effects and the Optimal Quantity of the Central Bank Prints

In this subsection I assume that besides corn, there is another universally accepted media of exchange, shells, which the central bank of the economy can produce costlessly. Therefore, entrepreneurs can use shells, besides corn or corn certificates, to hire workers.

To expand the money supply, the central bank lends to each banker S units (i.e. $S \times N$ kg) shells at $t = 0$ and demands her to pay back at $t = 2$ to it S units shells or their equivalent, with 1 kg shells equivalent to 1 kg of corn. At $t = 0$, then bankers lend to entrepreneurs both corn certificates or shells. I assume that an entrepreneur borrows only shells or only corn certificates, but not both of them, in order to avoid the complexity of which debt of the two is senior.[13]

A loan contract for corn certificates is as was in the basic model: If an entrepreneur borrows from a banker certificates of E kg corn at $t = 0$, then at $t = 2$ he has duty to repay the banker with either corn or the corn certificates issued by her, in total $E(1+r)$, with x kg of corn equivalent to the certificates of this notional amount.

A loan contract for shells is similar. If an entrepreneur borrows from a banker E_s kg shells at $t = 0$, then at $t = 2$ he has duty to repay her with shells or corn, in total $E_s(1+r_s)$ kg, with x kg shells equivalent to x kg of corn. As will be shown, the optimal monetary policy entails $r_s > 0$ in equilibrium.

Note that in contrast to corn certificates, shells are not any bankers' liability: a holder of shells loaned out by a banker has no right to ask the bank to redeem the shells with corn; indeed, it is because shells are not bankers' liability that issuance of shells enlarges money supply while keeps bankers in solvency. A worker accepts shells because he knows he can use them to buy corn from any entrepreneur at $t = 2$. The amount of corn which 1 kg shells exchanges at $t = 2$, denoted by p_2, is 1 in the good state: $p_2 \leq 1$ always as entrepreneurs can offset debt of 1 kg shells with 1 kg corn; and if $p_2 < 1$, each shell-borrowing entrepreneur would want only shells, $E_s(1 + r_s)$ kg of them, to repay what he has borrowed, E_s, thus the total demand to be $1 + r_s$ times of the total supply, not in equilibrium. In the bad state, when entrepreneurs default, $p_2 < 1$. Then shell-borrowing entrepreneurs want to use only shells to clear their debts.

[13] That is, when the entrepreneur defaults, should he do his best to first clear the debt of corn certificates or the debt of shells?

As they default, all their output is spent to buy shells. Therefore, p_2 is equal to the ratio of the total output of all such entrepreneurs over the total issue of shells, which, by a reasoning similar to that leading to condition (12), equals $\underline{A}(1+r_s)/(\overline{A}\alpha)$. Therefore, The price of shells at $t = 0$ is:

$$\delta_s = q + (1-q) \cdot \frac{\underline{A}(1+r_s)}{\overline{A}\alpha}. \tag{25}$$

Alternatively, this equation can be derived from equation (13), by noting that shells are backed by none of the banker's wealth (and only by the loans), namely, replacing G with 0, and replacing r with r_s.

Now move on to the properties of the equilibrium, which is characterized in the proof of the equilibrium. Let

$$\underline{S}(G) \equiv \frac{\overline{A}\alpha}{q(\overline{A}\alpha - \underline{A})}(G^{SB} - G).$$

And note that (24) gives G as a function of R, which is denoted by $G(R)$. Then let $G^{FB} \equiv G(R^{FB})$, namely the level of bankers' wealth at which bank issue draws the first best amount of labour to entrepreneurs without inducing bank default. And let $\underline{G} \equiv G(\frac{1}{\alpha}R^{SB}) > 0$, where $\frac{1}{\alpha}R^{SB} = \arg\max_{\widehat{R}} \widehat{\pi}(\widehat{R})$, with $\widehat{\pi}(\widehat{R})$ given by (19), namely, $\frac{1}{\alpha}R^{SB}$ is the level of the equilibrium real interest rate where a banker gets the maximum profit from a single entrepreneur of her. With (9) and (10), $\frac{1}{\alpha}R^{SB} > R^{FB}$. Thus $\underline{G} < G^{FB}$, as $G(R)$ decreases with R by Proposition 4 (ii).

Proposition 5 *If $S \leq \underline{S}(G)$, then in the unique equilibrium,*

(i) the real interest, $\widehat{R}$, is pinned down by the central bank issuance, S, through

$$S = (\frac{\underline{A}}{w^\alpha})^{\frac{1}{1-\alpha}} \frac{1-(1-q)\theta}{q\theta^{\frac{1}{1-\alpha}}} - \frac{1-(1-q)\theta}{q(1-\theta)}G, \text{ with } \theta \equiv \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}. \tag{26}$$

And $\widehat{R}$ *decreases with* S *and at* $S = \underline{S}(G)$ *equals* R^{SB}*, the real interest in the second best allocation, given by (20);*

(ii) the interest rate of lending shells is given by

$$r_s = \frac{(q\overline{A}\alpha + (1-q)\underline{A})\widehat{R} - 1}{\overline{A}\alpha - (1-q)\underline{A}\widehat{R}}, \tag{27}$$

which decreases with S *and equals 0 at* $S = \underline{S}(G)$*;*

(iii) if $G \geq \underline{G}$*, bankers' profit decreases with* S *always, and if* $G < \underline{G}$*, it increases with* S *for* $S \leq S(\frac{1}{\alpha}R^{SB})$ *and decreases with* S *otherwise, where function* $S(R)$ *is given by (26).*

Proof. See Appendix. ■

Here are comments and intuitions as for the proposition.

First, *the central bank printing affects the real interest rate,* $\widehat{R}$*, thus efficiency, if and only if* $\widehat{R}$ *is high in the sense* $\widehat{R} > R^{SB}$ (due to the bank issue anchored by inadequate wealth of bankers). The "if" part is given by result (i) here. Intuitively, it lowers $\widehat{R}$, the price of borrowing means of payment, namely money, to entrepreneurs, by supplementing the money supply with its prints, and therefore, the more it prints, the lower is $\widehat{R}$ and r_s. As for the "only if" part, note that by Proposition 2 (iii), $\widehat{R}$ can be dragged down below R^{SB} by no means, or bankers stop lending, not a case in equilibrium.

Second, even when the central bank printing can lower $\widehat{R}$, the effect has a limit: the rate can never be lower than R^{SB} as $\widehat{R} \geq R^{SB}$ always. Indeed, at $S = \underline{S}(G)$, by results (i) and (ii) here, $\widehat{R} = R^{SB}$ and $r_s = 0$, meaning the profit margin of lending both inside and fiat money is 0. Bankers are thus indifferent with any quantity of money circulated, which, therefore, is pinned down by the real side of the economy. Any further printing by the central bank, namely $S > \underline{S}(G)$ will not raise the quantity of money circulated, but crowd out the inside money or stay in the vaults of bankers.

Third, result (iii) states that while the supply of fiat money always benefits the real sector (i.e. entrepreneurs) by lowering the real interest rate, it may very likely make the banking sector worse-off, though it looks like giving bankers free cash. Intuitively, this policy, by enlarging the lending capacity of all bankers, generates two opposing effects. One, it increases the scale of bankers' business enabling entrepreneurs to hire more workers. The other, it diminishes the profit margin of bank lending, as measured by $\widehat{R}$, by subjecting bankers to fiercer competition. By result (iii), the latter, negative, effect dominates the former if $\widehat{R} < \frac{1}{\alpha} R^{SB}$. Note, however, that even when the banking sector as whole is worse-off by the central bank funds, an individual banker has incentives to request them, because she takes prices $\widehat{R}$ and r_s as given, and into no account the effect of her request on the prices.

With the effects of central bank printing made clear, now we consider the optimal quantity of printing. The efficiency is solely determined by $\widehat{R}$, which, by result (i), decreases with S through (26). If at $S = 0$, namely without central bank intervention, $\widehat{R} < R^{FB}$, then too many workers have been hired by entrepreneurs compared to the first best allocation and any reduction in $\widehat{R}$ lowers efficiency. Therefore, the optimal quantity of printing is 0. If $\widehat{R} \geq R^{FB}$ at $S = 0$, the central bank can implement $\widehat{R} = R^{FB}$, namely the first best allocation, by printing $S = S(R^{FB})$, where function $S(R)$ is given by (26). Note that $\widehat{R} < R^{FB}$ at $S = 0$ if and only if $G > G^{FB}$. Then the following proposition is self-evident.

Proposition 6 *The optimal quantity of the central bank printing is $S = 0$ if $G \geq G^{FB}$*

and $S = S(R^{FB})$ *if* $G < G^{FB}$, *with function* $S(R)$ *given by (26). Therefore, when the fiat money is printed, namely,* $S > 0$, *the interest rate of borrowing it is* $r_s(R^{FB}) > 0$, *where function* $r_s(R)$ *is given by (27).*

Note that if $G < G^{FB}$, in the absence of Friction 3, as was shown in section 3, the market only implements the second best, with which the central bank printing can do nothing, but in its presence, the central bank achieves the first best, thus more than can be achieved by the competitive market troubled by fewer frictions.

5 Discussion: What Does G represents: Bankers' Wealth or Reserve?

In the paper, the corn stock of bankers, G, may be regarded as representing both bankers' wealth and their reserves for redemption at $t = 1$. In this section I underline that the former is the case. To see this point clearly, let me change the model a little bit. Suppose now and here at $t = 0$ not only bankers have corn, but so do some workers of whose stocks the sum total is $G_w \times N$ units. And these workers, consuming at $t = 2$ but worried about the safety of cellaring their corn from $t = 0$ to then, want to deposit it with bankers in exchange for their corn certificates, D'. Assume these deposits flow to bankers equally. Then, the wealth of each banker is G, while the corn reserve for redemption at $t = 1$ is $G + G_w$.

Carry out the analysis in a parallel way, and the balance sheet of a banker at $t = 1$

is as follows:

Asset	**Liability**
corn Reserve $(G + G_w)$	Equity
Loans to entrepreneurs (Y)	Debt to holders of certificates loaned out (D)
	Debt to depositors (D')

Table 2: The balance sheet of bank with additional corn reserves

The banker does not default in the bad state if and only if

$$D + D' \leq G + G_w + \underline{Y}.$$

When she does not default, her issues is worth its notional value. Therefore,

$$D' = G_w.$$

Then in the bad state, the non-default condition becomes

$$D \leq G + \underline{Y},$$

the same as (14), which, in the same logical development, leads to (12), from which, in turn, all the subsequent results are derived, that is, none of them needs to change with the the introduction of G_w, the new source of reserves.

6 Conclusion

Banks, unlike another other business enterprises, have the privilege of lending by issuing liability, as their liability is accepted as means of payment (when their soundness is not in doubt), by which the money supply depends critically on the balance sheet operations by banks. This privilege of banks is the focus of the paper. It gives a general equilibrium analysis of the quantity and efficiency implications of bank issue, and based on it, discusses principles for the foundation of central banking, with the power of competition between banks underline. Its main findings are as follows.

First, due to competition, banks do not earn a lot with their privilege of lending by issuing. Indeed, if the competition is sufficient, which is the case in the paper either because no friction anchors the quantity of bank issue to banks' wealth, or because banks have abundant wealth to support enough bank issue without inducing default, then the profit margin of bank issue is 0 and the equilibrium quantity of bank issue circulated is determined by the real side of the economy, and therefore, no efficiency is lost due to the friction that drives the real sector to borrow money from the banking sector.

Second, if banks' wealth anchors bank issue through some friction and is scanty, then it matters for efficiency and becomes a public concern, because, the smaller the banks' wealth, the less the quantity of inside money (i.e. bank issue) circulated, and the higher the real interest rate, which harms the real sector.

Third, if (and only if) the real interest rate is higher than a threshold, there is room for the central bank to affect the real interest rate by printing fiat money and lending it at zero interest rate to *all* banks. This policy benefits the real sector always as it lowers the real interest rate. However, it may very likely make the banking sector worse-off, although it looks like the policy gives banks free cash, because of, again, the power of competition, which is made fiercer by the policy.

Appendix: The Proofs

Of Lemma 1:

In equilibrium, only one wage, denoted by F, as will be shown, prevails on the market. Competitive equilibrium is thus defined as a profile of (F, L), such that:

(a) Given the posted wage prevailing on the market, F, the optimal labor demand of each entrepreneur is L;

(b) Given each entrepreneur demands L workers, F clears the labor market.

For (a): Given F, the representative entrepreneur chooses L to maximize $q(\overline{A}L^{\alpha} - FL) + (1-q)\max(\underline{A}L^{\alpha} - FL, 0)$, where the max in the latter term captures the possibility that in the bad state the entrepreneur might default, that is, all her output is not sufficient to pay wages she had promised and thus goes to her workers. That, under Assumption (1), is in fact the case: Otherwise, she would hire $(\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}}$ workers; then in the bad state, her output is $\underline{A}((\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}})^{\alpha}$, which, if $\underline{A} < A_e\alpha$ (assumed in 1), is smaller than $F \cdot (\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}}$, the promised wage outlay, and hence she defaults, a contradiction. Given she defaults and gets 0 in the bad state, she only cares for the good state profit and thus maximizes $\overline{A}L^{\alpha} - FL$. Equilibrium condition (i) is, therefore, summarized by:

$$L = (\frac{\overline{A}\alpha}{F})^{\frac{1}{1-\alpha}} \tag{28}$$

For (b): As there are much more workers than entrepreneurs, the labor market is cleared by making workers indifferent between autarky and hired by an entrepreneur, namely, by giving them expected wage income w. In the good state, the workers hired get the promised wage, F; in the bad state, each entrepreneur defaults, as was shown, and the workers equally share the output and each gets $\frac{\underline{A}L^{\alpha}}{L} = \underline{A}L^{\alpha-1}$. Therefore, equilibrium condition (ii) is summarized by:

$$qF + (1-q)\underline{A}L^{\alpha-1} = w \tag{29}$$

Combine these two equations, and the amount of labor hired in equilibrium, namely, the second best amount of labor for each entrepreneur, is given by (3).

Now I show only one F prevails on the market. If an entrepreneur posts F, then by (28) he hires $L = (\frac{\overline{A}\alpha}{F})^{\frac{1}{1-\alpha}}$ workers, to whom the wage income is F in the good state and $\underline{A}L^{\alpha-1} = \frac{\underline{A}}{\overline{A}\alpha}F$, both increasing with F. Therefore workers only go to entrepreneurs who post the highest F and in equilibrium only one F prevails.

Q.E.D.

Of Lemma 2:

Suppose otherwise entrepreneurs do not default. Then their problem is:

$$\max_E q(\overline{A}L^\alpha - E(1+r)) + (1-q)(\underline{A}L^\alpha - E(1+r)) \text{ s.t. } (4)$$

From the constraint, $E = wL/\delta$. Substitute it into the objective and let $\gamma \equiv w(1+r)/\delta$. Then entrepreneurs' problem becomes

$$\max_L A_e L^\alpha - \gamma L.$$

The solution satisfies

$$L^{\alpha-1} = \frac{\gamma}{A_e \alpha}. \tag{30}$$

At this level of labor, entrepreneur will default in the bad state, and thus the supposition made at the beginning leads to a self-contradiction: Default means $\underline{A}L^\alpha < E(1+r)|_{E=wL/\delta;\gamma\equiv w(1+r)/\delta} \Leftrightarrow \underline{A}L^\alpha < \gamma L \Leftrightarrow \underline{A}L^{\alpha-1} < \gamma \Leftrightarrow |_{(30)}\underline{A} \times \frac{\gamma}{A_e\alpha} < \gamma \Leftrightarrow \underline{A} < A_e\alpha$, which is assumed in (1).

Q.E.D.

Of Proposition 1:

(i): As the entrepreneur borrowers all default in the bad state and each hands over his whole output, $\underline{y} = \underline{A}L^{\alpha}$, to the banker, the value of the banker's loans in the bad state, $\underline{Y}$, thus equals $\underline{y}$ times the number of entrepreneurs she finances, D/E. With (7) (which determines L) and (6) (which determines E), we find

$$\frac{\underline{y}}{E} = \frac{\underline{A}(1+r)}{\overline{A}\alpha}. \tag{31}$$

As $\underline{Y} = \underline{y} \cdot D/E = D \cdot \underline{y}/E$, $\underline{Y} = \underline{A}(1+r)/\overline{A}\alpha \cdot D$. Substitute it into (14), which then becomes (12). Thus no bank run occurs if and only if (12) holds true.

(ii): By Proposition 1, if condition (12) holds true, no bank run will arise and the banker is always redeem her papers according to the face value. Therefore, the papers are valued at no discount, that is, $\delta = 1$.

I am thus left with the case where condition (12) is violated. Consider what happens at $t = 1$. If $\widetilde{A} = \underline{A}$, the banker defaults. That is, all her assets, which are worth $G + \underline{Y}$, are all used to redeem the papers the overall notional value of which is D. Given the certificate holders are all risk neutral, the discount factor of the certificates in the bad state, $\underline{p_1}$, is thus $(G + \underline{Y})/D$. With $\underline{Y}$ given by (11),[14]

$$\underline{p_1} = \frac{G}{D} + \frac{\underline{A}(1+r)}{\overline{A}\alpha}.$$

On the other hand, if $\widetilde{A} = \overline{A}$, then the entrepreneurs do not default and thus the banker does not default. Thus her papers is not discounted, namely, $\overline{p_1} = 1$.

[14] Note that the derivation of (11) is independent of $p_c = 1$ (i.e the non-default of the banker) and applicable to the case where the banker defaults in the bad state.

At $t = 0$, it is anticipated that the good state will occur with probability q, the bad state $1 - q$. Thus, the price of the banker's papers, $q\overline{p_1} + (1-q)\underline{p_1}$, is:

$$\delta = q + (1-q)(\frac{G}{D} + \frac{\underline{A}(1+r)}{\overline{A}\alpha}). \tag{32}$$

This is the price of the papers if (12) is violated and the banker defaults in the bad state. If the condition holds true, the banker will never defaults and $\delta = p$.

The case of default and that of non-default put together, (13) follows and the lemma is proved.

Q.E.D.

Of Proposition 2:

Put differently, the banker's objective function is:.

$$\Pi(D, r) = \left\{ \begin{array}{c} G + \frac{q\overline{A}\alpha + (1-q)\underline{A}}{\overline{A}\alpha}(r - r^{SB})D, \text{ if } D(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}) \leq G \\ q(G + Dr), \text{ otherwise} \end{array} \right\}, \tag{33}$$

where

$$r^{SB} \equiv \frac{(1-q)(\overline{A}\alpha - \underline{A})}{q\overline{A}\alpha + (1-q)\underline{A}} = R^{SB} - 1, \tag{34}$$

namely, the net interest rate at the second best level.

Note that, given D, the banker wants an interest rate as high as possible and hence (**??**) is binding, which implies:

$$r = \widehat{R} \cdot \delta - 1 \tag{35}$$

Furthermore, if the banker does not default, and thus $\delta = 1$, then $r = \widehat{R} - 1$. Therefore, $r \geq r^{SB}$ if and only if $\widehat{R} - 1 \geq r^{SB}$, or equivalently,

$$\widehat{R} \geq R^{SB}. \tag{36}$$

To summarize, the representative bankers' problem is that given the prevailing real interest $\widehat{R}$, she chooses (D,r) to maximize her profit in (33), subject to the constraint in (35).

Lemma 3A: if $\widehat{R} \geq \frac{\overline{A}\alpha}{\underline{A}}$, then $D = \infty$, $r = \widehat{R} - 1$, $\delta = 1$, and $\Pi = \infty$.

Proof. First, if $\delta = 1$, by (35) $r = \widehat{R} - 1 > r^{SB}$, given $\widehat{R} \geq \frac{\overline{A}\alpha}{\underline{A}}$. Moreover $1 + r \geq \frac{\overline{A}\alpha}{\underline{A}}$. It follows that the condition for no default, (12) is honored for any D; intuitively, at such a high real interest rate, paper issuance can be completely backed by the loan assets it finances and is thus not constrained by the stock of corn reserve. Therefore, $\delta = 1$ indeed. Since $r > r^{SB}$ and the banker never defaults, the objective function in (33) picks value from the upper branch and is maximized by $D = \infty$, which gives $\Pi = \infty$. ■

Lemma 3B: if $\frac{\overline{A}\alpha}{\underline{A}} > \widehat{R} > \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}}$, then $D = \infty$, $r = \frac{(q\overline{A}\alpha + (1-q)\underline{A})\widehat{R} - \overline{A}\alpha}{\overline{A}\alpha - (1-q)\underline{A}\widehat{R}}$, $\delta = \frac{q\overline{A}\alpha}{\overline{A}\alpha - (1-q)\underline{A}\widehat{R}} < 1$ and $\Pi = \infty$.

Proof. In this case, by (35) and $\delta \leq 1$, $1 + r < \widehat{R} < \frac{\overline{A}\alpha}{\underline{A}}$. Then, the condition for no default, (12), is equivalent to $D \leq G(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha})^{-1}$. Consider first what is the optimal decision and the profit of the banker if she choose not to default, namely chooses $D \leq G(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha})^{-1}$. In this case, $\delta = 1$, which by (35) implies $r = \widehat{R} - 1$; and her objective takes the value firm the upper branch of (33), with r^{SB} defined in (34). Note that $r - r^{SB} = \widehat{R} - 1 - r^{SB} > \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}} - 1 - \frac{(1-q)(\overline{A}\alpha - \underline{A})}{q\overline{A}\alpha + (1-q)\underline{A}} = 0$. Therefore, given r, the bankers wants D as big as possible subject to no default, namely, she chooses $D = G(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha})^{-1}$. Then, her profit under the optimization is $\Pi^{nd} = G + \frac{q\overline{A}\alpha + (1-q)\underline{A}}{\overline{A}\alpha}(r - r^{SB})(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha})^{-1}G$, with $r = \widehat{R} - 1$, which is finite.

On the other hand, I am going to check that the banker gets $\Pi = \infty$ by choosing to default, namely, $D > G(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha})^{-1}$. In case of default, by the lower branch of (13),

$$D = \frac{(1-q)G}{\delta - q - (1-q)\frac{\underline{A}(1+r)}{\overline{A}\alpha}}. \tag{37}$$

And by the lower branch of (33), the profit of the bank is $qG + qrD$. Substitute (37) for D, and her profit, up to an affine transformation, is

$$\frac{r}{\delta - q - (1-q)\frac{\underline{A}(1+r)}{\overline{A}\alpha}}.$$

With r substituted with (35) and proper rearrangement, this profit objective becomes a function of δ alone, as follows:

$$\frac{\widehat{R}\cdot\delta - 1}{\delta(1-(1-q)\frac{\underline{A}}{\overline{A}\alpha}\widehat{R}) - q}. \tag{38}$$

Note first that $1-(1-q)\frac{\underline{A}}{\overline{A}\alpha}\widehat{R}) > q > 0$, since it is equivalent to $\widehat{R} < \frac{\overline{A}\alpha}{\underline{A}}$. Therefore, it is feasible for the banker to choose

$$\delta = \frac{q}{1-(1-q)\frac{\underline{A}}{\overline{A}\alpha}\widehat{R}}, \tag{39}$$

as it is between 0 and 1. At this value for δ, the denominator in (38) equals 0, while the nominator in (38) is positive: $\widehat{R}\cdot\delta - 1 = \frac{\widehat{R}q}{1-(1-q)\frac{\underline{A}}{\overline{A}\alpha}\widehat{R}} - 1 > 0 \Leftrightarrow \widehat{R}q > 1-(1-q)\frac{\underline{A}}{\overline{A}\alpha}\widehat{R} \Leftrightarrow \widehat{R}(q + (1-q)\frac{\underline{A}}{\overline{A}\alpha}) > 1$, which is assumed in the case. Therefore, the banker gets an infinitely large profit.

Overall, in this case the banker gets $\Pi = \infty$ by choosing δ as in (39), which implies, through (35), $r = \frac{(q\overline{A}\alpha+(1-q)\underline{A})\widehat{R}-\overline{A}\alpha}{\overline{A}\alpha-(1-q)\underline{A}\widehat{R}}$, and through (37), $D = \infty$. ■

These two lemmas imply result (i). Result (ii) is proved as the following lemma.

Lemma 3C: if $\widehat{R} = \frac{\overline{A}\alpha}{q\overline{A}\alpha+(1-q)\underline{A}}$, then the banker obtains $\Pi = G$ and is indifferent with any (D, r), where r is settled by D through (35), namely

$$\frac{1+r}{\delta(D,r)} = \widehat{R},$$

with $\delta(D,r)$ given by (13).

Proof. For this case, if the banker chooses not to default, so $\delta = 1$, then $r = \widehat{R} - 1 = \widehat{r}$ at the particular value of $\widehat{R}$. Thus by the upper branch of (33), her profit is G for any $D \leq (1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha})^{-1}G$ (so that she does not default). Therefore she is indifferent with any D in the range. If she chooses to default in the bad state, namely, chooses $D > (1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha})^{-1}G$, then her profit is $qG + qrD$, which, with D substituted with (37), becomes

$$qG + (1-q)G\frac{rq}{\delta - q - (1-q)\frac{\underline{A}(1+r)}{\overline{A}\alpha}},$$

Substitute δ with $(1+r)/\widehat{R}$ (by 35) and rearrange, and it becomes

$$qG + (1-q)G\frac{qr}{r(\frac{1}{\widehat{R}} - (1-q)\frac{\underline{A}}{\overline{A}\alpha}) + \frac{1}{\widehat{R}} - (1-q)\frac{\underline{A}}{\overline{A}\alpha} - q}, \tag{40}$$

which equals G at $\widehat{R} = \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}}$, where $\frac{1}{\widehat{R}} - (1-q)\frac{\underline{A}}{\overline{A}\alpha} - q = 0$ and $\frac{1}{\widehat{R}} - (1-q)\frac{\underline{A}}{\overline{A}\alpha} = q$. That is, the banker gets $\Pi = G$ and is indifferent with any D with which she is to default in the bad state.

Overall, the banker gets $\Pi = G$ in either case and is indifferent with (D,r) that satisfies (35) with $\delta(D,r)$ given by (13). ■

Result (iii) is proved as the following lemma.

Lemma 3D: if $\frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}} > \widehat{R}$, then $D = 0$, namely, lending to entrepreneurs is not profitable to the banker.

Proof. For this case, if the banker does not default, then $r = \widehat{R} - 1 < r^{SB}$ and thus the optimal choice of her is $D = 0$ by which her profit is G. If the banker chooses to default, then by the discussion above, her profit is represented in (40). As $\frac{1}{\widehat{R}} - (1-q)\frac{\underline{A}}{\overline{A}\alpha} - q > 0$ in this case, the fraction in (40) is smaller than $q/(\frac{1}{\widehat{R}} - (1-q)\frac{\underline{A}}{\overline{A}\alpha})$, which in turn is smaller than 1, since $\frac{1}{\widehat{R}} - (1-q)\frac{\underline{A}}{\overline{A}\alpha} > q$. Therefore, the profit term in (40) is strictly smaller than G, and worse than what the banker can gets with $D = 0$. Overall, therefore, her optimal choice is $D = 0$, that is, the market conditions are too tough for the banker to make any lending. ■

The whole proposition is thus proved. Q.E.D.

Of Proposition 3:

(i): It has been shown in the main text.

(ii): If in the symmetric equilibrium bankers default, which means that if all but zero measure of bankers issue the most without going default, then the aggregate bank issue falls below the second best level, then in any other equilibrium, a positive measure of bankers default, because, by result (i) if, the value of the aggregate bank issue is always at the second best level. To prove result (ii), therefore, it suffices to show that bankers default in the symmetric equilibrium if and only if (21) is honored, for which I characterized the symmetric equilibrium below by solving simultaneous equations of (18), (13), $\widehat{D} = \widehat{E}$, and (6). The last two together give

$$\widehat{D} = (\frac{\overline{A}\alpha}{1+\widehat{r}})^{\frac{1}{1-\alpha}}(\frac{\widehat{\delta}}{w})^{\frac{\alpha}{1-\alpha}}. \tag{41}$$

With now $\widehat{R} = R^{SB} = \frac{\overline{A}\alpha}{q\overline{A}\alpha+(1-q)\underline{A}}$, (18) becomes

$$\frac{1+\widehat{r}}{\widehat{\delta}} = \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}}. \tag{42}$$

Suppose now first bankers do not default – I will check later it is indeed the case so long as (21) is not satisfied. Then the bankers's papers are not discounted, that is,

$$\widehat{\delta} = 1. \tag{43}$$

This together with (42) and (41) implies the following values of $(\widehat{r}, \widehat{D}, \widehat{\delta})$:

$$\widehat{r} = \frac{(1-q)(\overline{A}\alpha - \underline{A})}{q\overline{A}\alpha + (1-q)\underline{A}} \tag{44}$$

$$\widehat{D} = \frac{(q\overline{A}\alpha + (1-q)\underline{A})^{\frac{1}{1-\alpha}}}{w^{\frac{\alpha}{1-\alpha}}} \tag{45}$$

$$\widehat{\delta} = 1; \tag{46}$$

Now it is straightforward to very that with these values of D and r, indeed bankers do not default in the bad state, namely, condition (12) is honored, if and only if (21) is not honored.

Q.E.D.

By Proposition 2, bankers earn a positive profit if $\widehat{R} > R^{SB}$, which is the case in the equilibrium as Friction 3 and (22) are sustained. Otherwise, by

Of Proposition 4:

Note that as there is no default, $\delta = 1$ and $1 + r = \widehat{R}$.

(i): First, in the equilibrium, $\widehat{R} > R^{SB}$. Otherwise, by Proposition 2, $\widehat{R} = R^{SB}$ (as $\widehat{R} < R^{SB}$ would discourage bank issue at all); the demand of bank issue by this rate, by (the proof of) Proposition 3 (ii), would be so high that to meet it entails bank default, which is disallowed due to Friction 3. Second, as $\widehat{R} > R^{SB}$, then by Proposition 2, the profit margin of bank issue is positive. Third, in the presence of Friction 3, bankers' problem is (17) with the additional constraint to ensure non-default, namely, (12), which, with $\Pi(D, r)$ given by (33), $r = \widehat{R} - 1$, and $r^{SB} = R^{SB} - 1$, becomes as follows:

$$\max_{D,R} G + \frac{q\overline{A}\alpha + (1-q)\underline{A}}{\overline{A}\alpha}(R - R^{SB})D, \text{ s.t. } R \leq \widehat{R} \text{ and } D \leq G(1 - \frac{\underline{A}}{\overline{A}\alpha}R)^{-1}.$$

It is easy to see the solution is $R = \widehat{R}$ and

$$D = \frac{G}{1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}}.$$

The latter means that bankers want to issue as much as possible, to the point beyond which they will default, that is, until constraint (12) is binding at the optimum. As each banker's issuance is uniquely determinant, there is a unique equilibrium.

The equilibrium real interest rate, $\widehat{R}$, is pinned down by equalizing this supply with the demand, which, by (6) and $1 + r = \widehat{R}$ and $\delta = 1$, is:

$$\widehat{E} = (\frac{\overline{A}\alpha}{\widehat{R}})^{\frac{1}{1-\alpha}}(\frac{1}{w})^{\frac{\alpha}{1-\alpha}}.$$

(24) follows from $\widehat{D} = \widehat{E}$.

(ii): $\widehat{R}(G)$ is determined by (24) and is thus the inverse function of

$$G(\widehat{R}) = (\overline{A}\alpha)^{\frac{1}{1-\alpha}}/w^{\frac{\alpha}{1-\alpha}} \cdot \frac{1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}}{\widehat{R}^{\frac{1}{1-\alpha}}},$$

which is obviously a decreasing function of $\widehat{R}$. Thus the inverse function, $\widehat{R}(G)$, is decreasing.

By (23) and (24) together,

$$\widehat{D} = (\overline{A}\alpha)^{\frac{1}{1-\alpha}}/w^{\frac{\alpha}{1-\alpha}} \cdot \frac{1}{\widehat{R}^{\frac{1}{1-\alpha}}},$$

which decreases with $\widehat{R}$ and therefore increases with G.

(iii): With $R = \widehat{R}$ for each banker, the number of workers hired in equilibrium, by (7), is $\widehat{L} = (\frac{\overline{A}\alpha}{w\widehat{R}})^{\frac{1}{1-\alpha}}$. Thus, $\widehat{L}$ increases with G, since $\widehat{R}$ decreases with it, and $\widehat{L} < L^{SB}$ as $\widehat{R} > R^{SB}$.

Of Proposition 5:

First I characterize the equilibrium if to each banker the central bank lends $S \leq S(G)$ units shells. Entrepreneurs' demand of shells, E_s, solves the same problem as the demand of corn certificates, given by (5), except r is replaced with r_s and δ with δ_s. Therefore, the real interest rate for shell-borrowing entrepreneurs is $(1 + r_s)/\delta_s$, while the real interest rate of borrowing bank-issued certificates is $1 + r$ (since $\delta = 1$ given bankers do not default). As was noted, entrepreneurs' profit depends only on the real interest of the deal. The real interest rate of borrowing shells must equal that of borrowing corn certificates, namely,

$$\frac{1 + r_s}{\delta_s} = 1 + r \equiv \widehat{R}. \tag{47}$$

By (7), the labor hired in equilibrium is:

$$\widehat{L} = (\frac{\overline{A}\alpha}{w})^{\frac{1}{1-\alpha}} \widehat{R}^{\frac{-1}{1-\alpha}}. \tag{48}$$

The total wage for hiring this amount of labor, $w\widehat{L}$, is paid either with shells whose value is $\delta_s S$, or with the corn certificates whose quantity and thus value (as $\delta = 1$), by (23), is $\widehat{D} = G/(1 - \frac{A}{\overline{A}\alpha}\widehat{R})$, that is,

$$w\widehat{L} = \delta_s S + \frac{G}{1 - \frac{A}{\overline{A}\alpha}\widehat{R}}. \tag{49}$$

These four equations (note 47 has two) together with (**??**) (which settles δ_s) determines $(\delta_s, r_s, r, \widehat{R}, \widehat{L})$ and characterizes the equilibrium.

Now the proposition is proved as follows.

(i): By (47), $1 + r_s = \widehat{R}\delta_s$. Substituting it into (25) and rearranging, we have:

$$\delta_s = \frac{q}{1-(1-q)\frac{A}{\underline{A}\alpha}\widehat{R}}. \tag{50}$$

Substitute it and (48) into (49)), rearrange, let $\theta \equiv \frac{A}{\underline{A}\alpha}\widehat{R}$, and we have

$$\underline{A}^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}} \theta^{\frac{-1}{1-\alpha}} = \frac{q}{1-(1-q)\theta} S + \frac{G}{1-\theta}.$$

Rearrange it further, and we have

$$S = (\frac{\underline{A}}{w^\alpha})^{\frac{1}{1-\alpha}} \frac{1-(1-q)\theta}{q\theta^{\frac{1}{1-\alpha}}} - \frac{1-(1-q)\theta}{q(1-\theta)} G \equiv F(\theta),$$

which is (26).

To show $\widehat{R}$ decreases with S, it suffices to show so does θ, or equivalently its inverse function $F(\theta)$ is decreasing. To show that, it suffices to show $\frac{1-(1-q)\theta}{q(1-\theta)}$ increases with θ, which is straightforward: $(\frac{1-(1-q)\theta}{q(1-\theta)})' = \frac{1}{(1-\theta)^2} > 0$. And to show $\widehat{R} = R^{SB}$ at $S = \underline{S}$, it suffices to show that $F(\frac{A}{\underline{A}\alpha}R^{SB}) = \underline{S}$, which is straightforward.

(ii): Equations (47) and (50) put together, $1 + r_s = \frac{q\widehat{R}}{1-(1-q)\frac{A}{\underline{A}\alpha}\widehat{R}}$, from which (27) follows. It is straightforward to see that r_s increases with $\widehat{R}$ which decreases with S by part (i). Therefore, r_s decreases with S. And it is also straightforward to see that $r_s = 0 \Leftrightarrow \widehat{R} = R^{SB}|_{\text{part (i)}} \Leftrightarrow S = \underline{S}(G)$.

(iii): In equilibrium each banker serves N entrepreneurs, from each of who she gets $\widehat{\pi}$ given by (19). Therefore, bankers' profit is $N\widehat{\pi}$. To prove the properties of the result for bankers' profit, therefore, it suffices to prove these properties for $\widehat{\pi}$. for With (19) $\widehat{\pi}$ increases with $\widehat{R}$ for $\widehat{R} \in [R^{SB}, \frac{1}{\alpha}R^{SB}]$ and decreases with it for $\widehat{R} > \frac{1}{\alpha}R^{SB}$. If $G \geq \underline{G}$, then $\widehat{R} \leq \frac{1}{\alpha}R^{SB}$ by definition of $\underline{G}$ at $S = 0$. Therefore, with S increasing, $\widehat{R}$ decreases by result (ii) and thus all the time below $\frac{1}{\alpha}R^{SB}$, and then $\widehat{\pi}$ decreases. If $G < \underline{G}$, then $\widehat{R} \geq \frac{1}{\alpha}R^{SB}$ for $S \leq S(\frac{1}{\alpha}R^{SB}) \equiv F(\frac{A}{\bar{A}\alpha^2}R^{SB})$. As $\widehat{R}$ always decreases with S, $\widehat{\pi}$ increases with S for $S \leq S(\frac{1}{\alpha}R^{SB})$ and increases with it for $S \geq S(\frac{1}{\alpha}R^{SB})$.

References

[1] Araujo, L. and Minetti, R. (2011), "On the Essentiality of Banks", *International Economic Review,* 52, 679 - 691.

[2] Cavalcanti, R., Erosa, A., and Temzelides, T. (1999), "Private Money and Reserve Management in a Random Matching Model," *Journal of Political Economy*, 107, 929–945.

[3] Cavalcanti, R. and Wallace, N. (1999), "Inside and Outside Money as Alternative Media of Exchange," *Journal of Money, Credit, and Banking*, 31, 443–457.

[4] Chari, V. V., and Jagannathan, R. (1988), "Banking Panics, Information, and Rational Expectations Equilibrium," *Journal of Finance*, 43, 749 - 763.

[5] Champ, B., Smith, D., and Williamson, S. (1996) "Currency Elasticity and Banking Panics: Theory and Evidence," *Canadian Journal of Economics*, 29, 828–64.

[6] Clapham, J. (1944), *The Bank of England: A History*, Cambridge at the University Press.

[7] Diamond, D. and P. Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91, 401–419.

[8] Gertler, M. and Karadi, P (2011), "A model of unconventional monetary policy," Journal of Monetary Economics, 58, 17–34.

[9] Gertler, M. and Kiyotaki, N. (2010), "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*, V. 3A, ed. B. M. Friedman and M. Woodford, North-Holland.

[10] Goldstein, I. (2010), "Fundamentals or Panic: Lessons from the Empirical Literature on Financial Crises," mimeo, University of Pennsylvania, the Wharton School.

[11] Jacklin, C. J., and S. Bhattacharya (1988), "Distinguishing Panics and Information-based Bank Runs: Welfare and Policy Implications," *Journal of Political Economy,* 96, 568–92.

[12] Hart O., and L. Zingales, (2011) "Inefficient Provision of Money," working paper.

[13] Hart O., and L. Zingales, (2011) "Inefficient Provision of Liquidity," working paper.

[14] Kiyotaki, N. and Moore, J. (2001), "Evil is the Root of All Money ", Clarendon Lecture 1, Oxford, 2001.

[15] Kiyotaki, N. and Wright, R. (1989), "On Money as a Medium of Exchange," *Journal of Political Economy,* 97, 927–54.

[16] Kreps, D. and J. Scheinkman (1983), "Quantity precommitment and Bertrand competition yields Cournot outcomes," *Bell journal of Economics*, 14, 326-337.

[17] Mackenzie, A. (1953), *The Bank of England Note*, Cambridge at the University Press.

[18] Marimon, R., Nicolini, J., and Teles, P. (2011), "Money is an Experience Good: Competition and Trust in the Private Provision of Money," EUI working paper.

[19] Ostroy, J. and Starr, R. 1990), "The Transactions Role of Money," in Benjamin Friedman and Frank Hahn, eds., *Handbook of Monetary Economics*, V. 1. New York: North-Holland, 1990, pp. 3-62.

[20] Samuelson, P. A. (1958), "An Exact Consumption Loan Model of Interest, with or without the Social Contrivance of Money" *Journal of Political Economy,* 66, 467-82.

[21] Stein, J. (2012), "Monetary Policy as Financial Stability Regulation," *Quarterly Journal of Economics*, 127, 57 - 95.

[22] Wallace, N. (1980), "The Overlapping Generations Model of Fiat Money," In *Models of Monetary Economies*, ed. J.H. Kareken and N. Wallace. Minneapolis: Federal Reserve Bank of Minneapolis.

[23] Wright, R. (2008), "Search-and-Matching Models of Monetary Exchange." The New Palgrave Dictionary of Economics. 2nd Edition. Eds. S. Durlauf and L. Blume. Palgrave Macmillan.

[24] Wallace, N. and Zhu, T. (20007), "Float on a Note," *Journal of Monetary Economics,* 54, 229 - 246.

[25] Wang, T. (2010), "Risk Sharing, (Over)Leverage, and the Dimensions of Competition", mimeo.

[26] Zeng, Z., "A Theory of the Non-Neutrality of Money with Banking Frictions and Bank Recapitalization," *Economic Theory*, forthcoming.