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A non-monotonic relationship between public debt and economic growth: the effect of financial monopsony

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Abstract:

There is some evidence of a non-monotonic relationship between public debt and economic growth. With reference to the Diamond (1965) OLG model, we provide a rationale for this possibility; (i) where the financial sector is monopsonistic; (ii) where the acquisition of its equity constitutes a form of non-productive saving; (iii) where public debt has a fixed price form. As a competing asset, the issuance of debt reduces financial profits and equity values and, possibly, over an initial range, the sum of non-productive saving, comprising public debt and financial sector equity, thereby leading to a net crowding-in effect.

1. Introduction

Despite its many uses subsequently, Diamond's (1965) model of overlapping generations first addressed an issue of public debt to show how an increase could be Pareto-improving by correcting the over-saving externality that arises with non-altruistic and finite-lived households and with exogenous growth.[1] While, conceptually, this could entail quite small amounts of public debt, present concerns are with levels that appear to be inordinately high; and ominously so, if indeed economic growth is not exogenous but inversely related.

Equilibrium theory generally attests to a globally negative relationship between economic growth and public debt, due both to the effects of asset crowding-out and to those of any distortionary taxes needed to service it. In an AK version of a representative and infinite-lived agent model Greiner (2013) shows that economic growth is monotonically decreasing in the ratio of public debt to GDP. In models such as this with no place for intergenerational redistribution effects and where debt can rolled be over indefinitely, the effects are indirect and due to distortionary, debt-servicing taxes. Alternatively, in an OLG model, the direct asset effects can cause severe crowding-out to the extent there may even be a loss of equilibrium – in terms of levels in Rankin and Roffia (2003) and of growth in Braeuninger (2005).

However, recent empirical research in Reinhart and Rogoff (2010) and (2011) and in Chercherita and Westphal (2012) points to the possibility of a non-monotonic relationship, where debt becomes dangerous for growth only when it reaches about 90% of GDP. In contrast, the method employed by Eberhardt and Prebitero (2013) generates results that endorse negative monotonicity. While not attempting to review this strand of the empirical literature, but pointing to Panizza and Presbitero (2013) that does, we present a theoretical model to demonstrate an inverse-U shaped relationship may not exceed the bounds of possibility.

This is predicated on a monopsonistic financial sector which leads to a higher rate of return on public debt. The effect of issuing public debt not only crowd-outs out monopsony saving, but also reduces the profit and the equity value of this sector. As the acquisition of financial equity, like that of debt, is deemed to constitute a non-productive form of saving, the question is whether debt crowds-out financial equity by a coefficient that is *greater* or *less* than unity? We find that under some circumstances, particularly, also that the stock of public debt is sufficiently small, it may be *greater*, so that an inverse-U-shaped relationship emerges, once a mechanism for growth has been incorporated into the model. The remaining four sections, in turn, present the model and the main analysis and offer some further discussion and a conclusion.

2. The model

2.1 Main features

There are two types of household, the customers and the owners of the financial sector with the relative weights, $1-\varpi$ and ϖ.[2] This division allows us to circumvent the issue arising in general equilibrium pertaining to the appropriate objective of a profit-making (here, finance) firm when its customers are also its owners. It is also expedient to the present analysis, which requires that one part of the economy receives a very low rate of return, while the other part a return that exceeds the economic growth rate in order for a determinate, forward-looking price for financial sector equity. Furthermore, it makes the model easier to solve, since it transpires that no household would choose to hold more than two assets. Lastly, it allows greater descriptive realism than the standard case of a single representative individual per generation, thus causing the results to have distributional as well as macroeconomic implications.

[1] This no longer holds, if growth is endogenous, as shown by Saint-Paul (1992)

[2] They may self-select on the basis of their willingness to pay a putative fixed transaction cost for acquiring equity, although this decision is not a part of the analysis.

The customers of the financial sector obtain a low, monopsonistic factor of interest, R^M, while the owners are privileged by receiving the higher, competitive interest factor, R^C, on their deposits, which is also the firms' cost of borrowing, R^F, their marginal product of capital. In addition, the owners receive R^E, as the return factor on their holdings of flexible-price financial equity, which is equalized through an arbitrage process with their competitive return, R^C, on deposits. A key assumption is of no gains from further investment in the financial sector, so that its fixed stock of equity is held just as a store value to be exchanged only within the household sector, thus constituting a form a non-productive saving.

The government issues a fixed-price public debt and sets the factor of return, R^B, between the monopsonistic and competitive interest factors.[3] To summarise, the structure of rate of return dominance is $R^M < R^B < R^C (= R^E = R^F)$, which means that the customers have an over-riding incentive to acquire public debt in place of deposits, while the owners have none at all. Acquiring public debt is another form of non-productive saving, but one undertaken by the customers not the owners. The question is whether this form of non-productive saving crowds-out the other form by a factor which is greater or less than unity?

2.2 The aggregate conditions

Investment, leading to an increase in the following period's capital stock depends on the aggregate deposit saving of all households. Under the assumptions of 100% depreciation and of no population growth, capital accumulation in per capita terms is given by

$$k_{t+1} = d_t = (1-\varpi)d_{1,t} + \omega d_{2,t} \tag{1}$$

Furthermore in each period the government raises tax revenue τ_t and borrows b_t in order to pay off both the principal and interest, R_t^B, on its existing debt, the amount

[3] In the UK this would be constituted by non-tradeable *National Savings Certificates*, which traditionally have been bought over *Post Office* counters.

borrowed in the preceding period, b_{t-1}. Taxes are endogenous, and in a steady state, where everything grows by the same factor, G, they are determined as

$$\tau = (G^{-1}R^{B} - 1)b, \tag{2}$$

As Woodford (1990) also notes, the inequality $R^{B} < G$ implies that public debt leads to budget surpluses, where households receive subsidies instead of tax demands.

2.3. The customers, type-one households

Both types of households have a two period utility function, $U^{Y}_{j,t} = \ln(1-\theta)c^{Y}_{j,t} + \theta \ln c^{M}_{j,t+1}$, $j = 1,2$. Each household works in the first period and is retired in the second. Type-one households receive a wage, $w_{1,t}$, which is lower than the type-two household wage, $w_{1,t} < w_{2,t}$. In order to simplify the analysis[4], we assume a progressive taxation system, where low wage, type-one households do not pay tax at all. They may save by holding both financial deposits and public debt, $s_{1,t} = d_{1,t} + b_{1,t}$, while the inequality, $R^{B}_{t+1} > R^{M}_{t+1}$, means they would, if they could, only hold public debt. However, the supply of public debt is deemed to be fixed at, b_t, so that the average customer can only acquire the amount $(1-\varpi)^{-1}b_t$, leaving a residual demand for deposits,

$$d_{1,t} = \theta\left(w_{1,t} - (1-\varpi)^{-1}b_{1,t}\right) - (1-\theta)(1-\varpi)^{-1}b_t R^{B}_{t+1} \big/ R^{M}_{t+1}. \tag{3}$$

Public debt crowds out customer deposit saving at least by a factor of one,

$$\partial d_{1,t} \big/ \partial\left((1-\varpi)^{-1}b_{1,t}\right) = -\theta - (1-\theta)\left(R^{B}_{t+1} \big/ R^{M}_{t+1}\right) \le -1 \quad \text{as} \quad R^{B}_{t+1} \ge R^{M}_{t+1}.$$

2.4 The owners, type-two households

[4] If the customers also pay taxes, the growth factor then appears in the square root solution for the monopsonistic deposit interest factor, which, in turn, enters the growth equation both as a level and as an inverse.

Apart from receiving higher wages, type-two households also obtain the higher competitive return, $R_{t+1}^{C} > R_{t+1}^{B}$, and thus choose not to hold debt at all, $b_{2,t} = 0$, so that in aggregate

$$b_t = (1-\omega) b_{1,t} \tag{4}$$

Under the progressive system, they alone pay taxes τ_t^Y and τ_{t+1}^O over the two periods of their lives, thus saving $s_{2,t} = \theta\left(w_{2.t} - \tau_t^Y\right) - (1-\theta)\,\tau_{t+1}^O \big/ R_{t+1}^C$. In order to simplify further, the per period taxes are set such that $\tau_t^O = \left(\theta/(1-\theta)\right)(R_{t+1}^C \big/ G_{t+1})\tau_t^Y$ to give

$$s_{2,t} = \theta w_{2,t} \tag{5}$$

This assumption of tax neutrality may generally be regarded as restrictive, but not necessarily within the present context where the return on public debt, R^B, is also to be set rather than determined. Thus, setting R^B arbitrarily close to G, according to equation (2), is an alternative route for obtaining equation (5). Setting it below G – provided that R^M is even lower – amounts to a policy of relatively subsidizing the young, where $\left|\tau_t^O\right| < \left(\theta/(1-\theta)\right)(R_{t+1}^C \big/ G_{t+1})\left|\tau_t^Y\right|$, which actually enhances the positive effects of *low levels* of public debt on growth, thus adding to rather than taking away from the main result of the model. Conversely, giving the inequality the opposite sign would undermine the results, so the equality assumption may here be regarded as neutral.

The financial equity price is determined from the following no-arbitrage,

$$R_{t+1}^C = R_{t+1}^E \equiv (\pi_{t+1} + v_{t+1}) \big/ v_t \tag{6}$$

where π_{t+1} is future financial dividends or profits. In a steady state where profits grow by the same factor G, there is a determinate, steady state and a forward-looking solution for financial equity at $v_t = \pi_{t+1} \big/ (R^C - G)$, provided that $R^C > G$ The deposit saving of each type-two household is its total saving less its share, ϖ^{-1}, of a normalised unit of equity,

$$d_{2,t} = \theta w_{2,t} - \varpi^{-1} \pi_{t+1} \big/ (R^C - G) \qquad (7)$$

2.5 The financial sector

The financial sector maximizes the profit to be made through intermediation,

$$\pi_{t+1} = (1-\varpi)\left(R^F_{t+1} - R^M_{t+1}\right) d_{1,t} \qquad (8)$$

Firms have an inelastic demand for loans at the interest factor, R^F_{t+1}. As the financial firm does not profit from its customers, $R^C_{t+1} = R^F_{t+1}$, its profits base is limited to customer deposits, $(1-\varpi)d_{1,t}$, and its rate of profit is the spread between the competitive and monopsonistic returns, $R^C_{t+1} - R^M_{t+1}$. Profit is maximized where

$$R^M_{t+1} = \max\left(R^{M*}_{t+1}, \overline{R}\right), \text{ where } R^{M*}_{t+1} = \sqrt{\frac{(1-\theta)}{\theta}\left(\frac{R^B_{t+1}(1-\varpi)^{-1}b_t}{w_{1,t} - (1-\varpi)^{-1}b_t}\right)R^K_{t+1}}$$

$$\partial R^M_{t+1} \big/ \partial \overline{b}_t > 0 \text{ and } \partial R^M_{t+1} \big/ \partial R^B_{t+1} > 0, \qquad (9)$$

As negative values for the nominal interest rate are ruled-out and as inflation is assumed to be sufficiently low, the solution for the real interest factor could be constrained at the minimum $\overline{R}$. However, for a sufficiently large debt, there will be an interior solution, R^{M*}_{t+1}, which gives rise to the customer deposit demand,

$$d_{1,t} = \theta\left(w_{1,t} - (1-\varpi)^{-1}b_t\right) - \sqrt{\theta(1-\theta)\left(w_{1,t} - (1-\varpi)^{-1}b_t\right)\left(\frac{R^B_{t+1}(1-\varpi)^{-1}b_t}{R^K_{t+1}}\right)},$$

$$\partial d_{1,t} \big/ \partial b_{1,t} < 0 \qquad (10)$$

2.6 Discussion

At this point, it is possible to give the basic intuition of the results before completing the model. Capital accumulation depends on total deposit saving. The deposits of the owners comprise the total amount they save less their acquisitions of financial sector equity, which are determined in direct proportion of customer deposits, as these constitute the financial profit base. Equations (1) and (5)-(8) give

$$k_{t+1} = \varpi s_{2,t} + (1-\varpi)\left(\frac{R^M - G}{R^C - G}\right)d_{1,t}$$

The final term is customer deposits minus the steady state value of financial equity. Apart from the positive effect, $\partial R^M / \partial b > 0$, which is due to public debt raising the interest elasticity of demand for customer deposits, there is the more direct effect, $\partial k / \partial d_1$, which acquires a negative sign if the monopsony interest factor is lower than the growth factor, $R^M < G$, which in turn implies that $\partial k / \partial b > 0$. This result obtains because the effect of a *permanent* change in debt on the value of discounted financial profits over the infinite future is very powerful. However, this sign may only obtain for small amounts of public debt, which, of course, is the rationale for an inverted-U.

2.7 Production

The assumption of different wages levels, made for expositional rather than substantive purposes, is motivated by assuming human capital levels, $H_{1,t} < H_{2,t}$.[5] The output of a production firm, indexed z is

$$Y_t(z) = \left(\left(H_1 L_{1,t}(z) + H_2 L_{2,t}(z)\right)k_t\right)^{1-\alpha} K_t(z)^{\alpha}, \text{ where } 0 < \alpha < 1$$

Constant returns in the factors internal to the firms, labour and capital, $L_{1,t}(z)$, $L_{2,t}(z)$, $K_t(z)$, and also capital, both internal and external, $K_t(z)$ and k_t, $k_t \equiv K_t / L_t$, give rise to the possibility of endogenous economic growth as expounded by Romer (1986) and exemplified by Lucas (1988). Production profits are maximized where

$$R_t^F = \alpha\left(\left(H_1 L_{1,t}(z) + H_2 L_{2,t}(z)\right)k_t\right)^{1-\alpha} K_t(z)^{\alpha-1}$$

$$w_{j,t}(z) = (1-\alpha)H_1\left(\left(H_1 L_{1,t}(z) + H_2 L_{2,t}(z)\right)k_t\right)^{-\alpha} K_t(z)^{\alpha} \qquad j = 1,2 .$$

In a symmetric equilibrium where $K_t(z)/L_t(z) = k_t$, $\forall z$,

[5] In order to explain the division in terms of wage differences, it would also be necessary to transform the original utility function, so that utility is not a logarithmic function of wealth and so that the interest cost of monopsony is not additively separable from incomes.

$$y_t = Ak_t\,; \qquad R_t^F = \alpha A\,; \qquad w_{j,t} = (1-\alpha)A_j k_t\,, \quad j = 1{,}2\,;$$

where $A_j \equiv H_j\big((1-\varpi)H_1 + \omega_1 H_2\big)^{-\alpha}$, $j = 1{,}2$; $A \equiv (1-\varpi)A_1 + \varpi A_2$ (11)

3. The analysis

The model comes together in a single equation for the economic growth factor,

$$G = \theta\big(\varpi(1-\alpha)A_2\big) + \left(\frac{R^M - G}{R^C - G}\right)\left((1-\varpi)\theta(1-\alpha)A_1 - \left(\theta + (1-\theta)\frac{R^B}{R^M}\right)\beta A\right), \qquad (12)$$

where β, $\beta \equiv b/y$, is the public debt to GDP ratio, *plus* equation (9) for the monopsony interest factor. Quantifying its parameters as $\theta = 1/3$ $\alpha = 1/3$, $\varpi = 1/4$, $A_1 = 12$, $A_2 = 18$, $\overline{R} = 1$, generates the following *table* of values to demonstrate the effects of public debt on the monopsony interest and economic growth factors under three debt interest regimes.

Table: The public debt-GDP ratio, the monopsony deposit interest factor and the economic growth factor

$\theta = 1/3$ $\alpha = 1/3$, $\varpi = 1/4$, $A_1 = 12$, $A_2 = 18$

Policy One: $R^B = 2$			Policy Two: $R^B = R^C = 4.5$			Policy Three: $R^B = \hat{R}^M + \varepsilon$		
β	R^M	G	β	R^M	G	β	R^M	G
0.0233	1.000	1.0000	0.0107	1.0000	1.0000			
0.03	1.1415	1.0406	0.02	1.3814	1.1070			
0.04	1.3342	1.0913	0.04	2.0014	1.2367	0.044	1.0000	1.0000
0.05	1.5105	1.1324	0.06	2.5141	1.2910	0.06	1.4046	1.1050
0.06	1.6761	1.1661	0.08	2.9817	1.2915	0.08	1.9756	1.2134
0.07	1.8344	1.1935	0.10	3.4290	1.2474	0.10	2.6129	1.2702
0.08	1.9878	1.2153	0.12	3.8703	1.1657	0.12	3.3288	1.2433
			0.14	4.3156	1.0532	0.14	4.1387	1.0983
			0.148	4.5000	1.0000	0.148	4.5000	1.0000

The first policy, $R^B = 2$ is of setting a low fixed return on public debt. Increasing the debt-GDP ratio raises the values of both endogenous factors, until the monopsony interest rate overtakes the debt interest rate, at which point the policy break downs, because the customers would cease to hold the debt. The second policy is to fix the debt interest rate at the competitive level, $R^B = 4.5$, leading to an evidently inverted-U-shape between public debt and economic growth. The third case allows for variation in the debt interest rate, where it is pre-set arbitrarily close to the anticipated outcome for the deposit interest rate at each level of public debt. Again, an inverse-U arises.

Note, however, that the interior growth maxima occur at very low values of the ratio of public debt to GDP – very far from a putative 90%. There are two answers to this point. First, it reflects the feature of a two period OLG model that the denominator represents an income flow over the half-life period – say 30 years– rather than the single year on which customary measures of this ratio are compiled Second, related work in Roberts (2014) shows that parsimoniously generalizing to *three* overlapping generations allows for substantial and empirically plausible debt-GDP ratios, because households may then save over two periods instead of one, allowing a differentiation between asset stocks and flows.[6] Investment crowding-out, caused by *flows* of debt, may be commensurately small while coexisting with large *stocks* of public debt.

4. Further discussion

The result depends on a requisite combination of flexible-price financial equity and fixed-price public debt. The former allows the powerful equity valuation effects that arise from a fall in discounted financial profits over an infinite horizon in response to the anticipation of permanent changes in the amount of public debt.[7] If public debt too were flexibly priced, it would enter the broad asset class as financial equity, thus invalidating the present structure, but here insulating customer deposit demand from its effects. If,

[6] We anticipate that the interest elasticity effect would then become more important.

[7] This point is made in Roberts (2009) with reference to various regimes for returning profits.

instead, the financial customers held flexible-price debt, a process of arbitrage would ensure the equalization of their interest rates on terms set by the financial monopsonist. Thus, choosing this alternative instrument for administering the public debt is tantamount to the policy-maker surrendering the scope to act as a first-mover in what is actually an interest setting game.[8]

5. Concluding comments

Some empirical work has tentatively suggested a possibly non-monotonic relationship between economic growth and public debt, something which has not been supported by mainstream theory. Our insight is that if low household interest rates are caused not by over-saving, as in the original analysis of Diamond, but by monopsony behaviour in the saving sector, the issuance of public debt might provide a partial corrective in providing an alternative means of saving. The model shows that this may be also captured by *initial* gains to economic growth, thus leading to a non-monotonic relationship, while the assumption that households are differentiated by type as well as by age, allows for intra-generational distributional as well as macroeconomic effects.[9]

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[8] This form of public debt here captures an aspect of social security by causing *de facto* transfers to individuals in the latter part of their lives. Similar results were obtained for the operation of a pay-as-you-go pension in Roberts (2003).

[9] Political economy considerations, where the customers form a voting majority, suggest very large public debts.

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