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Structure of Banking Liabilities

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Abstract

The paper argues that systemic risk must be taken into account when designing optimal bankruptcy procedures in general, and priority rules in particular. Allowing for endogenous formation of links in the interbank market we show that the optimal policy depends on the distribution of shocks and the severity of fire sales.

Keywords: Banks; Priority rules; Systemic Risk
JEL: G21, G28

1 Introduction

An important issue that the design of any bankruptcy procedure must resolve is the allocation of priority rights among the various claimants of the corresponding entity's assets. By definition the value of the assets of a bankrupt entity is below the value of its liabilities and, therefore, a rule is needed for allocating those assets to the various claimants. Often the allocation of priority gives rise to a complex hierarchy among the creditors having at the top holders of secured debt and right down at the bottom the providers of equity. There is an extensive literature in financial economics[1] that studies the optimal design of bankruptcy procedures. The main aim of the design is to make investment attractive to creditors by shifting sufficient risk on those agents who have control over decision-making. In a setting with multiple stages of financing by multiple creditors the design of priority rights takes the form of a hierarchical structure.

A policy area that has attracted a lot of attention, especially in the aftermath of the 2008 crisis, is the design of priority rules for banks. What is striking is the variety of rules applied around the globe (Lenihan, 2012; Wood, 2011). The arguments supporting the rules in place are mainly about the incentives that these rules provide to depositors and other creditors to monitor the activities of banks. The lower a creditor is on the priority list, the less likely is that she will receive (at least full) compensation in the case of bankruptcy and therefore the stronger her incentives to ensure that the borrower does not take excessive risks. While nobody disagrees about the validity of the last statement there is substantial variation in opinions about which is the most suitable party to perform the monitoring service.

In this paper, we argue that systemic risk is another aspect of banking that must be taken into account when designing bankruptcy procedures. In general, the design of optimal priority rules is focused on the welfare of a bank's creditors, namely, its depositors, other debt holders and equity owners. However, when systemic risk is a concern the design must also take into consideration the welfare of third parties and, in particular, all those who provided funds to the rest of the banking system. The degree to which a bank's credit providers will be affected when the bank becomes insolvent it will depend on their position on the priority ladder. When among those creditors that are not getting paid in full are other banks, as long as these banks stay solvent the losses will be absorbed by equity holders. However, when these banks become insolvent the losses will be absorbed by their creditors and this process will continue till either the system clears or all banks become bankrupt. The total systemic losses will depend on the value of assets that can be recovered when banks become insolvent and thus go into liquidation. The sale of assets in depressed markets, 'fire sales' as known in the literature, further deteriorates balance sheets and thus enhances the fragility of the financial system.[2] It is straightforward to show that if the value of assets is fully recovered under liquidation, that is the market value is equal to the book value, then the seniority structure does not matter. This is because the total

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[1] See Berglöf *et al.* (2010) for a review of the relevant literature.
[2] See Shleifer and Vishny (2010) for a review of the related literature.

losses are limited to the value of initial losses.[3] With this in mind, fire sales are going to be important in our work.

Recent work on systemic risk, models contagion in banking following a network approach.[4] One important finding of this literature is that the structure of the network matters. Thus far, this structure is exogenously given. This can be fine as long as the seniority structure of liabilities is fixed.[5] However, our aim is to compare the welfare implications of two alternative priority structures, namely, depositor seniority and bank seniority, and to do so we need to take into consideration the impact of seniority structure on the formation of the interbank network. Again, it is straightforward to show that if the network structure of the intertbank market could not be affected by the choice of the seniority structure then the optimal option would be to allocate seniority to banks. The reason is that as more of the losses are absorbed by depositors the lower will be the losses absorbed by the interbank market and thus the lower the risk of further insolvencies.[6] However, this is not anymore true when the network structure is endogenous. The problem is that allowing for endogenous network formation complicates significantly the analysis and therefore we will ignore the incentives for monitoring that each seniority structure offers. Clearly, this is an important issue and any related policy debate needs to include it. We are going to offer our thoughts in the concluding section of the paper.

We will analyze a model of the banking system where banks finance their investments by two types of borrowing, namely, retail deposits and loans from other banks. We focus on the optimal lending policy of a bank that has some excess liquidity. The bank has three lending options: the first one is to offer the loan to a bank that has zero net obligations to the rest of the banking system, the second option is to offer the loan to a bank that is a net borrower, and the third option is to offer the loan to a net lender. We derive the lender bank's profit maximizing choice under both priority rules and then we derive the corresponding social optimum choice. Given that priority rules only matter when a bank fails our results will be sensitive to (a) the size of the shock and (b) the probability distribution of shocks across the banking system. We carry out the analysis under two alternative scenaria regarding the distribution of shocks. In one case, we assume that each bank is hit by a shock with the same probability but bigger banks are hit by proportionally bigger shocks while in the other case we assume that the probability that a bank is hit by a shock is proportional to its size but the size of the shocks are the same for each bank.

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[3] The only thing affected by the seniority structure is the distribution of losses between depositors and bank owners. However, the seniority structure should minimize total losses leaving their distribution to other policy instruments.

[4] For reviews of the literature see Allen and Babus (2009) and Bougheas and Kirman (2015a).

[5] In the literature so far the analysis is carried out under the supposition that in the case of bankruptcy depositors have priority. See, for example, Acemoglu *et al.* (2015).

[6] We are intentionally ignoring concerns about the impact of the priority structure on the likelihood of liquidity runs that can lead to insolvencies and hence systemic risk. The reason is that there are other instruments, such as deposit insurance, that are more appropriate to deal with such concerns (see, Diamond and Dybvig, 1983).

The former scenario corresponds to the case where a bank's various asset returns are strongly correlated while the latter scenario corresponds to the case where bank portfolios are well diversified.[7]

Our main result is that, in general, under bank seniority the profit maximizing network structure is also the one that maximizes social welfare. However, there is an important exception when the joint likelihood of (a) extreme high initial losses, (b) catastrophic fire sales, (c) low profitability, and (d) lack of asset diversification, is very high. These four conditions provide a fair characterization of the status of the US banking system before the 2007 financial crisis.

We organize our work as follows. After a brief review of the related theoretical literature in Section 2 we describe our model. In Section 3 we present our results and we conclude in the final section. All derivations are provided in the Appendix.

Related Literature   Responding to the 1980s Savings and Loans crisis, the US Congress enacted the 1991 Federal Deposit Insurance Corporation Improvement Act followed by the 1993 Depositor Preference Act. The introduction of these Acts motivated a long debate among financial economists, legal scholars and policymakers that is reviewed in Bougheas and Kirman (2015b). Here we restrict our attention to some theoretical developments that are more closely related to this present work.

Some experts argue that non-depositor priority rights provide strong incentives to depositors to discipline the banks. Actually, Calomiris and Kahn (1991) have suggested that by its very nature demandable debt, which allows depositors to withdraw their funds at will, offers the required market discipline device.

Others believe that banks and other creditors are more suitable monitors. For example, Rochet and Tirole (1996) argue that interbank exposures generated through transactions in the interbank market provide strong incentives for banks to monitor other banks and therefore interbank loans should be junior to deposits. Kaufman (2014) has challenged this claim by arguing that how effective the banks are as monitors depends on their beliefs about the likelihood that the government would intervene in times of crises in which case banks would consider transactions in the interbank market as bearing low risk. Birchler (2000) also supports depositor preference on the grounds that banks have an informational advantage relative to a large number of small depositors.

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[7]Correlation here refers to the asset returns of a bank's portfolio. Our intention is to study how the priority structure of banking libilities affects the transmission of shocks from one bank to another and thus we are ignoring correlations of asset returns across the banking system where multiple banks fail simultaneously. As long as the correlation of returns across the banking system is not perfect our main argument is still valid although the results become weaker. Of course, as Acharya (2009) has demonstrated any design of new policies need to consider the incentives that these policies offer to banks to have correlated portfolios and thus enhancing systemic risk. The argument is that if banks expect that it is more likely that they will be bailed out during systemic events then they have an incentive to increase correlation. However, our arguments related to the choice of priority structure aim directly at reducing systemic risk and thus minimizing the total cost of these events.

Moreover, he argues that raising funds by offering to depositors a standardized product with priority rights is a more efficient than having each depositor sign a bilateral contract with a bank. Thus, the introduction of a priority list reduces the amount of resources devoted to socially inefficient information gathering and such an arrangement seems to be ideal for banks that raise funds from a large number of uninformed investors.

In contrast to the above mentioned studies, Freixas *et al.* (2004) offer a mixed view. In their model banks provide two services, namely, screening and monitoring. By screening potential applicants they improve the pool of loans that they offer while by monitoring firms that have been offered loans banks ensure that these firms perform well. The optimal seniority structure depends on which of the two moral hazard problems associated with the two services is more pressing.

Most of the work on the seniority status of bank loans has focused on the interbank market where such loans are not secured. However, on the liability side of their balance sheet banks have other claims by financial institutions that are secured and therefore have top priority. Bolton and Oehmke (2015) analyse the seniority status of some types of derivatives and conclude that while these claims provide risk sharing opportunities, their position on the top of the priority ladder can lead to inefficiencies as it transfers risk to other bank creditors such as depositors.[8]

2   The Banking Network

In order to assess the welfare implications of each of the two alternative priority rights allocation policies we need first to understand how each of these options would affect the structure of the interbank network which in turn would depend on the profit maximizing decisions of each bank in the network. The dynamic formation of the interbank market network is a difficult problem that has recently attracted some attention (Babus, 2015; Cohen-Cole *et al.*, 2010). This work takes the priority ladder of banking liabilities as exogenously given. Our work is further complicated by the dependence of the network structure on the priority ladder that requires making very complex welfare comparisons. With that in mind we will examine a very simple banking network that will allow us to make such comparisons and then we will consider its relevance for more realistic environments.

There are four banks: I, II, III, IV. There are two types of risk-neutral agents: bank owners and depositors. We keep bank balance sheets very simple. On the asset side bank i has customer loans, L_i, and may have loans offered to other banks. Let $l_i^j (j \neq i)$ denote loans from bank i to bank j. On the liability side bank i has customer deposits, D_i, may have deposits from other banks and equity E_i. Let $d_i^j (j \neq i)$ denote deposits in bank i from bank j. Balance sheets

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[8] There are some proposals in favor of subordinated debt but Blum (2000) casts some doudt on their efficacy.

must satisfy the constraints

$$L_i + \sum_{j \neq i} l_i^j = D_i + \sum_{j \neq i} l_i^j + E_i, \quad \forall i$$

and he interbank market must satisfy the constraints

$$l_i^j = d_j^i, \quad \forall i \text{ and } \forall j.$$

Table 1 shows the initial balance sheets of the four banks:

Table 1: Initial Bank Balance Sheets

$L_I = 1$	$D_I = 1$
$Assets = 1$	$Liabilities = 1$

$L_{II} = 1$	$D_{II} = 2$
$l_{II}^{III} = 1$	
$Assets = 2$	$Liabilities = 2$

$L_{III} = 2$	$D_{III} = 1$
	$d_{III}^{II} = 1$
$Assets = 2$	$Liabilities = 2$

$L_{IV} = 1$	$D_{IV} = 1$
$Assets = 1$	$Liabilities = 1$

Each bank has funded one unit of loans with its own deposits. In addition, bank II had an extra unit of deposits that it loaned to bank III that used it to finance an extra unit of loans. The net interest rate on consumer loans is equal to $z < 1$, the interest rate on deposits is equal to 0 and interbank interest rate is also equal to 0.[9]

Suppose that bank IV obtains an extra unit of deposits that is willing to loan to another bank. All other three banks can fund an extra unit of consumer loans. In what follows we provide answers to the following four questions:

- Assuming that depositors have priority to which other bank will bank IV offer the loan to maximize its profits?

- Assuming that depositors have priority which bank should receive the loan so that social welfare is maximized?

- Assuming that banks have priority to which other bank will bank IV offer the loan to maximize its profits?

- Assuming that banks have priority which bank should receive the loan so that social welfare is maximized?

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[9] Given the interest rate on consumer loans, the interbank rate in equilibrium could take any value in the interval $[0, z]$. It will become clear that the exact value is not important for our conclusions and, therefore, to keep things simple we set it equal to 0.

The answers to the four questions only matter when there is a banking crisis and in particular when a bank other than IV goes into liquidation. Thus, we assume that one of the other three banks has to write off some of its assets. The answers will also depend on many other modeling choices such as the likelihood and size of shocks and the expectations of bank IV about future changes in each bank's balance sheets including changes in the interbank network. We will begin by analyzing a benchmark case that provides simple and intuitive answers. Later we will discuss how our results might be affected if we move to more complex variations of our model. Our benchmark model satisfies the following restrictions:

Assumption 1 *(Myopic Expectations) After bank IV offers the loan to one of the other three banks it does not expect any further changes in any of the balance sheets.*

Assumprion 2 *(Catastrophic Fire Sales) Any subsequent balance sheet shocks completely wipe out the value of customer loans on the bank's balance sheet.*

Clearly, the answers to the four questions will also depend on the beliefs that bank IV has about the size distribution of the initial shock. Suppose that bank i is the one inflicted by the initial shock. Then, let ψ_i denote bank i's liquidation value of customer loans. Thus, the size of the shock is given by $L_i - \psi_i$. We will compare the answers to the four questions under two alternative scenaria related to the distribution of shocks across the banking system.[10]

Proportional Shocks The probability that any of the banks I or II or III becomes insolvent is equal to $\frac{1}{3}$. Shocks are proportional to the value of the inflicted bank's customer loans ($\frac{\psi_i}{L_i} = \psi \; \forall i$).

Identical Shocks $L_i - \psi_i = L_j - \psi_j \leqslant 1$, for every bank i or j; The probability that a bank becomes insolvent is proportional to the value of its customer loans.

The above two scenaria regarding the distribution of shocks across the banking system capture two polar cases. With proportional shocks the underlying assumption is that scale does not lead to diversification. Put differently, as a bank grows (in terms of customer loans) it replicates its existing portfolio (extreme specialization). Thus, as a bank's customer loans grow in size so does the bank's exposure to the risks associated with the particular sector financed by the bank. Under the asumption that each sector is equally likely to be inflicted by a negative shock and thus the corresponding firms become unable to meet their obligations with their bank, shocks are proportional. In contrast, the implication of the assumption of identical shocks is extreme diversification. Thus, as a bank doubles in size (again, in terms of customer loans) it also doubles the number of sectors it finances. Under the same supposition as above, that is each sector is equally likely to be inflicted by a negative shock, now all shocks have

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[10]For the moment we do not impose any restrictions on the joint distribution of z and ψ_i.

the same size (assuming each sector has similar financial needs) but as a bank doubles the number of sectors that it finances it also doubles the probability that it will be inflicted by a shock.

3 Results

The solution of the model proceeds in two steps. Firstly, we derive for each priority case and for each option that bank IV has for offering the loan, bank IV's profits and social welfare. The latter is defined as total bank profits plus total available deposits after any bank resolution.[11] This exercise is completed for all admissible values of profits, z, and liquidation values, ψ_i. Secondly, we use the calculations from the first step to derive bank IV's profit maximizing choice and also the loan offer that maximizes welfare. The analysis is completed by choosing the priority rights policy that would maximize welfare conditional on bank IV's profit maximizing choice. This step is repeated for each of the two restrictions on the distribution of shocks across the banking system.

The detailed derivations can be found in the Appendix. The following Propositions summarize the results.

Proposition 1 *The structure of the network is affected by the priority rights policy choice.*

It is clear that, keeping the structure of the network fixed, bank IV's expected profits are higher under bank priority for any of the three loan offer options as more of the losses are absorbed by depositors. However, the expected profit maximizing choice under bank priority is not the same as under depositor priority. An easy way to understand the last statement is by considering bank IV's expected profits when it offers the loan to bank II that has already offered a loan to bank III. Under depositor priority bank IV is potentially exposed to failures of either bank II or bank III. In contrast, under bank priority there is a buffer of deposits at bank II protecting bank IV.

Thus, even if both bank IV's expected profits and social welfare are higher under bank priority when the structure of the network is fixed, the profit maximizing choice in not necessarily the same as the social welfare maximizing choice when the structure of the network is affected by the allocation of priority rights. Next, we identify the conditions when the two choices diverge.

Proposition 2 *(Proportional Shocks):*
(a) Under depositor seniority, for any values of z and ψ, it is never optimal for bank IV to offer the loan to bank II.

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[11] To simplify our analysis we have assumed linear utility. The absence of any curvature matters only for one important case where we carefully discuss the various trade-offs. We have also assumed equal weighting between equityholders and depositors. Our model does not distinguish between bank managers and equityholders and as we have argued above there is no *a priori* for the social planner to favor depositors over equityholders. If one of the aims is to protect depositors in order to guarantee adequate liquidity then this can be achieved by alternative policy instruments such as deposit insurance.

(b) Under bank seniority, for any values of z and ψ, offering the loan to bank II weakly dominates the alternative two options.

(c) Offering the loan to bank II maximizes welfare for any values of z and ψ except the worst case scenario of very high initial losses and very low profitability.

Parts (a) and (b) make it clear, as discussed above, why the priority structure has an effect on bank IV's profit maximizing choice. We also know that as, long the network structure is fixed bank seniority maximizes welfare. This is because having depositors absorb the losses prevents the spread of the crisis to other banks. Ignoring for the moment the worst case scenario, we also find that offering the loan to bank II is also the social welfare maximizing case. Form the point of view of social welfare we care about both depositors and equityholders. Given that bank II has a higher value of deposits, under bank seniority, offering the loan to this bank reduces the likelihood that the crisis spreads. Certainly, this would mean that depositors suffer most of the losses. But as aggregate losses are low this is only a distributional issue.

The above argument is not true for the worst case scenario. In that case offering the loan to bank II does not maximize welfare. When the shocks are high it is better for the network not to be too connected (Acemoglu *et al.*, 2015a). Low connectivity reduces contagion. However, from the point of view of bank IV offering the loan to bank II is not dominated by the other two choices. Given that bank IV does not take into account the effect of its choice on depositors there is a conflict between the equilibrium profit-maximizing choice and the one that maximizes social welfare. Of course, from an *ex ante* point of view everything depends on the relative likelihood of these extreme events (fat tails).

Proposition 3 *(Identical Shocks):*

(a) Under depositor seniority the choice of bank IV would depend on the distribution of shocks.

(b) Under bank seniority bank IV will be indifferent across the three choices.

(c) Expected welfare is maximized by offering the loan either to bank I or bank II.

Now the size of the shocks is relatively small and therefore bank priority offers a strong protection to bank IV so that it is indifferent between the three available options. The profit maximizing choice under depositor priority is more complicated and depends on the particular distribution of the shocks. Lastly, social welfare is maximized by avoiding offering the loan to bank III. This is because in the latter case there is a high concentration of loans in bank III and thus when this bank fails there is a higher reduction in profits.

We regard the two alternative restrictions that we have imposed on the distribution of shocks across the banking system as two polar cases of a much broader space of such distributions. Considering together the results of Propositions 2 and 3 we find that under bank seniority bank IV would offer the loan to bank II. In general, this would also be the social welfare maximizing choice.

The only exception would be if the joint likelihood of (a) extreme high initial losses, (b) catastrophic fire sales, (c) low profitability, and (d) lack of asset diversification, is very high. These four conditions provide a fair characterization of the status of the US banking system before the 2007 financial crisis. Our results suggest that while under most circumstances bank seniority would offer more protection against systemic risk, the higher connectivity that it encourages, might exacerbate the systemic consequences of extreme events.

4   Conclusion

Our work has not delivered an unconditional optimal policy recommendation. This is not too surprising given that we already know that while some types of network structures are better at protecting the system during mild episodes the same structures can prove catastrophic during extreme events (Acemoglu *et al.*, 2015a).

In order to keep our analysis simple, we have completely ignored other reasons for supporting one or another priority rights policy. However, there is a substantial body of work that has examined the incentives that such policies offer to various types of creditors to monitor banks. What has been absent from this discussion so far are their potential systemic risk implications. Keeping the analysis tractable also meant that we had to impose a couple of strong restrictions on our model.

The analysis in the paper was carried out under the assumption that fire sales are catastrophic. We have also carried the analysis for the case of fire sales that are not catastrophic and it turns out that the general message does not change. As we have pointed out in the Introduction, in the absence of fire sales priority rights are irrelevant for the social welfare implications of systemic risk. For values of fire sales in the intermediate range we get many more cases to consider making the presentation of the results cumbersome. However, given that we are interested in comparing choices *ex ante* much of the intermediate variation vanishes out.

What is more worrisome is our assumption of myopic expectations. What it implies is that bankers do not expect any further changes in the network structure. Although, recently there has been some progress in the direction of endogenizing the formation of banking networks (Babus, 2015; Cohen-Cole *et al.* 2010), the complexity of the issues that we have attempted to address in this paper it does not allow us to use their methods. We chose our network structure as it captures the three alternative types of borrowers that a lending bank might meet (positive exposures, negative exposures, zero exposures). Any other lending bank would be facing similar options. Clearly, as the network structure gets more complicated a banks decisions will not depend only on the net exposures of their potential lenders but also on the exposures further down the line.

In summary, our results have been derived from a very simple network structure under naive behavioral assumptions. Having said that we believe that are

very intuitive and at the very least are a good starting point for addressing important policy issues.

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5 Appendix

Throughout our analysis we assume that when a bank becomes insolvent its assets are distributed to its creditors according to priority rules and all parties at the same priority level share their allocated assets in proportion to their corresponding claims. Let Π_i denote bank i's profits and C_i final withdrawals (consumption) by the depositors of bank i. As a result of potential bank liquidations we have $C_i \leqslant D_i$.

We begin by deriving bank IV's profits, Π_{IV}, and social welfare (post-bankruptcy customer deposits plus total bank profits), W, for each of the two priority cases and for each of the three options of bank IV, for all admissible values of z and ψ_i. Then we compare bank IV's profit maximizing choice with the social welfare maximizing choice for each seniority structure and for each of the two restrictions on the distribution of shocks across the banking system.

5.1 Preliminary Derivations

5.1.1 Depositor Seniority

(a) Loan from Bank IV to Bank I

Table 2 shows the new balance sheets of banks I and IV.[12]

Table 2: Loan from Bank IV to Bank I

$L_I = 2$	$D_I = 1$
	$d_I^{IV} = 1$
$Assets = 2$	$Liabilities = 2$
$L_{IV} = 1$	$D_{IV} = 2$
$l_{IV}^I = 1$	
$Assets = 2$	$Liabilities = 2$

We consider separately the three cases of initial insolvencies:

Case 1 *Bank I goes bankrupt*

In this case the shock only affects bank I (direct hit) and its creditor bank IV. Depending on the value of $\psi_I \in [0, 2]$ we need to consider two cases:

(a) $\psi_I \leqslant 1$. Depositor seniority implies that $C_I = \psi_I$. Thus, bank IV will lose its deposits at bank I, go bankruptcy itself and, by Assumption 2, $C_{IV} = 0$. Thus, we have

$$\Pi_{IV} = 0 \text{ and } W = 3(1 + z) + \psi_I$$

given that $C_{II} = 2$, $\Pi_{II} = z$, $C_{III} = 1$ and $\Pi_{III} = 2z$.

(b) $\psi_I > 1$. In this case the depositors of bank I get all their deposits back, $C_I = 1$, and bank IV recovers $\psi_I - 1$ of its loan to bank I. There are two cases to consider depending on whether or not bank IV remains solvent:

(i) $z + \psi_I < 2$. In this case the bank gets liquidated as the value of its assets that is the sum of the value of its loans $1 + z$ plus the funds that managed to

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[12] The balance sheets of banks II and III are not directly affected by this transaction.

recover from bank I are less than the value of its liabilities which is equal to 2. The depositors of the bank will get $C_{IV} = \psi_I - 1$ and thus we have as in the previous case

$$\Pi_{IV} = 0 \text{ and } W = 3(1+z) + \psi_I$$

but now the higher value of ψ_I is shared between the depositors of the two affected banks.

(ii) $z + \psi_I \geqslant 2$. Now bank IV is solvent and thus

$$\Pi_{IV} = z + \psi_I - 2 \text{ and } W = 4(1+z) + \psi_I$$

where for the derivation of welfare we notice that the depositors of all banks got their funds back and the sum of the profits of the two unaffected banks is equal to $3z$.

Case 2 *Bank II goes bankrupt*

The bankruptcy only affects bank II whose assets include a loan of 1 unit to bank III. Therefore, its depositors will receive $\psi_{II} \in [0, 1]$ plus a deposit of 1 unit at bank III. Given that all banks other than III have not been affected, we have:

$$\Pi_{IV} = z \text{ and } W = 5(1+z) + \psi_{II}$$

Case 3 *Bank III goes bankrupt*

The bankruptcy will also affect bank II that has offered a loan to bank III. Symmetry implies that the welfare results in this case are exactly the same as those derived from the case when bank I goes bankrupt. When bank I receives the new loan from bank IV there is a symmetric network structure, namely, two banks offering a loan and two banks receiving a loan. Moreover, banks I and III are the two banks receiving the loans. The only difference is that in this case bank IV is solvent. Thus, we have the following cases:

(a) $\psi_{III} \leqslant 1$.

$$\Pi_{IV} = z \text{ and } W = 3(1+z) + \psi_{III}$$

(b) $\psi_{III} > 1$.
(i) $z + \psi_{III} < 2$.

$$\Pi_{IV} = z \text{ and } W = 3(1+z) + \psi_{III}$$

(ii) $z + \psi_{III} \geqslant 2$.

$$\Pi_{IV} = z \text{ and } W = 4(1+z) + \psi_{III}$$

(b) Loan from Bank IV to Bank II

Table 3 shows the new balance sheets of banks II and IV.

Table 3: Loan from Bank IV to Bank II

$L_{II} = 2$	$D_I = 2$
$l_{II}^{III} = 1$	$d_{II}^{IV} = 1$
$Assets = 3$	$Liabilities = 3$
$L_{IV} = 1$	$D_{IV} = 2$
$l_{IV}^{II} = 1$	
$Assets = 2$	$Liabilities = 2$

Case 1 *Bank I goes bankrupt*

Given that other banks are not affected, we have:

$$\Pi_{IV} = z \text{ and } W = 5(1 + z) + \psi_I$$

where $\psi_I \in [0, 1]$.

Case 2 *Bank II goes bankrupt*

We need to consider two cases depending of the value of $\psi_{II} \in [0, 2]$:

(a) $\psi_{II} < 1$. The depositors of bank II receive ψ_{II} plus a deposit of 1 unit at bank III. Bank IV's loan to bank II will not be repaid and therefore bank IV will go bankrupt and, by Assumption 2, $C_{IV} = 0$. Thus,

$$\Pi_{IV} = 0 \text{ and } W = 3(1 + z) + \psi_{II}$$

(b) $\psi_{II} > 1$. In this case the depositors of bank II are fully compensated by receiving ψ_{II} plus $2 - \psi_{II}$ of deposits at bank III. Bank IV recovers $\psi_{II} - 1$ of its loan to bank II and there are two cases to consider depending on whether or not bank IV remains solvent. The analysis is exactly the same as in the case when bank IV offers the loan to bank I and the latter becomes insolvent. The only difference is that we need to replace ψ_I with ψ_{II}. Thus,

(i) $z + \psi_{II} < 2$.

$$\Pi_{IV} = 0 \text{ and } W = 3(1 + z) + \psi_{II}$$

(ii) $z + \psi_I \geqslant 2$.

$$\Pi_{IV} = z + \psi_I - 2 \text{ and } W = 4(1 + z) + \psi_{II}$$

Case 3 *Bank III goes bankrupt*

The bankruptcy will also affect bank II that has offered bank III a loan and potentially bank IV that has offered bank II a loan. Note that $\psi_{III} \in [0, 2]$.

(a) $\psi_{III} < 1$. The payoff of the depositors of bank III is given by $C_{III} = \psi_{III}$ and bank II's loan is not repaid. There are two cases to consider depending on whether or not bank II remains solvent.

(i) $z < \frac{1}{2}$. Given that the assets of bank II are equal to $2(1 + z)$ and the liabilities are equal to 3, the inequality implies that bank II also goes bankrupt.

The bankruptcy implies that that $C_{II} = 0$ which, in turn, implies that bank IV goes bankrupt and thus $C_{IV} = 0$. Only the isolated bank I survives.

$$\Pi_{IV} = 0 \text{ and } W = 1 + z + \psi_{III}$$

(ii) $z \geqslant \frac{1}{2}$. Bank II is solvent and $C_{II} = D_{II} = 2$ and $\Pi_{II} = 2z - 1$. Bank IV is not affected.

$$\Pi_{IV} = z \text{ and } W = 4(1 + z) + \psi_{III}$$

(b) $\psi_{III} \geqslant 1$. Now the depositors of bank III are fully paid, $C_{III} = 1$ and bank II recovers $\psi_{III} - 1$. Once more, there are two cases to consider depending or not bank II remains solvent. The only difference with the above case is that now the assets of bank II are larger by $\psi_{III} - 1$. Thus, now we have
(i) $2z + \psi_{III} < 2$.

$$\Pi_{IV} = 0 \text{ and } W = 1 + z + \psi_{III}$$

(ii) $2z + \psi_{III} \geqslant 2$.

$$\Pi_{IV} = z \text{ and } W = 4(1 + z) + \psi_{III}$$

(c) Loan from Bank IV to Bank III

Table 4 shows the new balance sheets of banks III and IV.

Table 4: Loan from Bank IV to Bank III

$L_{III} = 3$	$D_{III} = 1$
	$d_{III}^{II} = 1$
	$d_{III}^{IV} = 1$
$Assets = 3$	$Liabilities = 3$
$L_{IV} = 1$	$D_{IV} = 2$
$l_{IV}^{III} = 1$	
$Assets = 2$	$Liabilities = 2$

Case 1 *Bank I goes bankrupt*

This is similar to the case when bank IV offers the loan to bank II. Given that other banks are not affected, we have:

$$\Pi_{IV} = z \text{ and } W = 5(1 + z) + \psi_{I}$$

where $\psi_{I} \in [0, 1]$.

Case 2 *Bank II goes bankrupt*

The payoff to depositors of bank II is equal to ψ_{II}, where $\psi_{II} \in [0, 1]$, plus a deposit of 1 unit at bank III. All other banks are not affected.

$$\Pi_{IV} = z \text{ and } W = 5(1 + z) + \psi_{II}$$

Case 3 *Bank III goes bankrupt*

We need to consider the following two cases depending on the value of $\psi_{III} \in [0,3]$.

(a) $\psi_{III} < 1$. The payoff of depositors of bank III is given by $C_{III} = \psi_{III}$ and the loans to banks II and IV are not repaid and thus both banks go bankrupt with only bank I surviving.

$$\Pi_{IV} = 0 \text{ and } W = 1 + z + \psi_{III}$$

(b) $\psi_{III} \geqslant 1$. Now the depositors of bank III are fully paid, $C_{III} = D_{III} = 1$, and banks II and IV each recover $\frac{\psi_{III}-1}{2}$ from their one 1 unit of loan to bank III. What happens to the two banks depends on the value of ψ_{III}. Each of these two banks have assets equal to $1 + z + \frac{\psi_{III}-1}{2} = z + \frac{1}{2} + \frac{\psi_{III}}{2}$ and liabilities equal to 2. Thus, we have two cases to consider:

(i) $z + \frac{\psi_{III}}{2} < \frac{3}{2}$. Bank II and IV are insolvent.

$$\Pi_{IV} = 0 \text{ and } W = 1 + z + \psi_{III}$$

(ii) $z + \frac{\psi_{III}}{2} \geqslant \frac{3}{2}$. Bank II and IV are solvent.

$$\Pi_{IV} = z + \frac{\psi_{III}}{2} - \frac{3}{2} \text{ and } W = 3(1+z) + \psi_{III}$$

5.1.2 Bank Seniority

(a) Loan from Bank IV to Bank I

See Table 1 for the balance sheets of banks II and III and Table 2 for the balance sheets of banks I and IV.

Case 1 *Bank I goes bankrupt*

Depending on the value of $\psi_I \in [0,2]$ we need to consider two cases:

(a) $\psi_I < 1$. Given that banks have seniority $C_I = 0$ and bank IV receives a payoff equal to ψ_I and depending on its value we need to consider tow cases:

(i) $z < 1 - \psi_I$. Bank IV goes bankrupt and $C_{IV} = \psi_I$.

$$\Pi_{IV} = 0 \text{ and } W = 3(1+z) + \psi_I$$

(ii) $z \geqslant 1 - \psi_I$. Bank IV is solvent $C_{IV} = 2$ and $\Pi_{IV} = z + \psi_I - 1$.

$$\Pi_{IV} = z + \psi_I - 1 \text{ and } W = 4(1+z) + \psi_I$$

(b) $\psi_I \geqslant 1$. $C_I = \psi_I - 1$ and bank IV's loan to bank I is fully repaid.

$$\Pi_{IV} = z \text{ and } W = 4(1+z) + \psi_I$$

Case 2 *Bank II goes bankrupt*

Given that the only liabilities of bank II are customer deposits, the outcome is exactly the same as the case with depositor seniority.

$$\Pi_{IV} = z \text{ and } W = 5(1+z) + \psi_{II}$$

Case 3 *Bank III goes bankrupt*

The bankruptcy will also affect bank II that has offered a loan to bank III. Symmetry implies that the welfare results in this case are exactly the same as those derived from the case when bank I goes bankrupt. When bank I receives the new loan from bank IV there is a symmetric network structure, namely, two banks offering a loan and two banks receiving a loan. Moreover, banks I and III are the two banks receiving the loans. The only difference is that in this case bank IV is solvent. Thus, we have the following cases:

(a) $\psi_{III} \leqslant 1$.
(i) $z < 1 - \psi_I$.

$$\Pi_{IV} = z \text{ and } W = 3(1+z) + \psi_{III}$$

(ii) $z \geqslant 1 - \psi_I$.

$$\Pi_{IV} = z \text{ and } W = 4(1+z) + \psi_I$$

(b) $\psi_{III} > 1$.

$$\Pi_{IV} = z \text{ and } W = 4(1+z) + \psi_{III}$$

(b) Loan from Bank IV to Bank II

See Table 1 for the balance sheets of banks I and III and Table 2 for the balance sheets of banks II and IV.

Case 1 *Bank I goes bankrupt*

All other banks are not affected by the shock. $C_I = \psi_I$.

$$\Pi_{IV} = z \text{ and } W = 5(1+z) + \psi_I$$

Case 2 *Bank II goes bankrupt*

The creditor bank IV has its loan repaid by obtaining 1 unit of deposits at bank III. The payoff of depositors of bank II is equal to $C_{II} = \psi_{II}$.

$$\Pi_{IV} = z \text{ and } W = 4(1+z) + \psi_{II}$$

Case 3 *Bank III goes bankrupt*

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The bankruptcy will also affect bank II that has offered bank III a loan and potentially bank IV that has offered bank II a loan. Note that $\psi_{III} \in [0, 2]$.

(a) $\psi_{III} < 1$. The payoff of the depositors of bank III is given by $C_{III} = 0$ and bank II receives ψ_{III}. There are two cases to consider depending on whether or not bank II remains solvent.

(i) $z < \frac{1 - \psi_{III}}{2}$. Given that the assets of bank II are equal to $2(1 + z) + \psi_{III}$ and the liabilities are equal to 3, the inequality implies that bank II also goes bankrupt. The bankruptcy implies that that $C_{II} = 0$ and bank IV receives ψ_{III}. However, the inequality implies that bank IV with assets equal to $1 + z + \psi_{III}$ and liabilities equal to 2 also goes bankrupt, hence $C_{IV} = \psi_{III}$.

$$\Pi_{IV} = 0 \text{ and } W = 1 + z + \psi_{III}$$

(ii) $z \geqslant \frac{1 - \psi_{III}}{2}$. Bank II is solvent and hence bank IV is not affected.

$$\Pi_{IV} = z \text{ and } W = 4(1 + z) + \psi_{III}$$

(b) $\psi_{III} \geqslant 1$. Bank II's loan is fully repaid and $C_{III} = \psi_{III} - 1$.

$$\Pi_{IV} = z \text{ and } W = 4(1 + z) + \psi_{III}$$

(c) Loan from Bank IV to Bank III

See Table 1 for the balance sheets of banks I and II and Table 2 for the balance sheets of banks III and IV.

Case 1 *Bank I goes bankrupt*

As in the case when bank IV offers the loan to bank II the only bank affected is bank I.
$$\Pi_{IV} = z \text{ and } W = 5(1 + z) + \psi_I$$

Case 2 *Bank II goes bankrupt*

The payoff to depositors of bank II is equal to ψ_{II} plus 1 unit of deposits at bank III.
$$\Pi_{IV} = z \text{ and } W = 5(1 + z) + \psi_{II}$$

Case 3 *Bank III goes bankrupt*

Depending on the value of $\psi_{III} \in [0, 3]$ we need to consider two cases:

(a) $\psi_{III} < 2$. In this case banks II and IV each receive $\frac{\psi_{III}}{2}$ and $C_{III} = 0$. What happens to banks II and IV depends on the value of ψ_{III}. Each bank's assets are equal to $1 + z + \frac{\psi_{III}}{2}$ while the corresponding liabilities are equal to 2. Then, once more, we need to consider tow cases:

(i) $z < 1 - \frac{\psi_{III}}{2}$. Both banks go bankrupt and $C_{II} = C_{IV} = \frac{\psi_{III}}{2}$.

$$\Pi_{IV} = 0 \text{ and } W = 1 + z + \psi_{III}$$

(ii) $z \geqslant 1 - \frac{\psi_{III}}{2}$. Banks II and IV are solvent, $C_{II} = C_{IV} = D_{II} = D_{IV} = 2$, and $\Pi_{II} = \Pi_{IV} = z + \frac{\psi_{III}}{2} - 1$.

$$\Pi_{IV} = z + \frac{\psi_{III}}{2} - 1 \text{ and } W = 3(1 + z) + \psi_{III}$$

(b) $\psi_{III} \geqslant 2$. The loans of banks II and IV are fully repaid and $C_{III} = \psi_{III} - 2$.

$$\Pi_{IV} = z \text{ and } W = 3(1 + z) + \psi_{III}$$

5.2 Comparing Bank Seniority to Depositor Seniority (Proportional Shocks)

5.2.1 The Optimal Choice of Bank IV

Depositor Seniority For the case of depositor seniority and under the restriction that shocks are proportional, Table 5 below shows the expected profits of bank IV, $E[\Pi_{IV}]$, for each of the 3 loan offer options and for all possible values of per unit of loan profits, z, and liquidation value per unit of loan, ψ. The last column of the table indicates the optimal choice, that is the one that maximizes bank IV's expected profits, under the supposition that shocks per unit of loans and profits per unit of loan can only take the value that corresponds to that particular row of the table.

Table 5: Bank IV's Optimal Choice: Depositor Seniority; Proportional Shocks

Table 5A: $\psi < \frac{1}{3}$

z	$E[\Pi_{IV}]$	OC
$z < \frac{1}{2}$	$I : \frac{2}{3}z$	$*$
	$II : \frac{1}{3}z$	
	$III : \frac{2}{3}z$	$*$
$z \geqslant \frac{1}{2}$	$I : \frac{2}{3}z$	$*$
	$II : \frac{2}{3}z$	$*$
	$III : \frac{2}{3}z$	$*$

Table 5B: $\frac{1}{3} \leqslant \psi < \frac{1}{2}$

z	$E[\Pi_{IV}]$	OC
$z < \frac{1}{2}$	$I : \frac{2}{3}z$	$*$
	$II : \frac{1}{3}z$	
	$III : \frac{2}{3}z$	$*$
$\frac{1}{2} \leqslant z < \frac{3}{4}$	$I : \frac{2}{3}z$	$*$
	$II : \frac{2}{3}z$	$*$
	$III : \frac{2}{3}z$	$*$
$\frac{3}{4} \leqslant z < 1; \psi < 1 - \frac{2}{3}z$	$I : \frac{2}{3}z$	$*$
	$II : \frac{2}{3}z$	$*$
	$III : \frac{2}{3}z$	$*$
$\frac{3}{4} \leqslant z < 1; \psi \geqslant 1 - \frac{2}{3}z$	$I : \frac{2}{3}z$	
	$II : \frac{2}{3}z$	
	$III : z + \frac{1}{2}(\psi - 1)$	$*$

z	$E[\Pi_{IV}]$	OC
$z < 1 - \psi$	$I : \frac{2}{3}z$	$*$
	$II : \frac{1}{3}z$	
	$III : \frac{2}{3}z$	$*$
$1 - \psi \leqslant z < \frac{3}{2}(1-\psi)$	$I : \frac{2}{3}z$	$*$
	$II : \frac{2}{3}z$	$*$
	$III : \frac{2}{3}z$	$*$
$\frac{3}{2}(1-\psi) \leqslant z < 2(1-\psi)$	$I : \frac{2}{3}z$	
	$II : \frac{2}{3}z$	
	$III : z + \frac{1}{2}(\psi - 1)$	$*$
$2(1-\psi) \leqslant z$	$I : z + \frac{2}{3}(\psi - 1)$	$*$
	$II : z + \frac{2}{3}(\psi - 1)$	$*$
	$III : z + \frac{1}{2}(\psi - 1)$	

The restriction that shocks are proportional implies that, for example, the probability that a bank with 2 units of customer loans will be inflicted by a shock of size x is equal to the probability that a bank with 3 units of customer loans will be inflicted by a shock of size $\frac{3}{2}x$. Then, in deriving the above table if, for example, bank I has 2 units of customer loans and thus $\psi_I \in [0,2]$ we have set $\psi_I = 2\psi$, where $\psi \in [0,1]$, in the derivations of the previous section.

Consider Table 5A. For the first three rows we have $\psi < \frac{1}{3}$ and $z < \frac{1}{2}$. For the derivation of the first row we focus on sub-section 2.1 that is when bank IV offers the loan to bank I. From case 1(a) we find that if bank I goes bankrupt $\Pi_{IV} = 0$. Similarly, from case 2(a) we find that if bank II goes bankrupt $\Pi_{IV} = z$ and if bank III goes bankrupt $\Pi_{IV} = z$. Under the supposition of proportional shocks each bankruptcy event is equiprobable and thus we conclude that $E[\Pi_{IV}] = \frac{2}{3}z$. Similarly, for the derivation of the second row we focus on sub-section 2.2 that is when bank IV offers the loan to bank II and in particular cases 1, 2(a) and 3(a)(i) and for the derivation of the second row we focus on sub-section 2.3 that is when bank IV offers the loan to bank II and in particular cases 1, 2 and 3(a). Comparing the three rows we find that under the supposition that liquidation values and profits satisfy $\psi < \frac{1}{3}$ and $z < \frac{1}{2}$ bank IV would be indifferent between offering the loan to banks I and III. For the next three rows of the table we now have $z \geqslant \frac{1}{2}$. The only difference between this case and the one considered above is that when bank IV offers the loan to bank II we need to consider case 3(a)(ii) instead of case 3(a)(i). Now all three choices result in the same expected profits.

Next, consider Table 5B. When $\frac{1}{3} \leqslant \psi < \frac{1}{2}$ and bank IV offers the loan to bank III, if the latter goes bankrupt the liquidation value of its assets is higher than its obligations to depositors. Therefore, its remaining assets will be equally distributed to its two creditor banks, namely, II and IV, and what happens to these banks it depends on the values of both z and ψ. Given that $\psi < \frac{1}{2}$, as long as $z < \frac{3}{4}$, we have $\psi < 1 - \frac{2}{3}z$ and thus the relevant case is 3(b)(i). The derivation of expected profits follows exactly the same logic as the one used for the derivations of Table 5A. In contrast, when $z \geqslant \frac{3}{4}$ what happens

depends on the relative values of z and ψ and the results are captured by the last six rows of the table. Notice that in the last row the expected profits of bank IV are boosted by the fact that it recovers a sufficient amount of funds when bank III goes bankrupt to stay solvent. In fact the inequalities imply that $\frac{2}{3}z \leqslant z + \frac{1}{2}(\psi - 1) < z$.

Lastly, consider Table 5C. We compare the first three rows of the table and the next three rows for the case when bank IV offers the loan to bank II. All depends on whether bank II stays solvent in which case bank IV also stays solvent (compare cases 3(b)(i) and 3(b)(ii)). Next, focusing on the middle six rows of the table we observe that there is a difference between the first three rows of this group and the last three rows when bank IV offers the loan to bank III. This is exactly the same case discussed in the previous paragraph and the results depend on whether or not bank IV can remain solvent after the bankruptcy of bank III. Finally, comparing the last six rows of the table we find that there is a difference between the first three rows of this group and the last three rows when bank IV offers the loan to either bank I or bank II. It all depends on whether or not bank IV remains solvent after the bank that was offered the loan went into bankruptcy.

Bank Seniority   Table 6 shows the expected profits of bank IV for the case of bank seniority.

Table 6: Bank IV's Optimal Choice: Bank Seniority; Proportional Shocks

Table 6A: $\psi < \frac{1}{2}$

z	$E[\Pi_{IV}]$	OC
$z < \frac{1}{2} - \psi$	$I : \frac{2}{3}z$	*
	$II : \frac{2}{3}z$	*
	$III : \frac{2}{3}z$	*
$\frac{1}{2} - \psi \leqslant z < 1 - 2\psi$	$I : \frac{2}{3}z$	
	$II : z$	*
	$III : \frac{2}{3}z$	
$1 - 2\psi \leqslant z < 1 - \frac{3}{2}\psi$	$I : z + \frac{2\psi-1}{3}$	
	$II : z$	*
	$III : \frac{2}{3}z$	
$1 - \frac{3}{2}\psi \leqslant z$	$I : z + \frac{2\psi-1}{3}$	
	$II : z$	*
	$III : z + \frac{1}{2}\psi - \frac{1}{3}$	

Table 6B: $\frac{1}{2} \leqslant \psi < \frac{2}{3}$

z	$E[\Pi_{IV}]$	OC
$z < 1 - \frac{3}{2}\psi$	$I : z$	*
	$II : z$	*
	$III : \frac{2}{3}z$	
$z \geqslant 1 - \frac{3}{2}\psi$	$I : z$	*
	$II : z$	*
	$III : z + \frac{1}{2}\psi - \frac{1}{3}$	

Table 6C: $\frac{2}{3} \leqslant \psi$

$E[\Pi_{IV}]$	OC
$I : z$	*
$II : z$	*
$III : z$	*

Consider Table 6A. Focusing on the first three rows we observe that when the level of profits z is very low, bank IV always becomes insolvent when the bank to which it offered the loan becomes insolvent. Thus, with probability $\frac{2}{3}$ the bank remains solvent. Comparing the first three rows with the next three rows we find that the expected profits of bank IV increase when it offers the loan to bank II. Comparing cases 3(a)(i) and 3(a)(ii) we find that for sufficiently high values of z bank II remains solvent after the bankruptcy of bank III. Next, we compare the middle six rows and find that there is a difference depending on whether or not bank IV becomes insolvent when it offers the loan to bank I and the latter becomes insolvent (cases 1(a)(i) and 1(a)(ii)). Moving to the last six rows we find that there is a difference in expected profits when bank IV offers the loan to bank III. As z moves above the threshold that separates the last three rows from the three rows above them bank IV remains solvent despite the bankruptcy of bank III.

Differences in the other two tables also capture how further increases in ψ and z affect the expected profits of bank IV when it offers the loan to bank III and the latter goes bankrupt. In Table 6B we have exactly the same results as for the case above. In contrast, in Table 6C, ψ is sufficiently high for bank IV to fully recover its loan to bank III.

5.2.2 Social Welfare

Depositor Seniority For the case of depositor seniority and under the restriction that shocks are proportional, Table 7 below shows the expected social welfare, $E[W]$, for each of the 3 loan offer options and for all possible values of per unit of loan profits, z, and liquidation value per unit of loan, ψ. The last column of the table indicates the optimal choice, that is the one that maximizes expected social welfare, under the supposition that shocks per unit of loans and profits per unit of loan can only take the value that corresponds to that particular row of the table.

Notice that the second term of all expressions is the same. The reason is that at any time there are 5 units of consumer loans in the books of banks I, II and III and under proportional shocks $\frac{5}{3}\psi$ equals the expected liquidation value of these loans. Thus, from now on we focus on the first term. For each pair of values for z and ψ and for each possible loan offer by bank IV we derive the first term by adding the welfare results for the corresponding three bankruptcy cases divide by 3 given that each of the three banks becomes insolvent with the same probability. The various cutoffs are the same as the ones derived for the derivation of bank IV's expected profits under depositor seniority. As an example, consider the first entry of Table 7A. The loan is offered to bank I and,

therefore, we focus on Section 2.1 and take the average of the welfare values of cases 1(a), 2 and 3(a). Notice that in this case $\psi_I = 2\psi$, $\psi_{II} = \psi$ and $\psi_{III} = 2\psi$ and thus, as we pointed above, the average liquidation value is equal to $\frac{5}{3}\psi$. As another example, consider the last entry of Table 7C. The loan is offered to bank III and, therefore, we focus on Section 2.3 and take the average of the welfare values of cases 1, 2 and 3(b)(ii). Notice that in this case $\psi_I = \psi$, $\psi_{II} = \psi$ and $\psi_{III} = 3\psi$ and thus, once more, the average liquidation value is equal to $\frac{5}{3}\psi$.

Table 7: Expected Social Welfare: Depositor Seniority; Proportional Shocks

Table 7A: $\psi < \frac{1}{3}$

z		$E[W]$	OC
$z < \frac{1}{2}$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$II:$	$\frac{9}{3}(1+z) + \frac{5}{3}\psi$	
	$III:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	$*$
$z \geqslant \frac{1}{2}$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$II:$	$\frac{12}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	

Table 7B: $\frac{1}{3} \leqslant \psi < \frac{1}{2}$

z		$E[W]$	OC
$z < \frac{1}{2}$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$II:$	$\frac{9}{3}(1+z) + \frac{5}{3}\psi$	
	$III:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	$*$
$\frac{1}{2} \leqslant z < \frac{3}{4}$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$II:$	$\frac{12}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$\frac{3}{4} \leqslant z < 1; \psi < 1 - \frac{2}{3}z$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$II:$	$\frac{12}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$\frac{3}{4} \leqslant z < 1; \psi \geqslant 1 - \frac{2}{3}z$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$II:$	$\frac{12}{3}(1+z) + \frac{5}{3}\psi$	
	$III:$	$\frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$

Table 7C: $\frac{1}{2} \leqslant \psi$

z		$E[W]$	OC
$z < 1 - \psi$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$II:$	$\frac{9}{3}(1+z) + \frac{5}{3}\psi$	
	$III:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	$*$
$1 - \psi \leqslant z < \frac{3}{2}(1-\psi)$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$II:$	$\frac{12}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$\frac{3}{2}(1-\psi) \leqslant z < 2(1-\psi)$	$I:$	$\frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$II:$	$\frac{12}{3}(1+z) + \frac{5}{3}\psi$	
	$III:$	$\frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
$2(1-\psi) \leqslant z$	$I:$	$\frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$II:$	$\frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III:$	$\frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$

Bank Seniority   Table 8 below shows the expected social welfare for the case of bank seniority.

Table 8: Expected Social Welfare: Bank Seniority; Proportional Shocks

Table 8A: $\psi < \frac{1}{2}$

z	$E[W]$	OC
$z < \frac{1}{2} - \psi$	$I: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$II: \frac{10}{3}(1+z) + \frac{5}{3}\psi$	
	$III: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	$*$
$\frac{1}{2} - \psi \leqslant z < 1 - 2\psi$	$I: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$II: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$1 - 2\psi \leqslant z < 1 - \frac{3}{2}\psi$	$I: \frac{12}{3}(1+z) + \frac{5}{3}\psi$	
	$II: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$1 - \frac{3}{2}\psi \leqslant z$	$I: \frac{12}{3}(1+z) + \frac{5}{3}\psi$	
	$II: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$

Table 8B: $\frac{1}{2} \leqslant \psi < \frac{2}{3}$

z	$E[W]$	OC
$z < 1 - \frac{3}{2}\psi$	$I: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$II: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$z \geqslant 1 - \frac{3}{2}\psi$	$I: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$II: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
	$III: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$

Table 8C: $\frac{2}{3} \leqslant \psi$

$E[W]$	OC
$I: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
$II: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$
$III: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	$*$

The cut-off values correspond to those of Table 6. For the derivations we have followed the same steps as for the case of depositor seniority but this time we used the results of Section 3.

5.2.3   Depositor vs Bank Seniority

Proof of Proposition 1 (Part 1)   The proof follows from a direct comparison of Tables 5 and 6. Table 5 shows that under depositor seniority it is never optimal for bank IV to offer the loan to bank II. For any values of z and ψ by offering the loan to bank I, bank IV makes at least as high profits as when offering the loan to bank II. Then, bank IV optimal choice will be either to offer the loan to bank I or to bank III with the optimal choice depending on the distributions of z and ψ. In contrast, under bank seniority, Table 6 shows

that offering the loan to bank II is the dominant choice for any values of z and ψ.

Proof of Proposition 2 The proof follows from the proof of Proposition 1 and the following observations: According to Table 8, bank seniority that provides incentives to bank IV to offer the loan to bank II, maximizes welfare in all cases but the top one which corresponds to the worst case scenario of very high initial losses and very low profitability. For this particular case, welfare would be maximized by the depositor seniority option that provides incentives to bank IV to offer the loan to either bank I or bank III.

5.3 Comparing Bank Seniority to Depositor Seniority (Identical Shocks)

With identical shocks we have $L_i - \psi_i = L_j - \psi_j \leqslant 1$, for every bank i or j. Then, we define $\psi \equiv \psi_i - L_i + 1$. Thus, if $L_i = 1$, $\psi = \psi_i$, if $L_i = 2$, $\psi = \psi_i - 1$ and if $L_i = 3$, $\psi = \psi_i - 2$. Put differently, given that the losses are restricted to be at most equal to 1 unit of consumer loans, ψ equals 1 minus these losses and is identical across banks.

5.3.1 The Optimal Choice of Bank IV

Depositor Seniority For the case of depositor seniority and under the restriction that shocks are identical, Table 9 below shows the expected profits of bank IV, $E[\Pi_{IV}]$, for each of the 3 loan offer options and for all possible values of per unit of loan profits, z, and ψ values defined above. The last column of the table indicates the optimal choice, that is the one that maximizes bank IV's expected profits, under the supposition that shocks per unit of loans and profits per unit of loan can only take the value that corresponds to that particular row of the table.

Table 9: Bank IV's Optimal Choice: Depositor Seniority; Identical Shocks

z	$E[\Pi_{IV}]$	OC
$z < \frac{1-\psi}{2}$	$I : \frac{3}{5}z$	*
	$II : \frac{1}{5}z$	
	$III : \frac{3}{5}z$	*
$\frac{1-\psi}{2} \leqslant z < 1-\psi$	$I : \frac{3}{5}z$	
	$II : \frac{3}{5}z$	
	$III : z + \frac{3}{5}\left(\frac{\psi-1}{2}\right)$	*
$1-\psi \leqslant z$	$I : z + \frac{2}{5}(\psi - 1)$	*
	$II : z + \frac{2}{5}(\psi - 1)$	*
	$III : z + \frac{3}{5}\left(\frac{\psi-1}{2}\right)$	

Consider the first row of the table that is derived using the results of Section 2.1. Given that the losses cannot exceed one unit and banks I and III have

two units of loans each, the relevant cases are 1(b)(i), 2 and 3(b)(i) . Thus, for the derivations we let $\psi_I = \psi_{III} = 1 + \psi$ and $\psi_{II} = \psi$ and the entry in the first column follows from $z + \psi_I < 2 \Leftrightarrow z < 1 - \psi$ and from $z < \frac{1-\psi}{2} \Rightarrow z < 1 - \psi$. The total number of customer loan units on the books of banks I, II and III is equal to 5 and, thus, the probability that bank I will go bankrupt is equal to $\frac{2}{5}$, the corresponding probability for bank II is equal to $\frac{1}{5}$ and the corresponding probability for bank III is equal to $\frac{2}{5}$. Then, $E[\Pi_{IV}] = \frac{2}{5} \times 0 + \frac{1}{5} \times z + \frac{2}{5} \times z = \frac{3}{5}z$.

For the second row we use the results of Section 2.2 and in particular cases 1, 2 and 3(b)(i). In this case we have $\psi_{II} = \psi_{III} = 1 + \psi$ and $\psi_I = \psi$. For the case 2(b)(i) the cut-off value for z is the same as above but for case 3(b)(i) we have a new cut-off value given by $\frac{1-\psi}{2}$. Lastly, the probabilities that banks I, II and III become insolvent are equal to $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{2}{5}$, respectively. For the third row we use the results of Section 2.3 and in particular cases 1, 2(b)(i) and 3(b)(i). In this case we have $\psi_I = \psi_{II} = \psi$ and $\psi_{III} = 2 + \psi$. For the case 3(b)(i) the cut-off value for z is given by $2z + \psi_{III} < 2 \Leftrightarrow z < 1 - \psi$. Lastly, the probabilities that banks I, II and III become insolvent are equal to $\frac{1}{5}$, $\frac{1}{5}$ and $\frac{3}{5}$, respectively.

Following the same steps we have completed the table.

Bank Seniority Table 10 shows the expected profits of bank IV for the case of bank seniority.

Table 10: Bank IV's Optimal Choice: Bank Seniority; Identical Shocks

$E[\Pi_{IV}]$	OC
$I : z$	*
$II : z$	*
$III : z$	*

Bank seniority completely protects bank IV form the insolvencies of any other bank when the size of the shocks are restricted to be less than 1.

5.3.2 Social Welfare

Depositor Seniority Table 11 shows the social welfare results for the depositor seniority case. For the derivation of the entries we follow exactly the same steps as those that we followed for the derivation of Table 9. The only difference is that now we use the results for W instead of $E[\Pi_{IV}]$.

Table 11: Expected Social Welfare: Depositor Seniority; Identical Shocks

z		$E[W]$	OC
$z < \frac{1-\psi}{2}$	$I : \frac{21}{5} + \frac{17}{5}z + \psi$		*
	$II : \frac{17}{5} + \frac{13}{5}z + \psi$		
	$III : \frac{19}{5} + \frac{13}{5}z + \psi$		
$\frac{1-\psi}{2} \leqslant z < 1 - \psi$	$I : \frac{21}{5} + \frac{17}{5}z + \psi$		
	$II : \frac{23}{5} + \frac{19}{5}z + \psi$		
	$III : 5 + \frac{19}{5}z + \psi$		*
$1 - \psi \leqslant z$	$I : 5 + \frac{21}{5}z + \psi$		*
	$II : 5 + \frac{21}{5}z + \psi$		*
	$III : 5 + \frac{19}{5}z + \psi$		

Then the result of the row is obtained from cases 1(b)(i), 2 and 3(b)(i) of Section 2.1. In particular, we have $\frac{2}{5}(3(1+z)+\psi_I) + \frac{1}{5}(5(1+z)+\psi_{II}) + \frac{2}{5}(3(1+z)+\psi_{III}) = \frac{2}{5}(3(1+z)+1+\psi) + \frac{1}{5}(5(1+z)+\psi) + \frac{2}{5}(3(1+z)+1+\psi) = \frac{21}{5} + \frac{17}{5}z + \psi$. As another example, consider the last row of the table where the relevant cases are 1, 2 and 3(b)(ii). In this case we have $\frac{1}{5}(5(1+z)+\psi_I) + \frac{1}{5}(5(1+z)+\psi_{II}) + \frac{3}{5}(3(1+z)+\psi_{III}) = \frac{1}{5}(5(1+z)+\psi) + \frac{1}{5}(5(1+z)+\psi) + \frac{3}{5}(3(1+z)+2+\psi) = 5 + \frac{19}{5}z + \psi$.

Bank Seniority Table 12 below shows the expected social welfare for the case of bank seniority.

Table 12: Expected Social Welfare: Bank Seniority; Identical Shocks

	$E[W]$	OC
$I : 5 + \frac{21}{5}z + \psi$		*
$II : 5 + \frac{21}{5}z + \psi$		*
$III : 5 + \frac{19}{5}z + \psi$		

In all 3 entries of the table the total welfare of depositors is equal to $5 + \psi$. There are 6 units of deposits in the banking network and each time a bank defaults its depositors lose $1 - \psi$. Profits are lower when bank IV offers the loan to bank III because of the concentration of loans in one bank.

5.3.3 Depositor vs Bank Seniority

Proof of Proposition 1 (Part 2) This follows directly from Tables 9 and 10.

Proof of Proposition 3 From Table 12, it is clear that as long as bank IV does not offer the loan to bank III expected welfare will be maximized. Table 9 shows that under depositor seniority the choice of bank IV would depend on the distribution of shocks. In contrast, Table 10 shows that under bank seniority bank IV will be indifferent across the three choices.