Intertemporal Distribution of Well-Being and Integrated Assessment

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Abstract: We explore how alternative social objectives on the intertemporal distribution of well-being affect the integrated assessment of climate change. In contrast to the literature that studies alternative parameterizations of a particular social welfare function, we shift the focus and directly assume a parametric form for the intertemporal distribution of well-being. Specifically, we consider a growth rate that linearly decreases to zero in some given time horizon. Maximizing the short-run growth rate in the deterministic modeling structure of Nordhaus' latest version of DICE, we find that the social cost of carbon increases convexly with the time horizon over which positive, but attenuating, growth in well-being is maintained. While the social cost of carbon in 2015 is US$ 11 for growth over a time horizon of 150 years, it is US$ 140 for a time-horizon of 300 years. This transparently shows how concerns for intergenerational distributive justice determine the social cost of carbon.

JEL-Classification: Q01, Q54, H21, C61, D31, D91

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1 Introduction

Avoiding dangerous anthropogenic interference in the climate system has been an important international policy goal at least since the United Nations Framework Convention on Climate Change in 1992. Economic studies of optimal climate policy typically use integrated assessment models (IAMs) to determine an optimal path of emission abatement by means of a cost-benefit analysis (Stern 2007, Nordhaus 2008, Golosov et al. 2014). Different studies arrive at remarkably different estimates for the optimal tax rate on carbon emissions into the atmosphere, i.e. the social cost of carbon (Table 1). The main reason for these differences are different assumptions about how well-being – i.e. an indicator for the situation under which humans live within a generation (Asheim 2010) – should be intertemporally distributed. These assumptions are typically embodied in an intertemporal social welfare function (SWF) used for the evaluation of climate policies (Botzen and van den Bergh 2014). Specifically, these differences are largely attributable to the specific parametrization of the SWF in terms of the so called ethical parameters, namely the time preference rate and the intertemporal elasticity of substitution.

Table 1: Selected estimates of the optimal carbon tax, quoted after Golosov et al. (2014), and the resulting gross external cost of carbon.

Study	Optimal tax	Gross external cost of carbon[1]
Nordhaus (2010)	30 US$/tC	275 billion US$
Golosov et al. (2014)	60 US$/tC	550 billion US$
Stern (2007)	250 US$/tC	2,300 billion US$

Starting with Ramsey (1928), the long lasting economic and philosophical discussion on which type of intertemporal SWF should be applied (Buchholz and Schymura 2011,

[1]Global marginal external costs as in Muller et al. (2011) resulting from enhanced climate change due to emissions of 9,167 gigatons of carbon in 2010 (Boden et al. 2013)

Botzen and van den Bergh 2014) mostly focuses on the "correct" parametrization of the SWF within the standard discounted utilitarian framework. However, there is a growing literature considering and developing alternative social welfare criteria (Asheim 2010, Zuber and Asheim 2012, Fleurbaey and Zuber 2014). Llavador et al. (2010, 2011) study the implications of alternative social welfare criteria on intergenerational well-being in a dynamic framework with exogenous emissions development. They find that both intergenerational maximin and a sustainable growth path are feasible and yield higher utility levels for the first generation than their reference value in 2000. Moreover, they find that in case of the sustainable growth path, the trade-off for the first generation in terms of consumption sacrifice is small compared to the prospect of sustained future growth in well-being. Yet, applications of alternative social welfare criteria in well-known IAMs are still relatively rare (Botzen and van den Bergh 2014).

In this paper we shift the focus and directly assume a parametric form for the intertemporal distribution of well-being rather than studying the parametrization of a particular SWF. Specifically, we assume that the growth rate of per capita well-being linearly decreases to zero in some given time horizon τ, with zero growth thereafter. Among the so-defined set of development paths, we determine the one that maximizes the initial growth rate, subject to feasibility constraints imposed by the IAM. The IAM we use is the deterministic modeling structure of Nordhaus' latest version of DICE (Nordhaus and Sztorc 2013). The maximization of the initial growth rate requires a full-fledged dynamic optimization, as an increase in the initial growth rates affects patters of investment in man-made capital, as well as carbon emissions into the atmosphere, both of which have long-term consequences that fully have to be taken into account.

By varying the time horizon until which the global economy is growing, we study how the desire for growth affects the social cost of carbon. This approach allows us to directly assess the effect of concerns for intergenerational distribution on the social cost of carbon.

Our approach to directly define intertemporal distributions of well-being is useful for three reasons. First, for society and policy-makers it might be more transparent to agree on a certain intertemporal distribution of well-being over time than to argue about specific parameter values for a particular SWF. In a recent survey, Drupp et al. (2015) elicit expert opinion on the value of the long-term social discount rate. One of the responses to their open-ended question for comments was the following:

"Instead of imposing a SWF and calculate the corresponding optimum, it is 'better' to depict a set of feasible paths of consumption, production, temperature, income distribution, etc. and let the policy maker make a choice" (Drupp et al. 2015, p.17).

Thereby, in a sense, different intertemporal distributions of well-being could be understood as being part of a "map" that could be used by policy-makers to "navigate" among different policy options, a metaphor that has recently been proposed by Edenhofer and Minx (2014).

Second, our approach makes the effects of different objectives with respect to intergenerational justice on efficient climate policy much more obvious. Last but not least, this alternative approach gives additional insights into the functioning of popular IAMs such as the DICE model.

We find that the social cost of carbon in 2015 is a convex function of the time horizon until positive growth in well-being is sustained. While for 150 years of positive – but linearly decreasing – growth gates of well-being the social cost of carbon in 2015 is US$ 10.63, it is US$ 140.44 for a time-horizon of 300 years (all in 2005 prices). The convex relationship between the distribution of growth in well-being and the social cost of carbon implies that continued growth for about 180 years (until 2194) results in social cost of carbon of US$ 19 in 2015, which is equivalent to Nordhaus and Sztorc (2013) specification of the standard social welfare function. Sustaining growth for about 290 years (until 2303) results in social cost of carbon of US$ 93 in 2015, equivalent to the discounting implied by the specification of Stern (2007).

We conclude that normative conceptions of intergenerational distributive justice crucially determine the social cost of carbon. The more society finds it just to make an ongoing development in well-being possible also for distant future generations, the higher the resulting social cost of carbon will be today. Therefore specifying the societal goal with respect to the intertemporal distribution of well-being should be the starting point of any climate change related cost-benefit analysis.

The paper is organized as follows: Section 2 formally introduces the efficient development of the climate-economy system applied in the paper, section 3 presents the results of the dynamic optimization, before section 4 discusses the results and concludes.

2 Efficient development of the climate-economy system

In the following we sketch the generic framework of the integrated climate-economy model according to the discrete-time DICE structure (Nordhaus and Sztorc 2013). Functional forms and parameter specifications can be found in the Appendix.

We use K_t to denote the economy's capital stock in period t, and $Y(L_t, K_t, E_t, T_t^{AT}, t)$ to denote the production function. Output depends positively on labor L_t, which changes over time due to population growth, the capital stock K_t, and carbon emissions E_t. Output decreases with the global mean atmospheric temperature, T_t^{AT}, which determines climate damages. Productivity increases over time due to exogenous technical progress. Aggregate consumption in period t is $C_t = L_t\, c_t$, where L_t is population size and c_t is per capita consumption. Using δ^K to denote the proportional rate of capital depreciation, the national accounting equation reads

$$K_{t+1} = (1-\delta^K)\, K_t + Y(L_t, K_t, E_t, T_t^{AT}, t) - L_t\, c_t \qquad \text{accounting} \quad (1a)$$

Emissions are generated by using up the stock of carbon resources, governed by the cake-eating equation

$$S_{t+1} = S_t - E_t \qquad \text{carbon resources} \quad (1b)$$

The atmospheric temperature develops according to

$$T_{t+1}^{AT} = T_t^{AT} + \chi^1 \left[F_{t+1} - \chi^2 T_t^{AT} - \chi^3 \left(T_t^{AT} - T_t^{LO}\right)\right] \quad \text{atmospheric temperature} \quad (1c)$$

where

$$F_{t+1} = \kappa \left[\frac{\log\left(M_{t+1}^{AT}/M_{EQ}^{AT}\right)}{\log 2}\right] + F_{t+1}^{EX} \qquad (1d)$$

is radiative forcing, which depends on the atmospheric stock of Carbon, M_t^{AT}. Atmospheric temperature also reacts to the lower ocean temperature, which, in turn, develops according to

$$T_{t+1}^{LO} = T_t^{LO} + \chi^4 \left(T_t^{AT} - T_t^{LO}\right) \qquad \text{lower ocean temperature} \quad \text{(1e)}$$

The carbon cycle is modeled by the following three-box model which stocks of carbon in the atmosphere, M_t^{AT}, the upper ocean, M_t^{UP}, and the lower ocean, M_t^{LO}. Carbon emissions enter into the atmosphere. The entire carbon cycle is described by

$$\begin{aligned}
M_{t+1}^{AT} &= E_{t+1} + \phi^{11} M_t^{AT} + \phi^{21} M_t^{UP} && \text{atmospheric carbon stock} \quad \text{(1f)}\\
M_{t+1}^{UP} &= \phi^{12} M_t^{AT} + \phi^{22} M_t^{UP} + \phi^{32} M_t^{LO} && \text{upper ocean carbon stock} \quad \text{(1g)}\\
M_{t+1}^{LO} &= \phi^{23} M_t^{UP} + \phi^{33} M_t^{LO} && \text{lower ocean carbon stock} \quad \text{(1h)}
\end{aligned}$$

Given the initial states of capital, K_0, resource, S_0, carbon stocks, M_0^{AT}, M_0^{UP}, M_0^{LO}, and temperatures, T_0^{AT} and T_0^{LO}, and given population and technology developments, the set of equations (1) define all feasible consumption/emission paths. The question is which among all feasible paths should be chosen. The answer to this question determines the social cost of carbon, i.e. the shadow price of carbon emissions into the atmosphere.

One natural restriction would be to choose only among the dynamically efficient consumption/emissions paths. Assuming that well-being depends only on consumption, a feasible path $c_t = (c_0, c_1, \ldots)$ is dynamically efficient if no other feasible path $c'_t = (c'_0, c'_1, \ldots)$ exists with $c'_t \geq c_t$ for all t and $c'_t > c_t$ for at least one period t. The dynamically efficient investment/emission path is found by maximizing per capita consumption c_{t_m} at one period in time t_m, keeping consumption at all other points in time at some pre-specified feasible minimum levels, $c_t \geq \bar{c}_t$ for all $t \neq t_m$. Defining $\pi_{t_m} = 1$

and $\bar{c}_{t_m} = 0$, the Lagrangian for this optimization problem can be compactly written as

$$
\begin{aligned}
L = \sum_{j=0}^{\infty} \pi_j \, L_j \, (c_j - \bar{c}_j) \\
+ \lambda_j^K \, \left((1-\delta^K)\, K_j + Y(L_j, K_j, E_j, T_j^{AT}, j) - L_j \, c_j - K_{j+1}\right) \\
+ \lambda_j^S \, (S_j - E_j - S_{j+1}) \\
+\lambda_j^{TAT} \left(T_j^{AT} + \chi^1 \left[\kappa \left[\frac{\log\left(M_{j+1}^{AT}/M_{EQ}^{AT}\right)}{\log 2} \right] + F_{j+1}^{EX} - \chi^2 T_j^{AT} - \chi^3 \left(T_j^{AT} - T_j^{LO}\right) \right] - T_{j+1}^{AT} \right) \\
+ \lambda_j^{TLO} \, \left(T_j^{LO} + \chi^4 \left(T_j^{AT} - T_j^{LO}\right) - T_{j+1}^{LO}\right) \\
+ \lambda_j^{MAT} \, \left(E_{j+1} + \phi^{11} \, M_j^{AT} + \phi^{21} \, M_j^{UP} - M_{j+1}^{AT}\right) \\
+ \lambda_j^{MUP} \, \left(\phi^{12} \, M_j^{AT} + \phi^{22} \, M_j^{UP} + \phi^{32} \, M_j^{LO} - M_{j+1}^{UP}\right) \\
+ \lambda_j^{MLO} \, \left(\phi^{23} \, M_j^{UP} + \phi^{33} \, M_j^{LO} - M_{j+1}^{LO}\right), \quad (2)
\end{aligned}
$$

where π_j is the Lagrangian multiplier for the constraint $c_t \geq \bar{c}_t$; λ_j^K for the capital accumulation constraint (1a); λ_j^S for the carbon resource constraint (1b); λ_j^{TAT} for atmospheric temperature (1c); λ_j^{TLO} for the temperature of the lower ocean (1e); and λ_j^{MAT}, λ_j^{MUP}, and λ_j^{MLO} for the carbon stocks in the atmosphere, upper and lower ocean, respectively.

The first-order conditions describing an efficient development of the climate-economy system can be written as follows. The conditions for the dynamically efficient consumption and emission levels are

$$\frac{\partial L}{\partial C_t} = 0 \quad \Leftrightarrow \quad \pi_t = \lambda_t^K \tag{3a}$$

$$\frac{\partial L}{\partial E_t} = 0 \quad \Leftrightarrow \quad \lambda_t^K \, Y_{E_t} + \lambda_{t-1}^{MAT} = \lambda_t^S \tag{3b}$$

and the conditions for the efficient intertemporal allocation of capital and the resource are

$$\frac{\partial L}{\partial K_t} = 0 \quad \Leftrightarrow \quad \lambda_t^K \, \left(1 - \delta^K + Y_{K_t}\right) = \lambda_{t-1}^K \tag{3c}$$

$$\frac{\partial L}{\partial S_t} = 0 \quad \Leftrightarrow \quad \lambda_t^S = \lambda_{t-1}^S \tag{3d}$$

The conditions for the efficient temperature dynamics capture the coupled dynamics of the two temperature boxes, as well as damage caused by the atmospheric temperature on production output,

$$\frac{\partial L}{\partial T_t^{AT}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{TAT} = \lambda_t^{TAT} \left(1 - \chi^1 \left(\chi^2 + \chi^3\right)\right) \quad + \lambda_t^{TLO} \chi^4 \qquad + \lambda_t^K Y_{T_t^{AT}} \tag{3e}$$

$$\frac{\partial L}{\partial T_t^{LO}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{TLO} = \lambda_t^{TAT} \chi^1 \chi^3 \qquad + \lambda_t^{TLO} \left(1 - \chi^4\right). \tag{3f}$$

Finally, the efficient dynamics of the carbon cycle are characterized by

$$\frac{\partial L}{\partial M_t^{AT}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{MAT} = \lambda_t^{MAT} \phi^{11} + \lambda_t^{MUP} \phi^{12} \qquad + \lambda_{t-1}^{TAT} \frac{\chi^1 \kappa}{\log 2} \frac{1}{M_t^{AT}} \tag{3g}$$

$$\frac{\partial L}{\partial M_t^{UP}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{MUP} = \lambda_t^{MAT} \phi^{21} + \lambda_t^{MUP} \phi^{22} + \lambda_t^{MLO} \phi^{23} \tag{3h}$$

$$\frac{\partial L}{\partial M_t^{LO}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{MLO} = \lambda_t^{MUP} \phi^{32} + \lambda_t^{MLO} \phi^{33}, \tag{3i}$$

capturing the dynamics of carbon flows between the three boxes and the effect of atmospheric carbon on the atmospheric temperature.

Using (3b) and (3c) in (3d), and rearranging, we obtain the following modified Hotelling rule

$$\frac{Y_{E_t} - Y_{E_{t-1}}}{Y_{E_{t-1}}} - \frac{\lambda_{t-1}^{MAT} - \lambda_{t-2}^{MAT}}{\lambda_t^K Y_{E_{t-1}}} = Y_{K_t} - \delta^K. \tag{4}$$

This condition prescribes how to use the carbon resource efficiently. In the absence of climate damages, we would have $\lambda_t^{MAT} = 0$ for all t. In this case, condition (4) is the simple Hotelling rule familiar from the Dasgupta and Heal (1979) model. The non-renewable resource should be used such that the growth rate of the marginal product of the resource equals the net capital interest rate. The second term on the left-hand side of (4) corrects this efficiency condition taking the future climate damages of current emissions into account.

3 Intertemporal Distributional Objectives and the Social Cost of Carbon

Conditions (3) together with (1) characterize any Pareto-efficient dynamic path. The initial social cost of carbon, measured in units of consumption, along the Pareto-efficient path are given by the ratio of the Lagrangian multiplier of atmospheric carbon, λ_0^{MAT}, and of consumption at $t = 0$, λ_0^K. Clearly, the pre-specified consumption levels $\bar{c}_j$ will have a major influence on the social cost of carbon. Thus, a central question is how to distribute per-capita consumption over time, i.e. which among the many Pareto-efficient paths to choose.

In the following we first briefly review the literature that uses a social welfare function to capture social preferences with respect to intertemporal distribution, before we turn to our approach of directly considering a specific functional form for the intertemporal distribution of well-being.

3.1 Distributional objectives embodied in a social welfare function

Nordhaus and Sztorc (2013), like most integrated assessment models, ranks paths of per capita consumption by means of the social welfare function (SWF)

$$W_0(c_0, c_1, c_2, \dots) = \sum_{t=0}^{60} \frac{1}{(1+\rho)^t} L_t \frac{c_t^{1-\eta}}{1-\eta}, \tag{5a}$$

which they interpret as the utility function of a representative, infinitely-lived agent (ILA), weighted by population size L_t. The parameters of the welfare function are the time preference rate, ρ, and the preference for consumption smoothing over time, η, with $1/\eta$ being the constant intertemporal elasticity of substitution of consumption.

Maximizing (5a) subject to (1) leads to conditions (3), but with (3a) replaced by

$$(1+\rho)^{-t} c_t^{-\eta} = \lambda_t^K \quad \overset{(3c)}{\Leftrightarrow} \quad (1+\rho)\left(1 + \frac{c_t - c_{t-1}}{c_{t-1}}\right)^{\eta} = 1 + Y_{K_t} - \delta^K, \tag{5b}$$

which is the discrete-time version of the well-known Ramsey rule (Dasgupta 2008). This condition characterizes the intertemporal distribution of consumption that is optimal according to the social objectives with respect to intertemporal distribution embodied in (5a).

The influential work of Nordhaus (2008) calls ρ and η “unobserved normative parameters”. Much of the recent economic debate on the social costs of carbon focuses on how a society should choose the values for the parameters ρ and η. Interpreting the social welfare function (5a) as the utility function of a representative ILA, these parameters can be derived from observed behavior on markets reflecting opportunity costs of capital (Arrow et al. 1996, Buchholz and Schymura 2011). In this vein, Nordhaus (2008) argues that short-term time preferences should be in line with historical consumption choices. He thus uses the Ramsey equation (5b) to determine ρ and η from inferred values of real market interest rates and the consumption growth rate.

Other studies interpret the intertemporal SWF (5a) as the (discounted) Utilitarian objective. According to this point of view, ethical considerations regarding intergenerational trade-offs of consumption should guide the choice of ρ and η (Arrow et al. 1996, Aldy et al. 2010) and focus more on long-term climate impacts (Stern 2007). Already Ramsey (1928) and Pigou (1932) argued for a zero rate of social pure time preference on ethical grounds. In this approach, the rate at which future consumption is discounted falls considerably below the opportunity cost of capital and these studies arrive at much higher estimates for the social cost of carbon. In that vein, (Stern 2007) uses a very small value of $\rho = 0.001$, merely to reflect a positive probability that mankind may become extinct at some future date, and arrives at an optimal carbon tax, which exceeds the one recommended by Nordhaus (2010) by almost one order of magnitude.

3.2 Direct specification of the intertemporal distribution of well-being

In the following we propose to shift the focus and directly assume a parametric form for the intertemporal distribution of well-being.

At least since the Brundtland Report (WCED 1987) sustainability is a key concept in the environmental discussion. Also the United Nations Framework Convention on

Climate Change (United Nations 1992) formulated the objective achieving sustainable economic development for reasons of climate protection. Asheim et al. (2001) showed that under plausible ethical assumptions sustainability calls for non-decreasing and efficient paths of well-being.

Against this background one possible intertemporal distribution of well-being is obtained by applying the maximin criterion, which maximizes consumption of the worst-off generation. Solow (1974) first applied a version of the maximin criterion to the Dasgupta-Heal-Solow growth model where an exhaustible resource is modeled being continuously substituted with produced capital. However, by construction, maximin dismisses any investment into the well-being of future generations above the level of the present generation. Consequently, a strict application hinders economic growth leading to stagnation (Rawls 1971). Yet, growth in human well-being may be desirable for a manifold of reasons. For example parents may wish their children to have a higher quality of live compared to them or society wants to make an on-going development of mankind's achievements, like increasing average life expectancy, also possible in the far future (Llavador et al. 2011). Economic growth in a carbon-dependent economy comes at a cost, since emitting one additional ton of carbon into the atmosphere causes future climate damage, which society *should* (in a normative sense) consider in form of the social cost of carbon. According to the DICE modeling structure society can either invest in man-made capital or in the natural capital of the climate system. The social cost of carbon associated with each intertemporal distribution of well-being is measured by the shadow price of carbon reflecting the "green" investments into the climate system, which are necessary to implement the respective path.

In this paper we study the trade-off between growth in human-well being and the social cost of carbon today by parametrizing the following intertemporal paths of consumption per capita. Let τ be some arbitrary, but fixed, time-horizon, measured in number of years, for which positive but linearly decreasing growth in consumption per capita is maintained. Under this assumption, consumption per capita evolves according to

$$c_t = \left(1 + \max\left\{0, g^c \cdot \left(1 - \frac{t}{\tau}\right)\right\}\right) \cdot c_{t-1} \text{ for all } t > 0, \text{ with } c_0 \text{ given.} \tag{6a}$$

Thus, we have growth, $c_t > c_{t-1}$, for all $t < \tau$ and $c_t = c_{t-1}$ thereafter. The corresponding time path of consumption per capita is logistically shaped reaching its satiation level c_τ at $t = \tau$, which can be understood as being similar to the concept of carrying capacity in ecology. It is the level of consumption per capita, which will be sustained until the end of the modeling horizon. Such a limit to growth is contained in the DICE model, which, in the long run, is identical to the Dasgupta-Heal-Solow-Stglitz model discussed above. The time path of consumption per capita is constructed similar to the one obtained under Nordhaus and Sztorc (2013).

The red dashed curves in Figure 1 illustrate the parametrized intertemporal distribution of consumption per capita. The upper panel shows the growth rate, which is linearly decreasing from g^c to zero within the time horizon τ, and remains zero after τ. The lower panel in Figure 1 shows the resulting consumption per capita, as described by (6a).

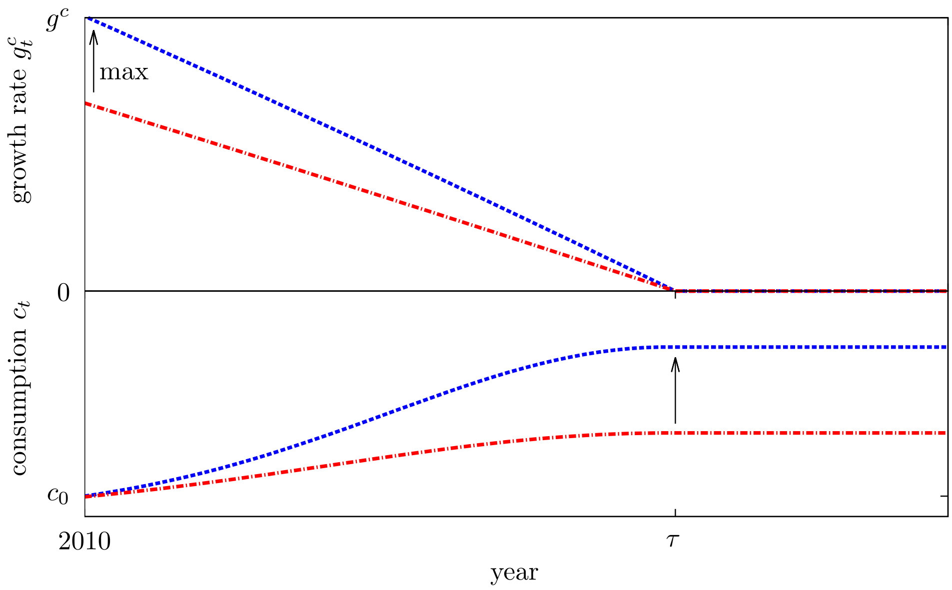

Figure 1: consumption per capita c_t and its growth rate g_t^c, for an arbitrary time-horizon τ where $g_\tau^c = 0$. While the red dotted line shows a sub-optimal time path, the blue dotted line depicts the optimized scenario we use.

A consumption path described by (6a) is feasible only if g^c is sufficiently small. Indeed, the longer the time horizon τ, the smaller g^c must be to render the consumption path (6a) feasible given the economic and climate constraints (1).

The time-horizon τ could also be thought of as a measure of sustainable growth in human well-being, i.e. how many generations can sustain a positive growth rate. The more we shift τ into the future, the more evenly growth in well-being will be intertemporally distributed and thus, the more weight society puts on intergenerational distributive justice. In the limit case ($\tau = \infty$) g_t^c will be the maximal constant growth rate of consumption per capita, which can be sustained for the total modeling horizon of 300 years given the DICE economic and climate constraints. However, we are interested in how the social cost of carbon varies with τ. Hence, we exogenously change τ and evaluate the corresponding optimal social cost of carbon in 2015 as it is defined in DICE (see Appendix).

Among all feasible paths described by (6a), we shall consider the Pareto efficient one, which is characterized as the solution to the following optimization problem, and illustrated by the blue curves in Figure 1,

$$\max_{\{c_t, e_t\}} g^c \text{ subject to (1) and (6a).} \tag{6b}$$

Using φ_t to denote the Lagrangian multiplier for the constraint

$$c_t \geq \bar{c}_t \equiv c_0 \prod_{j=0}^{t} \left(1 + \max\left\{g^c \cdot \left(1 - \frac{j}{\tau}\right)\right\}\right) \tag{7}$$

derived from (6a), everything else as in (2), we find that the first-order conditions for this dynamic optimization problem are formally identical to (3), except that (3a) is replaced by $\varphi_t = \lambda_t^K$ for all t.

Thus, we can interpret the expression $\Delta(t;\tau) \equiv \frac{\varphi_t}{\varphi_0}$ as a consumption discount factor, which depends on τ for obvious reasons.

4 Quantitative Results for the DICE Model

In this section we present the numerical dynamic optimization results. They have been calculated using the Knitro solver (version 8.1.1) together with the AMPL optimization software. The AMPL code is provided in the Appendix.

Figure 2 shows the time paths for consumption per capita for different time horizons $\tau = \{50, 100, 150, 200, 250, 300\}$ for which positive growth takes place, under maximized initial growth rates g^c. Figure 3 shows the social cost of carbon in 2015 as a function of

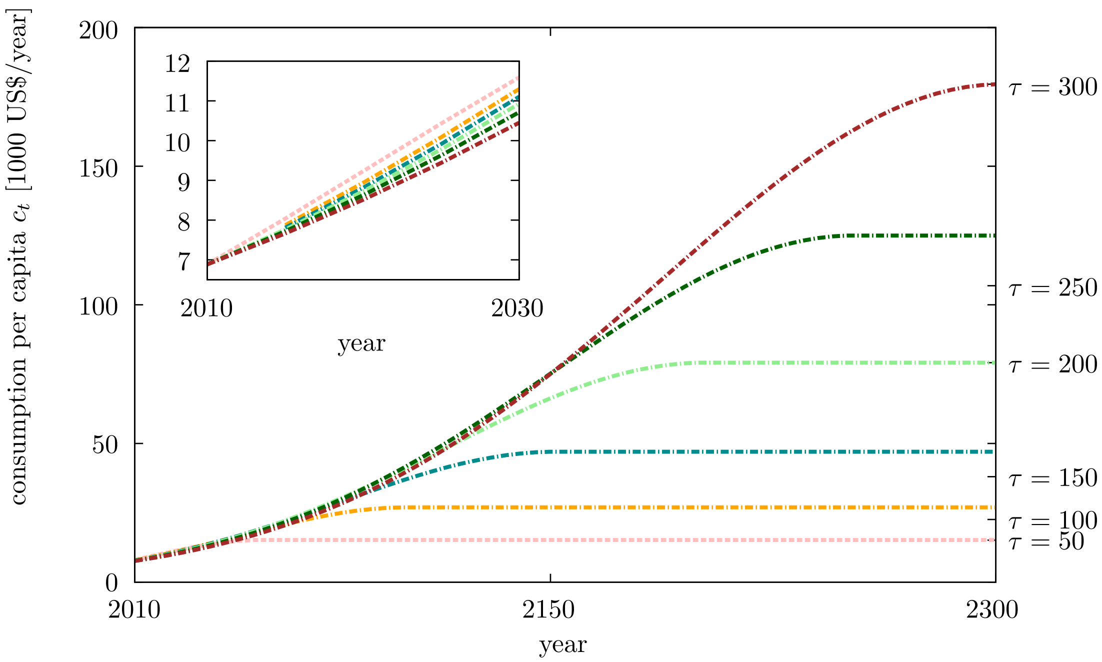

Figure 2: Time paths (2010-2300) of consumption per capita for $\tau = \{50, 100, 150, 200, 250, 300\}$, with initial growth rate g^c optimized.

the time horizon τ.

We find that the social cost of carbon in 2015 is a convex function of the time horizon for which positive growth in consumption per capita is sustained. While the social cost of carbon in 2015 is US$ 10.63 (at 2005 prices) for a time horizon of 150 years, it is US$ 140.44 for a time-horizon of 300 years. The convex relationship between the time horizon and the social cost of carbon shows how strongly the social cost of carbon depends on the desired prospects for growth. When thinking of τ as a measure of sustainable growth in human well-being, i.e. how many generations can sustain a positive growth rate, the trade-off between future growth in well-being and the social cost of carbon becomes the

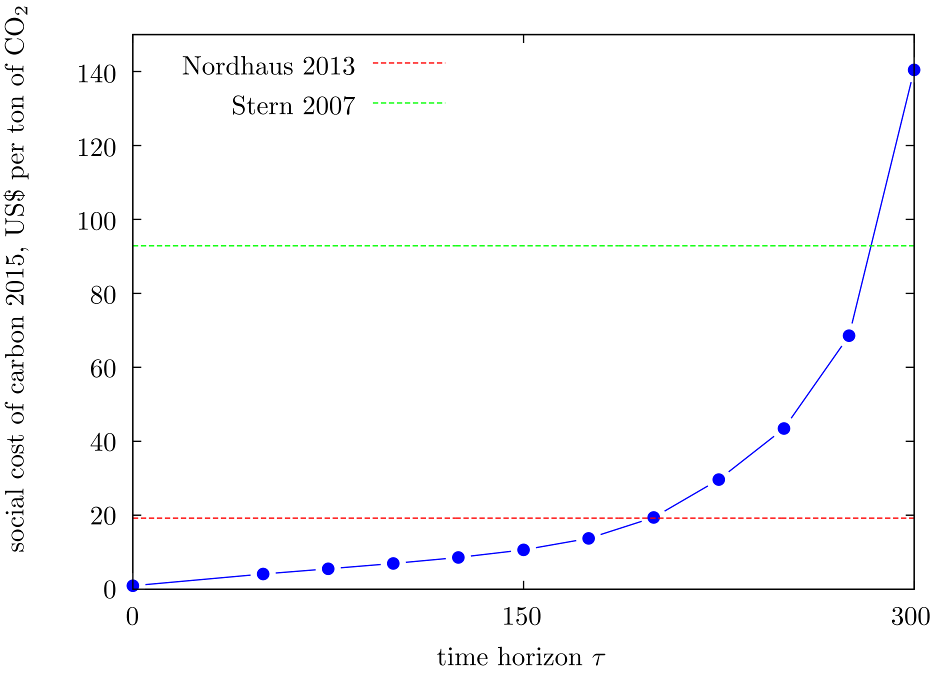

Figure 3: Social cost of carbon in 2015 as a function of the time horizon τ.

more severe the more sustainable a society wants to grow and thus, the more a weight society puts on intergenerational distributive justice.

We can also compare the results of our approach with the previous literature. The scenario of Nordhaus and Sztorc (2013) which leads to a social cost of carbon of US$ 19.24 in 2015 (red vertical line) is equivalent to a time horizon of growth until year 2194, whereas a time horizon until year 2303 leads to a social cost of carbon equivalent to the one obtained when considering Stern discounting (Stern 2007), which implies a social cost of carbon of US$ 92.88 in 2015.

As illustrated in figure 2 in the sub-graph, future growth in consumption per capita can only be implemented at the expense of lower near term growth rates (2010-2030). The earlier the point τ of zero growth is reached, the higher are the initial levels of consumption per capita from 2010 to 2030. The point of consumption catch-up, i.e. the point in time where the consumption-per capita path with the next higher growth horizon cuts the next lower consumption per capita path, is displayed in figure 4 (left panel).

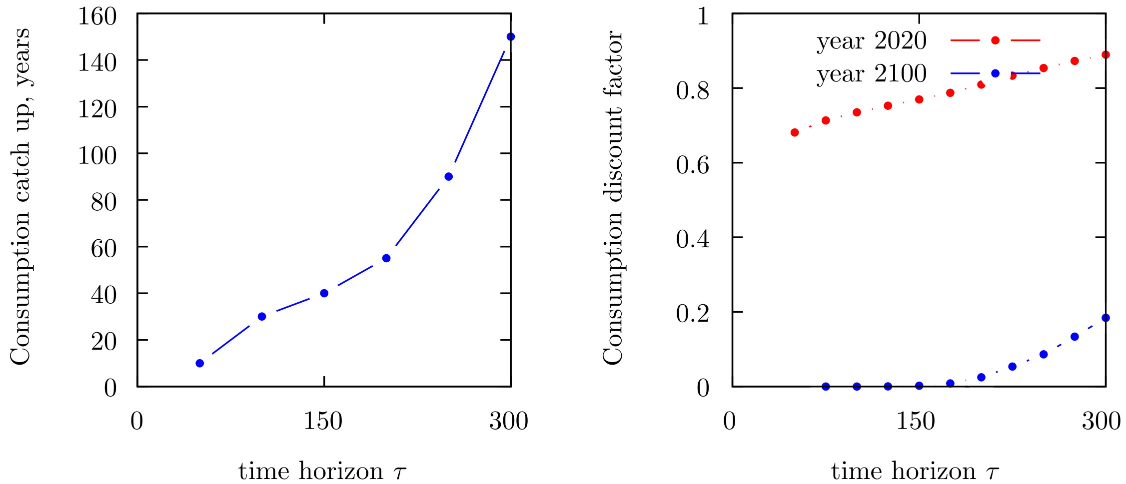

Figure 4: Consumption catch-up time (left panel) and endogenous consumption discount factor (right panel) as a function of τ.

The graph makes obvious that the more sustainable the growth rate of consumption per capita is distributed over time, the longer it takes for the respective consumption path to catch up with a path where the distribution of growth is shifted towards earlier generations. The right panel in figure 4 tells the same story: It depicts the endogenous discount factor for consumption per capita $\Delta(t;\tau)$ at two points in time, $t = 2020$ and $t = 2100$, as a function of τ. The red line points show $\Delta(2020;\tau)$; the blue ones $\Delta(2100;\tau)$. Both are increasing functions of τ reflecting that the longer the growth horizon, the more it is dynamically efficient to attach higher weights to consumption levels of future generations.

In short, our analysis makes the intergenerational trade-off in the light of anthropogenic climate change very transparent: Increasing τ and thereby letting g_t^c be more evenly intertemporally distributed, raises both the social cost of carbon in 2015 and the consumption sacrifice for early generations.

5 Discussion and Conclusions

Our paper explores how alternative social objectives on the intertemporal distribution of well-being affect the integrated assessment of climate change. Specifically, it departs from the lively debate on which parametrization of an intertemporal SWF should be applied, thus shifting the focus from the "correct" parametrization of the SWF to a more direct analysis. We use the deterministic modeling structure of Nordhaus' latest version of DICE (Nordhaus and Sztorc 2013) and assume that the growth rate of per capita well-being linearly decreases to zero in some given time horizon.

Maximizing the short-run growth rate by means of dynamic optimization, we obtain that the social cost of carbon in 2015 is a convex function of the time horizon for which positive growth in well-being is sustained. While the social cost of carbon in 2015 is US$ 11 for growth over a time horizon of 150 years, it is US$ 140 for a time-horizon of 300 years. The convex relationship between the distribution of growth in well-being and the social cost of carbon implies that continued growth for about 180 years (until 2194) results in social cost of carbon of US$ 19 in 2015, which is equivalent to Nordhaus and Sztorc (2013) specification of the standard social welfare function. Sustaining growth for about 290 years (until 2303) results in social cost of carbon of US$ 93 in 2015, equivalent to the discounting implied by the specification of Stern (2007).

The lengths of the time horizon with positive growth in human well-being can be interpreted as a measure of 'sustainability' in growth and thus the weight society puts on intergenerational distributive justice. In this respect, we find that a more evenly (sustainable) intertemporally distributed growth rate of well-being more than strongly raises the social cost of carbon and increases the consumption sacrifice for early generations. Consequently the earlier the point of zero growth is reached, implying a more uneven distribution of growth in well-being over time, the higher are the initial levels of per capita well-being from 2010 to 2030.

We decided to use DICE for that exercise, a model that combines a one sector optimal economic growth model with a simple climate module. One reason for choosing DICE is that it is one of the most popular IAMs. Its advantages of clarity, transparency and accessibility without compromising too much realism of the complex climate-economy feedbacks produce a large audience being familiar with its philosophy (Traeger 2014). DICE makes it possible to obtain quantitative estimates of the social cost of carbon while it is still possible to understand what actually drives these results. Hence, DICE is suited particularly well to inform a wide audience on the implications of contrasting perceptions of intergenerational distributive justice on the social cost of carbon.

By making the intergenerational trade-off in the light of anthropogenic climate change more transparent, the results show that normative conceptions of intergenerational distributive justice crucially determine the social cost of carbon. Therefore the knowledge of the overall societal goal in terms of the intertemporal distribution of well-being should be the starting point of any climate change related cost-benefit analysis. Policy-makers may then choose among different feasible paths of well-being, which we find more transparent as opposed to discussing the "correct" parameters of the respective intertemporal SWF.

Appendix

DICE 2013R Model

Table 2: DICE 2013R, Data (1)

Parameter	Unity	Equation
Preferences		
Time preference rate / year	-	$\rho = 0.015$
Elasticity of marginal utility of consumption	-	$\eta = 1.45$
Population and technology		
Capital elasticity	-	$\gamma = 0.3$
Initial world population	Millions	$L_0 = 6838$
Population growth rate	-	$g^L = 0.134$
Depreciation rate of capital / year	-	$\delta^K = 0.1$
Initial world gross output	Trillions 2005 US$	$Y_0^{Gross} = 63.69$
Initial capital	Trillion 2005 US$	$K_0 = 135$
Initial level of total factor productivity (TFP)	-	$A_0 = 3.8$
Initial growth rate of TFP / period	-	$g_0^A = 0.079$
Decline rate of TFP / period	-	$\delta^A = 0.006$

Table 3: DICE 2013R, Data (2)

Parameter	Unity	Equation
Emissions		
Initial industrial emissions	Gigatons CO_2	$E_0^{Ind} = 33.61$
Initial emissions control rate	-	$\mu_0 = 0.039$
Emissions control rate 2010-2150	-	$\mu = [0, 1]$
Emissions control rate 2150-2310	-	$\mu = [0, 1.2]$
Initial land emissions	Gigatons CO_2	$E_0^{Land} = 3.3$
Initial cumulative emissions	Gigatons CO_2	$E_0^{Cum} = 90$
Initial carbon intensity	Kilograms CO_2	$\sigma_0 = \frac{E_0^{Ind}}{Y^{Gross}(1-\mu_0)}$
Initial growth of carbon intensity	-	$g_0^{sigma} = -0.01$
Decline rate of carbon intensity / period	-	$\delta^{\sigma} = -0.001$
Carbon cycle		
Initial concentration atmosphere	Gigatons carbon	$M_0^{AT} = 830.4$
Initial concentration upper oceans	Gigatons carbon	$M_0^{UP} = 1527$
Initial concentration deep oceans	Gigatons carbon	$M_0^{LO} = 10010$
Equilibrium concentration atmosphere	Gigatons carbon	$M_{EQ}^{AT} = 588$
Equilibrium concentration upper oceans	Gigatons carbon	$M_{EQ}^{UP} = 1350$
Equilibrium concentration deep oceans	Gigatons carbon	$M_{EQ}^{UP} = 10000$
Flow atmosphere to atmosphere	-	$\phi_{11} = 1 - \phi_{12}$
Flow upper oceans to atmosphere	-	$\phi_{21} = \phi_{12} \frac{M_{EQ}^{AT}}{M_{EQ}^{UP}}$
Flow atmosphere to upper oceans	-	$\phi_{12} = 0.088$
Flow upper oceans to upper oceans	-	$\phi_{22} = 1 - \phi_{21} - \phi_{23}$
Flow deep oceans to upper oceans	-	$\phi_{32} = \phi_{23} \frac{M_{EQ}^{UP}}{M_{EQ}^{LO}}$
Flow upper oceans to deep oceans	-	$\phi_{23} = 0.0025$
Flow deep oceans to deep oceans	-	$\phi_{11} = 1 - \phi_{12}$

Table 4: DICE 2013R, Data (2)

Parameter	Unity	Equation
Climate model		
Equilibrium climate sensitivity	°C increase /doubling of CO_2	$\nu = 2.9$
Forcing due to equilibrium CO_2 doubling	Watts / square meter	$\kappa = 3.8$
2010 forcing of non-CO_2 GHG	Watts / square meter	$F_0^{EX} = 0.25$
2100 forcing of non-CO_2 GHG	Watts / square meter	$F_18^{EX} = 0.7$
Initial atmospheric temperature change	°C from 1750	$T_0^{AT} = 0.8$
Initial deep oceans temperature change	°C from 1750	$T_0^{LO} = 0.0068$
Speed of adjustment atmospheric temperature	-	$\chi_1 = 0.098$
Equilibrium forcing /doubling of CO_2	-	$\chi_2 = \kappa/\nu$
Heat loss from atmosphere to deep oceans	-	$\chi_3 = 0.088$
Heat gain of deep oceans	-	$\chi_4 = 0.025$
Climate change abatement costs		
Damage quadratic term	-	$\psi = 0.00267$
Initial abatement costs	Trillions 2005 US$	$\Lambda_0 = 0$
Exponent of abatement cost function	-	$\Theta = 2.8$
Initial backstop price	2005 US$	$p_0^{Back} = 344$
Decline rate of backstop price / period	-	g^{Back}

Table 5: DICE 2013R, Exogenous equations

Parameter	Unity	Equation
Total population	Millions	$L_t = L_{t-1}\left(\frac{10500}{L_{t-1}}\right)^{g^L}$
Total factor productivity (TFP)	-	$A_t = \frac{A_{t-1}}{1-g^A_{t-1}}$
Growth rate TFP per period	-	$g^A_t = g^A_0 e^{-5t\delta^A}$
Total factor productivity (TFP)	-	$A_t = \frac{A_{t-1}}{1-g^A_{t-1}}$
Carbon intensity	Kilograms CO_2 / output	$\sigma_t = \sigma_{t-1} e^{-5 g^{\sigma}_{t-1}}$
Growth rate of carbon intensity / period	-	$g^{\sigma}_t = \frac{g^{\sigma}_{t-1}}{(1+\delta^{\sigma})^5}$
External forcing	Watts / square meter	$F^{EX}_t = F^{EX}_0 + \frac{1}{18}\left(F^{EX}_{18} - F^{EX}_0\right)(t+2)$
Backstop price	2005 US$ / tonne of CO_2	$p^{Back}_t = p^{Back}_{t-1}\left(1 - g^{Back}\right)$
Adjusted cost factor for backstop	2005 US$ / tonne of CO_2	$\hat{p}_t^{\,Back} = \frac{p^{Back}_t \sigma_t}{1000\ \theta}$

Table 6: DICE 2013R, Endogenous equations

Parameter	Unity	Equation
Welfare under Nordhaus	-	$W_0(c_0, c_1, c_2, \dots) = \sum_{t=0}^{60} \frac{1}{(1+\rho)^{5t}} L_t \frac{c_t^{1-\eta}}{1-\eta}$
Total emissions	Gigatons CO_2	$E_t = E_t^{Ind} + E_t^{Land}$
Carbon concentration atmosphere	Gigatons carbon	$M_t^{AT} = \frac{5}{3.666} E_t + \phi^{11} M_{t-1}^{AT} + \phi^{21} M_{t-1}^{UP}$
Carbon concentration upper oceans	Gigatons carbon	$M_t^{UP} = \phi^{12} M_{t-1}^{AT} + \phi^{22} M_{t-1}^{UP} + \phi^{32} M_{t-1}^{LO}$
Carbon concentration lower oceans	Gigatons carbon	$M_t^{LO} = \phi^{23} M_{t-1}^{UP} + \phi^{33} M_{t-1}^{LO}$
Total radiative forcing	Watts / square meter	$F_t = \kappa \left[\frac{\log\left(M_t^{AT} / M_{EQ}^{AT}\right)}{\log 2} \right] + F_t^{EX}$
Atmospheric temperature change	°C from 1750	$T_t^{AT} = T_{t-1}^{AT} + \chi \left[F_t - \chi^2 T_{t-1}^{AT} - \chi^3 \left(T_{t-1}^{AT} - T_{t-1}^{LO} \right) \right]$
Upper ocean temperature change	°C from 1750	$T_t^{LO} = T_{t-1}^{LO} + \chi^4 \left(T_{t-1}^{AT} - T_{t-1}^{LO} \right)$
Capital	Trillions 2005 US$	$K_t = \left(1 - \delta^K\right)^5 K_{t-1} + 5 I_{t-1}$
Gross output	Trillions 2005 US$	$Y^{Gross} = A_t \left(\frac{L_t}{1000} \right)^{1-\gamma} K_t^{\gamma}$
Industrial emissions	Gigatons CO_2	$E_t^{Ind} = \sigma_t \left(1 - \mu_t\right) Y^{Gross}$
Cumulative emissions	Gigatons carbon	$E_t^{Cum} = E_{t-1}^{Cum} + \frac{5}{3.666} E_{t-1}^{Ind} \leq 6000$
Abatement costs	Trillions 2005 US$	$\Lambda_t = Y_t^{Gross} \hat{p}_t^{Back} \mu_t^{\Theta}$
Damage fraction	-	$\Omega_t = \Psi (T_t^{AT})^2$
Net output	Trillions 2005 US$	$Y_t = \left[Y_t^{Gross} \left(1 - \Omega_t\right) \right] - \Lambda_t$
Consumption	Trillions 2005 US$	$C_t = Y_t - I_t$
Consumption per capita	Thousands 2005 US$	$c_t = \frac{1000 \ C_t}{L_t}$
Social cost of carbon	2005 US$ / tonne of carbon	$p_t^c = p_t^{Back} \mu_t^{\Theta - 1}$

AMPL programming code

```
# AMPL code (mod-file) for Hänsel and Quaas (2015)
#Intertemporal Distribution of Well-Being in DICE

# PARAMETERS
# modelling horizon
param T:=60;
# population and technology
param gamma:=0.3;  # capital elasticity in production function
param L0:=6838; # initial world population (millions)
param gL:=0.134; # initial growth rate of world population per perio

param L {t in 0..T}>=0;
let L[0]:=L0;
let {t in 1..T} L[t]:=L[t-1]*((10500/L[t-1])^gL);

param deltaK:=0.1; #depreciation rate on capital per year
param Qgross0:=63.69; #initial world gross output (trillions 2005 USD)
param K0:=135; #initial capital value (trillions 2005 USD)
param A0:=3.8; #initial level of total factor productivity (TFP)
param gA0:=0.079; #initial growth rate for TFP per period
param deltaA:=0.006; #decline rate of TFP per period

param gA {t in 0..T}>=0; # growth rate for TFP per period
let {t in 0..T} gA[t]:=gA0*exp(-deltaA*5*(t));

param A {t in 0..T}>=0; # TFP
let A[0]:=A0;
let {t in 1..T} A[t]:=A[t-1]/(1-gA[t-1]);
```

```
# emission parameters
param gsigma0:=-0.01; #initial growth rate of sigma (coninuous per year)
param deltasigma:=-0.001; #decline rate of decarbonization per period
param ELand0:=3.3; # initial land emissions(GtCO2)
param deltaLand:=0.2; #decline rate of land emissions per period
param EInd0:=33.61; # initial industrial emissions(GtCO2)
param Ecum0:=90; #initial cumulative emissions (GtC)
param mu0:=0.039; # initial emissions control rate
param Lambda0:=0; # initial abatement costs(trillions 2005 USD)
param sigma0:=EInd0/(Qgross0*(1-mu0)); #initial carbon intensity(kgCO2 per output)

param gsigma {t in 0..T}; # growth rate of carbon intensity per period
let gsigma[0]:=gsigma0;
let {t in 1..T} gsigma[t]:=gsigma[t-1]*((1+deltasigma)^5);

param sigma {t in 0..T}>=0; # carbon intensity(kgCO2 per output of 2005 USD)
let sigma[0]:=sigma0;
let {t in 1..T} sigma[t]:=sigma[t-1]*exp(gsigma[t-1]*5);

param ELand {t in 0..T}>=0; # land emissions per period (GtCO2)
let ELand[0]:=ELand0;
let {t in 1..T} ELand[t]:=ELand [t-1]*(1-deltaLand);
# carbon cycle
param MAT0=830.4; # initial concentration in atmosphere(GtC)
param MUP0:=1527; # initial concentration in upper ocean(GtC)
param MLO0:=10010; # initial concentration in deep oceans(GtC)
param MATEQ:=588; # equilibrium concentration in atmosphere (GtC)
param MUPEQ:=1350; # equilibrium concentration in upper ocean (GtC)
param MLOEQ:=10000; # equilibrium concentration in deep oceans(GtC)
```

```
# flow parameters (carbon cycle transition matrix)
param phi12:=0.088;
param phi23:=0.00250;
param phi11=1-phi12;
param phi21=phi12*MATEQ/MUPEQ;
param phi22=1-phi21-phi23;
param phi32=phi23*MUPEQ/MLOEQ;
param phi33=1-phi32;
# climate model parameters
param nu:=2.9; # equilibrium climate sensitivity (°C per doubling CO2)
param kappa:=3.8; # forcing of equilibrium CO2 doubling (Wm-2)
param Fex0:=0.25; # 2010 forcing of non-CO2 GHG (Wm-2)
param Fex18:=0.70; # 2100 forcing of non-CO2 GHG (Wm-2)
param Fex {t in 0..T}=Fex0+1/18*(Fex18-Fex0)*(2+t); # external forcing (Wm-2)
param TLO0:=0.0068; # initial temperature change of upper ocean (°C from 1750)
param TAT0:=0.80; # initial atmospheric temperature change (°C from 1750)
param xi1:=0.098; # speed of adjustment parameter for atmospheric temperature
param xi2=kappa/nu; # climate model parameter
param xi3:=0.088; # coefficient of heat loss from atmosphere to upper oceans
param xi4:=0.025; # coefficient of heat gain by deep oceans
# climate damage parameters
param Psi:=0.00267; # damage quadratic term
# abatement cost
param Theta:=2.8; # exponent of abatement cost function
param pback0:=344; # initial backstop price(2005 USD per tCO2)
param gback:=0.025; # decline rate of backstop price per period

param pback {t in 0..T}>=0; # backstop price(2005 USD per tCO2)
let pback[0]:=pback0;
let {t in 1..T} pback[t]:=pback[t-1]*(1-gback);
```

```
# adjusted cost factor for backstop
param phead {t in 0..T}=pback[t]*sigma[t]/Theta/1000;

# VARIABLES # Upper and lower bounds for stability according to Nordhaus (2013)
# capital(trillions 2005 USD)
var K {t in 0..T}>=1;
# aggregate consumption(trillions 2005 USD)
var C {t in 0..T}>=2;
# Investment(trillions 2005 USD)
var I {t in 0..T}>=0;
# per capita consumption (1000s 2005 USD]
var c {t in 0..T}>=0.01;
# Gross output(trillions 2005 USD)
var Qgross {t in 0..T}>=0;
# carbon atmosphere (GtC)
var MAT {t in 0..T}>=10;
# carbon upper ocean (GtC)
var MUP {t in 0..T}>=100;
# carbon lower ocean (GtC)
var MLO {t in 0..T}>=1000;
# total radiative forcing (Wm-2)
var F {t in 0..T}=kappa*((log(MAT[t]/MATEQ))/log(2))+Fex[t];
# atmospheric temperature change (°C from 1750)
var TAT {t in 0..T}>=0,<=40;
# ocean temperature change (°C from 1750)
var TLO {t in 0..T}>=-1, <=20;
# damage fraction
var Omega {t in 0..T}=Psi*(TAT[t])^2;
# damages(trillions 2005 USD)
var damage {t in 0..T}=Omega[t]*Qgross[t];
```

```
# output net of damages and abatement(trillions 2005 USD)
var Q {t in 0..T};
# emission control rate
var mu {t in 0..T}>=0 <=1;
# abatement costs as fraction of output(trillions 2005 USD)
var Lambda {t in 0..T}=Qgross[t]*phead[t]*(mu[t]^Theta);
# industrial emissions (GtCO2)
var EInd {t in 0..T}=sigma[t]*Qgross[t]*(1-mu[t]);
# total emissions (GtCO2)
var E {t in 0..T}=EInd[t]+ELand[t];
# maximum cumulative extraction fossil fuels (GtC)
var Ecum {t in 0..T}<=6000;
# Marginal cost of abatement (social cost of carbon)(2005 USD per tCO2)
var cprice {t in 0..T}=pback[t]*mu[t]^(Theta-1);

# OBJECTIVE FUNCTION AND CONSTRAINTS
param tau default 10; # Time-horizon until zero growth: To be varied up to tau=60
var g>=0; # growth rate per period
let g:=((240.632802/6.886393)^(1/59))-1; # CAGR Nordhaus (2013)
maximize objective_function: g;
subject to constr_c {t in 1..T}: c[t]=(1+max(0,g*(1-t/tau)))*c[t-1];

subject to constr_capital_dynamics {t in 1..T}:
K[t]=(1-deltaK)^5*K[t-1]+5*I[t-1];
subject to constr_output_gross {t in 1..T}:
Qgross[t]=A[t]*((L[t]/1000)^(1-gamma))*(K[t]^gamma);
subject to constr_output_net {t in 0..T}:
Q[t]=(Qgross[t]*(1-Omega[t]))-Lambda[t];
subject to constr_accounting {t in 0..T}:
C[t]=Q[t]-I[t];
```

```
subject to constr_consumtionpercapita {t in 0..T}:
c[t]= 1000*C[t]/L[t];
subject to constr_cumulativeemissions {t in 1..T}:
Ecum[t]=Ecum[t-1]+(EInd[t-1]*5/3.666);
subject to constr_atmosphere {t in 1..T}:
MAT[t]=E[t]*(5/3.666)+phi11*MAT[t-1]+phi21*MUP[t-1];
subject to constr_upper_ocean {t in 1..T}:
MUP[t]=phi12*MAT[t-1]+phi22*MUP[t-1]+phi32*MLO[t-1];
subject to constr_lower_ocean {t in 1..T}:
MLO[t]=phi23*MUP[t-1]+phi33*MLO[t-1];
subject to constr_atmospheric_temp {t in 1..T}:
TAT[t]=TAT[t-1]+xi1*((F[t]-xi2*TAT[t-1])-(xi3*(TAT[t-1]-TLO[t-1])));
subject to constr_ocean_temp {t in 1..T}:
TLO[t]=TLO[t-1]+xi4*(TAT[t-1]-TLO[t-1]);
# Initial conditions
subject to initial_C: C[0]=47.09;
subject to initial_capital: K[0] = K0;
subject to initial_netoutput: Qgross[0] = Qgross0;
subject to initial_Ecum: Ecum[0]=Ecum0;
subject to initial_MAT: MAT[0]=MAT0;
subject to initial_MUP: MUP[0]=MUP0;
subject to initial_MLO: MLO[0]=MLO0;
subject to initial_TLO: TLO[0]=TLO0;
subject to initial_TAT: TAT[0]=TAT0;
subject to initial_mu: mu[0]=mu0;
subject to control1 {t in 1..28}: mu[t]<=1;
subject to control2 {t in 29..T}: mu[t]<=1.2;

# OTHER
# savings rate
```

```
var srate {t in 0..T}=I[t]/Q[t];
# yearly growth rate (CAGR) of consumption per capita
var grate {t in 1..T}=(((c[t]/c[t-1])^(1/5))-1)*100;
```

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