# THE CLIMATE-PENSION DEAL: AN INTERGENERATIONAL BARGAIN

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ABSTRACT. We augment a conventional climate-economy model with a Diamond overlapping generations structure to assess the intergenerational impacts of climate change and climate policies. Efficient outcomes involve the usual marginal condition: the carbon price equals the sum of discounted future marginal damage costs. We introduce an intergenerational bargain which picks out a particular efficient point, grounded on laissez-faire utilities instead of discount rates, leaving all generations better off. This is achieved by intergenerational transfers compensating for past mitigation. We calibrate our model to the DICE model. Discounted welfare maximisation under market-derived discount rates or under zero discounting both leave early generations worse off. The Pareto-improving bargain involves substantial mitigation, with carbon prices roughly twice as high as under welfare maximisation with market-derived discount rates.

#### 1. Introduction

Carbon dioxide, the most important greenhouse pollutant, resides in the atmosphere for millennia, making the relevant decision horizon of climate change economics exceptionally long compared to most other economic problems. Because of this, climate policy evaluation has to contend with distributional issues not only within a generation, but also across generations. Indeed, the prominent climate economist William Nordhaus suggests, in his popular book, that such issues present an obstacle to climate change mitigation: "Most countries must wait at least half a century to reap the fruits of [mitigation]. From a practical point of view, this raises a thorny problem in generational politics. People often resist making sacrifices for future generations. (...) Asking present generations to shoulder large abatement costs for future generations (...) is difficult to sell." (Nordhaus, 2013).

Climate policies are typically evaluated in a discounted utilitarian framework: for example, Nordhaus's analysis above is based on such an evaluation. In such a framework, questions of intergenerational distribution are compressed into the

choice of the discount rate (Heal, 2009, forthcoming).<sup>1</sup> However, the infinitely-lived agent employed in the discounted utilitarian approach obscures a simple insight: that climate change involves an externality, and thus the potential for Pareto-improving policies. As the externality is intergenerational, these policies require that later generations compensate their predecessors for their efforts to reduce emissions. Such intertemporal resource transfers are feasible. They can be implemented by government debt (Bovenberg and Heijdra, 1998), or using a system of pension transfers.

In this paper, we make three contributions. First, we tackle the issue of Pareto-improving climate policies in the canonical climate-economy modeling framework, characterising the entire Pareto frontier of the economy. Second, we propose a novel, intuitive approach to picking a 'reasonable' point on this frontier. Recent normative work on climate economics has extensively debated the choice of discount rate: i.e. which Pareto-efficient point is appropriate. We take a reverse approach, emphasising the normative relevance of the 'business-as-usual' (BAU) utilities. Third, we calibrate our model and compare the distributional implications of our normative approach to the focal points of the discount rate debate: a rate of pure time preference inferred from observed market rates of interest, or mandated to near-zero on ethical grounds.<sup>2</sup> Our approach is Pareto-improving, while the other two are not. Our approach suggests a carbon tax twice as high as under discounting based on market interest rates.

The basic principle of how Pareto-improving policies work is simple. Monetary transfers paid by later generations (when young) to concurrently living earlier generations (when old) compensate the latter for mitigating climate damages, and for any compensation they have paid to even earlier generations. This is illustrated in Figure 1. Consider a reduction in the externality, lowering aggregate consumption in early periods, but increasing it in later periods, measured in present value terms (red bars on the first line). Each generation lives for two periods, overlapping with a contemporary group. Even though net aggregate benefits only accrue after the death of the first generation  $\mathcal{G}_0$ , the sequence of green transfers from young to old

<sup>&</sup>lt;sup>1</sup>For canonical examples of models in a discounted utilitarian framework, see Stern (2006); Nordhaus (2008); Golosov et al. (2014). Dennig et al. (2015) deal with intragenerational equity in such a framework.

<sup>&</sup>lt;sup>2</sup>How to map observed market rates of interest to a rate of pure time preference of the underlying agents also depends on the generational structure (Schneider et al., 2012).

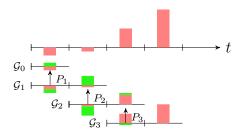


FIGURE 1. Disaggregation of period consumption and transfers

make sure that even they can obtain a (net) benefit, as do all future generations. Such a Pareto improving policy exists if the net present value of of the aggregate benefits minus costs is positive (so that the red bars sum up to a positive value) for a marginal reduction in the externality.

The counterpart of the transfers is lower saving: the policy of mitigation plus transfers implies roughly unchanged consumption for the early generations, but decreased output, so that investment must fall. In other words, the policy causes the economy to shift assets from physical capital to natural capital (in the form of a more stable climate).

That Pareto-improving climate policies exist is not an entirely novel point. John and Pecchenino (1994) point out that Pareto-improving policies exist in a Diamond OLG model augmented with a renewable resource, interpretable as an unpolluted environment, even if physical capital is accumulated in a dynamically efficient manner. Bovenberg and Heijdra (1998) use a Blanchard-Yaari 'perpetual youth' model, with a stock pollutant, to show that government debt can be used to achieve a particular Pareto-efficient point which dominates BAU. Gerlagh and Keyzer (2001) consider a Diamond OLG model without physical capital, showing that a trust fund associated with a stock of non-renewable natural capital can similarly ensure efficiency while also maintaining sustainability. Foley (2007) has also made the argument in a stylised conceptual model without explicitly talking about intergenerational transfers; Broome (2010) has promoted the idea in the philosophical literature. None of these papers characterise the entire Pareto frontier. Moreover, they are difficult to map to the workhorse climate-economy framework. An exception is provided by Howarth (1996), who shows that the representative-agent DICE framework can be interpreted as a reduced-form representation of a Diamond OLG framework, given a particular set of welfare weights. Howarth's analysis remains couched in a discounted utilitarian framework. We employ a Diamond OLG set-up in an economy which otherwise closely corresponds to the conventional frameworks employed by Nordhaus (2008) and Golosov et al. (2014), without limiting the welfare metric to discounted utility.

We propose a Pareto-efficient policy which Pareto dominates the BAU outcome. Our proposal is normative, akin to choosing welfare weights on the lifetime utilities of different generations; but, crucially, we treat the BAU endowments as normatively relevant. Consider a "deal" between two consecutive generations, the current old and current young. The old will not live to experience the benefits of abatement, which is costly to them. They are compensated by the young just enough to make them indifferent to no deal (BAU). The young then benefit if the mitigation gains (the climate improvements monetized into their retirement income) are greater than the cost of mitigation (monetized into their wage income) plus the cost of compensating the old, in present value terms. We take the lifetime utilities resulting from this deal as a new baseline. We then include the subsequent generation and consider a second deal between all three generations. This deal will allow more mitigation in the first and second period, with the first two generations just compensated, and any additional surplus given to the third generation. We iterate, adding an arbitrary number of generations. The outcome is Pareto superior to BAU by construction. Further, the iterative updating of baseline utilities is a nod towards recognising that past mitigation represents 'sunk investments'.

We calibrate by matching the BAU outcome and social optimum under market-derived discount rates with DICE-2013R. We then compare our normative proposal to the focal points of this "market discounting" run (corresponding conceptually to the approach of Nordhaus (2008)), and the optimisation of welfare given effectively uniform Pareto weights (corresponding to Stern (2006)). In the absence of transfers, both of these focal points involve losses for the early generations. In contrast, the intergenerational bargain Pareto-dominates BAU by construction and involves substantial mitigation: the discounted utilitarian optimum yields a current carbon price of 110\$/tC and our outcome yields a current price of 230\$/tC. The gross transfers are at most 2% of GDP: important, but manageable.

Pareto-improving climate policies may also be achieved by alternative mechanisms. Rezai et al. (2012) do not explicitly model different generations, but achieve a period by period increase in consumption by coupling abatement with a savings

reduction in an infinitely-lived agent model. The mechanism requires heavy climate damages, so that under BAU, agents save a large fraction of their income in expectation of a future disaster; tackling the externality allows them to cut back on saving, so that all members of a long-lived dynasty experience a Pareto gain.

In Karp and Rezai (2014), the benefits of future climate policy are capitalised into the value of long-lived capital assets, with monetary transfers ruled out. Future generations are made worse off through friction in asset prices. The Pareto improvement is achieved by mitigating and transferring resources to the young. A situation like this can be captured by our theoretical model, but is excluded in the numerical implementation, which follows Nordhaus' DICE model in using a neo-classical (Cobb-Douglas) production function.

Rangel (2003) and Boldrin and Montes (2005) show that public goods provision can be tied to pension provision in a time-consistent manner. These papers only consider short-lived externalities such as education of the young, and care services for the old. Unlike the present paper, these papers consider the dynamically consistent equilibrium, in which public good provision is supported by the threat of punishment in old age.

1.1. Organisation of the paper. In Section 2 we describe the OLG model and its equilibrium. In Section 3 we solve the social planner's problem. In Section 4 we introduce the notion of an inter-generational contract, and solve for the simplest contract, to which only contemporary generations are parties. Building on the framework derived in Section 4, in Section 5 we extend these contracts to include future generations. We derive the conditions for efficient contracts and relate these to the planner's problem. In Section 6 we provide an algorithm for choosing a particular efficient policy on the efficient frontier, and relate this approach to welfare-maximization. Finally, in Section 7 we compare the two approaches in a numerical model calibrated to the well known DICE model before concluding in Section 8.

#### 2. The model

Individuals live for two periods and are grouped into homogeneous overlapping generations. Individuals get utility from a single consumption good in both periods. They supply labour inelastically during the first period and allocate consumption between periods by saving in a competitive capital market. Competitive firms with identical technology convert capital, labour, and energy into output. The energy used in production is transformed into emissions, and these accumulate as a stock of greenhouse gases (GHGs) in the atmosphere. This stock affects (negatively) future production possibilities.

The model is effectively an overlapping generations (OLG) production economy à la Diamond, augmented by a stock externality. See Diamond (1965) on the former and Ploeg and Withagen (1991) on the latter.

2.1. Consumers and generations. Denote by  $\mathcal{G}_t$  the generation of individuals born in period t. It is composed of a **population** of  $L^t$  identical individuals with **utility function**  $U^t(c^{M,t}, c^{O,t+1})$ . We take the population levels as exogenously given. Bar the subscript on generations, time will be denoted by superscripts. Subscripts will be used for derivatives, indicating the variable(s) with respect to which the derivative is being taken. As evident in the notation for consumption, we will be distinguishing between the young (**minors**) and the **old** by the letters  $\mathbf{M}$  and  $\mathbf{O}$ .

Individuals supply labour inelastically during youth for which they earn a wage rate  $w^t$ . A pension policy is an aggregate amount  $P^t$  that generation  $\mathcal{G}_t$  transfers to  $\mathcal{G}_{t-1}$ . From the point of view of an individual in  $\mathcal{G}_t$  this is an exogenous cost of  $P^t/L^t$ . The **net youth income** of such an individual is given by

$$(1) m^t = w^t - P^t/L^t$$

Individuals can save at an **interest rate**  $r^{t+1}$ . Defining the **per capita pension** received during retirement by  $p^{t+1} = P^{t+1}/L^t$ , we can write the inter-temporal choice problem of each such individual in  $\mathcal{G}_t$ 

(2) 
$$\max_{c^{M,t},c^{O,t+1}} U^t \left( c^{M,t}, c^{O,t+1} \right)$$

$$c^{M,t} = m^t - s^t$$

$$c^{O,t+1} = \left[ 1 + r^{t+1} \right] s^t + p^{t+1}.$$

The interior solutions to (2) are given by a consumption Euler equation and implicitly define a savings function

(3) 
$$s^t(m^t, r^{t+1}, p^{t+1}).$$

 $<sup>\</sup>overline{{}^{3}$ The letter Y is unfortunately unavailable, as it is conventionally used for output.

The total capital saved by  $\mathcal{G}_t$  is  $K^{t+1} = L^t s^t$ . It will be useful to define the aggregate youth and retirement incomes of  $\mathcal{G}_t$  as

$$(4) M^t := w^t L^t - P^t$$

(5) 
$$O^{t+1} := r^{t+1}K^{t+1} + P^{t+1}.$$

2.2. **Production, emissions, and damage.** Aggregate production in period t is a function  $F^t(K^t, L^t, E^t, T^t)$ , which we take to have constant returns to scale in capital, labour and **energy**:  $K^t, L^t$ , and  $E^t$ . Energy (in emission units) is extracted at a cost  $q^t$ .

Let  $t_0$  be the first period of the industrial era during which GHGs are emitted. Given a vector of past emissions  $\vec{E}_t = \{E^{t_0}, E^{t_0+1}, \dots, E^{t-2}, E^{t-1}\}$  the current **temperature increase** is given by a function

(6) 
$$T^t = \tilde{T}(\vec{E}_t).$$

This very general specification will suffice for the purpose of our theoretical results. The important quantity for the determination of efficient emissions is the derivative  $\tilde{T}_{E^i}^t$ , which refer to a the temperature response to past emissions.

Past emissions affect the production possibilities of the economy via temperature. We parameterise this relationship in a **production function**,  $F^t(K^t, L^t, E^t, T^t)$ . We assume that  $F^t$  is increasing, concave and homogeneous of degree one in  $K^t, L^t, E^t$ . It is decreasing and concave in  $T^t$ . Concavity in  $T^t$  amounts to assuming that climate damages are *convex* in temperature.

It will be convenient for interpretation to think of all three following cross derivatives  $F_{KT}^t$ ,  $F_{LT}^t$  and  $F_{ET}^t$  as negative. This is not a consequence of the assumptions above and not necessary for the technical results, but it embodies the intuitive feature that climate change negatively affects the productivity of *all* factors of production. Most models use multiplicative damages, for which this certainly holds.

**Net output** is given by technological output minus the cost of extraction.

(7) 
$$Y^{t} = F^{t}(K^{t}, L^{t}, E^{t}, T^{t}) - q^{t}E^{t}$$

<sup>&</sup>lt;sup>4</sup>We could equivalently assume that the temperature reduces utility directly, rather than indirectly through reduced consumption. This would change some technical aspects of our model, but the flavour of the results would remain the same.

2.3. Competitive equilibrium. Many competitive firms own the production technology described above.<sup>5</sup> The perfectly competitive interest rates, wage rates, and extraction costs are equal the marginal products of the corresponding factors of production:

(8) 
$$F_K^t(K^t, L^t, E^t, T^t) = r^t$$

(9) 
$$F_L^t(K^t, L^t, E^t, T^t) = w^t$$

(10) 
$$F_E^t(K^t, L^t, E^t, T^t) = q^t.$$

That is to say, firms rent capital and labour from individuals at rates that equal the marginal products, (8) and (9), and choose a quantity of emissions so that the marginal product at the chosen level is equal to the exogenous extraction cost, (10).

In equilibrium, prices satisfy conditions (8) - (10). Since labour is supplied inelastically, the remaining fixed point condition for the **competitive equilibrium** is

(11) 
$$s^{t}(m^{t}, F_{K}^{t+1}(\underline{K}^{t+1}, L^{t+1}, \underline{E}^{t+1}, T^{t+1}), 0)L^{t} - \underline{K}^{t+1} = 0$$

where  $s^t$  is the savings function defined in (3). The equilibrium described by (8)–(11) is the **business-as-usual** (BAU) outcome, in which there are zero pensions,<sup>6</sup> and the emission rate is uncontrolled (i.e. the marginal product of emissions equals the extraction cost). This stands in contrast to the outcome we will describe in Sections 4 and 5, in which generations contract with each other to achieve a reduction of the emission rate along with pension transfers.

2.4. The value function. The value function of  $\mathcal{G}_t$  is the equilibrium utility of a representative individual in that generation. If there is there is no policy in period t+1, the emission level is determined endogenously, and the value can be

<sup>&</sup>lt;sup>5</sup>We do not specify the structure of firm ownership by individuals as is done in the standard Arrow-Debreu framework, as our assumptions result in zero profits in the competitive equilibrium. In the policy equilibrium described later there will, in fact, be profits even for these competitive firms since the resource stock will be controlled to below the competitive level. We still dispense with an explicit ownership structure, as we conceive of the emission control policy as taxing or appropriating this rent.

<sup>&</sup>lt;sup>6</sup>In reality pay-as-you-go transfers already exist in many countries, so non-zero pensions would have to feature in the BAU. We abstract from this, as it does not change the analysis, other than altering the reference position from which to consider Pareto improvements.

seen as a function of the variables  $M^t, T^{t+1}, L^t, L^{t+1}$ .

(12) 
$$V^{t} = V^{t}(M^{t}, T^{t+1}, L^{t}, L^{t+1})$$

$$= U^{t} \left(M^{t}/L^{t} - \frac{K^{t+1}}{L^{t}}, \left[1 + F_{K}^{t+1}(K^{t+1}, L^{t+1}, E^{t+1}, T^{t+1})\right] K^{t+1}/L^{t}\right)$$

We distinguish this from the value function (equilibrium utility) when policy determines the period t + 1 emission level and pension. The policy variables are chosen to benefit the generations involved, but are taken as parametric by individuals making their savings decisions. The effect on the equilibrium is that condition (10) may not necessarily hold, as the policy generally sets emissions at a level such that the marginal product is different from the extraction cost.<sup>8</sup> Furthermore, condition (11) will become

(13) 
$$s^{t}(m^{t}, F_{K}^{t+1}(\underline{K^{t+1}}, L^{t+1}, E^{t+1}, T^{t+1}), P^{t+1}/L^{t})L^{t} - \underline{K^{t+1}} = 0.$$

The difference between (11) and (13) is due to the fact that in the policy equilibrium the emission level is a policy variable and, in general, there will be a non-zero pension.

We will distinguish between the **competitive value**, depending on variables  $M^t, T^{t+1}, L^t, L^{t+1}$ , and the **policy value**, additionally depending on  $E^{t+1}$  and  $P^{t+1}$ , by denoting the latter using the hat notation.

$$\hat{V}^{t}(M^{t}, T^{t+1}, E^{t+1}, P^{t+1}, L^{t}, L^{t+1})$$

Of course, if  $E^{t+1}$  is set to its competitive value, and  $P^{t+1} = 0$ , then  $V^t = \hat{V}^t$ .

2.5. The abatement cost and the resource rent. An important quantity is the competitive resource use. This is how much the economy emits when left to its own devices without any externality-correcting policy. We denote this emission level by  $^*E^t$ . Define the marginal cost of abatement as

(15) 
$$\tau^{t} = F_{E}^{t}(K^{t}, L^{t}, E^{t}, T^{t}) - q^{t}.$$

By (10),  $\tau^t = 0$  at the competitive emission level  $^*E^t$ . Due to concavity of  $F^t$  with respect to  $E^t$ ,  $\tau^t > 0$  whenever  $E^t < ^*E^t$ . If  $E^t = ^*E^t$  the marginal cost of abatement in period t is zero.

<sup>&</sup>lt;sup>7</sup>Note that the value function is not defined as a function of state variables; we do not use dynamic programming in the present paper.

<sup>&</sup>lt;sup>8</sup>In general, Pareto efficient policy will have a marginal product of emissions that is higher than the extraction cost.

Whichever the policy instrument (tax or quota), there will be a quantity,  $\tau^t E^t$ , that is not directly benefiting either the young or the old. This is interpretable as either a resource rent, due to the restriction  $E^t < {}^*E^t$ , or a tax revenue, due to the tax rate  $\tau^t$ . As our focus is the distribution of costs and benefits of emission reduction policy across generations, we take the distribution of the resource rent as a matter of policy.

Consider the marginal change in the resource rent due to a "small" reduction in emissions,  $-dE^t < 0$ . The rent would change by

(16) 
$$d\left(\tau^{t}E^{t}\right) = -\left[F_{EE}^{t}E^{t} + F_{E}^{t} - q^{t}\right]dE^{t}$$

This term will be important in Section 4, as we calculate the distribution of costs of policies controlling resource use.

## 3. The social planner's solution

The Pareto frontier of the economy is typically mapped out by maximizing the utilitarian (linear) social planner's objective function with varying welfare weights subject to the resource constraint. We focus on the dual approach, which treats the relative weights as outcomes, with the corresponding primitives being the utility levels of all but one generation.

We maximize the utility of the representative agent of a *single* generation, subject to meeting given utility levels for the agents of the remaining generations. Consider the problem determining the Pareto frontier amongst the individuals in generations  $\{\mathcal{G}_t\}_{t=n-1}^{n+i}$ , for some  $i \geq 0$ . Define the set of **choice variables** of the problem as<sup>10</sup>

(17) 
$$\Pi = \{c^{M,t}, c^{O,t}, E^t, K^{t+1}, T^{t+1}\}_{t=n}^{n+i}, c^{O,n+i+1}\}$$

<sup>&</sup>lt;sup>9</sup>For example, the resource rent might show up as positive profits, distributed according to shares held in the company. Since our pension instrument allows for arbitrary lump-sum transfers in a given period, this would only redefine the baseline wealth distribution.

<sup>&</sup>lt;sup>10</sup>Notice that we are excluding  $c^{M,n+i+1}$  from the choice set, as it doesn't benefit any generation under consideration and would automatically be chosen as naught.

The k-problem of the social planner is the constrained maximization

$$\max_{\Pi} \quad U^k(c^{M,k},c^{O,k+1}) \quad \text{s.t.}$$

(18) 
$$U^t(c^{M,t}, c^{O,t+1}) \ge \bar{U}^t$$
  $t \ne k, n-1 \le t \le n+i$ 

(19) 
$$T^{t+1} = \tilde{T}^{t+1}(\vec{E})$$
  $n \le t \le n+i$ 

(20) 
$$F^t - q^t E^t + K^t \ge L^t c^{M,t} + L^{t-1} c^{O,t} + K^{t+1}$$
  $n \le t \le n+i$ 

(21) 
$$F^{n+i+1} - q^{n+i+1}E^{n+i+1} + K^{n+i+1} \ge L^{n+i}c^{O,n+i+1}$$

The constraints are, respectively, the requirement that other generations' utilities each meet their given minimum level, taken as primitives; the temperature evolution equation; the material balance constraint for all periods but the last; and the same for the last period. Denote by  $\{\alpha^t\}_{t\neq k}$ ,  $\{\mu^t\}_{t=n}^{n+i}$  the multipliers to (18) and (19) and by  $\{\lambda^t\}_{t=n}^{n+i+1}$  the multipliers to (20) and (21).

The first order conditions to this problem can be written to yield consumption Euler equations

(22) 
$$\frac{\lambda_t}{\lambda_{t+1}} = 1 + F_K^{t+1} = \frac{U_{C_M^t}}{U_{C_Q^{t+1}}}$$

and efficient emission conditions

(23) 
$$F_E^t - q^t = -\sum_{j=1}^{t-n+i+1} \prod_{k=1}^j \frac{\tilde{T}_{E^t}^{t+j}}{1 + F_K^{t+k}} F_T^{t+j}$$

for t = n, n + 1, ..., n + i. Given our functional form assumptions, these are necessary as well as sufficient conditions for internal solutions to the problem.

The important thing to notice is that the competitive equilibrium described in subsection 2.4 does not attain the planners solution: simply compare (10) to (23). This is hardly surprising, since the GHG emissions from fossil fuel use present an uncorrected externality.

#### 4. Contract between contemporaneous generations

We will consider *inter-generational contracts* in the spirit of pay-as-you-go pensions systems such as social security in the United States of America and similar unfunded pension schemes in other OECD countries. In this section we consider

<sup>&</sup>lt;sup>11</sup>These conditions are derived in Howarth and Norgaard (1992)

FIGURE 2. Generational structure: private and public variables

contracts in which only variables for a single period are set jointly, considering only the gains to the two generations alive in that period. These are the types of contracts studied by Dao et al. (2015) in a similar framework. In the next section we will consider contracts covering policy variables across many periods and gains to an arbitrary number of generations.

4.1. Private/public variables and the contract curve. Figure 2 is a schematic of the generational structure in our model. Generations earn aggregate income  $M^t$  during youth and  $O^t$  in old age and substitute between the two periods by their savings decision. In considering a departure from competitive equilibrium, emissions  $E^t$  and aggregate pension transfer  $P^t$  (paid by  $\mathcal{G}_t$  to  $\mathcal{G}_{t-1}$ ) are policy variables, which can be decided upon jointly by the generations alive during period t. Figure 2 draws them in curly brackets between the generations who can come to agreement on the value these policy variables take.

Suppose that period n is when the economy starts considering serious mitigation policy. The generations  $\mathcal{G}_{n-1}$  and  $\mathcal{G}_n$  could come to an agreement improving upon the competitive equilibrium (BAU) from the point of view of their constituent

<sup>&</sup>lt;sup>12</sup>A repeated application of such contracts in all periods would lead to policy in all periods, but such a sequence of policies would not be internalising all the benefits that are achievable.

individuals. At the moment of the policy choice, capital  $K^n$  and temperature  $T^n$  are already given.

Suppose the two generations are considering mandating an emission level  $E^n \leq {}^*E^n$  (or implementing by carbon tax) as well as some pension level  $P^n$ . Whether this is collectively rational for those two generations depends on whether they could mutually do better. Consider a small change  $-dE^n < 0$  from the provisional choice  $E^n$ . If the individuals in both generations can be made better off by such an additional emission reduction (coupled with the correct change in the redistributive pension), then  $(E^n, P^n)$  is suboptimal. Policies that cannot be improved upon in this sense will be said to be on  $\mathcal{G}_{n-1}$  and  $\mathcal{G}_n$ 's **contract curve**, which we denote by  $\mathcal{C}_{n-1}^n \subset \mathbb{R}^+ \times \mathbb{R}$ . This is the set of locally efficient (internal) policy vectors.

4.2. A small policy change and the distribution of costs. The marginal change  $-dE^n$  would lead to a change in total output during period n of

$$dY^n = -[F_E^n - q^n]dE^n < 0$$

From the discussion in subsection 2.5 we know the term in brackets as the marginal abatement cost, or the per unit carbon tax that would be required to attain the underlying emission level.

Using Euler's Theorem for homogenous functions and (7),

$$Y^{n} = F_{K}^{n}K^{n} + F_{L}^{n}L^{n} + [F_{E}^{n} - q^{n}]E^{n}$$

Differentiating the identity above by  $E^n$  yields

(25) 
$$F_E^n - q^n = F_{KE}^n K^n + F_{LE}^n L^n + F_{EE}^n E^n + F_E^n - q^n$$

The terms on the right hand side of (25) can be seen as the constituent components of the marginal abatement cost. Recalling that the interest and wage rates are  $F_K^n$  and  $F_L^n$  respectively (see (8) and (9)) we can see that the first two terms are the marginal cost shares of the private labour and capital shares of output. The remaining term is exactly the marginal change in the resource rent (or tax revenue), given in (16).

Consider the following convention. We will denote by  $P^n$  the total transfer that  $\mathcal{G}_{n-1}$  receives in addition to its capital rent  $F_K^nK^n$ . Notice that if  $E^n < *E^n$ ,  $P^n$  is not the deduction from the aggregate wage of  $\mathcal{G}_n$ : in addition to the private wealth  $F_K^nK^n$  of the old and  $F_L^nL^n$  of the young, the resource rent  $[F_E^n - q^n]E^n$ 

must be allocated. It is at the joint disposal of the contracting generations in that it is separated from private wealth by public agreement on an emission level  $E^n < *E^n$ : the agreement will stipulate how to allocate this additional wealth. Thus, in order to ensure that the old get  $P^n$  more than their private income, the young must pay an amount  $P^n - [F_E^n - q^n] E^n$  from their private income. We express this convention in redefining the term (4), so that, for all t

(26) 
$$M^{t} = F_{L}^{t}L^{t} + \left[F_{E}^{t} - q^{t}\right]E^{t} - P^{t}$$

$$(27) O^t = F_K^t K^t + P^t$$

It is important to note that the above is a convention on the definition of  $P^n$ , rather than an assignment of the property right to the resource rent to  $\mathcal{G}_n$ .

It is easy to see from (27) that if  $dP^n = F_{KE}^n K^n dE^n$  then<sup>13</sup>

$$dO^n = -F_{KE}^n K^n dE^n + dP^n = 0$$

and  $\mathcal{G}_{n-1}$  is indifferent to the "small" change. The burden on  $\mathcal{G}_n$  is

$$dM^{n} = -\left[F_{LE}^{n}L^{n} + F_{EE}^{n}E^{n} + F_{E}^{n} - q^{n} + F_{KE}^{n}K^{n}\right]dE^{n}$$

$$= -\left[F_{E}^{n} - q^{n}\right]dE^{n}$$
(28)

were the first equality is simply the derivative of (26) and the second is the identity (25). Having only just compensated the old for their share of the additional abatement cost,  $\mathcal{G}_n$  bears the full marginal cost of abatement (see (24)).

4.3. Future benefit and the policy trade-off. In period n+1 the stock of carbon is reduced by the period n emission reduction, which leads to an improvement in the economic conditions then, and an *increase* in the old age income  $O^{n+1}$  of  $\mathcal{G}_n$ .

The emission reduction  $-dE^n$  leads to a reduction in the n+1 temperature of

(29) 
$$dT^{n+1} = -\tilde{T}_{E^n}^{n+1} dE^n$$

This leads to a first order increase in n+1 output of

$$\mathrm{d}Y^{n+1} = -F_T^{n+1}\tilde{T}_{E^n}^{n+1}\mathrm{d}E^n$$

<sup>&</sup>lt;sup>13</sup>We consider a policy intervention at a point in time when the past savings decision has been made and the current capital stock has been formed. We will later consider the effect on savings of an expected policy intervention of this sort.

and the corresponding increase in the period n+1 capital rent of  $\mathcal{G}_n$  of

(30) 
$$dO^{n+1} = -F_{KT}^{n+1} K^{n+1} \tilde{T}_{E^n}^{n+1} dE^n$$

In a partial equilibrium model (or a small open economy in which individuals borrow from abroad, thus substituting between periods without affecting the interest rate), efficiency would be achieved when

$$dM^n = -\frac{dO^{n+1}}{1 + r^{n+1}}$$

That is, when

(31) 
$$F_E^n - q^n = -\frac{\tilde{T}_{E^n}^{n+1} F_{KT}^{n+1} K^{n+1}}{1 + r^{n+1}}$$

This is a consequence of the consumption Euler equation. Individuals in  $\mathcal{G}_n$  substitute between periods until the ratio of the marginal utilities equals  $1 + r^{n+1}$ .

4.4. General equilibrium and the efficient policies. Given that the individuals adjust their savings decision (via the savings function (3)) to optimally allocate consumption between the two periods of life, the change in  $M^n$ ,  $O^{n+1}$  and  $T^{n+1}$  (and the corresponding changes in  $m^n$  and  $r^{n+1}$ ) change the parameters of the equilibrium conditions (10) and (11). This leads to changes in the equilibrium capital stock and emission levels in period n+1, as they respond to changes in the previous net income of the young,  $M^n$ , and the current temperature  $T^{n+1}$ . These responses, in turn, affect the equilibrium utility of the individuals in generation  $\mathcal{G}_n$ . Switching the notation to a generic period, t, Lemma 1 characterises these marginal effect on the competitive value function, as a function of the changes in capital and emissions.

**Lemma 1.** Let the savings function be defined as in (3) and the competitive value function as in (12), and assume that the period t+1 equilibrium capital and emissions satisfy conditions (11) and (10). The derivatives of the competitive value function with respect to  $M^t$  and  $T^{t+1}$  (normalized by first period marginal utility) are given by

$$\begin{split} V_M^t L^t &= 1 + \frac{F_{KK}^{t+1} K_M^{t+1} K^{t+1} + F_{KE}^{t+1} E_M^{t+1} K^{t+1}}{1 + F_K^{t+1}} \\ V_T^t L^t &= \frac{F_{KT}^{t+1} K^{t+1} + F_{KK}^{t+1} K_T^{t+1} K^{t+1} + F_{KE}^{t+1} E_T^{t+1} K^{t+1}}{1 + F_K^{t+1}} \end{split}$$

The second and third terms in the numerators are due to the equilibrium changes in the interest rate due to changes in capital and emissions respectively, which we compute in Lemma 2 in the appendix.

*Proof.* This is simply an application of the chain rule to the definition of  $V^t$ . Consider  $V_M^t$ :

$$V_{M}^{t} = \frac{U_{C^{M}}^{t}}{L^{t}} \left[ 1 - K_{M}^{t+1} \right] + \frac{U_{C^{O}}^{t}}{L^{t}} \left[ K_{M}^{t+1} (1 + F_{K}^{t+1}) + K_{M}^{t+1} F_{KK}^{t+1} K^{t+1} + E_{M}^{t+1} F_{KE}^{t+1} K^{t+1} \right]$$

The consumption Euler equation states that  $U_{C^M}^t/U_{C^O}^t = 1 + r^{t+1}$ . So dividing by  $U_{C^M}^t$  and multiplying by  $L^t$  we get

$$V_M^t L^t = 1 - K_M^{t+1} + \frac{K_M^{t+1}(1 + F_K^{t+1}) + K_M^{t+1}F_{KK}^{t+1}K^{t+1} + E_M^{t+1}F_{KE}^{t+1}K^{t+1}}{1 + r^{t+1}}$$

Recognizing that  $r^{t+1} = F_K^{t+1}$  in equilibrium allows us to cancel out the first two terms containing  $K_M^{t+1}$  (this is the content of the Envelope Theorem in this context).

The derivative with respect to  $T^{t+1}$  is done similarly.

With this notation we see that the change in value of  $\mathcal{G}_n$  is given by

$$\mathrm{d}V^n = V_M^n \mathrm{d}M^n + V_T^n \mathrm{d}T^{n+1}$$

With  $dM^n$  given by (28) and  $dT^{n+1}$  given by (29), we get that the contract curve between  $\mathcal{G}_{n-1}$  and  $\mathcal{G}_n$  satisfies

(32) 
$$F_E^n - q^n = -\tilde{T}_{E^n}^{n+1} \frac{V_T^n}{V_M^n}.$$

The difference between (31) and (32) is the fact that the latter takes into account the changes in the interest rate (the  $F_{KX}^{n+1}K^{n+1}$  for  $X \in \{M, T\}$  terms in  $V_M^n$  and  $V_T^n$ ) that result from the equilibrium changes in  $K^{n+1}$  and  $E^{n+1}$ ).

We have proven the following proposition.

**Proposition 1.** The contract curve  $C_{n-1}^n$  between  $G_{n-1}$  and  $G_n$  is the path in  $(E^n, P^n)$ -space defined by (32).

#### 5. Non-present generations

If in period n a level of abatement corresponding to condition (32) is agreed upon between  $\mathcal{G}_{n-1}$  and  $\mathcal{G}_n$ , no further mutual gains are achievable for the individuals

in those generations. That is the definition of the contract curve.<sup>14</sup> However, there are benefits accruing to  $\mathcal{G}_{n+1}$  that are not internalized at the contract curve  $\mathcal{C}_{n-1}^n$ . These include the effects of lower temperatures on returns to both labour (when young, and net of the effect of the changed capital stock) and capital (when old), with some general equilibrium adjustments. That is to say, for  $(E^n, P^n) \in \mathcal{C}_{n-1}^n$ , there will be mutual improvements to  $\mathcal{G}_{n-1}$ ,  $\mathcal{G}_n$  and  $\mathcal{G}_{n+1}$ , since  $\mathcal{G}_{n+1}$  can compensate its two predecessors with the instrument  $P^{n+1}$ .

5.1. Contracts and non-present parties. Of course  $\mathcal{G}_{n+1}$  is not present during period n, when the choice of  $E^n$  must not only be contracted, but also implemented. Any contract internalizing the benefits to  $\mathcal{G}_{n+1}$  must be decided upon as if  $\mathcal{G}_{n+1}$  was present along with  $\mathcal{G}_{n-1}$  and  $\mathcal{G}_n$ . Let us call this triumvirate  $G_{n-1}^{n+1}$ . If they were together at a table,  $\mathcal{G}_{n+1}$  could agree to pay  $\mathcal{G}_n$  so that  $\mathcal{G}_n$  could pay  $\mathcal{G}_{n-1}$  and still have some left over so that both are compensated for the cost of an additional unit of reduction of  $E^n$ . If the cost to  $\mathcal{G}_{n+1}$  of this compensation is less than the benefit it attains from the resulting emission reduction, this sequence of policies is mutually beneficial to all three generations. The triumvirate should only agree to policies that cannot be thus improved upon.

The actual contract achieved if  $\mathcal{G}_{n+1}$  were actually at a table with the other two would be more complicated than that. The choice of period t+1 emissions,  $E^{n+1}$ , interacts with the costs and benefits in the sequence above, and so it must be taken into account. The set of policies the coalition  $G_{n-1}^{n+1}$  cannot find a local mutually beneficial improvement over is the **contract surface**  $C_{n-1}^{n+1}$  in  $(E^n, P^n, E^{n+1}, P^{n+1})$ -space.

Proposition 2 gives the conditions that the equilibrium variables must satisfy at the polices on this contract surface.

<sup>&</sup>lt;sup>14</sup>Whether there is a Pareto improvement *not* covered by the marginal conditions depends on whether the conditions describe a global maximum. Convexity of  $\tilde{T}^{n+1}(\vec{E})$  along with concavity of  $V^n$  in  $M^n$  and  $T^{n+1}$  would be sufficient, but certainly not necessary.

**Proposition 2.** The contract surface  $C_{n-1}^{n+1}$  between  $G_{n-1}$ ,  $G_n$  and  $G_{n+1}$  is the part of  $(E^n, P^n, E^{n+1}, P^{n+1})$ -space satisfying

(33) 
$$F_E^n - q^n = -\frac{\tilde{T}_{E^n}^{n+1}}{1 + F_K^{n+1}} F_T^{n+1} - \frac{\tilde{T}_{E^n}^{n+2}}{1 + F_K^{n+1}} \frac{V_T^{n+1}}{V_M^{n+1}}$$

(34) 
$$F_E^{n+1} - q^{n+1} = -\tilde{T}_{E^{n+1}}^{n+2} \frac{V_T^{n+1}}{V_M^{n+1}}$$

*Proof.* In the Appendix.

The proof follows a similar approach to that of Proposition 1, with one key difference: when considering mitigation over two periods, the second-period mitigation will be foreseen by the savers in  $\mathcal{G}_n$ , who will thus adjust their saving in response; this changes the impact on  $\mathcal{G}_{n+1}$ . However, an emission reduction policy in which the old individuals of  $\mathcal{G}_n$  are fully compensates for mitigation in period n+1 still costs the young of  $\mathcal{G}_{n+1}$  exactly the marginal abatement cost.

Notice that the first term on the right hand side of (33) is simply the total period n + 1 benefit from period n emissions reductions, discounted by the rate of interest. There is no correction for the equilibrium adjustment of variables as in the other two right hand side terms or the right hand side term in (32). As we shall see later, the general equilibrium corrections drop out in all but the last summand corresponding to the benefit to the last generation party to the contract.

5.2. More generations and the contract curve. The result in Proposition 2 can be generalized to an arbitrary number of generations. Before we do so, we introduce the necessary terminology. For all i, the **i-coalition** from period n is the set of all adjacent generations between period n and n+i, denoted by  $G_{n-1}^{n+i} = \{\mathcal{G}_t\}_{t=n-1}^{n+i}$ . The contract curve is the set of locally efficient points amongst them. It is a subset of  $\{E^t, P^t\}_{t=n}^{n+i}$  and we denote it by  $\mathcal{C}_{n-1}^{n+i}$ .

**Theorem 1.** For all  $i \geq 0$ , the contract surface  $C_{n-1}^{n+i}$  of the i-coalition  $G_{n-1}^{n+i}$  satisfies the following conditions. For all  $t = n, n + 1, \ldots, n + i$ 

(35) 
$$F_E^t - q^t = -\sum_{i=1}^{t-n+i} \prod_{k=1}^j \frac{\tilde{T}_{E^t}^{t+j}}{1 + r^{t+k}} F_T^{t+j} + \prod_{k=1}^{t-n+i} \frac{\tilde{T}_{E^t}^{t+i}}{1 + F_K^{t+k}} \frac{V_T^{n+i}}{V_M^{n+i}}$$

*Proof.* The relationship (50) holds for any period t, whenever the indifference condition is satisfied for  $\mathcal{G}_t$ . By recursively substituting  $dM^{t-1}$  into the expression for

 $dM^t$  we get that

$$\mathrm{d} M^{t+i} = F_T^{t+i} \mathrm{d} T^{t+i} + \sum_{j=1}^{i-1} F^{t+j} \mathrm{d} T^{t+j} \prod_{k=j+1}^{t+i} \left[ 1 + F_K^k \right] + \mathrm{d} M^t \prod_{k=1} \left[ 1 + F_K^{t+k} \right]$$

Putting this condition into the indifference condition of  $\mathcal{G}_{n+i}$ , as in the last step of the proof of Proposition 2, yields the result.

5.3. **Time-consistent contracts.** If a policy is in  $C_{n-1}^{n+i}$ , none of the participating generations can be made better off without making another worse off. So in a world of full commitment, it should be easy to argue that emission and pension levels corresponding to a point on the contract curve should be implemented. But if the current young expect that the deal assured them by the contract may be changed to their detriment in the next period, they would hesitate to make the necessary cuts today.

This commitment problem arises in many political economics models of debt or pensions. If there is no population growth a model with a median voter equilibrium provides the necessary stability. If the intergenerational policies require a supermajority for repeal, then stability can even be provided in a model with population growth. Currently developed economies find it very hard to repeal pay-as-you-go-pensions, even though less than a third of the population are beneficiaries. Intergenerational altruism, intragenerational heterogeneity and the high voter turn-out of retirees, none of which we explicitly model here, all play a role in this empirical stability. We do not explicitly model the features needed in order to ensure stability in the model, but mention that stability of pensions is well supported in the political economics literature (see Tabellini (1991), Tabellini (2000), Kotlikoff and Svensson (1988), Cukierman and Meltzer (1989), Grossman and Helpman (1998), Becker (1983) and Dixit and Londregan (1998)).

5.4. Social planner with general equilibrium continuation. Comparison between (35) and (23) reveals that they are identical bar the last summand, which in (35) involves the general equilibrium terms from Proposition 1, that are not present in (23). Here we provide a modification of the planner's maximization problem that has (35) as its first order conditions.

Define the set of choice variables of the modified problem as

(36) 
$$\Delta = \{c^{M,t}, K^{t+1}\}_{t=n}^{n+i-1}, \{c^{O,t}, E^t, T^{t+1}\}_{t=n}^{n+i}, M^{n+i}\}.$$

Notice that the difference between (17) and (36) is that the former includes the variable  $M^{n+1}$  and excludes  $\{c^{M,n+i}, K^{n+i+1}, c^{O,n+i+1}\}$ . As you will see below, this is because this version of the planner does not directly control the consumptions of the last generation and last period capital, but rather controls the amount of first period wealth,  $M^{n+i}$  that the last generation is endowed with.

The problem of the **social planner with equilibrium continuation** (SPEC) is the constrained maximisation

(37) 
$$\max_{\Lambda} V^{n+i}(M^{n+i}, T^{n+i+1}, L^{n+i}, L^{n+i+1})$$
 s.t.

(38) 
$$U^t(c^{M,t}, c^{O,t+1}) \ge \bar{U}^t$$
  $n-1 \le t \le n+i-1$ 

$$(39) \quad T^{t+1} = \tilde{T}^{t+1}(\vec{E}) \qquad \qquad n \le t \le n+i$$

$$(40) F^t - q^t E^t + K^t \ge L^t c^{M,t} + L^{t-1} c^{O,t} + K^{t+1} n \le t \le n+i-1$$

$$(41) F^{n+i} - q^{n+i}E^{n+i} + K^{n+i} > L^{n+i-1}c^{O,n+i} + M^{n+i}.$$

This planner then maximises the competitive value function (12) of the last generation, rather than it's utility. She sets aside an amount  $M^{n+i}$ , which results in an equilibrium value for the individuals in  $\mathcal{G}_{n+i}$ , and trade's that off against the resources used for the consumption of the previous generations. This trade-off is embodied in the last constraint (41), which is distinct from (21), reflecting the different choice set.

We can now state the theorem that will make points on the contract curve computable as a single constrained maximisation, rather than as a system of i+1 equilibrium conditions.

**Theorem 2.** The first order conditions of the social planner with equilibrium continuation are given by the conditions (35), along with the Euler equations. Furthermore, if  $V^{n+i}$  is quasi-concave in  $M^{n+i}$  and  $T^{n+i+1}$ , and if  $\tilde{T}^t(\vec{E})$  is convex for all t, these are necessary and sufficient conditions for the this planner's optima.

Given a set of parameters  $\{\bar{U}^t\}_{t=n-1}^{n+i-1}$  the first order conditions pick an allocation in  $\Delta$  that corresponds to the contract curve  $\mathcal{C}_{n-1}^{n+i}$ . Given this, it is straightforward to back out the transfers that implement this contract in the decentralized way it was derived in the preceding sections.

If the Lagrange multipliers associated with the utility constraints (38) are given by  $\{\alpha^t\}_{t=n-1}^{n+i-1}$ , then these can be interpreted as the utility weights of the equivalent

maximisation in which there are no constraints (38), but the planner maximises the sum of utilities, weighted by  $\{\alpha^t\}_{t=n-1}^{n+i-1}$ , and with a weight of one on the last included generation. This interpretation holds both for the social planner with equilibrium continuation, as well as the social planner in Section 3.

## 6. CLIMATE POLICY

We have described the Pareto frontier that is achievable as a contract amongst any set of adjacent generations. What we had set out to show was that by taking advantage of contemporaneous transfers from young to old we could devise policy that is a Pareto improvement over BAU. This is quite clearly the case. However, in order to pick a particular policy on this frontier it is necessary to make trade-offs.

The standard welfarist approach picks utility weights for the different generations and picks the optimal policy with respect to the weighted sum of utilities. In representative agent models the debate over these weights is simply the prominent debate over discount rates. In fact, as we show below there is an isomorphic mapping between the representative agent's discount rate and the utility weights in an OLG model.

As we indicated with our alternative approach to the specification of the social planner, there is another way to arrive at a particular Pareto efficient point, which we find more appropriate in the intergenerational context. Rather than choosing weights to trade off between generations, we extend the notion that the outcome ought to be a Pareto *improvement* over the business-as-usual outcome. The basic idea is based on an algorithm that considers a sequence of contracts, always adding an additional generation to the previous set already in the contract. When a generation gets added, that contract gets chosen which gives the newly added generation all of the gains available from the contract, subject to giving all the other (previously already in a contract with each other) generations the utility they had in the previous contract. The first step of this recursion is fixed by letting the very first (old) generation simply get its business-as-usual utility.

6.1. Reservation utilities and a stationary contract. Consider aiming for a contract between the I+1 generations alive between period n and period n+I. Given Theorem 2, what we need is a list of reservation utilities  $\{\bar{U}^t\}_{t=n-1}^{n+I-1}$ . Given these, the theorem provides for a unique point on the contract curve.

We know what we want the outcome to be superior to BAU, so denoting by  $U_{BAU}^{n-1}$  the BAU utility of  $\mathcal{G}_{n-1}$ , we set  $\bar{U}^{n-1} = U_{BAU}^{n-1}$ . To get the reservation utility of  $\mathcal{G}_n$  maximise the social planner with equilibrium continuation of Subsection 5.4 for i = 0. The resulting utility of  $\mathcal{G}_n$  is the (constrained) maximum of the objective,  $\bar{U}_{max}^n = \max(V^n)$ .

Now set  $\bar{U}^n = \bar{U}^n_{max}$ , keep  $\bar{U}^{n-1} = U^{n-1}_{BAU}$ , and maximise the same problem for i = 1. Now repeat the process, always setting the reservation utility of the most recently added generation to the maximum, until i = I.

The result is certainly on the contract curve, as it is the result of the equivalent maximisation. It is tautologically Pareto superior to BAU from the perspective of  $\mathcal{G} + n - 1$  and  $\mathcal{G}_n$  – one is made indifferent to BAU, and the other will do at least as well, since more instruments are at their disposal than if they just saved optimally for retirement at competitive emission rates.

We will show in numerical examples below that this approach is also a Pareto improvement for all subsequent generations. The reason for this is simple. The result, say, of the first agreement between  $\mathcal{G}_{n-1}$  and  $\mathcal{G}_n$  is a reduction in the emissions (and therefore future temperature) and a reduction in the capital accumulated in period n+1. Thinking of temperature as the opposite of natural capital, the contract substitutes physical capital for natural capital. If the next generation benefits more from the added natural capital than from the (general equilibrium) loss in physical capital, then it will already be benefiting from previous contracts in this algorithm, to which it is not even party. Of course this is not generally the case for every choice of parameters, but it is the case in our numerical implementation.

## 7. Numerical results

In this section, we present a calibrated version of our model. This calibration serves two purposes. The main purpose is to assess the distributional implications of our normative proposal, comparing it to the outcomes under discounted welfare maximisation. In particular, we will demonstrate the intergenerational equity implications of our model and of two alternative benchmarks. The first corresponds conceptually to a 'market discounting' approach, with the utility of future generations discounted at a rate of pure time preference inferred from market rates (Nordhaus, 2008). The second corresponds to a 'no discounting' approach, with no utility discounting (Stern, 2006).

The second purpose is to give some indication of carbon prices suggested by our normative approach, to the extent one places credence in numbers derived from integrated assessment models. Even if one is suspicious of such exercises (Pindyck, 2013; Stern, 2013), the results give a qualitative indication of how carbon prices might look under our intergenerational bargain, relative to the benchmark approaches.

7.1. Calibration. We explain our basic approach to calibration here, relegating the details to the Appendix. We take most of the parameters and functional forms from DICE-2013R, but simplify the model structure. First, we leave out the carbon cycle. Instead, we use a linear relationship between cumulative emissions since the beginning of the Industrial Revolution, and mean surface temperature deviation over the preindustrial era. Second, we explicitly model energy use and production. Third, we do not explicitly model the backstop technology, instead incorporating it into a parameter measuring the carbon intensity of energy. We calibrate particular time paths for total factor productivity (TFP), for carbon intensity of energy, and for the cost of producing energy. We choose these parameters to match DICE time paths of output (net of extraction costs and damages), of energy use, and of the temperature deviation. Finally, we choose the planner's discount rate so that the social welfare maximisation problem, with exponentially decreasing Pareto weights, yields a carbon tax path close to the optimal path in DICE-2013R. The period length is 25 years.

We use logarithmic utility in our model.<sup>17</sup> The consumers' utility discount rate and the planner's 'generational discount rate' are both calibrated to 2.4%. The calibrated TFP growth rate starts out at 1.8% per year, decreasing to .9% in 2115 and to .4% by 2240. Energy production costs grow at a rate of 1% per year until 2115, after which the growth rate rises to 2.6% per year by 2215. This increase in

<sup>&</sup>lt;sup>15</sup>DICE-2013R has been made available on William Nordhaus's website (http://www.econ.yale.edu/~nordhaus/homepage/Web-DICE-2013-April.htm).

<sup>&</sup>lt;sup>16</sup>The linear relationship accounts for two countervailing effects. The effect of marginal emissions is decreasing with cumulative emissions due to the saturation of the 'infrared window' with higher atmospheric concentrations. However, higher concentrations lead to stronger positive carbon cycle feedbacks due to a weakening of terrestrial carbon sinks, offsetting this saturation (Matthews et al., 2009).

<sup>&</sup>lt;sup>17</sup>Howarth (1996) shows that, with logarithmic utility, a representative agent model can be reinterpreted as an underlying Diamond OLG model's aggregate behaviour, reflecting a particular set of welfare weights.

costs takes place concurrently with a decarbonisation of energy production, with the carbon intensity of primary energy falling by roughly .9% until 2190, after which there is a rapid decarbonisation by 2240.<sup>18</sup>

We run the simulations for 12 periods, or 300 years, including all generations in the social welfare function in the benchmark cases, and considering a bargain between the first 11 generations in our normative approach. However, we report the results only for the first nine periods. Full exogenous decarbonisation also occurs in period nine; implicitly, we assume the economy will adjust to the ultimate climate end state over a century, so that after that the marginal damages due climate change fall to zero. This matters for the no-discounting case only; for the market discounting case, increasing the length of the horizon to more than one century has minor effects only.

7.2. **Results.** We illustrate the carbon taxes in our calibration in Figure 3. The dashed black line illustrates carbon taxes from the DICE-2013R optimal run, i.e. our calbration target. The thin orange line displays the taxes in our 'market discounting' benchmark case. DICE yields a carbon tax of \$72/tC in the period starting 2015, while the corresponding run in our model yields a tax of \$111/tC.<sup>19</sup>

The thin blue line illustrates the benchmark in which all generations' utilities are given equal weight in the social welfare function. Carbon taxes are much higher under uniform Pareto weights: starting out at \$1010/tC in 2015, and peaking at close to \$2360/tC in 2115. Note that the social cost of carbon in the benchmarks is calculated as according to (23), in which the Pareto weights do not appear directly. However, the weights have an indirect effect via the lump-sum transfers: these determine capital accumulation, and thus the consumption discount rate.

The intergenerational bargain we propose yields carbon taxes between these benchmark cases, with 2015 carbon taxes starting at \$232/tC, or roughly double the 'market discounting' benchmark, but well below the 'no discounting' case. Thus, our intergenerational bargain allows for more abatement than the 'market discounting' benchmark, as it implicitly places much more weight on the welfare

<sup>&</sup>lt;sup>18</sup>The calibration cannot be perfect due to the differing model structures, as shown in the next section. The deviation between DICE-2013R and our corresponding benchmark run is, however, of second-order importance, as our main interest is in comparing the benchmark runs conducted using our model structure with our normative proposal. In particular, beyond their use in calibration, we will not compare DICE-2013R results with our model runs.

<sup>&</sup>lt;sup>19</sup>We put excess weight on matching the carbon taxes in 2015.

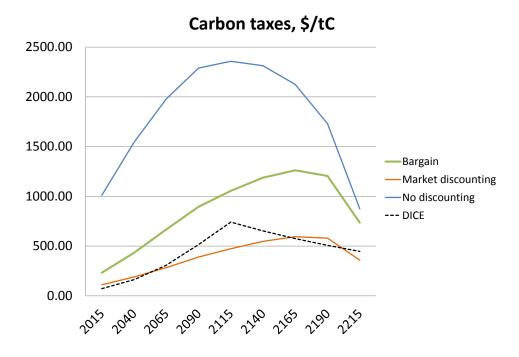


FIGURE 3. Carbon taxes.

of later generations. On the other hand, abatement falls far short of the 'no discounting' benchmark, as the interests of the first generations also count.

The pension transfers associated with the three optima are shown in Figure 4. In our bargain (green bars), transfers from later to earlier generations are used to compensate previous generations for their abatement efforts. The gross transfers are not negligible, peaking at 2% of GDP. However, these transfers are initially small compared to the 'carbon rent' created by climate policy. We illustrate this is Figure 5, showing the pension transfer (from young to old) and the share of the resource rent refunded to the young, as a fraction of the income of the young.<sup>20</sup> For the first century, the compensating transfer is covered by the young generation's share of the tax proceeds (or permit auction revenues). Only after this would the government need to collect a part of the young generation's wage income, with the amount collected always less than 1% of income. These flows of compensation do not seem extraordinary, nor politically unacceptable, in magnitude.

 $<sup>^{20}</sup>$ Resource rents are refunded to the young and the old in the ratio of labour share of GDP to the capital share.

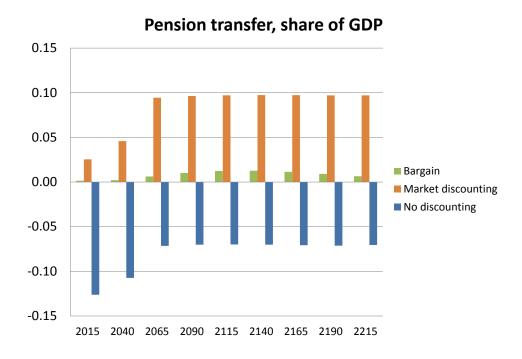
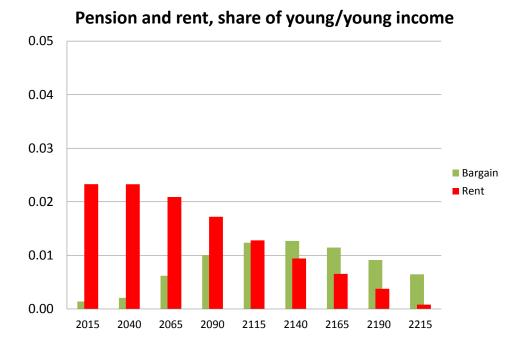


FIGURE 4. Pension transfers.

Observe that the two benchmark cases involve very large lump-sum transfers, with consumption brought forward in the 'market discounting' benchmark (orange bars), and pushed to the future under the 'no discounting' benchmark (blue bars). The gross transfer flows are of the order of 5-10% of GDP. A consistent social planner who has access not only to instruments for controlling emissions, but also for implementing lump-sum transfers between generations, will not restrict herself to just cutting emissions. After all, a planner who puts more weight on the welfare of earlier generations will not only have them abate less, but also transfers income to them. Conversely, a non-discounting planner wants to transfer resources into the future. Explicit modeling of the generational structure makes it clear that the choice of discount rate—the Pareto weights imposed on different generations—affects much more than emission control rates.

Such consumption transfers are clearly not the intended outcome of studies looking at optimal abatement under discounted utility—reflected, for example, in William Nordhaus discussing the early generations reducing their emissions as losers, rather than as the big winners they would be under a social planner using



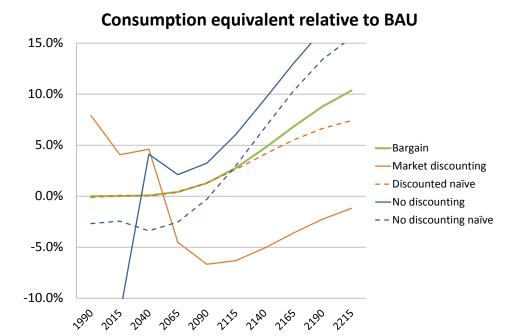
## FIGURE 5. Pensions vis-a-vis rents.

a relatively high discount rate and having access to intergenerational transfers. Thus, to assess welfare gains, we also consider a 'naive' implementation of the benchmark cases, in which only the resulting taxes are implemented, while the associated transfers are set to zero.<sup>21</sup>

The effects on welfare are displayed in Figure 6. We measure welfare by taking BAU welfare as the benchmark, and considering how much each generation's consumption would have to be increased (uniformly over their lifetimes) to achieve the same welfare improvements.

Our intergenerational bargain (thick green line) is, by construction, Pareto-improving, so that the policy yields no generation a negative consumption-equivalent. Most of the welfare improvements are pushed into the far future. To see why, note that the utility levels of the first two generations are constrained to whatever efficiency improvements they could achieve between themselves. Due to the long lifetime of carbon emissions, this is but a small fraction of the total gains from abatement. With each subsequent generation, the surplus achieved by adding a

<sup>&</sup>lt;sup>21</sup>Note that the 'naive' equilibrium is clearly inefficient, and not even a second-best outcome.



## FIGURE 6. Welfare relative to BAU.

new generation always goes to that generation, at the same time increasing the baseline utility level for all future generations. This could be seen as increasing the political sustainability of the bargain. Future generations, who will keep paying sizable transfers long after the original mitigation investments (by then sunk) have been undertaken, could be seen to be less likely to renege on the compensation payments if they have experienced a large benefit due to these investments.

The other solid lines display the welfare gains in the benchmarks. It is apparent that both involve substantial distributional implications, with earlier generations the winners under 'market discounting' (solid orange line), later generations under 'no discounting' (solid blue line). These changes result from the planner using the lump-sum transfers to shift consumption. The outcomes look stark, but they

are the logical conclusion of the social planner having access to intergenerational transfers.<sup>22</sup>

The dashed lines show the respective welfare outcomes if the benchmark taxes are implemented 'naively', without imposing the associated pensions. Welfare effects are more modest, but the first old still lose out, with welfare losses equivalent to a .1% fall in consumption, the first young losing roughly half that amount. The 'no-discounting' benchmark, even naively implemented, leads to much larger welfare losses early on. All the policies considered yield substantial benefits, compared to no mitigation, for generations living in the 22nd century.

The welfare costs we display here are large. Even the losses accrued by the first old under the naive market discounting outcome are not trivial, as the notional fall in consumption lasts for 25 years. Note that a typical non-financial recession involves aggregate income falling by 2%, for one year only (Jordà et al., 2013). It is thus not surprising that climate policies without compensation mechanisms, even ones based on discount rates inferred from market behaviour, generate political opposition. Similarly, the benefits accruing to future generations under all the policies are very large.

Finally, we can back out the generational discounting structure which would replicate our bargaining outcome. The term structure of the discount rates has an inverted-U shape, with the generational discount rate bracketed between 1% and 1.2% (Figure 7).

## 8. Conclusion

Climate change and climate policy have substantial intertemporal equity implications, due to the extremely long horizon over which the costs of climate change unfold. Understanding these implications requires explicit modeling of the generational structure. The majority of models used in climate change economics abstract from this, compressing different generations into a single infinitely-lived agent. This ubiquitous modeling choice is an imperfect tool for assessing questions of intergenerational distribution.

<sup>&</sup>lt;sup>22</sup>The non-monotonicities in the welfare gains for the generation born in 2040 are related to population growth flattening out in this period. Thus, the reinterpretation of results of an infinitely-lived agent model as an outcome of an underlying OLG model is problematic, and may mask large intergenerational transfers, or alternatively reflect suboptimal behaviour by the planner.



FIGURE 7. Generational discounting.

We have shown, in an otherwise standard climate-economy model, that intergenerational transfers allow Pareto-improving climate policies to be crafted. Such Pareto-improving policies require transfers to be paid from later to earlier generations, in compensation for past mitigation. Our calibrated model shows that these transfers, which could be implemented as either government debt or as a system of pension transfers, need not be of prohibitive magnitude. We have shown that other prominent policy proposals are not Pareto-improving, instead featuring winners and losers. A policy which lowers the welfare of the generations responsible for its implementation will be unpopular. With appropriately designed policies, including compensating transfers, this does not present a political obstacle.

We have introduced a normative proposal on how to determine a goal for climate policy which Pareto-dominates the business-as-usual outcome. Much of the recent discussion on discount rates has focused on the appropriate choice of discount rate. We make a contribution to this debate, approaching the question from the opposite angle: which types of policies make all generations weakly better off? Our iterative intergenerational bargain involves carbon taxes substantially higher than

a policy determined from discounted utility maximisation, with Pareto weights decreasing at individuals' pure rate of time preference (as inferred from market transactions). However, the carbon taxes are well below taxes which would follow from a zero-discounting approach.

The intergenerational bargain we propose is meant to demonstrate that climate policy need not involve intergenerational conflict over the *de facto* reference point of the business-as-usual outcome. It should be noted, however, that this baseline could be seen as ethically questionable, were the impacts of unchecked climate change so terrible that future generations' welfare levels would be very low. Our bargain could be amended to account for this by the requirement that reference utilities meet a given minimum standard.

# APPENDIX A. MATHEMATICAL APPENDIX AND DESCRIPTION OF THE NUMERICAL MODEL

## A.1. General equilibrium effects on the interest rate.

**Lemma 2.** Let the savings function be defined as in (3) and assume that the period t+1 equilibrium capital and emissions satisfy conditions (11) and (10). Then the equilibrium changes due to marginal changes in  $M^t$  and  $T^t$  are given by

(42) 
$$K_M^{t+1} = \frac{s_m^t}{1 - s_r^t \Psi} \qquad K_T^{t+1} = \frac{s_r^t \Phi}{1 - s_r^t \Psi}$$

$$(43) E_M^{t+1} = -\frac{F_{EK}^{t+1}}{F_{EE}^{t+1}} K_M^{m+1} E_S^{t+1} = -\frac{F_{ET}^{t+1}}{F_{EE}^{t+1}} - \frac{F_{EK}^{t+1}}{F_{EE}^{t+1}} K_T^{t+1}$$

where

(44) 
$$\Psi = \left(F_{KK}^{t+1} - F_{KE}^{t+1} \frac{F_{EK}^{t+1}}{F_{EE}^{t+1}}\right) L^{t} \qquad \Phi = \left(F_{KT}^{t+1} - F_{KE}^{t+1} \frac{F_{ET}^{t+1}}{F_{EE}^{t+1}}\right) L^{t}$$

Furthermore, if in period t+1 emissions and pensions are set as a matter of policy and capital satisfies (11), then the equilibrium changes to capital due to marginal changes in  $M^t$ ,  $E^{t+1}$ ,  $T^{t+1}$  and  $P^{t+1}$  are given by

$$\begin{split} \hat{K}_{M}^{t+1} &= \frac{s_{m}^{t}}{1 - s_{r}^{t} F_{KK}^{t+1} L^{t}} \\ \hat{K}_{S}^{t+1} &= \frac{s_{r}^{t} F_{KK}^{t+1} L^{t}}{1 - s_{r}^{t} F_{KK}^{t+1} L^{t}} \\ \hat{K}_{E}^{t+1} &= \frac{s_{r}^{t} F_{KK}^{t+1} L^{t}}{1 - s_{r}^{t} F_{KK}^{t+1} L^{t}} \\ \end{split} \qquad \qquad \hat{K}_{P}^{t+1} &= \frac{s_{p}^{t}}{1 - s_{r}^{t} F_{KK}^{t+1} L^{t}} \end{split}$$

*Proof.* The proof is a straightforward application of the implicit function theorem on the equilibrium conditions (11) and (10).

A.2. **Proof of Proposition 2.** Before proving this proposition, we provide a further lemma.

**Lemma 3.** Let the savings function be defined as in (3) and assume the period t+1 equilibrium capital satisfies (11). Assume the period t+1 emissions are fixed at some level, and the policy value function is defined as in (14). The derivatives of the policy value function with respect to  $M^t, T^{t+1}, E^{t+1}$  and  $P^{t+1}$  (normalized by first period marginal utility) are given by

(45) 
$$\hat{V}_M^t L^t = 1 + \frac{\hat{K}_M^{t+1} F_{KK}^{t+1} K^{t+1}}{1 + F_K^{t+1}}$$

(46) 
$$\hat{V}_T^t L^t = \frac{F_{KT}^{t+1} K^{t+1} + \hat{K}_T^{t+1} F_{KK}^{t+1} K^{t+1}}{1 + F_K^{t+1}}$$

(47) 
$$\hat{V}_{E}^{t}L^{t} = \frac{F_{KE}^{t+1}K^{t+1} + \hat{K}_{E}^{t+1}F_{KK}^{t+1}K^{t+1}}{1 + F_{K}^{t+1}}$$

(48) 
$$\hat{V}_{P}^{t}L^{t} = \frac{1 + \hat{K}_{P}^{t+1} F_{KK}^{t+1} K^{t+1}}{1 + F_{K}^{t+1}}$$

As for the corresponding derivatives of the competitive value given in Lemma 1, the second terms in the numerator are due to the equilibrium adjustment of capital, which we calculate explicitly in Lemma 2.

The proof is similar to that of Lemma 1: a straightforward application of the chain rule to the definition of  $\hat{V}^t$ .

Now we can prove Proposition 2.

Proof of Proposition 2. The proof of condition (34) is analogous to that of condition (32), applied to the subsequent period. The only difference is that this is now a foreseen change from the point of view of the savers in  $\mathcal{G}_n$  and will therefore lead to a change in the capital stock. This, in turn, affects the impact on  $\mathcal{G}_{n+1}$  in a way that  $\mathcal{G}_n$  was not affected by the unforeseen changes in  $\{E^n, P^n\}$  that lead to condition (32).

Suppose that a small change  $-dE^{n+1}$  in  $E^{n+1}$  is proposed. This would lead to a change in the utility of individuals in  $\mathcal{G}_n$  of  $\hat{V}_E^n dE^{n+1}$ . In order to compensate

them for this a pension  $dP^{n+1}$  would be required so that

$$d\hat{V}^{n} = -\hat{V}_{E}^{n} dE^{n+1} + \hat{V}_{P}^{n} dP^{n+1} = 0$$

In anticipation of these policies individuals in  $\mathcal{G}_n$  would adjust their savings behaviour resulting a change in the equilibrium capital given by

$$dK^{n+1} = \hat{K}_P^{n+1} dP^{n+1} - \hat{K}_E^{n+1} dE^{n+1}$$

Substituting in (47) and (48) into the indifference condition we get

$$\frac{\mathrm{d}P^{n+1} - F_{KE}^{n+1}K^{n+1}\mathrm{d}E^{n+1} + \left[\hat{K}_{P}^{n+1}\mathrm{d}P^{n+1} - \hat{K}_{E}^{n+1}\mathrm{d}E^{n+1}\right]F_{KK}^{n+1}K^{n+1}}{L^{n}\left[1 + F_{K}^{n+1}\right]} = 0$$

and thus that

(49) 
$$dK^{n+1}F_{KK}^{n+1}K^{n+1} = \left[F_{KE}^{n+1}K^{n+1}dE^{n+1} - dP^{n+1}\right]$$

This gives us an expression for the change in the equilibrium capital stock as a function of the changes in  $E^{n+1}$  and  $P^{n+1}$  when these changes are designed to keep  $\mathcal{G}_n$  indifferent.

The costs (and benefits) of  $\{dE^{n+1}, dP^{n+1}, dK^{n+1}\}$  on  $\mathcal{G}_{n+1}$ , are contained entirely in their effects on  $M^{n+1}$  and  $T^{n+2}$ . Therefore, no more gains to  $\mathcal{G}_{n+1}$  will be possible when  $dV^{n+1} = V_M^{n+1} dM^{n+1} + V_T^{n+1} dT^{n+2} = 0$ .

First, recall that, by the convention (26) on who is assigned the resource rent

$$M^{n+1} = F_L^{n+1} L^{n+1} + \left[ F_E^{n+1} - q^{n+1} \right] E^{n+1} - P^{n+1}$$

Therefore

$$dM^{n+1} = -\left[F_{LE}^{n+1}L^{n+1} + F_{EE}^{n+1}E^{n+1} + F_{E}^{n+1} - q^{n+1}\right]dE^{n+1} - dP^{n+1} + \left[F_{LK}^{n+1}L^{n+1} + F_{EK}^{n+1}E^{n+1}\right]dK^{n+1}$$

Since  $F^{n+1}$  has constant returns to scale in its three factor inputs<sup>23</sup>

$$F_{LK}^{n+1}L^{n+1} + F_{EK}^{n+1}E^{n+1} = -F_{KK}^{n+1}K^{n+1}$$

 $<sup>\</sup>overline{^{23}\text{To see this}}$ , differentiate the Euler identity with respect to  $K^{n+1}$ .

Substituting (49) into  $dM^{n+1}$  we get

$$\begin{split} \mathrm{d} M^{n+1} &= - \left[ F_{LE}^{n+1} L^{n+1} + F_{EE}^{n+1} E^{n+1} + F_{E}^{n+1} - q^{n+1} \right] \mathrm{d} E^{n+1} \\ &- \mathrm{d} P^{n+1} - \left[ F_{KE}^{n+1} K^{n+1} \mathrm{d} E^{n+1} - \mathrm{d} P^{n+1} \right] \end{split}$$

The terms in  $dP^{n+1}$  cancel out, and the terms in second derivatives of the production function add up to zero<sup>24</sup> with the result that

$$dM^{n+1} = -\left[F_E^{n+1} - q^{n+1}\right] dE^{n+1}$$

So we get that, even when the policies are expected and the equilibrium capital is adjusted, an emission reduction policy in which the old get fully compensated costs the young exactly the marginal abatement cost. What has happened here is that, despite the fact that capital accumulation is discouraged by the policies, the net cost of keeping the predecessor indifferent to an emissions reduction (by virtue of a compensatory pension) is simply the marginal cost of abatement assuming fixed capital. The reason for this is in equation (49). Even though there is an additional cost to  $\mathcal{G}_{n+1}$  due to the lowering of the capital stock and the consequent reduction in wages, the compensatory pension must not be as large as the direct cost of emission reductions ( $P^{n+1} < F_{KE}^{n+1} K^{n+1} dE^{n+1}$ ) since the there is an increase in the interest rate ( $F_{KK}^{n+1}$ ) that just offsets this effect.

The policy results in a reduction in period n+2 temperature by  $\mathrm{d}T^{n+2}=-\tilde{T}^{n+2}_{E^{n+1}}\mathrm{d}E^{n+1}$ , and so, since  $\mathrm{d}V^{n+1}=V^{n+1}_M\mathrm{d}M^{n+1}+V^{n+1}_T\mathrm{d}T^{n+2}$  must equal zero at the efficient contract, we get the condition

$$F_E^{n+1} - q^{n+1} = -\tilde{T}_{E^{n+1}}^{n+2} \frac{V_T^{n+1}}{V_M^{n+1}}.$$

This establishes (34).

The proof of condition (33) requires an additional argument, as the internalisation of benefits in periods n+1 as well as n+2 will require a sequence of pensions, with  $\mathcal{G}_{n+1}$  both paying and receiving a pension.

As in Section 4 above, the idea is to consider a marginal change  $-dE^n$  reducing  $E^n$ , and sequentially adding the pensions that will ensure generations are made indifferent, starting with  $\mathcal{G}_{n-1}$ , then  $\mathcal{G}_n$ , and finally requiring that  $\mathcal{G}_{n+1}$  be indifferent to the policies that keep all its predecessors indifferent.

 $<sup>\</sup>overline{^{24}\text{To see this}}$ , differentiate the Euler identity with respect to  $E^{n+1}$ .

We are considering changes in  $E^n, P^n, E^{n+1}$ , and  $P^{n+1}$ . By the conditions in Lemmata 1 and 3,  $\mathcal{G}_{n-1}$  is indifferent when

$$dO^n = -F_{KE}^n K^n dE^n + dP^n = 0$$

 $\mathcal{G}_n$  is indifferent when

$$d\hat{V}^{n} = \hat{V}_{M}^{n} dM^{n} + \hat{V}_{T}^{n} dT^{n+1} + \hat{V}_{P}^{n} dP^{n+1} = 0$$

and  $\mathcal{G}_{n+1}$  is indifferent when

$$dV^{n+1} = V_M^{n+1} dM^{n+1} + V_T^{n+1} dT^{n+2} = 0$$

We already know that the indifference condition for  $\mathcal{G}_{n-1}$  implies that the direct cost (mitigation and compensation) on  $\mathcal{G}$  is  $dM^n = -[F_E^n - q^t] dE^n$ . What we are looking for is an expression for the direct cost  $dM^{n+1}$  on  $\mathcal{G}_{n+1}$  as a function of  $dM^n$ , conditional on the indifference condition on  $\mathcal{G}_n$ . This will have to take into account the resulting endogenous changes in  $K^{n+1}$  and  $T^{n+1}$ , as well as the pension  $P^{n+1}$  paid in order to keep  $\mathcal{G}_n$  indifferent.

Plugging (45) (46) and (48) into  $\mathcal{G}_n$ 's indifference condition, and collecting the terms due to the change in the capital stock we get

$$-\mathrm{d} K^{n+1} F_{KK}^{n+1} K^{n+1} = \left[1 + F_K^{n+1}\right] \mathrm{d} M^n + F_{KT}^{n+1} K^{n+1} \mathrm{d} T^{n+1} + \mathrm{d} P^{n+1}$$

Recalling the convention (26) for  $M^{n+1}$ , we get that

$$\mathrm{d} M^{n+1} = \left[ F_{LT}^{n+1} L^{n+1} + F_{ET}^{n+1} E^{n+1} \right] \mathrm{d} T^{n+1} + \underbrace{\left[ F_{LK}^{n+1} L^{n+1} + F_{EK}^{n+1} E^{n+1} \right]}_{-F_{KK}^{n+1} K^{n+1}} \mathrm{d} K^{n+1} - \mathrm{d} P^{n+1}$$

Replacing the term in  $dK^{n+1}$  with the line above we get

(50) 
$$dM^{n+1} = \underbrace{\left[F_{LT}^{n+1}L^{n+1} + F_{ET}^{n+1}E^{n+1} + F_{KT}^{n+1}K^{n+1}\right]}_{F_T^{n+1}} dT^{n+1} + \left[1 + F_K^{n+1}\right] dM^n$$

Finally, we plug this into the indifference condition of  $\mathcal{G}_{n+1}$  and get

$$V_M^{n+1} \left[ F_T^{n+1} \mathrm{d} T^{n+1} + \left[ 1 + F_K^{n+1} \right] \mathrm{d} M^n \right] + V_T^{n+1} \mathrm{d} T^{n+2} = 0$$

Rearranging yields

$$-dM^{n} = -\frac{dT^{n+1}}{1 + F^{n+1}}F_{T}^{n+1} - \frac{dT^{n+2}}{1 + F_{K}^{n+1}}\frac{V_{T}^{n+1}}{V_{M}^{n+1}}$$

Substituting for  $dM^n = -[F_E^n - q^n]dE^n$  and  $dT^{n+i} = \tilde{T}_{E^n}^{n+i}dE^n$  yields condition (33).

A.3. **Numerical model.** We briefly outline the numerical model here. The period length is set at 25 years.

**Consumers.** Utility is logarithmic. The consumers' annual discount rate is 2.4%. Net output is used in consumption by the young and the old, and in investment by the young.

**Production.** Production is Cobb-Douglas, with constant returns to scale in capital, labour and energy. Only the young generation works. The capital share is .3, and the energy share is .05. Aggregate TFP growth is exogenous and calibrated as described in the main text. The temperature deviation T hits output, with output net of damages being a factor  $1 - .00266375T^2$  of gross output (as in DICE-2013R). From the gross output, we also subtract the real extraction cost of energy, to get net output (which we match to DICE output net of damages).

Climate. The climate model is simple: cumulative carbon emissions (in GtC) translate into temperature deviation linearly, with one GtC causing 1.33 degrees of warming. Energy use causes carbon emissions, but the carbon intensity of energy decreases over time exogenously (calibrated as discussed in the main text).

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