Trade-Induced Industrialization and Economic Growth

by

Jee-Hyeong Park*

Department of Economics

Seoul National University

Seoul, Korea

First Draft: June, 2010

Revised Draft: October, 2010

Abstract

Based on a modified Heckscher-Ohlin model of Deardorff and Park (2010), this paper develops a dynamic model of trade-induced industrialization and economic growth. It shows that a developing country may grow out of its autarky steady state with no industrialization into a new steady state with full industrialization by opening to trade with a large industrialized country, exporting the labor-intensive intermediate input in exchange for the capital-intensive intermediate input for the modern good. Even when the developing country is on its path toward complete industrialization under autarky, free trade may induce it to grow faster with its return to capital being raised and sustained at a level that is higher than its autarky level during its industrialization process. Once it completes its industrialization process by having all of its resources in the modern sector, then diminishing returns to capital come back to accompany further capital accumulation, slowing down the growth of the economy. This trade-induced industrialization and economic growth, having an expansion of international trade both in its absolute value and in its ratio to the size of the developing country, correspond well with the dynamic profiles of East Asian Miracle countries' economic growth based on their export-oriented industrialization strategy.

JEL Classification Code: F11; Q14; Q24; Q41

Keywords: Industrialization; Economic Growth; Export-Oriented Industrialization

---

* Address: Department of Economics, Seoul National University, Seoul, 151-746, Korea; email: j-hpark@snu.ac.kr. For constructive comments, I would like to thank Alan Deardorff, Keun Lee and Keunkwan Ryu and seminar participants at Seoul National University. I am also thankful to the editor and an anonymous referee for their helpful comments. I would like to gratefully acknowledge financial support from the Department of Economics at the Seoul National University with the grant provided by the Seoul National University Foundation.

Trade-Induced Industrialization and Economic Growth

First Draft: June, 2010
Revised Draft: October, 2010

Abstract

Based on a modified Heckscher-Ohlin model of Deardorff and Park (2010), this paper develops a dynamic model of trade-induced industrialization and economic growth. It shows that a developing country may grow out of its autarky steady state with no industrialization into a new steady state with full industrialization by opening to trade with a large industrialized country, exporting the labor-intensive intermediate input in exchange for the capital-intensive intermediate input for the modern good. Even when the developing country is on its path toward complete industrialization under autarky, free trade may induce it to grow faster with its return to capital being raised and sustained at a level that is higher than its autarky level during its industrialization process. Once it completes its industrialization process by having all of its resources in the modern sector, then diminishing returns to capital come back to accompany further capital accumulation, slowing down the growth of the economy. This trade-induced industrialization and economic growth, having an expansion of international trade both in its absolute value and in its ratio to the size of the developing country, correspond well with the dynamic profiles of East Asian Miracle countries' economic growth based on their export-oriented industrialization strategy.

JEL Classification Code: F11; Q14; Q24; Q41

Keywords: Industrialization; Economic Growth; Export-Oriented Industrialization

1. Introduction

The success of export-oriented industrialization by developing countries in East Asia, such as Hong Kong, Taiwan, South Korea, and Singapore in 1960s and 70s, and more recently by China in 1980s and 90s, has put forward such a strategy as a right formula for developing countries' industrialization and economic growth, replacing the old strategy of import-substituting industrialization. While these East Asian countries' rapid growth in exports (and imports) of manufactured goods has accompanied rapid economic growth, there exist skepticisms about the view that attributes the economic success of these countries to "outward-oriented" trade policies. One of main reasons for such skepticisms comes from the fact that these East Asian countries, except Hong Kong, have pursued trade policies that are far from free trade.[1] Another reason is the lack of an economic model that can explain the industrialization and growth experiences of these East Asian countries, placing the "outward-oriented" trade policies as the main cause for their economic success.

Based on a modified Heckscher-Ohlin model of Deardorff and Park (2010), this paper develops a dynamic model of trade-induced industrialization and economic growth. Deardorff and Park modify the standard Heckscher-Ohlin model in two ways. First, a *modern* (manufactured) good requires two intermediate inputs for its production with one intermediate input being more capital intensive than the other and both intermediate inputs being more capital intensive than a *traditional* good. Second, they assume that consumers perceive the modern good as a perfect substitute for the traditional one. This paper imbeds such a static model in a Ramsey growth model in which capital accumulation is endogenously determined by a representative consumer's optimal saving behavior.

The analysis of this paper shows that a developing country may grow out of its autarky steady state with no industrialization (producing only the traditional good) into a new steady state with full industrialization (producing no traditional good) by opening to trade with a large industrialized country. Even when the developing country is on its path toward complete industrialization under autarky, free trade with the industrialized country may induce it to grow faster throughout its industrialization process during which its return to capital is raised compared to its autarky return to capital during industrialization. Under free trade, however, the developing country will end up specializing in the production of the labor-intensive intermediate input for the modern good, exporting it in exchange for importing the capital-intensive intermediate input from the (more) industrialized large country.

This model of trade-induced industrialization and economic growth is useful in explaining the

[1] "The East Asian Miracle: Economic Growth and Public Policy" of World Bank (1993) compares average tariff rates of several groups of developing countries, showing that these East Asian countries have been less protectionist than others. However, they have not employed trade policies that are anything close to free trade with their average rate of protection being 24 percent in the year of 1985. Discussing such data, for example, Krugman and Obstfeld (2006) present the following skepticism: "while there is a correlation between rapid growth in exports and rapid overall economic growth, the correlation does not necessarily demonstrate that free trade policy have been the main reason for the high growth."

experiences of developing countries in East Asia that have pursued an export-oriented industrialization strategy. In addition to sharing the experience of creating a heavily export-oriented manufacturing sector, these developing countries also share the following economic profiles: a rapid economic growth accompanying a rapid expansion of imports of intermediate inputs from developed countries; a high return to capital sustained for an extended period of time; an expansion of international trade, both in its absolute value and in its ratio to the size of the economy, accompanying a rise in the capital-labor ratio.[2] These economic profiles of East Asian Miracle countries correspond well with this model's implication for a developing country's trade-induced industrialization and economic growth.

With regard to the above-mentioned skepticism about attributing the economic success of these countries to their trade policies, the analysis of this paper also generates the following result. If the developing country's tariff on the capital-intensive intermediate input is zero, then the resulting competitive equilibrium will be equivalent to the one under complete free trade regardless of its tariff levels on other goods. Any positive tariff on the modern final good would simply induce the small country to import the capital-intensive intermediate input (instead of importing the final good) so that it can produce the final good by itself, without causing any distortional loss.[3] While East Asian Miracle countries' trade policies were far from free trade during their export-oriented industrialization process, for example, Korea has allowed its producers to have tariff-free access to intermediate inputs that are required for producing modern manufactured goods to promote exporting such products.[4] According to this paper's analysis, such an "outward-oriented" trade policy would generate the effects on the small developing country's industrialization and economic growth that are similar to those under free trade.

There exist numerous studies that link trade and economic growth. Because providing a comprehensive discussion of such studies is beyond the scope of this paper, I focus on discussing a few well-known studies in the context of explaining the East Asian Growth miracle. One of most influential studies on East Asian miracle is the empirical study by Young (1995).[5] The growth accounting exercise done by Young on Hong Kong, Taiwan, South Korea, and Singapore in 1960s, 70s and 80s suggests that the combination of high investment rates and rapidly improving education levels can explain a large fraction, possibly almost all, of the rapid growth in these East Asian

[2] Deardorff and Park (2010) provide a detailed discussion of these characteristics of trade-induced industrialization.

[3] Because there exist more goods than production factors in the static trade model of Deardorff and Park (2010), the trade pattern is indeterminate. This indeterminacy in trade patterns enables the small country to bypath any tariff on the modern final good via importing the capital-intensive intermediate input that is required to produce the final good. See footnote 18 for a more detailed discussion on indeterminacy in trade patterns.

[4] As discussed in Section 3 of Krueger (1979), Korean government provided "tariff exemptions on imports of raw materials and spare parts" and "Tariff and tax exemptions granted to domestic suppliers of exporting firms" as well as other types of export promotion schemes.

[5] As acknowledged by Young (1995), there exist similar studies on these East Asian countries done prior to his study, such as Lau and Kim (1994) and Pyo and Kwon (1991).

countries. According to Young's analysis, the total factor productivity growth has played either a very limited or possibly no role in yielding the rapid growth of these East Asian countries. This implies that the endogenous growth models in which trade leads to an increase in the long-run growth by encouraging R&D activities, such as Romer (1990), Grossman and Helpman (1991), and Rivera-Batiz and Romer (1991), might not be a proper model to explain East Asian growth miracle.

While the much-publicized article of Krugman (1994) cites the empirical study of Young (1995) to emphasize the limitation of rapid economic growth of these East Asian countries that is largely based on factor accumulation, one still needs to explain why such rapid factor accumulation was possible in these economies that have pursed export-oriented industrialization. As potential models of East Asian Miracle, Lucas (1993) suggests models of growth and trade that emphasize learning by doing in which trade allows learning to occur in a large scale, such as Stokey (1988, 1991), Young (1991), and Matsuyama (1991). In contrast with these models that emphasize the scale effect associated with international trade, this paper assumes no such scale effect in explaining the relationship between a developing country's rapid growth and its trade.

As a potential explanation for the East Asian Growth miracle, Ventura (1997) emphasizes that international trade may enable small developing countries to sustain their economic growth without experiencing diminishing returns to capital as small countries do not affect product prices in the world market. In contrast with Ventura's model in which a small country can avoid diminishing returns to capital indefinitely, the return to capital is fixed only during the industrialization process in the model of this paper. As a result, the growth of a small developing country jump-started by international trade will slow down once it completes its industrialization, a feature that does not rise in the model of Ventura.[6]

There is another feature of the model of this paper that distinguishes it from Ventura's and other growth models based on a standard Heckscher-Ohlin trade model. Note that the growth of a small developing country that is faster than the world average requires its time preference to be more saving-oriented than the rest of the world in Ventura's model. On the contrary, free trade can cause a rapid growth of a small developing country even when its time preference is less saving-oriented than the rest of the world in this paper's model. As shown by Bajona and Kehoe (2008) and Song (2008), standard dynamic Heckscher-Ohlin models with an identical-time preference across countries predict that free trade is likely to be detrimental to the growth of a developing economy. In a standard

[6] A slowdown in the growth rate after 20-30 years of rapid industrialization has been a common phenomenon in East Asian countries. As a potential explanation for such slowdowns, Acemoglu and Ventura (2002) emphasize that specialization and trade introduce de facto diminishing returns for countries that accumulate capital faster than average as they experience declining prices of export products. In contrast with their model, the diminishing returns occur in this paper's model in the absence of declining prices of export products: once all the resources are moved away from the traditional sector into the production of the labor-intensive-intermediate input production for the modern good under free trade, the usual diminishing returns in production technology come into play without any change in the price of the intermediate input.

Heckscher-Ohlin model, the return to capital in a labor-abundant developing country would fall as a result of opening to trade with a large capital-abundant developed country, discouraging the developing country's incentive to save as a result. In the modified Hecksher-Ohlin model of this paper, the developing country's return to capital during its industrialization process is higher under free trade than under autarky, which in turn may induce it to grow faster throughout its industrialization process under free trade than under autarky.

The rest of the paper proceeds as follows. Section 2 explains the basic setup of the model, characterizing the industrialization and growth under autarky. After describing the competitive equilibrium under free trade with fixed factor endowments, Section 3 demonstrates how opening to trade with a large industrialized country can induce the industrialization and economic growth of a developing country. Section 4 then concludes with a brief discussion of possible extensions of the model.

2. Industrialization and Growth under Autarky

This section develops a dynamic model of industrialization and economic growth based on the modified Heckscher-Ohlin model of Deardorff and Park (2010), focusing on the equilibrium under autarky. It demonstrates that two types of autarky steady states emerge from such a dynamic model: a non-industrialized steady state and a completely-industrialized one.

2.1. The Basic Setup

Following Deardorff and Park (2010), consider an economy in which there are two types of industries, a traditional industry and a modern one. The traditional industry produces good X and the modern one produces good Y. Good X is more labor-intensive than good Y. Good Y is assembled costlessly from two intermediate inputs, M and N, which differ in capital intensity and are both more capital-intensive than good X. Production technologies are Cobb-Douglas for all industries, for simplicity:

$$
\begin{aligned}
X &= F_x(K_x, L_y) = A_x K_x^{\alpha_x} L_y^{1-\alpha_x}, \\
Y &= F_y(M, N) = A_y' M^{\nu} N^{1-\nu}, \\
M &= F_m(K_m, L_m) = A_m K_m^{\alpha_m} L_m^{1-\alpha_m}, \text{ and} \\
N &= F_n(K_n, L_n) = A_n K_n^{\alpha_n} L_n^{1-\alpha_n},
\end{aligned}
\tag{1}
$$

where K_i and L_i respectively denote the amount of capital and the amount of labor employed in industry *i*, with $1 > \alpha_n > \alpha_m > \alpha_x > 0$ and $\nu \in (0, 1)$.[7] Thus, the capital intensity in production is in

[7] The results of this paper continue to hold under a wide range of production technologies, such as any production function that is homogenous of degree 1, as long as the production functions have the same factor-intensity ordering as those in (1) without any factor intensity reversal. The use of Cobb-Douglas functions is mainly for expositional simplicity and the numerical analysis in the following section.

the order of input N, good Y, input M, and good X. Denote the wage rate for labor and rental rate for capital by w and r, respectively. Let p_M and p_N represent prices of two intermediate inputs, M and N.

Capital stocks (K) depreciate at the constant rate $\delta \in [0, 1]$ and the supply of labor (L) increases at the constant rate n, $L(t) = e^{nt}$ with t denoting time.[8] $k(t) \equiv K(t)/L(t)$, denoting the capital per capita. Capital and labor are homogenous and perfectly mobile across industries but not mobile across countries.

Consumers perceive good X and good Y as perfect substitutes, as assumed by Deardorff and Park (2010).[9] With regard to consumer's utility maximization problem, thus I can simply focus on the total consumption of good X and good Y. $C(t)$ denotes the total consumption in time t, having $c(t) \equiv C(t)/L(t)$ represent the consumption per capita. A representative consumer maximizes overall utility, U, as given by

(2) $$U = \int_0^\infty \{[c(t)^{(1-\theta)} - 1]/(1-\theta)\} e^{-(\rho-n)t} dt,$$

where θ (> 0) is the elasticity of marginal utility and ρ is the time preference. The intertemporal elasticity of substitution for this utility function is constant, denoted by $\sigma = 1/\theta$. A representative consumer can use its final good (good X or good Y) either for its consumption or for its investment. Taking the final good as a numeraire (i.e., setting the price of good X and good Y to be 1), the budget constraint of a representative consumer is

(3) $$c(t) + \dot{k}(t) + nk(t) = [r(t) - \delta]k(t) + w(t),$$

where $\dot{k} \equiv dk/dt$, with a dot over a variable denoting differentiation with respect to time. A representative consumer's total expenditure (consumption plus investment) on the left side of the equality in (3) equals its total income (the sum of capital and labor income) on the right side of the equality in (3).

The competitive equilibrium of this economy consists of a sequence of prices and quantities that result from optimization behaviors and market clearings at any point in time. As in a typical neoclassical model, the competitive market will generate an equilibrium at time t in which the value of its final good output is maximized given its resource and technology constraints. For any given level of capital per capita, k, thus prices of inputs and outputs are determined in the same way as in the static model developed by Deardorff and Park (2010). Given these prices, a representative consumer supplies labor and capital inelastically and chooses the path for c and k that maximizes (2)

[8] Thus, we normalize the labor supply at time 0 at unity.

[9] One may model good Y of the modern sector to be a superior substitute for the traditional good X, consumers having the instantaneous utility function at time t, $u(x_t, y_t) = x_t + \lambda y_t$ with $\lambda > 1$ and lower case letters representing the amounts of consumption of the corresponding goods at time t. Assuming $\lambda > 1$ (or even $\lambda < 1$) instead of $\lambda = 1$, however, does not affect the qualitative results of the following analysis. For simplicity of exposition, I assume that $\lambda = 1$.

subject to (3). The first-order conditions for this optimization problem yields:

(4) $r - \delta = \rho + \theta(\dot{c}/c)$,

(5) $\dot{k} = (r - \delta - n)k + w - c$, and

(6) $\lim_{t\to\infty} k \exp\left[-\int_0^t [r(s) - \delta - n] ds \right] = 0$,

where time variable t is omitted for simplicity. Equation (4) states that the benefit of saving equals the cost of forgone consumption, which consists of a time preference term plus a correction factor that depends on the steepness of the consumption path multiplied by the elasticity of the marginal utility (i.e. the inverse of the intertemporal elasticity of substation). Equation (5) is a restatement of the budget constraint, and Equation (6) is the transversality condition.

2.2. Two Types of Steady States under Autarky

Prior to analyzing the effect of trade on growth and industrialization, this subsection describes what happens under autarky. While production of good Y requires intermediate inputs M and N, it is possible to represent the production technology for good Y simply as a function of capital and labor. More specifically, I can obtain the following result:

Lemma 1.

When firms are subject to identical factor prices, it is possible to represent the production technology of good Y as a Cobb-Douglas function of L and K. Given technologies specified in (1), the production function of good Y takes the following form:

(7)
$$Y = F_y(K_y, L_y) = A_y K_y^{\alpha_y} L_y^{1-\alpha_y},$$
$$\alpha_y = \nu\alpha_m + (1-\nu)\alpha_n, \text{ and}$$
$$A_y = A_y^{\prime} A_m^{\nu} A_n^{1-\nu} \left(\frac{1-\alpha_m}{\alpha_m}\right)^{(1-\alpha_m)\nu} \left(\frac{1-\alpha_n}{\alpha_n}\right)^{(1-\alpha_n)(1-\nu)} \left(\frac{\alpha_n(1-\nu)}{\alpha_m \nu}\right)^{(1-\nu)} \frac{\alpha_m \nu}{1-\alpha_y} \left(\frac{1-\alpha_y}{\alpha_y}\right)^{\alpha_y}.$$

Proof) See Appendix for the Proof

Note that $\alpha_n > \alpha_y > \alpha_m > \alpha_x$, thus the modern good Y is more capital-intensive than the traditional good X.

Because firms are subject to identical factor prices under autarky, *Lemma 1* simplifies the analysis of the autarky economy by transforming the 4-good, 2-factor model into a 2-good (X and Y), 2-factor (L and K) model with Cobb-Douglas production technologies. As mentioned earlier, the competitive market will generate an equilibrium at time t in which the value of its final good output is maximized given its resource and technology constraints. While the resource constraints may change over time by population growth and endogenous choice of investment, L and K are fixed at a specific time.

For any endowment combination of (L, K) at time t, thus the Lerner Diagram in Figure 1 can describe the equilibrium production and factor prices that maximize the value of its final good output.[10] Figure 1 depicts unit (and hence unit value) isoquants for X and Y in solid black curves. There is a unique common tangent line to the unit-value isoquants for X and Y, creating two tangency points. Denote the capital-labor ratios defined by these tangency points on the X and Y isoquants by k_x and k_y, respectively.

As is usual in the familiar Lerner Diagram, a country with factor endowment such as point E_2, which lies between the rays k_x and k_y , will produce both goods. It will have factor prices $\tilde{w}$ and $\tilde{r}$ given by the (reciprocals of) the intercepts of the tangent line. Because the capital intensity in sector X (Y) will be equal to k_x (k_y) under these factor prices, each sector's factor employment that is compatible with full employment is identified as X_2 and Y_2, respectively for sector X and sector Y, in Figure 1. It will produce more of good Y, and less of good X, the closer is the endowment point to ray k_y.

A country that is endowed with less capital per capita than ray k_x, such as at point E_3, will produce only the more labor-intensive good X. Its factor prices (not shown in Figure 1) will be given by a line tangent to the X isoquant at the capital-labor ratio of its endowment, the reciprocals of the intercepts of the tangent line, $1/w_3$ and $1/r_3$, thus a lower wage than $\tilde{w}$ and a higher rental than $\tilde{r}$. A country that is endowed with more capital per capita than k_y will similarly specialize, this time in good Y.

For any given level of capital per capita, k ($\equiv K/L$) at time t, the following lemma characterizes the equilibrium production and factor prices that maximize the value of its final good output, replicating Proposition 1 of Deardorff and Park (2010) with production technologies being specified by (1) and (7):

Lemma 2. Under autarky

a) if $k \leq k_x$, a country will produce and consume only good X, with $r = \alpha_x A_x k^{\alpha_x - 1}$ and $w = (1-\alpha_x) A_x k^{\alpha_x}$, thus the return to capital (labor) falling (rising) in response to a rise in k;
b) if $k_x < k < k_y$, a country will produce and consume both goods X and Y, with $r = \tilde{r}$ and $w = \tilde{w}$, thus the return to capital (labor) being fixed in response to a rise in k; and
c) if $k \geq k_y$, a country will produce and consume only good Y, with $r = \alpha_y A_y k^{\alpha_y - 1}$ and $w = (1-\alpha_y) A_y k^{\alpha_y}$, thus the return to capital (labor) falling (rising) in response to a rise in k;

[10] The following analysis using the Lerner Diagram in Figure 1 directly comes from Deardorff and Park (2010).

where $k_x \equiv [\alpha_x/(1-\alpha_x)]\tilde{w}/\tilde{r}$,

$k_y \equiv [\alpha_y/(1-\alpha_y)]\tilde{w}/\tilde{r}$,

$$\tilde{r} \equiv \alpha_x A_x \left\{ \frac{\alpha_x A_x}{\alpha_y A_y} \left[\frac{\alpha_x(1-\alpha_y)}{\alpha_y(1-\alpha_x)} \right]^{\alpha_y - 1} \right\}^{\frac{\alpha_x - 1}{\alpha_y - \alpha_x}}$$

$$\tilde{w} \equiv (1-\alpha_x) A_x \left\{ \frac{\alpha_x A_x}{\alpha_y A_y} \left[\frac{\alpha_x(1-\alpha_y)}{\alpha_y(1-\alpha_x)} \right]^{\alpha_y - 1} \right\}^{\frac{\alpha_x}{\alpha_y - \alpha_x}},$$

Proof) See Appendix for the Proof

Having the equilibrium production and factor prices characterized as in *Lemma 2* for any given *k* at time *t*, a representative consumer chooses the path for *c* and *k* that maximizes its overall utility, *U* in (2) subject to its budget constraint in (3). *Lemma 2* implies that the income per capita, *rk* + *w*, or equivalently, the (final) output per capita under autarky,[11] denoted by $f_A(k)$, takes the following form:

(8)
$$\begin{aligned} f_A(k) &= F_x(K,L)/L = A_x k^{\alpha_x} \text{ if } k \le k_x, \\ &= \tilde{r}k + \tilde{w} \text{ if } k_x < k < k_y, \text{ and} \\ &= F_y(K,L)/L = A_y k^{\alpha_y} \text{ if } k \ge k_y. \end{aligned}$$

Using $f_A(k)$ defined in (8), the first order conditions of the dynamic optimization, (4), (5), and (6), can be rewritten into

(9) $$\dot{c}/c = [f_A'(k) - \delta - \rho]/\theta,$$

(10) $$\dot{k} = f_A(k) - c - (\delta + n)k, \text{ and}$$

(11) $$\lim_{t\to\infty} k \exp\left[-\int_0^t [f_A'(k) - \delta - n]ds\right] = 0,$$

respectively, where $f_A'(k) \equiv \partial f_A(k)/\partial k$.

Based on these first order conditions, the following analysis using phase diagrams in Figure 2a, 2b, and 2c demonstrates that only two types of steady states will rise as a result of economic growth under autarky: a steady state with *no industrialization* (i.e. *X* > 0 and *Y* = 0) and a steady state with *complete industrialization* (i.e. *X* = 0 and *Y* > 0). Figure 2a shows the phase diagram when the consumer's valuation of current consumption relative to future consumption (ρ) is higher than $\tilde{r} - \delta$. In Figure 2a, the combinations of (*k*, *c*) that satisfy $\dot{c} = 0$ is a vertical line located at k_A^S. If $\dot{c} = 0$, (9) implies $f_A'(k) - \delta = \rho$. Because $f_A'(k) - \delta$ strictly decreases in $k \in [0, k_x]$, reaching $\tilde{r} - \delta$ at k_x, the unique value of *k* that satisfies $f_A'(k) - \delta = \rho\ (> \tilde{r} - \delta)$, denoted by k_A^S in Figure 2a, is strictly less than k_x. The combinations of (*k*, *c*) that satisfy $\dot{k} = 0$ is a concave curve that is

[11] Recall that the prices of good X and good Y are set to be 1, thus *rk* + *w* is the real income per capita.

divided into three distinctive ranges: it is strictly concave both in the range of no industrialization ($k \leq k_x$) and in the range of complete industrialization ($k \geq k_y$), but it has a linear slope over the range of *partial industrialization* ($k_x < k < k_y$ with X > 0 and Y > 0). If $\dot{k} = 0$, (10) implies $c = f_A(k) - (\delta + n)k$, and the slope of $\dot{k} = 0$ shown in Figure 2a is a direct result of the concavity of $f_A(k)$ defined in (8).

Because $\dot{c} = 0$ and $\dot{k} = 0$ must hold in a steady state,[12] the intersection of these two curves in Figure 2a, identifies the steady state under autarky, denoted by E_A^S. If $\rho > \tilde{r} - \delta$, the competitive equilibrium under autarky will always entail a steady state with no industrialization. The preceding discussion on $\dot{c} = 0$ curve has already proven this by showing that the unique value of k that satisfies $\dot{c} = 0$ and (9), denoted by k_A^S in Figure 2a, is strictly less than k_x. In addition, a standard argument can be applied to establish that the system will exhibit saddle-path stability.[13] If the initial endowment is k_0 in Figure 2a, for example, the competitive economy will choose its initial consumption level at c_0, then follow the stable saddle path, denoted by $c(k)$, toward the steady state, E_A^S.

For the case in which $\rho < \tilde{r} - \delta$, Figure 2b indentifies both the steady state under autarky, E_A^S, and the stable saddle path, $c(k)$, that describes transitional dynamics of the economy toward the steady state. If $\rho < \tilde{r} - \delta$, note that the competitive equilibrium under autarky will always entail a steady state with complete industrialization (i.e. X = 0 and Y > 0). The following argument proves this result. Once again, $\dot{c} = 0$ and (9) require that $f_A'(k) - \delta = \rho$ in a steady state. Because $f_A'(k) - \delta$ strictly decreases in $k \geq k_y$, from $\tilde{r} - \delta$ at $k = k_y$, the unique value of k that satisfies $f_A'(k) - \delta = \rho$ $(< \tilde{r} - \delta)$, denoted by k_A^S in Figure 2b, is strictly greater than k_y.

It remains to analyze what happens if $\rho = \tilde{r} - \delta$. First, note that there exists a range of k values, more specifically $k \in [k_x, k_y]$, with which $\dot{c} = 0$ and (9) are satisfied: for $k \in [k_x, k_y]$, $f_A'(k) - \delta = \rho$ is satisfied as $f_A'(k) - \delta = \rho = \tilde{r} - \delta$. This implies that there exists a range of stead states that may rise under autarky. These steady states are the combinations of (k, c) on $\dot{k} = 0$ curve that lie between k_x and k_y, as shown by $\hat{E}_A^S$ in Figure 2c. Among these steady states, note that only E_A^S is on the stable saddle path, $c(k)$ for $k \leq k_x$. If the initial endowment of the economy is less than k_x, such as k_0 in Figure 2c, then the economy will move along this stable

---

[12] For example, see section 2.5 of Barro and Sala-i-Martin (1995) for a derivation of this result.

[13] See section 2.6 of Barro and Sala-i-Martin (1995) for such an argument.

saddle path $c(k)$ toward the steady state E_A^S, a steady state with no industrialization (i.e. $Y = 0$).

Following Proposition 1 characterizes the steady state and transitional dynamics toward it under autarky, summarizing the above analysis based on phase diagrams in Figure 2:

Proposition 1. Under autarky, if the initial endowment of capital is not enough to allow immediate industrialization with $k_0 \leq k_x$,

a) a country will grow along a stable saddle path toward a steady state, experiencing an increase in the per capita consumption and the return to labor, but a decrease in the return to capital; and,
b) the resulting steady state will entail either no industrialization (i.e. $X > 0$ and $Y = 0$) if $\rho \geq \tilde{r} - \delta$ or complete industrialization (i.e. $X > 0$ and $Y = 0$) if $\rho < \tilde{r} - \delta$.

The characteristics of economic growth described by Proposition 1(a) are very similar to those under a typical neoclassical growth model. This is not surprising because the model of this paper is also a neoclassical growth model with the capital being subject to diminishing returns. Proposition 1(b), however, emphasizes that only two extreme types of steady states will emerge with regard to the degree of industrialization, eliminating the possibility of having a steady state with partial industrialization (i.e. $X > 0$ and $Y > 0$).

Why would this economy not settle in a state with partial industrialization? Once the industrialization process is initiated by enough capital accumulation (i.e. $k > k_x$) with a low enough time preference (i.e. $\rho < \tilde{r} - \delta$), then further accumulation of capital will not induce diminishing returns to capital until the country completes its industrialization process with $k \geq k_y$. This is because an increase in capital will be absorbed by an increase in demand for capital created by a change in the country's industry composition that will accompany the capital accumulation: with the relative price of good Y being fixed, an increase in capital in the country induces its resources to move from the labor-intensive traditional sector to the capital-intensive modern sector, following Rybczynski theorem.[14] Because the return to capital does not fall during this industrialization process, the country will continue to save (thus, grow) to complete the industrialization. Once the industrialization process finishes with $k \geq k_y$, then further capital accumulation will be subject to diminishing returns and the country will eventually reach a steady state with complete industrialization.

There are three additional points worth mentioning with regard to Proposition 1(b). First, it is a result that is not likely to rise in a growth model with a fixed saving rate, such as Solow's. In the Solow growth model, the steady state with partial industrialization may arise easily with an

[14] In contrast with original Rybczynski Theorem in which the relative prices are fixed due to the small open economy assumption, note that the relative price of good Y (in term of X) is fixed because X and Y are perfect substitutes in this paper.

intermediate (i.e. neither too low nor too high) saving rate. Second, it is a dynamic phenomenon that results from dynamic optimization: recall that the static model of Deardorff and Park (2010) does not preclude the possibility of partial industrialization (i.e. $X > 0$ and $Y > 0$). Finally, Proposition 1(b) implies that two countries with a very small difference in their time preferences may end up reaching two distinct steady states that are very different in their income per capita. For example, consider the case in which one country's time preference is equal to $\tilde{r} - \delta - \varepsilon$ and the other country's is equal to $\tilde{r} - \delta + \varepsilon$, with $\varepsilon > 0$. Then, the latter country will reach a steady state with no industrialization and the former one will reach a steady state with complete industrialization regardless of how small the gap that these countries have for their time preferences, 2ε. The following Corollary 1 to Proposition 1 summarizes this last point:

Corollary 1. Even when countries have time preferences that are almost identical, such countries may end up reaching steady states that are very different from each other under autarky: a totally unindustrialized one and a completely industrialized one, with a non-negligible (quite possibly significant) per capita income gap across such steady states.

Due to diminishing returns to capital, a typical neoclassical growth model requires a large cross-county difference in time preferences to explain a large cross-country income differences in steady states. In contrast, the model of this paper does not require a large difference in time preferences in explaining such income gaps. The following section analyzes the effect of free trade on the economic growth as well as checking the robustness of Corollary 1 against opening up free trade.

3. Trade-induced Industrialization and Economic Growth

Under free trade, the competitive equilibrium of the world economy consists of a sequence of prices and quantities that result from optimization behaviors and market clearings at any point in time. Then, the competitive market under free trade will generate an equilibrium at time *t* in which the value of each country's final good output is maximized given its resource and technology constraints. For any given factor endowments of the world economy, thus prices of inputs and outputs are determined in the same way as in the static model developed by Deardorff and Park (2010). Because a representative consumer in each country will solve its dynamic optimization problem with the competitive equilibrium at any time *t* being determined in the same way as in the static model, the following subsection first describes the free trade equilibrium with fixed factor endowments.

3.1. Competitive Equilibrium under Free Trade with Fixed Factor Endowments

Following Deardorff and Park (2010), consider a free-trade world in which a large capital-abundant country with $k > k_y$ trades with a small country. The prices of goods prevailing under free trade will be identical to the large country's autarky prices, with the exception of good X that is not produced in the large country. This is because a small country is too small to affect such prices by assumption. Assume that the large country is in its steady state with its steady-state capital-labor ratio, denoted by k^*, being greater than k_y, as shown by E^* in Figure 3. As discussed in the previous section, then the large country will produce only good Y, and its factor prices are given by a line tangent to the Y isoquant at the capital-labor ratio of its endowment, the reciprocals of the intercepts of the tangent line, $r_N^/$ and $w_N^/$. Note that the large country's prices of M and N, denoted by p_M^* and p_N^*, should be determined so that unit-value isoquants for M and N are tangent to this unit-value isocost line of the large country, to satisfy zero profit conditions of the sectors producing M and N. In Figure 3, the large country's unit-value isoquants for M and N, gray curves denoted by $M = 1/p_M^*$ and $N = 1/p_N^*$, respectively, are indeed drawn to tangent to the isocost line. Under these factor prices, the large country's capital intensities in sector N and M, denoted by $k_N^/$ and $k_M^{//}$, have the expected relative sizes with $k_N^/ > k_M^{//}$ in Figure 3.

Because a small country can exchange M, N, and Y based on the large country's autarky prices for these goods, by employing any combination of labor and capital on the $M = 1/p_M^*$ curve or on the $N = 1/p_N^*$ curve in Figure 3, a small country can obtain 1 unit of Y under free trade with the large country. Thus, the small country's unit-value isoquant for producing Y under free trade is the convex hull of the large country's unit-value isoquants for M and N, as depicted by a black bold curve Y_I^F in Figure 3. Note that Y_I^F is located strictly below the small country's unit-value isoquant under autarky, denoted by the $Y = 1$ curve in Figure 3, except being tangent to the $Y = 1$ curve only at one point on the ray from the origin to E^*.

The comparison of Y_I^F and $Y = 1$ curves in Figure 3 thus reveals that the small country can obtain good Y more easily under free trade than under autarky, as if it has experienced a technological improvement in producing good Y. For example, if the small country were producing both X and Y under autarky, then p_M^*, the small country's free-trade price of M, will be higher than its autarky price of M, denoted by p_M^A. This is because the large country's wage (rental rate) is higher (lower) than the small country's wage (rental rate) under autarky with $w_n^/ > \tilde{w}$ ($r_n^/ < \tilde{r}$) as shown in Figure 3, and M is a labor intensive good relative to good Y. Such a change in the price of M caused by free trade shifts the small country's autarky unit-value isoquant for M, denoted by a dotted curve $M = 1/p_M^A$ in Figure 3, down to its free trade unit-value isoquant for M, enabling the small country to obtain 1 unit of Y by hiring less labor and capital under free trade than under autarky. The following lemma summarizes these observations:

Lemma 3.

Under free trade with a fully-industrialized (i.e. producing only good Y) large country, a small country's unit-value isoquant for good Y, denoted by Y_I^F, is the convex hull of the large country's unit-value isoquants for M and N. Compared with its autarky unit-value isoquant for Y defined by $F_y(K_y, L_y) = 1$, Y_I^F, being defined by the following implicit function of *L* and *K*, changes as if there is a technological improvement in producing good Y:

$$
(12)\quad
\begin{aligned}
&A_{ym}^F K^{\alpha_m} L^{1-\alpha_m} = 1 \text{ if } k \le k_m^{//},\\
&K + [(1-\alpha_m)/\alpha_m]k^* L = K_m^F + [(1-\alpha_m)/\alpha_m]k^* L_m^F \text{ if } k_m^{//} < k < k_n^{/}, \text{ and}\\
&A_{yn}^F K^{\alpha_m} L^{1-\alpha_m} = 1 \text{ if } k \ge k_n^{/},
\end{aligned}
$$

where $A_{yi}^F \equiv A_y \left(\dfrac{1-\alpha_y}{1-\alpha_i}\right)^{(1-\alpha_i)} \left(\dfrac{\alpha_y}{\alpha_i}\right)^{\alpha_i} (k^*)^{\alpha_y - \alpha_i}$, with $i = m$ or n,

$$
\begin{aligned}
&k_m^{//} \equiv k^*[\alpha_m(1-\alpha_y)]/[\alpha_y(1-\alpha_m)],\\
&k_n^{/} \equiv k^*[\alpha_n(1-\alpha_y)]/[\alpha_y(1-\alpha_n)],\\
&K_m^F \equiv (k^*)^{1-\alpha_y}\alpha_m/(A_y\alpha_y), \text{ and}\\
&L_m^F \equiv (k^*)^{-\alpha_y}(1-\alpha_m)/[A_y(1-\alpha_y)],
\end{aligned}
$$

with k and k^* ($>$ k_y) denoting the per capita capital of the small country and the large one, respectively.

Proof) See Appendix for the Proof.

Given the free-trade unit-value isoquant for good Y defined as in Lemma 3, the following discussion using the Lerner Diagram in Figure 4 describes the free trade equilibrium of a small country with any endowment.[15] The Lerner Diagram under free trade in Figure 4, are composed of isocost lines and unit-value isoquants for X and Y, depicted by solid black lines and curves, respectively. Note that there is a unique common tangent line to these unit-value isoquants for X and Y, creating two tangency points. Denote the capital-labor ratios defined by these tangency points on the X and Y isoquants by $k_x^{/}$ and $k_m^{/}$, respectively. Then, I can derive

$$
(13)\quad
\begin{aligned}
&k_x^{/} \equiv \omega\alpha_x/(1-\alpha_x),\\
&k_m^{/} \equiv \omega\alpha_m/(1-\alpha_m), \text{ and}\\
&\omega \equiv \left[\frac{\alpha_x A_x}{\alpha_y A_y}\left(\frac{\alpha_y}{1-\alpha_y}\right)^{(1-\alpha_y)}\left(\frac{\alpha_x}{1-\alpha_x}\right)^{(\alpha_x-1)}\right]^{\frac{1}{\alpha_m-\alpha_x}}\left(\frac{\alpha_y}{(1-\alpha_y)k^*}\right)^{\frac{\alpha_y-\alpha_m}{\alpha_m-\alpha_x}},
\end{aligned}
$$

[15] The following analysis using the Lerner Diagram in Figure 4 is largely a replication of the corresponding static analysis of Deardorff and Park (2010).

given production technologies described in (1) and (7).[16]

A small developing country whose per capita capital is below $k_x^{/}$, such as E_1 in Figure 4, will remain as an autarky economy. In autarky it would produce only the traditional good X, and it continues to do so under free trade, since its factor prices, given by a tangent to the X-isoquant at its factor ratio, make even good M too costly to produce. In principle, it could export good X in exchange for good Y, since the two are perfect substitutes and have the same price in both countries. I exclude such trade because nobody strictly gains from it.[17]

If a small country that has a factor endowment which places it between $k_x^{/}$ and $k_m^{/}$, like E_2 or E_3 in Figure 4, then it will produce both X and M under free trade, exporting M in exchange for importing N from the large country.[18] In this range of factor endowments, the small country's factor prices under free trade are fixed at $w_x^{/}$ and $r_x^{/}$, the reciprocals of the intercepts of the isocost line that is tangent to the unit-value isoquants for X and Y. With the relative prices of good X and M being fixed, this follows Rybczynski theorem in which an increase in capital of the small country induces its resources to move from the labor-intensive X sector to the capital-intensive M sector, preventing the diminishing returns to capital to occur.

If a small country has a factor endowment like E_2 in Figure 4, which places it between $k_x^{/}$ and k_x, then it produces only good X under autarky. But with trade its factor prices make it competitive in producing the intermediate input M, and it therefore reallocates some of its labor and more of its capital to the production of M, a modern-sector production. Thus it begins to "industrialize" as a result of trade. Similar results is obtained for a small country that has somewhat more capital, so that in autarky it did produce at least a little of M, N, and Y. Such a country, with endowment E_3 in Figure 4, will cease production of the most capital-intensive intermediate input N when it opens to trade, reallocate factors from both N and X to M, and export M in exchange for either N or Y. For the small country whose factor endowment is located between $k_x^{/}$ and $k_m^{/}$, note that the factor prices change against labor and in favor of capital with the move to free trade.

If a small country's factor endowment falls between $k_m^{/}$ and $k_m^{//}$, such as E_4 in Figure 4, then it ends up specializing in the production of M. In this range of factor endowments, an increase in

[16] Note that $\omega = w/r$ when the small country produces both M and X, with which cost-minimizing firms will produce M and X, satisfying $k_m^{/} = K_m / L_m = [\alpha_m /(1-\alpha_m)](w/r)$ and $k_x^{/} = K_x / L_x = [\alpha_x /(1-\alpha_x)](w/r)$. See Appendix for derivation of (13)

[17] This could be justified by the introduction of an infinitesimally small iceberg transport cost.

[18] It can also export good Y in exchange for good N because it can combine imported input N with domestically made input M to produce and export good Y. The exact pattern of production and trade is here indeterminate, as is often the case when there exist more goods than factors. Whichever pattern of production and trade that the small country ends up taking, in essence, it obtains the capital-intensive intermediate input N necessary for the production of good Y at the lower cost by producing extra units of good M and exchanging it for good N (or indirectly conducting such an exchange of intermediate inputs in terms of factor content of trade) in the world market.

capital will simply implies a more production of M in a more capital intensive way, resulting in a decrease (increase) in the return to capital (labor).

For a small country whose endowment is more capital abundant than $k_m^{//}$, such as E_5 in Figure 4, so that it would produce the most capital-intensive input N as well as M, then Rybczynski theorem will apply to the country again. With the relative prices of M, N, and Y being fixed, an increase in its capital endowment induces its resources to move from the relatively labor-intensive M sector to the more capital-intensive N sector. Thus, the small country's factor prices will be fixed at the large country's factor prices, $w_n^/$ and $r_n^/$, as long as its factor endowment falls between $k_m^/$ and $k_n^{//}$. If a small country specializes in the production of good N under free trade with its capital abundance being greater than $k_n^/$, then an increase in capital will implies a more production of N in a more capital intensive way, once again resulting in a decrease (increase) in the return to capital (labor).

For any given level of capital per capita, k ($\equiv K/L$) at time t, the following lemma characterizes the equilibrium production and factor prices that maximize the value of a small country's final good output under free trade, summarizing the results discussed above with production technologies being specified by (1) and (7):

Lemma 4. Under free trade with a fully-industrialized large country,

a) if $k \le k_x^/$, a small country will remain as an autarky economy, producing and consuming only good X, with $r = \alpha_x A_x k^{\alpha_x - 1}$ and $w = (1-\alpha_x) A_x k^{\alpha_x}$, thus the return to capital (labor) falling (rising) in response to a rise in k;

b) if $k_x^/ < k < k_m^/$, a small country will produce good X and input M, exporting M in exchange for importing N, with $r = r_x^/ \equiv \alpha_x A_x (k_x^/)^{\alpha_x - 1}$ and $w = w_x^/ \equiv (1-\alpha_x) A_x (k_x^/)^{\alpha_x}$, thus the return to capital (labor) being fixed in response to a rise in k;

c) if $k_m^/ \le k \le k_m^{//}$, a small country will produce only input M, exporting M in exchange for importing N, with $r = \alpha_m A_{ym}^F (k)^{\alpha_m - 1}$ and $w = (1-\alpha_m) A_{ym}^F (k)^{\alpha_m}$, thus the return to capital (labor) falling (rising) in response to a rise in k;

d) if $k_m^{//} < k < k_n^/$, a small country will produce inputs M and N, exporting M in exchange for importing N when $k < k^*$ but exporting N in exchange for importing N when $k > k^*$, with $r = r_n^/ \equiv \alpha_m A_{ym}^F (k_m^{//})^{\alpha_m - 1}$ and $w = w_n^/ \equiv (1-\alpha_m) A_{ym}^F (k_m^{//})^{\alpha_m}$, thus the return to capital (labor) being fixed in response to a rise in k; and

e) if $k \ge k_n^/$, a small country will produce only input N, exporting N in exchange for importing M, with $r = \alpha_n A_{yn}^F k^{\alpha_n - 1}$ and $w = (1-\alpha_n) A_{yn}^F k^{\alpha_n}$, thus the return to capital (labor) falling (rising) in response to a rise in k.

3.2. Industrialization and Economic Growth under Free Trade

Having the equilibrium production and factor prices under free trade characterized as in *Lemma 4* for any given *k* at time *t*, a representative consumer chooses the path for *c* and *k* that maximizes its overall utility, *U* in (2) subject to its budget constraint in (3). *Lemma 4* implies that the income per capita, $rk + w$, or equivalently, the (final) output per capita under free trade, denoted by $f_F(k)$, takes the following form:

$$
\begin{aligned}
(14)\qquad f_F(k) &= A_x k^{\alpha_x} \text{ if } k \le k_x^{/},\\
&= r_x^{/} k + w_x^{/} \text{ if } k_x^{/} < k < k_m^{/},\\
&= A_{ym}^F k^{\alpha_m} \text{ if } k_m^{/} \le k \le k_m^{//},\\
&= r_n^{/} k + w_n^{/} \text{ if } k_m^{//} < k < k_n^{/}, \text{ and}\\
&= A_{yn}^F k^{\alpha_n} \text{ if } k \ge k_n^{/}.
\end{aligned}
$$

It is easy to show that $f_F(k) = f_A(k)$ if $k \le k_x^{/}$ and $f_F(k) > f_A(k)$ if $k > k_x^{/}$, except having $f_F(k) = f_A(k)$ at $k = k^*$. Using $f_F(k)$ defined in (14), the first order conditions of the dynamic optimization, (4), (5), and (6), can be rewritten into

$$(15)\qquad \dot{c}_F / c_F = [\, f_F^{/}(k_F) - \delta - \rho]/\theta ,$$

$$(16)\qquad \dot{k}_F = f_F(k_F) - c_F - (\delta + n)k_F , \text{ and}$$

$$(17)\qquad \lim_{t\to\infty} k_F \exp\left[-\int_0^t [\, f_F^{/}(k_F) - \delta - n]ds\right] = 0 ,$$

respectively, where *F* subscripts denote variables under free trade with $f_F^{/}(k) \equiv \partial f_F(k)/\partial k$.

To analyze the effect of free trade on the industrialization and economic growth of a developing country, this subsection first considers a world in which there exist a large capital abundant country in a steady sate with complete industrialization, such as E_A^S in Figure 2b, and a small labor abundant country in a steady state with no industrialization, such as E_A^S in Figure 2a, prior to engaging in free trade. This subsection will also analyze the case in which countries have identical time preferences, considering a small developing country on its stable saddle path toward complete industrialization under autarky, such as *c*(*k*) with $k \le k_x$ in Figure 2b, prior to trading with a large industrialized country.

3.2.1. The Case of An Autarky Steady State with No Industrialization

As shown in Section 2, a country will reach an autarky steady state with no industrialization if its time preference parameter ρ is equal or higher than $\tilde{r} - \delta$. This subsection thus focuses on the case of a small developing country with $\rho \ge \tilde{r} - \delta$. Based on the first order conditions for the dynamic optimization of a representative consumer of such a small country under free trade, (15), (16), and (17), the following analyses using phase diagrams in Figure 5a and 5b demonstrate that free trade will initiate the small developing country's industrialization and economic growth toward complete industrialization as long as its time preference parameter is not too high.

Figure 5a shows the phase diagrams when the consumer's time preference parameter ρ is lower than $r_x^{/} - \delta$. The autarky steady state of the small country under consideration, E_A^S, is the intersection of $\dot{c} = 0$ and $\dot{k} = 0$, depicted by gray curves in Figure 5a. Opening to trade with a large capital-abundant country will raise the small country's output per capita from $f_A(k)$ to $f_F(k)$ for $k > k_x^{/}$, as discussed earlier. This shifts up its stationary capital curve in Figure 5a from $\dot{k} = 0$ to $\dot{k}_F = 0$ because (16) implies $c_F = f_F(k_F) - (\delta + n)k_F$ with $\dot{k}_F = 0$.

In addition, the stationary consumption line in Figure 5a shifts to the right from $\dot{c} = 0$ to $\dot{c}_F = 0$ under free trade. The combinations of (*k*, *c*) that satisfy $\dot{c}_F = 0$ is a vertical line located at $k_F^S \in [k_m^{/}, k_m^{//})$ in Figure 5a. If $\dot{c}_F = 0$, (15) implies $f_F^{/}(k_F) - \delta = \rho$. Because $f_F^{/}(k_F) - \delta$ is strictly higher than $r_x^{/} - \delta$ for $k_F \in [0, k_m^{/}]$ with $f_F^{/}(k_F) = r_x^{/}$ at $k_F = k_m^{/}$, the value of k_F that satisfies $f_A^{/}(k) - \delta = \rho$ $(< r_x^{/} - \delta)$ should be higher than $k_m^{/}$. Because $f_F^{/}(k_F) - \delta$ is strictly lower than $\tilde{r} - \delta$ for $k_F \geq k_m^{//}$ with $f_F^{/}(k_F) = r_n^{/}$ at $k_F = k_m^{//}$ and $\tilde{r} > r_n^{/}$, the value of k_F that satisfies $f_F^{/}(k) - \delta = \rho$ $(\geq \tilde{r} - \delta)$ should be lower than $k_m^{/}$. Finally, there should exist a unique value of $k_F \in [k_m^{/}, k_m^{//})$ that satisfies $f_F^{/}(k_F) - \delta = \rho$, such as k_F^S in Figure 5a, because $f_F^{/}(k_F) - \delta$ strictly and continuously decreases in k_F from $r_x^{/} - \delta$ at $k_F = k_m^{/}$ to $r_n^{/} - \delta$ at $k_F = k_m^{//}$.

As $\dot{c}_F = 0$ and $\dot{k}_F = 0$ must hold in a steady state under free trade, the intersection of these two curves in Figure 5a, identifies the small country's steady state under free trade, denoted by E_F^S. Once again, a standard argument can be applied to establish that the system will exhibit saddle-path stability. If the initial endowment is k_A^S in Figure 5a, the small country's endowment in its autarky steady state, then a representative consumer will initially reduce its consumption level from c_A^S to c_F^0 when it starts to trade with a large industrialized country, placing the economy on the new stable saddle-path toward the new steady state E_F^S, denoted by $c_F(k_F)$. Therefore, free trade induces the small developing country to grow out of the traditional sector (i.e. producing only the traditional good X) into the modern sector, specializing in the production of good M, the labor-intensive intermediate input for the modern good.

A small developing country, however, will not be able to realize this growth and industrialization as an outcome of trading with a large industrialized country if its time preference parameter is too high with $\rho \geq r_x^{/} - \delta$. While such a developing country will have an access to the same improved output per capita function $f_F(k)$ for $k > k_x^/$ under free trade, its time preference is too high for it to take advantage of such a potential gain from free trade. As shown in Figure 5b, the small country's output per capita indeed shifts up from $f_A(k)$ to $f_F(k)$ for $k > k_x^/$. If $\rho > r_x^/ - \delta$, however, the combinations of (*k*, *c*) that satisfy $\dot{c}_F = 0$ is a vertical line located at k_F^S that is smaller than $k_x^/$. Because $f_F^/(k_F) - \delta$ strictly decreases in $k_F \in [0, k_x^/]$, reaching $r_x^/ - \delta$ at $k_F = k_x^/$, the unique value of *k* that satisfies $f_F^/(k_F) - \delta = \rho\ (> r_x^/ - \delta)$, denoted by k_F^S in Figure 5b, is strictly less than $k_x^/$.

If $\rho = r_x^/ - \delta$, then any $k_F \in [k_x^/, k_m^/]$ will satisfy $f_F^/(k_F) - \delta = \rho$ with $f_F^/(k_F) = r_x^/$ for $k_F \in [k_x^/, k_m^/]$. However, the only free-trade steady state that will be reached starting from the autarky steady state is the one with $k_F = k_x^/$. Because the small country's autarky steady state is also its free trade steady state, it will stay in the same steady state: moving to other free-trade steady states with $k_F > k_x^/$ from this steady state would be costly for it, an action that is not on a stable saddle path toward a steady state. If $\rho \geq r_x^/ - \delta$, thus free trade will not affect the steady state of the small developing country, having $E_F^S = E_A^S$ as shown in Figure 5b. If the small country's relative valuation of its current consumption versus its future consumption is too high with $\rho \geq r_x^/ - \delta$, then the small country would not have an incentive to save to accumulate its capital to the level that is higher than $k_x^/$. Because $k_x^/$ is the critical level of capital endowment above which the small country starts to realize gains from trade with the large industrialized country, free trade is irrelevant for the small developing country with $\rho \geq r_x^/ - \delta$.

The following proposition summarizes the above results based on the phase diagram analysis in Figure 5:

Proposition 2. Assume that there exists a fully-industrialized large country that has reached its steady state with $\rho^* < \tilde{r} - \delta$, having its steady-state per capita capital $k^* > k_y$. For a small developing country in a non-industrialized autarky steady state with $\rho \geq \tilde{r} - \delta$, the following is the dynamic effect of opening to free trade with the industrialized large country:

a) If $\rho < r_x^/ - \delta$, then the small country will start to grow out the autarky steady state with no industrialization into a new steady state with complete industrialization. In the new steady state, it will be specialized in producing input M.
b) If $\rho \geq r_x^/ - \delta$, then the small country will stay in the same autarky steady state with no industrialization, remaining as an autarky economy.

For a small developing country, Proposition 2a establishes that industrialization and economy growth may result from opening to trade with a large industrialized country. When the small country jumps into a stable saddle path toward the new steady state, the return to capital will initially increase to a higher level, and stay high until it completes its industrialization with $X = 0$. This trade-induced rise in the return to capital stimulates capital accumulation, initiating the trade-induced economic growth that can be sustained for some extended period prior to completion of its industrialization process. Once all the resources move out of the traditional sector into the modern sector of producing input M, then the return to capital starts to fall until it reaches the new steady state, accompanying a slowdown in the economic growth. When the small developing country grows along the stable saddle path, it will experience increases in the per capita consumption, the return to labor, and the share of exports and imports to the gross domestic product.

Note that the above description of trade-induced economic growth accords well with the following economic profile of the Asian Tiger economies during the early decades of their export-oriented industrialization discussed in the introduction: a rapid expansion of imports of intermediate inputs from developed countries; a high return to capital sustained for an extended period of time; an expansion of international trade, both in its absolute value and in its ratio to the size of the economy, accompanying a rise in the capital-labor ratio.

Also recall that there exists some skepticism about claiming free or freer trade as a possible cause for the economic growth of the East Asian Miracle countries because of their relatively high tariffs on imports in the early period of their industrialization. While these countries had such tariffs on finals products, it is important to note that these East Asian countries had placed very low tariffs or sometime even import subsidies on intermediate inputs to promote their export-oriented industrialization, as discussed in the introduction. In the model of this paper, the trade policy combination of a high tariff on the final good Y and a zero tariff on the capital intensive intermediate input will be equivalent to free trade. Given the indeterminacy of trade patterns discussed in footnote 18 of Section 3.1, such asymmetric tariffs simply play the role of narrowing down the pattern of production and trade by inducing the developing country to import N (instead of importing Y) in exchange for exporting either M or Y, thus effectively avoiding any tariff being paid.

Proposition 2a also demonstrates a limitation of the economic growth and industrialization that is initiated by trading with an industrialized country: the developing country will not reach the level of development in which it would produce the most capital-intensive intermediate input, N. In the case of Proposition 2a, it is not free trade but a relatively high time preference parameter that keeps the developing country from reaching such a level of industrialization because it would have been stuck in the steady state with no industrialization under autarky. As shown by Proposition 2b, if the small country's time preference parameter is too high, then such a developing country cannot escape from its autarky steady state with no industrialization even after opening to trade with a large industrialized country.

Proposition 2a and 2b together imply that two developing countries with a very small difference in their time preferences may end up reaching two distinct steady states under free trade. For example, consider the case in which one country's time preference is equal to $r_x^l - \delta - \varepsilon$ and the other country's is equal to $r_x^l - \delta + \varepsilon$, with $\varepsilon > 0$. After opening to trade with a large industrialized country, then, the latter country will remain in its autarky steady state with no industrialization but the former one will start its trade-driven industrialization process, eventually reaching a new steady state with complete industrialization regardless of how small the gap that these countries have for their time preferences, $2\varepsilon\,(> 0)$. As a free trade counter-part to Corollary 1 under autarky, the following Corollary 2 to Proposition 2 summarizes this last point:

Corollary 2. Even when developing countries have time preferences that are almost identical, such countries may end up reaching steady states that are very different from each other after opening to trade with a large industrialized country: a totally unindustrialized one and a completely industrialized one, with a non-negligible (quite possibly significant) per capita income gap across such steady states.

3.2.2. The Case of Identical Time Preferences among Countries

In contrast with the case of an autarky steady state with no industrialization in which the developing country has a time preference that is different from the large industrialized country, this subsection analyzes the case in which countries have identical time preferences. With identical preference and technologies, the developing country under autarky will eventually reach the same steady state as the large industrialized country's. Would the small developing country reach the same steady state under free trade? How would opening to trade with the large industrialized country affect the growth path of the developing country? The analysis in this subsection provides answers to these questions.

As shown by gray curves in Figure 6, which replicate the autarky phase diagram in Figure 2b of Section 2, a small developing country with $\rho = \rho^* = r_n^l - \delta < \tilde{r} - \delta$ would be on its stable saddle path, $c(k)$, toward its an autarky steady state with complete industrialization prior to opening to trade with the large industrialize country in its steady state, E_A^S. Consider the case in which such a developing country has not started its industrialization process with $k = k_x^l < k_x$ under autarky, having its consumption level at $c = c_A^0$.

Under free trade with the large industrialized country, the small country's output function changes from $f_A(k)$ to $f_F(k)$ for $k > k_x^l$, shifting its stationary capital curve in Figure 6 from $\dot{k} = 0$ to $\dot{k}_F = 0$, which reflects its gains from international trade. In addition, note that the stationary consumption line in Figure 6 shifts to the left from $\dot{c}=0$ to $\dot{c}_F=0$ under free trade. While $\dot{c}_F=[f_F^{/}(k)-\delta-\rho]c_F/\theta=0$ is satisfied for any $k_F \in [k_m^{//}, k_n^{/}]$ with $f_F^{/}(k_F)=r_n^{/}$ for $k_F \in [k_m^{//}, k_n^{/}]$, the only free-trade steady state that will be reached starting from the autarky saddle path with $k < k_m^{//}$ is the one with $k_F = k_m^{//}$. This is because there exists no stable path that starts with $k < k_m^{//}$ then would reach any other free-trade steady states, the combinations of (k_F, c_F) on the $\dot{k}_F=0$ curve with $k_F \in (k_m^{//}, k_n^{/}]$. If there exists such a saddle path, then it should hit $\dot{c}_F=0$ at a point that is strictly below E_F^S when $k_F = k_m^{//}$, as shown by a dotted path in Figure 6. Once the small country hits such a point below E_F^S, then it will accumulate its capital with its consumption being fixed until k_F reaches $k_n^{/}$. After that it will start to head down toward $c_F=0$ as shown by the dotted path in Figure 6, invalidating such a path as a possible stable saddle path toward any other free-trade steady state.

The above analysis based on phase diagrams in Figure 6 establishes the following proposition 3:

Proposition 3. Assume that a small developing country has a time preference that is identical to the time preference of a large country that has reached its fully-industrialized steady state with $\rho=\rho^* < \tilde{r}-\delta$. Consider the small country that is either in a stage of no-industrialization or in an early stage of industrialization with $k \le k_m^{//}$ under autarky. Then, opening to trade with the large industrialized country leads it to grow into a free-trade steady state in which it stops accumulating its capital at $k = k_m^{//}$, a level that is strictly below its autarky-steady-state level, specializing in the production of the labor-intensive intermediate input M.

In contrast with the results in Section 3.1, Proposition 3 shows that free trade can negatively affect the small developing country's growth potential if it has the time preference that is identical to the large country's. This change in the steady state caused by free trade may seem surprising because the developing country will end up consuming permanently less in its steady sate under free trade than under autarky. However, note that the developing country does not lose from free trade. For example, consider the case in which the small country's *k* is equal to $k_m^{//}$. According to the gray transition path, denote by *c*(*k*) in Figure 6, the small country under autarky will initially choose to consume c_A^t, strictly less than its free-trade steady state consumption level, c_F^S. This thriftiness under autarky enables the small country to accumulate its capital to its autarky-steady-state capital, eventually attaining a higher steady-state consumption level than under free trade. The small country, however, would not choose to go though such thrift period if it had an access to the large country's market so that it could attain its free trade consumption level right away: its discounted payoff from staying at E_F^S will be higher than the discounted payoff obtained under autarky, which involves movement along the transition path, $c(k)$, from $k = k_m^{//}$ to $k = k_A^S$. Free trade allows the small country obtain such a higher level of consumption, c_A^S, by specializing in the production of M and obtaining the final good Y in exchange for exporting M to the more capital-abundant large country.

While free trade negatively affects the small developing country's long-term growth potential by inducing it to have a lower level of capital accumulation in its steady state, it is not clear how the same free trade would affect the small country's short-term growth rate and the speed of industrialization. Stable saddle paths shown in Figure 6 indicate the possibility of the short-term growth rate and the speed of industrialization being raised under free trade, at least initially. According to $c(k)$ and $c_F(k_F)$, if the initial level of capital is $k_x^/$, then free trade will induce it to consume at c_F^0 which is strictly less than its autarky consumption level, c_A^0. This in turn implies that the small country's growth rate is initially higher under free trade than under autarky. In addition, its industrialization will start almost right away under free trade if $k > k_x^/$, but the industrialization will be initiated after a certain period under autarky only after it accumulates enough capital with $k > k_x (> k_x^/)$. These short-term effects of free trade on the economic growth and industrialization, however, are not yet proven to exist because the phase diagram analysis in Figure 6 can only characterize steady states and qualitative nature of stable saddle paths toward them, providing no definite answer for the relative location of such saddle paths.

To analyze the short-term effects of free trade on the economic growth and industrialization, I conduct the following numerical analysis. By setting $\alpha_x = 0.1$, $\alpha_m = 0.25$, $\alpha_n = 0.35$, $A_x = A_m = A_n = 1$, $A_y^/ = 1.96$, $\rho = 0.07$, $\delta = 0.05$, and $n = 0.01$, the numerical analysis generates the phase diagrams in Figure 7a and demonstrates how free trade affects the return to capital and the short-term growth rate over time in Figure 7b and 7c, respectively.[19] As shown in Figure 7a, it is indeed possible to have the case in which free trade induces the small country to consume less with $c_F(k_x^/) < c(k_x^/)$, implying a higher growth rate and faster industrialization when $k = k_x^/$. In fact, $c_F(k) < c(k)$ even for $k < k_x^/$ in which no trade will occur between the countries under free trade. The expectation of realizing a higher return to capital in the future after it accumulates enough capital to start trading with $k > k_x^/$ induces it to save more prior to actual trading being initiated between the countries.

[19] While some of these parameter values, such as $\alpha_y = \nu\alpha_m + (1-\nu)\alpha_n = 0.3$, is from the previous studies on economic growth (for example, see Section 2.6.5 of Barro and Sala-i-Martin, 1995), other parameter values are chosen to ease the view of phase diagrams in Figure 7a. For a wide range of parameter values, however, the qualitative nature of the numerical analysis remain the same as the one shown in Figure 7.

Given that the small country has $k = k_x^l / 1.2$ at time 0, Figure 7b illustrates the change in its return to capital over time under free trade (autarky) by a black (gray) curve. Similarly, Figure 7c shows the change in its growth rate over time. Under autarky, note that the industrialization starts at time 1.4 and completes at time 12, as shown by the gray-colored return to capital being constant during that period in Figure 7b. As illustrated by the free-trade (black-colored) return to capital being constant over the period that starts prior to time 1.4 and ends at time 4.9, free trade not only allows the small country to initiate its industrialization earlier but also significantly shortens the period that takes to complete the industrialization.

Because the return to capital is higher under free trade than under autarky during the industrialization process, free trade induces the small country to save more and grow faster during that process, as shown by the black curve is located above the gray curve prior to its completion of industrialization at time 4.9 in Figure 7c. After time 4.9, the return to capital decreases due to diminishing return to capital in the production of good M and the free-trade growth rate of the small country starts to fall rapidly, eventually to rates that are lower than those that it would have attained under autarky. In summary, the numerical analysis in Figure 7 demonstrates that free trade can speed up the small developing country's industrialization, possibly raising its growth rate at least in an early period of its industrialization.

4. Conclusion

Regarding a mechanism through which that the export-oriented industrialization strategy may promote a developing country's economic growth and industrialization, this paper provides the following explanation. Opening to trade with an industrialized country enables a developing country to obtain the capital-intensive intermediate input required to produce the modern manufactured good more easily (i.e., at a price that is lower than the autarky price) by importing it in exchange for exporting the labor-intensive intermediate input for the modern good. Such trade promotes an expansion of the modern manufacturing sector that is more capital-intensive than the traditional sector, raising the return to capital. This higher return to capital caused by free(r) trade promotes the developing country's capital accumulation by encouraging its savings. Because the relative price of the modern good is fixed due to its perfect substitutability with the traditional one, such capital accumulation stimulated by free(r) trade policy does not incur diminishing returns to capital during the process of industrialization, simply shifting more resources from the labor-intensive traditional sector to the capital-intensive modern sector, following Rybczynski theorem. Such a sustained high return to the capital induces the developing country to complete its industrialization process once it is initiated by opening to trade with the industrialized country, which this paper refers as *trade-induced industrialization and economic growth*.

Once the developing country completes its industrialization by producing only the labor-intensive intermediate input for the modern good, then the usual diminishing returns to capital will accompany further capital accumulation, slowing down the growth of the economy until it reaches its steady state under free(r) trade. Given that the developing country has a time preference that is either identical to or less-saving-oriented than the industrialized country's, it will end up specializing in the production of the labor-intensive intermediate input, having capital per capita that is strictly lower than its industrialized trading partner's in its steady state. Throughout such a transition to a new steady state, the growth of the developing country will accompany an expansion of international trade, both in its absolute value and in its ratio to the size of the developing country.

This trade-induced industrialization and economic growth implied by the model of this paper correspond well with the economic profiles of East Asian Miracle countries during their export-oriented industrialization process described in the introduction. Compared with the existing explanations for East Asian Miracle, which often emphasize some sorts of increasing return to scale, the model here puts the traditional gains of international trade based on comparative advantage as the driving force behind the spectacular economic growth and speedy industrialization of miracle economies. This aspect of the model conforms to the empirical studies stressing that the growth of East Asian Miracle countries is largely driven by their rapid factor accumulation rather than by productivity improvements. It also emphasizes that tariffs on final goods would not affect this trade-driven industrialization and economic growth as long as the tariffs on intermediate inputs for producing manufactured goods are low enough, a common feature of the export-oriented industrialization strategy of East Asian Miracle countries.

While the above discussion suggests the simple dynamic model developed in this paper as a potential model of industrialization and economic growth of East Asian Miracle countries, formally testing it would require further development of the model. For example, even though the return to skilled labor was rising much more rapidly than the return to unskilled labor in the early stage of Korean industrialization as this paper's model would imply, the real return to unskilled labor was rising rather than falling during this period.[20] A possible reason for such a rise in the real wage of unskilled workers during the trade-induced industrialization of Korea is improvement in labor productivity that comes from utilizing imported intermediate inputs or machines that embody technological progress in developed countries. Incorporating such improvement in production

[20] The current model implies that the real return to capital would rise but the real return to labor would fall in a developing country as a result of trade-induced industrialization, with those returns staying at the changed levels throughout its industrialization process. By interpreting human capital embodied in skilled labor as a part of broadly-defined capital, one can draw an implication from the model that the return to skilled labor would increase more rapidly than the return to unskilled labor as a result of trade-induced industrialization. According to the 1976 Occupational Wage Survey of the Ministry of Labor of Korea, which started in 1970, the real wage of unskilled workers in modern manufacturing sector was rising at the annual rate of 5.5% from 1971 to 1976, a much lower rate than the corresponding growth rate for skilled workers and professionals, 11%.

technology of the modern manufacturing sector is a non-trivial extension of the current model when it entails a difference in the speed of technological progress across sectors. Such an extension may also generate the case in which free trade enables a developing country to grow out of its autarky steady state with no-industrialization into a steady state with full industrialization even when countries have an identical discount rate.[21]

The model here also has been a very particular and special example, designed for expositional simplicity and understanding rather than for generality. It remains to be seen what characteristics of a more general model, with more industries, a more general formulation of preferences, and possibly an endogenous human capital formation process, would be needed to generate comparable results. Once such a more general theoretical result could be established, it would then make sense to investigate the validity of the model in explaining East Asian Miracle.

[21] As emphasized by many studies on endogenous growth, such as Grossman and Helpman (1991), the technological progress in an integrated world economy can be faster than the technological progress of a closed economy. If technological progress of the modern manufacturing sector in a developing country is faster under free trade than under autarky as the endogenous growth literature implies, then such an extension of the current model may no longer relapse into the dynamic curse of free trade even with an identical discount rate across countries. The faster technological progress of the modern manufacturing sector under free trade can raise the return to capital beyond the level that is attainable under autarky throughout the industrialization process, generating a stronger incentive for a developing country to industrialize under free trade than under autarky.

References

Acemoglu, D., and Ventura, J., 2002, "The World Income Distribution," *Quarterly Journal of Economics* 117, 659-694.

Bajona, C., and Kehoe, T., 2008, "Trade, Growth, and Convergence in a Dynamic Heckscher-Ohlin Model," Federal Reserve Bank of Minneapolis Research Department Staff Report 378.

Barro, R. and Sala-i-Martin, X., 1995, *Economic Growth*, New York, NY: McGraw-Hill.

Deardorff, A. and Park, J-H., 2010, "A Story of Trade-induced Industrialization," *International Economic Journal*, forthcoming.

Grossman, G. M., and Helpman, E., 1991, *Innovation and Growth in the Global Economy*, Cambridge, Mass.: MIT Press.

Krueger, A., 1979, *The Developmental Role of the Foreign Sector and Aid, Studies in the Modernization of the Republic of Korea: 1945-1975*, Harvard East Asian Monograph, Harvard University Press.

Krugman, P. R., 1994, "The Myth of Asia's Miracle," *Foreign Affairs* 73 (6), 62-78.

Krugman P. R, and Obstfeld, M., 2006, *International Economics: Theory and Policy*, 7th Edition, Boston: Addison-Wesley.

Lau, L. J., and Kim, J-I., 1994, "The Sources of Economic Growth of the East Asian Newly Industrializing Countries," *Journal of the Japanese and International Economies* 8, 235-271

Lucas, R. E., Jr., 1993, "Making a Miracle," *Econometrica* 61 (2), 251-272.

Matsuyama, K., 1991, "Increasing Returns, Industrialization, and Indeterminacy of Equilibrium," *Quarterly Journal of Economics* 106, 617-650.

Pyo, H-K., and Kwon, H-Y., 1991, "Estimation of Real Factor Input and Factor Productivity in the Korean Private Sector," *Korean Economic Journal* 30, 147-94.

Rivera-Batiz, L. A., and Romer, P. M., 1991, "Economic Integration and Endogenous Growth," *Quarterly Journal of Economics* 106, 530-555.

Romer, P. M., 1990, "Endogenous Technological Change," *Journal of Political Economy* 98 (Part I): S71-S102.

Song, E. Y., 2008, "Trade and the Speed of Convergence," mimeo.

Stokey, N. L., 1988, "Learning by Doing and the Introduction of New Goods," *Journal of Political Economy* 96, 701-717.

Stokey, N. L., 1991, "The Volume and Composition of Trade Between Rich and Poor Countries," *Review of Economic Studies* 58, 63-80.

Ventura, J., 1997, "Growth and Interdependence," *Quarterly Journal of Economics* 112 (1), 57-84.

World Bank, 1993, *The East Asian Miracle: Economic Growth and Public Policy*, Oxford: Oxford

University Press.

Young, A., 1991, “Learning by Doing and the Dynamic Effects of International Trade,” *Quarterly Journal of Economics* 106, 369-406.

Young, A., 1995, “The Tyranny of Numbers,” *Quarterly Journal of Economics* 110, 641-680.

Appendix

Proof of Lemma 1

To prove Lemma 1, I use the following steps: (i) deriving p_M / p_N as a function of w/r from solving the cost minimization problems for producing M and N; (ii) deriving the amount of M (N) to produce 1 unit of Y in a cost-minimizing way as a function of w/r, denoted by m^1 (n^1); (iii) establishing that the production function of Y can be represented by a Cobb-Douglas function of the form $Y = F_y(K_y, L_y) = A_y K_y^{\alpha_y} L_y^{1-\alpha_y}$ with $\alpha_y = \nu\alpha_m + (1-\nu)\alpha_n$ by showing that $wL_y / rK_y = (1-\alpha_y)/\alpha_y$ results from solving cost minimizing problems for producing M, N, and Y; (iv) establishing that A_y is a function of α_m, α_n, α_y, ν, $A_y^/$, A_m, and A_n as shown in (7).

(i) Denote the amount of *K* and *L* to produce 1 unit of M (N) in a cost-minimizing way by K_m^1 (K_n^1) and L_m^1 (L_n^1), respectively. Using $A_j (K_j^1)^{\alpha_j} (L_j^1)^{1-\alpha_j} = 1$ and $K_j^1 / L_j^1 = \alpha_j w / [(1-\alpha_j) r]$ from cost minimization for *j*, I can obtain that $K_j^1 = A_j^{-1}\{\alpha_j w / [(1-\alpha_j) r]\}^{1-\alpha_j}$ and $L_j^1 = A_j^{-1}\{\alpha_j w / [(1-\alpha_j) r]\}^{-\alpha_j}$, with *j* = *m* or *n*. Denote the vertical intercept of a unit isocost line for producing M (N) in the space of *L* and *K* by $\overline{K}_m$ ($\overline{K}_m$). From $\overline{K}_j = (w/r)L_j^1 + K_j^1$ for *j* = *m* or *n* , I can obtain that

$$\text{(A1)} \qquad \frac{p_M}{p_N} = \frac{\overline{K}_m}{\overline{K}_n} = \frac{A_n}{A_m}\left(\frac{w}{r}\right)^{\alpha_n - \alpha_m} \frac{\left(\frac{1}{\alpha_m}\right)\left(\frac{\alpha_m}{1-\alpha_m}\right)^{1-\alpha_m}}{\left(\frac{1}{\alpha_n}\right)\left(\frac{\alpha_n}{1-\alpha_n}\right)^{1-\alpha_n}}.$$

(ii) Solving the cost minimization problem for producing 1 unit of Y subject to $A_y^/ (m^1)^\nu (n^1)^{1-\nu} = 1$ yields $m^1 / n^1 = \nu p_N / [(1-\nu) p_M]$, which in turn implies $m^1 = (A_y^/)^{-1}\{[(\nu/(1-\nu)](p_N / p_M)\}^{1-\nu}$ and $n^1 = (A_y^/)^{-1}\{[(\nu/(1-\nu)](p_N / p_M)\}^{-\nu}$. Using the expression for p_M / p_N in (A1) and these expressions for m^1 and n^1, I can easily obtain m^1 and n^1 as a function of w/r.

(iii) Denote the amount of *K* and *L* to produce 1 unit of Y in a cost-minimizing way by K_y^1 and L_y^1, respectively. Then,

$$\text{(A2)} \qquad K_y^1 = m^1 K_m^1 + n^1 K_n^1 \text{ and } L_y^1 = m^1 L_m^1 + n^1 L_n^1,$$

where m^1, n^1, K_m^1, K_n^1, L_m^1, and L_n^1 are all functions of w/r as derived in (i) and (ii), which in turn yields K_y^1 and L_y^1 in (A2) as a function of w/r. Denote these K_y^1 and L_y^1 obtained in (A2) by $K_y^1(w/r)$ and $L_y^1(w/r)$. It is straightforward to show that $wL_y^1(w/r)/rK_y^1(w/r) = (1-\alpha_y)/\alpha_y$, establishing that the production function of Y can be represented as a Cobb-Douglas function of a following form, $Y = F_y(K_y, L_y) = A_y K_y^{\alpha_y} L_y^{1-\alpha_y}$

(iv) Solving the cost minimization problem for producing 1 unit of Y subject to $A_y (K_y^1)^{\alpha_y} (L_y^1)^{1-\alpha_y} = 1$ yields $K_y^1 / L_y^1 = \alpha_y w/[(1-\alpha_y)r]$, which in turn implies that $K_y^1 = (A_y)^{-1}\{[(\alpha_y/(1-\alpha_y)](w/r)\}^{1-\alpha_y}$. From $K_y^1 = K_y^1(w/r)$,

$$A_y = \{[(\alpha_y/(1-\alpha_y)](w/r)\}^{1-\alpha_y} / K_y^1(w/r)$$

(A3)
$$= A_y^{/} A_m^{\nu} A_n^{1-\nu} \left(\frac{1-\alpha_m}{\alpha_m}\right)^{(1-\alpha_m)\nu} \left(\frac{1-\alpha_n}{\alpha_n}\right)^{(1-\alpha_n)(1-\nu)} \left(\frac{\alpha_n(1-\nu)}{\alpha_m \nu}\right)^{(1-\nu)} \frac{\alpha_m \nu}{1-\alpha_y} \left(\frac{1-\alpha_y}{\alpha_y}\right)^{\alpha_y}.$$

Proof of Lemma 2

(a) If $k \le k_x$, all the resources are employed in producing X. Because good X is a numeraire good, this implies $r = \partial F_x / \partial K = \alpha_x A_x k^{\alpha_x - 1}$ and $w = \partial F_x / \partial L = (1-\alpha_x) A_x k^{\alpha_x}$.

(c) If $k > k_y$, all the resources are employed in producing Y, having $r = \partial F_y / \partial K = \alpha_y A_y k^{\alpha_y - 1}$ and $w = \partial F_y / \partial L = (1-\alpha_y) A_y k^{\alpha_y}$ with good Y being a numeraire good.

(b) If $k_x < k < k_y$, a country will produce both goods X and Y, implying that $w/r = (K_x/L_x)[(1-\alpha_x)/\alpha_x] = (K_y/L_y)[(1-\alpha_y)/\alpha_y]$ from the cost minimization for producing goods X and Y. Continuity of w and r with respect to changes in k, then implies that $k_x[(1-\alpha_x)/\alpha_x] = k_y[(1-\alpha_y)/\alpha_y]$. Using this equality together with $r = \alpha_x A_x (k_x)^{\alpha_x - 1} = \alpha_y A_y (k_y)^{\alpha_y - 1}$, one can obtain that

(A4)
$$k_x = \left\{ \frac{\alpha_x A_x}{\alpha_y A_y} \left[\frac{\alpha_x (1-\alpha_y)}{\alpha_y (1-\alpha_x)} \right]^{\alpha_y - 1} \right\}^{\frac{1}{\alpha_y - \alpha_x}}, \text{ and}$$
$$k_y = \left\{ \frac{\alpha_x A_x}{\alpha_y A_y} \left[\frac{\alpha_x (1-\alpha_y)}{\alpha_y (1-\alpha_x)} \right]^{\alpha_x - 1} \right\}^{\frac{1}{\alpha_y - \alpha_x}}.$$

Having k_x and k_y as derived in (A4), the continuity of w and r with respect to changes in k implies that

(A5)
$$\tilde{r} = \alpha_x A_x (k_x)^{\alpha_x - 1} = \alpha_x A_x \left\{ \frac{\alpha_x A_x}{\alpha_y A_y} \left[\frac{\alpha_x (1-\alpha_y)}{\alpha_y (1-\alpha_x)} \right]^{\alpha_y - 1} \right\}^{\frac{\alpha_x - 1}{\alpha_y - \alpha_x}}, \text{ and}$$
$$\tilde{w} = (1-\alpha_x) A_x (k_x)^{\alpha_x} = (1-\alpha_x) A_x \left\{ \frac{\alpha_x A_x}{\alpha_y A_y} \left[\frac{\alpha_x (1-\alpha_y)}{\alpha_y (1-\alpha_x)} \right]^{\alpha_y - 1} \right\}^{\frac{\alpha_x}{\alpha_y - \alpha_x}}.$$

Given the expressions in (A4) and (A5), it is straightforward to show that $k_x = [\alpha_x/(1-\alpha_x)]\tilde{w}/\tilde{r}$ and $k_y = [\alpha_y/(1-\alpha_y)]\tilde{w}/\tilde{r}$.

Proof of Lemma 3

Because Y_I^F should be the convex hull of the large country's unit-value isoquants for M and N as explained in the text preceding *Lemma* 3, the following proof focuses on deriving the implicit function specified in (12). Using the same logical steps in deriving p_M / p_N in (A1) of the proof of *Lemma* 1 and the fact that $p_j = p_j^*$ and $p_Y = p_Y^*$ under free trade, one can derive the following expression for p_Y / p_j with *j* = *m* or *n*:

(A6) $$\frac{p_Y}{p_j} = \frac{\overline{K}_Y}{\overline{K}_j} = \frac{A_j}{A_y}\left(\frac{w^*}{r^*}\right)^{\alpha_j-\alpha_y} \frac{\left(\frac{1}{\alpha_y}\right)\left(\frac{\alpha_y}{1-\alpha_y}\right)^{1-\alpha_y}}{\left(\frac{1}{\alpha_j}\right)\left(\frac{\alpha_j}{1-\alpha_j}\right)^{1-\alpha_j}},$$

where $p_Y / p_j = 1/p_j$ because $p_Y = 1$ with Y being a numeraire good, and w^* and r^* represent the large country's wage and rental rates, respectively. Because $k^* > k_y$, $r^* = \partial F_y / \partial K = \alpha_y A_y (k^*)^{\alpha_y - 1}$ and $w^* = \partial F_y / \partial L = (1-\alpha_y) A_y (k^*)^{\alpha_y}$, implying that $w^* / r^* = (1-\alpha_y) k^* / \alpha_y$. Using this equality, one can rewrite $1/p_j$ in (A6) into:

(A7) $$\frac{1}{p_j} = \frac{A_j}{A_y}\left(\frac{\alpha_j}{\alpha_y}\right)^{\alpha_j}\left(\frac{1-\alpha_j}{1-\alpha_y}\right)^{1-\alpha_j}\left(k^*\right)^{\alpha_j-\alpha_y}.$$

As shown in Figure 3, Y_I^F, being the convex hull of the large country's unit-value isoquants for M and N , should satisfy $F_m(L,K) = A_m K^{\alpha_m} L^{\alpha_m - 1} = 1/p_M$ if $k < k_m^{//}$ and $F_n(L,K) = A_n K^{\alpha_n} L^{\alpha_n - 1} = 1/p_N$ if $k > k_n^{/}$. This implies that Y_I^F is the implicit function that satisfies $p_M A_m K^{\alpha_m} L^{\alpha_m - 1} = 1$ if $k < k_m^{//}$ and $p_N A_n K^{\alpha_n} L^{\alpha_n - 1} = 1$ if $k > k_n^{/}$, which in turn implies that $A_{yi}^F = p_i A_i$ with *i* = *m* or *n*, generating the corresponding expressions in (12). If $k_m^{//} \le k \le k_n^{/}$, Y_I^F, being the convex hull of the large country's unit-value isoquants for M and N , should be the line that is tangent to the large country's unit-value isoquants for M and N. This line has a slope that is equal to w^*/r^* and crosses an intersection of $A_{ym}^F K^{\alpha_m} L^{\alpha_m - 1} = 1$ and $K = k_m^{//} L$, as shown in Figure 3. With this intersection point being denoted by (L_m^F, K_m^F), one can easily obtain the expression for this tangent line in (12) for $k_m^{//} \le k \le k_n^{/}$.

Finally, the large country's firms produce both M and N subject to Cobb-Douglas technologies, having $k_m^{//} = K_m / L_m = \alpha_m w^* /[(1-\alpha_m) r^*]$ and $k_n^{/} = K_n / L_n = \alpha_n w^* /[(1-\alpha_n) r^*]$ as a result of their cost minimization, which in turn generate the expressions for $k_m^{//}$ and $k_n^{/}$ in (12).

***Derivation of* (13)**

When the small country produces both X and M, cost-minimizing firms subject to the Cobb-Douglas production technologies in (1) should satisfy the following equalities

(A8) $$\frac{w}{r} = \frac{(1-\alpha_x)}{\alpha_x}\frac{K_x}{L_x} = \frac{(1-\alpha_x)}{\alpha_x}k_x^{/} = \frac{(1-\alpha_m)}{\alpha_m}\frac{K_m}{L_m} = \frac{(1-\alpha_x)}{\alpha_x}k_m^{/}.$$

These inequalities in (A8) imply that $k_x^{/} = [(1-\alpha_m)/\alpha_m][\alpha_x/(1-\alpha_x)]k_m^{/}$. Using this equality together with $r = \partial F_x/\partial K = \alpha_x A_x (k_x^{/})^{\alpha_x - 1} = \partial F_m/\partial K = \alpha_m A_{ym}^F (k_m^{/})^{\alpha_m - 1}$, it is straightforward to obtain:

(A9) $$k_m^{/} = \left[\frac{\alpha_x}{\alpha_y A_{ym}^F}\left(\frac{(1-\alpha_m)\alpha_x}{\alpha_m(1-\alpha_x)}\right)^{(\alpha_x - 1)}\right]^{\frac{1}{\alpha_m - a_x}}.$$

Using (A9) and A_{ym}^F defined in (12), one can derive the small country's free-trade factor price ratio when it produces both good X and M, having $w/r = [(1-\alpha_x)/\alpha_x]k_m^{/} = \omega$. According to (A8), then $k_m^{/} = [\alpha_m/(1-\alpha_m)]\omega$ and $k_x^{/} = [\alpha_x/(1-\alpha_x)]\omega$.

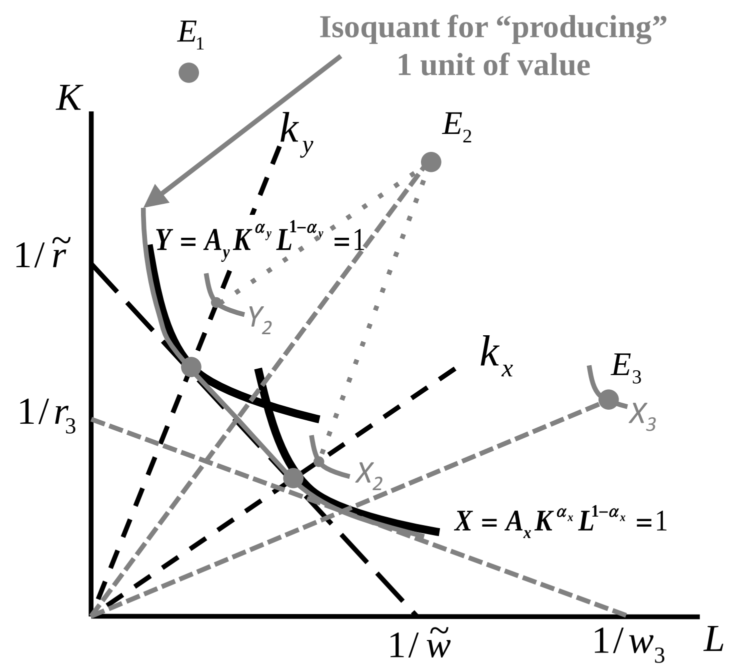

Figure 1. Autarky Equilibria, Lerner Diagram

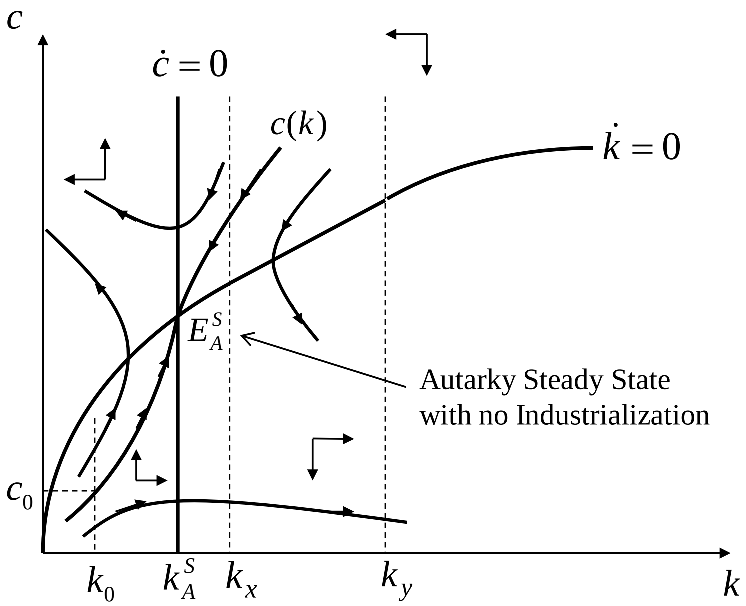

Figure 2a. Phase Diagram under Autarky with $\rho > \tilde{r} - \delta$

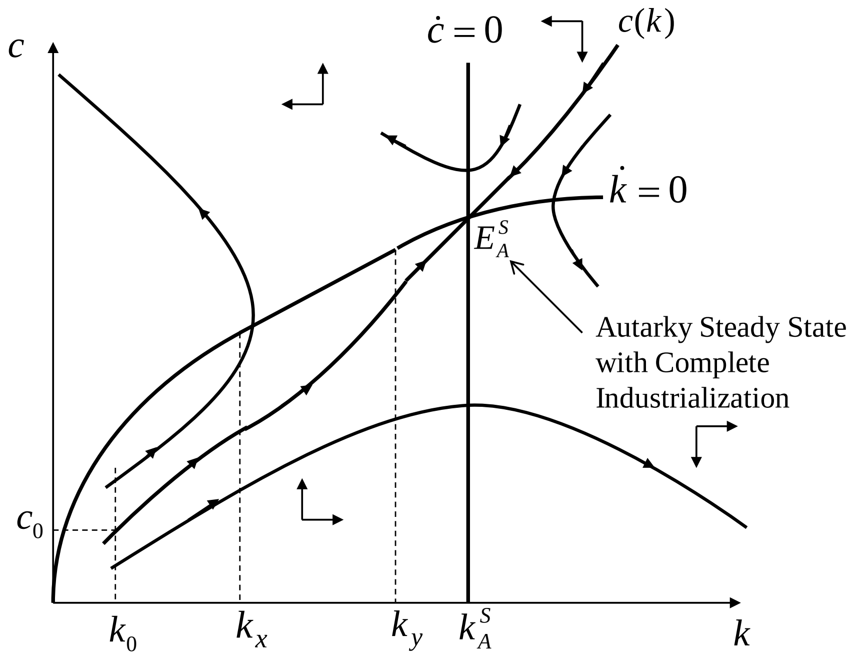

Figure 2b. Phase Diagram under Autarky with $\rho < \tilde{r} - \delta$

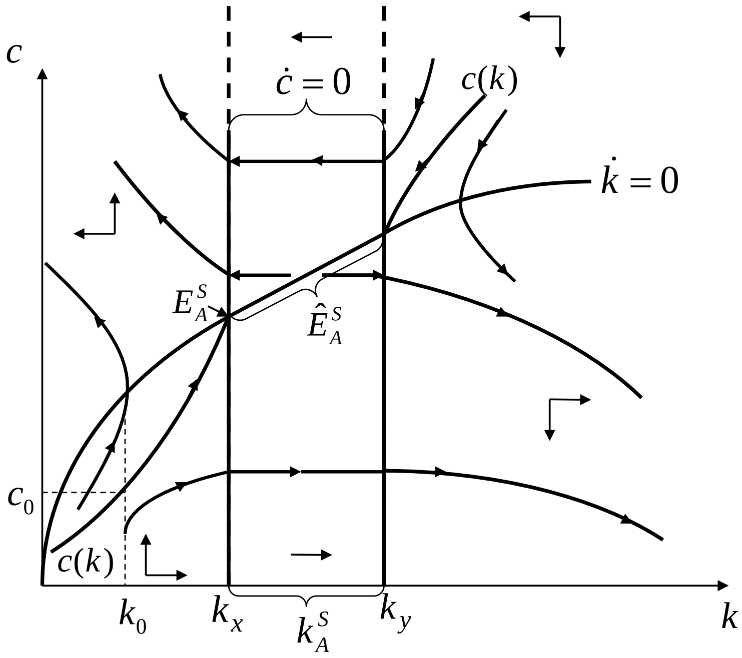

Figure 2c. Phase Diagram under Autarky with $\rho = \tilde{r} - \delta$

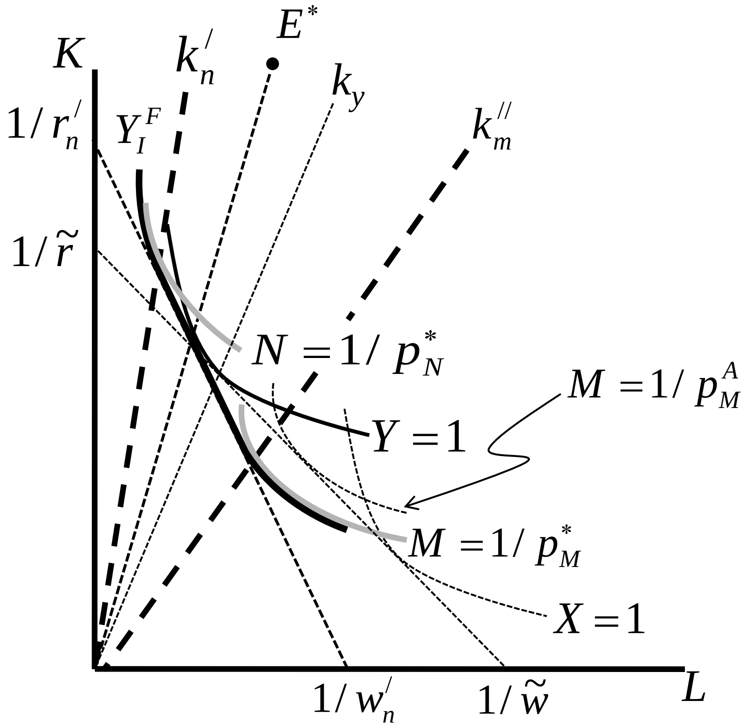

Figure 3. Unit-Value Isoquant for Y under Free Trade

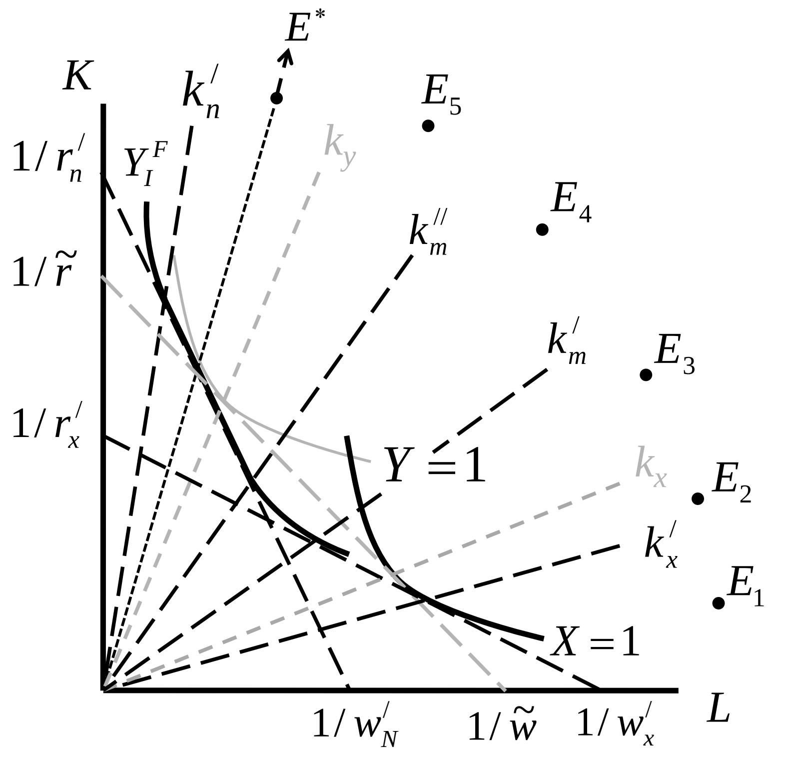

Figure 4. Free Trade Equilibrium for a Small Country, Lerner Diagram

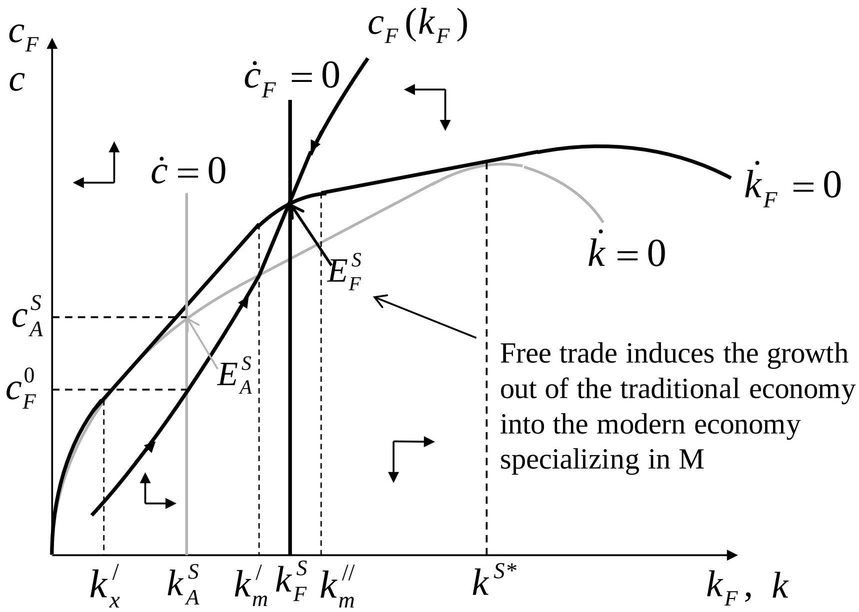

Figure 5a. Phase Diagram under Free Trade with $\tilde{r} - \delta \le \rho < r_x^/ - \delta$

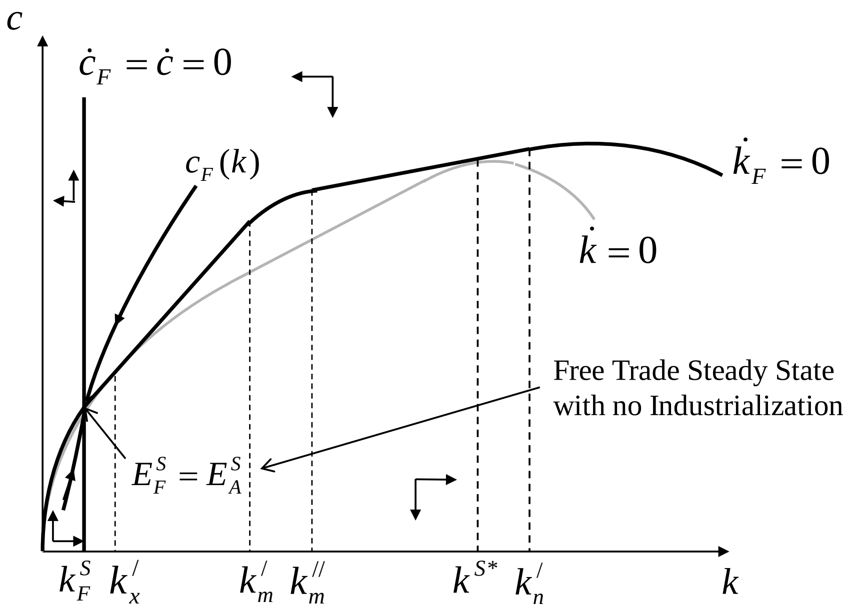

Figure 5b. Phase Diagram under Free Trade with $\rho \geq r_x^/ - \delta$

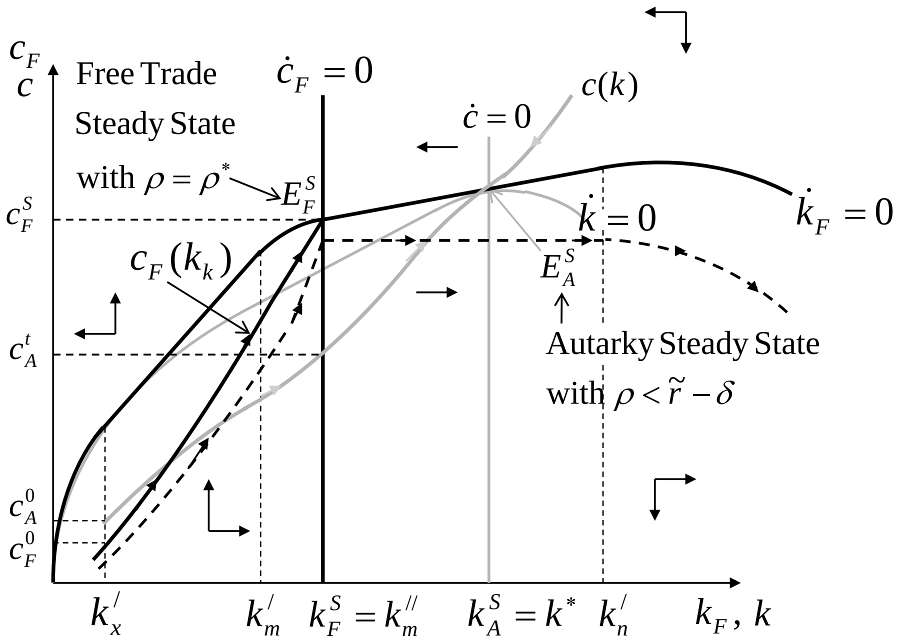

Figure 6. Phase Diagram under Free Trade with $\rho = \rho^* < \tilde{r} - \delta$

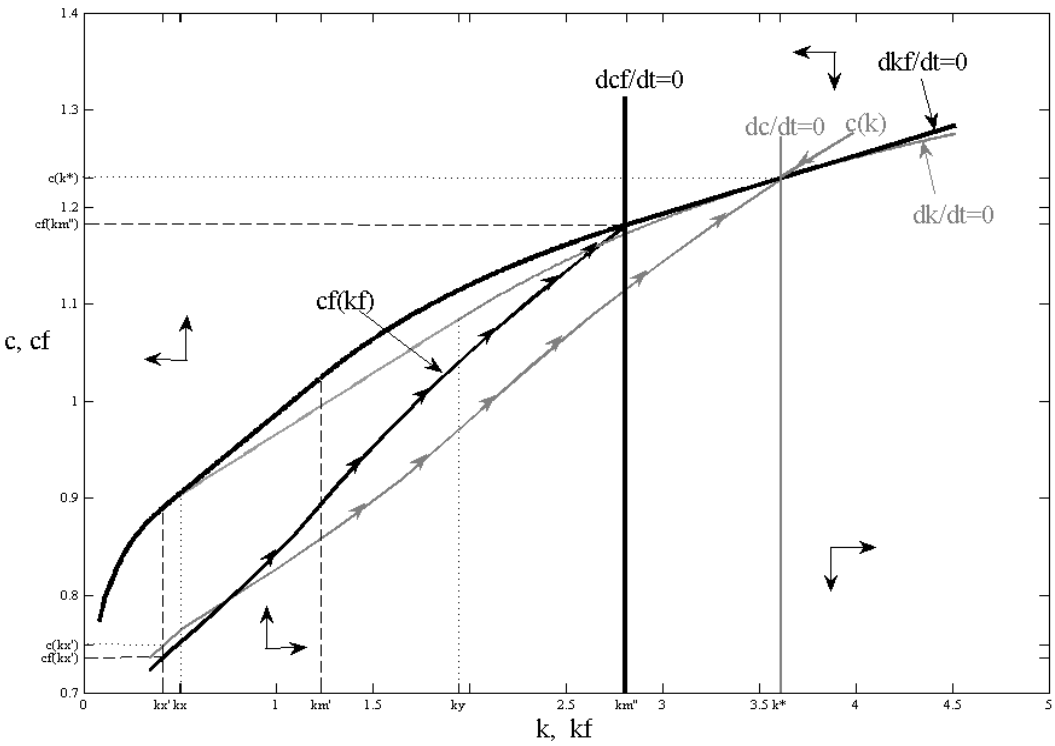

Figure 7a. Phase Diagrams from Numerical Analysis

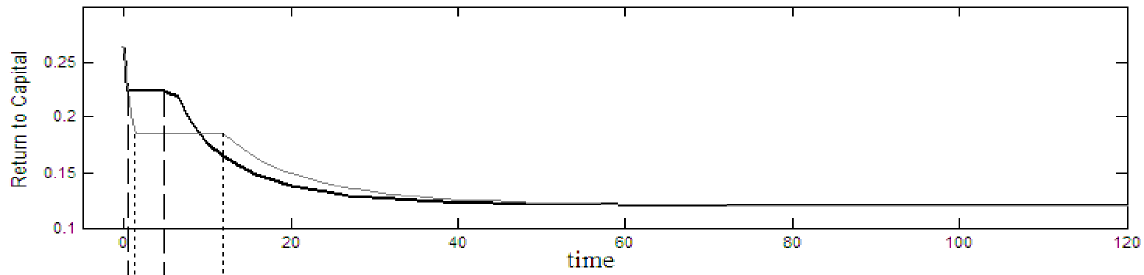

Figure 7b. Return to Capital from the Numerical Analysis

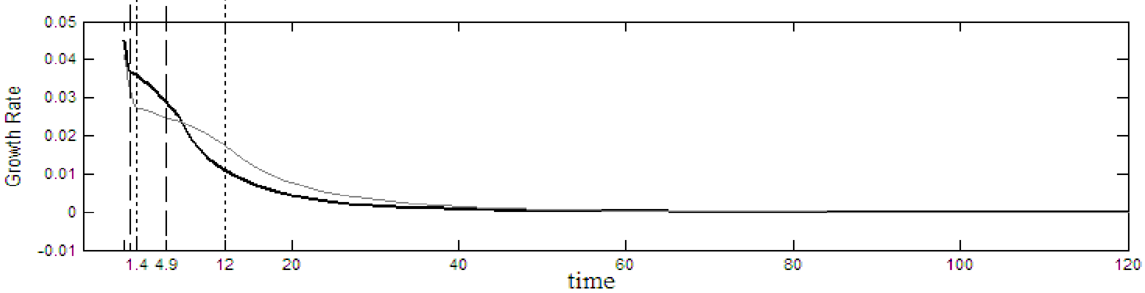

Figure 7c. Growth Rate from the Numerical Analysis

with $\alpha_x = 0.1$, $\alpha_m = 0.25$, $\alpha_n = 0.35$, $A_x = A_m = A_n = 1$, $A_y^{/} = 1.96$, $\rho = 0.07$, $\delta = 0.05$, and $n = 0.01$.