Eur. Phys. J. E **22**, 219–226 (2007)
DOI: 10.1140/epje/e2007-00001-6

THE EUROPEAN PHYSICAL JOURNAL E

Instabilities in vertically vibrated fluid-grain systems

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Received 25 October 2006/ Received in final form 29 November 2006
Published online: 17 January 2007 –

Abstract. When a bed of fluid-immersed fine grains is exposed to vertical vibration a wealth of phenomena may be observed. At low frequencies a horizontal bed geometry is generally unstable and the bed breaks spatial symmetry, acquiring a tilt. At the same time it undergoes asymmetric granular convection. Fine binary mixtures may separate completely into layers or patterns of stripes. The separated regions may exhibit instabilities in which they undergo wave-like motion or exhibit quasi-periodic oscillations. We briefly review these and a number of related behaviours, identifying the physical mechanisms behind each. Finally, we discuss the magneto-vibratory separation of binary mixtures which results from exposing each component to a different effective gravity and describe the influence of a background fluid on this process.

PACS. 45.70.-n Granular systems

1 Introduction

Not only are granular materials widespread in nature and in commerce, but many phenomena which result from the dynamical interaction of grains and fluids are very common. Sand dunes move slowly across the surface of the Earth under the influence of the wind [1,2], while the movement of the ocean waves leaves ripple patterns both undersea and upon the beach [3–5]. Others, such as the devastating pyroclastic flows resulting from a volcanic eruption are fortunately less common [6]. Here we restrict ourselves to describing a number of the phenomena which are observed when fine grains are vibrated vertically within a fluid such as water or air.

We will suppose that a bed of grains is held upon a horizontal platform which is being vibrated vertically as $z_p = A\sin(\omega t)$, and that the grains are also under the influence of Earth's gravity, g. If the amplitude of vibration is such that the parameter $\Gamma = A\omega^2/g$ is greater than unity, then the bed will be thrown from the supporting platform by the vibration. For a range of $\Gamma > 1$, the bed will undergo flight and will land and settle within the same vibration cycle. This is illustrated in Figure 1. The throwing, flight and settling will then be repeated in each successive cycle. We now suppose that the bed is immersed in a fluid. As it leaves the platform, fluid will be drawn downwards through the bed by the reduced pressure found below it. Later, as the bed approaches the platform, the overpressure below the bed will drive fluid upwards through it. These fluid motions are also illustrated in Figure 1. It is cyclic motion of fluid through the bed which is responsible for most of the phenomena which we shall describe here. Should Γ be sufficiently high that the bed has not fully settled before it is thrown again, then the bed will exhibit a range of related phenomena within this continuous granular dynamics regime. However, these will only be touched upon lightly in the present review.

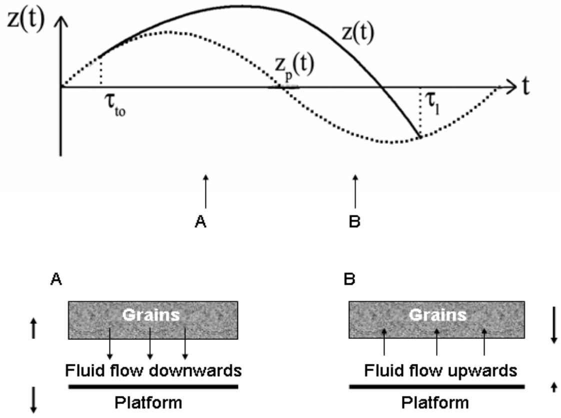

Fig. 1. (Upper) Schematic diagram showing the motion of a sinusoidally vibrating platform (dotted line) and the flight of a fluid-immersed granular bed thrown from it (continuous line). The bed leaves the platform at $\tau_{to} = (1/\omega)\sin^{-1}(1/\Gamma)$ and lands at τ_l. It then settles and is thrown again in the next cycle. (Lower) Illustrations showing that early in flight (A) fluid is drawn down through the bed, while later in flight (B) fluid is forced up through the bed.

Fluid-grain effects are conveniently observed over a range of particle diameters, d, which depends upon the particle density and on the fluid used. Comparison of the

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fluid drag forces to the inertial forces indicates that it is the combination $\eta/\rho_g d^2$ which principally determines the influence of the fluid upon the dynamics of the granular motion. Here η is the dynamic viscosity of the fluid and ρ_g is the density of the grain material. The need for adequate fluid-grain coupling sets an upper grain size limit of about 3 mm for granular systems immersed in water and 400 μm for systems immersed in air. However, since finer grains are more strongly influenced by the fluid, the fluid drag may become so large at sufficiently reduced grain sizes that extreme vibration must be applied to maintain relative granular motion. For typical grain densities this sets practical lower size limits of about 100 μm for water and 20 μm for air.

2 Observing and modelling fluid-grain behaviour

In the laboratory the granular response to vertical vibration is frequently studied using high speed photography. In air-immersed systems the grains are relatively small and this inhibits the observation of the motion of individual grains. Air immersed grains are also susceptible to the influence of electro-static charge which builds up in response to sustained vibration. Water immersed systems are expected to show similar phenomena for grains which are about seven times larger than for air [7]. Thus, in water immersed systems, individual grains are easier to observe. In water static charge is no longer a problem. However, observation alone is often not sufficient to identify the physical mechanisms behind a particular phenomenon. Fluid immersed beds under vibration are complex examples of non-linear systems and, as yet, there exists no general theoretical framework to aid understanding. However, considerable insight has come both from computer simulations, which allow the modification of parameters which are difficult to adjust in the laboratory, and from simple analytical models.

The effects which we shall describe usually involve both granular and fluid convection and it is necessary to carry out simulations involving at least two dimensions. In these dense particulate systems "molecular dynamics" (MD) rather than "event driven" codes are usually necessary and, due to the very large number of particles, the fluid-grain coupling is usually treated through an empirical mean-field approach rather than through the treatment of individual grains. In one successful scheme the grain-grain collisions are modelled using a soft-sphere spring-dashpot approach, while the behaviour of the fluid flow is reproduced using a modification of the Navier-Stokes' equations which include the local mean variables of the grains such as bed porosity and grain velocity [8]. The fluid-grain drag is incorporated into the Navier-Stokes' equations through use of an empirical "bed equation" such as that of Ergun [9] or Di Felice [10]. Even though they often ignore features such as particle rotation, these simulations are usually capable of reproducing the principal behaviours observed in the laboratory [11].

In vertically vibrated systems considerable insight is offered by applying the one-dimensional model of von Kroll [12] or applying modifications of that model [7]. In this approach a vibrated granular system is considered as a set of vertical columns, the motion of the fluid and grains within each column being treated in isolation. This is a good approximation if the vertical components of the grain and fluid velocities dominate over the horizontal components and if the frictional interactions between the columns and between columns and walls may be ignored. The approach does not offer direct estimates of the horizontal motion, but, importantly, it does offer an approximate method of estimating the pressure gradients which lead to that motion.

Von Kroll considered a granular bed as a rigid plug of constant porosity moving purely in the vertical dimension within an incompressible mass-less fluid [12]. Fluid-grain coupling was introduced through the use of Darcy's law. The method offers the following equation of motion for the bed

$$\dot{u} + \gamma u + g = -\ddot{z}_p, \tag{1}$$

where u is the rate of change of the relative position of the bed with respect to the platform. The deviation of the pressure below a bed of thickness h from the ambient pressure is given by

$$\Delta P = -hKu. \tag{2}$$

In these equations K is the Darcy's law constant, and the parameter $\gamma = K/\rho_g(1-\phi)$ describes the grain-fluid coupling, ϕ being the bed porosity. Note that the equation of motion of bed flight does not depend upon the bed thickness, but the pressure variations scale linearly with thickness. The flight of the bed may be obtained directly by integration once $u(t)$ is known. Equations (1) and (2) may readily be modified to include non-linear terms in the fluid-grain velocity, for example through the use of the Ergun equation. In an incompressible fluid two separate beds do not influence each other and equation (1) may be applied separately to each as long as they are not in contact. Gutman has modified the von Kroll model to take into account the effects of air compressibility which become important for deeper beds and higher frequencies [13].

In the case of a dense incompressible liquid such as water there are important effective mass corrections and modifications due to buoyancy. The flight equation then becomes

$$\epsilon\dot{u} + \gamma u + \alpha g = -\alpha\ddot{z}_p. \tag{3}$$

Here $\epsilon = 1 + \rho_f(1-\phi)/\rho_g\phi$ is an effective mass correction and $\alpha = (\rho_g - \rho_f)/\rho_g$ is a buoyancy correction [7]. This equation, too, may readily be adapted to include non-linear terms in the fluid-grain velocity. Once again the flight equation does not contain the bed depth but the pressure variation ΔP scales linearly with bed depth. Since the liquid may be treated as being incompressible, the flight equation (3) of the modified Kroll model, too, may be applied individually to two or more separated beds while they are not in contact [7].

In many fluid-immersed vertically vibrated beds the vertical motion within each cycle of vibration is indeed

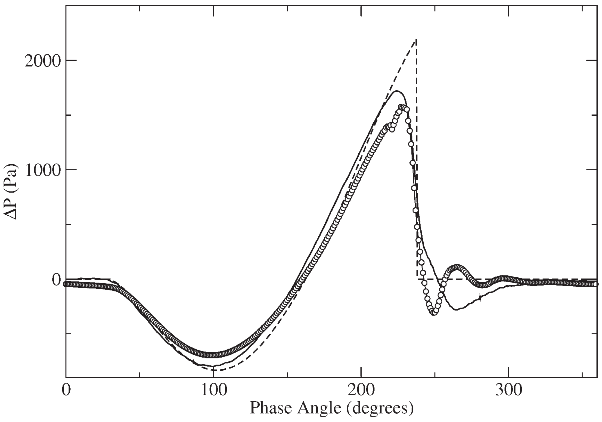

Fig. 2. The pressure variation ΔP, as a fuction of the vibration phase angle, under a water-immersed bed of 700 μm diameter bronze spheres vertically vibrated at 20 Hz and with $\Gamma = 2$. The experimental data, shown as a continuous line, is compared with the predictions of the modified Kroll model (broken line) and predictions from Navier-Stokes simulation (circles). The pressure variation due to the vertical acceleration of the water column in which the grains are immersed has been subtracted in each case.

appreciably larger than the corresponding horizontal motion. Also the fluid damping lowers the granular temperature within the bed and this often results in an almost constant porosity throughout the bulk of the bed. The von Kroll class of models may then offer a semi-quantitative description of the vertical motion of any columnar section of the bed, and of the corresponding pressure variations.

Figure 2 shows experimental data for the pressure variation under a wide vertically vibrated water immersed bronze bed. The data have been obtained by using semiconductor diaphragm pressure transducers fitted to the top and bottom of the cell. The lower transducer is separated from the grains by a mesh. Figure 2 also shows, for comparison, the predictions of the modified von Kroll model which include the non-linear term of the Ergun bed equation and the predictions of the Navier-Stokes model which includes fluid-grain interactions through use of the Ergun equation [8,11]. It is seen that there is good agreement in flight between the experimental data and the predictions of these models, but after the bed landing at about 240° there are some detailed discrepancies.

However, despite this agreement the von Kroll models have the limitation that they do not directly describe the *response* of the bed to horizontal pressure gradients. As we shall see, this response may in some situations be estimated approximately from simple mechanics. However, often we must resort to MD simulation or to the use of constitutive relations.

3 Faraday tilting and fluid-driven convection

Following preliminary observations by Chladi, Faraday carried out a detailed study of the way in which the presence of air causes fine grains which are thrown vertically from a horizontal platform to group together to form conical piles [14]. Grains are seen to cascade down the outer surfaces of these "Faraday piles", the shape being maintained by movement within the pile. Correspondingly, a body of fine grains being vertically vibrated within a container will spontaneously tilt until the tilt angle reaches a dynamic angle of repose. Grains are continually seen to cascade down the slope, the angle of which is maintained by movement within the bulk.

After much debate, studies of "Faraday tilting" have unambiguously identified the phenomenon as due to the effect on the grains of the fluid being driven backwards and forwards through the bed by the vibration [11–21]. The basic mechanism may be understood through consideration of the von Kroll model. The bed may be modelled as a set of columns, each undergoing predominantly vertical motion. Within this approximation each column undergoes the same flight and the magnitude of the fluid pressure variations depend linearly upon the depth below the upper surface. The model therefore predicts horizontal pressure gradients in a tilted bed, directed towards the deeper parts of the bed early in flight, and towards the shallower parts of the bed later in flight.

The Faraday tilting effect results from the response of the bed to these pressure gradients. Even if a bed is only slightly tilted, the grains throughout the bulk of the bed undergo tilt-enhancing horizontal acceleration early in flight, while later in flight they are slowed in their horizontal motion by reverse acceleration. The bed tilt builds until the bulk tilt enhancement early in flight is balanced by two effects, the bulk tilt detraction upon landing and the tilt detraction caused by surface grains cascading down the upper slope once the tilt angle approaches the angle of repose. This explanation for Faraday tilting has been confirmed both by simulations of water-immersed granular beds and by corresponding experimental observations [11]. A typical Faraday tilting response to vertical vibration is illustrated in Figure 3 which shows, from experiment, a bed of water-immersed bronze grains.

The Faraday tilting effect weakens if the frequency is raised since the fluid-grain coupling also then weakens. The tilting effect also weakens if Γ is sufficiently large that the bed dynamics become continuous, there being insufficient time for the grains to settle within each vibration cycle [11]. If the fluid-grain coupling becomes extreme, for example by reduction of the grain diameter, then grain-grain motion within the bed is inhibited, but Faraday piles may be observed on the upper surface, the body of the bed now acting as a platform.

A vertically vibrated granular bed is normally observed to exhibit granular convection due to the effects of wall friction. However, a fluid-grain system which undergoes Faraday tilting is observed to show strong granular convective flow even if the wall friction is low. The mechanism for this convection and the mechanism for the Faraday tilting are one and the same. The horizontal pressure gradients in a tilted bed produce a net flow of bulk grains towards the deeper side of the bed. Since the tilt angle cannot exceed the angle of repose, this flow is eventually

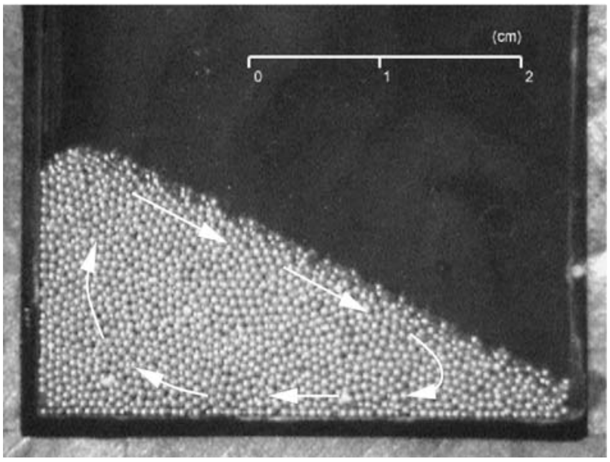

Fig. 3. The Faraday tilting response, from experiment, of a water-immersed bed of 655 μm diameter bronze spheres subjected to vertical vibration of 25 Hz and $\Gamma = 2.5$. Granular convection is indicated by the white arrowed lines. The container in which these grains are shaken has a horizontal aspect ratio of 4:1, being 40 mm by 10 mm in horizontal section. In such vertically vibrated systems the dominant tilting and convection takes place in the most extensive vertical plane, and for an aspect ratio considerably greater than unity, as here, there is relatively little tilting or convective motion in the minor plane (here into the figure). The mean granular and fluid motion is almost entirely two-dimensional and in the major plane shown.

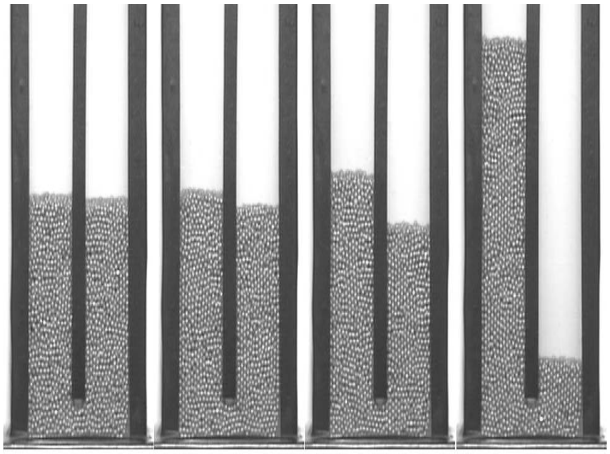

Fig. 4. The behaviour, from experiment, of water-immersed 700 μm diameter bronze spheres held in a partitioned container. The two halves are linked by a hole in the partition near to the base. The system is subject to vertical vibration at 25 Hz and with $\Gamma = 3.0$. The images have been taken, from left to right, at time intervals of 30 s. The acceleration of the transfer of grains into the highest column is clearly evident.

balanced by surface flow down the slope. There is thus a convective circulation down the slope, the grains returning up-slope within the bulk. In Figure 3 the convective flow in response to the vibration is indicated schematically by white arrowed lines. In this strongly coupled system the weaker counter-rotating convection cell resulting from wall friction which is sometimes found in the deepest part of the bed is completely absent.

4 U-tube and partition instabilities

It has been reported that the majority of the grains held in a u-tube move into one arm under vertical vibration [22]. The majority of the grains held in two or more containers which are linked near their base by holes or channels will move into just one of the containers [23,24]. While geometric asymmetries or the existence of a horizontal component of the vibration may sometimes be responsible for these effects the presence of a fluid within a fine granular bed provides a powerful mechanism for this form of spatial instability.

Figure 4 shows the behaviour under vertical vibration of a collection of water-immersed bronze grains held in a partitioned container. The two sides are linked at the bottom by a small hole. Fluid may also circulate through a second hole positioned higher in the partition (not shown). Even if attempts are made to start with equal numbers of grains on each side of the partition, the system breaks symmetry, grains passing to which ever column is highest, at an increasing rate. The von Kroll model may once again be used to provide the basis for an explanation. In this model the flight of the beds in each container will be similar, but the under pressure found early in flight will be most extreme for the bed with the greatest depth. This produces a fluid pressure gradient directed from the shallowest to the deepest bed. Grains influenced by this gradient will then move towards the deepest bed, being slowed in their motion by the reverse gradient which occurs later in each cycle. Since the pressure gradients will be proportional to the difference in bed heights, the model predicts that the granular movement towards the deeper bed will increase in speed as the difference in bed heights increases. The von Kroll model provides of course only a crude approximation. In reality, the two columns are strongly coupled by the holes and the von Kroll model cannot with accuracy be applied to each column in isolation. Nevertheless the model is able to provide the basic mechanism for the instability, and is able to qualitatively predict the dependence of the rate of granular movement.

5 The fluid-enhanced "Brazil nut" effect

The simplest example of a granular mixture consists of a host bed containing one intruder. It is well-known from early studies in the coal industry that, under vibration, a larger particle will rise to the surface of a bed of smaller particles, this now being known as the "Brazil nut" effect [25,26]. Studies have shown that the presence of fluid in a fine granular bed may greatly enhance the Brazil nut effect [27–29]. A larger or heavier intruder will be less influenced by fluid drag and will fly higher than the host bed particles which are strongly influenced by the downwards flow of fluid early in flight. Host particles will move into

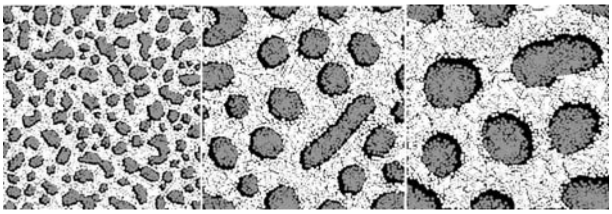

Fig. 5. The simulated separation of a fluid-immersed binary mixture of bronze and glass spheres shaken within a two-dimensional container in the absence of gravity. Here the sinusoidal vibration is of fixed amplitude but is applied within each cycle in a random direction. Due to the differential influence of the fluid, grains of the same type cluster (left) and the cluster sizes grow until the clusters no longer touch each other in their motion (right).

the resulting void under the intruder and the intruder will fall upon them later in each cycle. Thus the intruder is ratcheted upwards in each vibration cycle. In a strongly fluid controlled system the intruder must have a larger $\rho_g d^2$ product than that of the host particles in order for it to rise within the bed.

6 The patterning and separation of fine binary mixtures

Dramatic fluid driven separation effects have been observed in fluid immersed binary mixtures of bronze and glass when vertical vibration is applied [7,30,31]. In the absence of gravity a tube initially filled with a fluid immersed homogeneous granular mixture will form stripes when the tube is shaken along its axis [32]. The stripes reduce in number until the increased space provided by the coarsening is sufficient to prevent the separated regions from touching during the vibratory motion. The similar separation of a binary mixture shaken within a two-dimensional cell is shown in Figure 5. The grains form clusters of the same grain type, which coarsen until no further contact between separated regions occurs.

In the presence of both Earth's gravity and low frequency vertical vibration, grains of similar type are seen to group together. The groups then coarsen and, following convection, move until an almost pure region of the more dense bronze lies above an almost pure region of the less dense glass. The system exhibits Faraday tilting but with separate convection cells in the two regions. An example of such a separated system is shown in Figure 6, which shows, from simulation, the separation of a water-immersed mixture of 1000 μm glass ($\rho_g = 2500$ kg m^{-3}) and bronze ($\rho_g = 8900$ kg m^{-3}) spheres subjected to vibration at 20 Hz and with $\Gamma = 2.5$. The convection flows are indicated by black arrowed lines. It is noteworthy that the underside of the bronze over-layer is moving up-hill, despite the down-hill flow of the nearby glass grains.

The explanation for this behaviour is as follows [7,32]. Fluid is driven backwards and forwards through the bed by the vibration. Grains of the same type will be similarly influenced by the fluid forces and will tend to stay together. Grains of different types will however be differently influenced by the fluid forces and will move apart. This leads to the formation of coarsening clusters of bronze and of glass grains [33]. These clusters move under fluid-driven convection. The bronze clusters, being more dense, have a greater tendency to move upwards. When bronze-rich and glass rich regions of sufficient purity have developed, they will be thrown differently by the vibration. This enables a gap to develop between these regions during those periods of time when they are thrown by the vibration, times when the beds are most kinetically active. The formation of a gap enables relative motion without mixing and the formation of distinct convection cells [7]. The presence of both strong convection and the absence of mixing enables the purity of the two regions to increase further, and in some cases the separation may be complete. The grains at the bottom of the upper bronze layer are able to move uphill, since they are influenced by the reduced pressure found under the bronze bed early in flight. They therefore acquire a horizontal velocity component towards the deeper side of the Faraday tilted glass bed and are displaced in that direction during each cycle.

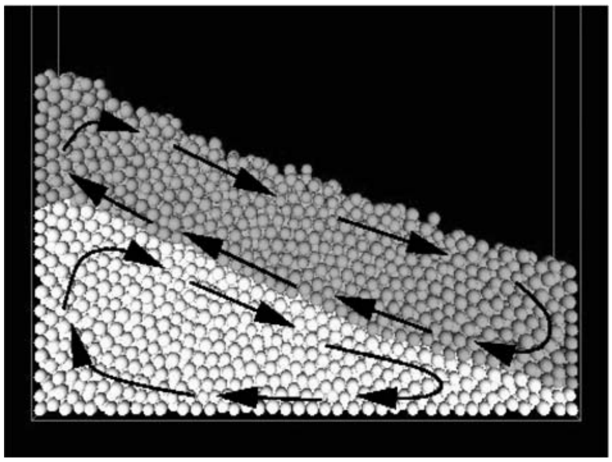

Fig. 6. The separation, from simulation, of a water-immersed mixture of 1000 μm glass (light) and bronze (dark) spheres subjected to vibration at 20 Hz. and with $\Gamma = 2.5$. The convection flows are indicated by black arrowed lines. It is noteworthy that the underside of the bronze over-layer is moving up-hill, despite the down-hill flow of the nearby glass grains.

At higher frequencies the binary mixtures are also observed to separate almost completely under vertical vibration, both under the influence of water [7] and of air [30,31]. At these frequencies, however, the initial separation is into the form of a sandwich with a bronze layer between a lower and an upper glass layer. From experimental studies of water immersed systems this appears to be a metastable configuration which changes into the low-frequency "bronze on top" configuration after a considerable time of vibration [7]. In some air-immersed systems the sandwich configuration may be stable or it may exhibit temporal and spatial instabilities involving quasi-periodic oscillations of form [31]. This behaviour

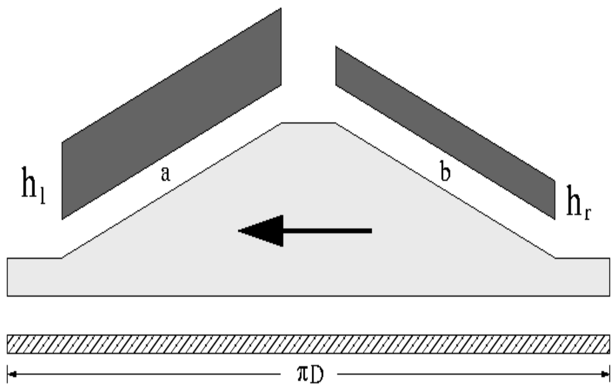

Fig. 7. Schematic illustration of the *unwrapped* geometry found when a water-immersed glass-bronze mixture is vertically shaken in the space between two co-axial cylinders of mean diameter D. The common axis of the cylinders is in the vertical direction. The bronze layers are shown heavily shaded and are of unequal vertical thicknesses h_l and h_r. The whole system moves (rotates) in the direction indicated, that is in the direction of the thicker bronze over-layer.

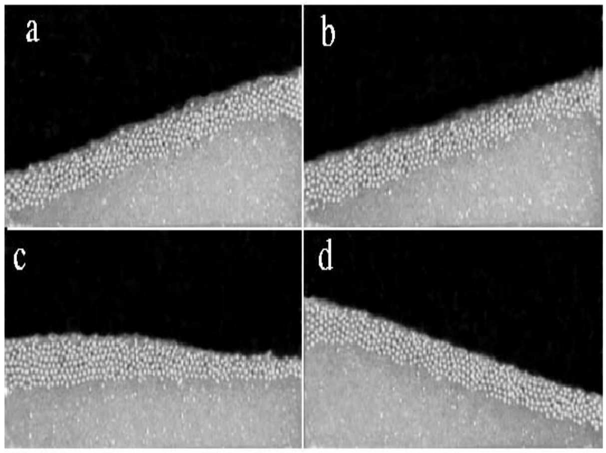

Fig. 8. Images, from experiment, showing one half cycle of the periodic behaviour of a separated water-immersed 25%:75% bronze-glass mixture under vertical vibration. The mean grain diameter is 655 μm and a frequency of 50 Hz and $\Gamma = 3.5$ have been used. The images from top left to bottom right were taken after 0, 3, 6 and 11 s of vibration. One complete period of oscillation occupies 22 s.

is not well understood despite extensive simulation studies [34].

The criterion for fluid driven separation in these systems is that the separating species must be differentially influenced by the fluid in which they are immersed. The principal determining factor for the separation of two species a and b is that the parameter $S = \rho_a d_a^2/\rho_b d_b^2$ should be sufficiently far from unity [7]. This prediction is somewhat modified by the corrections for buoyancy and for effective mass if the fluid has appreciable density. While some separation will occur with S quite close to unity, $S > 1.5$ is usually needed for the creation of the clear gap between separated beds which leads to almost complete or total separation.

7 Temporal and spatial instabilities of separated systems

The mechanisms behind a number of the temporal instabilities exhibited by separated fluid-immersed binary systems under vertical vibration are understood. An example is the behaviour observed when a water immersed bronze-glass mixture is held in the annular gap between two coaxial cylinders [35]. The mixture separates with the bronze uppermost. At the same time the system breaks spatial symmetry, developing a single Faraday pile, the bronze lying above a glass hill. This system is unstable against the bronze thicknesses on the two slopes becoming unequal since the hill moves in the direction of the thicker bronze layer, maintaining that layer as the thicker of the two. The geometry of this arrangement is shown in Figure 7. The basis for this behaviour may once again be understood through application of the von Kroll model. The mixture separates with the bronze uppermost and develops a Faraday pile through the mechanisms we have already discussed. Early in flight the under-pressure below a thicker bronze overlayer (a in Fig. 7) is more extreme than the under-pressure below a thinner bronze over-layer (b in Fig. 7). This results in a horizontal pressure gradient across the glass pile which drives it in the direction shown by the arrow. The system as a whole rotates about a vertical axis under purely vertical vibration. Again it is noteworthy that the bottom of each bronze layer moves uphill, grains being returned downhill by movement at the upper surface. This is once again the result of the negative pressures early in flight below each bronze bed. For this particular system it has proved possible to obtain an approximate analytical expression for the rotation period in terms of frequency, Γ and the system parameters. This has been achieved by estimating the response of the glass pile to the pressures across it provided by the motion of the bronze beds. The modified von Kroll model was used to provide these pressures. The expression for the rotation period is in reasonable quantitative agreement with the experimental observations [35].

A further instability is illustrated in Figure 8. A bronze-glass mixture which has separated with the bronze uppermost is commonly observed to exhibit Faraday tilting. The system parameters may be chosen, however, so that the tilt oscillates backwards and forwards between the two equivalent angles in a periodic fashion. In such a system the bronze over-layer is not of uniform thickness. Since the pressure is lower under the thicker bronze regions early in flight the glass is drawn to those regions. The thicker bronze regions are lifted until the dynamical angle of repose of the bronze is reached (8a). The bronze then avalanches down the slope, its inertia taking it to the bottom (8b). This region is now the thickest and it begins to rise as glass is drawn underneath it (8c). The tilt is drawn towards the other side of the box (8d). This process repeats indefinitely in a periodic fashion.

8 Separation under differential effective gravity

It is possible to separate granular mixtures by vibrating them within an inhomogeneous magnetic field [36,37]. This may have the effect of making the different types of grain behave as if they are exposed to different strengths of gravity. Let us consider a diamagnetic or paramagnetic grain of volume V. In a strong vertical magnetic field, B, it will acquire a magnetic moment equal to $\chi VB/\mu_0$, where χ is the volume magnetic susceptibility. If the field is spatially inhomogeneous, having a vertical gradient equal to dB/dz, then the grain will be subject to a vertical force given by

$$F = \frac{\chi V}{\mu_0} B \frac{dB}{dz}. \tag{4}$$

Here z is again the vertical coordinate, considered as positive upwards. The magnetic force acts in addition to the normal force of gravity, and in general the grain may be considered to experience an effective gravity, $\tilde{g}$, given by

$$\tilde{g} = g - \frac{\chi}{\rho_g \mu_0} B \frac{dB}{dz}. \tag{5}$$

Here g is considered conventionally as a positive quantity. For the weakly magnetic materials which we are considering here, $\tilde{g}$ does not depend upon either the volume or the shape of the grain, but only upon its susceptibility and density and upon BdB/dz. For diamagnetic materials χ is negative, and strong magnetic fields with a negative field gradient have been used to reduce $\tilde{g}$ to zero for a range of materials, causing levitation [39–41].

Granular mixtures of components which have distinct values of the ratio χ/ρ_g, may be separated by the application of both an inhomogeneous magnetic field and vertical vibration, even though the magnetic force is insufficient to levitate any component [36]. Vertical vibration will be sufficient to throw individual grains from a sinusoidally vibrating platform if $\tilde{\Gamma} = A\omega^2/\tilde{g}$ is appreciably greater than unity. A single grain with a lower value of $\tilde{g}$ will be thrown higher, and land later, than a single grain with a higher value of $\tilde{g}$. Although the grains within a mixture will impede each others' motion, it is reasonable to expect that a periodically thrown mixture of components with sufficiently distinct values of $\tilde{g}$ will show a tendency to separate even for a $|BdB/dz|$ product considerably lower than that needed to fully levitate any one component.

If this process is carried out in the absence of a fluid, through the use of a vacuum, it is found that excellent separation of bismuth grains (ρ_g = 9800 kg m^{-3}, $\chi = -1.65 \times 10^{-4}$) and bronze grains ($\rho_g$ = 8900 kg m^{-3}, $\chi = -5.5 \times 10^{-6}$) of similar size may be obtained [36]. Such grains cannot be well separated under the influence of vibration alone. However, under the influence of a negative field gradient, the more magnetic bismuth experiences a reduced effective gravity while the gravity experienced by the weakly magnetic bronze particles is close to normal Earth gravity. Under this influence the bismuth separates above the bronze and purities of better than 0.1% can be obtained by the application of $BdB/dz = -500\ T^2$ m^{-1} provided that Γ is suitably chosen. However, it is noteworthy that the degree of separation is a smooth and continuous function of the separation parameters Γ and BdB/dz [36].

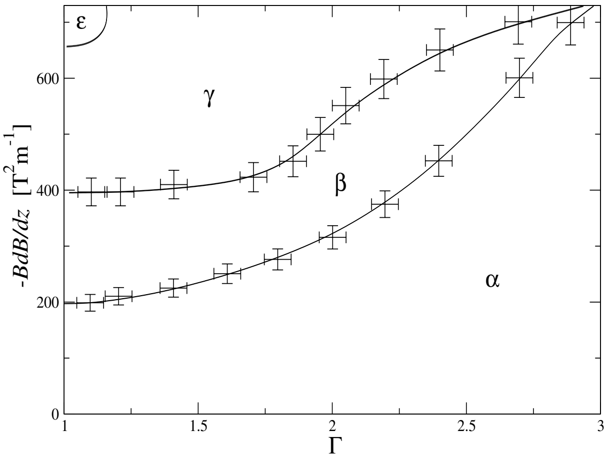

Fig. 9. A schematic diagram describing the behaviour of an equal mixture of 75–90 μm bismuth and bronze grains when vibrated in air in the presence of an inhomogeneous magnetic field. The regions corresponding to a mixed state (α), a partially separated "bismuth on top" state (β) and a completely separated "bismuth on top" state (γ) are shown. ϵ indicates a region influenced by magnetic cohesion. The granular configuration and the associated convection flows change abruptly from one state to another if the boundaries are crossed, there being little hysteresis. Further details may be found in [37,38].

This behaviour contrasts strongly with that found if similar experiments are conducted in the presence of air [37,38]. A bismuth-bronze mixture is found to separate into a number of different spatial forms, each having a distinctive convection pattern. Depending upon Γ and BdB/dz, mixed states, partially separated states and well separated states are found, each form occupying its own area of the Γ, BdB/dz plane. If the boundary between these regions is crossed, the change in form is abrupt. In the mixed state, global convection is evident, while in the states showing near-perfect separation, the separated regions have individual convection flows, well defined gaps appearing between the separated regions during bed flight. The partially separated states exhibit more than one convection cell, but the lack of clearly defined gaps during bed flight causes interaction which maintains the partially mixed state.

Such behaviour is illustrated in Figure 9 which shows schematically the regions of the Γ, BdB/dz plane where the different behaviours of a 50%:50% by volume mixture of 75–90 μm bismuth and bronze grains are found when vibrated in air. Regions corresponding to a mixed state (α), a partially separated "bismuth on top" state (β) and a completely separated "bismuth on top" state (γ) are shown. The granular configuration and the associated convection flows change abruptly from one state to another if the boundaries are crossed, there being little hysteresis.

In the case of bismuth and bronze grains of unequal size, the parameter S may be sufficiently far from unity that fluid driven separation occurs even the absence of a magnetic field, with well separated bismuth and bronze regions having distinct convection flows. The application of an appropriate magnetic field may then be used to exactly counteract this tendency, causing an abrupt transition into a well mixed state with a single global convection roll.

9 Conclusions

We have reviewed a wide range of the behaviours exhibited by fluid immersed fine granular beds when subject to vertical sinusoidal vibration. These include a number of instabilities in which symmetry is broken or in which the granular configuration is unstable in space and time. The application of the von Kroll models to these complex nonlinear systems often enables us to identify the principles behind these behaviours. They frequently offer a reasonably good prediction of the vertical flight of the grains and of the corresponding fluid pressures. However, only in exceptional circumstances is it possible to use these models directly to estimate the response of the fluid and grains to the pressure gradients. MD simulations usually offer a more accurate and flexible approach to estimating this response, particularly for processes involving substantial granular convection.

Finally we note that many of these phenomena do not require sinusoidal vibration or even the application of periodic disturbances. Many, such as granular separation and partition instabilities will occur even if occasional random taps are applied.

We are grateful to the Engineering and Physical Sciences Research Council for support, to Makin Metal Powders Ltd. for their gifts of powders, and to the workshop staff of the School of Physics and Astronomy for their skills and enthusiasm.

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