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The influence of air on the magneto-vibratory separation of binary granular mixtures

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Abstract The application of an inhomogeneous magnetic field to a binary granular mixture composed of diamagnetic or paramagnetic particles exposes the two species to different effective forces of gravity. Vertical vibration may then cause the mixture to separate through the action of this differential gravity. Here we consider the influence of a background fluid, such as air, on this magneto-vibratory separation. We show that air-driven tilting and granular convection greatly influence the separation dynamics. We identify the magnetic and vibratory conditions for mixed, partially separated and very well separated states. Each corresponds to a distinct pattern of granular convection, the transition from one state to another being abrupt as the experimental conditions are altered. Magneto-vibratory separation may enhance air-driven separation, may cause inverted separation, or may be adjusted to counteract the air-driven separation and maintain the mixed state. We use computer simulations to give insight into these separation processes. We show that many of the phenomena are associated with the gaps which open up between separated regions over those fractions of the vibration cycle in which the convective motion takes place. It is these gaps which enable the abrupt transition between separated states and which prevent the convection within the separated regions from causing mixing.

Keywords Granular · Magnetic · Vibration · Separation · Air

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1 Introduction

The application of both an inhomogeneous magnetic field and vertical vibration to a granular mixture may cause separation. This paper concerns the effect of an ambient fluid such as air on this process.

The idea of diamagnetic levitation may be traced back to Lord Kelvin, but levitation was first demonstrated experimentally by Braunbeck [1,2]. Let us consider a diamagnetic or paramagnetic grain of volume V. In a strong vertical magnetic field, B, it will acquire a magnetic moment equal to $\chi VB/\mu_o$, where χ is the volume magnetic susceptibility. If the field is spatially inhomogeneous, having a vertical gradient equal to dB/dz, then the grain will be subject to a vertical force given by

$$F = \frac{\chi V}{\mu_o} B \frac{dB}{dz}. \tag{1}$$

Here z is the vertical coordinate, considered as positive upwards. The magnetic force acts in addition to the normal force of gravity, and in general the grain may be considered to experience an *effective gravity*, $\tilde{g}$, given by

$$\tilde{g} = g - \frac{\chi}{\rho \mu_o} B \frac{dB}{dz}. \tag{2}$$

Here g is the acceleration due to Earth's gravity, considered conventionally as a positive quantity, and ρ is the density of the grain material. For the weakly magnetic materials which we are considering here, $\tilde{g}$ does not depend upon either the volume or shape of the grain, but only upon its susceptibility and density and upon BdB/dz. For diamagnetic materials χ is negative, and strong magnetic fields with a negative field gradient have been used to reduce $\tilde{g}$ to zero for a range of materials, causing levitation [3,4].

Granular mixtures having components with distinct values of the ratio χ/ρ, may be separated by the application of both an inhomogeneous magnetic field and vertical vibration, even though the magnetic force is insufficient to levitate any component [5]. Vertical vibration will be sufficient to throw individual grains from a sinusoidally vibrating platform if the parameter $\tilde{\Gamma} = a\omega^2/\tilde{g}$ is appreciably greater than unity. Here a is the amplitude of vibration and $\omega = 2\pi f$ is the angular frequency of vibration. We will also use $\Gamma = a\omega^2/g$ to characterise the amplitude of vibration. A single grain with a lower value of $\tilde{g}$ will be thrown higher, and land later, than a single grain with a higher value of $\tilde{g}$. Although the grains within a mixture will impede each others' motion, it is reasonable to expect that a periodically thrown mixture of components with sufficiently distinct values of $\tilde{g}$ will show a tendency to separate even for a $|BdB/dz|$ product considerably lower than that needed to fully levitate any one component. Catherall et al. [5] have studied, in vacuo, the separation of binary mixtures composed either of diamagnetic grains or of diamagnetic and paramagnetic grains and have investigated how the separation depends upon BdB/dz and Γ. The degree of separation was found by these authors to vary smoothly with these parameters, but they observe almost complete separation of a number of mixtures if the conditions are suitably chosen.

For fine grains, the influence of an ambient fluid such as air introduces two important additional effects. The first of these is fluid-driven granular convection. At low frequencies, a bed of fine grains having an initially horizontal surface breaks symmetry and tilts when vibrated vertically within a container, the bed developing strong fluid-driven granular convection. These effects were first studied by Faraday, who noted that fine particles formed erupting piles when vibrated vertically upon a horizontal platform [6] due to their interaction with the air. It was more recently confirmed that piling and tilting result from the interaction between the grains and the background fluid [7]. The detailed mechanism for these effects has since been clarified [8,9]. As a bed is thrown from a platform during vibration, fluid is drawn down through the bed into the gap which opens up below it. The fluid is later driven up through the bed as it returns towards the platform. Any deviation from horizontality of the bed leads to a horizontal component of the fluid flow. This flow causes granular movement which leads to enhancement of the tilt. If the vibration is continued, the tilt grows until the surface achieves a dynamic "angle of repose". There is then a balance between the granular motion which enhances tilt, this occurring earlier in bed flight, and the granular motion which reduces tilt. This motion principally occurs throughout the bed at times close to bed impact and through grains avalanching down the upper surface of the bed over a larger fraction of the vibration cycle [9]. Collectively, the granular movements produce granular convective circulation, the grains cascading down the tilted upper surface and returning up-slope within the bed. This "Faraday tilting" effect weakens at high frequencies, and for values of Γ high enough that the granular dynamics spill over from one cycle to the next [9]. In the present work, we shall see that Faraday tilting and its associated convection play an important role since low frequencies are used and the granular beds are thrown and substantially settle within each cycle of vibration.

In the second effect, a fluid such as air may itself induce granular separation as it is draw downwards through the bed early in bed flight and forced upwards later in flight [10,11]. If the two species of a binary mixture are influenced differently by the fluid drag, similar grains which for any reason come together will tend to move together, while dissimilar grains will be drawn apart by the fluid flow. The separating grains coarsen into fewer and fewer regions of each type. The species with the larger value of the product ρd^2 has a greater ratio of inertia to fluid drag. Through the same mechanism as that of the air-enhanced Brazil nut effect [12] this leads, at low frequencies, to that species forming a region lying above a region rich in the species having the lower value of ρd^2. Here d is the effective grain diameter. For mixtures of grains of type a and b for which the parameter

$$S = \frac{\rho_a d_a^2}{\rho_b d_b^2} \tag{3}$$

is appreciably larger than unity, almost complete separation may be found, with a above b. The boundary between the upper region rich in a and the lower region rich in b may be distinct on the length scale of a single grain [11,13]. This air-driven separation may act to assist magnetic separation or it may act in an opposite sense. We shall see that there are circumstances where magnetic separation may be adjusted to cancel the air-driven separation, ensuring a well mixed state.

Catherall et al. [14] have published a short note on the influence of air on the separation of mixtures of bronze and bismuth grains which results from the application of both inhomogeneous magnetic fields and vibration. In such systems, the relative strength of the magnetic and air-driven separation mechanisms may readily be varied through adjustment of the magnetic field. These systems therefore provide an excellent test of our understanding of the interplay of magnetic and air-driven separation and of the role of Faraday tilting, convection and the gaps which may appear between separated beds during vibration. It is also important to obtain an proper understanding of these magneto-vibratory techniques since they provide a practical means of obtaining the almost complete separation of many granular mixtures, or of creating and maintaining a mixed state.

Here, we report a detailed study of the influence of air on the vibro-magnetic separation process. We study binary systems of grains which separate under the influence of air and vibration alone and systems of grains which do not. Insight is gained into the separation dynamics through the use of two computer simulation models. The first is a simplified model which utilises one-dimensional fluid movement; the second uses a Navier–Stokes' treatment which allows two-dimensional fluid flow. Comparison between the predictions of these models further highlights the role of air-driven convection.

2 Experimental techniques

The magnetic fields used in these experiments were produced by an Oxford Instruments superconducting magnet with a closed-cycle cooling system, an arrangement specifically designed for levitation experiments. The magnet has a 5 cm diameter bore with a vertical axis, and is capable of producing a maximum magnetic field of 17 T, and a maximum field-field gradient product, $|BdB/dz|$, of 1,470 $\mathrm{T^2\,m^{-1}}$. This product may be controlled by varying the current in the superconducting coils. A mixture under investigation was shaken in a rectangular soda-glass container, of internal dimensions 10 mm by 20 mm and of height 50 mm. In most experiments a granular bed depth of 15 mm was used. Within such a geometry, the granular motion is observed to be close to two dimensional, the principal granular motion occurring parallel to the larger faces of the box. The motion is almost identical when observed through either of the large faces. The container was mounted in the bore of the magnet, on a plinth connected by a thin walled stainless-steel connecting rod to a long throw loudspeaker [5,14]. A sinusoidal waveform from a signal generator was fed to the loudspeaker via a power amplifier in order to vibrate the rod and container vertically. The rod was constrained to move in the vertical direction by slide bearings. A cantilever capacitance accelerometer attached to the bottom of the rod was used to measure the amplitude of vibration. Two lengths of connecting rod were used. The longer length positions the container in the upper part of the bore where BdB/dz is maximally negative. A shorter length was used to position the container in the symmetric lower part of the bore where BdB/dz is maximally positive. In each of these positions there is a local maximum in $|BdB/dz|$ In either region $|Bdb/dz|$ falls on moving in a radial direction, but less severely than it does on moving away along the positive or negative z axis. For a bed centred on either of these regions $|BdB/dz|$ falls from its maximum value by at most 4% within the region occupied by the bed during vibration. In this report we will use the maximum value of $|BdB/dz|$, that at the centre of the bed.

3 The experimental observation of magneto-vibratory separation

3.1 The separation of mixtures of bismuth and bronze grains of equal sizes

Firstly we shall consider the dynamics of a mixture of fine bronze and bismuth grains, the grains of each species being close to spherical. The density of bismuth is 9,800 $\mathrm{kg\,m^{-3}}$ and its magnetic susceptibility, χ, is -165×10^{-6} while the corresponding figures for bronze are 8,900 $\mathrm{kg\,m^{-3}}$ and -5.5×10^{-6} respectively. The low magnetic susceptibility of bronze ensures that its effective gravity is hardly modified over the range of fields used in these experiments; $\tilde{\Gamma}_{Br}$ is therefore close to Γ. Bismuth is much more strongly influenced by a magnetic field and it levitates when $-BdB/dz = 730\ \mathrm{T^2\,m^{-1}}$.

Here, we consider a mixture containing equal volumes of bismuth and bronze in which both species have the same size range, 75–90 μm. Preliminary studies of this mixture were briefly presented in ref. [14]. In zero magnetic field, significant air-driven separation is not expected since the parameter S, eqn. 3, is close to unity, and indeed such a mixture is observed not to show appreciable separation under a wide range of vibratory conditions. The initial well mixed state shown in Fig. 1a is visually indistinguishable from the state obtained after a considerable period of vibration in zero field. In a magnetic field, however, very good separation may be obtained as Fig. 1 shows. Here $f = 10\,\mathrm{Hz}$, $\Gamma = 1.5$ and $-BdB/dz = 500\ \mathrm{T^2\,m^{-1}}$. Granular convection is indicated by the overlaid arrowed black lines. In this magnetic field the effective gravities of the bronze and bismuth components are 9.7 and 3.1 $\mathrm{ms^{-2}}$, respectively. Having a lower effective gravity than the bronze, the bismuth migrates towards the top of the bed, whilst the bronze grains migrate downwards. It may be seen that after 15 s of vibration the upper part of the bed has become rich in bismuth while the lower region has become rich in bronze. The upper bismuth region develops a vigorous convection cell, while the lower region initially contains two convection cells. As separation

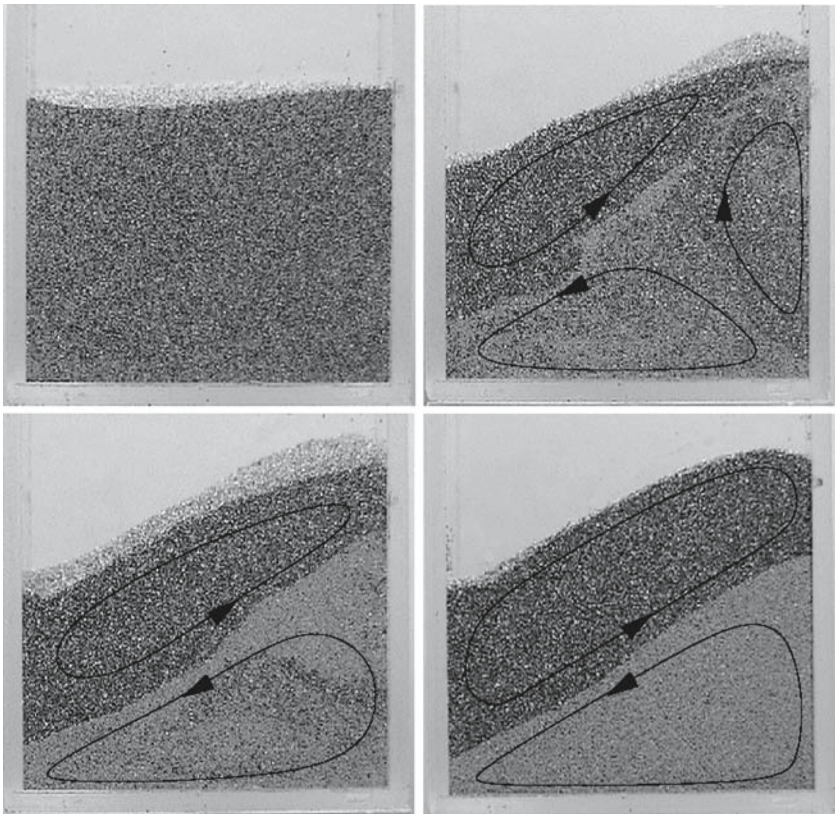

Fig. 1 The separation of a 50%:50% mixture by volume of bismuth (*darker*) and bronze (*lighter*) grains of diameters 75–90 μm. $BdB/dz = -500\ \mathrm{T^2\,m^{-1}}$, $\Gamma = 1.5$ and $f = 10\,\mathrm{Hz}$. The images are taken at **a** 0s; **b** 15s; **c** 30s and **d** 120s of vibration. The *arrowed black lines* indicate granular convection

develops one of these lower cells weakens and disappears completely after about 25 s. Soon after one minute, an equilibrium is reached in which there is one convection cell in an upper region almost pure in bismuth and a second convection cell in a lower region almost pure in bronze, Fig. 1d.

During the development of the separation shown in Fig. 1, there is little evidence for the local clustering and coarsening characteristic of the strong air-driven separation which would be present if the parameter S was far from unity. However, air causes the development of the Faraday tilting and the associated air-driven convection which is evident in the figure; these features are not present if the box is evacuated. A Faraday tilted bed composed of a single type of fine grain usually exhibits one principal convection cell. Here, the differential gravity separation mechanism leads, eventually, to two separate strong convection cells even though the air drag on the two species of grain is very similar. Since these two convection cells have the same sense there appears to be intense granular shear at the sharp interface between the bronze and bismuth regions (Fig. 1d). At the same time no granular mixing is apparent. Once the convection cells are established the granular circulation in both the upper and lower regions enables the magnetic separation to be completed more speedily as minority grains are flushed out of each region.

The influence of varying Γ and BdB/dz on the separation dynamics of this mixture was investigated at the fixed vibrational frequency of 10 Hz. The mixture was observed to evolve into one of three definite equilibrium states depending upon the values of these two parameters. We describe as *well separated* a state in which almost complete separation into two tilted regions occurs. In this state the purity of each region is typically better than 99.9% and the boundary between the two regions is sharp to within 1-2 grains. An example of such a state is shown in Fig. 1d. A second state of separation we describe as *partially separated*. This equilibrium state is similar in appearance to the transient state shown in Fig. 1b. There are two distinct tilted beds, an upper one consisting mainly of bismuth and a lower one mainly of bronze. A strong convection cell exists within each bed, but there is also a somewhat weaker third cell linking the upper and lower beds near the upper part of the boundary between the two layers. This third cell is responsible for a degree of mixing which limits the purity of each bed to approximately 80%. A third type of equilibrium state we describe as *mixed*. Under vibration the bed tilts, but little or no separation is apparent and the equilibrium state of the vibrated mixture exhibits only one large global convection cell.

Figure 2 shows the conditions for the different equilibrium states. We generally used an initially well mixed initial state and vibration was applied until no further changes in the configuration occurred. Clear regions of the *mixed* (α), *partial* (β) and *well separated* (γ) configuration are evident. The boundaries between these regions are abrupt, each region corresponding to quite distinct convection configurations. Within the *well separated* region, γ, the quality of the separation is virtually independent of Γ and BdB/dz. These observations are significantly different from those reported for vacuum conditions where the degree of separation was observed to vary gradually with Γ and BdB/dz [5]. In the *mixed* region, α, global air-driven convection overcomes any tendency to segregate due to the differences in effective gravity of the two species. For values of $-BdB/dz$ close to that for levitation of the bismuth grains, 730 $\mathrm{T}^2\,\mathrm{m}^{-1}$, the granular bed was observed to be influenced by magnetic cohesion at lower values of Γ. In this region, ϵ, the cohesive effect of the magnetic dipole-dipole interactions between the bismuth grains is sufficient to trap bronze grains within cages of attracting bismuth grains at the lower amplitudes of vibration insufficient to overcome this effect. This cohesion slows both the rate of separation and the final quality of separation in this region.

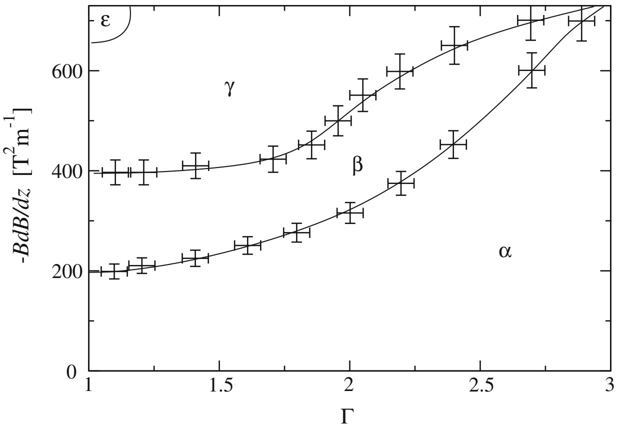

Fig. 2 A diagram indicating the conditions for the observation of each of the three equilibrium states for a 50%:50% mixture by volume of 75–90 μm bismuth and bronze grains, vibrated at 10 Hz. The regions of the *mixed* state, α, of the *partially separated* "bismuth on top" state, β, and of the *well separated* "bismuth on top" configuration, γ, are shown, as is the region influenced by appreciable magnetic cohesion, ε. The error bars indicate the reproducibility of the location of the transitions between these regions. The transitions usually occur over a range of variables appreciably smaller than the error bars. Unlike the other boundaries in this figure, the boundary of the cohesive region ϵ is not sharp. The line is only an indication of the region where cohesive effects are noticeable

Although most of our investigations used a well mixed initial state, the equilibrium state reached after an appreciable period of vibration was found quite generally to depend only upon Γ and BdB/dz and not upon the initial state.

The influence of frequency on the separation dynamics was investigated at a number of fixed values of Γ and $-BdB/dz$. It was observed that the conditions for the onset of the *partially separated* "bismuth on top" and of the *well separated* "bismuth on top" configurations were little affected by changes in frequency over the range investigated, 10–40 Hz. However, the time-scale for reaching equilibrium from an initially well-mixed state was greatly influenced by both frequency and Γ. The equilibrium separation time was obtained by closely analysing and comparing images taken after successive 10s periods of vibration until no further development was observed to occur. Although this method relies on human judgement the results were reproducible to about ±10%. They showed that, within the *well separated* region, the time to equilibrium varies as $\omega^{2.1\pm0.3}$ and as $1/(\Gamma\text{-}1)^{2.8\pm0.5}$. Since the more rapid separation occurs at lower frequencies, most of the experiments reported here were carried out at 10 Hz, towards the lower limit of our equipment.

Altering the particle size of both components, while keeping their mean size equal, is found to affect the conditions for the onset of both the *well separated* and *partially separated* configurations. In each case, reducing the particle size increases the $|BdB/dz|$ product required for that type of separation to occur. The minimum $|BdB/dz|$ product required for the *well separated* state, that found at values of Γ just above unity (see Fig. 2), is found to have the form $-BdB/dz = p + q/d^2$, where $p = 270 \pm 10\ \mathrm{T}^2\,\mathrm{m}^{-1}$ and $q = (9.2 \pm 1.2) \times 10^{-7}\ \mathrm{T}^2\,\mathrm{m}$.

The separation phenomena which we have observed result from a complex interplay of the effects of Earth's gravity, magnetic forces, air damping forces and intergranular collisions; an analytical treatment is not currently available. However, reducing the particle sizes increases the influence of air-damping with respect to the influence of magnetic forces. It is then reasonable to suppose that larger values of $|BdB/dz|$ are required for separation to be observed. Either species of grain is influenced by the sum of the magnetic forces, and the air drag forces. The magnetic forces are proportional to $VBdB/dz$ and therefore to d^3BdB/dz, while the air drag forces are proportional, for laminar fluid flow, to $\kappa d\eta u$. Here η is the dynamic viscosity of the fluid, u is the fluid velocity with respect to the grains, and κ is a factor containing the granular porosity of the bed. Separation depends upon the differences in behaviour of the two species of grain. The equation of motion for the relative acceleration of the two components contains a drag term, proportional to $1/d^2$ and a magnetic term, proportional to BdB/dz. If we suppose that the onset of separation occurs when the relative acceleration is sufficiently large to overcome collisional mixing, the observed dependence of BdB/dz on d follows.

For mixtures of grains of equal sizes, reducing the size of the particles not only raises the magnitude of $|BdB/dz|$ required for the onset of the *well separated* state, but also extends the range of influence of magnetic cohesion, since the effect of the magnetic dipole-dipole interaction is larger for smaller particles [5].

3.2 The separation of mixtures of bronze and bismuth grains of unequal sizes

We now consider the effect of varying the size ratio of the bronze and bismuth grains. Firstly we shall consider a mixture composed of 75–90 μm bismuth grains and 200–250 μm bronze grains in the ratio 50%:50% by volume. Here, the bronze grains are less effected by air than the smaller bismuth grains, since the parameter $S = \rho_{Br}d_{Br}^2/\rho_{Bi}d_{Bi}^2 \approx 6.8$ for this mixture. Under vertical vibration air-driven "bronze on top" separation is expected in the absence of a magnetic field [10,11] and indeed this is the case. Figure 3a shows the equilibrium state of the mixture, reached after 60s of vibration at 10 Hz and $\Gamma = 1.5$. The mixture separates with a very pure bronze layer above a very pure bismuth region. The finer bismuth grains exhibit a greater tendency to Faraday tilting than the coarser bronze grains [9]; the convection is correspondingly stronger in the former. In zero magnetic field, separation with "bronze on top" is obtained over a wide range of vibrational conditions. By applying a magnetic field such that $-BdB/dz = 600\ \mathrm{T^2\,m^{-1}}$ at 10 Hz and $\Gamma = 1.5$ the differential gravity separation mechanism dominates the air-driven mechanism, causing the mixture to separate with the bismuth above the bronze, as is shown in Fig. 3d. Interestingly, it is possible to balance the competition between the two mechanisms and to cause the system to remain in a mixed state under vibration, or to mix an already separated system. Figure 3b shows that the equilibrium separation state of a mixture vibrated under $-BdB/dz = 250\ \mathrm{T^2\,m^{-1}}$ is well mixed throughout its bulk, although there is a very thin layer of bronze grains at the upper surface. This experiment was repeated with a number of initial states including the separated granular configurations of Fig. 3a and d. The final state was observed to be independent of the initial configuration. Indeed this is true of all of the experiments reported here. The final equilibrium state is independent of the initial state.

A schematic diagram displaying the vibratory and magnetic conditions for the different equilibrium states is shown in Fig. 4. Regions of the *well separated* and *partially separated* configurations and of the *mixed* state are found which are similar in general form to those for equal sized particles. At higher values of $|BdB/dz|$, in region γ, a *well separated* "bismuth on top" configuration is found, each region having a single convection cell. However, in these unequal sized mixtures the purity of the *well separated* state falls short of that for the equal sized mixtures, reaching about 99% by volume rather than better than 99.9% purity. In the region α of

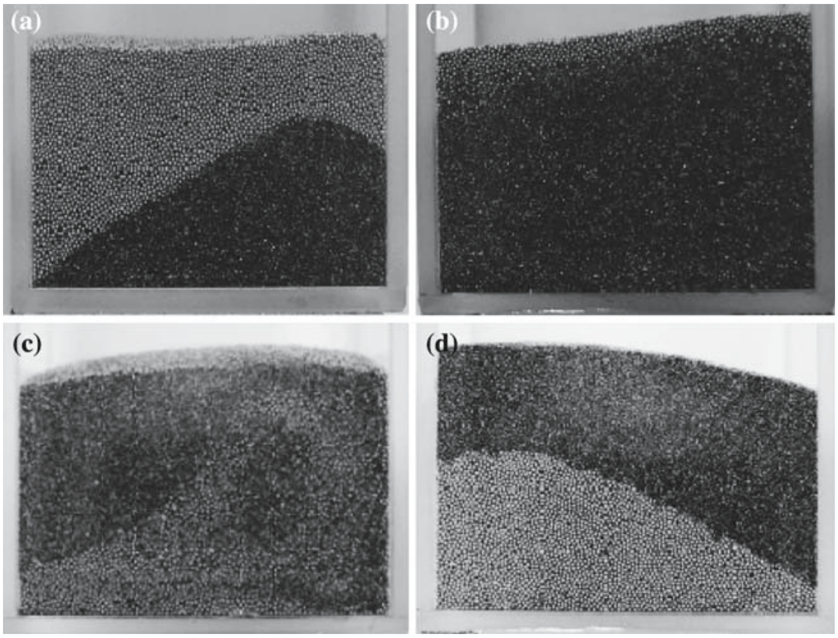

Fig. 3 The separation of a 50%:50% mixture by volume of 75–90 μm bismuth grains (darker) and 200–255 μm bronze grains (lighter), when vibrated at $\Gamma = 1.5$ and $f = 10$ Hz. The images were taken after the system had reached equilibrium taken at (a) $BdB/dz = 0\ \mathrm{T^2\,m^{-1}}$, (b) $-BdB/dz = 250\ \mathrm{T^2\,m^{-1}}$, (c) $-BdB/dz = 400\ \mathrm{T^2\,m^{-1}}$ and (d) $-BdB/dz = 600\ \mathrm{T^2\,m^{-1}}$

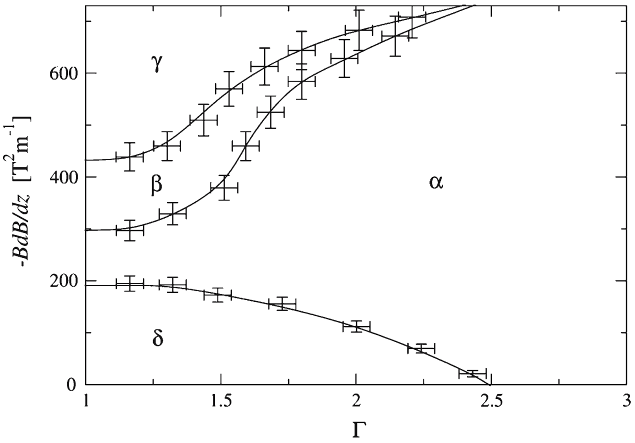

Fig. 4 A diagram indicating the conditions for the observation of each of the equilibrium states for a 50%:50% mixture by volume of 75–90 µm bismuth grains and 200–255 µm bronze grains, vibrated at 10 Hz. The regions of the *mixed* state, α, of the *partially separated* "bismuth on top" state, β, of the *well separated* "bismuth on top" configuration, γ, and of the *well separated* "bronze on top" configuration, δ, are shown

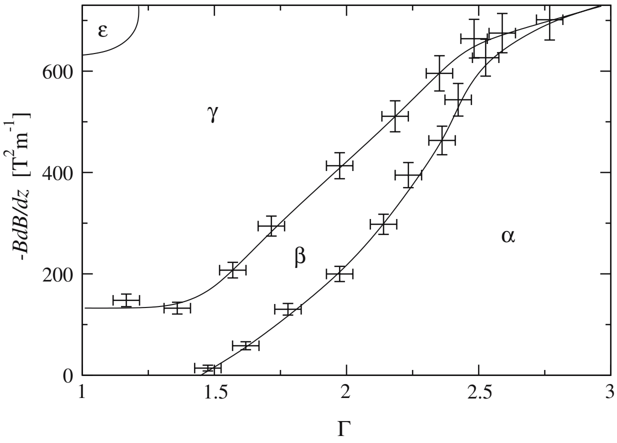

Fig. 5 A diagram indicating the conditions for the observation of each of the equilibrium states for a 50%:50% mixture by volume of 75–90 µm bismuth grains and 38–53 µm bronze grains, vibrated at 10 Hz. The regions of the *mixed* state, α, of the *partially separated* "bismuth on top" state, β, and of a *well separated* "bismuth on top" configuration, γ, are shown as is the region influenced by appreciable magnetic cohesion, ε

Fig. 4 the *mixed* state is observed, there being little or no segregation. In region β a *partially separated* "bismuth on top" configuration is found. Here appreciable separation is evident with a tilted bismuth rich layer forming above a bronze rich layer. The bismuth region has a single convection cell, while the lower bronze region has a principal convection cell and a second weaker cell close to one wall which causes a degree of mixing between the bronze-rich and bismuth rich beds. In the region δ, a *well separated* "bronze on top" configuration is found again with a single convection cell in each region. We observe no appreciable region of *partial separation* between the regions α and δ. We also observe little sign of magneto-cohesion in this system. The attraction between bismuth grains here is insufficient to cage the far larger bronze particles and the effects of magnetic cohesion are very weak.

A second mixture of unequal sizes which we investigated consisted of 75–90 µm bismuth grains and 38–53 µm bronze grains in the ratio 50%:50% by volume. Here, the influence of air will tend to cause the mixture to separate with the bismuth uppermost under vibration alone since $S = \rho_{Br}d_{Br}^2/\rho_{Bi}d_{Bi}^2 \approx 0.28$. Figure 5 shows, schematically, the conditions of Γ and of $-BdB/dz$ for the various forms of separation. Region α indicates a region where no appreciable separation occurs, while β indicates where a *partially separated* configuration may be found. γ is a region of a *well separated* "bismuth on top" configuration, while ε indicates a region where the separation is appreciably restricted by magnetic dipole-dipole forces.

The similarity between the principal boundaries of Figs. 2, 4 and 5 should be noted. Increasing the size of the bronze grains from 75–90 µm to 200–250 µm, while keeping the bismuth grain sizes fixed, increases the values of $|BdB/dz|$ at which each boundary occurs, increasingly so at lower values of Γ. The boundary between the mixed state and the "bronze on top" state is drawn from positive values of BdB/dz to the negative values shown in Fig. 4. Decreasing the size of the bronze grains from 75–90 µm to 38–53 µm has the opposite effect, pushing the boundaries towards more positive values of BdB/dz. Figure 6 shows the minimum value of $|BdB/dz|$ required for the onset of a *well separated* "bismuth on top" configuration, plotted as a function of the bronze grain diameter for $f = 10$ Hz, and $\Gamma = 1.4$. Here, the bismuth particle diameters remain fixed within the range 75–90 µm. The data of Fig. 6 may be fitted to a relationship of the form $-BdB/dz = r - s/d^2$, where $r = 472 \pm 24$ T^2 m^{-1} and $s = -(6.7 \pm 0.6) \times 10^{-7}$ T^2 m.

We suppose that increasing the size of the bronze particles reduces the effect of air damping on their motion, causing them to be thrown higher during each vibration cycle. Consequently a larger magnetic force is required for the bismuth to segregate towards the upper surface of the mixture. The observed dependence on d_{Br} follows from the tentative arguments given in Sect. 3.1.

In these mixtures of unequal sizes, where the air-driven separation mechanism operates, there is some evidence for clustering and coarsening during the progress to equilibrium. This has been previously reported as a feature of fluid-driven separation [9–11].

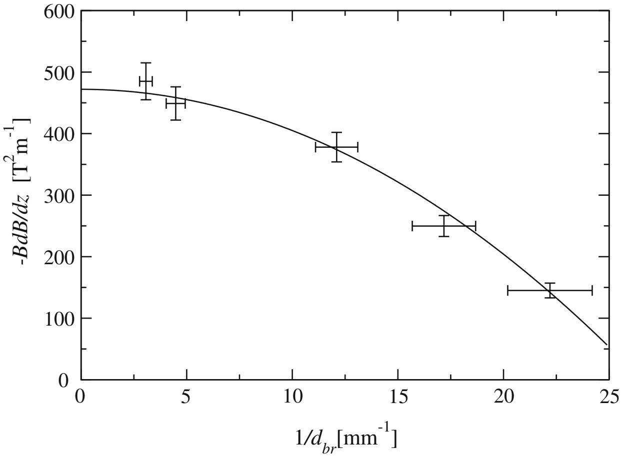

Fig. 6 The minimum field-gradient product, $-BdB/dz$ required to produce a *well separated* "bismuth on top" configuration at $f = 10$ Hz. and $\Gamma = 1.4$, plotted against inverse bronze particle diameter, for mixtures containing bismuth particles in the range 75–90 µm. Bronze grains with a mean size ranging from 45 to 327 µm were used in these experiments. The *solid line* is the fit to the data described in the text

3.3 The separation of mixtures of bismuth and glass

Bismuth and bronze have similar densities. A number of experiments were conducted on mixtures of bismuth grains and soda-glass ballotini ($\rho = 2{,}500\,\mathrm{kg\,m^{-3}}$ and $\chi = -5.5 \times 10^{-6}$) in order to examine the behaviour of systems of grains where the components have appreciably different densities and it is therefore possible to obtain a value for S which is appreciably different from unity, while having similar grain sizes.

We considered a 50%:50% mixture, in which both species have diameters in the range 75–90 µm. For such a mixture $S = \rho_{Bi} d_{Bi}^2 / \rho_{Gl} d_{Gl}^2 \approx 3.5$; it will tend to separate with the bismuth uppermost under vibration alone. An image showing the equilibrium state of separation under $f = 10$ Hz and $\Gamma = 1.5$ is shown in Fig. 7a. Applying a negative field gradient reduces the effective gravity of the bismuth and the air-driven and differential gravity separation mechanisms act in concert, leading to enhanced separation on a shortened time scale. For example, with $\Gamma = 1.5$ the mixture takes about 10 min to reach equilibrium separation to a purity of about 95% in zero magnetic field while, in $-BdB/dz = -500\, T^2\, m^{-1}$, 99.9% purity is reached within 5 minutes. Attempts to cancel the air-driven and differential gravity separation mechanisms against each other produced, for this mixture, mixed beds of which an example is shown in Fig. 7c for $\Gamma = 1.5$ at $BdB/dz = 200\, T^2\, m^{-1}$. Some slight structure is still evident. The application of a large positive BdB/dz further enhances the effective gravity of the bismuth to produce a *well separated* "glass on top" state. An example is shown in Fig. 7d for $\Gamma = 2.0$.

A diagram showing the magnetic and vibratory conditions for obtaining the various equilibrium states is shown in Fig. 8, for positive values of BdB/dz. In the region labelled γ, and also for negative values of BdB/dz, a *well separated* "bismuth on top" configuration is observed, with purities better than 99.9%. The dashed line indicates the condition $\tilde{\Gamma}_{Bi} = 1$. To the left of

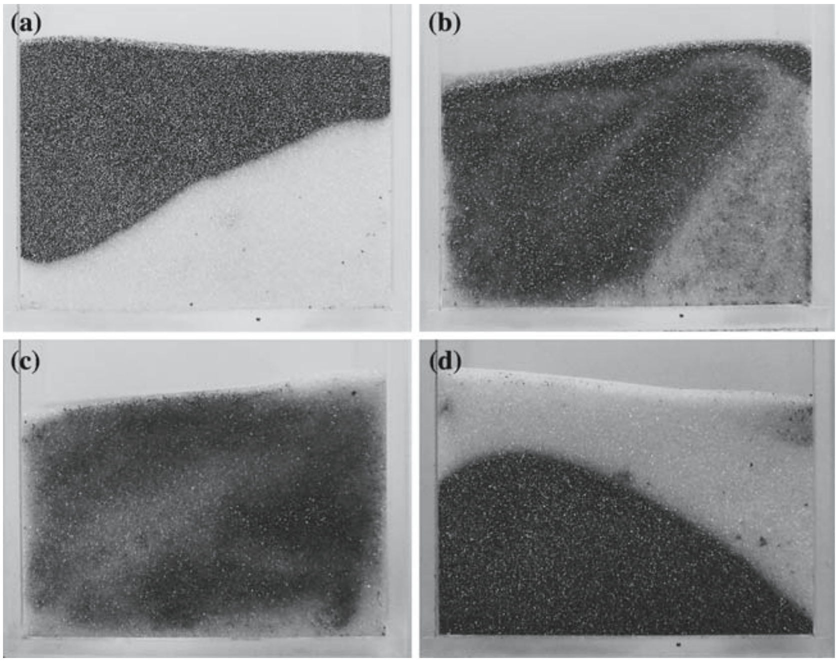

Fig. 7 Images showing the equilibrium separation of 50%:50% by volume 75–90 µm bismuth and glass mixtures after vibration in air at 10 Hz. **a** $\Gamma = 1.5$ and $BdB/dz = 0\ \mathrm{T^2\,m^{-1}}$, **b** $\Gamma = 2.2$ and $BdB/dz = 200\ \mathrm{T^2\,m^{-1}}$, **c** $\Gamma = 1.6$ and $BdB/dz = 200\ \mathrm{T^2\,m^{-1}}$ and **d** $\Gamma = 2.0$ and $BdB/dz = 500\ \mathrm{T^2\,m^{-1}}$

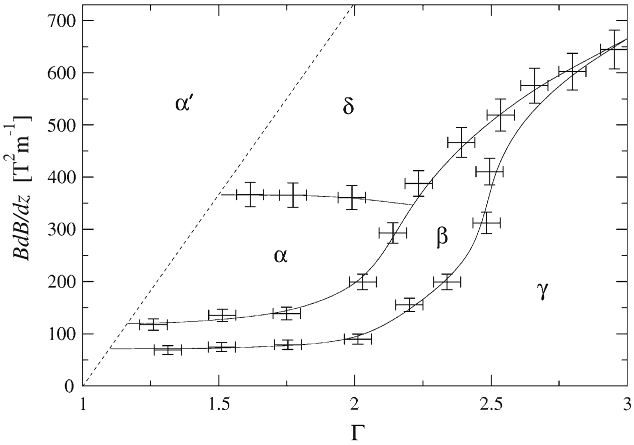

Fig. 8 A diagram indicating the conditions for the observation of each of the equilibrium states for a 50%:50% mixture by volume of 75–90 μm diameter bismuth and glass grains, vibrated at 10 Hz. The regions of the *mixed* state, α, of the *partially separated* "bismuth on top" state, β, of a *well separated* "bismuth on top" configuration, γ, and of a *well separated* "glass on top" configuration, δ, are shown. In the region α' the bed is inactive and an initially mixed bed will remain mixed

this line, in the region α', the granular bed is inactive; an initially well mixed bed will remain mixed. As this region is approached from higher values of Γ, the dynamics slow and within the region itself, the relative motion ceases altogether. In the region β we observe a *partially separated* configuration. Here there is a bismuth-rich layer above a glass-rich layer, each having a principal convection cell, but there is mixing at the interface with additional convective activity in that region. As a result the degree of separation falls far short of that of the *well separated* state (Fig. 7b shows an example). In the region α, the system remains in the well mixed state with a single convection cell which continually mixes the glass and bismuth components. Figure 7c shows an example of such a state. At sufficiently high values of $|BdB/dz|$, in the region δ, the system separates into a *well separated* "glass on top" state; an example is shown in Fig. 7d. With the exception of the boundaries with α', we observe that the boundaries between the various regions are abrupt and correspond to quite distinct systems of vibration induced granular convection.

4 Simulation techniques

We now describe numerical simulations of the separation. In these simulations we do not aim to reproduce every feature found in experiment. Indeed this is not possible with the computing facilities at our disposal. For example, the number of grains in our simulations (thousands) is far lower than in experiment (many millions). Consequently, the predicted time-scales for separation are shorter than in reality. Rather, the intention is to obtain insight into features of the experiments which are difficult to investigate through direct observation.

We have used two models for the fluid-grain interactions. In both models, each grain is treated as a sphere which undergoes inelastic collisions with other grains and with the container walls. The collisions are modelled using a linear spring-dashpot in the normal direction and Coulomb friction in the tangential direction [15]. The interaction of the particles with the magnetic field is treated through a modified acceleration due to gravity, Eqn. 2. The model parameters are shown in Table 1. The properties of the fluid, air, are also shown in the table.

The first model, referred to here as SD, uses a simple Stokes' drag force [16] which is applied to each particle and is dependent on the particle size and velocity relative to the container base. The fluid is treated as incompressible, a satisfactory approximation for the low frequencies and shallow beds used here [17]. This model couples the grains to a fluid that is being driven by the vibrated container. Within this model, the fluid moves with the container in the direction of vibration, as it would in the absence of the grains. Although simple, this model reproduces many features of the air-driven separation phenomenon observed experimentally [16].

The second, principal, modelling method, referred to as NS, involves the simulation of the fluid dynamics using the two-dimensional Navier–Stokes' equations. The velocity field $\mathbf{v}$ and pressure field P are assumed to obey the incompressible Navier–Stokes' equation,

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\mathbf{v} + \mathbf{g} + f(\mathbf{v} - \mathbf{u}), \tag{4}$$

where ρ is the fluid density, ν is the kinematic viscosity of the fluid, $\mathbf{g}$ the acceleration due to gravity and $\mathbf{u}$ is the mean granular velocity. The forcing term $f(\mathbf{v} - \mathbf{u})$ models the coupling between the fluid and grains through

Table 1 Particle parameters used in the simulations

Parameter	Value
Number of bronze particles	1,250
Number of bismuth particles	1,250
Particle diameter	75–90 μm
Density of bronze	8,900 kg m^{-1}
Density of bismuth	9,800 kg m^{-1}
Magnetic susceptibility of bronze	-5.5×10^{-6}
Magnetic susceptibility of bismuth	-165×10^{-6}
Container dimensions (mm)	2.0, 0.5, 5.0
Spring constant	1,000 N m^{-1}
Friction coefficient	0.2
Density of Air	1.2 kg m^{-3}
Kinematic viscosity of air	1.5 10^{-5} m^2 s^{-1}

the use of Ergun's bed equation [20], treating the grains locally as a porous medium. Equation 4 is solved numerically in conjunction with the fluid-grain continuity equation [18,19]. The fluid equations are discretized on a staggered grid [21] and time integrated in a similar manner to the equation of motion of each particle. A two-dimensional treatment of the fluid is appropriate for the box geometry used here, for which experimental observations show that the granular and fluid motion are close to two dimensional. Further details of the model can be found in references [22,23].

The two-dimensional modelling of the fluid in the NS method introduces an additional degree of freedom, motion in the horizontal direction. The presence of the grains influences the flow of the fluid and so changes how the fluid affects the flow of the grains. A similar model has been used to investigate Faraday tilting in vibrated granular beds [9].

5 Simulation results

We consider the behaviour of 1,250 bismuth and 1,250 bronze spheres with diameters in the range 75–90 μm, vibrated in a rectangular box of dimensions 0.5 mm by 2mm in the horizontal plane. Figure 9a shows the time development of this mixture under $\Gamma = 1.5$ at 10 Hz, and $BdB/dz = -400\ \mathrm{T}^2\,\mathrm{m}^{-1}$, using the SD method to model the influence of air. Soon after vibration is applied, the top of the bed becomes very rich in bismuth and the bottom of the bed very rich in bronze. Eventually the mixture separates into a horizontal upper bismuth layer over a horizontal lower bronze layer, with a sharp boundary between the two. However, there is a notable absence of Faraday tilting and there is almost no convective circulation of air or of grains within the bed. Figure 9b shows the application of the NS model to the same system, also for $\Gamma = 1.5$ at 10 Hz, but in the absence of a magnetic field. The bed breaks symmetry and in response to horizontal air movements within the bed forms a distinctive tilt after about 0.5s of vibration. At the same time, a convection cell is established within the bed, grains cascading down the upper slope and returning up-slope within the bulk. No separation occurs. These features are very similar to those observed experimentally and the angle of tilt, about 22°, is also in good agreement with the experiment [9]. Figure 9c shows the behaviour predicted by the NS model when a magnetic field with $BdB/dz = -400\ \mathrm{T}^2\,\mathrm{m}^{-1}$ is applied under the same vibratory conditions. Soon after vibration begins, the bed breaks symmetry, developing a tilt due to the Faraday piling effect. At the same time convection drives

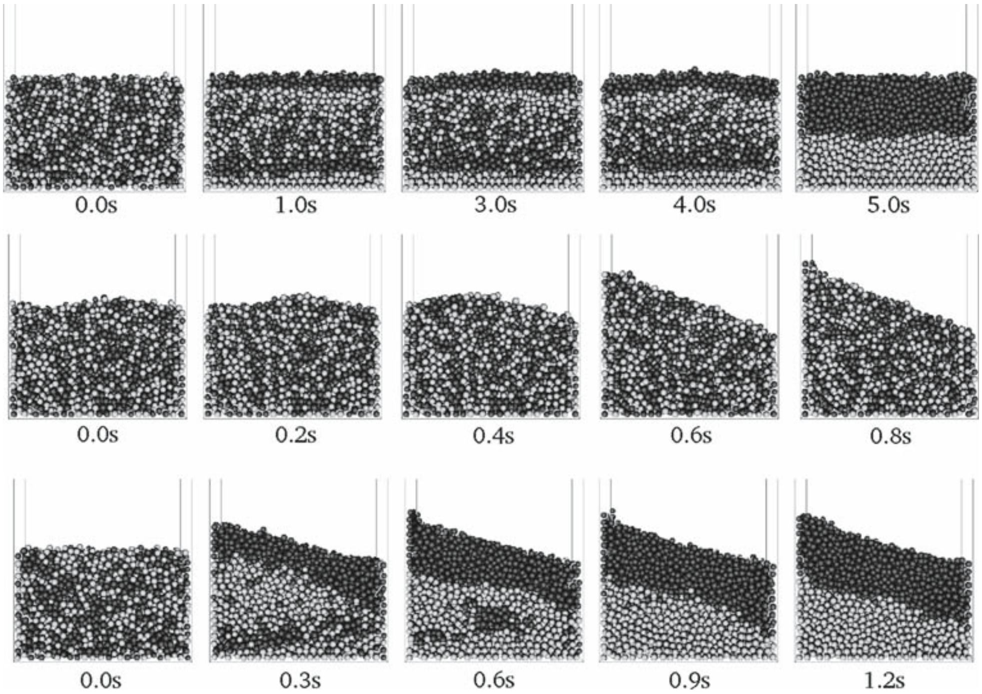

Fig. 9 The time development of the granular behaviour, when vibrated in air at 10 Hz and with $\Gamma = 1.5$, of a collection of 1,250 bismuth (*darker*) and 1,250 bronze (*lighter*) spheres of diameters in the range 75–90 μm, observed in numerical simulations.**a** with $-BdB/dz = 400\ \mathrm{T}^2\,\mathrm{m}^{-1}$ using the SD model, **b** in zero field using the NS model and **c** with $-BdB/dz = 400\ \mathrm{T}^2\,\mathrm{m}^{-1}$ using the NS model

grains around the bed, clockwise in the example shown. Bismuth grains begin to concentrate on the upper surface, forming a separate layer with its own convective flow after approximately 0.6s. Further vibration leads to an almost complete separation, with a pure bismuth bed above a pure bronze bed, each having its own convection cell.

These comparisons show that a model allowing two dimensional fluid flow is necessary to correctly describe the development of the tilt and granular convection which are such important features of the experiments. The separation times predicted by the SD model are far slower than those of the NS model, since the NS model allows the strong convective circulation which speeds separation. These findings complement those of Milburn et al. [9], on the influence of convection on air-driven granular separation. Using a similar method to Milburn et al. to quantify the rates of convection, we find that the convection rates of the NS model increase rapidly from those of the SD model as Γ is increased until they are typically two orders of magnitude greater between $\Gamma = 1.4$ and $\Gamma = 3$. Similar large factors are also found in the presence of a magnetic field. We shall now only consider simulations using the NS model.

We have noted that when beds separate as in Fig. 9c, there is strong but independent granular circulation in the upper and lower layers. This circulation is in the same sense, and there is therefore apparent granular shear at the interface; yet we observe no mixing. Simulations using the NS model make the reason for this clear. The majority of the granular convective motion within each bed starts and stops during each cycle of vibration. Almost all of the convective motion occurs during parts of the vibration cycle when gaps exist between the separated beds, as they are both thrown. It is these gaps which prevent mixing. Such a gap is shown in Fig. 10. If, as in the figure, the two grain species have the same size this gap is very clear. However, if one species (usually the upper) is far larger than the other, then small grains may be found in the gap, which becomes a region of lower granular density, rather than a clearly defined void. The separation will then be less complete as we have observed in our experiments on grains of unequal sizes. We find in simulations that excellent separation is always associated with well defined gaps during vibration. Gaps will only develop between two regions if they have different flight paths through differences in composition. If these differences become sufficient to develop an appreciable gap, then different granular circulation patterns develop in the two regions and the separation will greatly improve. This, in turn, enables the gap to become even more distinct and almost complete separation may result.

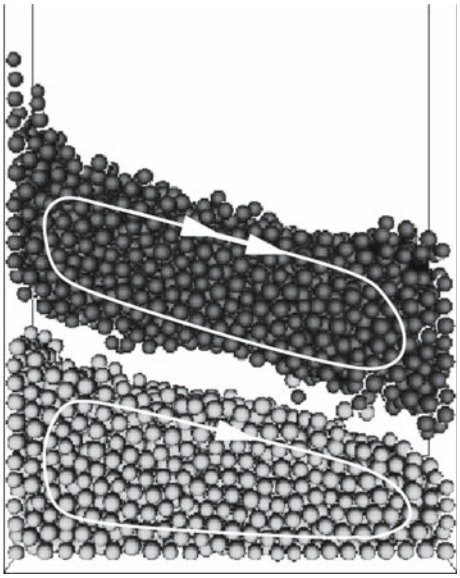

Fig. 10 Image obtained through the use of the NS model showing the presence of a gap between the separated bismuth (*darker*) and bronze (*lighter*) beds during their flight. Here $f = 10$ Hz, and $\Gamma = 2.5$. The *arrowed lines* show granular convection

In fine granular systems these gaps are very small and are therefore extremely difficult to observe in experiments, although they have been seen in water immersed systems of large grains [13].

In our experiments we have seen that distinct regions of behaviour are observed as BdB/dz and Γ are changed (Figs. 2, 4, 5). Each region corresponds to a distinctive pattern of convection and degree of separation; the boundary between the regions is well defined unlike the situation for vacuum systems [5]. The use of the NS model enables us to study these regions and to examine the sharpness of their boundaries. Figure 11 shows the equilibrium separation of 1,250 bismuth and 1,250 bronze particles, as $|BdB/dz|$ is increased, both species having sizes within the range 75–90 μm. Here $\Gamma = 1.5$. At $-BdB/dz = 0$ and 50 $\mathrm{T}^2\,\mathrm{m}^{-1}$ global convection mixes the two species. Close to $BdB/dz = -100$ $\mathrm{T}^2\,\mathrm{m}^{-1}$ separation into an upper bismuth and a lower bronze layer occurs, as the diagram shows. Where this happens, distinct granular convection in the two regions is established and gaps open up between the two beds in flight. However, the separation here is far from complete. For stronger fields for which $BdB/dz = -150$ $\mathrm{T}^2\,\mathrm{m}^{-1}$ and -200 $\mathrm{T}^2\,\mathrm{m}^{-1}$ the separation is substantially complete. Very well defined gaps occur in during bed flight, with distinct convection cells within the separated regions.

Quantitatively the simulations display a transition between the mixed state and the well separated state at a substantially lower value of $|BdB/dz|$ than in experiment (Fig. 2). The transition is broader in simulation

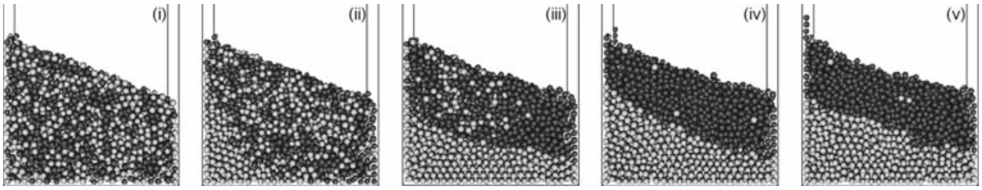

Fig. 11 The behaviour, when vibrated in air at 10 Hz and $\Gamma = 1.5$, of a collection of 1,250 bismuth and 1,250 bronze spheres of diameters in the range 75–90 μm observed in numerical simulations using the NS model. The figure shows the equilibrium configurations for the following values of BdB/dz in $\mathrm{T}^2\,\mathrm{m}^{-1}$) (*i*) 0, (*ii*) −50, (*iii*) −100, (*iv*) −150 and (*v*) −200

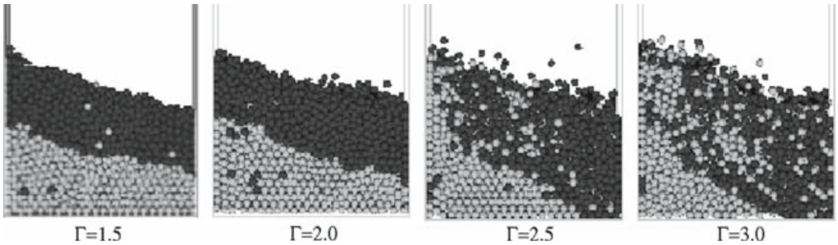

Fig. 12 The behaviour when vibrated in air at 10 Hz, of a collection of 1,250 bismuth and 1,250 bronze spheres of diameters in the range 75–90 μm observed in numerical simulations using the NS model. The figure shows the configurations after 5s of vibration with $BdB/dz = -150\ \mathrm{T}^2\,\mathrm{m}^{-1}$ and at the values of Γ shown

than in experiment, and, in the simulations which we have just described, a *partially separated* state with a third convection cell is not well developed. However, by varying the number of particles in our simulations we find that the abruptness of the transition increases with particle number as does the value of $|BdB/dz|$ at the transition. We conjecture that, were we able to simulate realistic particle numbers, we would be better able to obtain quantitative agreement with experiment, and better able to study the state of *partial separation*.

Figure 12 shows images of the system of 1,250 bismuth and 1,250 bronze particles after 5 s of vibration when exposed to $BdB/dz = -150\ \mathrm{T}^2\,\mathrm{m}^{-1}$ at various values of Γ. As Γ is increased the gap deteriorates and the convection passes from being local within each separated region to being global, this leading to strong mixing. These observations are qualitatively in agreement with the behaviours represented in Fig. 2. However, the quantitative agreement is again only reasonable. The transition from a well separated to a mixed state is not as abrupt as in experiment, and the existence of a well defined *partially separated* state is not clearly evident. The values of $|BdB/dz|$ are lower in simulation than in experiment. It is likely that these discrepancies too, are due to using an insufficiently large number of particles in simulation.

6 Discussion

The application of vertical vibration plus an inhomogeneous magnetic field is an effective way of separating a wide range of weakly magnetic materials. Our motivation for studying these systems is twofold. Not only do they provide an excellent test of our understanding of the physical processes involved, but they provide a flexible tool for manipulating the separation of many granular mixtures. The separation results from the grains being thrown by the vibration, the different species flying differently since they experience different effective gravitational forces. It may well be desirable to conduct the separation in air. However, for fine particles, air appreciably influences the dynamics, introducing effects which include tilting, enhanced convection and air-driven separation. These effects will be important for particles of diameters appreciably smaller than about 1 mm. In this paper we have discussed the behaviour of fine binary granular mixtures under the joint influence of air and the tendency to separation through differential effective gravity.

We have examined a mixture in which air-driven separation is not itself expected since the grains have much the same density and the same size range. For such mixtures air introduces the additional features of Faraday tilting and air-driven convection, features which speed magneto-vibratory separation through circulating the separating mixture. We have also studied mixtures in which air-driven separation is expected since the granular species have appropriate differences in size or density. Here, air not only introduces Faraday tilting and air-driven convection, but also air-driven separation, acting as an additional separation mechanism to that provided by the inhomogeneous magnetic field. The two may act in concert, in opposition or may be adjusted to cancel each other.

In general, separation will only occur under vibration of sufficient amplitude to cause movement of the grains with respect to one another. We have seen that a necessary condition for this is that $\tilde{\Gamma}$ for both species of grain must appreciably exceed unity. If this is not the case the inactive grains cage the active grains inhibiting separation. Once both species move under vibration, then either or both of the separation mechanisms may produce regions distinctly different in composition. If the differences in composition are sufficient, then the regions may be thrown differently by the vibration and a region deficient in particles may open up between them in flight. There may then be an abrupt transition into a *well separated* state where there is a distinct convection cell within each separated region and a very distinct gap, which prevents mixing. This transition has some of the qualities of a true phase transition.

The development of a *well separated* state may be inhibited by a number of factors including the intense granular activity which may be found at high Γ and sufficiently disparate grain sizes. Gaps may then not fully develop, a feature observed in simulations. It might be supposed that the larger the value of the parameter S, Eqn. 3, the more complete and more rapid the air-driven separation. However, if this large value of S has been achieved through large differences in diameter rather than through large differences in density, the boundary between separated regions will be very rough on the length scale of the smaller grains and the gaps which open up between the separated regions during vibration may be less well defined. Also, small grains close to the boundary may more easily pass through the spaces between large grains during parts of the vibration cycle, particularly at lower frequencies when the amplitude of vibration is large. For these reasons a large difference in grain size may therefore detract from the separation process, leading to regions less pure in the individual species.

We have also observed clearly defined states of *partial separation*, with associated characteristic convection cells. Here clear gaps between the region containing the third convection cell and the other regions are not fully developed, leading to the partial mixing which is observed.

We have studied the values of Γ and BdB/dz for which the *mixed* state and *partially separated* and *well separated* states occur for bismuth–bronze mixtures of various relative grain sizes. It is found that decreasing the grain sizes while keeping the bronze and bismuth grain sizes equal, raises the values of $|BdB/dz|$ at which the transition behaviours are found. However, decreasing the bronze grain sizes while keeping the bismuth grain sizes fixed, lowers the values of $|BdB/dz|$ at which the transition behaviours are found. We have suggested arguments based on the size dependences of the magnetic and damping forces to explain these results.

Our simulation studies have enabled us to examine the influence of Faraday tilting and air-driven convection on the separation processes and to observe the development of gaps between the separated regions. However, we note that the abrupt boundaries between the behaviours found in different regions of the BdB/dz, Γ plane are not found for very small particle numbers, our simulations suggesting that the abruptness is associated with larger particle number than we have used here.

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