Separation of binary granular mixtures under vibration and differential magnetic levitation force

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(Received 29 March 2004; revised manuscript received 8 November 2004; published 23 February 2005)

The application of both a strong magnetic field and a magnetic field gradient to a diamagnetic or paramagnetic material can produce a vertical force that acts in concert with the force of gravity. We consider a binary granular mixture in which the two components have different magnetic susceptibilities and therefore experience different effective forces of gravity when subjected to an inhomogeneous magnetic field. Under vertical vibration, such a mixture may rapidly separate into regions almost pure in the two components. We investigate the conditions for this behavior, studying the speed and completeness of separation as a function of differential effective gravity and the frequency and amplitude of vibration. The influence of the cohesive magnetic dipole-dipole interactions on the separation process is also investigated. In our studies insight is gained through the use of a molecular dynamics simulation model.

DOI: 10.1103/PhysRevE.71.021303 PACS number(s): 45.70.Mg, 85.70.Rp

I. INTRODUCTION

The effective separation of mineral particulates into two or more categories is a crucial aspect of many industrial processes. Particulate mixtures may be separated through their differences in physical properties such as size, density, conductivity, etc., or through some combination of these properties [1]. Often vibration is added, either as an intrinsic element of the process, as in agitated sieving, or, for example, to prevent the cohesive aggregation of fine particulates.

In recent years the physics community has become interested in a number of related problems including the "Brazil nut effect" in which, under vertical vibration, a large and heavy intruder rises to the top of a fine granular bed [2,3]. There are also conditions under which an intruder moves to the bottom of the bed, the "reverse Brazil nut effect" [4–7]. The corresponding problem of multiple intruders has been studied both in experiment and simulation [8–10]. However, identification of the conditions for the intruders to separate to the top or bottom of the bed is still a topic of active debate. The presence of air in vibrated fine granular beds has been noted to enhance the Brazil nut effect [11] and to cause excellent separation of fine binary mixtures under a wide range of conditions [12,13].

Separation through differences in magnetic properties has been widely used in industry for ferromagnetic materials [1]. Separation through differences in density, by selective flotation in a ferrofluid via the magneto-Archimedes effect, has found less widespread application. Recently, separation by magnetic levitation in pressurized oxygen gas, a paramagnetic medium, has been demonstrated for a range of materials [14–16]. This method requires high magnetic fields and separates particulates through differences in density and diamagnetic susceptibility.

Here, we describe the dynamics of binary granular mixtures which are subjected to both magnetic forces resulting from an inhomogeneous magnetic field and to mechanical vibration. Components of differing magnetic susceptibility are subject to different effective forces of gravity. Vertical vibration sufficient to throw the granular bed may then produce spatial separation of the different components. We study the conditions for this behavior and investigate the effectiveness of the separation for different frequencies and amplitudes of vibration. In many cases, we observe separation for magnetic fields of far lower magnitude than those typically required to produce full diamagnetic levitation. A simulation model is used to gain insight into the separation dynamics. Both in experiment and in simulation we restrict ourselves to conditions where air-driven Faraday tilting [17,18] and air-driven separation effects [12,13] play no appreciable role.

II. SEPARATION DUE TO MAGNETIC FORCES AND VIBRATION

Let us consider a single component granular material of magnetic susceptibility, χ, subject to a vertical magnetic field of magnitude B. Each grain will acquire an induced magnetic dipole moment parallel to the field. In the presence of a magnetic field gradient, dB/dz, where z is the vertical coordinate defined as positive upwards, a grain will experience a force of magnitude

$$F = \frac{\chi V}{\mu_0} B \frac{dB}{dz}, \tag{1}$$

where μ_0 is the permeability of free space and V is the volume of the grain. This force acts in concert with that due to gravity. In general a grain may be considered to experience an effective gravitational acceleration, $\tilde{g}$, given by

$$\tilde{g} = g - \frac{\chi}{\mu_0 \rho} B \frac{dB}{dz}, \tag{2}$$

where ρ is the material density and g is the normal Earth's gravity, treated as a positive quantity (9.81 m s^{-2}). If the product of B and dB/dz is negative, as in many experiments, then the magnetic force is directed downwards for paramagnetic materials and $\tilde{g} > g$. However, for diamagnetic materials, the magnetic force is directed upwards, opposing the force due to Earth's gravity; $\tilde{g} < g$. If the magnetic force is sufficient to equal the force due to Earth's gravity, then the body may be levitated in midair. This was first demonstrated by Braunbeck in 1939 using graphite [19]. Later workers levitated diamagnetic objects including water and a frog [20,21].

If a grain is now placed on a platform vibrating vertically as $A\sin(\omega t)$, it will be thrown from the platform if the dimensionless parameter $\tilde{\Gamma}$ exceeds unity. $\tilde{\Gamma}$ is the ratio between the maximum acceleration due to vibration and the effective gravitational acceleration,

$$\tilde{\Gamma} = \frac{A\omega^2}{\tilde{g}}, \quad (3)$$

where A is the amplitude of vibration, and ω is the angular frequency. A grain initially resting on the platform will leave the platform at a phase of $\sin^{-1}(1/\tilde{\Gamma})$ and, if uninfluenced by air damping, will be thrown to a height, h, given by

$$h = \frac{A}{2}(\tilde{\Gamma} + 1/\tilde{\Gamma}). \quad (4)$$

If the components of a granular mixture have differing values of χ/ρ, then they will experience different effective gravitational accelerations, $\tilde{g}$, when placed in a vertical inhomogeneous magnetic field, even in cases where their densities and sizes are identical. When vibrated vertically, the grains experiencing a lower effective gravity will have a higher value of $\tilde{\Gamma}$. If unimpeded by their neighbors in flight, they will be thrown higher than the grains with the greater effective gravity [Eqs. (3) and (4)] and will tend to land upon them. However, in a thrown granular bed, grains will interact strongly with their neighbors. Nevertheless, over many cycles of vibration, some degree of separation may occur.

It is the purpose of this paper to examine the dynamical behavior and the degree of separation of a number of binary mixtures as a function of the magnetic and vibratory conditions. The principal parameters of the problem are BdB/dz, the vibrational frequency, ω, and the vibrational amplitude, A, conventionally expressed as $\Gamma = A\omega^2/g$. BdB/dz determines the effective gravity acting on each component through Eq. (2). From this effective gravity, the corresponding parameter $\tilde{\Gamma}$ may be obtained from $\tilde{\Gamma} = \Gamma g/\tilde{g}$.

If $\tilde{\Gamma}$ is less than unity for both components, the granular bed will not be thrown and separation cannot occur. If the $\tilde{\Gamma}$ for one component exceeds unity, while that for the other does not, the grains of greater effective gravity may act as "cages" for the grains with lower effective gravity, impeding relative motion. For effective separation to occur, $\tilde{\Gamma}$ must appreciably exceed unity for both components. On the other hand, if $\tilde{\Gamma}$ is sufficiently high for either component, then that component will not be thrown, land and settle within each cycle of vibration; the dynamics will spill over from one cycle into the next. Equations such as (4) are not then valid and the influence of continuous dynamics on the separation process requires clarification. The effect of varying the frequency while maintaining $\tilde{\Gamma}$ constant also requires investigation.

At higher magnetic fields, magnetic cohesion forces due to magnetic dipole-dipole interactions may act to inhibit granular sorting. For many diamagnetic and paramagnetic materials, the magnetic moments induced by the applied magnetic field are sufficiently weak that magnetic grain-grain interactions do not play an important role in their static and dynamic behavior. However, for some materials these interactions become appreciable for sufficiently high magnetic fields and for sufficiently small particles. We may suppose that the moments induced by the applied field are not influenced by the moments of the neighboring grains. The energy, E, associated with the magnetic dipole-dipole interaction between two spheres 1 and 2 in a magnetic field is then given by

$$E = \frac{\mu_0}{4\pi}\left(\frac{\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2}{r^3} - \frac{3(\boldsymbol{\mu}_1 \cdot \mathbf{r})(\boldsymbol{\mu}_2 \cdot \mathbf{r})}{r^5}\right), \quad (5)$$

where

$$\boldsymbol{\mu} = \frac{\chi V \mathbf{B}}{\mu_0} \quad (6)$$

is the induced magnetic dipole moment of a grain, and r is the distance between the centers of grains 1 and 2. Grains may tumble in flight, but the induced magnetic moments remain closely parallel to the external field since the longitudinal magnetic relaxation times are orders of magnitude smaller than the typical time between collisions. If a conducting grain moves in an inhomogeneous field, electrical eddy currents induce an additional magnetic moment. This moment is negligible compared with μ for the fine conducting grains used here and under our conditions of vibration.

The energy associated with a pair of grains may be positive (repulsive) or negative (attractive) depending upon the orientation of $\mathbf{r}$ with respect to the direction of the applied field. For grains in close proximity, $r \sim d$, the granular diameter. The mean force between the grains in a granular bed is generally attractive but depends in magnitude upon the detailed granular ordering. It is, however, of order

$$F \approx \frac{3\chi^2 V^2 B^2}{2\pi\mu_0 d^4}. \quad (7)$$

The influence of cohesive attraction on the static stability of a granular bed is determined by the ratio of this force to the gravitational force. This dimensionless ratio, R, is given by

$$R = \frac{F}{m\tilde{g}} \approx \frac{\chi^2 B^2}{8\mu_0 \rho d\tilde{g}}. \quad (8)$$

It is observed, for example, that an applied magnetic field increases both the void fraction of a collection of soft iron spheres [22] and the static angle of repose [23,24]. In each case the observed changes are determined by the ratio of the magnetic cohesive force to the weight, R. Here, however, we study the separation dynamics resulting from vibration and it is the relative influence of cohesive and vibrational forces which is important. The vibrational forces, too, will scale with the particle mass and the ratio of the cohesive to the vibrational forces will also depend on B^2/d. It is therefore expected that cohesion will act to inhibit separation for suf-

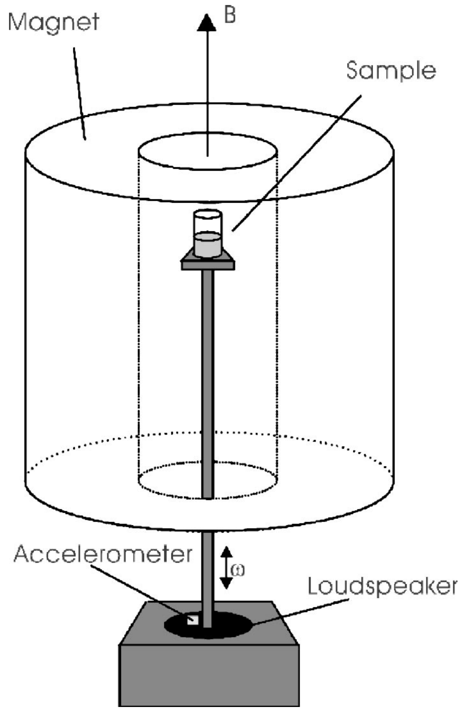

FIG. 1. Schematic diagram showing the magnet, the upper position of the sample in the magnet bore, and the vibration arrangements.

ficiently small particles placed in strong magnetic fields. The influence of cohesion will be stronger at lower values of $\tilde{\Gamma}$ when the vibrational forces are weaker.

III. EXPERIMENTAL METHOD

The magnetic fields used in these experiments were produced by an Oxford Instruments superconducting magnet with a closed-cycle cooler, a system specifically designed for levitation experiments. The magnet has a 5 cm diameter bore, and is capable of producing a maximum magnetic field of 17 T, and a maximum field-field gradient product, $|BdB/dz|$, of 1470 $T^2\ m^{-1}$. This can be controlled by varying the current in the superconducting coils.

A mixture under investigation was shaken in a cylindrical glass container, of internal diameter 15 mm and height 50 mm. In most experiments a granular bed depth of 15 mm was used. The container was mounted in the bore of the magnet, on a plinth connected to a long throw loudspeaker via a thin walled stainless-steel connecting rod, an arrangement illustrated in Fig. 1. The product BdB/dz varied by less than 10% within the volume occupied by the mixture. A sinusoidal waveform from a signal generator was fed to the loudspeaker via a power amplifier in order to vibrate the rod and container vertically. The rod was constrained to move in the vertical direction by slide bearings, not shown in the figure. A cantilever capacitance accelerometer attached to the bottom of the rod was used to measure the amplitude of vibration. Two lengths of connecting rod were used. The longer length positions the container in the upper part of the bore where BdB/dz is maximally negative. A shorter length was used to position the container in the symmetric lower part of the bore where BdB/dz is maximally positive.

If required, the glass container holding the granular sample could be evacuated through a flexible tube to pressures well below 100 Pa. It is known that at such pressures air does not influence the granular dynamics for the finest particle sizes used here [25].

IV. INITIAL OBSERVATIONS OF SEPARATION

The excellent separation which may be obtained by vibrating binary granular mixtures in an inhomogeneous magnetic field may be demonstrated using examples of mixtures which do not separate well under vibration alone. To avoid geometric separation effects, we have selected a system in which the two components have the same size. We use a 50%:50% mixture, by volume, of millet seed and small insulating beads. Both the beads and millet seed have mean diameters of about 2 mm. Both species are weakly diamagnetic; the susceptibility of the beads is -7.0×10^{-6} and that of the millet -10.0×10^{-6}. The mean density of the millet is 1400 kg/m^3. The density of the bead material is 2400 kg/m^3.

Figure 2(a) shows an image after vertical vibration at 10 Hz for 300 s in zero field using a vibration amplitude corresponding to $\Gamma=1.5$. Since the magnetic field is zero, $\tilde{g}=9.8\ m\ s^{-2}$ for both components. The separation is very poor indeed for the wide range of vibration conditions which we have investigated, including those of the figure. Figure 2(b) shows the seeds and beads after they have been vibrated at 10 Hz for 30 s in the upper part of the magnet bore at $BdB/dz=-730\ T^2\ m^{-1}$. Here $\tilde{\Gamma}=1.7$ ($\tilde{g}=8.1\ m\ s^{-2}$) for the insulating beads and $\tilde{\Gamma}=2.4$ ($\tilde{g}=5.6\ m\ s^{-2}$) for the millet seed. The separation is complete, with all of the millet seed lying above the beads. The component with the lower effective gravity lies above the component with the higher effective gravity. Figure 2(c) shows the seeds and beads after they

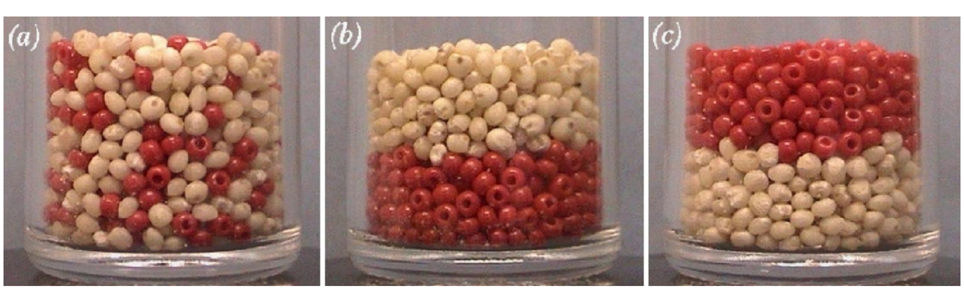

FIG. 2. The separation of millet and insulating beads. (a) After vibration at 10 Hz for 300 s in zero magnetic field. Middle; (b) after vibration for 30 s with $BdB/dz=-730\ T^2\ m^{-1}$; (c) after vibration for 30 s with $BdB/dz=+1000\ T^2\ m^{-1}$.

have been vibrated at 10 Hz for 30 s in the lower part of the magnet bore at $BdB/dz=+1000\ \mathrm{T^2\,m^{-1}}$. Here $\tilde{\Gamma}=2.3$ ($\tilde{g}=12.1\ \mathrm{m\,s^{-2}}$) for the insulating beads and $\tilde{\Gamma}=1.8$ ($\tilde{g}=15.5\ \mathrm{m\,s^{-2}}$) for the millet seed. Again the separation is complete but now the millet seed has separated as a layer lying below the beads. Once again the component with the lower effective gravity lies uppermost.

Under appropriate conditions we have observed almost complete separation for a wide range of mixtures, including mixtures of paramagnetic and diamagnetic materials. It is clear that the use of vibration and a differential magnetic force can provide a very quick and effective method of separation. However, the vibratory and magnetic conditions for optimal separation need to be clarified.

V. SEPARATION OF BRONZE AND BISMUTH MIXTURES

To further investigate separation, we have carried out a detailed study of the conditions for separation using a 50%:50% mixture by volume of fine bismuth and bronze spheres, each with diameters in the range 75–90 μm. This mixture was chosen since bismuth and bronze have similar densities but very different diamagnetic susceptibilities. We observe that this mixture cannot be appreciably separated by vibration in the absence of a magnetic field, due to the similarity in both the density and the size of the components. In these experiments the upper part of the magnet bore was used, BdB/dz being negative. For these fine grains the experiments were conducted in vacuo to avoid air effects. Bismuth has a density of $9800\ \mathrm{kg\,m^{-3}}$ and a susceptibility of -165×10^{-6}, whilst bronze has a density of $8900\ \mathrm{kg\,m^{-3}}$ and a volume susceptibility of -5.5×10^{-6}. The bronze is little affected by the magnetic field, so that the effective gravity seen by the bronze, $\tilde{g}_{Bnz}$, is close to g. Correspondingly, $\tilde{\Gamma}_{Bnz}\approx\Gamma$. However, the bismuth is much affected by the field and $\tilde{g}_{Bis}$ becomes zero at $|BdB/dz|=730\ \mathrm{T^2\,m^{-1}}$.

Effective separation of this mixture may be demonstrated using a frequency of 10 Hz, $\Gamma=1.5$ and a magnetic field such that $|BdB/dz|$ is $500\ \mathrm{T^2\,m^{-1}}$, giving $\tilde{\Gamma}_{Bis}=4.6$ ($\tilde{g}=3.1\ \mathrm{m\,s^{-2}}$), and $\tilde{\Gamma}_{Brz}=1.6$ ($\tilde{g}=9.6\ \mathrm{m\,s^{-2}}$). Images of the sample after 0, 20, 60, and 90 s are shown in Fig. 3. It may be seen that the mixture separates very quickly under these conditions, leaving two almost pure discrete layers, with the more strongly diamagnetic bismuth above the bronze. The boundary between the two components is very sharp indeed.

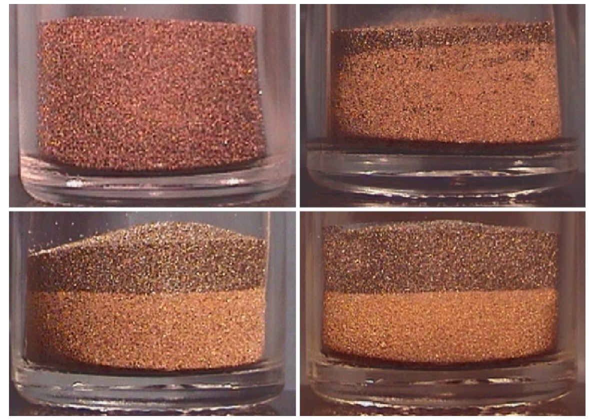

FIG. 3. Separation of a mixture of fine bronze and bismuth grains in vacuum. Vibration at 10 Hz with an amplitude of 3.7 mm and $BdB/dz=-500\ \mathrm{T^2\,m^{-1}}$ was used. The images shown are after 0, 20, 60, and 90 s of vibration.

Figure 4 shows the effects of applying three different values of $|BdB/dz|$ at 10 Hz and with $\Gamma=2.5$. The images shown were taken after 20 min vibration, long after the granular system has reached equilibrium separation. Figure 5 shows the measured degree of separation of the bismuth and the bronze as a function of $|BdB/dz|$. The lower curve shows the percentage of grains which are bronze taken from a sample at 75% of the bed height. The upper curve shows the percentage of grains which are bismuth taken from a sample at 25% of the bed height. It may be seen that the degree of separation increases remarkably with $|BdB/dz|$. This is quite generally true; both the degree of separation and the sharpness of the boundary improve with increased $|BdB/dz|$ at fixed Γ. Figure 5 also shows that the purity of the upper bismuth-rich region is always greater than that of the lower bronze-rich region. This too is generally true, irrespective of Γ.

The results of a detailed study of the quality of separation as a function of both $|BdB/dz|$ and Γ are shown schematically in Fig. 6 for a frequency of 10 Hz. All other system parameters are as described above. In presenting our results we define three degrees of separation, "poor," "good," and "excellent," found in the regions b, c, and d of Fig. 6, respectively. The boundary between "poor" and "good" separation corresponds to about 2% bronze in the bismuth and 10% bismuth in the bronze, these figures being taken from samples from the centers of the respective regions. At this boundary the interface between the bronze-rich and the bismuth-rich regions wanders spatially, is diffuse, and is about 10 grains wide. The boundary between "good" and "excellent" separation (c and d of Fig. 6) corresponds to about 0.1% bronze in the bismuth and 1% bismuth in the bronze. Here the interface between the bronze-rich and the bismuth-rich region is spatially smoother and about 4 grains wide.

Figure 4(a) is taken at $|BdB/dz|=180\ \mathrm{T^2\,m^{-1}}$ and $\Gamma=2.5$, well within the "poor" separation region b; there is about 8% bronze in the bismuth and 25% bismuth in the bronze. Figure 4(b) is taken close to the boundary between "poor" and "good" separation, while Fig. 4(c) corresponds to the boundary between "good" and "excellent" separation. Within the region d of Fig. 6, it is possible to obtain separation with about 0.01% bronze within the bismuth and 0.1% bismuth within the bronze, for example, at $|BdB/dz|=600\ \mathrm{T^2\,m^{-1}}$ and $\Gamma=2.5$ (see Fig. 5). The boundary between the two regions is then smooth and 1–2 grains wide.

Figure 6 shows that, as might be expected, no separation occurs for $\Gamma\lesssim1.1$, the region a. The condition $\Gamma\gtrsim1.1$ ensures that both species are in granular motion. "Poor" separation is found at lower values of $|BdB/dz|$, within the region shown as b. It should be noted that the boundary between the regions b and c first rises and then falls in $|BdB/dz|$ as Γ is increased. For sufficiently large $|BdB/dz|$ and Γ, separated

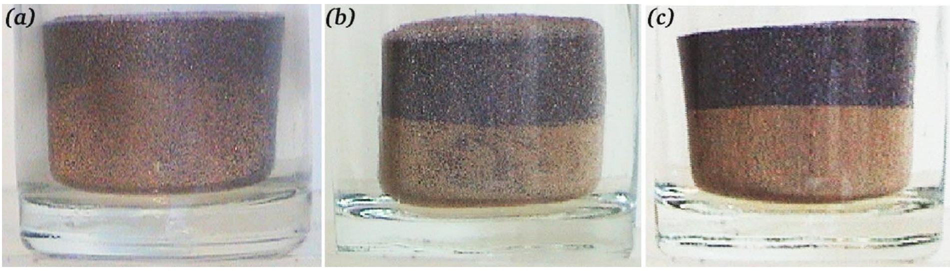

FIG. 4. Separation of bronze (light) and bismuth (dark) vibrated at 10 Hz, $\Gamma=2.5$ for over 20 min; (a) poor separation at $-BdB/dz=180\ \mathrm{T}^2\,\mathrm{m}^{-1}$; (b) good separation at $-BdB/dz=300\ \mathrm{T}^2\,\mathrm{m}^{-1}$; (c) excellent separation at $-BdB/dz=500\ \mathrm{T}^2\,\mathrm{m}^{-1}$.

bismuth grains will collide with the top of the container and rebound. The conditions for the onset of collisions with the top are shown as the broken line in Fig. 6. This line has been obtained from observations and from calculations, the two being in close agreement. It may be seen that the high Γ boundary of the region b follows this line over a range of $|BdB/dz|$. It therefore appears that collisions with the top of the container enhance separation. We further tested this supposition by repeating some of the experiments using a taller box to avoid collisions with the roof. We then observed no enhancement in the quality of separation, the boundary between b and c increasing steadily with increasing Γ.

Figure 6 shows that the upper limit of the region c increases rapidly with Γ. "Excellent" separation is found within the region d, which is limited at its upper extent by magnetic cohesion and by the condition for the levitation of the bismuth, $|BdB/dz|=730\ \mathrm{T}^2\,\mathrm{m}^{-1}$. It is interesting to observe what happens as this limit is approached. If magnetic cohesion is unimportant, the effect of the vibration on the separated bismuth weakens as levitation is approached, since flight for the bismuth extends over more and more periods of vibration. At full levitation, vibration has no influence on the levitated bismuth component; however the vibration continues to affect the lower bronze bed, tending to shake bismuth grains free.

Within the region e appreciable magnetic cohesion binds the bismuth together, trapping bronze within it. At sufficiently high $|BdB/dz|$, rather than a pure bismuth bed levitating above a pure bronze bed, the levitated component consists of bismuth containing a high proportion of bronze. Full levitation occurs at a somewhat higher field than for bismuth alone, due to the bismuth containing bronze as "ballast."

In general the time scale for separation to occur depends only weakly upon $|BdB/dz|$. However, it is observed to decrease with increasing Γ, the time scale for separation at $|BdB/dz|=500\ \mathrm{T}^2\,\mathrm{m}^{-1}$ varying from about 2 min at $\Gamma=1.25$ to about 40 s at $\Gamma=2.5$.

The information used to present Fig. 6 was obtained at the fixed frequency of 10 Hz. However, the influence of the frequency of vibration was investigated for a number of fixed values of Γ and $|BdB/dz|$. Over the range investigated, 10 Hz to 60 Hz, the form of Fig. 6 does not depend strongly upon frequency, except for the high Γ boundary between regions b and c which coincides with the broken line. It may be shown from simple mechanics that the value of Γ for the onset of collisions with the top of the container increases approximately linearly with the frequency, for any particular value of $|BdB/dz|$. Indeed this is what we observe experimentally.

Within the regions to the left of the broken line, increasing the frequency of vibration simply increases the time scale for separation to occur. For example, at $|BdB/dz|=500\ \mathrm{T}^2\,\mathrm{m}^{-1}$ and $\Gamma=1.5$ the time for separation increases from about 90 s at 10 Hz, to about 20 min at 40 Hz. The time scale varies approximately inversely with the square of the frequency. This appears to be true over wide areas of the phase diagram.

It is interesting to note that the schematic diagram, Fig. 6, shows no change in behavior as the dynamics pass from

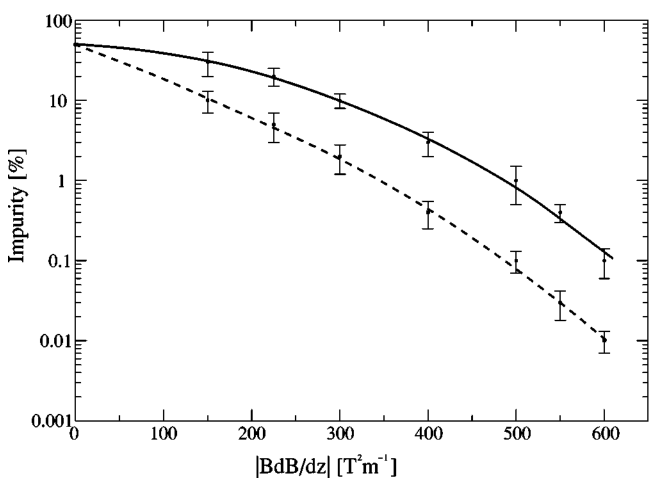

FIG. 5. Measured concentration of impurities in the bronze (solid line) and in the bismuth (dashed line) layers after equilibrium has been reached, under vibration at $\Gamma=2.5$ and a frequency of 10 Hz.

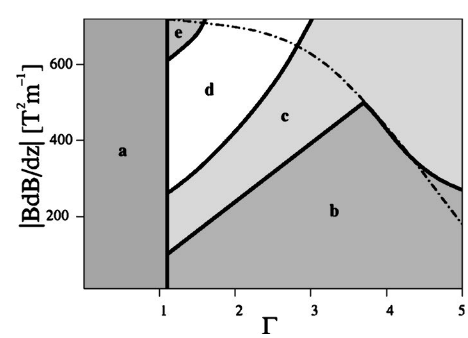

FIG. 6. Schematic diagram showing the separation behavior of fine bronze and bismuth grains in vacuum as a function of Γ and $|BdB/dz|$ at $f=10$ Hz. The labeled areas represent regions of no separation (a), poor separation (b), good separation (c), excellent separation (d), and magnetic cohesion (e). The dashed line indicates the onset of collisions of the bed with the roof of the container.

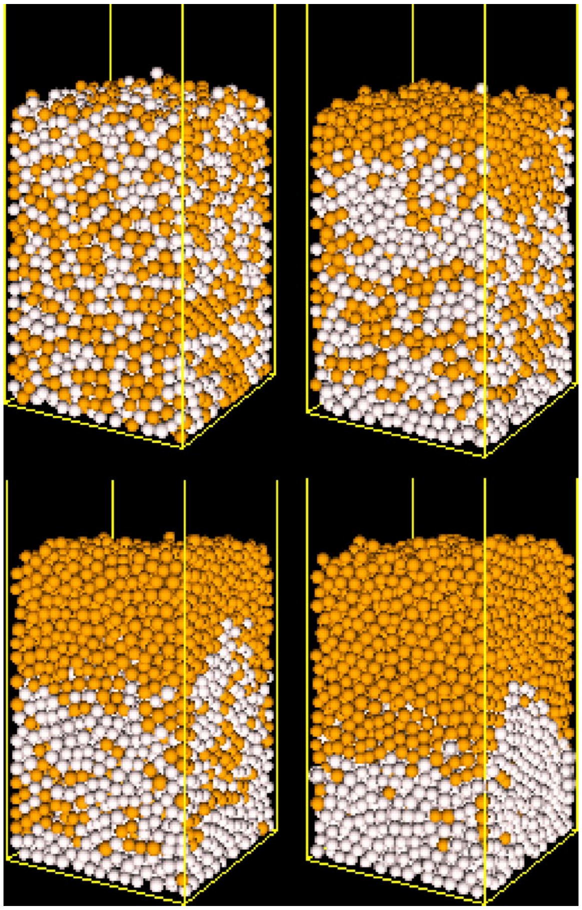

FIG. 7. Simulated separation of fine bronze (light) and bismuth (dark) particles. The images shown from top left to bottom right are after 0, 0.1, 0.5, and 2.0 s of vibration at 10 Hz, and $|BdB/dz| = 600\ \mathrm{T^2\,m^{-1}}$.

single cycle to continuous motion. The boundary between these two types of behavior lies along the line joining $\Gamma = 3.29, |BdB/dz| = 0$ and $\Gamma = 0, |BdB/dz| = 730\ \mathrm{T^2\,m^{-1}}$. Single cycle behavior lies to the left of this line.

VI. COMPUTER SIMULATION OF THE SEPARATION PROCESS

Computer simulation offers the ability to easily change many system properties not readily adjustable in experiment, and to investigate details of the dynamics which are otherwise difficult to study. To confirm that differences in effective gravity can be solely responsible for the separation observed, and to gain further insight into the separation dynamics, we have carried out a number of molecular dynamics simulations. We have simulated the motion of a binary mixture of 2500 bronze and 2500 bismuth particles using a simple 3D soft-sphere linear spring model [26]. Coulomb friction between particles and with the walls of the container has been included, but air damping has been ignored. The particles have a mean diameter of 100 μm with a size spread of ±10% to avoid crystallization effects. They are vibrated vertically in a rectangular box measuring 1400 μm by 1400 μm in the horizontal plane. Their coefficient of normal restitution is set equal to 0.95, and the particle-particle and particle-wall coefficient of friction are set equal to 0.2 and 0.3, respectively. A random arrangement is allowed to settle under equal gravity. Different effective gravities are then applied to the two species and sinusoidal vertical vibration commenced.

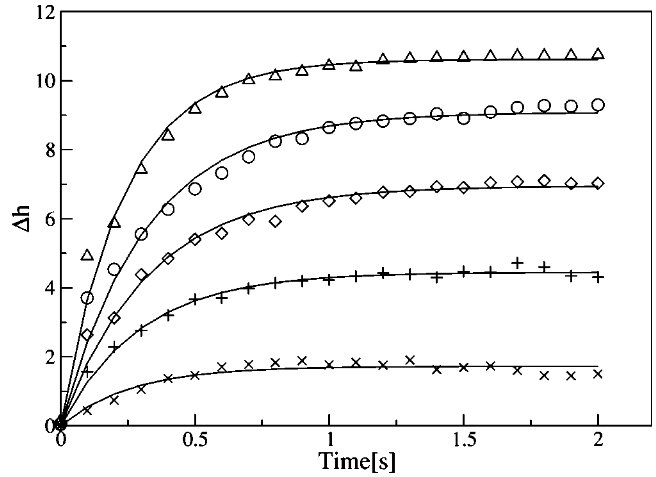

FIG. 8. The height difference Δh between the centers of mass of bismuth and bronze components, in units of particle diameters as a function of time at a frequency of 10 Hz, a Γ of 2.5 at several values of $-BdB/dz$: 656 $\mathrm{T^2\,m^{-1}}$ (△); 506 $\mathrm{T^2\,m^{-1}}$ (∘); 365 $\mathrm{T^2\,m^{-1}}$ (◇); 206 $\mathrm{T^2\,m^{-1}}$ (+); 106 $\mathrm{T^2\,m^{-1}}$ (×).

Figure 7 shows the behavior when $\Gamma = 2.5$ and $|BdB/dz| = 600\ \mathrm{T^2\,m^{-1}}$, under vertical sinusoidal excitation at a frequency of 10 Hz. Soon after vibration begins, the top of the bed becomes rich in bismuth and the very bottom devoid of bismuth. As vibration continues bismuth moves up through the bronze until the majority of the bismuth particles lie above the bronze particles. The quality of separation was measured by calculating the vertical distance, Δh, between the centers of mass of each component; the maximum Δh being half the depth of the bed, approximately 11 particle diameters for our simulations. Figure 8 shows the development of Δh as a function of time for different values of $|BdB/dz|$ when $\Gamma = 2.5$.

It can clearly be seen in Fig. 8 that the quality of separation increases with increasing $|BdB/dz|$ at constant Γ. This was observed over a wide range of Γ values, in good agreement with experimental observations. The quality of separation was also observed to decrease with increasing Γ, again agreeing well with observations.

The data points in Fig. 8 are well fitted by

$$\Delta h = \alpha(1 - e^{-t/\tau}), \tag{9}$$

where α is a measure of the completeness of the separation, and τ is the associated time constant of the separation process. Simulations indicate that this time constant is almost independent of $|BdB/dz|$, as we observe experimentally. For the separation data shown in Fig. 8 it is approximately 0.3 s. We note that this number is sensitive to the values used for the friction parameters; reducing friction tends to speed up the separation process. Simulations also indicate that the time for the mixture to reach equilibrium separation decreases with increasing Γ. This is because increasing Γ increases the velocity of the granular motion accordingly, quickening the time for equilibrium separation to occur.

In agreement with experiment, the quality of separation was observed to improve when the bismuth component collided with the roof of the container. These collisions tended to compact the bismuth bed. Upon collision with the bronze bed, little mixing occurs between the bronze and bismuth beds, the compaction preventing relative motion.

The simulations described above capture the principal features observed experimentally. Specifically, they confirm that differential gravity and vertical vibration alone are sufficient to cause separation and that mixing results from collisions between dilated beds. However, the simulations cannot be compared to the experiments quantitatively due to computational limitations. In the experiments there are the order of 10^7 grains while we are limited to simulations involving a maximum of only a few thousand particles. For fine particles of this reduced number, the vibration amplitude is of the same order of magnitude as the depth of the bed, inhibiting excellent separation.

VII. DISCUSSION

The vibration of granular mixtures in the differential gravity environment resulting from inhomogeneous magnetic fields offers a method of separation relevant to particulate separation. Separation occurs through differences in the ratio of magnetic susceptibility to density. We have observed almost complete separation for a number of systems at magnetic fields substantially lower in magnitude than those required to fully levitate any one of the components. The mechanism for separation has been confirmed both through experiments and through simulations and we have investigated the effect of changing the amplitude and frequency of vibration and the magnitude of the magnetic force.

The quality of separation was found to improve with increasing $|BdB/dz|$ both in simulation and experiment. As $|BdB/dz|$ is increased, the ratio between the effective gravitational accelerations of the components increases. This again confirms that differential gravity is the mechanism for separation.

The quality of separation was found to worsen with increasing Γ. When in flight a granular bed will tend to dilate; this dilation increases with increasing flight time. At low Γ, collisions between beds will not promote mixing since the beds will remain fairly compact, and the relative impact velocities will be low. As Γ is increased, the beds will become more dilate, and the relative impact velocities during collisions between the beds will also increase, promoting mixing. Convection induced by wall friction was observed to be very weak both in experiment and simulation. This suggests that the mechanism for the mixing found at high Γ is based on the collisions between dilated beds. Global convection plays a minor role.

In our studies we note that magnetic cohesion forces due to magnetic dipole-dipole interactions may act to inhibit granular sorting. For many diamagnetic and paramagnetic materials, the magnetic moments induced by the applied magnetic field are sufficiently weak that magnetic grain-grain interactions do not play an important role in their static and dynamic behavior. However, for some materials these interactions become appreciable for sufficiently high magnetic fields and for sufficiently small particle sizes. Separation methods involving somewhat lower magnetic fields and the application of vibration substantially reduce the effect of such cohesion, provided that the kinetic energy resulting from the vibration is appreciable compared with the energy of cohesion. Nevertheless, this grain-grain cohesion ultimately limits our ability to separate very fine grains by these magnetic methods. We only lightly touch upon these effects in the present paper, principally presenting results where magnetic cohesion does not play a major role. However, we hope to report in the near future a more detailed study of the effects of magnetic cohesion on both the static and dynamic behavior of granular systems.

ACKNOWLEDGMENTS

We are grateful to the Engineering and Physical Sciences Research Council for support, to Makin Metal Powders Ltd. for their gifts of powders, and to the workshop staff of the School of Physics and Astronomy for their skills and enthusiasm. We thank Rio Tinto plc. for support. P.S. is grateful to the ORS Scheme for providing financial support.

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