The effects of fluid-driven convection on the separation of binary mixtures

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ABSTRACT: Fluid-driven separation of a binary mixture of fine grains has recently been studied experimentally and in simulation. Under vertical vibration such a system can also exhibit fluid-driven convection. Here, we describe an investigation into the effect of this convection on the separation. By comparing simulations using two distinct fluid modelling techniques, we are able to show that, at low-frequencies, fluid-driven convection strengthens the separation mechanism.

1 INTRODUCTION

In many industrial processes, the mixing and separation of granular particulates is of major importance. Under certain conditions, mixtures can spontaneously separate into component-rich regions, for example under vibration. In some applications separation is required, but in many others complete mixing is desired. Consequently, the conditions under which separation or mixing occurs are of great current interest.

In the case of fine particulates the influence of an interstitial fluid becomes important. Fluid effects include surface patterning, enhancement of the Brazil Nut effect and fluid-driven separation phenomena (Burtally et al. 2003; Biswas et al. 2003). Here we are concerned with the related issue of fluid-driven convection. At low frequencies, fluid driven through a fine granular bed by vibration produces Faraday tilting and associated granular convection (Faraday 1831). Recently, simulations have been used to further the understanding of this fluid based effect (Milburn et al. 2005), but how this fluid-driven convection influences binary separation has not yet been studied.

In this paper, we describe numerical simulations of granular convection and separation using two different models for the fluid-grain interactions. In both models the fluid is coupled to the motion of the container. One model, the Stokes' drag model (SD), considers the fluid interaction with the grains as a modified Stokes' drag force. In the second model, referred to as NS, the fluid is treated using the incompressible Navier-Stokes equations.

2 SIMULATION TECHNIQUES

In both models, each grain is treated as a sphere which undergoes inelastic collisions with other grains and with the container walls. The collisions are modelled using a linear spring-dashpot in the normal direction and Coloumb friction in the tangentially direction (Cundall and Strack 1979). A single set of collision parameters was used throughout all the simulations described here. These values are given in Table 1. The grains were subjected to sinusoidal vertical vibration of amplitude, A, fixed at five times the diameter of an average particle. The dimensionless acceleration, $\Gamma = A\omega^2/g$, was varied by adjusting the angular frequency ω. Here g is the acceleration due to gravity.

Parameters	*Bronze*	*Glass*
Spring Constant (N/m)	3000	1800
Damping (kg/s)	12×10^{-5}	8×10^{-5}
Material Density (kg/m^3)	8900	2500
Diameter range (μm)	90-120	
Friction coefficient (same species)	0.2	0.15
Friction coefficient (other species)	0.18	
Friction coefficient (grain-wall)	0.18	0.15

Table 1: Particle collision parameters

The SD model uses a simple Stokes' drag force (Biswas et al. 2003) that is applied to each particle and that is dependent on particle size and relative velocity to the container base. This model couples the grains to a fluid that is being driven by the vibrated container. Within this model, the fluid moves with the container in the direction of vibration, as it would in the absence of the grains. Although simple, it reproduces many features of the separation phenomenon observed experimentally (Biswas et al. 2003).

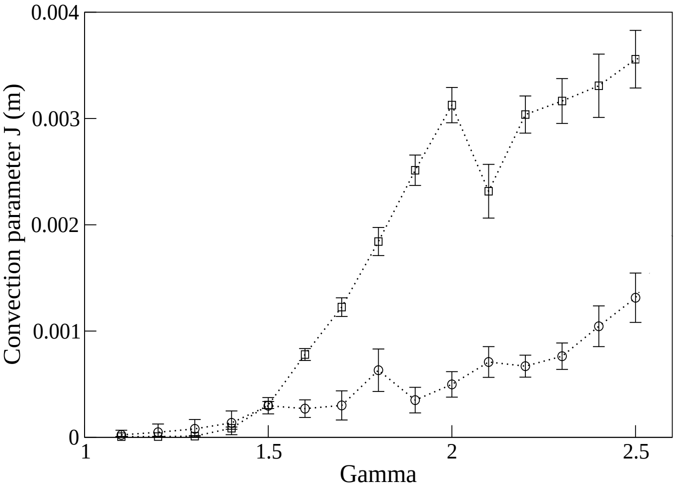

Figure 1: Convection activity for the two fluid models over a range of Γ. The SD model results are plotted as circles and the NS model results are plotted as squares.

The second modelling method (NS) involves the simulation of the fluid dynamics using two-dimensional Navier-Stokes equations for the velocity and pressure fields (Kuipers et al. 1993). These equations are used in conjunction with fluid-grain continuity equations. The fluid equations are time integrated in a similar manner to the equation of motion of each particle. Fluid-grain coupling is incorporated through the use of Ergun's bed equation. The two-dimensional modelling of the fluid in the NS method introduces an additional degree of freedom, motion in the horizontal direction. The presence of the grains influences the flow of the fluid and so changes how the fluid affects the flow of the grains.

In both the models the fluid was taken to be air, with a density of 1.2 kg m^{-3} and a viscosity of 1.8×10^{-5}Pa s.

3 CONVECTION

Convective motion within a vibrated system of particles has been investigated experimentally and by using numerical simulations (Knight et al. 1996; Taguchi 1992). Most studies have used large particles for which the influence of air can be neglected. The convective motion results from both granular collisions with other particles and interactions with container walls. Here we show that, when the particulates are small enough, fluid plays a major role in the resulting convective motion.

We consider a specific system of spherical particles and compare the convective motions predicted by the two models described above. As a measure of convective motion during a vibratory cycle, we use a slight modification to the averaged cell-to-cell flow measure J introduced by Taguchi (Taguchi 1992). The simulation space is divided into cells labelled by r, which are fixed relative to the container. If a particle has moved from one cell to another during a period of vibration T, half of its displacement is added to the totals, $J(r)$, kept for both the cell left and the cell entered. Averages are taken over many cycles. The time averaged totals are given by

$$\langle \mathbf{J}(r) \rangle = \left\langle \sum_{i=1}^{N} \frac{1}{2} |\delta(r_i(t) - r) - \delta(r_i(t-T) - r)| \right.$$
$$\left. \{\mathbf{x}_i(t) - \mathbf{x}_i(t-T)\} \right\rangle_t \quad (1)$$

where $\mathbf{x}_i$ is the position vector of particle i. Here, $\delta(r_i(t) - r)$ equals one if particle i is in cell r at time t, but zero otherwise. Our measure of the total displacement over the time period T is given by

$$J = \sqrt{\sum_r |\mathbf{J}(r)|^2} \quad (2)$$

In each simulation, 1000 bronze particles, contained in a glass box of horizontal dimensions 0.5×3.5 mm, were subjected to vertical sinusoidal vibration. The amplitude of vibration was fixed throughout all simulations to five times the diameter of a typical particle. Figure 1 shows that for the NS model, there is a greater amount of granular convection than in the SD model, over the range of accelerations shown. The extra degree of freedom in the NS model allows Faraday tilting, a configuration dominated by a single convection loop. Grains falling down the tilted slope, coupled to the fluid, enhance convective flow.

This finding then poses a further question; does this fluid-driven convection influence the separation of a binary granular mixture?

4 SEPARATION OF MIXTURES

The separation of binary mixtures has recently been studied by simulations using the SD model (Biswas et al. 2003). Separation results from the differential influence of the fluid on the two species of grain, as it is driven through the bed by vibration. Here, we compare the separation produced by this model to that using the more complex fluid treatment of the NS model. Each run involved 1500 bronze and 1500 glass particles, initially in a completely mixed state. The results of vibrating for 15 seconds at a number of values of Γ are shown in Figure 2. It is seen that separation happens far more readily in the NS model. It is clear that, within the container geometry used for the mixtures, the extra degree of freedom given to the fluid results in more rapid separation. For accelerations of $\Gamma \leq 2.5$, Faraday tilting is also observed.

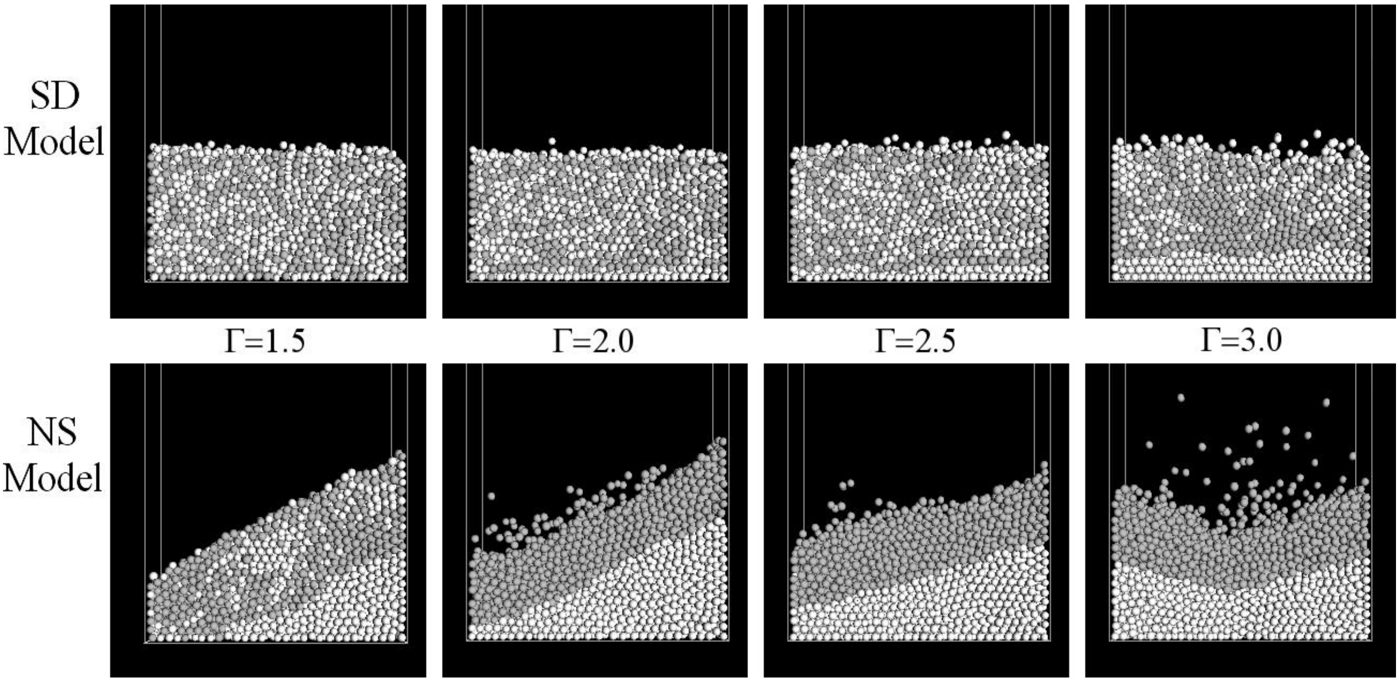

Figure 2: Snapshots of mixtures after 15 seconds of simulated real time for a range Γ. The bronze particles are shaded darker than the glass. The top row of pictures are from the SD model and the bottom row are from the NS model. The dimensionless accelerations, Γ, are given in the figure.

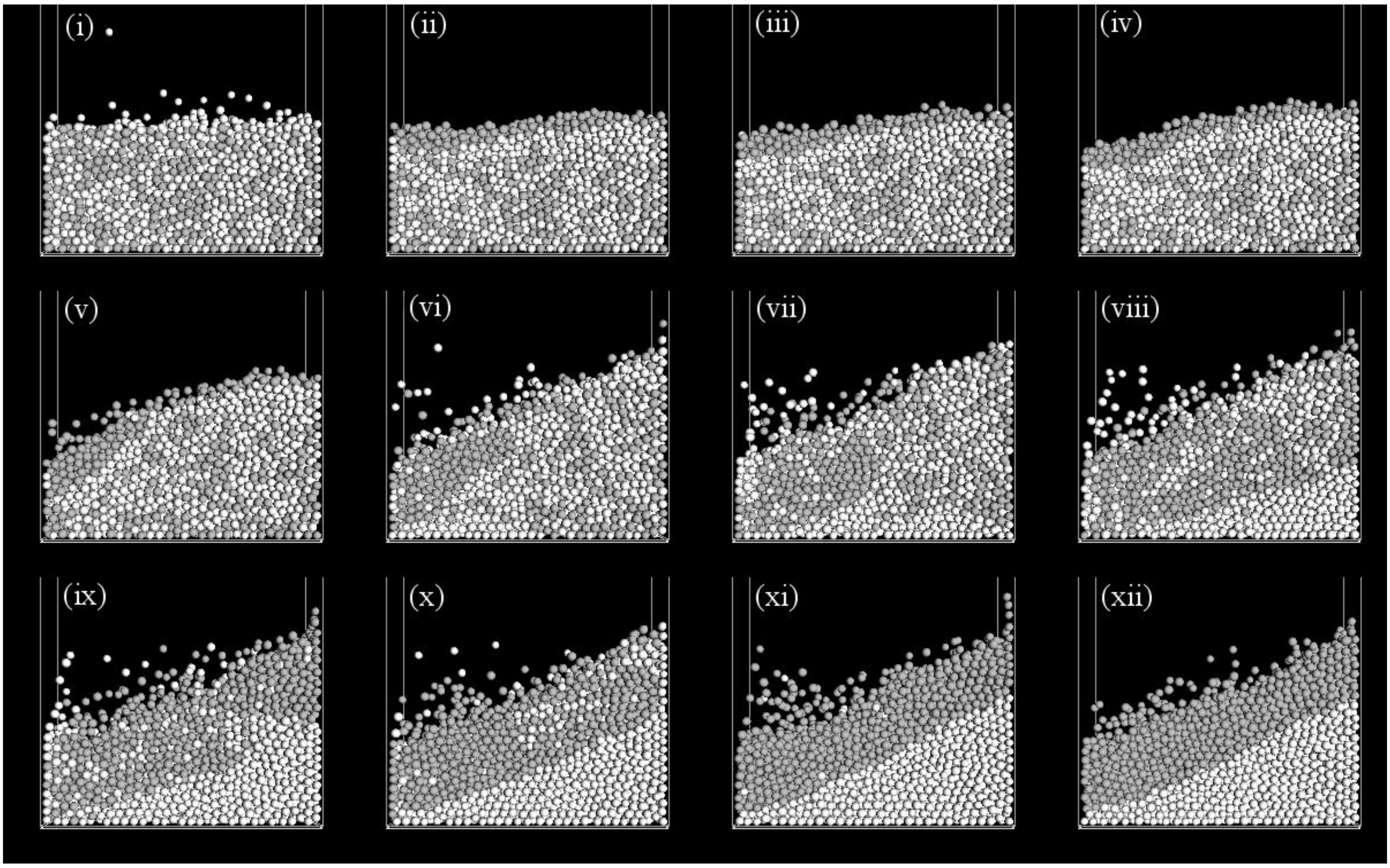

Figure 3: Snapshots of vibrated mixture with a dimensionless acceleration, $\Gamma = 2.0$. The pictorial evolution of the separation is shown from top left to bottom right.

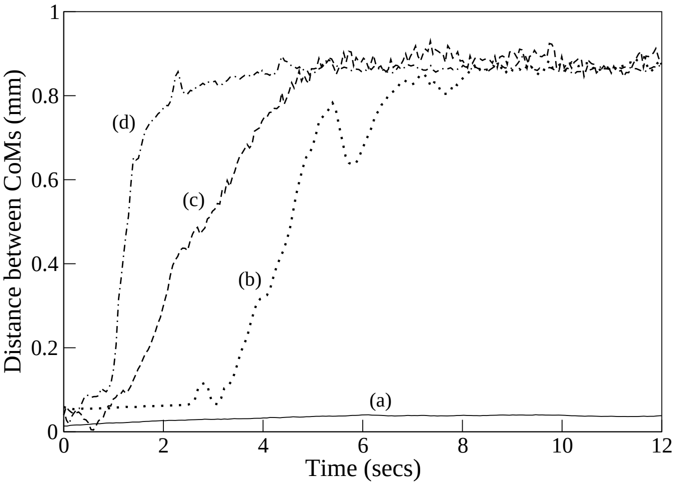

Figure 4: Time evolution of separation, showing the distance between the centre of mass of each component. Lines (a) and (b) are for $\Gamma = 2.0$, lines (c) and (d) are for $\Gamma = 3.5$. Lines (a) and (c) are for the SD model, lines (b) and (d) are for the NS model.

We now discuss how the fluid-driven convection, present in the NS model, contributes to the separation process. Figure 3 shows some snapshots of a mixture as it separates under vibration. Pictures (i)-(iv) span the first 2.5 seconds. Here we can see that initially a thin layer of bronze forms on top of the mixture. The middle row of pictures, (v)-(viii), shows progress to 4.0 seconds, where it can be seen that the bed has now developed a Faraday tilt. Within this configuration, there are now strong local convection currents in both the upper mixed layer and the lower glass region. Due to these convection currents, glass particles in the upper mixed layer are deposited in the lower glass-rich region while the bronze particles tend to flow up towards the top of the tilt. This process aids separation. Pictures (ix)-(xii) show the subsequent evolution up to 15.0 seconds. As can be seen, complete separation of the mixture now results. The bronze particles continue to be more active and flow up over the glass region and tumble down the upper slope.

The description above is for a system that separated while forming a tilted configuration. The tilt formation and convection present in the NS model allows the mixture to separate much more readily than in the SD model. At higher accelerations, however, separation is achieved in both models. A quantitative measure of the quality of separation is the distance between the centres of mass of the two components. This is plotted against time in Figure 4. Lines (a) and (b) are for $\Gamma = 2.0$ and for the SD and NS models respectively. They show the lack of separation in the SD model compared to the other. Lines (c) and (d) are for $\Gamma = 3.5$ and show the separation for both models. In the SD model, separation occurs but at a slower rate than in the NS model. At higher accelerations, fluid-driven convection becomes less of an influence due to the more energetic vibration.

5 CONCLUSIONS

We have investigated the influence of a fluid on granular convection and the separation of binary granular mixtures. With the use of two modelling techniques, we have shown that fluid motion can have a major influence on granular convection, and that this convection aids the separation process. It should be noted that in many cases, for example in the absence of a fluid, convection usually causes mixing.

The present work confirms that the separation mechanism is due to the differential influence of the fluid drag on the two species, as proposed in (Biswas et al. 2003; Burtally et al. 2003). This mechanism does not, however, explain why separated groups of particles of the same species come together to form a single region. In the present paper we have now shown that fluid-driven convection provides the mechanism for this to occur.

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