The separation of fine binary granular mixtures subjected to differential gravity and to vertical vibration in air

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ABSTRACT: We consider binary granular mixtures with components having similar densities but different magnetic susceptibilities and therefore experience different effective forces of gravity when subjected to a vertical magnetic field with a field gradient. Under vertical vibration and in vacuum, such a mixture may separate into regions almost pure in the two components. Air influences the dynamics of vertically vibrated fine binary granular mixtures, causing separation, Faraday tilting and enhanced convection. We investigate the influence of air on the dynamics of separation in the differential gravity environment for a range of vibratory conditions, magnetic field strengths and particle sizes. Separation is strongly influenced by air-driven tilting and convection and under different conditions these effects can enhance separation or result in complete mixing. We observe that magnetic cohesion ultimately limits the size of particles which can be separated using this technique.

1 INTRODUCTION

The separation of granular materials is currently a topic of great interest to the physics and engineering communities. Separation of mixtures may occur due to differences in properties such as size or density. The separation of materials due to differences in their magnetic properties has been widely used for ferromagnetic materials. However, the use of magnetic fields for the separation of diamagnetic materials has found less widespread applications due to the large magnetic fields required.

The magneto-buoyancy provided by a paramagnetic fluid has found a number of applications. Weakly diamagnetic materials such as KCl and NaCl have been separated through magneto-Archimedean levitation in pressurized oxygen gas in a closed vessel at ambient temperature (Ikezoe et al. 2002). Cooled gaseous oxygen and liquid oxygen have also been used to separate a wider range of materials, including diamond, gold and platinum, in an open vessel at ambient pressure (Catherall et al. 2003).

More recently, however, we have shown that the application of vibration in the differential gravity environment provided by an inhomogeneous magnetic field can been used to separate a wide range of granular mixtures (Catherall et al. 2005). Excellent separation may be achieved in a dry environment with magnetic field strengths far less than those required to fully levitate any of the components. These experiments were performed under vacuum conditions, or with large particles, so that the presence of air did not influence the granular dynamics.

Air is known to strongly influence the dynamics of fine grains subject to vertical vibration. At low frequencies, the horizontal components of air flow lead to Faraday tilting and rapid granular convection (Faraday 1831). For binary mixtures of distinct ρd^2, where ρ is the granular density and d is the grain diameter, excellent separation may occur due to the grain-fluid interaction (Burtally et al. 2002). Like particles are dragged equally by the forced air flow, while dissimilar grains are dragged apart. Local convection is found within each separated region. Global convection which would cause mixing is avoided due to gaps which open up between the separated regions over much of the vibration cycle. In this paper, we discuss the influence of air on the dynamics of binary granular mixtures subjected to vertical vibration in the differential gravity environment provided by an inhomogeneous magnetic field.

2 DIFFERENTIAL GRAVITY

Let us consider a grain of magnetic susceptibility χ subject to an inhomogeneous vertical magnetic field B. The vertical force on the grain due to the magnetic field is given by

$$F = \frac{\chi V}{\mu_0} B \frac{dB}{dz}, \tag{1}$$

where V is the volume, μ_0 is the permeability of free space and z is the vertical coordinate. The magnetic force acts in concert with the gravitation force. In general the grain may be considered to experience an effective gravity, $\tilde{g}$, given by

$$\tilde{g} = g - \frac{\chi}{\mu_0 \rho} B \frac{dB}{dz}, \tag{2}$$

where g is the acceleration due to Earth's gravity (9.81 ms^{-1}). The effective gravity is independent of the size of the grain and depends only on its magnetic properties, its density and the product $B\, dB/dz$. For diamagnetic materials, for which the magnetic susceptibility is negative, the effective gravity will be reduced if the product $B\, dB/dz$ is negative, and increased if this product is positive. If the magnetic force exactly opposes the weight of the grain it may be levitated (Beaugnon & Tournier 1991).

If the grain is now placed on a horizontal platform which is undergoing vertical sinusoidal vibration it will be thrown provided that the dimensionless parameter $\tilde{\Gamma}$ exceeds unity, where

$$\tilde{\Gamma} = \frac{A\omega^2}{\tilde{g}}. \tag{3}$$

Here, A and ω are the amplitude and angular frequency of the vibration. The parameter $\tilde{\Gamma}$ is the ratio of the maximum acceleration of the platform to the effective gravitational acceleration. If a binary mixture of grains is vibrated in a vertical inhomogeneous magnetic field the two components will experience different effective gravity provided that they have different values of χ/ρ. If $\tilde{\Gamma} > 1$ for both components, the component with the lower effective gravity will be thrown higher than the component with the greater effective gravity. There are conditions where over many cycles of vibration, the component with the lower effective gravity will tend to migrate to the surface of the bed (Catherall et al. 2005). We now consider the influence of air on this process.

3 OBSERVATIONS

Initially we studied mixtures for which the similarity in density of the components maintains the mixed state in the absence of a magnetic field even in air. In all experiments, we use mixtures of 15mm depth contained within a rectangular glass box 50 mm high and of internal dimensions 20 mm × 10 mm. We chose binary mixtures of 50%:50% by volume of bismuth and bronze grains. Bismuth has $\rho = 9800 \text{ kgm}^{-3}$ and $\chi = -165 \times 10^{-6}$ while for bronze $\rho = 8900 \text{ kgm}^{-3}$ and $\chi = -5.5 \times 10^{-6}$. The low magnetic susceptibility of bronze ensures that its effective gravity is hardly modified, $\tilde{g}_{brz} \approx g$, over the range of magnetic fields used in our experiments, $0\,\text{T}^2\text{m}^{-1} \leq |B\, dB/dz| \leq 730\,\text{T}^2\text{m}^{-1}$ (B ranging from 0 - 10 T). Consequently $\tilde{\Gamma}_{brz} \approx \Gamma$, where Γ is the dimensionless acceleration of the platform ($A\omega^2/g$). Bismuth is much more strongly affected by the magnetic field and can be levitated when $-B\, dB/dz = 730\,\text{T}^2\text{m}^{-1}$.

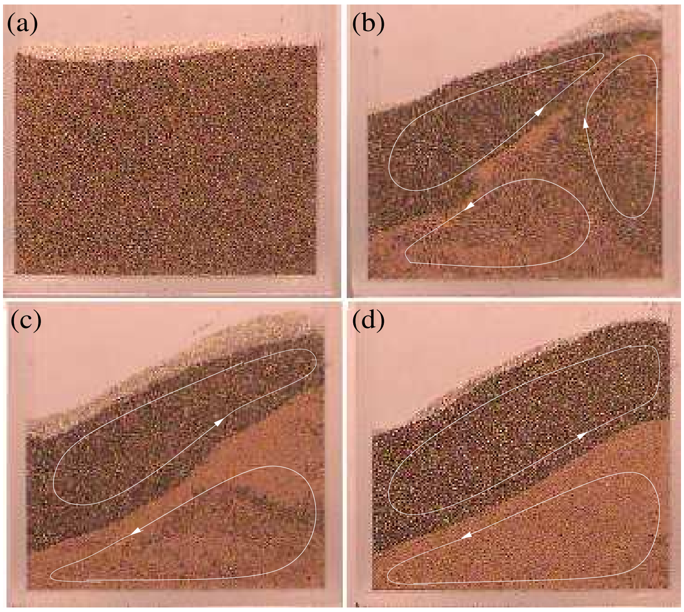

Figure 1. The separation of fine bronze (light) and bismuth (dark) grains through differential gravity in air. Here $-B\, dB/dz = 500\,\text{T}^2\text{m}^{-1}$ and the vibratory conditions are Γ = 1.5 and f = 10 Hz. Images are taken after (a) 0s; (b) 15s; (c) 30s; (d)120s.

Figure 2. A close-up view of the separated boundary between bronze and bismuth particles (75 - 90 μm) following vibration in air at Γ = 1.5, f = 10Hz, $-B\, dB/dz = 500\,\text{T}^2\text{m}^{-1}$ for 120s

3.1 *Bronze and bismuth of the same mean size*

We have investigated the dynamics of several mixtures of bronze and bismuth in air, where both components have the same narrow range of sizes, from 38 - 53 μm to 90 - 125 μm. The excellent separation which may be achieved is demonstrated in Figure 1. When $-B\, dB/dz = 500\,\text{T}^2\text{m}^{-1}$ the effective gravity of the bismuth is reduced to 3.1 ms^{-2}. Under sinusoidal vibration the mixture undergoes Faraday tilting due to the interaction of the granular bed with the air. The interaction with air also enhances convection in the granular bed. Having a lower effective gravity than the bronze, the bismuth grains gradually migrate towards the top of the bed, whilst the bronze grains migrate towards the bottom of the bed. After approximately 15s the system forms two distinct granular beds, the uppermost rich in bismuth, the lower bed consisting mainly of bronze. A strong convection cell is established within the upper bismuth-rich region,

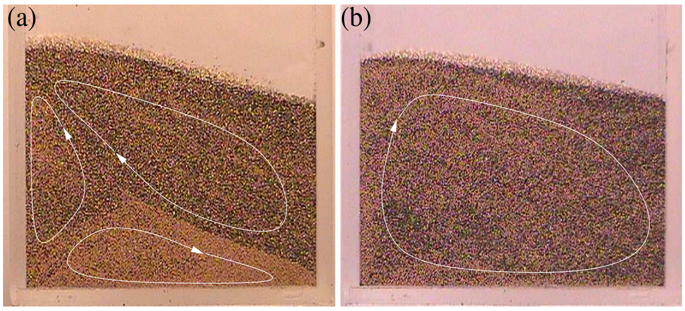

Figure 3. The separation of fine bronze and bismuth grains through differential gravity in air. Here $-B\,dB/dz = 500\,\mathrm{T}^2\mathrm{m}^{-1}$ and $f = 10\mathrm{Hz}$. The images are taken after the mixture had reached equilibrium; (a) $\Gamma = 2.0$; (b) $\Gamma = 2.5$

while two convection rolls are found within the lower bronze-rich region, Figure 1b. The upper of these two rolls weakens with time as the beds increase in purity and disappears completely after about 30s. The beds continue to purify under further vibration, reaching equilibrium after approximately 120s where each bed is greater than 99.9% pure. A magnified view the separation boundary is shown in Figure 2. The boundary between the layers is very sharp, typically on the scale of a single grain diameter.

We have investigated the influence of Γ and $B\,dB/dz$ on the dynamics of the mixtures and observe three granular regimes. We name the first regime "*excellent*" separation where two extremely pure beds are observed, Figures 1 and 2. The second class of behavior we describe as "*partial*" separation; Figure 3a is an example. Here there are two distinct beds, one mainly of bismuth and one mainly of bronze. However, there is a strong convection cell linking the two beds near one edge of the container causing a degree of mixing which limits purity of each bed to approximately 80%. Unlike the observation shown in Figure 1b, this convection cell does not weaken with time. We denote the third regime "*mixing*", where no separation is apparent and the granular bed exhibits only one large convection cell, Figure 3b.

A schematic diagram displaying the vibratory and magnetic conditions where the different behaviors occur is shown in Figure 4 for the particle sizes shown in Figures 1-3 (75 - 90 μm). In all experiments we used a well mixed initial state and vertical vibration was then applied. The change in behavior at the boundary between the regions of excellent and partial separation is abrupt. Only at values of $-B\,dB/dz$ above $400\,\mathrm{T}^2\mathrm{m}^{-1}$ and low values of Γ is *excellent* separation observed, region δ in Figure 4. Within the *excellent* separation region, the quality of the separation is virtually independent of Γ and $B\,dB/dz$. The convection that exists within each bed ensures that any impurity particle can reach the boundary, enabling it to migrate to the "correct" layer. This is contrary to the observations made in vacuum conditions, where the quality of the separation changes gradually with varying $B\,dB/dz$ and Γ (Catherall et al. 2005).

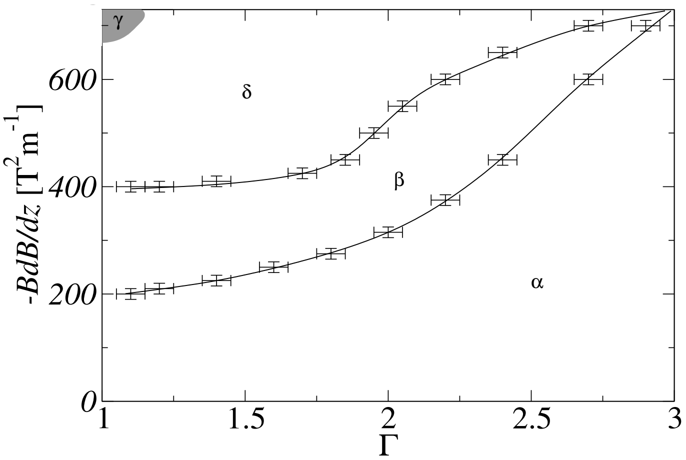

Figure 4. A schematic diagram indicating the conditions where various separation regimes are observed: *Mixing* (α); *partial* separation (β); *excellent* separation (δ); magnetically induced cohesion (γ).

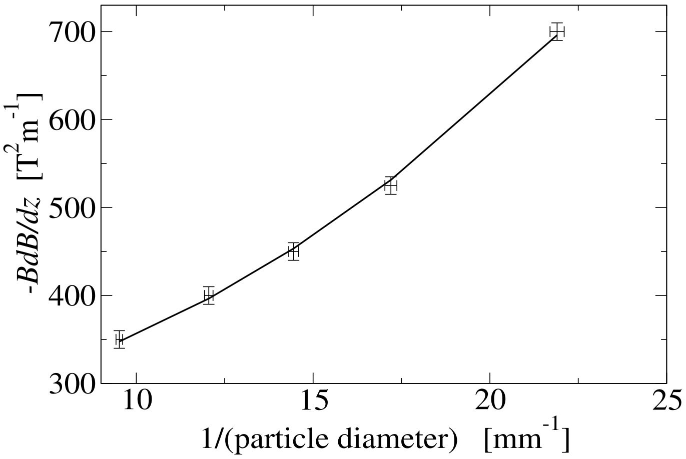

Figure 5. The minimum $-B\,dB/dz$ product required for the onset of *excellent* separation to occur in binary mixtures of bronze and bismuth vibrated at 10Hz and $\Gamma = 1.2$ as a function of particle size.

At high values of Γ and low values of $-B\,dB/dz$, region α of Figure 4, we observe no separation. In this region the mixing caused by global convection overcomes any tendency to segregate due to the differences in the effective gravity of the species.

The shaded region in Figure 4 indicates a region where the quality of the separation deteriorates. Here, cohesion between bismuth grains due to the induced dipole-dipole interaction causes some bronze grains to remain trapped within the bismuth layer. Increasing the acceleration of the platform substantially reduces this effect.

Altering the size of the particles was observed to appreciably affect the conditions required for the onset of *excellent* separation. Reducing the size of the particles increases the influence of air and a larger $-B\,dB/dz$ product is therefore required for separation to occur. The minimum $-B\,dB/dz$ product required for *excellent* separation as a function of particle size is shown in Figure 5. It is found to have the form $-B\,dB/dz = p + q/d^2$, where $p = 270\,\mathrm{T}^2\mathrm{m}^{-1}$. This value for p is in good agreement with the observations made under vacuum conditions (Catherall et al. 2005).

3.2 *Bronze and bismuth of dissimilar size*

We have investigated the separation of mixtures where the bronze and bismuth grains are of dissimilar mean size and are therefore affected differently by air (Burtally et al. 2002). Firstly, we used bismuth grains with diameters in the range 75-90 μm and bronze grains with diameters in the range 38-53 μm. Due to the differential influence of air on the components and to the percolation of the smaller grains through the gaps between the larger grains, a small degree of separation is observed with the bismuth segregating uppermost when this mixture is vibrated in air in zero magnetic field. Applying a negative $B\, dB/dz$ substantially improves the quality of the separation. By applying a positive $B\, dB/dz$ of approximately $200\,\mathrm{T}^2\mathrm{m}^{-1}$, thus increasing the effective gravity of the bismuth, the migration of the bismuth to the surface could be prevented leaving the system in a mixed state. Using higher field strengths, typically $400\,\mathrm{T}^2\mathrm{m}^{-1}$, the situation could be completely reversed, the mixture separating with a bronze on top configuration.

For bronze grains appreciably larger than the bismuth grains, the effect of air is to position the bronze uppermost. Here, the bronze particles were 200 - 250 μm, and the bismuth 75 - 90 μm. In zero field under vertical vibration, this mixture was observed to show some separation with the bronze concentrating uppermost. Applying a positive $B\, dB/dz$ forced the bismuth towards the bottom of the bed and improved the quality of the separation. Applying a negative $B\, dB/dz$ of approximately $100\,\mathrm{T}^2\mathrm{m}^{-1}$ reduced the effective gravity of the bismuth which prevented the separation of the bronze to the surface causing the system to remain in a mixed state. Further increasing the $-B\, dB/dz$ to greater than $400\,\mathrm{T}^2\mathrm{m}^{-1}$ caused a the mixture to separate with a bismuth layer uppermost.

4 CONCLUSIONS

We have studied the influence of air on the dynamics of fine binary mixtures subject to vertical vibration in the differential gravity environment provided by an inhomogeneous magnetic field. At low frequencies, air causes Faraday tilting linked to enhanced granular convection and this convection strongly influences the separation dynamics. For bronze and bismuth mixtures of the same mean size we find regions of frequency and Γ where complete separation, partial separation or complete mixing are found, corresponding to different patterns of convection within the grains. The boundaries between these regions are abrupt since the regions correspond to distinct convection patterns. We find that the minimum $-B\, dB/dz$ required for separation increases as the particle size is reduced. Excellent separation is achieved quickly since convection within each bed helps minority grains to join the other bed. Cohesion due to the dipole-dipole interaction limits the quality of the separation for very fine particles at high magnetic fields and low values of Γ.

We observe that mixtures of fine grains with dissimilar size show a tendency to segregate in zero field under vertical vibration in air, in agreement with previous observations (Burtally et al. 2002). In an inhomogeneous magnetic field, we can either enhance the quality of this separation, or by reversing the field gradient we can cause mixing by preventing separation from occurring. This could find applications in the minerals and powders industries.

REFERENCES

Beaugnon, E. & Tournier, R. 1991. Levitation of organic materials. *Nature* 349: 470.

Burtally, N., King, P. J., & Swift, M. R. 2002. Spontaneous air-driven separation in vertically vibrated fine granular mixtures. *Science* 295: 1877.

Catherall, A., King, P. J., Eaves, L., & Booth, S. R. 2003. Floating gold in cryogenic oxygen. *Nature* 422: 579.

Catherall, A., López-Alcaraz, P., Sanchez, P., Swift, M. R., & King, P. J. 2005. The separation of binary granular mixtures subject to vertical vibration and differential magnetic levitation force. *Phys. Rev. E* 71: 021303.

Faraday, M. 1831. On a peculiar class of acoustical figures; and on certrain forms assumed by groups of particles upon vibrating elastic surfaces. *Phil. Trans. Roy. Soc.* 52: 299.

Ikezoe, Y., Kaihatsu, T., Sakae, S., Uetake, H., Hirota, N., & Kitazawa, K. 2002. Separation of feeble magnetic particles with magneto-archimedes levitation. *Energy conservation and management* 43: 417.