The influence of magnetic cohesion on the stability of granular slopes

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We use a molecular dynamics model to simulate the formation and evolution of a granular pile in two dimensions, in order to gain a better understanding of the role of magnetic interactions in avalanche dynamics. We find that the angle of repose increases only slowly with magnetic field; the increase in angle is small even for inter-grain cohesive forces many times stronger than gravity. The magnetic forces within the bulk of the pile partially cancel as a result of the anisotropic nature of the dipole-dipole interaction between grains. However, we show that this cancellation effect is not sufficiently strong to explain the discrepancy between the angle of repose in wet systems and magnetically cohesive systems. In our simulations we observe shearing deep within the pile, and we argue that it is this motion that prevents the angle of repose from increasing dramatically. We also investigate different implementations of friction with the front and back walls of the container, and conclude that the nature of the friction dramatically affects the influence of magnetic cohesion on the angle of repose.

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I. INTRODUCTION

There is much current interest in the dynamics of granular systems [5]. Recent research has focussed on understanding the stability of granular piles and the characterization of dense granular flows [18].

An important measure of pile stability is the angle of repose. If grains are poured onto a flat surface they form a cone with a well-defined angle characteristic of the material. The angle of repose is also known to depend on the preparation method and the confining geometry. If a granular pile is tilted, or if more material is added to the top, the angle increases until the pile becomes unstable. An avalanche then occurs and the system relaxes through rearrangement of grains within the pile. If the rate of adding grains is increased, fully developed granular flows result. Both the slope angle and the avalanche dynamics are known to depend upon the dimensionality of the pile [20] and the confining geometry [22].

The majority of investigations into granular flows have focused on dry granular media in which the interaction between grains is only through collisions. In many systems cohesion is exists due to the presenece of a interstitial liquid, the influence of magnetic or electrostatic forces or, for fine powders, the effects of van der Waals forces. Cohesion in granular piles can affect the slope angle [12], the packing fraction [2] and the flow dynamics (ref). In particular, magnetic cohesion can be used as a control parameter to modify macroscopic properties of the pile. Another field in which cohesion is important is in the separation of binary mixtures. Recent research in magnetic separation techniques has enabled the vibration-driven separation of binary mixtures to be enhanced by using strong inhomogeneous magnetic fields [6]. However, a detailed understanding of the effects of magnetic cohesion is still lacking.

In this paper we describe molecular dynamics simulations carried out to investigate the influence of magnetic cohesion on the stability of granular slopes. In section II we briefly discuss the angle of repose as a measure of cohesion and review recent literature. Section III provides the details of our simulation and in section IV we present and discuss our results. We find that the weak dependence of the repose angle on magnetic cohesion can be understood in terms of the dynamics of the pile and the influence of friction with the front and back walls of the container.

II. ANGLE OF REPOSE AS A MEASURE OF COHESION

A generally-used measure of the effect of cohesion is the angle of repose of a granular pile. As the particles become more cohesive, the angle of a granular slope might be expected to increase. It is possible to define a cohesion strength R [8, 9] as the ratio of the maximum cohesive force F_v between two particles in contact, and the particle weight. For non-cohesive particles $R = 0$. When $R > 1$, the cohesive force is greater than the particle's weight and one particle can be suspended from another. As R increases from zero, the slope of a pile will be increasingly affected by cohesion, and one might expect the slope angle to approach $90°$ for $R > 1$.

Forsyth et al. [8] have carried out a series of experiments investigating the influence of magnetic cohesion on repose angle. They poured steel ball-bearings into a narrow box to measure the angle of repose θ_r in a uniform vertical magnetic field. They found that θ_r increased slowly and linearly with the magnetic field strength. The increase in θ_r was only a few degrees even when the interparticle cohesive forces were many times greater than the particles' weight.

Fazekas et al. [9] used a two-dimensional molecular dynamics model to simulate the experiments of Forsyth et al., treating the particles as point dipoles aligned with a uniform vertical magnetic field. The results showed a slow increase in angle of repose with magnetic field strength, at a rate of 0.5° per unit R. Even though the experiments were in three dimensions and the simulations were in two dimensions, there was good quantitative agreement in the rate of increase of θ_r. However, the value of θ_r in the absence of a magnetic field was substantially lower (19° rather than 31°) in the simulations. This discrepancy was attributed to the effects of friction between the particles and the front and back walls of the container.

The angle of repose of dry spheres is generally measured as about 23° (see [10] and references therein). The value of 31° obtained by Forsyth et al. is rather high, and this can be attributed to the narrowness of the container (width 5 particle diameters). Forsyth et al. found that the repose angle decreased when they used a wider container. A detailed experimental investigation of the influence of sidewalls on the repose angle has been carried out by Taberlet et al. [11].

In contrast with magnetic systems, experiments on wet granular materials show a dramatic increase in angle of repose when a small quantity of liquid is added [10, 12, 13]. Liquid bridges have been observed to form between particles in contact, providing a cohesive force. It is, however, difficult to directly relate the quantity of liquid to the interparticle force. Albert et al. [10, 12] measured the angle of repose of spherical glass particles with varying amounts of oil added. They fitted their data using a model based on the stability of a particle on the surface of a pile, treating the volume of the liquid bridges as an unknown parameter. They found that the slope angle approached 90° at $R = 1$, and the rate of increase $d\theta_r/dR$ was 58° per unit R.

The increase in θ_r in magnetic systems is a very small effect; $d\theta_r/dR$ is two orders of magnitude smaller than in wet granular systems. One would intuitively expect magnetic cohesion to have a more dramatic effect on the system, as occurs with liquid-bridge cohesion, but this appears not to be the case. To our knowledge, nowhere in the literature has anyone offered a satisfactory explanation of this discrepancy.

III. DETAILS OF THE SIMULATIONS

symbol	parameter	value	unit
d	mean particle diameter	0.8	mm
L	container width	40	mm
k	spring constant in linear model	9425	Nm^{-1}
E	Young's modulus	0.015	GPa
ρ	density	7500	kgm^{-3}
g	acceleration due to gravity	9.81	ms^{-1}
e	coefficient of restitution	0.95	
μ	particle-particle coefficient of friction	0.5	
μ_w	particle-wall coefficient of friction	0.5	
λ	viscous friction coefficient	10	kgs^{-1}
Δt	integration time step	5	μs

TABLE I: Table of parameters used in simulation

Our two-dimensional molecular dynamics model follows the scheme of Cundall and Strack [14]. The particles were modelled as spheres with an approximately Gaussian distribution of diameters, with a mean value of $d = 0.8$mm and a standard deviation of σ=0.03 mm. The distribution was curtailed at 3.35σ, so that all diameters lay in the range 0.7-0.9 mm. The induced magnetic dipole moments were always aligned with the external field, though the particles themselves could rotate in the plane of the container. The simulation parameters are listed in Table I.

We used a Hertzian contact model, with a non-linear damping force in the normal direction to model the dissipation of energy in collisions. We varied the effective coefficient of restitution e between 0.6 and 0.95, and found that the value of e had no significant effect on our results. The friction force in the tangential direction was set to the minimum of μF_n and λv_t, where μ is the Coulomb friction coefficient, F_n is the normal contact force, λ is a viscous friction coefficient and v_t is the relative tangential velocity of the point of contact. The values of μ and λ are given in Table I. The timestep was chosen to be 5μs, significantly shorter than the typical duration of a collision. We also compared results using different contact and friction models: a Hertzian contact model with stick-slip friction; and a linear spring model with Coulomb friction. The choice of contact and friction models had no significant effect on the simulation results.

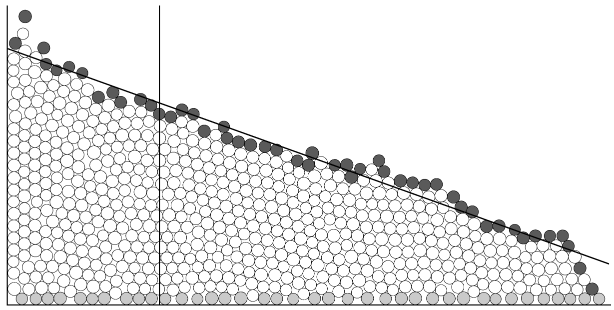

FIG. 1: A snapshot of a granular slope in the absence of a magnetic field. The diagonal line is a fit to the surface particles (darkly shaded). The lightly shaded particles adhere to the base of the container. The vertical line, a quarter of the container width from the left wall, is the position at which we evaluate the particles' velocities and magnetic forces as a function of depth in the pile, as discussed in section IV.

To simulate the formation of a granular pile, particles were introduced into the system, one every 3000 time steps. Each new particle was released with zero velocity on the left side of a container of width 50 particle diameters, at a height just greater than that of the highest existing particle in the pile. Hence newly-introduced particles had a low momentum and did not significantly disturb the pile upon impact. Particles colliding with the base of the container became stuck, forming an uneven surface upon which the pile was constructed. Particles reaching the right side were removed from the system.

Fig. 1 shows a snapshot of the simulation.

We used different methods to model the front and back walls of the container in our simulations. Firstly we compared our results to those of Fazekas et al. [9] in the absence of front and back container walls.

Secondly, we directed a small percentage, p, of each normal contact force F_n on each particle outwards, as if the particles were exerting a force pF_n on the front and back walls. Particles experienced friction $\mu_w p F_n$, where the particle-wall friction coefficient μ_w was set to 0.5. The percentage of force directed outwards was a parameter of the simulations.

Thirdly, we simulated the effect of front and back walls by treating the particles as sliding against the walls. The particles experienced a constant drag force αmg, which was proportional to the particle weight and opposed the direction of motion. Rotational drag was neglected. We treated the drag constant α as a variable parameter.

To determine the angle of repose of the pile, the width of the container was divided into bins, and the highest particle in each bin identified. A least-squares straight line fit was applied to these particles (see Fig. 1).

The particles in our simulations were spherical and weakly magnetic, with moments induced by and parallel to a uniform vertical magnetic field $\mathbf{B}$. It is well-known that the magnetic field due to a homogeneous sphere with total magnetic moment $\mathbf{m}$ in a uniform field is equal to that of a point dipole with the same magnetic moment located at the sphere's centre [15]. For weakly magnetic particles, the susceptibility χ is low ($\chi \ll 1$), such that the magnetic moment induced in each particle is too small to affect the uniformity of the field experienced by other particles. We therefore treat our spheres as a collection of interacting point dipoles.

The interaction energy E between two point dipoles of magnetic moment $\mathbf{m}$ separated by $\mathbf{r}$ is

$$E = \frac{\mu_0 |\mathbf{m}|^2}{4\pi |\mathbf{r}|^3}(1 - 3\cos^2\phi), \tag{1}$$

where ϕ is the angle between the direction of the magnetic field and the vector $\mathbf{r}$, and μ_0 is the permeability of free space [15]. The magnetic dipole-dipole force between two spheres has been measured [17], and found to be in good agreement with Eq. (1). The magnetic force is highly anisotropic; its sign changes depending on the relative position of the particles in the magnetic field. It is also relatively short-range, decaying as $1/|\mathbf{r}|^4$. In our simulations we use a cut-off of 6.25 diameters, beyond which we consider the magnetic forces to be negligible [16]. At this distance the magnetic forces are over three orders of magnitude smaller than for particles in contact.

Consider the interaction between two equal spheres in contact, with diameter d, volume V and magnetic dipole moment $\mathbf{m} = \chi V \mathbf{B}/\mu_0$. When $\mathbf{r}$ is parallel to $\mathbf{B}$, the particles attract with a maximum cohesive force of magnitude $F_v = \pi\chi^2 B^2 d^2/24\mu_0$. When $\mathbf{r}$ is perpendicular to the magnetic field, the particles repel with a force of half the magnitude, $F_h = F_v/2$. The cohesion strength R, defined as the ratio of the maximum cohesive force F_v between two particles in contact, and the particle weight, is given by

$$R = \frac{F_v}{mg} = \frac{\chi^2 B^2}{4\mu_0 \rho d g}, \tag{2}$$

where ρ is the density of the particles and g the acceleration due to gravity.

IV. RESULTS AND DISCUSSION

A. Validation of the Model

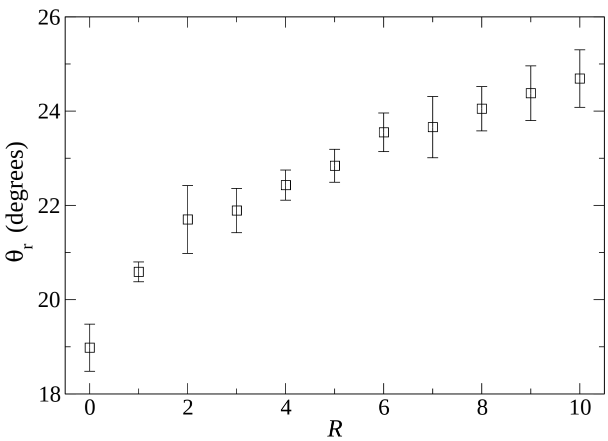

FIG. 2: Angle of repose as a function of cohesion strength R in a vertical magnetic field, in a container of width 50 particle diameters.

Firstly, we validate our model by repeating the simulations of Fazekas et al. [9] in a system with no front and back walls. We ran the simulation for 180 seconds (simulated time), during which 12000 particles were introduced into the system. We used a range of values of the cohesion strength R between 0 and 10. We consider that at higher field strengths than this, the slope is insufficiently smooth for a single angle to be a suitable parameter to describe the system. Fig. 2 shows the angle of repose as a function of the cohesion strength R. The angle increases slowly with cohesion, with an increase of only a few degrees even when the cohesive forces are ten times as great as the particle's weight. Our simulations show a linear dependence of the angle of repose on R, but with a non-linearity below R=2. Our simulations yielded a value of $d\theta_r/dR = 0.50°$ which is in good agreement with simulations of Fazekas et al. [9].

The effect of cohesion on θ_r is weak: when $R = 1$, magnetic and gravitational forces are of the same magnitude, and one might expect the angle of repose to be substantially greater than in the case of zero cohesion. It has been suggested that the weak dependence of θ_r on R is a result of the anisotropic nature of the cohesive force [8, 9]. We have discovered that, due to this anisotropy, the field-induced magnetic dipole-dipole forces in the bulk of the pile partially cancel each other out. In the following section, we investigate this cancellation effect to determine whether it provides a sufficient explanation for the weak dependence of repose angle on magnetic cohesion in our simulations.

B. Magnetic cancellation

The cohesion strength R overestimates the forces in the system. Because of the anisotropic nature of the magnetic dipole-dipole force, the average force between two particles in contact is less than the maximum force F_v. The force changes sign depending on the angle ϕ between $\mathbf{B}$ and $\mathbf{r}$, so the forces acting on a particle due to its surrounding particles can be either attractive or repulsive.

In three dimensions, magnetic forces can cancel exactly. We have calculated the net magnetic force on a point dipole above an infinite plane of magnetic material, in a uniform vertical magnetic field. The attraction of the dipole to material underneath is counteracted by repulsion from material to the sides. The forces cancel exactly and the dipole experiences no net force.

The analogous calculation in two dimensions (the net magnetic force on a point dipole due to an infinitely long line) demonstrates partial cancellation. The net force is non-zero but significantly less than F_v.

A dipole above an infinite layer of point dipoles arranged in a regular lattice also experiences partial cancellation, in both two and three dimensions. The magnetic dipole-dipole force is a short range force, decaying as $1/r^4$. Hence the force on a particle will depend very sensitively on the arrangement of its neighbouring particles, but only weakly on the arrangement of particles further away.

As a measure of the magnetic force on a particle in the bulk of a pile, we calculate the sum of the radial components of the magnetic forces due to all other particles. By differentiating the interaction energy Eq. (1), we obtain the radial component $\mathbf{F_r}$ of the magnetic dipole-dipole force between two particles:

$$\mathbf{F_r} = -\frac{\partial E}{\partial r}\hat{\mathbf{r}} = F_v \frac{d^4}{2r^4}(1 - 3\cos^2\phi)\hat{\mathbf{r}} \tag{3}$$

We use an algebraic sum of the magnitudes of the radial forces rather than a vectorial sum. A vectorial sum would be close to zero, but when the particle moves it will experience a greater cohesive force. An algebraic sum gives an estimate of the cohesion in the packing.

We calculate the total radial magnetic force $F_{\text{total}} = \Sigma|\mathbf{F_r}|$ on a particle due to its six nearest neighbours, assuming regular hexagonal packing. By summing the contributions from all six neighbours, we obtain the net cohesive force $F_{\text{total}} = 1.5F_v$. This is true for any orientation of the hexagon relative to the magnetic field direction. Now we add the contributions to the total radial force from the next-nearest neighbours. Consider another ring of particles added around the outside of our original hexagon. The total radial force including contributions from next-nearest neighbours is $F_{\text{total}} = 1.76F_v$.

We have also calculated the total radial magnetic force per particle in our simulations, and plotted the results in Fig. 3 as a function of vertical position in the pile. The simulation results show that F_{total} is approximately constant in the bulk of the pile, and agrees well with our calculated value of 1.76 F_v. Note that if the magnetic forces were always attractive, the corresponding net force would be 7.04 F_v. This suggests that the cohesion strength R overestimates the magnetic forces in the system by a factor of 4.

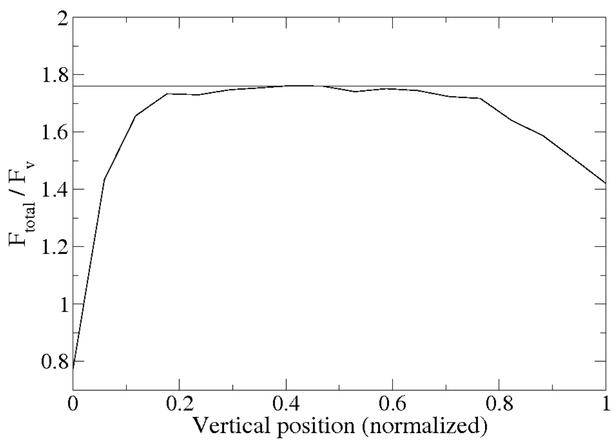

FIG. 3: Total radial magnetic force F_{total} per particle as a function of vertical position in the pile, measured at a horizontal position a quarter of the container width away from the left wall (the vertical line in Fig. 1). The horizontal line on the graph is 1.76 F_v. The particle positions are normalized so that the bottom of the pile is 0 and the top is 1.

Forsyth et al. [8] and Fazekas et al. [9] suggest that magnetic anisotropy could be an explanation for the two-orders-of-magnitude discrepancy between the size of the effects of cohesion on magnetic systems and wet systems. Our calculations suggest that this is not the case, and in the next section we outline an alternative explanation for the weakness of the effect of magnetic cohesion.

C. Avalanche dynamics

In steady fully-developed flows in three-dimensional piles, most of the motion occurs in a surface layer with a linear velocity profile, and there is creep motion in the bulk that decays exponentially [18, 19]. Aguirre et al. [20] report that in two-dimensional experiments in a slowly-tilted bed, the velocity profile is either purely exponential or a product of an exponential and a Gaussian.

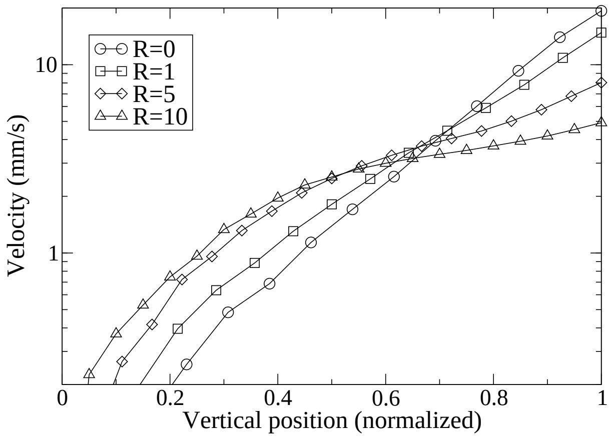

FIG. 4: The mean velocity per particle as a function of vertical position in the pile, measured at a horizontal position a quarter of the container width away from the left wall (the vertical line in Fig. 1). The particle positions are normalized so that the bottom of the pile is 0 and the top is 1. The addition of a magnetic field shifts the motion further down into the bulk of the pile.

Another system that exhibits a predominantly exponential velocity profile is a collection of monodisperse spheres in a slowly-sheared 3D Couette cell [21].

In the absence of a magnetic field, we have also observed shearing deep within the pile. In a magnetic field the surface of the heap is more rugged, and the size of surface irregularities increases with cohesion. Clusters of regularly-packed particles form and move as a block, both on the surface and in the bulk. Shear in the bulk occurs at the boundaries between clusters. The size of the clusters increases with cohesion, and contacts between neighbouring particles last for longer than in the absence of magnetic cohesion.

Fig. 4 plots the mean particle velocity in our simulations as a function of depth in the pile. In the absence of a magnetic field, the velocity decays exponentially with depth. The rate at which we add particles is slow enough that there is no constantly-moving surface layer, and the zero-field velocity profile is approximately exponential. On average, the tendency to slip at any given depth is proportional to the weight of particles above that depth. The frictional force that opposes slip, however, is also proportional to the weight of particles above. Hence the weight cancels out of the force balance equation and slip can occur at any depth in the pile.

In the presence of a magnetic field the motion shifts further down into the pile and the shape of the velocity profile changes, as can be seen in Fig. 4. This is because the interparticle cohesive forces in the bulk of the pile do not depend upon depth. Near the surface of the pile, cohesive forces can readily support the weight of the particles above, resulting in less shear than in the absence of cohesion. Further down in the pile the cohesive forces are less able to support the weight of the particles above, resulting in increased shear. It is this shear deep within the pile that prevents the angle of repose from increasing dramatically.

D. Effect of friction with front and back walls

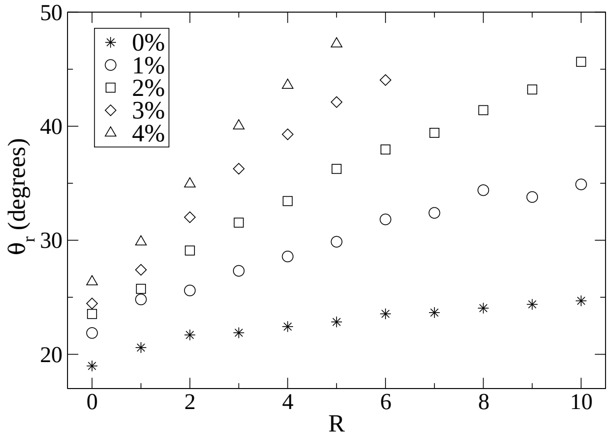

FIG. 5: Angle of repose as a function of cohesion strength R in a vertical magnetic field, in a container of width 50 particle diameters. Results are shown for different values of the friction percentage p.

So far we have considered an idealized two-dimensional system. Any real physical system will be influenced by the container walls. It is well-known that friction with confining walls can influence both the angle of repose and the velocity profile of avalanches in a narrow box [22, 23].

We introduce wall effects into our 2D simulations by using two different friction models, as described in Section III. In the first model, a percentage, p, of each normal contact force is directed towards the front and back walls. The frictional force depends upon the depth within the pile because a particle supports the weight of other particles resting on it. In the second model, a constant drag force proportional to the particle's weight is applied in the opposite direction to the particle's motion. In contrast with the first model, friction is independent of a particle's position in the pile.

Fig. 5 shows the angle of repose θ_r as a function of cohesion strength R for a range of values of p. The repose angle at zero field increases dramatically with p because the depth-dependent friction increasingly opposes motion further down in the pile. The variation of θ_r with R is much greater for higher values of p. In fact, the gradient $d\theta_r/dR$ increases linearly with p.

This behaviour can be understood by considering where the motion occurs in the pile. Fig. 6 plots the mean particle velocity as a function of depth. In the absence of front and back walls (p=0), increasing the magnetic field shifts the motion further down into the pile, as explained in section C. However, if p is non-zero, the frictional forces with the walls oppose motion in the bulk, causing the velocity profile to change shape, and the mo-

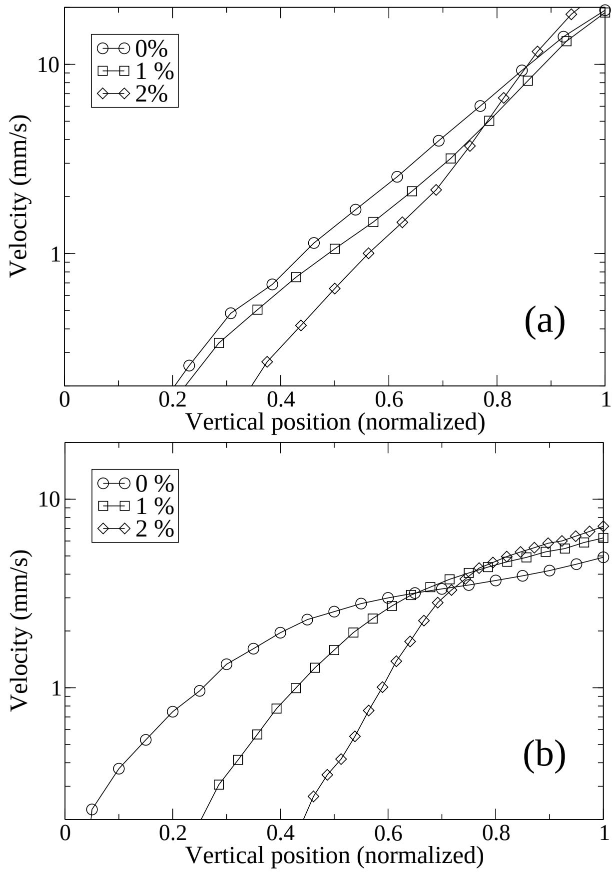

FIG. 6: The mean velocity per particle as a function of vertical position in the pile, measured at a horizontal position a quarter of the container width away from the left wall (the vertical line in Fig. 1). The particle positions are normalized so that the bottom of the pile is 0 and the top is 1. Results are shown for (a) $R = 0$ and (b) $R = 10$, at different values of the friction percentage p.

tion to shift closer to the surface of the pile. Hence there is less motion in the bulk and θ_r increases more quickly than in the absence of depth-dependent friction.

The effect of constant drag friction, however, is quite different. Fig. 7 shows the repose angle θ_r as a function of R, for different values of the drag constant α. The angle in zero field is about 24° for α=0.1, significantly higher than in the absence of front and back wall friction, and increases further for higher values of α. The repose angle θ_r increases by only a small amount with cohesion. In fact, the gradient $d\theta_r/dR = 0.5°$, the same as in the case with no front and back walls. Observing the simulations running, we can see that motion happens deep in the pile, not just near the surface.

The particle velocity profile as a function of depth in the pile is very similar to Fig. 4, demonstrating that this implementation of friction with the front and back walls does not change where slip occurs. Shear is still happening deep within the pile, preventing the angle of repose from increasing dramatically with magnetic cohesion.

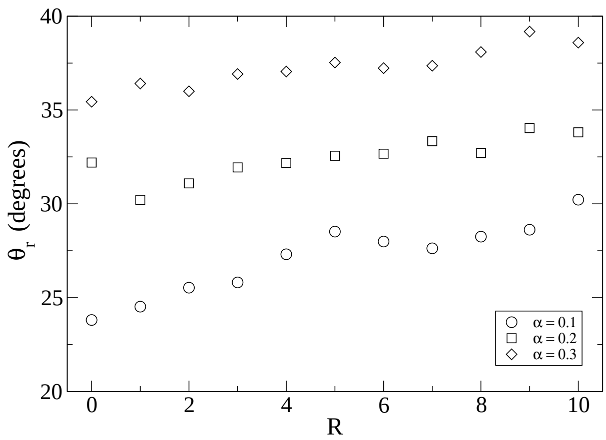

FIG. 7: Angle of repose as a function of cohesion strength R in a vertical magnetic field, in a container of width 50 particle diameters. Particles slide against the front and back walls of the container and are subject to a drag force αmg proportional to the particle weight. Results are shown for different values of the drag constant α.

E. Application to three-dimensional systems

The shear deep in the bulk of the pile in the absence of front and back walls explains why the dependence of repose angle on magnetic cohesion is weak. The presence of depth-dependent wall friction, however, results in a much larger gradient $d\theta_r/dR$. The inclusion of wall friction is an attempt to model the three-dimensional nature of many experimental geometries.

There is good agreement between $d\theta_r/dR$ in idealized two-dimensional simulations (both our own and those of Fazekas et al. [9]), and in the experiments of Forsyth et al. [8], but when wall friction effects are included in the simulations, there is no longer any agreement. One possible explanation is that the simulations were carried out using weakly magnetic particles, for which the point dipole approximation is valid, whereas Forsyth et al. used iron spheres, which are ferromagnetic and have a susceptibility $\chi \gg 1$.

Another possible explanation for the apparent discrepancy is that the simulations do not account for magnetic interactions in the third dimension, perpendicular to the front and back walls. We have estimated the magnetic force on a particle close to the front or back wall of a three-dimensional container, due to other particles in the container, assuming that all particles are weakly magnetic. The horizontal component of the magnetic dipole-dipole force between two point dipoles of moment $\mathbf{m}$ and separated by $\mathbf{r}$ is given by

$$\mathbf{F_x} = -\frac{\partial E}{\partial x}\hat{\mathbf{x}} = \frac{3\mu_0|\mathbf{m}|^{\mathbf{2}}}{4\pi|\mathbf{r}|^{\mathbf{4}}}\sin\phi(1 - 5\cos^2\phi)\hat{\mathbf{x}}, \qquad (4)$$

where $\hat{\mathbf{x}}$ is a unit vector in the direction perpendicular to the wall. We calculate the contributions to the horizontal force on a point dipole from all volume elements in the bulk, and integrate over the infinite half-space with $x > 0$ and $|\mathbf{r}| > d/2$. (The vertical component to the magnetic force cancels out due to symmetry.) We find that the dipole experiences an attractive force into the bulk of $0.147\ F_v$.

Thus, particles close to the front and back walls are attracted towards the bulk, and away from the walls. We speculate that this attraction will reduce the effect of wall friction, and that the system will behave more like our idealized two-dimensional simulations. This may be the cause of the weakness of the effect of magnetic cohesion on the angle of repose observed experimentally.

Recent experiments on granular avalanches in confined geometries subject to electric fields demonstrate that electric cohesion and wall interactions can significantly influence the repose angle [24]. It would therefore be interesting to investigate whether magnetic systems exhibit similar behaviour, as suggested by our calculations in this section.

V. CONCLUSION

We have used a two-dimensional molecular dynamics simulation to investigate the effect of magnetic cohesion on the repose angle of a granular pile. We found that the repose angle increases linearly with cohesion strength R. The effect is weak, even when magnetic forces are ten times as strong as gravity.

When a magnetic field is applied, the magnetic forces partially cancel out deep in the pile. Motion happens by shearing deep within the pile, in addition to motion close to the surface. We have shown that the slope angle has only a weak dependence on the magnetic field because shear deep in the pile prevents the angle of repose from increasing substantially.

We have investigated the effect of different implementations of friction with the front and back walls of the container. We conclude that the choice of friction model dramatically affects both the zero-field repose angle and its rate of increase with cohesion. Depth-dependent friction causes an increase in the zero-field repose angle and in the gradient $d\theta_r/dR$. Depth-independent friction causes an increase in the zero-field repose angle, but $d\theta_r/dR$ remains unchanged.

We have also demonstrated that in a three-dimensional system, particles near the front and back walls of the container experience a net attractive force pulling them towards the bulk of the pile and away from the walls. We suggest that this attraction will reduce the effect of wall friction on both the repose angle and its rate of increase with cohesion.

Acknowledgments

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