Busstepp 2011
Cosmology - Lecture 1

Ed Copeland -- Nottingham University

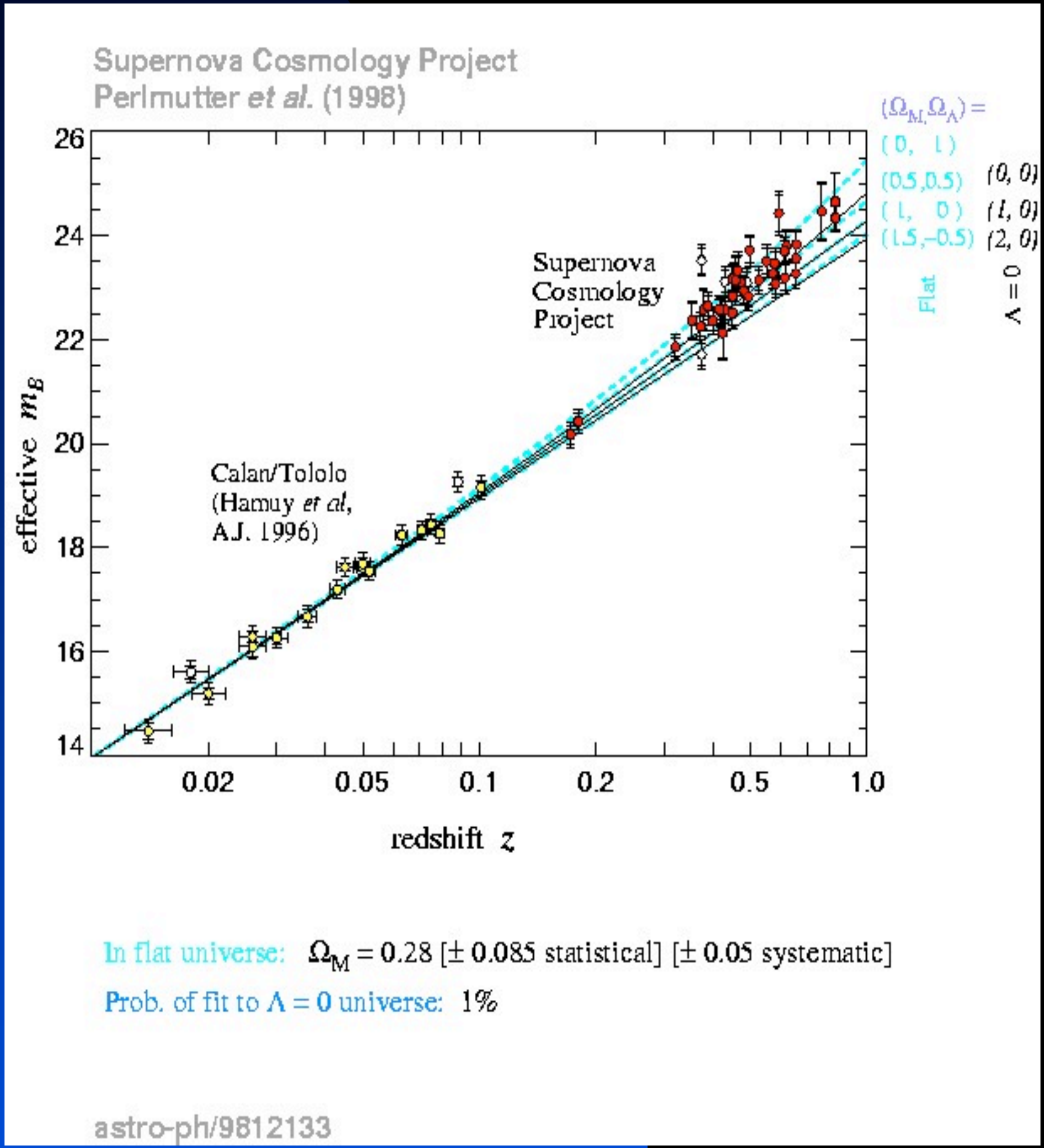
1. The general picture, evolution of the universe: assumptions and evidence supporting them.
2. Dark Energy - Dark Matter - Modified Gravity
3. Origin of Inflation and the primordial density fluctuations.
4. Scaling solutions applied to cosmology.

Aug 30th - Sep 2nd 2011

Nottingham

1. The Big Bang – (1sec → today)

The cosmological principle -- isotropy and homogeneity on large scales



Test 1

- **The expansion of the Universe $v = H_0 d$**

$H_0 = 74.2 \pm 3.6$ km s^{-1} Mpc^{-1}

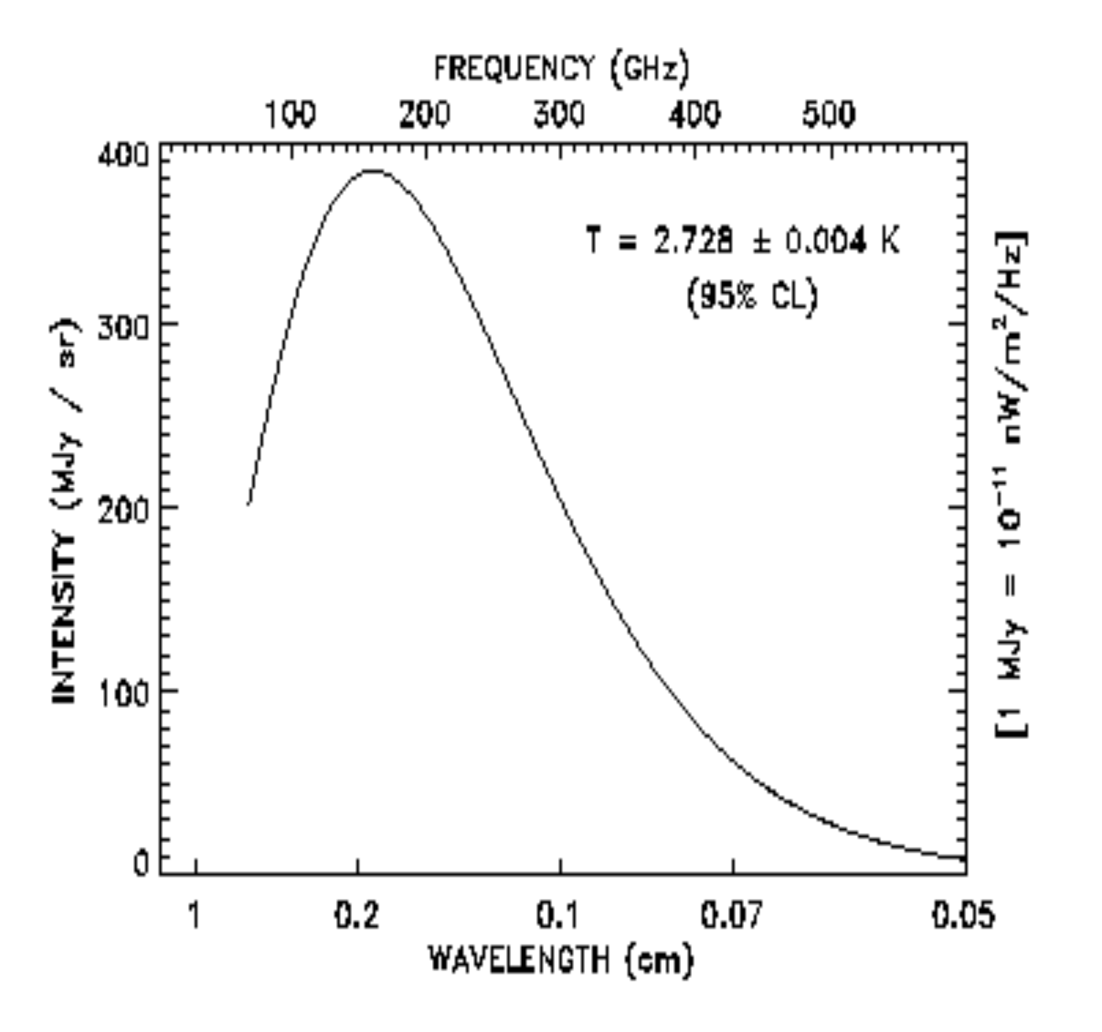
(Riess et al, 2009)

Distant galaxies receding with vel proportional to distance away.

Relative distance at different times measured by scale factor a(t) with

$$H = \frac{\dot{a}}{a}$$

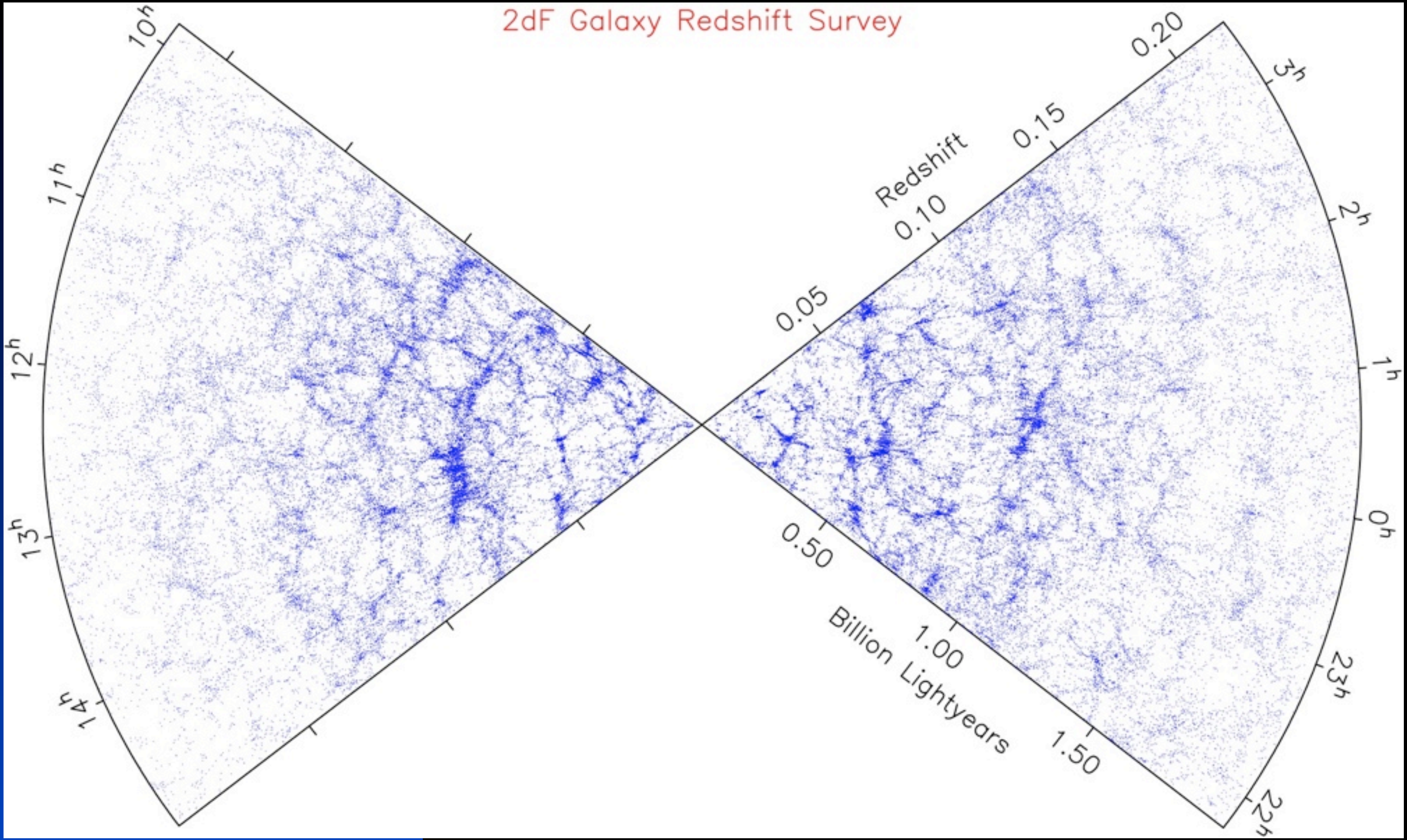
The Big Bang – (1sec → today)



Test 2

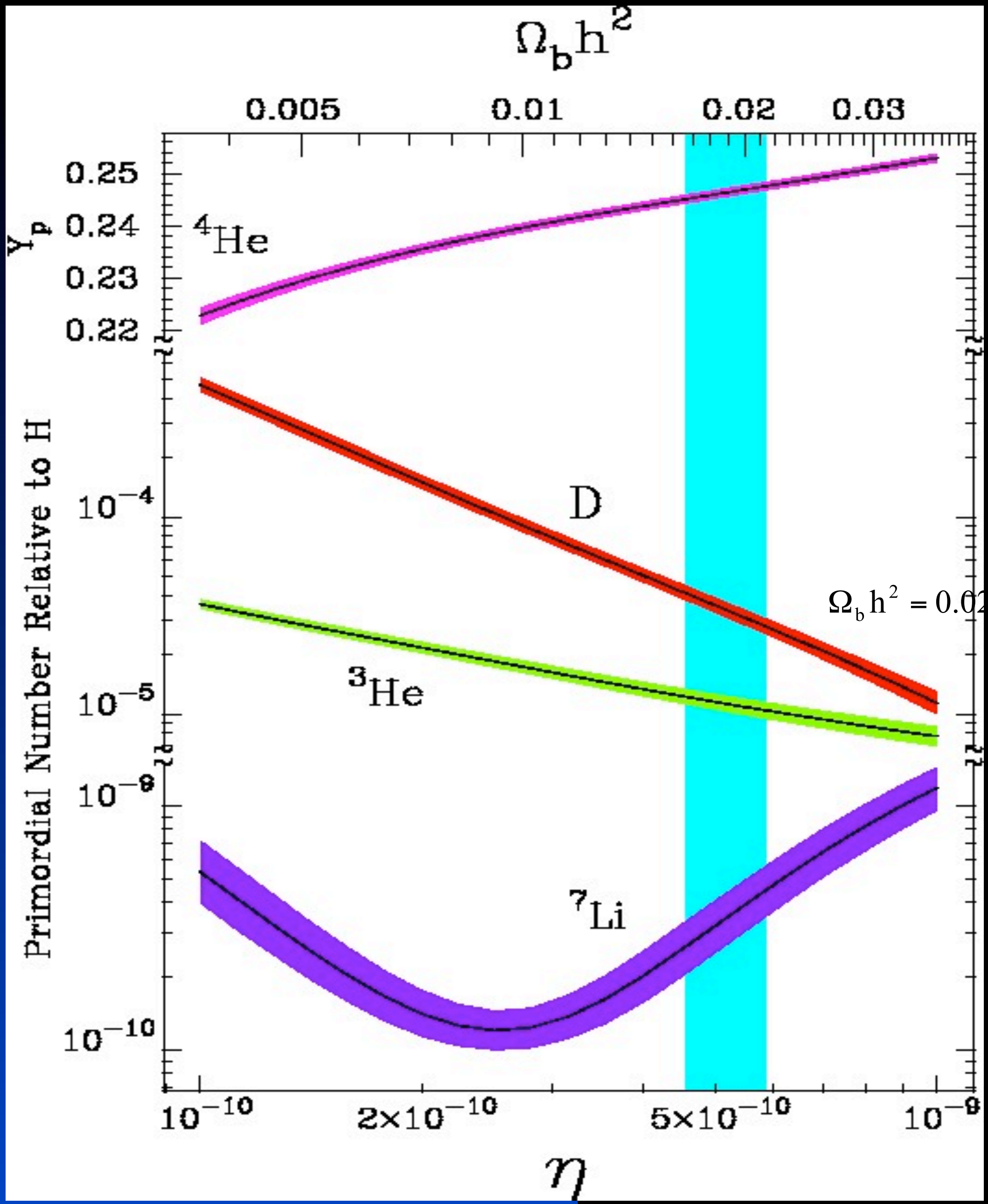
- **The existence and spectrum of the CMBR**
- **T_0=2.728 ± 0.004 K**
- Evidence of isotropy -- detected by COBE to such incredible precision in 1992
- Nobel prize for John Mather 2006

2dF Galaxy Redshift Survey



Homogeneous on large scales?

The Big Bang – (1sec → today)



Test 3

- **The abundance of light elements in the Universe.**
- **Most of the visible matter just hydrogen and helium.**

WMAP7 - detected effect of primordial He on temperature power spectrum, giving new test of primordial nucleosynthesis.

$$Y_P = 0.326 \pm 0.075$$

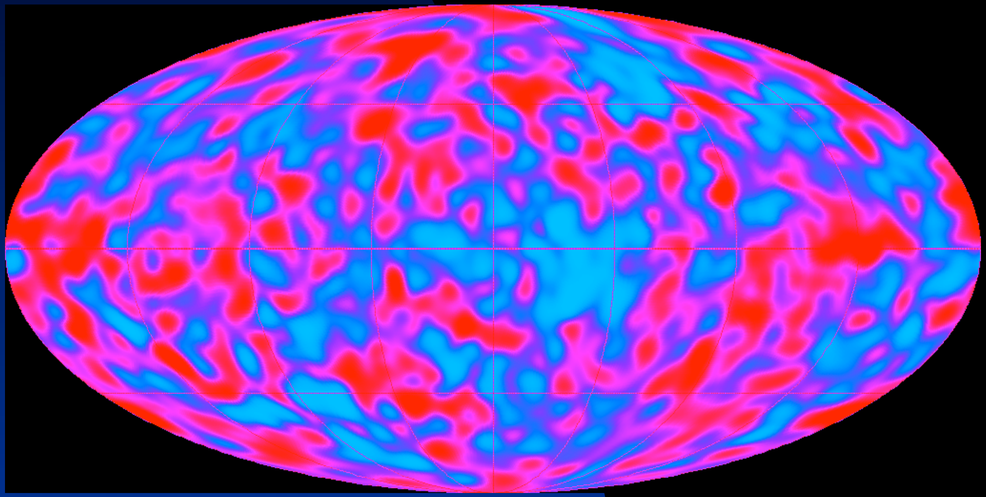
(Komatsu et al, 2010)

$$\Omega_b h^2 = 0.0225 \pm 0.0005 \ (68\% \ CL)$$

The Big Bang – (1sec → today)

Test 4

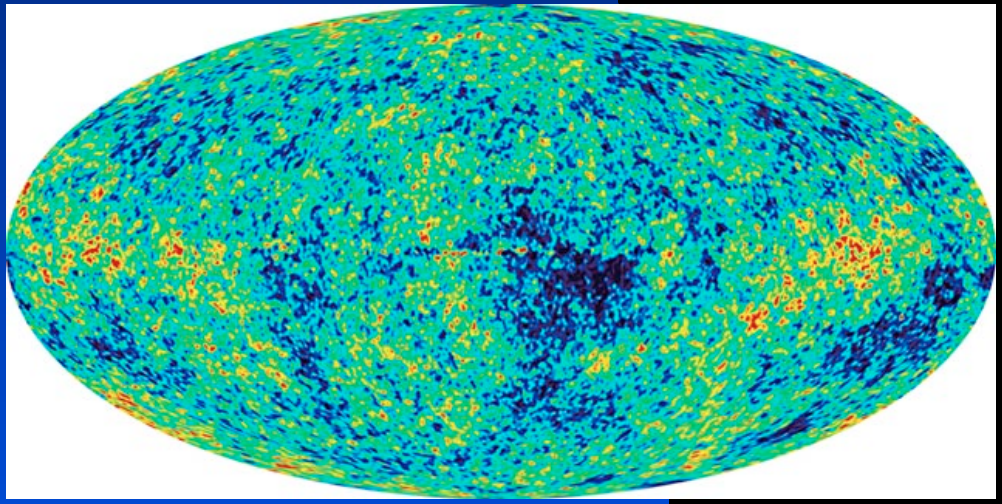
- **Given the irregularities seen in the CMBR, the development of structure can be explained through gravitational collapse.**



COBE - 1992, 2006

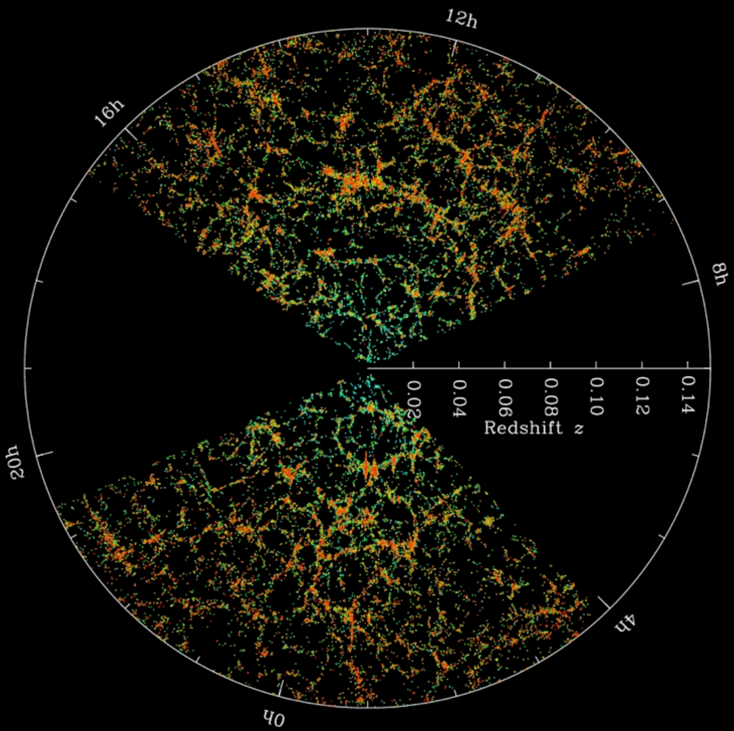
Nobel prize for

George Smoot



WMAP-2010

SDSS



The key equations

Einstein GR: $$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Geometry   Matter   Cosm const - could be matter or geometry

Relates curvature of spacetime to the matter distribution and its dynamics.

Require metric tensor $g_{\mu\nu}$ from which all curvatures derived indep of matter:

Invariant separation of two spacetime points ($\mu,\nu=0,1,2,3$):

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

Einstein tensor $G_{\mu\nu}$ -- function of $g_{\mu\nu}$ and its derivatives.
Energy momentum tensor $T_{\mu\nu}$ -- function of matter fields present.
For most cosmological substances can use perfect fluid representation for which we write

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu}$$

U^μ : fluid four vel = (1,0,0,0) - because comoving in the cosmological rest frame.
(ρ,p) : energy density and pressure of fluid in its rest frame

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

Reminder of curvatures

Christoffel symbols:

$$\Gamma^{\mu}_{\nu\sigma} = \frac{1}{2} g^{\mu\lambda} (g_{\sigma\lambda,\nu} + g_{\nu\lambda,\sigma} - g_{\sigma\nu,\lambda})$$

Riemann's curvature tensor:

$$R^{\mu}_{\nu\sigma\gamma} = \Gamma^{\mu}_{\nu\gamma,\sigma} - \Gamma^{\mu}_{\nu\sigma,\gamma} + \Gamma^{\mu}_{\alpha\sigma}\Gamma^{\alpha}_{\gamma\nu} - \Gamma^{\mu}_{\alpha\gamma}\Gamma^{\alpha}_{\sigma\nu}$$

Ricci tensor:

$$R_{\mu\nu} = R^{\sigma}_{\mu\nu\sigma}$$

Ricci scalar:

$$R = R^{\mu}_{\mu}$$

Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Not needed here -- maybe in the tutorials

Cosmology - isotropic and homogeneous FRW metric

Copernican Principle: We are in no special place. Since universe appears isotropic around us, this implies the universe is isotropic about every point. Such a universe is also homogeneous.

Line element $$ds^2 = -dt^2 + a^2(t)dx^2$$

$$dx^2 = \frac{1}{1-kr^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

t -- proper time measured by comoving (i.e. const spatial coord) observer. a(t) -- scale factor: k- curvature of spatial sections: k=0 (flat universe), k=-1 (hyperbolic universe), k=+1 (spherical universe)

Aside for those familiar with this stuff -- not chosen a normalisation such that $a_0=1$. We are not free to do that and simultaneously choose |k|=1. Can do so in the k=0 flat case.

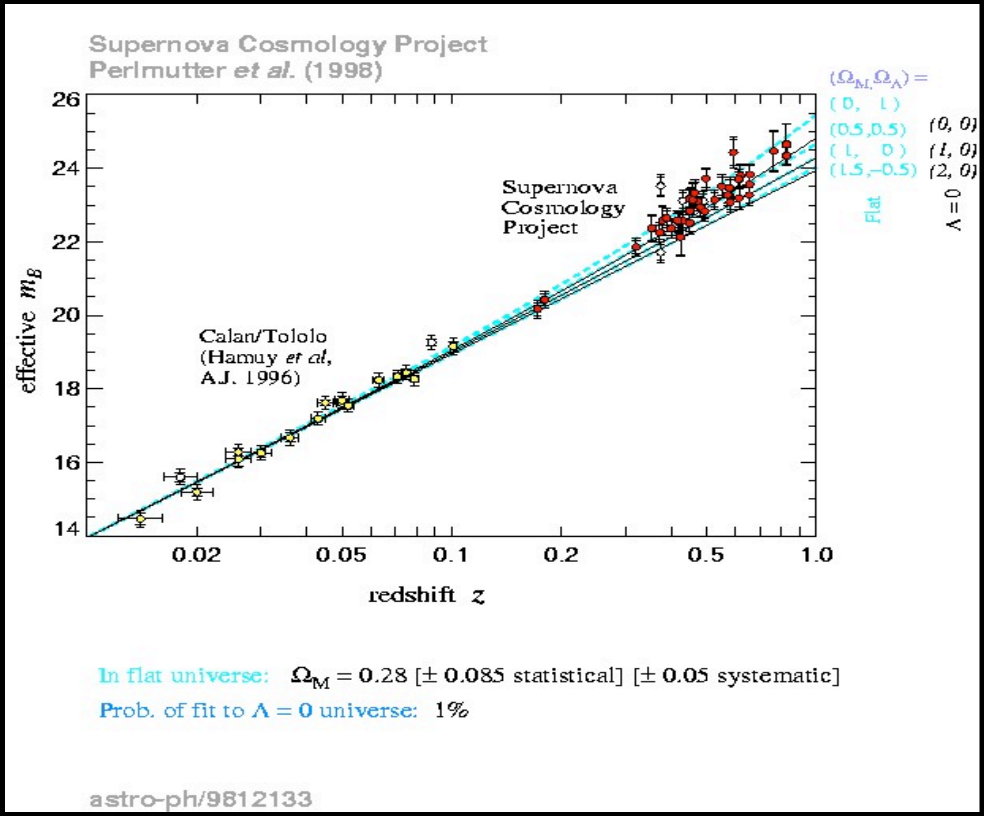
Intro Conformal time : τ(t) $\tau(t) \equiv \int^t \frac{dt'}{a(t')}$

Implies useful simplification : $ds^2 = a^2(\tau)(-d\tau^2 + dx^2)$

Hubble parameter :
(often called Hubble constant) $H(t) \equiv \frac{\dot{a}}{a}$

Hubble parameter relates velocity of recession of distant galaxies from us to their separation from us

$$v = H(t)r$$



$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$ applied to cosmology

Friedmann:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter, $\rho(t)=\Sigma_i\rho_i$ -- sum of all matter contributions, rad, dust, scalar fields ...

Energy density $\rho(t)$: Pressure $p(t)$

Related through : $p = w\rho$

Eqn of state parameters: w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

Eqns (Λ=0):

Friedmann + Fluid energy conservation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\nabla^\mu T_{\mu\nu} = 0$$

Combine Friedmann and fluid equation to obtain Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G(\rho + 3p) - - - Accn$$

$$If \;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

Inflation condition -- more later

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} \quad ; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

$$\mathrm{RD} \quad : \quad w = \frac{1}{3} : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-4} \;\; ; \; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$$

$$\mathrm{MD} \quad : \quad w = 0 : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3} \;\; ; \; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

$$\mathrm{VD} \quad : \quad w = -1 : \rho(t) = \rho_0 \;\; ; \; a(t) \propto e^{Ht}$$

Solutions with curvature in problem set.

A neat equation

$$\rho_c(t) \equiv \frac{3H^2}{8\pi G} \quad ; \quad \Omega(t) \equiv \frac{\rho}{\rho_c}$$

$$\Omega > 1 \leftrightarrow k = +1$$

$$\Omega = 1 \leftrightarrow k = 0$$

$$\Omega < 1 \leftrightarrow k = -1$$

Friedmann eqn

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

Ω_m - baryons, dark matter, neutrinos, electrons, radiation ...

Ω_Λ - dark energy ; Ω_k - spatial curvature

$$\rho_c(t_0) \equiv 1.88h^2 * 10^{-29}\,\mathrm{gcm}^{-3}$$

Critical density

Current bounds on H(z) -- Komatsu et al 2010 - (WMAP7+BAO+SN)

$$\mathbf{H^2(z) = H_0^2 \left(\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{de} \exp\left(3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right) \right)}$$

(Expansion rate) -- $H_0 = 70.4 \pm 1.3$ km/s/Mpc

(radiation) -- $\Omega_r = (8.5 \pm 0.3) \times 10^{-5}$

(baryons) -- $\Omega_b = 0.0456 \pm 0.0016$

(dark matter) -- $\Omega_m = 0.227 \pm 0.014$

(curvature) -- $\Omega_k < 0.008$ (95%CL)

(dark energy) -- $\Omega_{de} = 0.728 \pm 0.015$ -- Implying univ accelerating today

(de eqn of state) -- $1+w = 0.001 \pm 0.057$ -- looks like a cosm const.

If allow variation of form : $w(z) = w_0 + w' z/(1+z)$ then
$w_0 = -0.93 \pm 0.12$ and $w' = -0.38 \pm 0.65$ (68% CL)

How old are we?

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$where\ \rho = \rho_m + \rho_r + \rho_\Lambda$$

$$t = \int \frac{da}{\dot{a}} = \int \frac{da}{aH}$$

$$t_0 = H_0^{-1} \int_0^1 \frac{x\,dx}{\left[\Omega_{m0} x + \Omega_{r0} + \Omega_{\Lambda 0} x^4 + (1 - \Omega_0) x^2\right]^{1/2}}$$

$$where\ \Omega_0 = \Omega_{m0} + \Omega_{r0} + \Omega_{\Lambda 0}$$

$$Today: H_0^{-1} = 9.8 \times 10^9\ h^{-1}\ years; h = 0.7$$

$$H_0^{-1} - -\text{Hubble time}$$

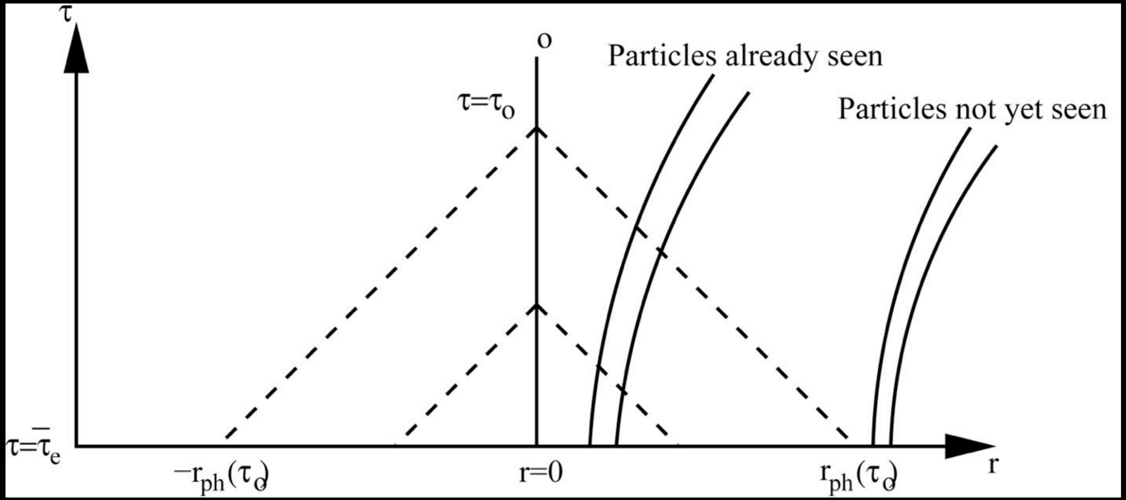
Useful estimate for age of universe

Ω_{m0}	Ω_{r0}	$\Omega_{\Lambda 0}$	t_0
1	0	0	9.4 Gyr
0.3	10^{-5}	0.7	13.4 Gyr
Open			
0.2	10^{-5}	0.2	12.4 Gyr
0.2	10^{-5}	0.6	13.96 Gyr
Closed			
0.3	10^{-5}	0.8	13.96 Gyr
0.4	10^{-5}	0.9	13.6 Gyr

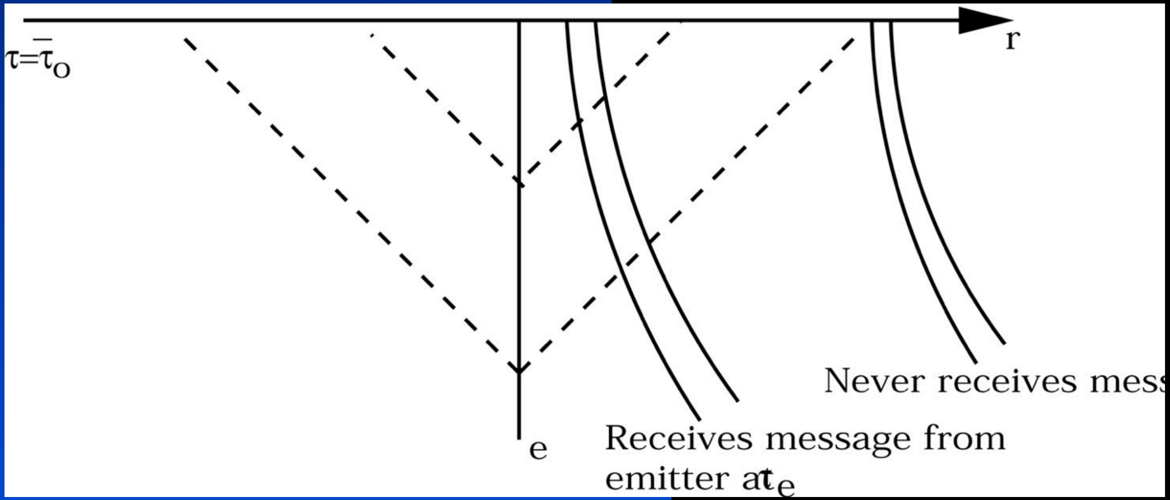
Horizons -- crucial concept in cosmology

a) Particle horizon: is the proper distance at time t that light could have travelled since the big bang (i.e. at which a=0). It is given by

$$d_p(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$



b) Event horizon: is the proper distance at time t that light will be able to travel in the future:



$$d_{\rm EH}(t) = a(t)\int_t^\infty \frac{dt'}{a(t')}$$

Trodden and Carroll 03

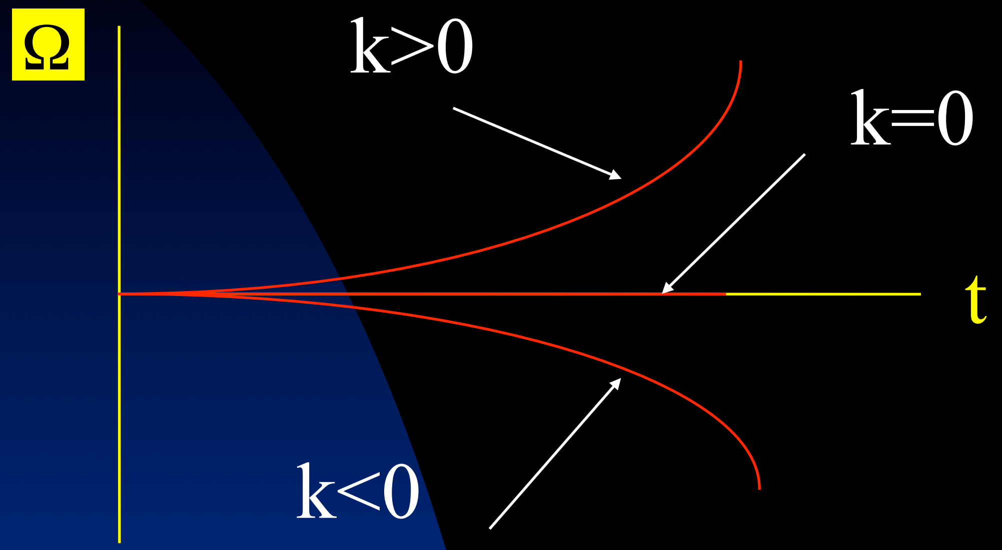
History of the Universe

10^{18} GeV	10^{-43} sec	10^{32} K	QG/String epoch
			Inflation begins (?)
10^{3} GeV	10^{-10} sec	10^{15} K	Electroweak tran
1 GeV	10^{-4} sec	10^{12} K	Quark-Hadron tran
1 MeV	1 sec	10^{10} K	Nucleosynthesis
1 eV	10^{4} years	10^{4} K	Matter-rad equality
	10^{5} years	3.10^{3} K	Decoupling → microwave bgd.
10^{-3} eV	10^{10} years	3K	Present epoch

The Big Bang – problems.

- Flatness problem – observed almost spatially flat cosmology requires fine tuning of initial conditions.
- Horizon problem -- isotropic distribution of CMB over whole sky appears to involve regions that were not in causal contact when CMB produced. How come it is so smooth?
- Monopole problem - where are all the massive defects which should be produced during GUT scale phase transitions.
- Relative abundance of matter – does not predict ratio baryons: radiation: dark matter.
- Origin of the Universe – simply assumes expanding initial conditions.
- Origin of structure in the Universe from initial conditions homogeneous and isotropic.
- The cosmological constant problem.

Flatness problem



Today: $\Omega < 1.1$

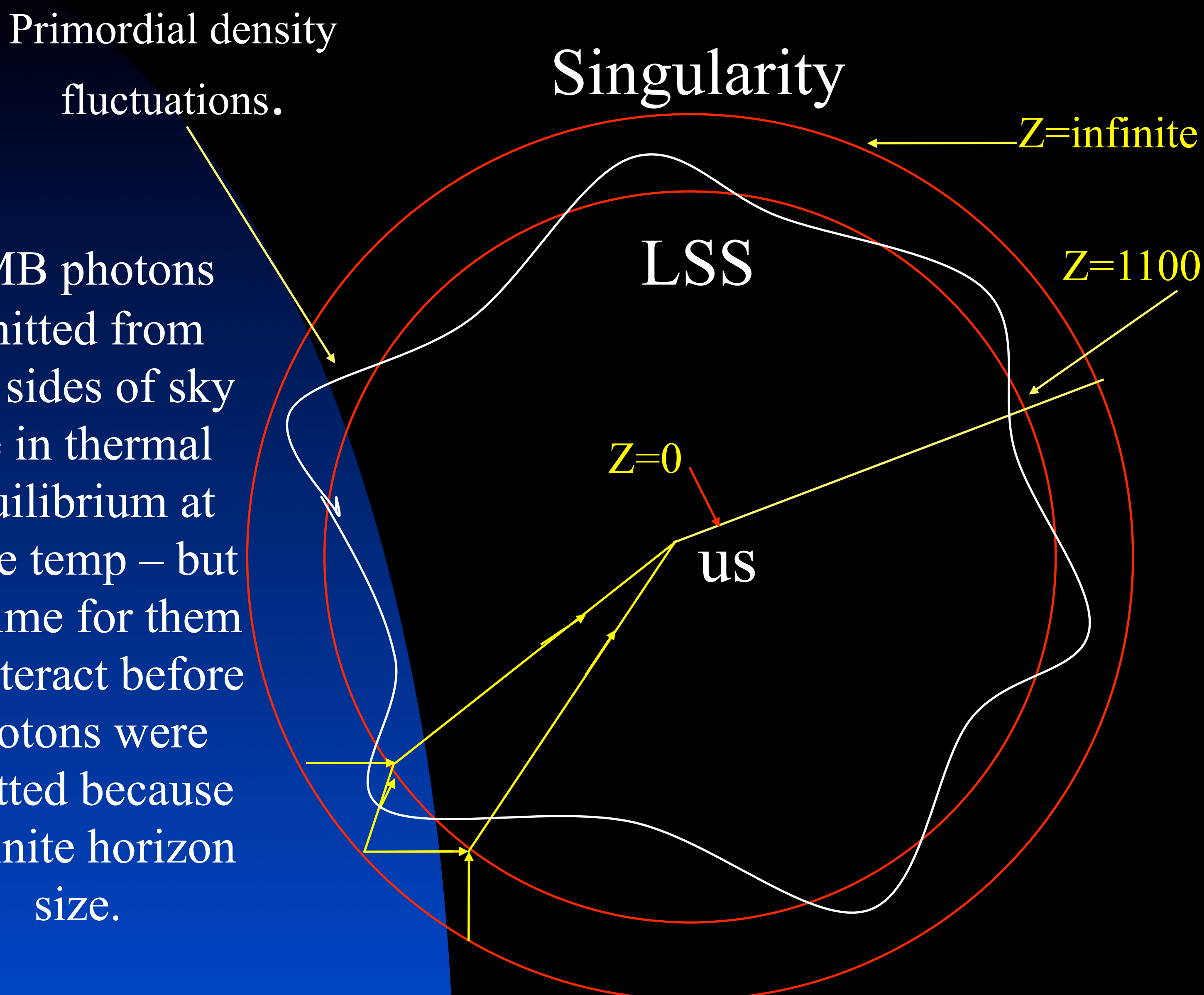
Why?

$$\Omega^{-1} = 1 - \frac{3k}{\kappa^2 \rho a^2} \propto \begin{cases} a & \text{mat} \\ a^2 & \text{rad} \\ a^{-2} & \Lambda \end{cases}$$

$$|\Omega(1\text{s}) - 1| = O(10^{-16})$$

Horizon problem

CMB photons emitted from opp sides of sky are in thermal equilibrium at same temp – but no time for them to interact before photons were emitted because of finite horizon size.



CMBR last interacted at 1+Z = 1100

300,000 yrs after big bang

Hubble radius was 2 degrees, 200 Mpc

LSS thickness – 15Mpc

Any region separated by > 2 deg – causally separated at decoupling.

Monopole problem

Monopoles are generic prediction of GUT type models.

They are massive stable objects, like domain walls and cosmic strings and many moduli fields.

They scale like cold dark matter, so in the early universe would rapidly come to dominate the energy density.

Must find a mechanism to dilute them or avoid forming them.

The big questions in cosmology today

a) What is dark matter? -- 23% of the energy density
b) What is dark energy? -- 73% of the energy density. Does dark energy interact with other stuff in the universe?
c) Is dark energy really a new energy form or does the accelerating universe signal a modification of our theory of gravity?
d) What is the origin of the density perturbations, giving rise to structures?
e) Is there a cosmological gravitational wave background?
f) Are the fluctuations described by Gaussian statistics? If there are deviations from Gaussianity, where do they come from?
g) How many dimensions are there? Why do we observe only three spatial dimensions?
h) Was there really a big bang (i.e. a spacetime singularity)? If not, what was there before?

A bit of thermodynamics - remember your stat mech

Gas -weakly interacting in kinetic eqm. Distribution function for particle species x, physical momentum p

$$f_x(p) = \frac{1}{e^{\frac{E_x - \mu_x}{T}} \pm 1}$$

- sign bosons, + sign fermions, μ chemical pot, T-temp: $E_x^2 = p^2 + m_x^2$

Include internal dof: i.e. spin by g_x (photons have g=2, neutrinos g=1)

number density: $$n_x = \frac{g_x}{(2\pi)^3} \int f_x(p) d^3p$$

energy density: $$\rho_x = \frac{g_x}{(2\pi)^3} \int E_x(p) f_x(p) d^3p$$

pressure: $$p_x = \frac{g_x}{(2\pi)^3} \int \frac{|p|^2}{3E_x(p)} f_x(p) d^3p$$

Non-Rel limit : m>>T

$$n_x \simeq g_x \left(\frac{m_x T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_x}{T}}$$

$$\rho_x \simeq m_x n_x \qquad p_x \simeq T n_x$$

Rel limit : m<<T -- BE and FD

$$n_x^{BE} = \frac{\zeta(3)}{\pi^2} g_x T^3 \qquad n_x^{FD} \simeq \frac{3}{4} n_x^{BE}$$

$$\rho_x^{BE} \simeq \frac{\pi^2}{30} g_x T^4 \qquad \rho_x^{FD} \simeq \frac{7}{8} \rho_x^{BE}$$

$$\zeta(3) = 1.202...$$

Friedmann eqn in early universe during rad dom: $\rho_{\rm rad} = \rho_{BE} + \rho_{FD} = \frac{\pi^2}{30} g_{\rm eff}(T) T^4$

Temp high so all particle species in therm eqm: for std model particles T>1TeV. Total num of dof for fermions (90), gauge and Higgs (28) so:

$$g_{\rm eff}(T = 1TeV) = 106.75$$

If the interaction rate between particles becomes smaller than the expansion rate, then those particles have a smaller temp than the photons (temp T) but might be relativistic. So, intro specific temp for each relativistic species.

$$g_{\rm eff}(T) = \sum_{i=\rm bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{j=\rm fermions} g_j \left(\frac{T_j}{T}\right)^4$$

Hence: $H = 0.33 \sqrt{g_{\rm eff}} \frac{T^2}{m_{\rm Pl}}$ and $t = 1.52 \frac{m_{\rm Pl}}{\sqrt{g_{\rm eff}} T^2}$

Kinetic Equilibrium - characterised by T - particles exchange energy, energy density constant:

$$X_1 + X_2 \leftrightarrow X_1 + X_2$$

Chemical Equilibrium - characterised by μ - species can change number, number density constant:

$$X_1 + X_2 \leftrightarrow X_3 + X_4 \quad \text{with} \quad \mu_1 + \mu_2 = \mu_3 + \mu_4$$

Equilibrium condition: interaction rate happens faster than the expansion rate of the universe. $\Gamma > H$

Now: $\Gamma = n < \sigma v >$

- n: Number density
- σ: Cross section
- v: Ave vel
- $<\,>$: Thermal Ave

Ex: Neutrino decoupling: $\gamma\gamma \leftrightarrow e^+e^- \quad \nu e \leftrightarrow \nu e \quad \nu\bar{\nu} \leftrightarrow \nu\bar{\nu}$

Cross section: $\sigma \simeq G_F^2 T^2 \rightarrow \Gamma \simeq G_F^2 T^5$

Hence: $$\frac{\Gamma}{H} = \left(\frac{T}{1\text{MeV}}\right)^3$$

So for T>1 MeV, neutrinos in thermal eqm with photons, but below 1MeV, interaction rate too low to maintain eqm with photon plasma.

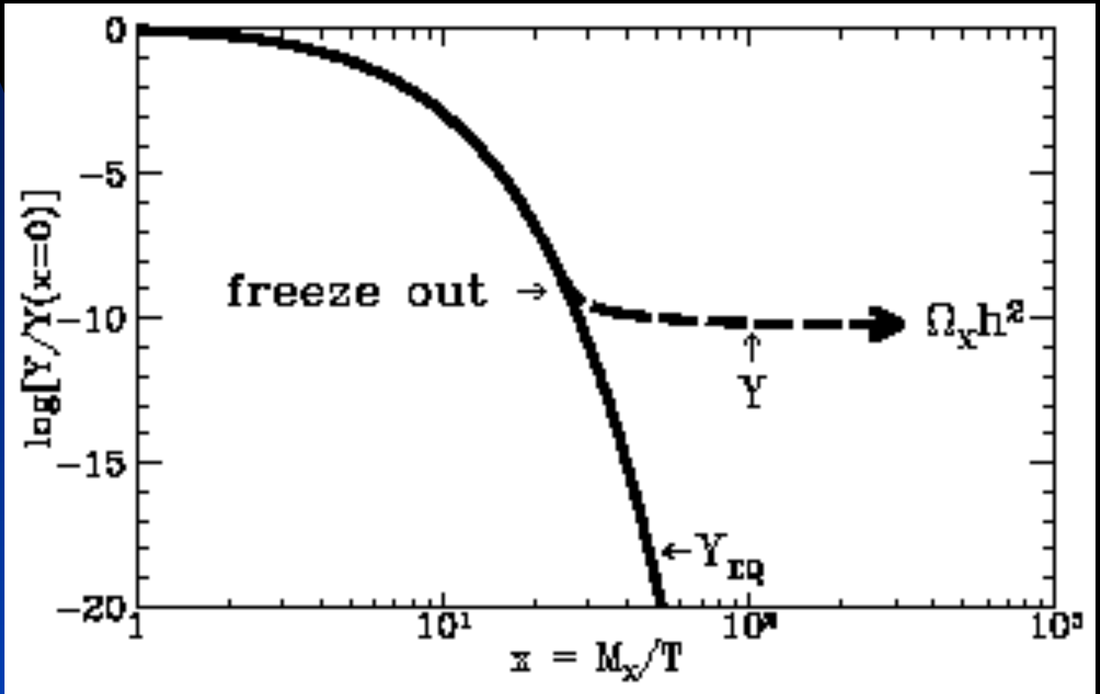
Decoupling: - departure from Kinetic Equilibrium

Freeze out: - departure from Chemical Equilibrium

Estimate decoupling or freeze out temp by Γ=H: $$n < \sigma v > \simeq \sqrt{g_{\rm eff}}\frac{T^2}{m_{\rm Pl}}$$

Note that for neutrinos with m<1 MeV, we have m<T hence relativistic. Such particles which are relativistic at freeze-out are hot-dark-matter candidates.

Weakly interacting particles tend to have m/T ~ 20, so non-relativistic particles and cold dark matter candidates.



Taken from http://nedwww.ipac.caltech.edu/level5/Kolb/Kolb5_1.html

Y - ratio of number density to entropy density

Turns out cold dark matter needed for structure formation. Doesn't match observations if it is hot.

Dark matter candidates:

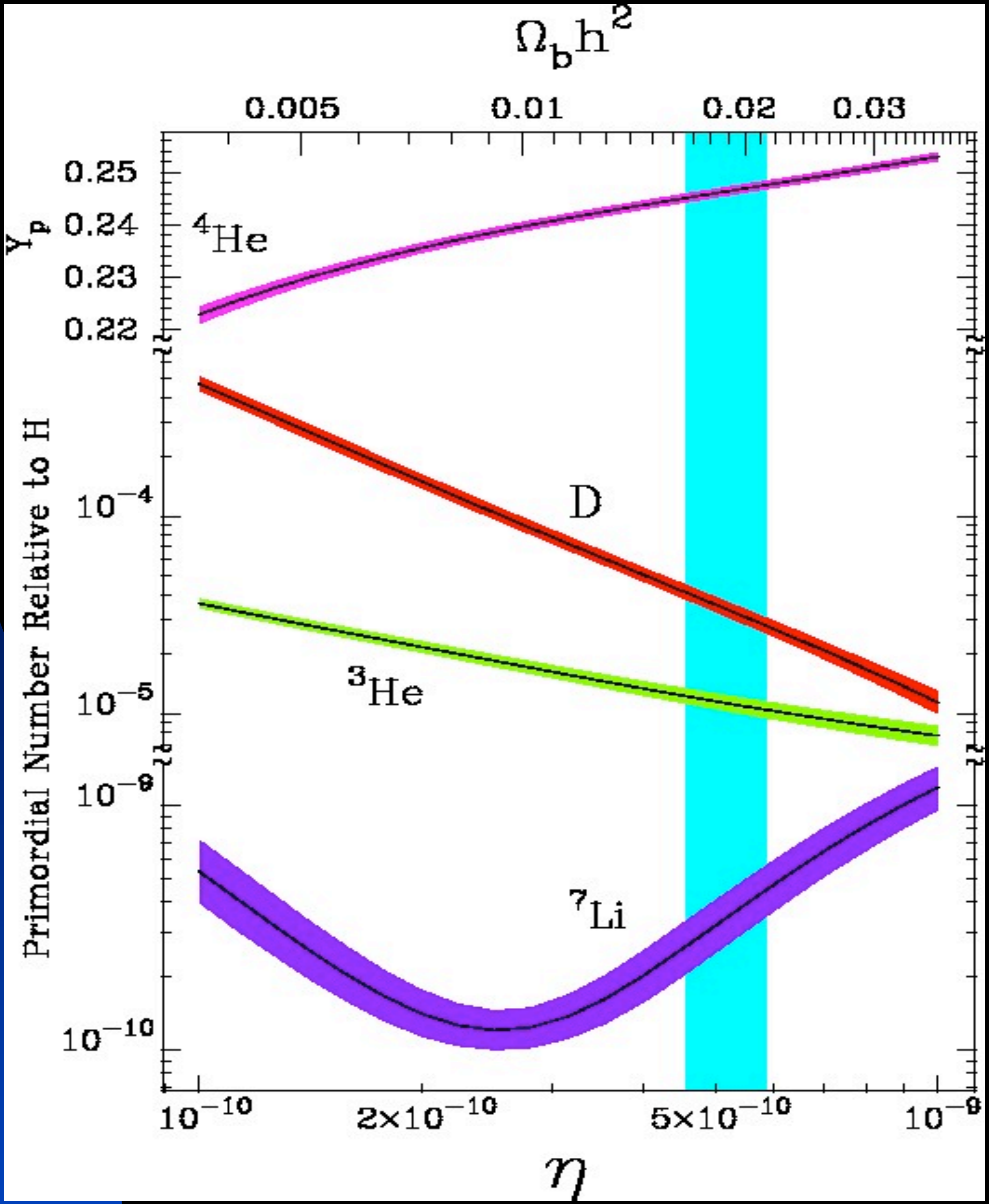
$$\Omega_m h^2 = 0.1369 \pm 0.0037$$

* Axion (solves CP problem of QCD)
* Neutrino – known to have mass, cannot be dominant dark matter.
* Neutralino – lightest supersymmetric particle.
* Gravitinos, Q-balls, WIMP-zillas…
* Kaluza-Klein dark matter
* Black holes
* …

Big Bang Nucleosynthesis -- formation of the lightest nuclei

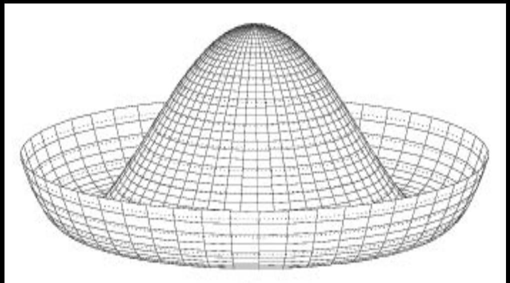
If the temperature is low enough, protons and neutrons can bind together to produce elements such as ^{4}He, D, ^{7}Li. For this to happen, the temperature must drop below about 1 MeV.

- Binding starts at T below the binding energy of the nuclei.
- During BBN the light elements are produced (in particular 3He, 4He, D, ^{7}Li). Heavier elements are created in stars at a much later time.
- Can predict the abundances as a function of the energy density in baryons-- a great success of the Hot Big Bang



$$\Omega_b h^2 = 0.0225 \pm 0.0005 \ (68\% \ CL)$$

Phase Transitions in the Early Universe -- could be vital!
Spontaneous symmetry breaking : Higgs, topological defects,
Finite temp effective potential:



$$V_T(\phi) = \left(-\frac{1}{2}m^2 + \frac{\lambda}{8}T^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4 + K$$

$$T > \frac{2m}{\sqrt{\lambda}} \quad \text{then} \quad m_{\text{eff}} > 0 \quad \text{and} \quad <\phi> = 0$$ symmetry restored

$$T < \frac{2m}{\sqrt{\lambda}} \quad \text{then} \quad m_{\text{eff}} < 0 \quad \text{and} \quad <\phi> \neq 0$$ symmetry broken

Example: GUT phase transition, Electroweak PT, QCD PT
Formation of topological defects such as cosmic strings, domain walls, monopoles, textures ...

I owe a great deal to cosmic strings -- they are neat and through cosmic superstrings could provide the first observational evidence for string theory.

Busstepp 2011
Cosmology - Lecture 2

Ed Copeland -- Nottingham University

1. Dark Energy - Dark Matter - Modified Gravity

Weighing the Universe

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

$1. \Omega_m$

a. Cluster baryon abundance using X-ray measurements of intracluster gas, or SZ measurements.

b. Weak grav lensing and large scale peculiar velocities.

c. Large scale structure distribution.

d. Numerical simulations of cluster formation.

$$\Omega_m h^2 = 0.1369 \pm 0.0037$$

$$\Omega_m << 1$$

(Komatsu et al, 2008) (WMAP5)

H_0=70.4±1.3 km s^{-1} Mpc^{-1}

2. Ω_b

BBN → $$\Omega_b h^2 = 0.0225 \pm 0.0005 \ (68\% \ CL)$$

↓

$\Omega_b << \Omega_m$ → **Require Dark matter !!**

Majority of baryonic matter dark.

Candidates: WIMPS (Neutralinos, Kaluza Klein Particles, Universal Extra Dimensions...)

Axinos, Axions, Axion-like light bosons, Sterile neutrinos, Q-balls, WIMPzillas, Elementary Black Holes...

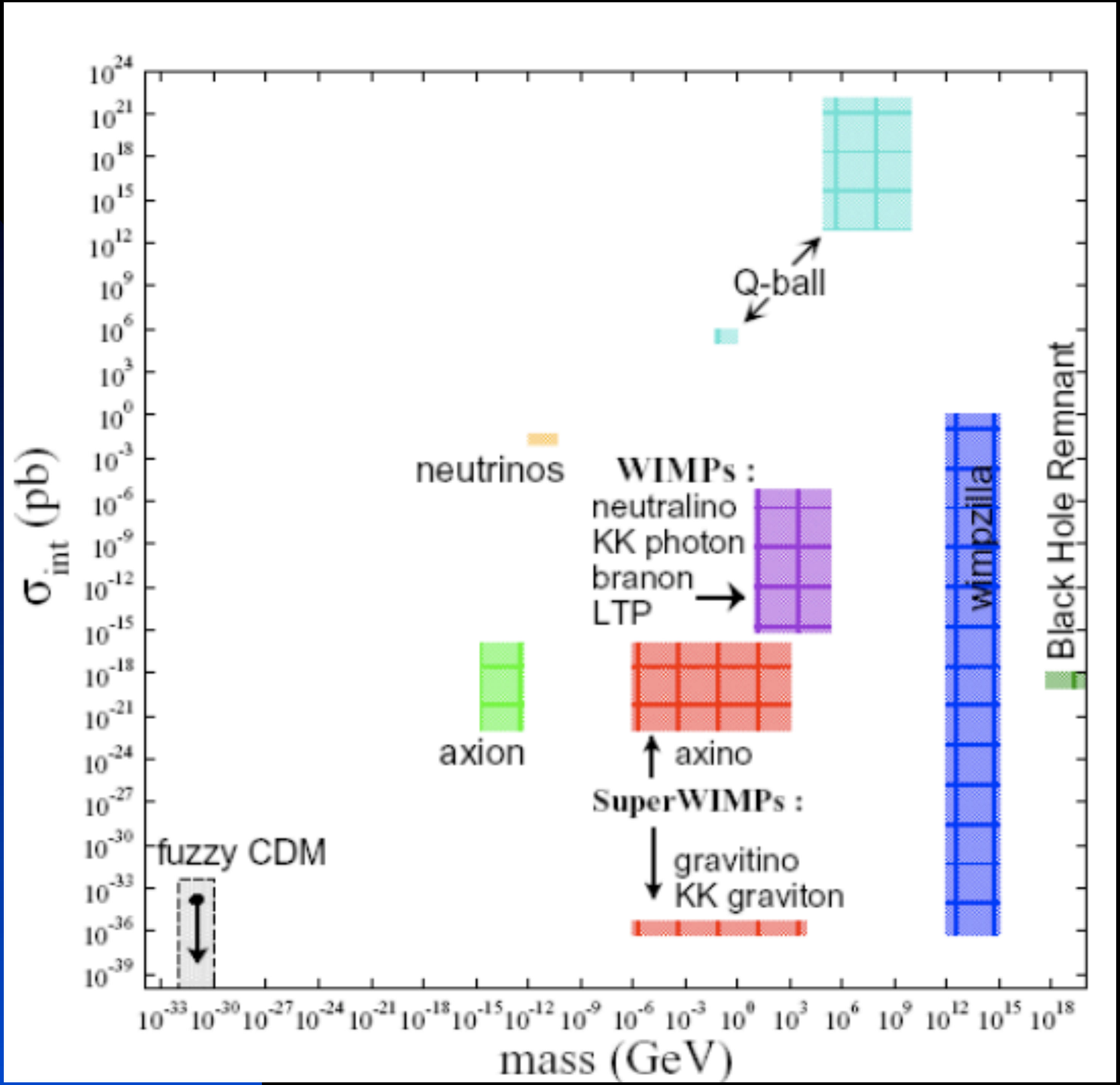
Search for them is on:

1. Direct detection -- 20 expts worldwide

2. Indirect detection -- i.e. Bullet Cluster !

3. LHC -- i.e. missing momentum and energy

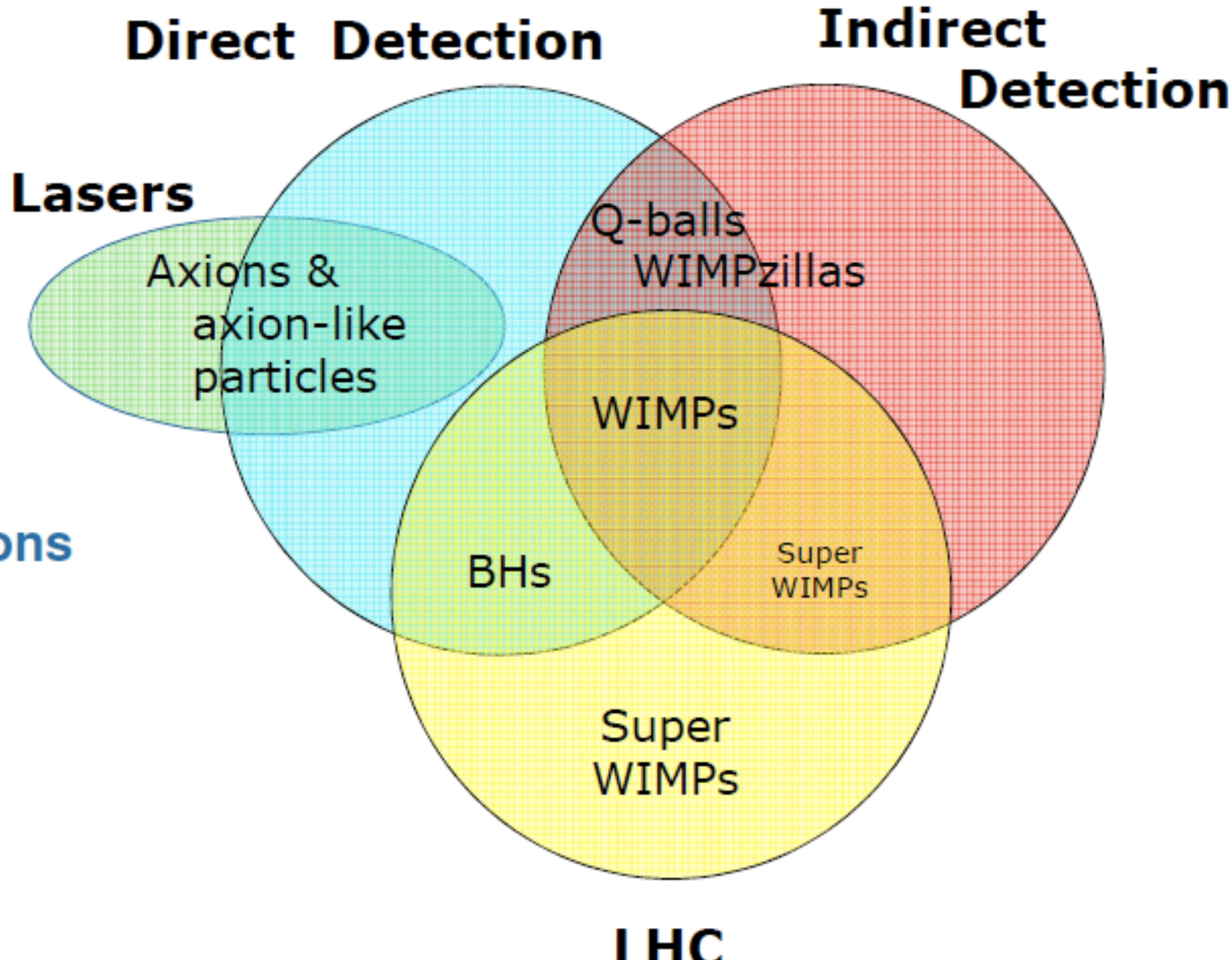
Dark Matter Candidates



C. Spiering, Cosmo 09

DM candidates:

- **WIMPs**
  - Neutralinos
  - Kaluza-Klein particles
  - ...
- **Axinos**
- **Super-WIMPs**
- **Axions**
- **Axion-like light bosons**
- **Sterile neutrinos**
- **Q-balls**
- **WIMPzillas**
- **Elementary BHs**
- **...**

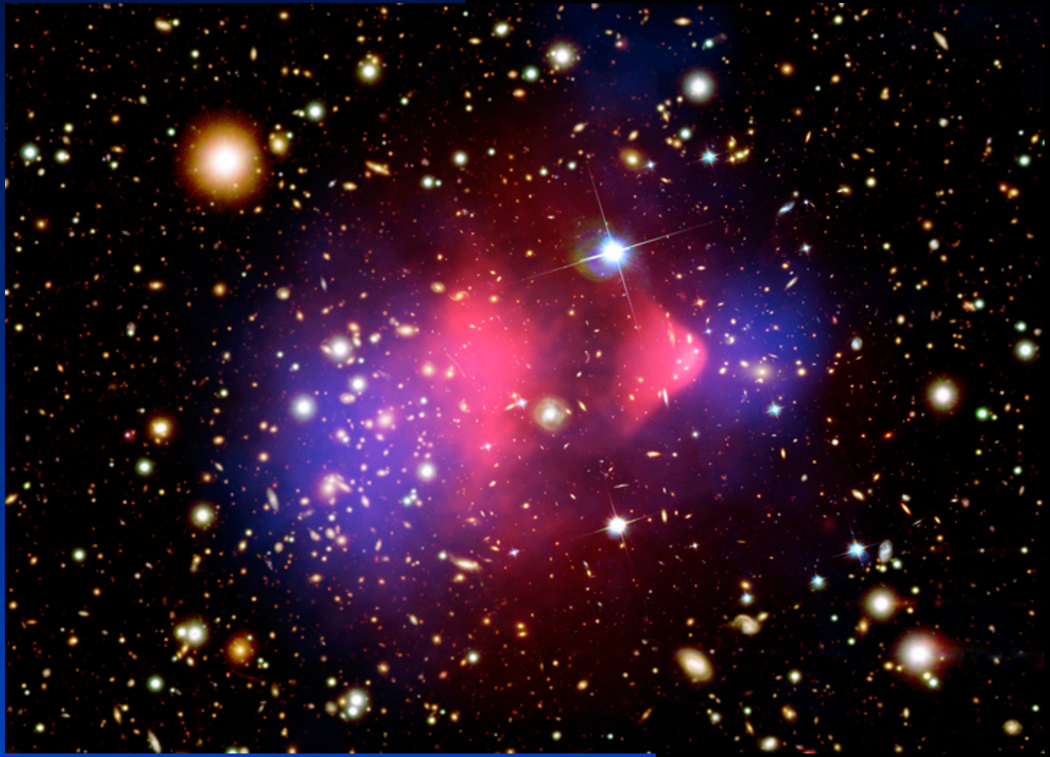


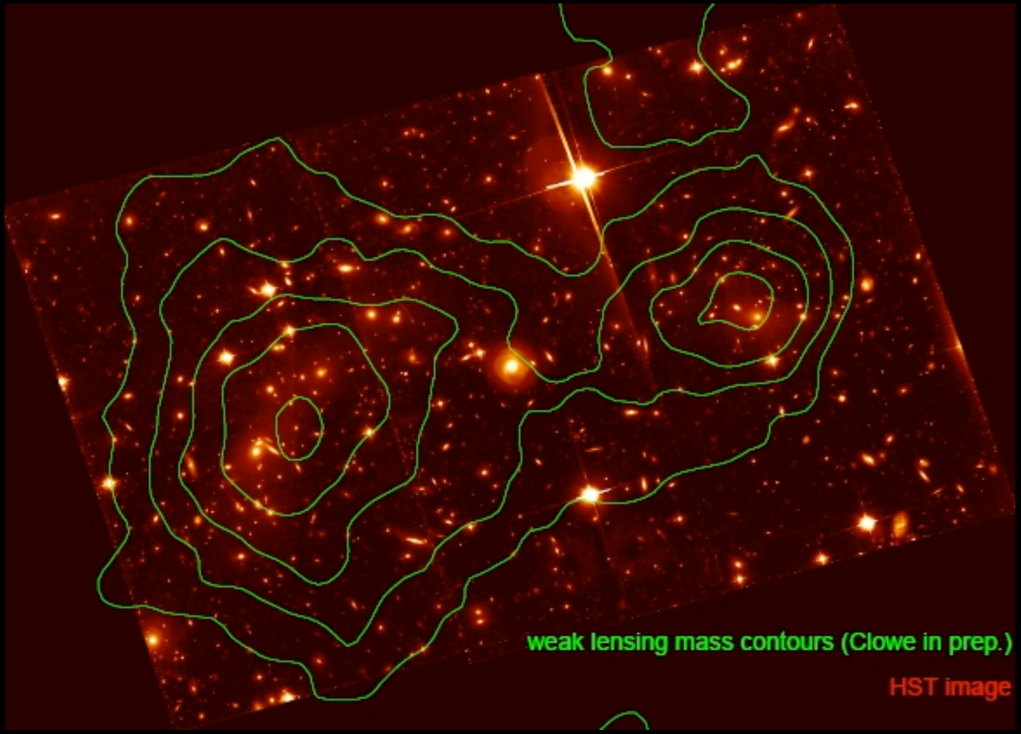
Indirect evidence for Dark Matter -- Bullet Cluster

Two clusters of galaxies colliding.

Dark matter in each passes straight through and doesn't interact -- seen through weak lensing in right image.

Ordinary matter in each interacts in collision and heats up -- seen through infra red image on left.





Clowe et al 2006

Evidence for Dark Energy?

$$3.\ \Omega_0 = \Omega_m + \Omega_\Lambda$$

Enter CMBR:

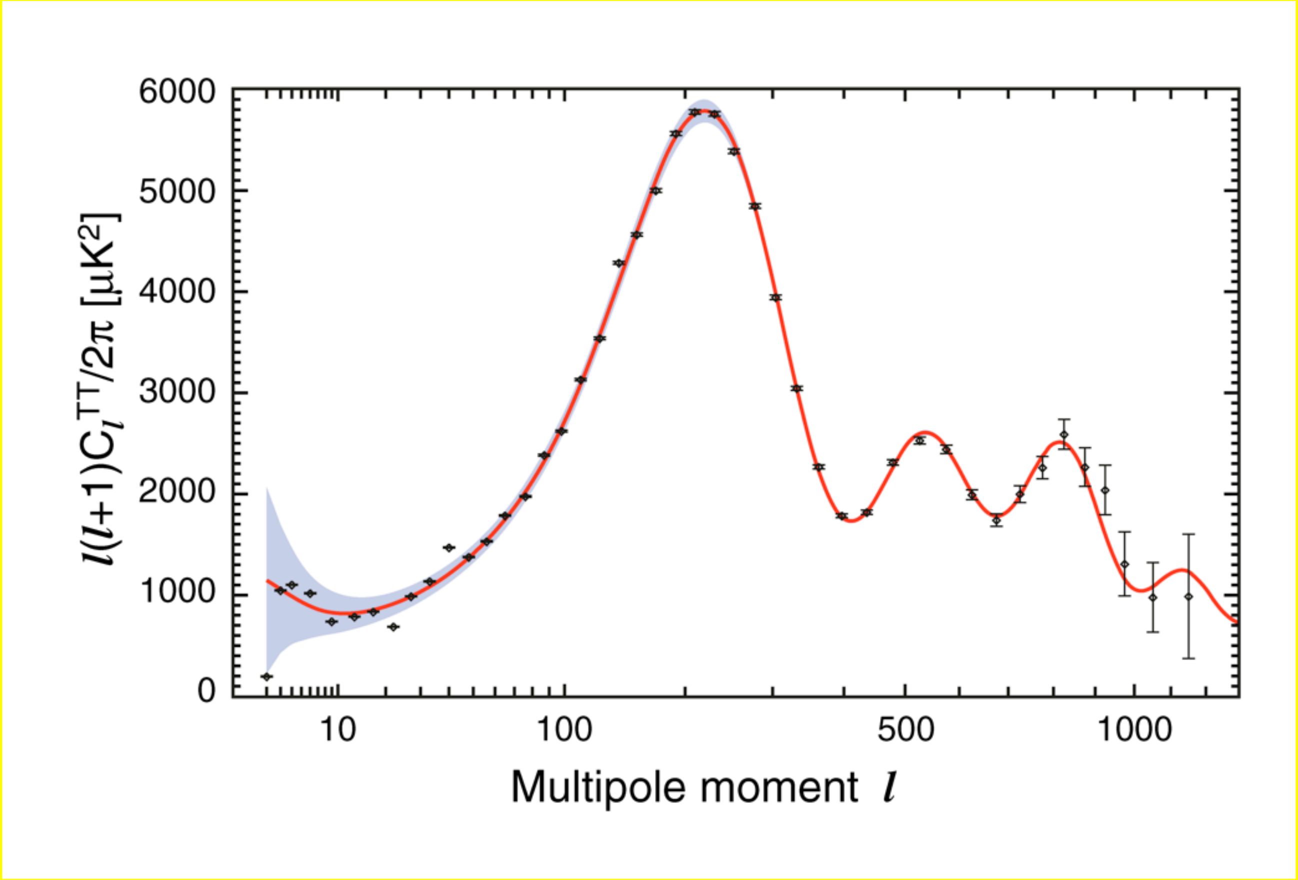
Provides clue. 1st angular peak in power spectrum.

$$l_{\text{peak}} \approx \frac{220}{\sqrt{\Omega_0}}$$

$$|1 - \Omega_0| = 0.03^{+0.026}_{-0.025}$$

WMAP3-Depends on assumed priors

Spergel et al 2006



$$-0.0175 < \Omega_k < 0.0085$$

Dunkley et al 2008 (WMAP5)

WMAP7 and dark energy

(Komatsu et al, 2010)

Assume flat univ +
+BAO+ SNLS:

$$w = -0.980 \pm 0.053$$

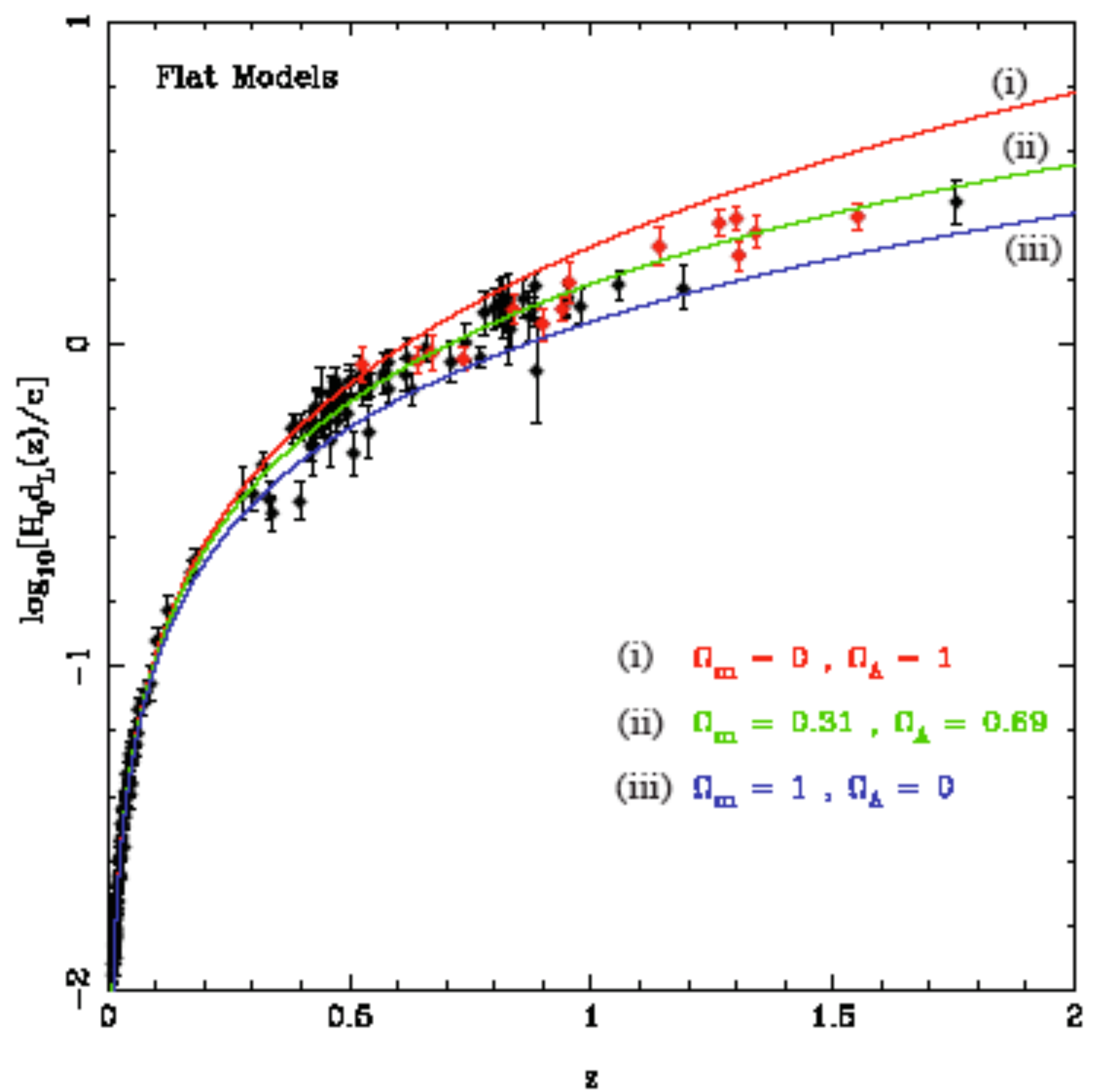
Drop prior of flat univ: WMAP + BAO + SNLS:

$$w = -0.999^{+0.057}_{-0.056} \quad \Omega_k = -0.0057^{+0.0067}_{-0.0068}$$

Drop assumption of const w but keep flat univ: WMAP + BAO + SNLS:

$$\begin{aligned} w_0 &= -0.93 \pm 0.12 \\ w_a &= -0.38^{+0.66}_{-0.65} \end{aligned}$$

Type Ia Luminosity distance v z [Reiss et al 2004]



Flat model
Black dots -- Gold data set
Red dots -- HST

$$(i)\Omega_m = 0,\ \Omega_\Lambda = 1 \quad (ii)\Omega_m = 0.31,\ \Omega_\Lambda = 0.69 \quad (iii)\Omega_m = 1,\ \Omega_\Lambda = 0$$

Coincidence problem – why now?

Recall: $$\frac{\ddot{a}}{a} \geq 0 <-> = (\rho + 3p) \leq 0$$

If: $$\rho_x = \rho_x^0 \, a^{-3(1+w_x)}$$

Universe dom by dark energy at: $$z_x = \left(\frac{\Omega_x}{\Omega_m}\right)^{\frac{1}{3w_x}} - 1$$

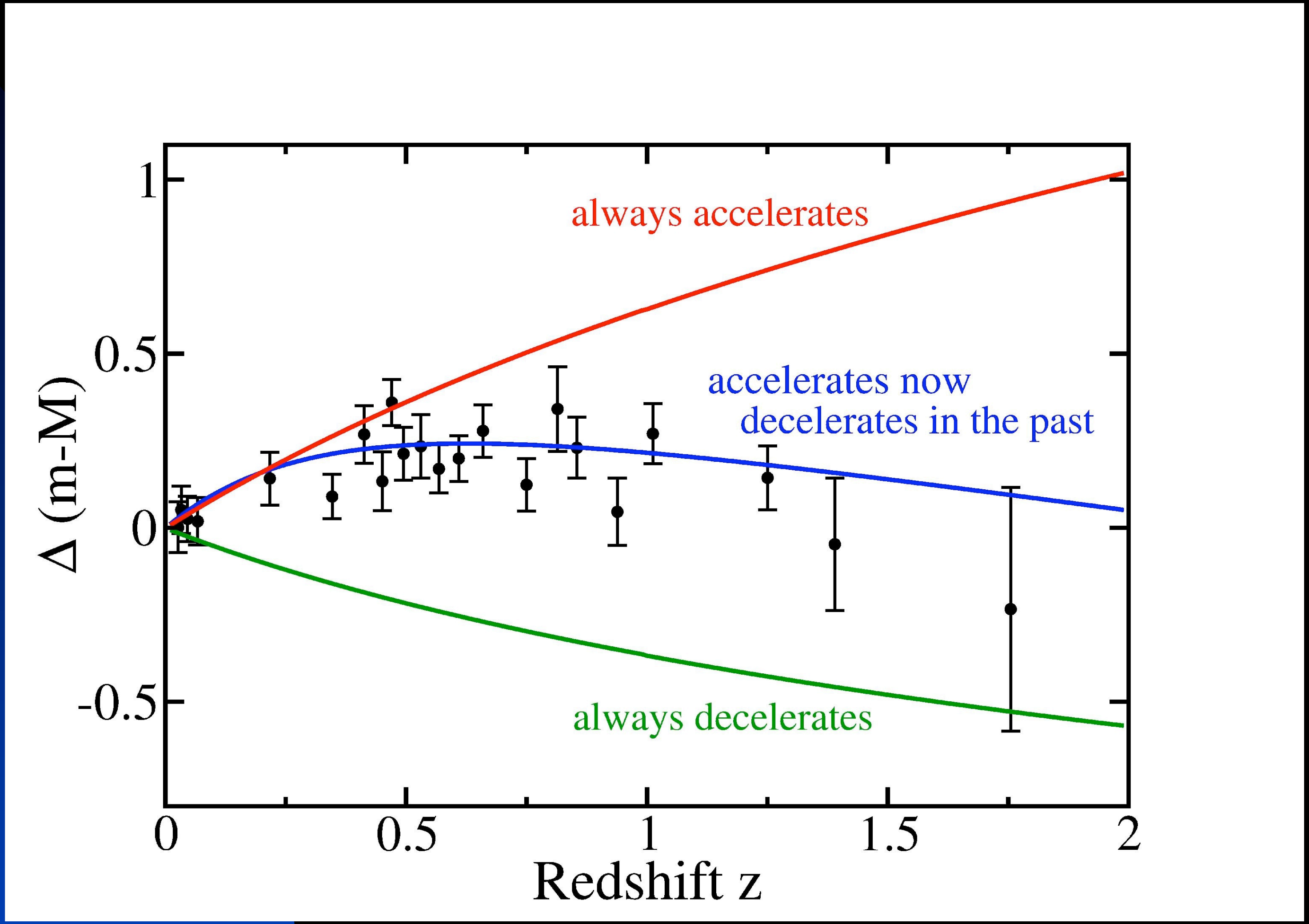
$$\left(\frac{\Omega_x}{\Omega_m}\right) = \frac{7}{3} \rightarrow z_x = 0.5,\ 0.3 \text{ for } w_x = -\frac{2}{3},\ -1$$

Univ accelerates at: $$z_a = \left(-(1+3w_x)\frac{\Omega_x}{\Omega_m}\right)^{\frac{-1}{3w_x}} - 1$$

$$z_a = 0.7,\ 0.5 \text{ for } w_x = -\frac{2}{3},\ -1$$

Constraint: $-0.11 < 1 + w < 0.14$ **Komatsu et al 2008 (WMAP5)**

The acceleration has not been forever -- pinning down the turnover will provide a very useful piece of information.



What is making the Universe accelerate?

Dark energy -- a weird form of energy that exists in empty space and pervades the universe -- also known as vacuum energy or cosmological constant.

Smoothly distributed, doesn't cluster.

Constant density or very slowly varying

Doesn't interact with ordinary matter -- only with gravity

Big problem though. When you estimate how much you expect there to be, from the Quantum world, the observed amount is far less than expected.

Theoretical prediction = 10^{120} times observation

The problem with the cosmological constant

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein (1917) -- static universe with dust

Not easy to get rid of it, once universe found to be expanding.

Anything that contributes to energy density of vacuum acts like a cosmological constant

$$< T_{\mu\nu} > = < \rho > g_{\mu\nu}$$

Lorentz inv

$$\lambda_{eff} = \lambda + 8\pi G < \rho >$$

or

$$\rho_V = \lambda_{eff}/8\pi G$$

Effective cosm const

Effective vac energy

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - < \rho > | \leq \frac{3H_0^2}{8\pi G}$$

Age Flat Non-vac matter

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - <\rho>| \leq \frac{3H_0^2}{8\pi G}$$

Hence: $\lambda_{eff} \leq H_0^2 \ or |\rho_V| \leq 10^{-29} gcm^{-3} \simeq 10^{-47} GeV^4$

Problem: expect $<\rho>$ of empty space to be much larger. Consider summing zero-point energies ($\hbar\omega/2$) of all normal modes of some field of mass m up to wave number cut off $\Lambda >> m$:

$$<\rho> = \int_0^\Lambda \frac{4\pi k^2 dk}{2(2\pi)^3}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$<\rho> = \frac{1}{2}\sum_{\text{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2+m^2}\frac{d^3k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where g_i are the dof of the field (+ for bosons, - for fermions).

Imagine just one field contributed an energy density $\rho_{cr} \sim (10^{-3}\text{ eV})^4$. Implies the cut-off scale $\Lambda < 0.01$ eV -- well below scales we understand the physics of.

Planck scale: $\Lambda \simeq (8\pi G)^{-1/2} \rightarrow <\rho> \simeq 2\times 10^{71} GeV^4$

But: $|\rho_V| = |<\rho> + \lambda/8\pi G| \leq 2\times 10^{-47} GeV^4$

Must cancel to better than 118 decimal places.

Even at QCD scale require 41 decimal places!

Very unlikely a classical contribution to the vacuum energy density will cancel this quantum contribution to such high precision

Not all is lost -- what if there is a symmetry present to reduce it? Supersymmetry does that. Every boson has an equal mass SUSY fermion partner and vice-versa, so their contributions to $<\rho>$ cancel.

However, SUSY seems broken today - no SUSY partners have been observed, so they must be much heavier than their standard model partners. If SUSY broken at scale M, expect $<\rho>\sim M^4$ because of breakdown of cancellations. Current bounds suggest M~1TeV which leads to a discrepancy of 60 orders of magnitude as opposed to 118 !

Still a problem of course -- is there some unknown mechanism perhaps from quantum gravity that will make the vacuum energy vanish ?

Different approaches to Dark Energy include amongst many:

- A true cosmological constant -- but why this value?
- Solid –dark energy such as arising from frustrated network of domain walls.
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments.
- Perhaps GR but Universe is inhomogeneous.

Early evidence for a cosmological constant type term.

1987: Weinberg argued that anthropically ρ_{vac} could not be too large and positive otherwise galaxies and stars would not form. It should not be very different from the mean of the values suitable for life which is positive, and he obtained $\Omega_{vac} \sim 0.6$

1990: Observations of LSS begin to kick in showing the standard $\Omega_{CDM} = 1$ struggling to fit clustering data on large scales, first through IRAS survey then through APM (Efstathiou et al).

1990: Efstathiou, Sutherland and Maddox - Nature (238) -- explicitly suggest a cosmology dominated today by a cosmological constant with $\Omega_{vac} < 0.8$!

1998: Type Ia SN show striking evidence of cosm const and the field takes off.

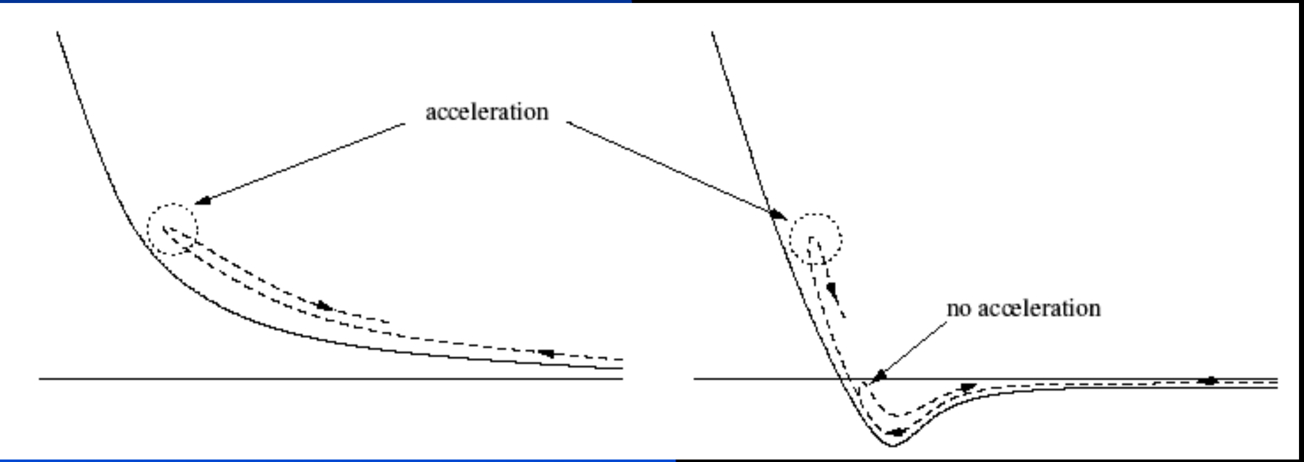
String/M-theory -- where are the realistic models?

`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

Recent extension: forbids four dimensional cosmic acceleration in cosmological solutions arising from warped dimensional reduction --[Wesley 08]

Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent, analogue of time-dependent scalar fields (radion)



Current realistic potentials are too steep

Models kinetic, not matter domination before entering accelerated phase.

Four form Flux and the cosm const: [Bousso and Polchinski]

Effective 4D theory from $M^4 x S^7$ compactification

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R + \Lambda_b - \frac{1}{2 \cdot 4!} F_4^2 \right)$$

Negative bare cosm const: $-\Lambda_b$

EOM: $\nabla_\mu \left(\sqrt{-g} F^{\mu\nu\rho\sigma} \right) = 0$ $\rightarrow F^{\mu\nu\rho\sigma} = c\epsilon^{\mu\nu\rho\sigma}$

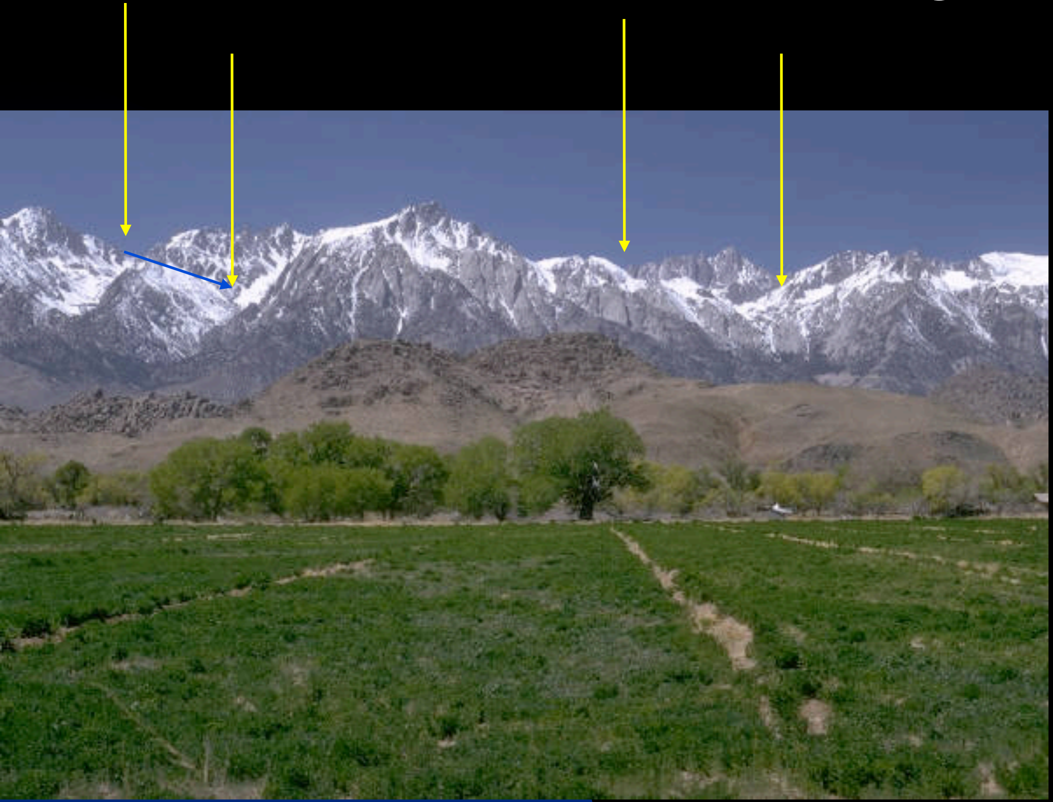
Eff cosm const: $\Lambda = -\Lambda_b - \frac{1}{48} F_4^2 = -\Lambda_b + \frac{c^2}{2}$

Quantising c and considering J fluxes $\Lambda = -\Lambda_b + \frac{1}{2} \sum_{i=1}^{J} n_i^2 q_i^2$

Observed cosm const with J~100

Still needed to stabilise moduli but opened up way of obtaining many de Sitter vacua using fluxes -- String Landscape in which all the vacua would be explored because of eternal inflation.

1.The String Landscape approach



Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes.

Many many vacua ~ 10^{500} !

Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunneling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

Landscape gives a realisation of the multiverse picture.

There isn’t one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

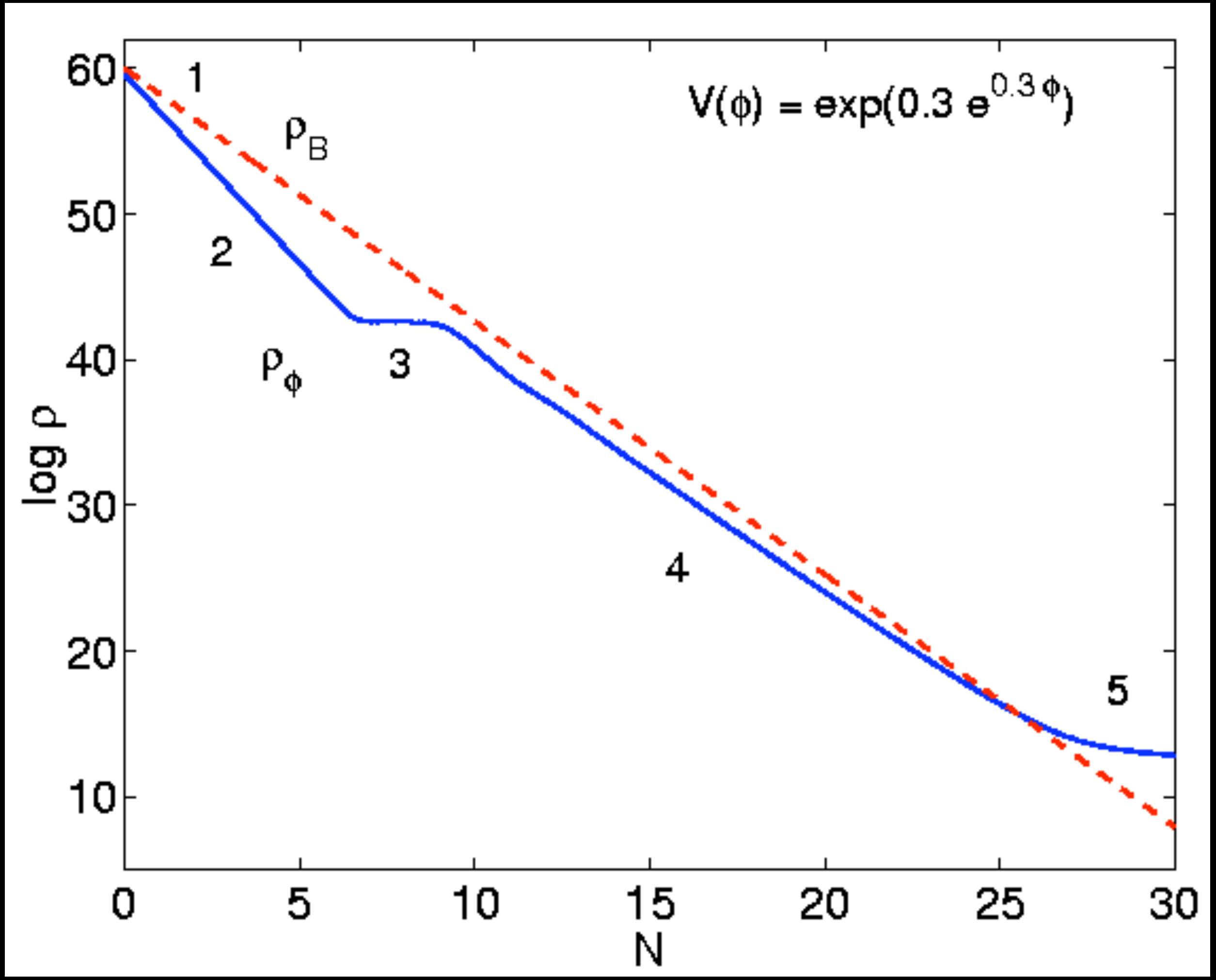
So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

This sounds like bad news, we will rely on anthropic arguments to explain it through introducing the correct measures and establishing peaks in probability distributions.

Or perhaps, it isn’t a cosmological constant, but a new field such as Quintessence which will eventually drive us to a unique vacuum with zero vacuum energy -- that too has problems, such as fifth force constraints, as we will see.

Slowly rolling scalar fields Quintessence - Generic behaviour

1. PE → KE
2. KE dom scalar field energy den.
3. Const field.
4. Attractor solution: almost const ratio KE/PE.
5. PE dom.



Nunes

Attractors make initial conditions less important

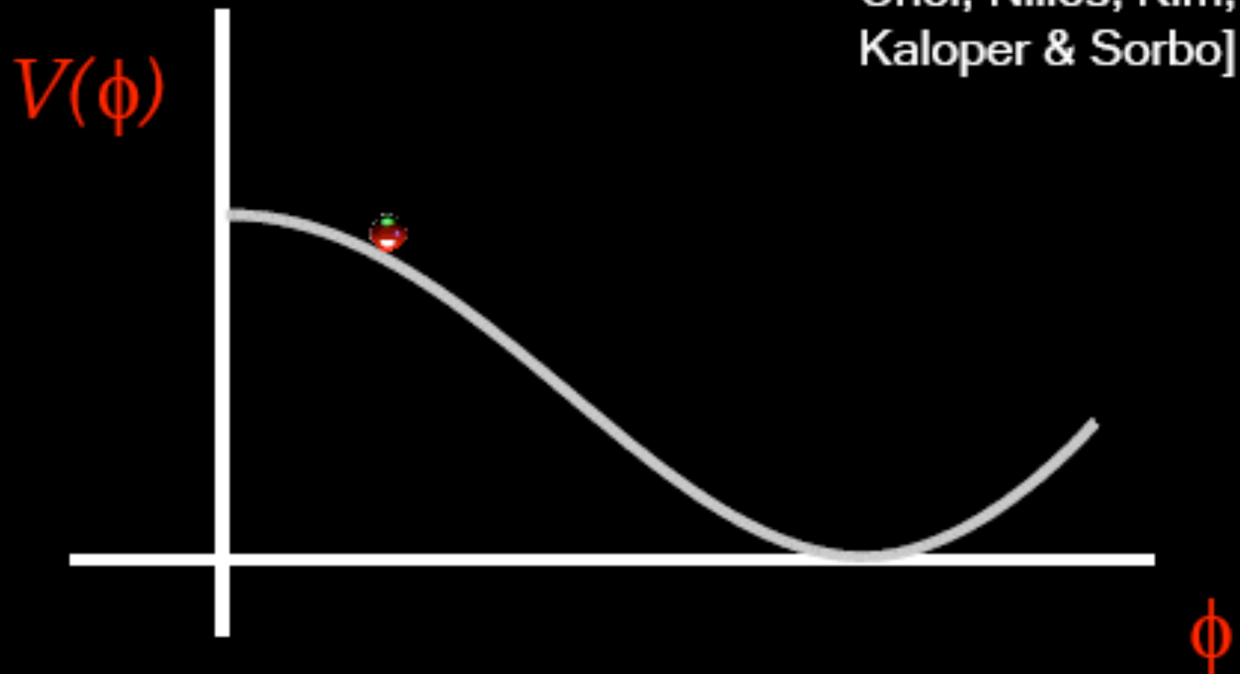
Particle physics inspired models?

Pseudo-Goldstone Bosons -- approx sym $\phi \rightarrow \phi + \text{const.}$

Leads to naturally small masses, naturally small couplings

[Hill, Freiman, et al; Choi; Nilles; Kim; Kaloper & Sorbo]

Barbieri et al



$$V(\phi) = \lambda^4 (1 + \cos(\phi / F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy.

1. Chameleon fields [Khoury and Weltman (2003) …]

Key idea: in order to avoid fifth force type constraints on Quintessence models, have a situation where the mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

2. Phantom fields [Caldwell (2002) …]

The data does not rule out $w<-1$. Can not accommodate in standard quintessence models but can by allowing negative kinetic energy for scalar field (amongst other approaches).

3. K-essence [Armendariz-Picon et al …]

Scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model?

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form. Similar fine tuning to Quintessence.

4. Interacting Dark Energy [Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + many others…]

Idea: why not directly couple dark energy and dark matter?

$$\begin{aligned} \text{Ein eqn} \quad &: \quad G_{\mu\nu} = 8\pi G T_{\mu\nu} \\ \text{General covariance} \quad &: \quad \nabla_\mu G^\mu_\nu = 0 \to \nabla_\mu T^\mu_\nu = 0 \end{aligned}$$

$$T_{\mu\nu} = \sum_i T^{(i)}_{\mu\nu} \to \nabla_\mu T^{\mu\,(i)}_\nu = -\nabla_\mu T^{\mu\,(j)}_\nu \ \text{ is ok}$$

Couple dark energy and dark matter fluid in form:

$$\begin{aligned} \nabla_\mu T^{\mu\,(\phi)}_\nu &= \sqrt{\frac{2}{3}}\kappa\beta(\phi)T^{\alpha\,(m)}_\alpha \nabla_\nu\phi \\ \nabla_\mu T^{\mu\,(m)}_\nu &= -\sqrt{\frac{2}{3}}\kappa\beta(\phi)T^{\alpha\,(m)}_\alpha \nabla_\nu\phi \end{aligned}$$

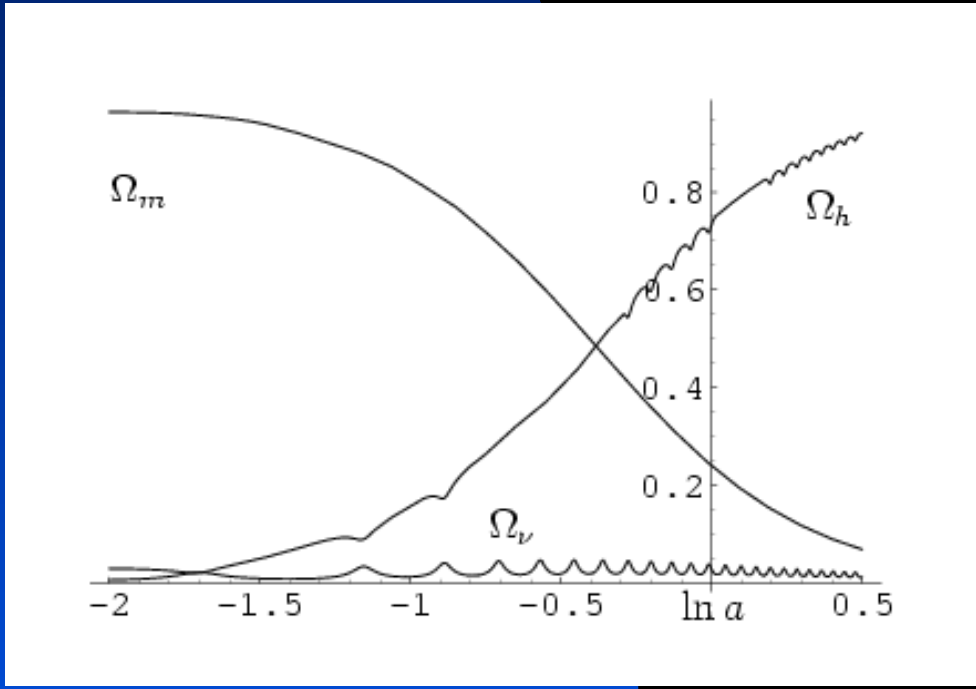
Including neutrinos -- 2 distinct DM families -- resolve coincidence problem [Amendola et al (2007)]

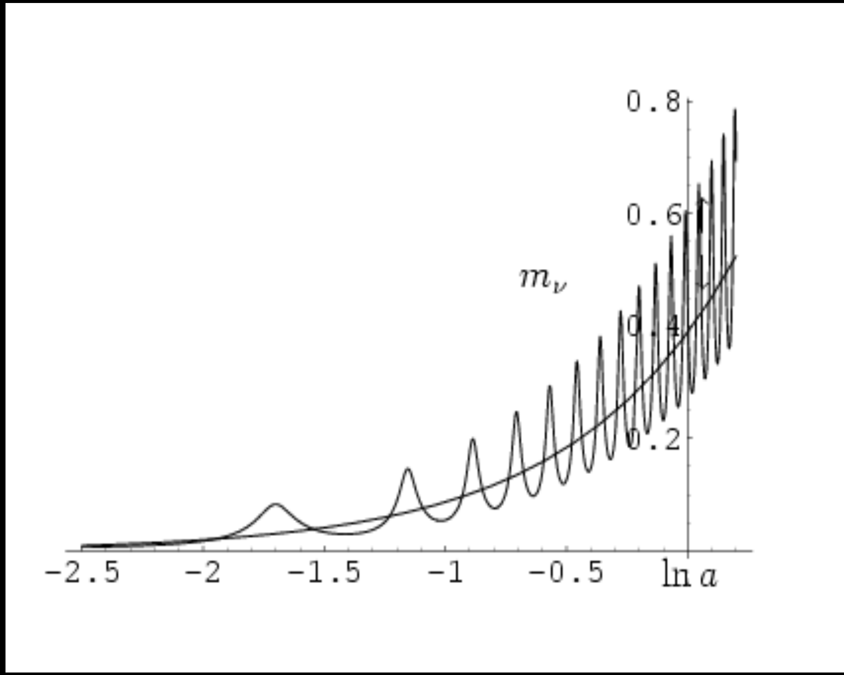
Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$





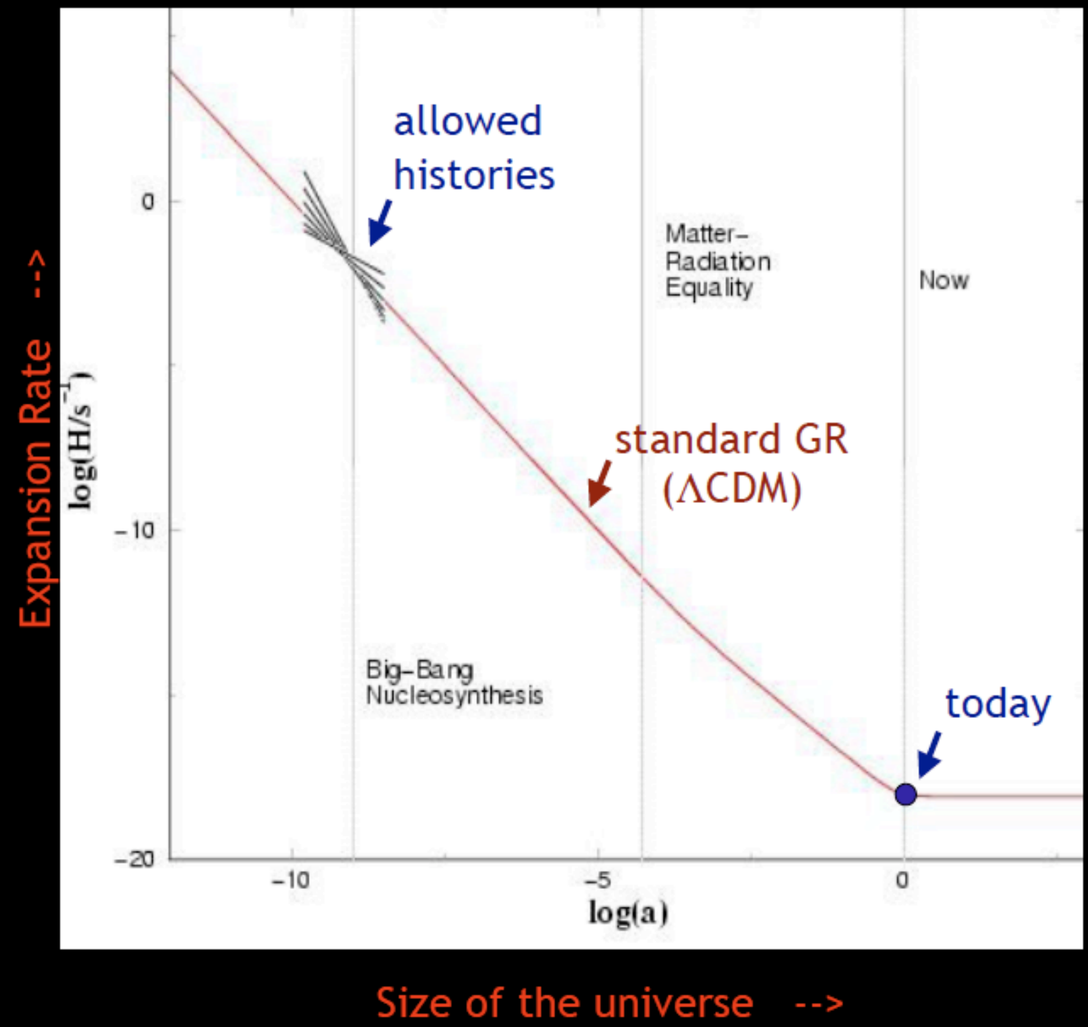
m_ν

Perhaps we are wrong -- maybe the question should be not whether dark energy exists, rather should we be modifying gravity?

Has become a big industry but it turns out to be hard to do too much to General Relativity without falling foul of data.

BBN occurred when the universe was about one minute old, about one billionth its current size. It fits well with GR and provides a test for it in the early universe.

Any alternative had better deliver the same successes not deviate too much at early times, but turn on at late times .



[Carroll & Kaplinghat 2001]

Any theory deviating from GR must do so at late times yet remain consistent with Solar System tests. Potential examples include:

- f(R) gravity -- coupled to higher curv terms, changes the dynamical equations for the spacetime metric.

[Starobinski 1980, Carroll et al 2003, ...]

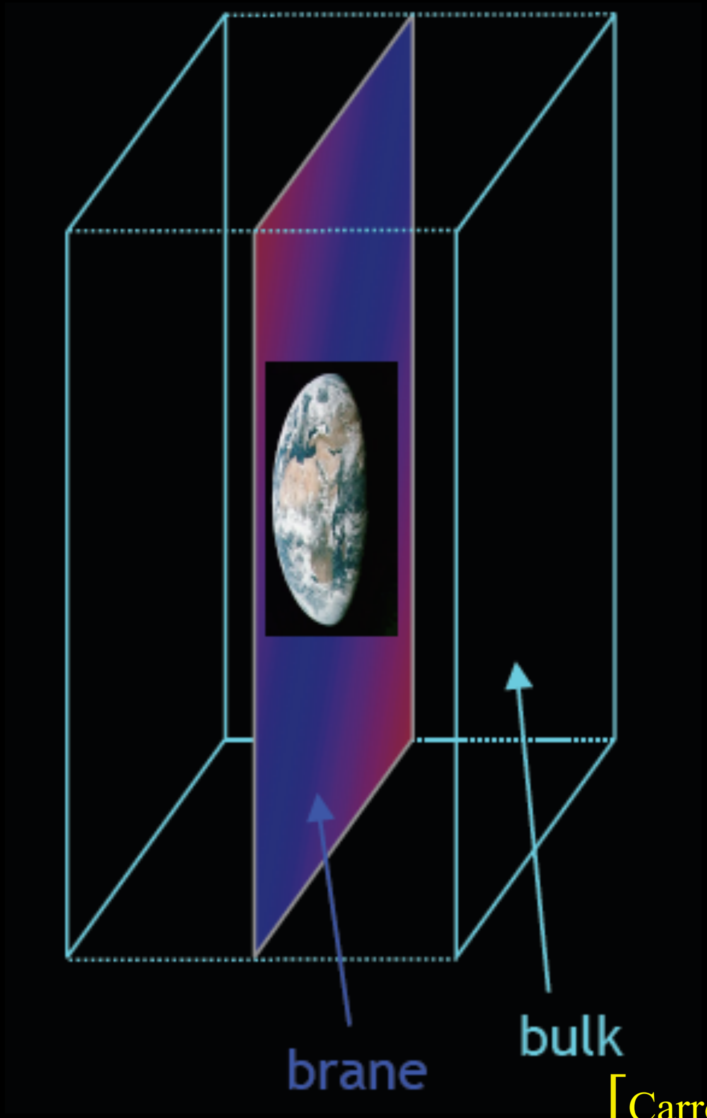
- Modified source gravity -- gravity depends on nonlinear function of the energy.

- Gravity based on the existence of extra dimensions -- DGP gravity

We live on a brane in an infinite extra dimension. Gravity is stronger in the bulk, and therefore wants to stick close to the brane -- looks locally four-dimensional.

Tightly constrained -- both from theory and observations -- ghosts !

Example of Galileon fields -- [Nicolis et al 08]



[Carroll]

Acc^n from new Gravitational Physics? [Starobinski 1980, Carroll et al 2003, ...]

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left(R - \frac{\mu^4}{R} \right) + \int d^4x \sqrt{-g}\, \mathcal{L}_M$$

Modify Einstein

Const curv vac solutions:

$$\nabla_\mu R = 0, \rightarrow R = \pm\sqrt{3}\mu^2$$

de Sitter or Anti de Sitter

Transform to EH action:

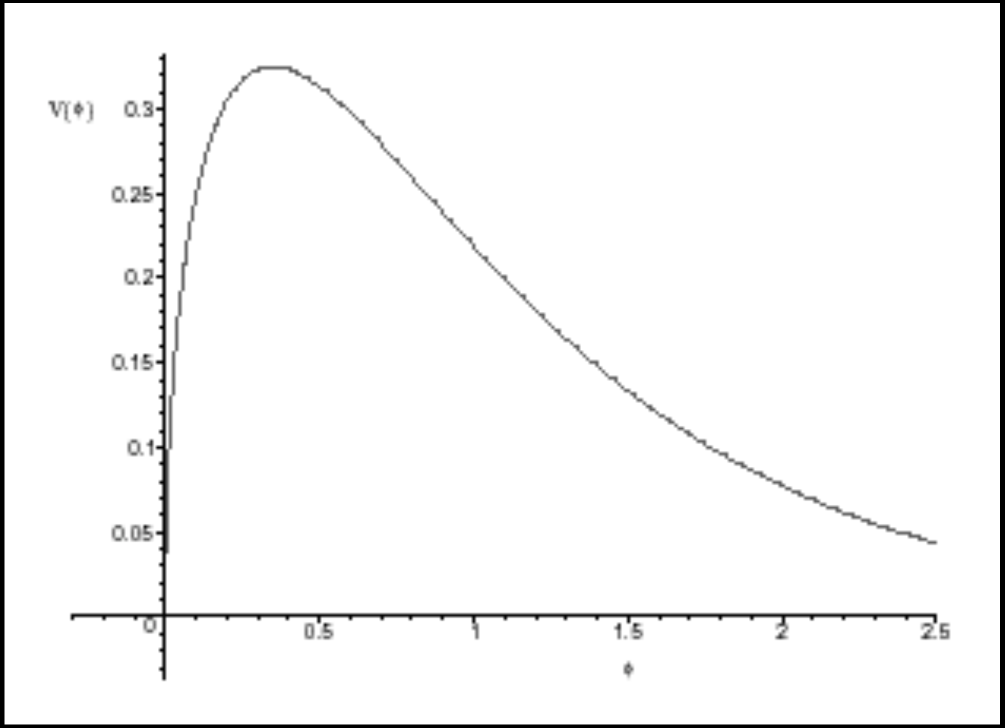
$$\tilde{g}_{\mu\nu} = p(\phi) g_{\mu\nu} \,, \quad p \equiv \exp\left(\sqrt{\frac{2}{3}} \frac{\phi}{M_P} \right) \equiv 1 + \frac{\mu^4}{R^2}$$

Scalar field minimally coupled to gravity and non minimally coupled to matter fields with potential:

$$V(\phi) = \mu^2 M_P^2 \frac{\sqrt{p-1}}{p^2}$$

Cosmological solutions:

1. **Eternal de Sitter - ϕ just reaches V_{max} and stays there. Fine tuned and unstable.**
2. **Power law inflation -- ϕ overshoots V_{max}, universe asymptotes with w_{DE}=-2/3.**
3. **Future singularity-- ϕ doesn't reach V_{max}, and evolves back towards ϕ=0.**



1.Fine tuning needed so acceleration only recently: $\mu \sim 10^{-33}$eV

2. Also, not consistent with classic solar system tests of gravity.

3. Claim that such R^{-n} corrections fail to produce matter dom era [Amendola et al, 06]

But recent results based on singular perturbation theory suggests it is possible [Evans et al, 07 -- see also Carloni et al 04]

More general f (R) models [Gurovich & Starobinsky (79); Tkachev (92); Carloni et al (04,07,09); Amendola & Tsujikawa 08; Bean et al 07; Wu & Sawicki 07; Appleby & Battye (07) and (08); Starobinsky (07); Evans et al (07); Frolov (08)…]

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{2\kappa^2} + \mathcal{L}_m \right]$$

No Λ

Usually f (R) struggles to satisfy both solar system bounds on deviations from GR and late time acceleration. It brings in extra light degree of freedom --> fifth force constraints.

Ans: Make scalar dof massive in high density solar vicinity and hidden from solar system tests by chameleon mechanism.

Requires form for f (R) where mass of scalar is large and positive at high curvature.

Issue over high freq oscillations in R and singularity in finite past.

In fact has to look like a standard cosmological constant [Song et al, Amendola et al]

To test GR on cosmological scales compare kinematic probes of dark energy to dynamical ones and look for consistency.

Kinematic probes: only sensitive to a(t) such as standard candles, baryon oscillations.

Dynamical probes: sensitive to a(t) and structure growth such as weak lensing and cluster counts.

Determining the best way to test for dark energy and parameterise the dark energy equation of state is a difficult task, not least given the number of approaches that exist to modeling it .

Dark Energy Task Force review: Albrecht et al : astro-ph/0609591

Findings on best figure of merit: Albrecht et al: arXiv:0901.0721

Busstepp 2011
Cosmology - Lecture 3

Ed Copeland -- Nottingham University

1. Origin of Inflation and the primordial density fluctuations.

Return to the beginning -- Inflation

A period of accelerated expansion in the early Universe

Small smooth and coherent patch of Universe size less than (1/H) grows to size greater than comoving volume that becomes entire observable Universe today.

Explains the homogeneity and spatial flatness of the Universe

and also explains why no massive relic particles predicted in say GUT theories

Leading way to explain observed inhomogeneities in the Universe

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G\,(\rho + 3p) --- Accn$$

$$If\;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

What is Inflation?

Any epoch of the Universe's evolution during which the comoving Hubble length is decreasing. It corresponds to any epoch during which the Universe has accelerated expansion.

$$\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \leftrightarrow \ddot{a} > 0$$

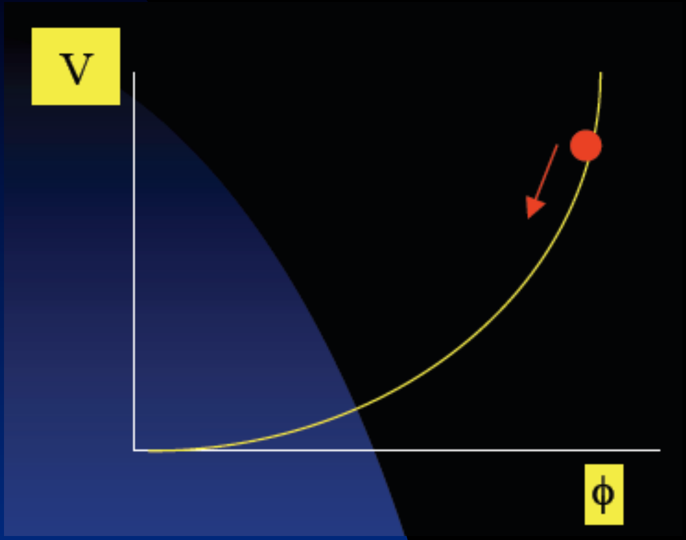
$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3}G\left(\rho + 3p\right) --- Accn$$

$$If \;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

For inflation require material with negative pressure. Not many examples. One is a scalar field!

Intro fundamental scalar field -- like Higgs

If Universe is dominated by the potential of the field, it will accelerate!



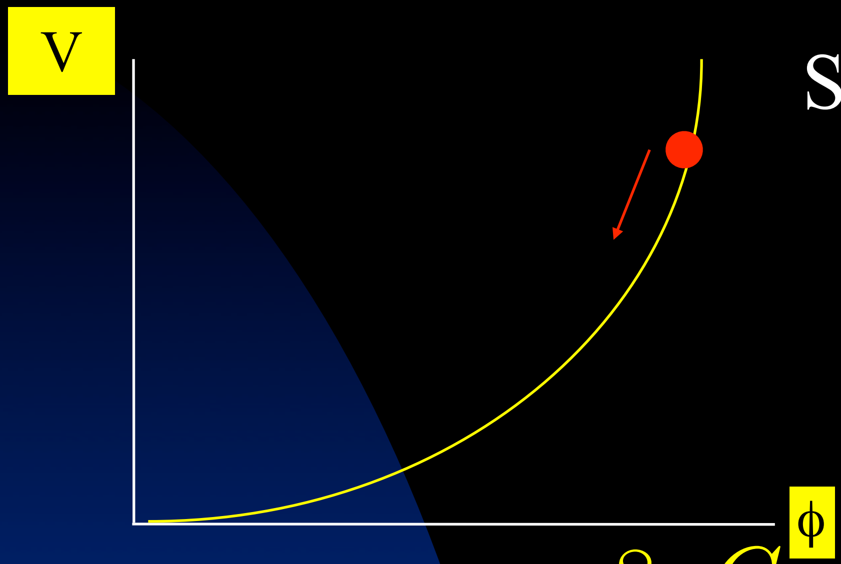
$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Of course no fundamental scalar field ever seen.

We aim to constrain potential from observations.

During inflation as field slowly rolls down its potential, it undergoes quantum fluctuations which are imprinted in the Universe. Also leads to gravitational wave production.

Examples of inflation



Simplest case – single scalar field

$$\rho_\phi = V(\phi) + \frac{\dot{\phi}^2}{2} \quad ; \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

EoM $$H^2 = \frac{8\pi G}{3}\rho_\phi \quad ; \quad \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Inflation $\ddot{a} > 0 \leftrightarrow (\rho + 3p) < 0 \leftrightarrow \dot{\phi}^2 \ll V(\phi)$ Slow roll approx

$$\Longrightarrow \quad H^2 = \frac{8\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Also: $$\dot{H} = -4\pi G\dot{\phi}^2,$$

So, define a quantity which specifies how fast H changes during inflation

Prediction -- potential determines important quantities

Slow roll parameters [Liddle & Lyth 1992]

$$\epsilon = \frac{1}{16\pi G}\left[\frac{V'(\phi)}{V(\phi)}\right]^2$$

$$\eta = \frac{1}{8\pi G}\left[\frac{V''(\phi)}{V(\phi)}\right]$$

Inflation occurs when both of these slow roll conditions are << 1

End of inflation corresponds to ε=1

How much does the universe expand? Given by number of e-folds

$$N \equiv \ln\left(\frac{a(t_{\rm end})}{a(t_i)}\right) = \int_{t_i}^{t_e} H\,dt \simeq \int_{\phi_i}^{\phi_e} \frac{V}{V'}\,d\phi$$

Last expression is true in the slow roll limit (for single field inflation).

Number of e-folds required

Solve say the Flatness problem:

Assume inflation until tend = 10^{-34} sec

Assume immediate radn dom until today, $t_0 = 10^{17}$ sec

Assume $$|\Omega \ (t_0) - 1| \leq 0.01$$

Now $$|\Omega - 1| = \frac{k}{a^2 H^2}; \quad RD: |\Omega - 1| \propto t$$

$$|\Omega (10^{-34}\ s) - 1| \leq 0.01 * 10^{-34} * 10^{-17} \leq 10^{-54}$$

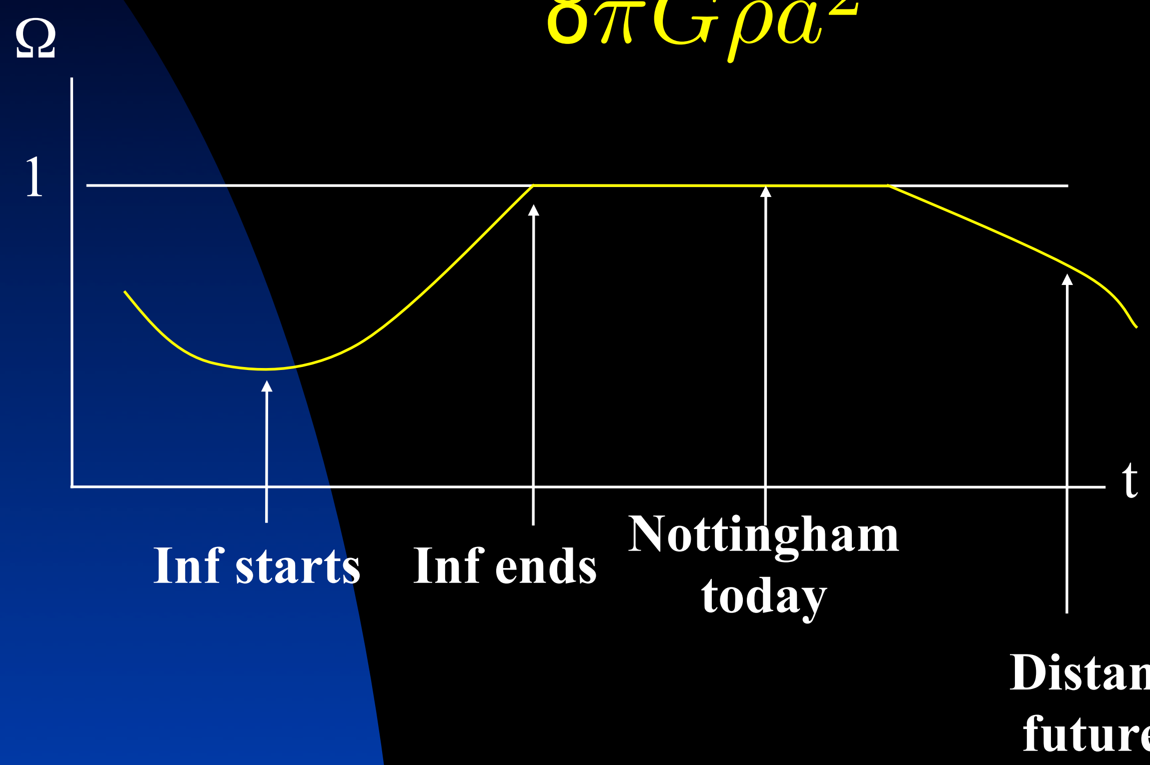
Inf $$|\Omega - 1| \propto 1/a^2$$ ⟶ $$\frac{|\Omega_{end} - 1|}{|\Omega_{ini} - 1|} = a_i^2 / a_e^2 = 10^{-54}$$

⟶ $$N = \ln \left[\frac{a_{tend}}{a_{tini}} \right] \approx 62$$

Solving the big bang problems

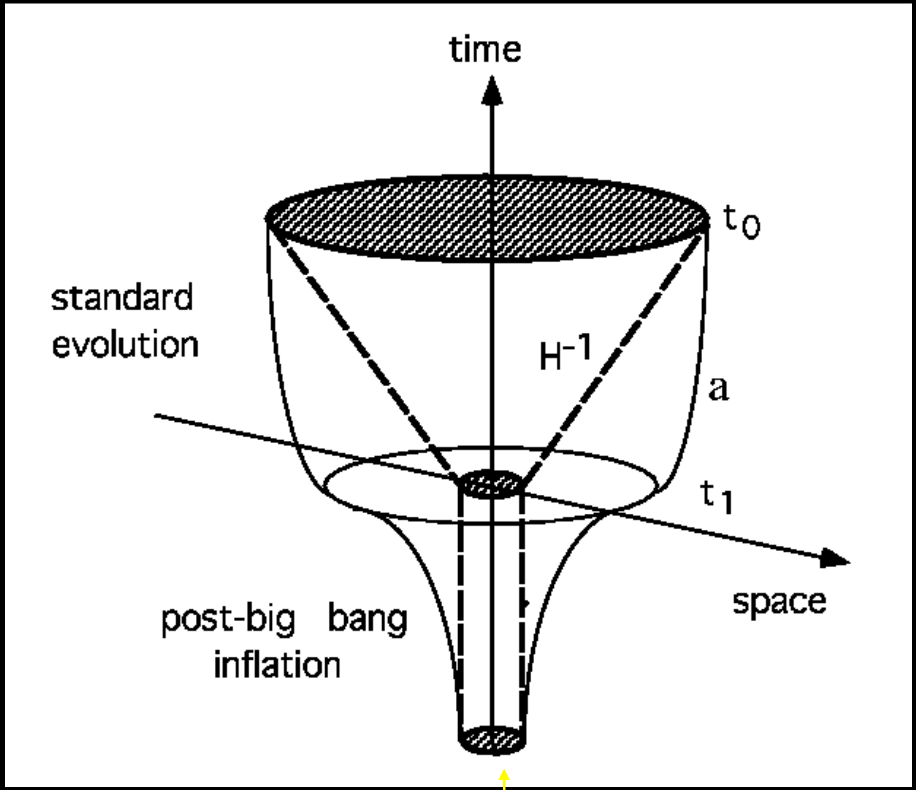
1. Flatness

$$\Omega^{-1} - 1 = -\frac{3k}{8\pi G\rho a^2} \propto a^{-2} \longrightarrow \exp(-2Ht)$$



2. Horizon problem:

Physical: H^{-1} const during inflation.



Initial causally connected region

3. Monopole problem: $\rho_{\rm mon} \propto a^{-3} \to 0$ rapidly during inflation

Everything infact diluted away except for the inflaton field itself.

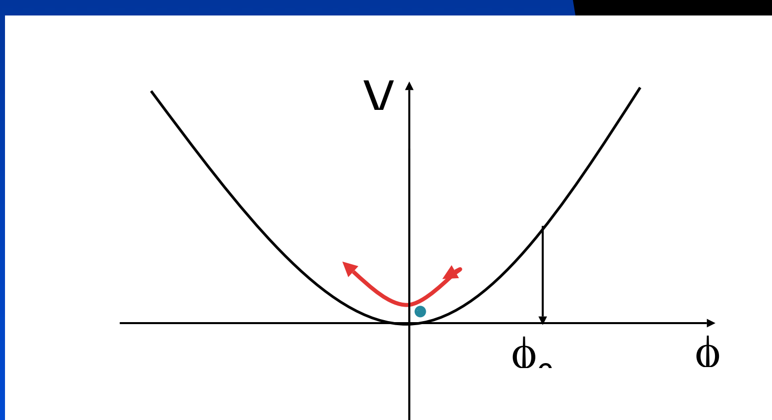
$T \propto a^{-1} \to 0$ rapidly during inflation

Hence need to reheat the universe at end of inflation

End of inflation

- Eventually SRA breaks down, as inflaton rolls to minima of its potential.

Experimental test of slow roll approximation – Aspen 2002



- Leaves a cold empty Universe apart from inflaton.
- Inflation has to end and the energy density of the inflaton field decays into particles. This is reheating and happens as the field oscillates around the minimum of the potential

End of inflation.

- Inflaton is coupled to other matter fields and as it rolls down to the minima it produces particles –perturbatively or through parametric resonance where the field produces many particles in a few oscillations.
- Dramatic consequences. Universe reheats, can restore previously broken symmetries, create defects again, lead to Higgs windings and sphaleron effects, generation of baryon asymmetry at ewk scale at end of a period of inflation.
- Important constraints: e.g.: gravitino production means : $T_{rh} < 10^9$ GeV -- often a problem!

Perturbative Reheating:

1. Instantaneous reheating where vac energy is converted immediately to radiation with $T_{RH.}$

2. Reheat by slow decay of ϕ with the zero modes comoving energy density decaying into particles which scatter and thermalise. Assume decay width for this is same as for free ϕ.

Expect small decay width, as flatness of potential requires weak coupling of ϕ to other fields. Also in SUGR if coupling not weak, overproduce gravitinos during reheating.

Boltzmann eqn:

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$$

$$\dot{\rho}_{\rm rad} + 4H\rho_{\rm rad} - \Gamma_\phi \rho_\phi = 0$$

T_{RH} – inflaton executes coherent oscillations about V_{min} after inflation.

$< \rho_\phi >_{osc} \propto a^{-3}$ Averaged over many coherent oscillations

$\rho_{\phi I}, a_I$ Values when coherent oscillations start.

Hubble expansion rate: $$H(a) = \sqrt{\frac{8\pi G}{3}\rho_{\phi I}\left(\frac{a_I}{a}\right)^3}$$

Equating: $H(a) = \Gamma_\phi$ gives $$\left(\frac{a_I}{a}\right) = \left(\frac{3G\Gamma_\phi^2}{8\pi\rho_{\phi I}}\right)^{1/3}$$

Assume at this moment all coherent energy density immediately transferred into radiation.

$$\rho_\phi = \rho_R \text{ where } \rho_\phi = \rho_{\phi I}\left(\frac{a_I}{a}\right)^3 \text{ and } \rho_R = \left(\frac{\pi^2}{30}\right)g_* T_{RH}^4$$

Hence: $$T_{RH} = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4}\sqrt{\Gamma_\phi M_P} = 0.2\left(\frac{200}{g_*}\right)^{1/4}\sqrt{\Gamma_\phi M_P}$$

Bound from Gravitino overproduction : $T_{RH} \leq 10^9 - 10^{10}\,\mathrm{GeV}$

Preheating: Traschen & Brandenberger; Kofman, Linde & Starobinsky

Non-perturbative resonant transfer of energy to particles induced by the coherent oscillations of ϕ -- can be very efficient!

Assume ϕ oscillating about min of potential.

$$V(\phi) = \frac{m^2\phi^2}{2}; \text{ Write } \phi(t) = \Phi(t)\sin mt$$

In expanding universe Φ decreases due to redshift of momentum.

Assume scalar field X coupled to ϕ

$$L_{int} = \frac{g^2 X^2 \phi^2}{2}$$

Mode eqn: $\chi_k = X_k a^{3/2}$: $\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\Phi^2(t)\sin^2(mt)\right)\chi_k = 0$

Minkowski space: Φ const

$$\chi_k'' + \left[A_k - 2q\cos(2z)\right]\chi_k = 0;$$

$$z = mt,\ \chi_k' \equiv d\chi_k/dz;\ q = \frac{g^2\Phi^2}{4m^2};\ A_k = 2q + \frac{k^2}{m^2}$$

Mathieu equation

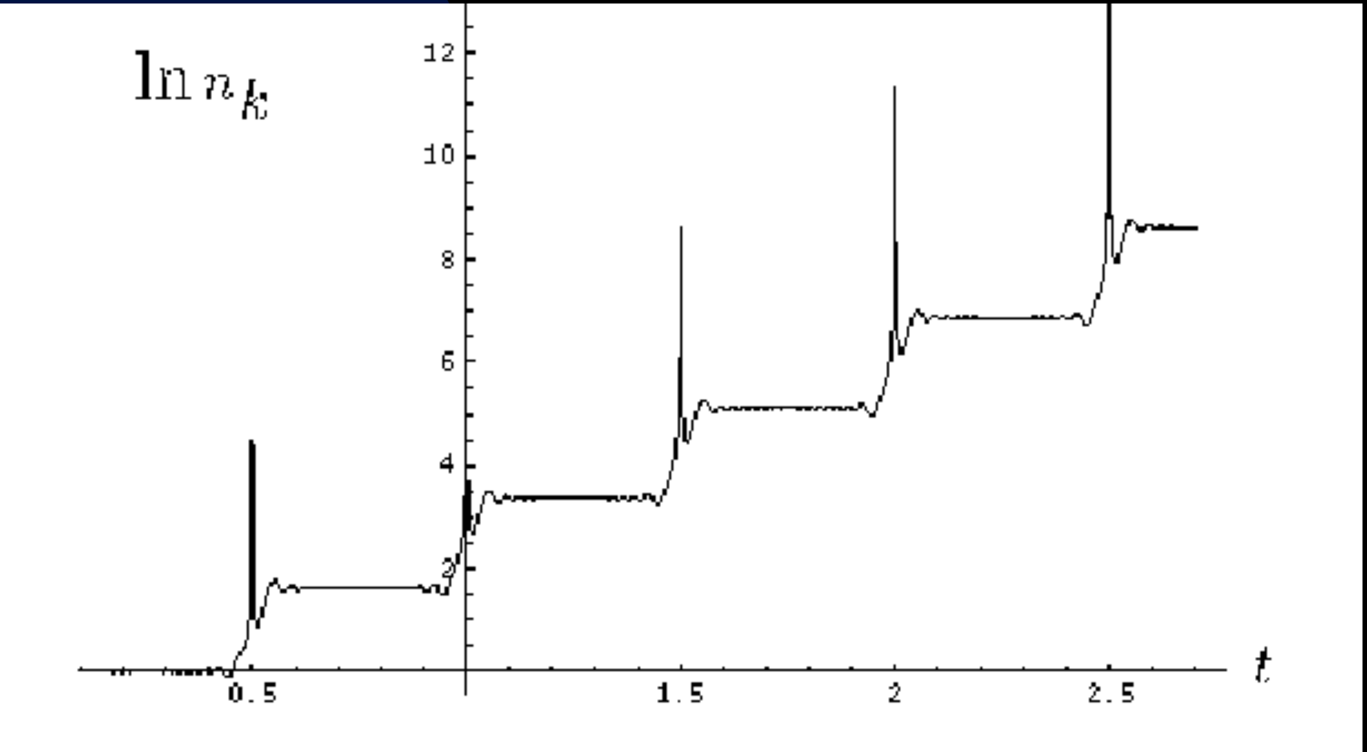
Exponential instability regions:

$$\chi_k \propto \exp(\mu_k z) \text{ where } \mu_k = \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{2k}{m} - 1\right)^2}$$

Max growth at 2k = m

Growth of modes leads to growth of occupation numbers of created particles

Number density = Energy of that mode/Energy of each particle (ω_k)

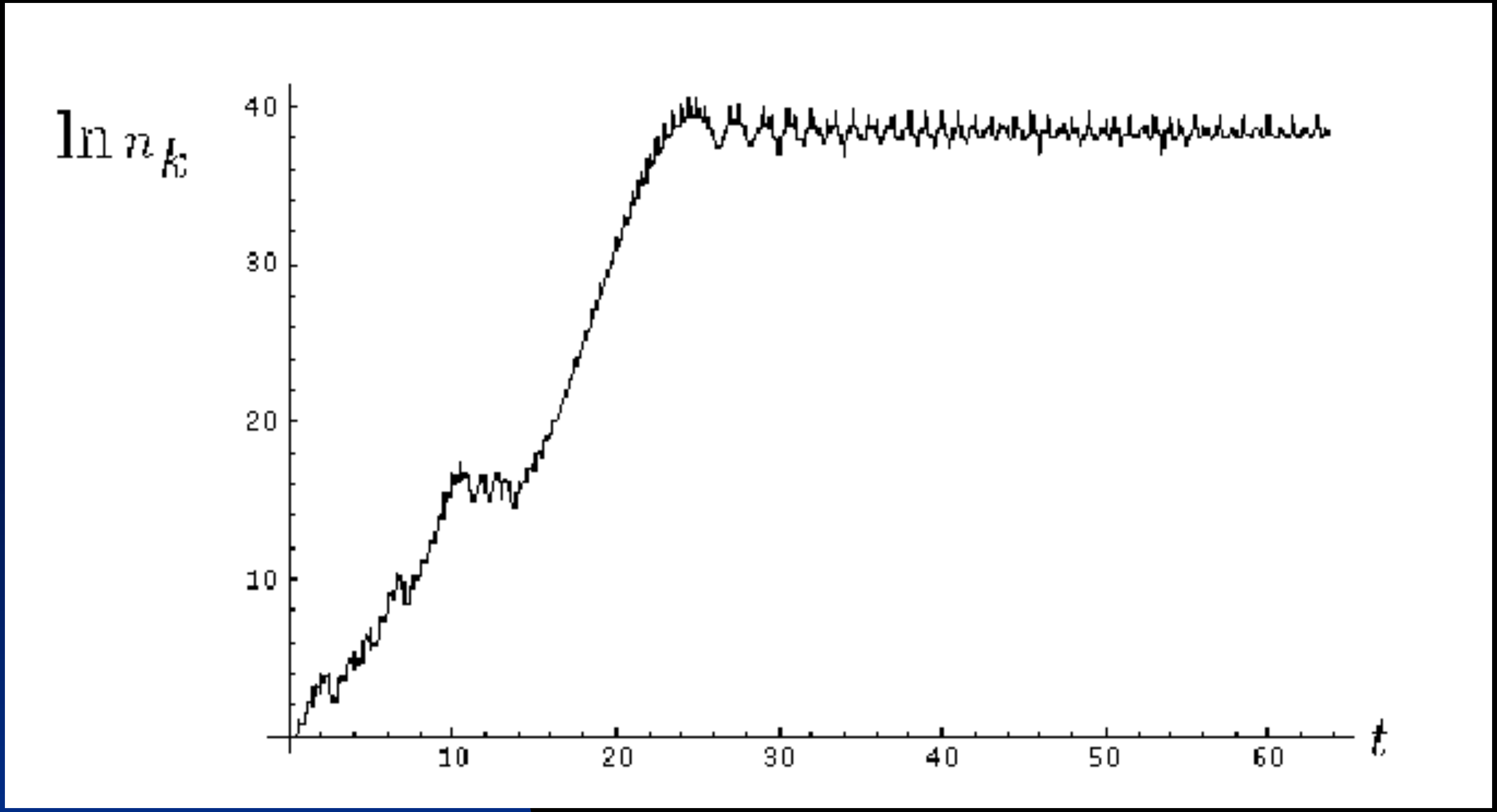


Kofman, Linde and Starobinsky (97)

Period of enhanced rate of energy transfer – preheating, because particles produced not in thermal eqm. Explosive growth every time $\phi(t)=0$.

$$n_k \propto \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2} \propto e^{2\mu_k z}$$

Still occurs when A,q not constant:



Kofman, Linde and Starobinsky (97)

Longer time evolution

This efficient quick transfer of energy means that can have large reheat temperatures, phase transitions, defect production and baryogenesis through production of particles with mass bigger than inflaton mass. Can also generate potentially obervable primordial gravitational waves from pre-heating.

The origins of perturbations -- the most important aspect of inflation

Idea: Inflaton field is subject to perturbations (quantum and thermal fluctuations). Those are stretched to superhorizon scales, where they become classical. They induce metric perturbations which in turn become later the first perturbations to seed the structures in the universe.

Also predict a cosmological gravitational wave background.

During inf $$\phi(\underline{x}, t) = \phi_0(t) + \delta\phi(\underline{x}, t)$$ ← Quantum fluc

Fourier modes: $$\delta\phi(\underline{x}, t) = \sum_k \delta\phi_k(t)\, e^{ikx}$$ → Generates fluc in matter and metric

$\delta_H^2(k)$ Scalar pertn – spectra of gaussian adiabatic density pertns generated by flucns in scalar field and spacetime metric. Responsible for structure formation.

$h_{\mu\nu}$ $A_G(k)$ Tensor pertn in metric– gravitational waves.

Key features

During inflation comoving Hubble length (1/aH) decreases.

So, a given comoving scale can start inside (1/aH), be affected by causal physics, then later leave (1/aH) with the pertns generated being imprinted.

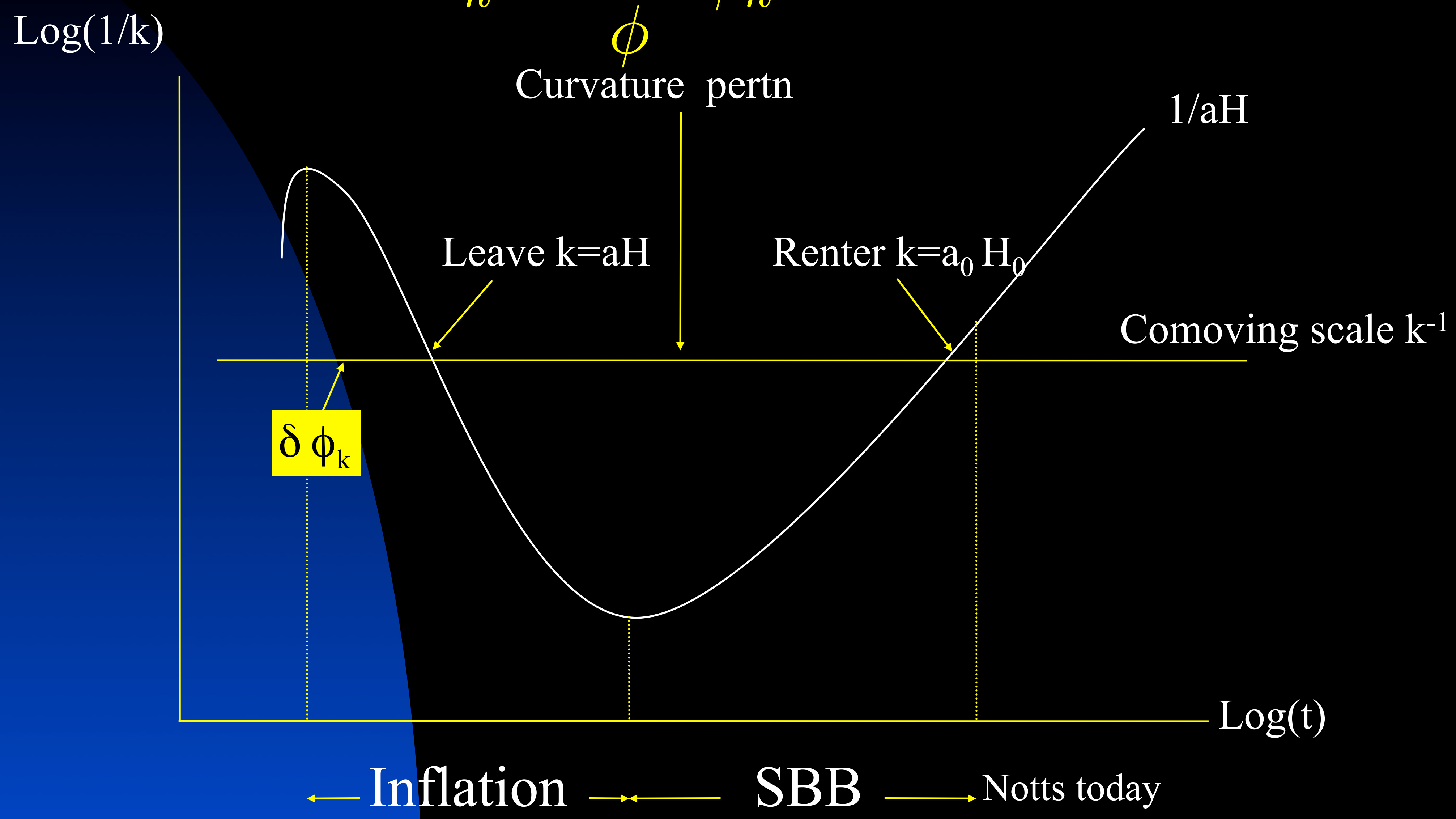
Quantum flucns in inflaton arise from uncertainty principle.

Pertns are created on wide range of scales and generated causally.

Size of irregularities depend on energy scale at which inflation occurs.

Pertn created causally, stretched by expansion.

$$\mathcal{R}_k = \frac{H}{\dot{\phi}}\delta\phi_k \simeq \text{const}$$



The power spectra

Focus on statistical measures of clustering.

Inflation predicts amp of waves of a given k obey gaussian statistics, the amplitude of each wave chosen independently and randomly from its gaussian. It predicts how the amplitude varies with scale — the power spectrum

Good approx -- power spectra as being power-laws with scale.

Density pertn

$$\delta_H^2(k) = \delta_H^2(k_0)\left[\frac{k}{k_0}\right]^{n-1}$$

Grav waves

$$A_G^2(k) = A_G^2(k_0)\left[\frac{k}{k_0}\right]^{n_G}$$

Four parameters

Some formulae

Power spectra $P_\phi(k) = \frac{k^3}{2\pi^2}\left\langle \left|\delta\phi_k\right|^2 \right\rangle$

Vacuum soln $\left\langle \left|\delta\phi_k\right|^2 \right\rangle = \frac{H^2}{2k^3}$ → $P_\phi(k) = \left|\frac{H}{2\pi}\right|^2 \Big|_{k=aH(\mathrm{Exit})}$

Amp of density pertn $\delta_H^2(k) = \frac{4}{25}\left(\frac{H}{\dot\phi}\right)^2\left(\frac{H}{2\pi}\right)^2_{k=aH}$

SRA $\delta_H(k) \propto \kappa^{3/2} \frac{V^{3/2}}{|V'|}\Big]_{k=aH}$ WMAP: 60 efolds before tend → $\delta_H(k) \approx 1.91 * 10^{-5}$

→ $V^{1/4}/\varepsilon \leq 10^{16}$ GeV -- Lyth

In other words the properties of the inflationary potential are constrained by the CMB

Tensor pertns : amp of grav waves. →

$$\left. A_G(k) \propto \kappa^2 V^{1/2} \right]_{k=aH}$$

Note: Amp of perts depends on form of potential. Tensor pertns gives info directly on potential but difficult to detect.

Observational consequences.

Precision CMBR expts like WMAP and Planck → probing spectra.

Standard approx – power law.

$$\delta_H^2(k) \propto k^{n-1}\ ;\ A_G^2(k) \propto k^{n_G}$$

$$n-1 = \frac{d\ln\delta_H^2}{d\ln k}\ ;\ n_G = \frac{d\ln A_G^2}{d\ln k}$$

Power law ok, only a limited range of scales are observable.

For range 1Mpc →10^4 Mpc : $\Delta \ln k \approx 9$

Crucial eqn $\frac{d\ln k}{d\phi} = \kappa\frac{V}{V'}$ ⟹ $n = 1 - 6\varepsilon + 2\eta\ ;\ n_G = -2\varepsilon$

$n=1$; $n_G=0$ – Harrison Zeldovich

CMBR → Measure relative importance of density pertns and grav waves.

$$R = \frac{C_2^{GW}}{C_2^{S}} \approx 4\pi\varepsilon$$

$$\text{where } \frac{\Delta T}{T} = \sum a_{lm} Y_m^l(\theta, \varphi), \; C_l = \left\langle |a_{lm}|^2 \right\rangle$$

C_l -- radiation angular power spectrum.

A unique test of inflation

$$R = -2\pi n_G$$

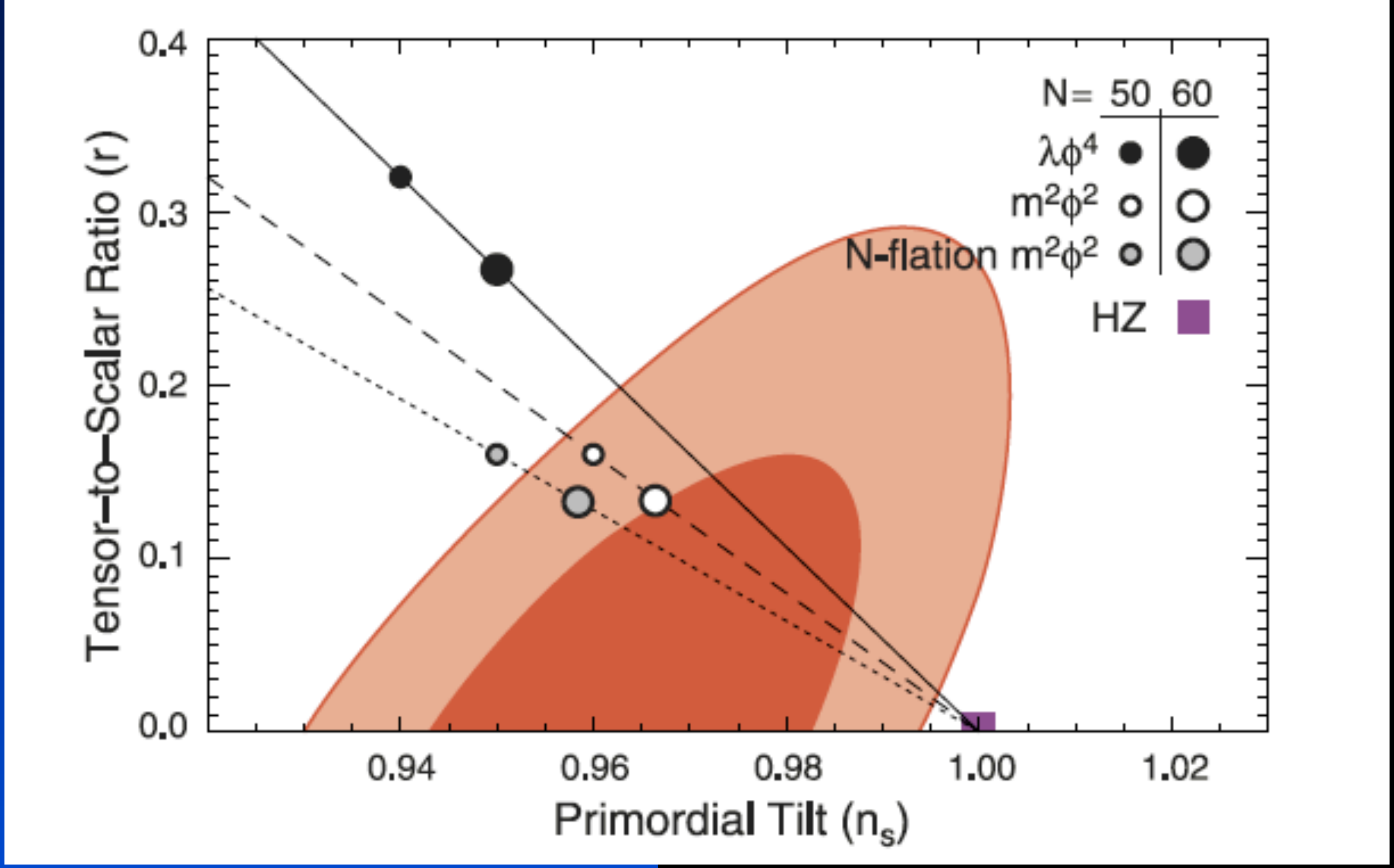
Indep of choice of inf model, relies on slow roll and power law approx. Unfortunately n_G too small for detection, but maybe Planck !

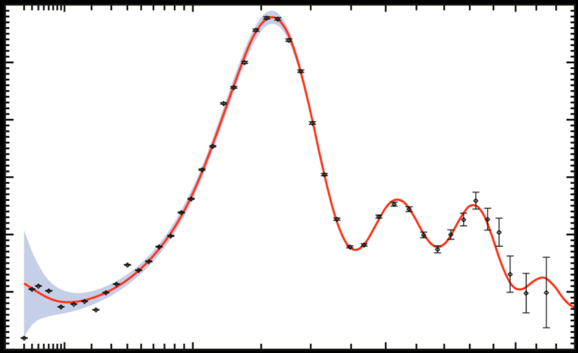
Example if include WMAP7+BAO+H0 constraints:

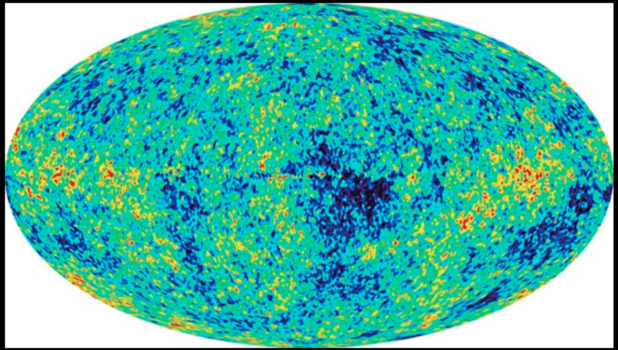
$$k_0 = 0.002 \mathrm{Mpc}^{-1}$$

No GW assumed: $n_s = 0.963 \pm 0.012$

Allow for GW:
$$\begin{aligned} n_s &= 0.973 \pm 0.014 \\ r &< 0.24 \ (95\% \ CL) \end{aligned}$$







(Komatsu et al, 2010)

Some examples – Chaotic Inflation

$$V(\phi) = \frac{m^2\phi^2}{2}$$ with $$\varepsilon = \frac{1}{2\kappa^2}\left[\frac{V'}{V}\right]^2 \quad ; \quad \eta = \frac{1}{\kappa^2}\left[\frac{V''}{V}\right]$$

Find:

$$\varepsilon = \frac{2}{\kappa^2\phi^2} = \eta$$

SRA:

$$H^2 = \frac{8\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Inf soln:

$$\phi(t) = \phi_i - \frac{\sqrt{2}mt}{\sqrt{3}\kappa} \quad ;$$

$$a(t) = a_i \exp\left[\frac{\kappa m}{\sqrt{6}}\left(\phi_i t - \frac{mt^2}{\sqrt{6}\kappa}\right)\right]$$

End of inflation: $\varepsilon = 1 \Rightarrow \phi_e = \frac{\sqrt{2}}{\kappa}$

Num of e-folds: $N(\phi) = -\kappa^2 \int_{\phi}^{\phi_e} \frac{V}{V'} d\phi = \frac{\kappa^2 \phi^2}{4} - \frac{1}{2}$

N=60: $\phi_{60} \approx \frac{16}{\kappa} > \phi_e$

Scale just entering Hubble radius today, COBE scale

Amp of den pertn: $\delta_H(k) = \frac{\kappa^3}{\sqrt{75\pi}} \frac{V^{3/2}}{|V'|} \Big]_{k=aH}$

Take to be 60 efolds before end of inflation.

Find: $\delta_H(k) = 12m\sqrt{G} \quad \text{where} \quad \kappa^2 \equiv 8\pi G$

Amp of grav waves: $A_G(k) = \sqrt{\frac{32}{75}} \, G V^{1/2} \Big]_{k=aH}$ 60 efolds before end of inflation.

Find: $A_G(k) \approx 1.4 m \sqrt{G}$

Normalise to COBE: $\delta_H(k) \approx 1.91 * 10^{-5}$

Find: $m = 2 * 10^{13}$ GeV **Constraint on inflaton mass!**

Spectral indices $n = 1 - 6\varepsilon + 2\eta \, ; \, n_G = -2\varepsilon$ Slow roll

Use values 60 e-folds before end of inflation.

$n = 0.97; \, n_G = -0.016$ Close to scale inv

2. Models of Inflation—variety is the spice of life.

(where is the inflaton in particle physics?)

(Lyth and Riotto, Phys. Rep. 314, 1, (1998), Lyth and Liddle (2009)

Field theory

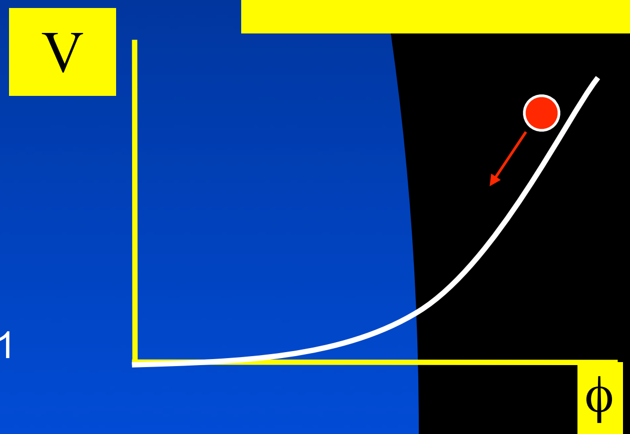
$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + M\phi^3 + \lambda\phi^4 + \sum_{d=5}^{\infty}\lambda_d M_P^{4-d}\phi^d$$

Quantum corrections give coefficients proportional to $\ln(\phi)$
and an additional term proportional to $\ln(\phi)$

1. Chaotic inflation .

$$V(\phi) \propto \phi^p;\ \ \phi >> M_P;\ \ n-1 = -(2+p)/2N;$$

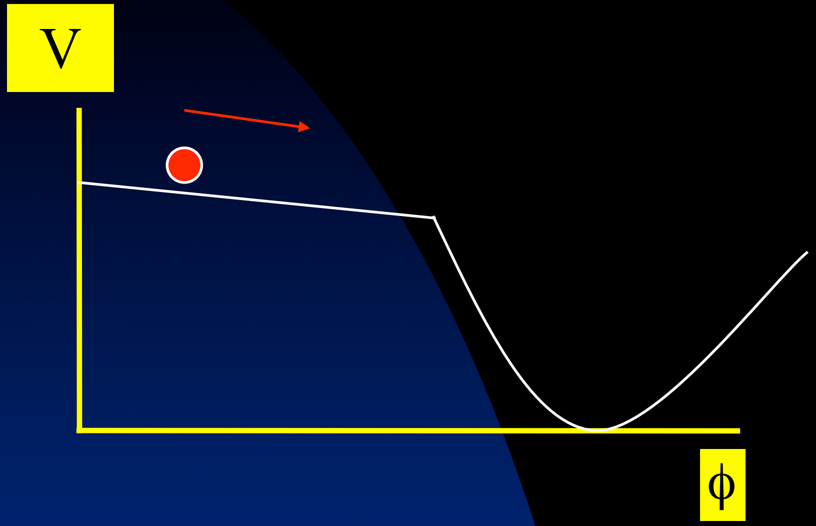
$$R = -2\pi n_G = {}^{3.1p}\!/_{N} \Rightarrow \text{sig grav waves.}$$



Inflates only for $\phi >> M_P$. Problem.
Why only one term? All other models inflate at $\phi < M_P$ and give negligible grav. waves.

2. New inflation

$$V(\phi) = V_0 - c\phi^p + \ldots;\ \ p \geq 3;\ \ n-1 = -\frac{2(p-1)}{(p-2)N}$$



$$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \ldots; \Rightarrow n-1 = -\frac{2M_P^2 m^2}{V_0}$$

$p = 2:$ modular, natural, quadratic inflation

3. Power-law inflation

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi}{m_P}\right);\ p > 1;\ \ n-1 = -\frac{2}{p}$$

1. Very useful because have exact solutions without recourse to slow roll. Similarly perturbation eqns can be solved exactly.

2. No natural end to inflation

4. Natural inflation

$$V(\phi) = V_0\left(1 + \cos\frac{\phi}{f}\right);$$

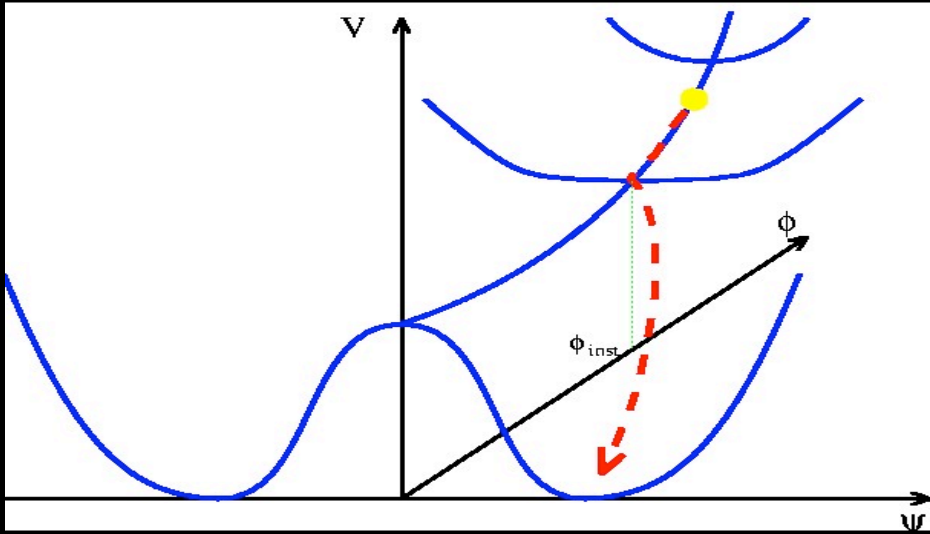
$$n - 1 < 0; \quad R - \text{negligible} - -\text{like New Inflation}$$

5. Hybrid inflation

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2;$$

$$n - 1 = \frac{2M_P^2 m^2}{V_0}$$

2 fields, inf ends when V_0 destabilised by 2nd non-inflaton field ψ

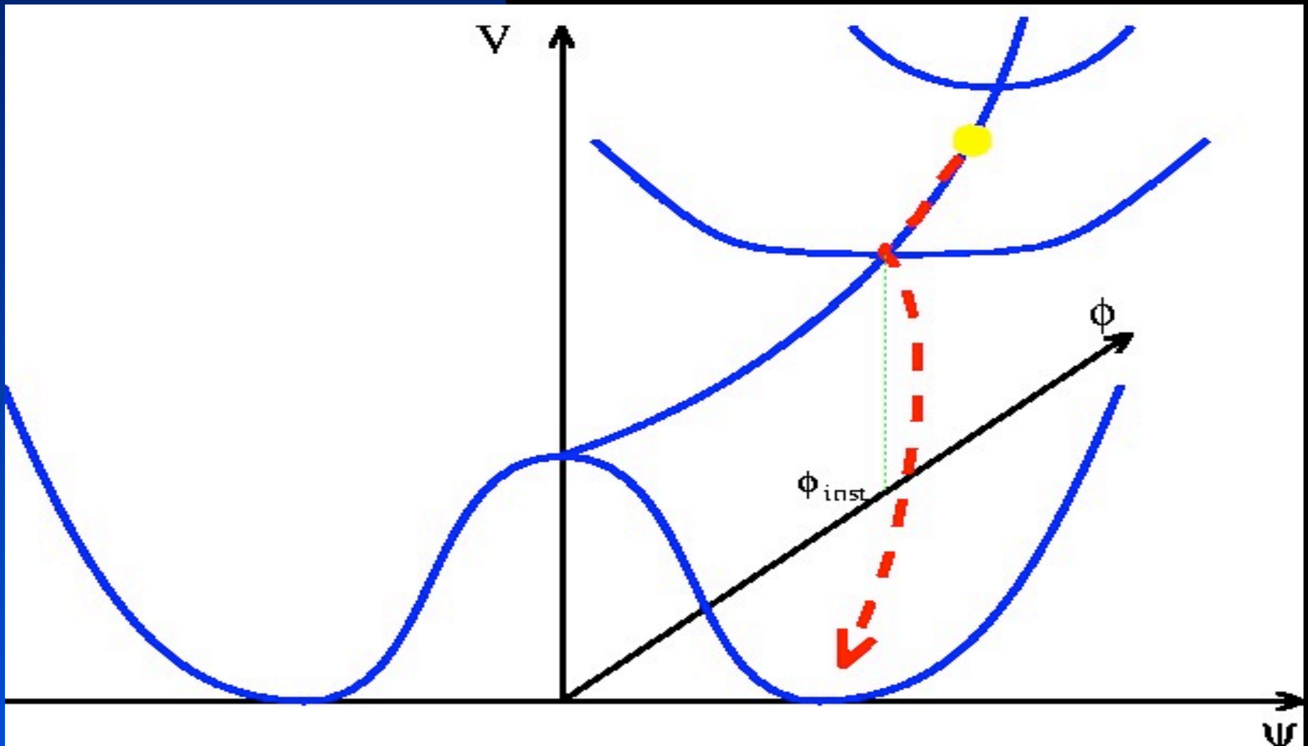


Two field inflation – more general

$$V(\phi,\psi) = \frac{1}{2}m_\phi{}^2\phi^2 + \frac{1}{2}g^2\phi^2|\chi|^2 + \frac{1}{4}\lambda\left(|\chi|^2 - \frac{m^2}{\lambda}\right)^2$$

Found in SUSY models.

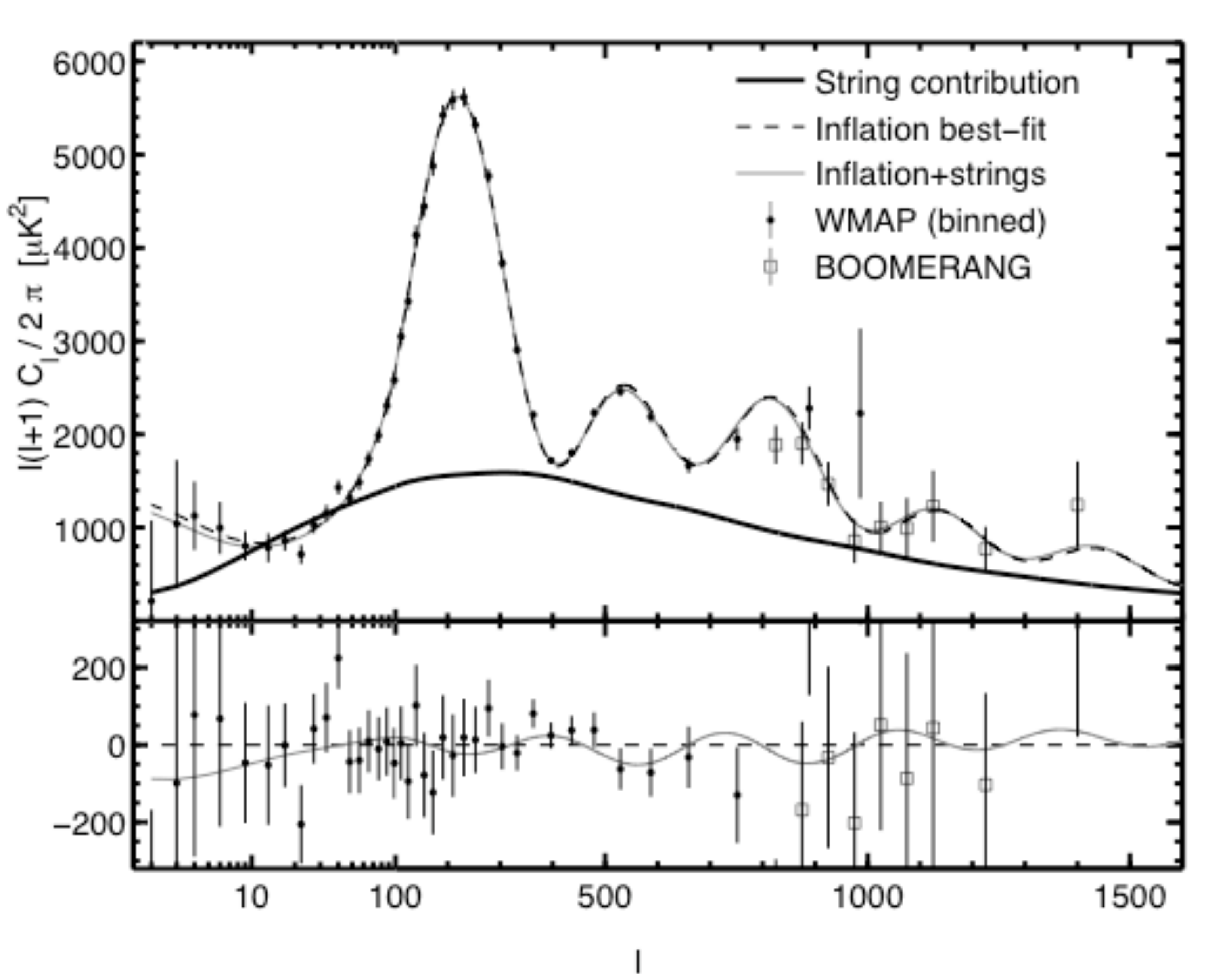
Better chance of success, plus lots of additional features, inc defect formation, ewk baryogenesis.



Inflation ends by triggering phase transition in second field.

Example of Brane inflation

Cosmic strings - may not do the full job but they can still contribute



Hybrid Inflation type models

String contribution < 11% implies $G\mu < 0.7 * 10^{-6}$.

Bevis et al 2007,2010.

Inflation model building today -- big industry

Multi-field inflation

Inflation in string theory and braneworlds

Inflation in extensions of the standard model

Cosmic strings formed at the end of inflation

The idea is clear though:

Use a combination of data (CMB, LSS, SN, BAO ...) to try and constrain models of the early universe through to models explaining the nature of dark energy today.

Inflation in string theory -- non trivial

The η problem in Supergravity -- N=1 SUGR Lagrangian:

$$\mathcal{L} = -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} + V_F,$$

with

$$V_F = e^{K/M_p^2}\left[K^{\varphi\bar{\varphi}}D_\varphi W\overline{D_\varphi W} - \frac{3}{M_p^2}|W|^2\right]$$

and

$$D_\varphi = \partial_\varphi W + \frac{1}{M_p^2}\partial_\varphi K$$

$$K(\varphi,\bar{\varphi}) = K_0 + K_{\varphi\bar{\varphi}}\varphi\bar{\varphi} + \ldots$$

Expand K about φ=0

$$\begin{aligned}\mathcal{L} &\approx -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} - V_0\left(1 + K_{\varphi\bar{\varphi}}|_{\varphi=0}\frac{\varphi\bar{\varphi}}{M_p^2} + \ldots\right) \\ &= -\partial\phi\partial\bar{\phi} - V_0\left(1 + \frac{\phi\bar{\phi}}{M_p^2} + \ldots\right),\end{aligned}$$

Canonically norm fields ϕ

Have model indep terms which lead to contribution to slow roll parameter η of order unity

$$\Delta\eta = M_p^2\frac{\Delta V''}{V_0} = 1.$$

So, need to cancel this generic term possibly through additional model dependent terms.

Ex 1: Warped D3-brane D3-antibrane inflation where model dependent corrections to V can cancel model indep contributions

[Kachru et al (03) -- KLMMT].

Find:

$$V(\phi) = V_0(\phi) + \beta H^2 \phi^2$$

β relates to the coupling of warped throat to compact CY space. Can be fine tuned to avoid η problem

Ex 2: DBI inflation -- simple -- it isn't slow roll as the two branes approach each other so no η problem

Ex 3: Kahler Moduli Inflation [Conlon & Quevedo 05]

Inflaton is one of Kahler moduli in Type IIB flux compactification. Inflation proceeds by reducing the F-term energy. No η problem because of presence of a symmetry, an almost no-scale property of the Kahler potential.

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{\mathcal{V}^2},$$

Inflaton moduli: τ_n

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{\mathcal{V}^2},$$

Find:

$$\begin{aligned} 0.960 &< n < 0.967, \\ -0.0006 &< \frac{dn}{d\ln k} < -0.0008, \\ 0 &< |r| < 10^{-10}, \end{aligned}$$

with large volume modulus

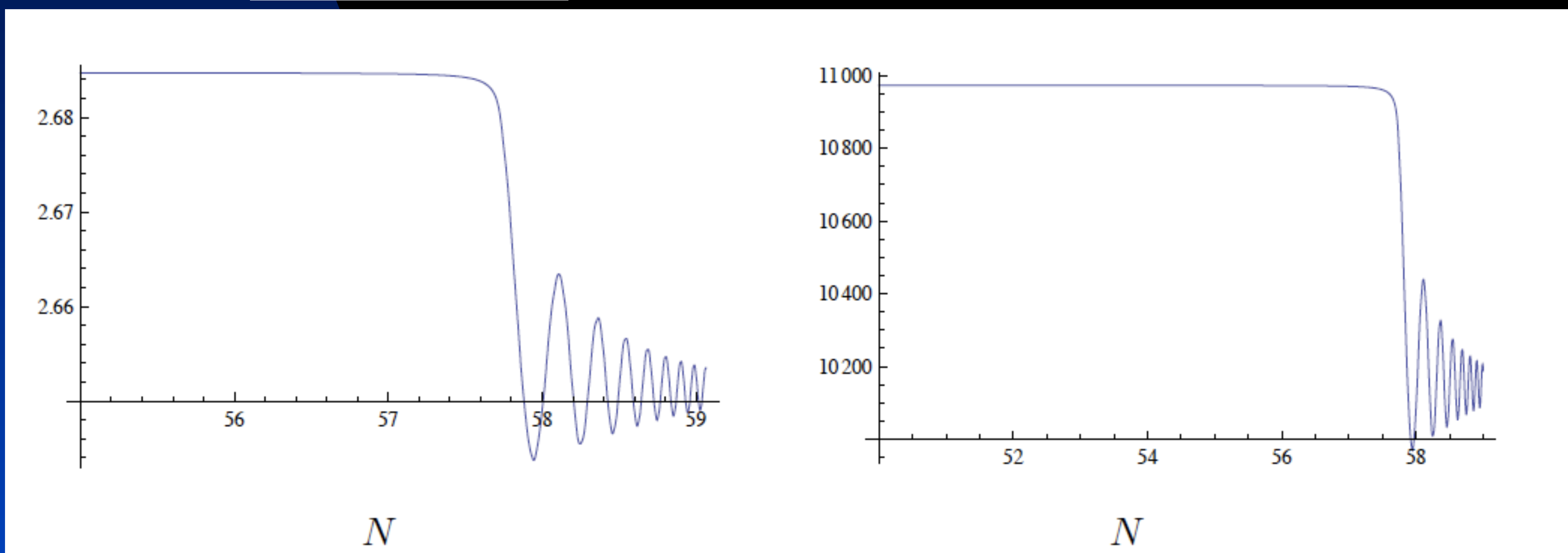
$$10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6,$$

and

$$\begin{aligned} \eta &\approx -\frac{1}{N_e}, \\ \epsilon &< 10^{-12}, \end{aligned}$$

for N_e≈50-60 efolds with low energy scale

$$V_{inf} \sim 10^{13}\text{GeV}.$$



Inflaton **[Blanco-Pillado et al 09]** **Volume modulus**

Can include curvaton as second evolving moduli -- Burgess et al 2010

Key inflationary parameters:

n: Perhaps Planck will finally determine whether it is unity or not.

r: Tensor-to-scalar ratio : considered as a smoking gun for inflation but also produced by defects and some inflation models produce very little.

dn/dln k : Running of the spectral index, usually very small -- probably too small for detection.

f_{NL}: Measure of cosmic non-gaussianity. Still consistent with zero, but tentative evidence of a non-zero signal in WMAP data which would provide an important piece of extra information to constrain models. For example, it could rule out single field models -- lots of current interest.

$G\mu$: string tension in Hybrid models where defects produced at end of period of inflation.

Also new perturbation generation mechanisms (e.g. Curvaton)

Perturbations not from inflaton but from extra field and then couple through to curvature perturbation

Things not explored - no time

1. Gravitational waves from pre-heating

2. Non-Gaussianity from multi-field inflation

3. Nature of perturbations (adiabatic v non-adiabatic)

4. Thermal inflation and warm inflation

5. Going beyond slow roll

6. Inflation model building -- how easy in string theory.

7. Where is the inflaton in particle physics ? How fine tuned is it?

8. Low energy inflation (i.e. TeV scale).

9. Singularity -- eternal inflation !

10. Impact of multiverse on inflation.

11. Alternatives: pre-big bang, cyclic/ekpyrotic, string cosmology, varying speed of light, quantum gravity ....

Busstepp 2011
Cosmology - Lecture 4

Ed Copeland -- Nottingham University

1. The power of scaling solutions in cosmology.

Aim -- to demonstrate the power of looking at cosmological systems using phase plane analysis, obtaining critical points and establishing conditions for the existence of attractor solutions.

1. Introduction

In cosmology as in many areas of physics we often deal with systems that are inherently described through a series of coupled non-linear differential equations.

Such systems often can not be solved analytically, yet they can be analysed through determining the late time behaviour of some combination of the variables, where they may approach some form of attractor solution, attractors in variables that are not always the basic variables the underlying equations describe.

By determining the nature of these attractor solutions (their stability for example) one can learn a great deal about the system in general.

Moreover the phase plane description of the system is often highly intuitive enabling easy analysis and understanding of the system.

In cosmology this is particularly useful. The universe is very old, and the existence of scaling solutions where a quantity becomes constant enables one to find the regime where scaling occurs, and then simply rescale the quantities to obtain their values today -- thereby avoiding doing a simulation for 13.7 Billion years !

Examples include the relative energy densities in scalar fields compared to the background radiation and matter densities, as well as the relative energy density in cosmic strings.

In general such a phase plane analysis reduces the order of the differential equations being investigated by introducing new variables which are themselves derivatives of the original variables.

Example in cosmology :

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\rho - \frac{k}{a^2} + \Lambda$$

Friedmann eqn

$$\dot{\rho} + 3H(\rho + p) = 0$$

Fluid eqn.

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho + 3p) + \frac{\Lambda}{3}$$

Acceleration eqn

where $\kappa^2 \equiv 8\pi G$

Note:

$$\frac{\ddot{a}}{a} \geq 0 <-> = (\rho + 3p) \leq 0$$

Tracker solutions

Wetterich,

Peebles and Ratra,

EJC, Liddle and Wands

Scalar field: $\phi: \ \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)\ ; \ \ p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$

EoM:

$$\dot{H} = -\frac{\kappa^2}{2}\left(\dot{\phi}^2 + \gamma\rho_B\right)$$

$$\dot{\rho}_B = -3\gamma H\rho_B$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}$$

+ constraint:

$$H^2 = \frac{\kappa^2}{3}\left(\rho_\phi + \rho_B\right)$$

Intro: $x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}$ $\quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}$ $\quad \lambda \equiv \frac{-1}{\kappa V}\frac{dV}{d\phi}$ $\quad \Gamma - 1 \equiv \frac{d}{d\phi}\left(\frac{1}{\kappa\lambda}\right)$

Eff eqn of state: $\gamma_\phi = \dfrac{\dot{\phi}^2}{V + \dfrac{\dot{\phi}^2}{2}} = \dfrac{2x^2}{x^2 + y^2}$

$$\Omega_\phi = \frac{\kappa^2 \rho_\phi}{3H^2} = x^2 + y^2$$

Friedmann eqns and fluid eqns become:

$$x' = -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$

$$y' = -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$

$$\lambda' = -\sqrt{6}\lambda^2(\Gamma - 1)$$

$$\frac{\kappa^2 \rho_\gamma}{3H^2} + x^2 + y^2 = 1$$

where $' \equiv d/d(\ln a)$

Note: $0 \le \gamma_\phi \le 2 : 0 \le \Omega_\phi \le 1$

Scaling solutions: (x`=y`=0)

No:	x_c	y_c	Existance	Stability	Ω_ϕ	γ_ϕ
1	0	0	$\forall\lambda,\gamma$	SP: $0<\gamma$ SN: $\gamma=0$	0	Undefined
2a	1	0	$\forall\lambda,\gamma$	UN: $\lambda<\sqrt{6}$ SP: $\lambda>\sqrt{6}$	1	2
2b	-1	0	$\forall\lambda,\gamma$	UN: $\lambda>-\sqrt{6}$ SP: $\lambda<-\sqrt{6}$	1	2
3	$\frac{\lambda}{\sqrt{6}}$	$\left(1-\frac{\lambda^2}{6}\right)^{1/2}$	$\lambda^2\le 6$	SP: $3\gamma<\lambda^2<6$ SN: $\lambda^2<3\gamma$	1	$\frac{\lambda^2}{3}$
4	$\left(\frac{3}{2}\right)^{1/2}\frac{\gamma}{\lambda}$	$\left[\frac{3(2-\gamma)\gamma}{2\lambda^2}\right]^{1/2}$	$\lambda^2\ge 3\gamma$	SN: $3\gamma<\lambda^2<\frac{24\gamma^2}{9\gamma-2}$ SS: $\lambda^2>\frac{24\gamma^2}{9\gamma-2}$	$\frac{3\gamma}{\lambda^2}$	γ

$$V=V_0e^{-\lambda\kappa\phi}$$

Late time attractor is scalar field dominated

$\lambda^2\le 6$

Field mimics background fluid.

$\lambda^2\ge 3\gamma$

Nucleosynthesis bound → $\lambda^2>20$

EJC, Liddle and Wands

$$V = V_0 e^{-\lambda\kappa\phi}$$

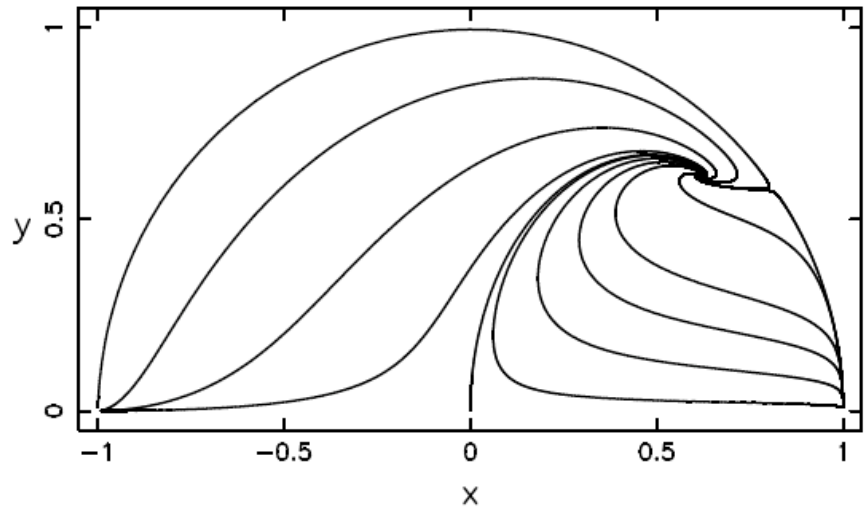


FIG. 3. The phase plane for $\gamma = 1$, $\lambda = 2$. The scalar field dominated solution is a saddle point at $x = \sqrt{2/3}$, $y = \sqrt{1/3}$, and the late-time attractor is the scaling solution with $x = y = \sqrt{3/8}$.

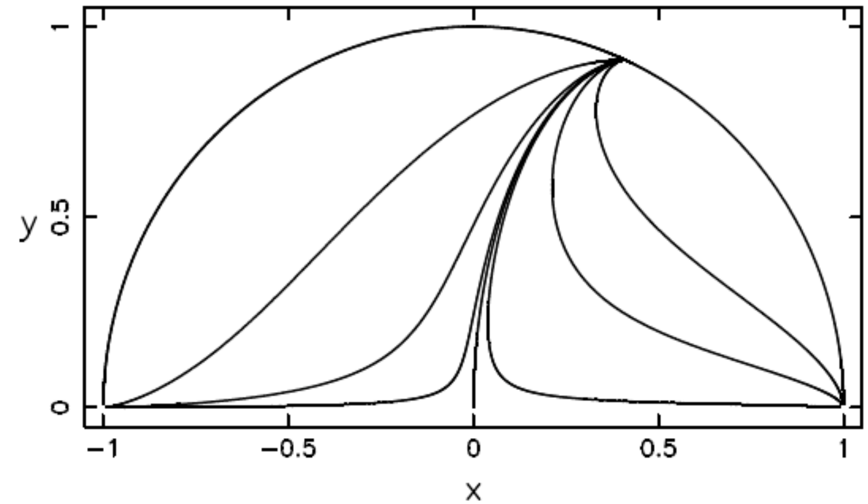


FIG. 2. The phase plane for $\gamma = 1$, $\lambda = 1$. The late-time attractor is the scalar field dominated solution with $x = \sqrt{1/6}$, $y = \sqrt{5/6}$.

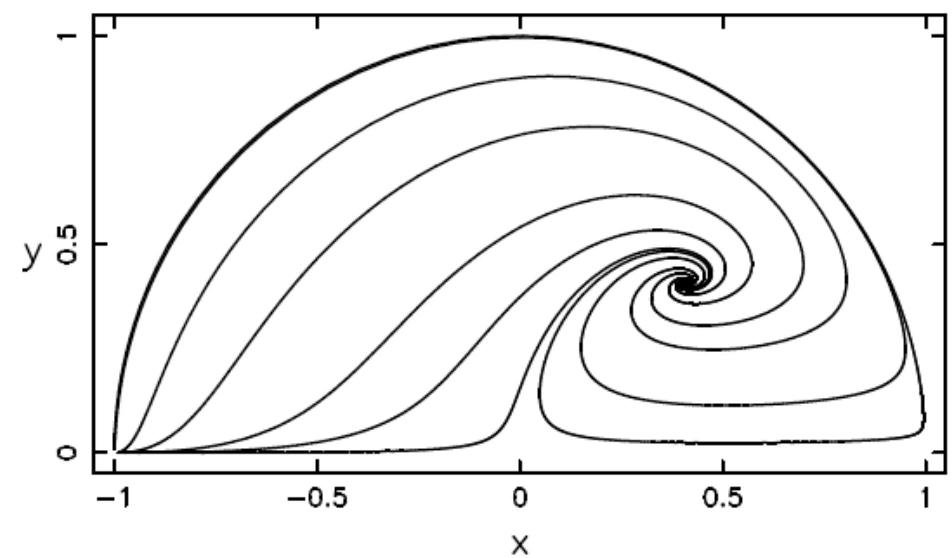


FIG. 4. The phase plane for $\gamma = 1$, $\lambda = 3$. The late-time attractor is the scaling solution with $x = y = \sqrt{1/6}$.

Stability criteria

Expand about critical points

$$x = x_c + u \,, \qquad y = y_c + v \,,$$

Sub into evoln eqns

$$x' = -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$
$$y' = -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right] \,,$$

Yields first order pertn eqns

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \mathcal{M}\begin{pmatrix} u \\ v \end{pmatrix}$$

General solution where $m_{\pm}$ are eigenvalues of M

$$u = u_+ \exp(m_+ N) + u_- \exp(m_- N)$$
$$v = v_+ \exp(m_+ N) + v_- \exp(m_- N)$$

Fluid-dominated solution:

$$m_- = -\frac{3(2-\gamma)}{2} \,, \qquad m_+ = \frac{3\gamma}{2} \,.$$

Kinetic-dominated solutions, $(x_c = \pm 1,\, y_c = 0)$:

$$m_- = \sqrt{\frac{3}{2}}\left(\sqrt{6} \mp \lambda\right) \,, \qquad m_+ = 3(2-\gamma)$$

Scalar field dominated solution:

$$m_- = \frac{\lambda^2 - 6}{2} \,, \qquad m_+ = \lambda^2 - 3\gamma$$

Scaling solution:

$$m_{\pm} = -\frac{3(2-\gamma)}{4}\left[1 \pm \sqrt{1 - \frac{8\gamma(\lambda^2 - 3\gamma)}{\lambda^2(2-\gamma)}}\right]$$

2. Applications in dark energy models

One approach to dark energy involves assuming the dark energy is dynamical, not due to an underlying cosmological constant. That is assumed to be zero from some as yet unknown symmetry argument and what we are left with is an evolving scalar field which came to dominate recently.

Depending on the underlying potential such a field can undergo a period of tracking where it mimics the background energy density before coming to dominate at late times.

All such models I am aware of require various degrees of fine tuning as we shall see

Coincidence problem – why now?

$$\frac{\ddot{a}}{a} \geq 0 <-> = (\rho + 3p) \leq 0$$

If: $$\rho_x = \rho_x^0\, a^{-3(1+w_x)}$$

Universe dom by Quintessence at:

$$z_x = \left(\frac{\Omega_x}{\Omega_m}\right)^{\frac{1}{3w_x}} - 1$$

$$\left(\frac{\Omega_x}{\Omega_m}\right) = \frac{7}{3} \rightarrow z_x = 0.5,\ 0.3 \text{ for } w_x = -\frac{2}{3},\ -1$$

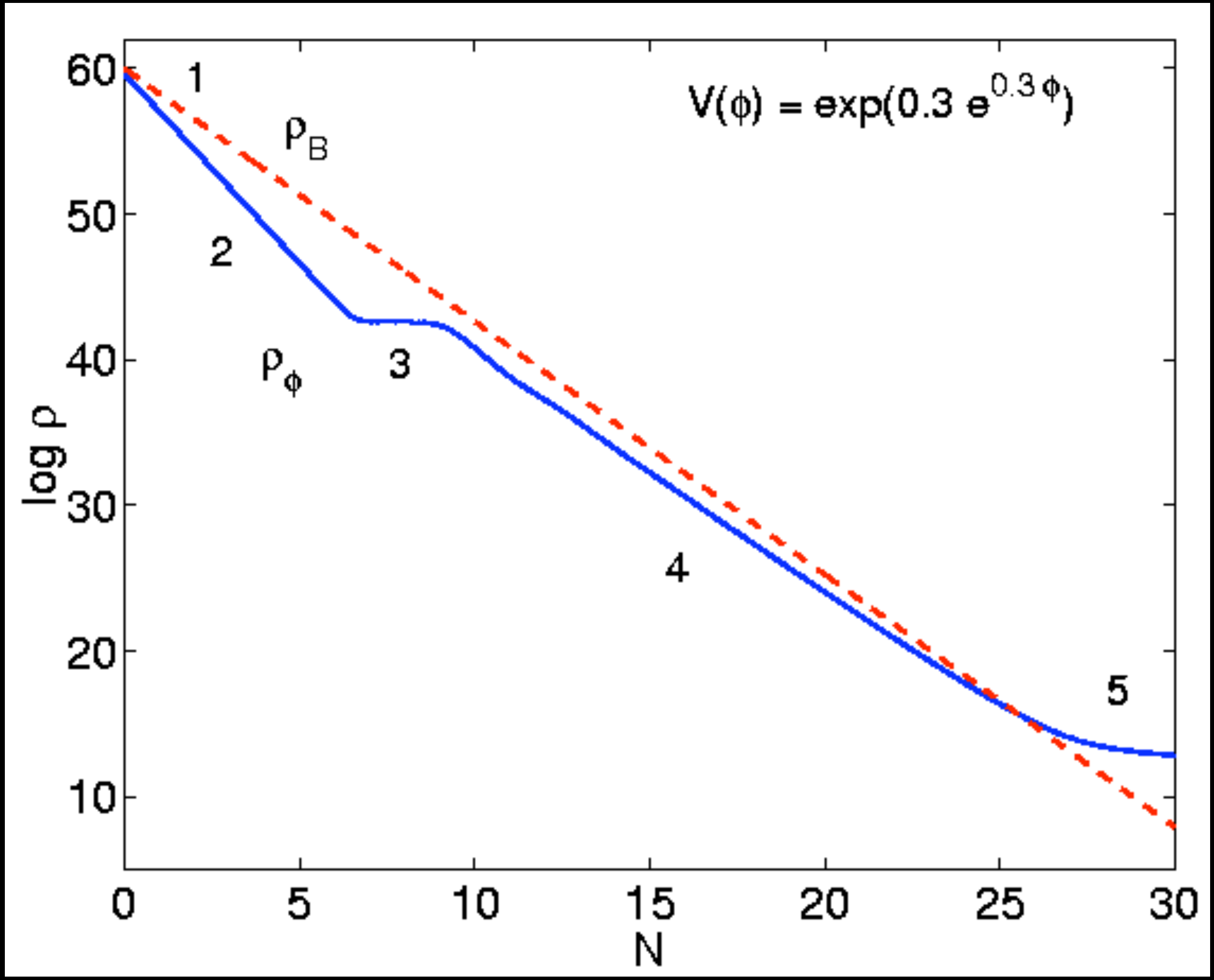
Univ accelerates at:

$$z_a = \left(-(1+3w_x)\frac{\Omega_x}{\Omega_m}\right)^{\frac{-1}{3w_x}} - 1$$

$$z_a = 0.7,\ 0.5 \text{ for } w_x = -\frac{2}{3},\ -1$$

Slowly rolling scalar fields
Quintessence - Generic behaviour

1. PE → KE
2. KE dom scalar field energy den.
3. Const field.
4. Attractor solution: almost const ratio KE/PE.
5. PE dom.

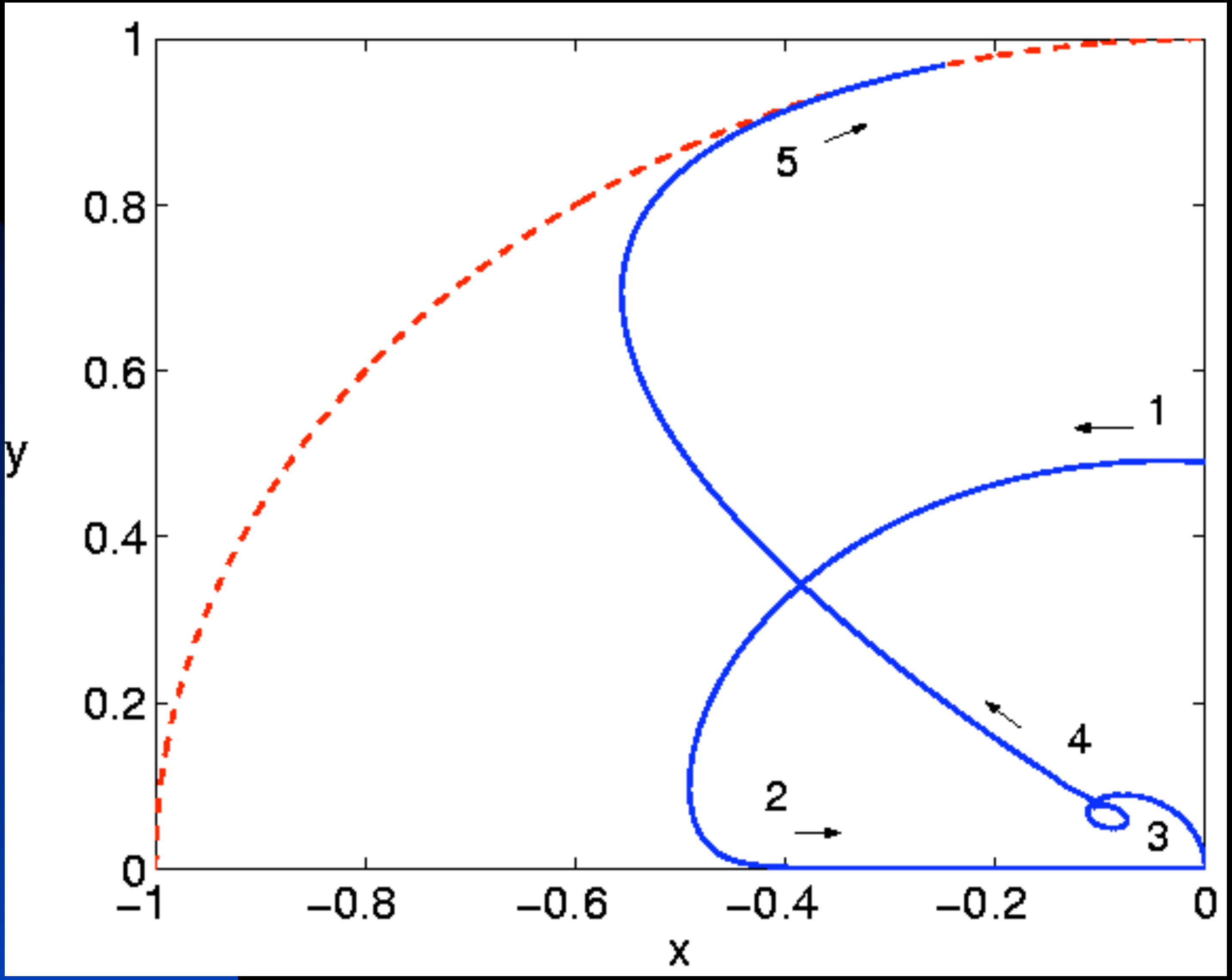


Nunes

Attractors make initial conditions less important

Phase Plane picture

Nunes



Typical example : Scaling solutions with exponential potentials. (EJC, Liddle and Wands)

$$V(\phi) = V_0 e^{-\kappa\lambda\phi}$$

Original Quintessence model

Peebles and Ratra;

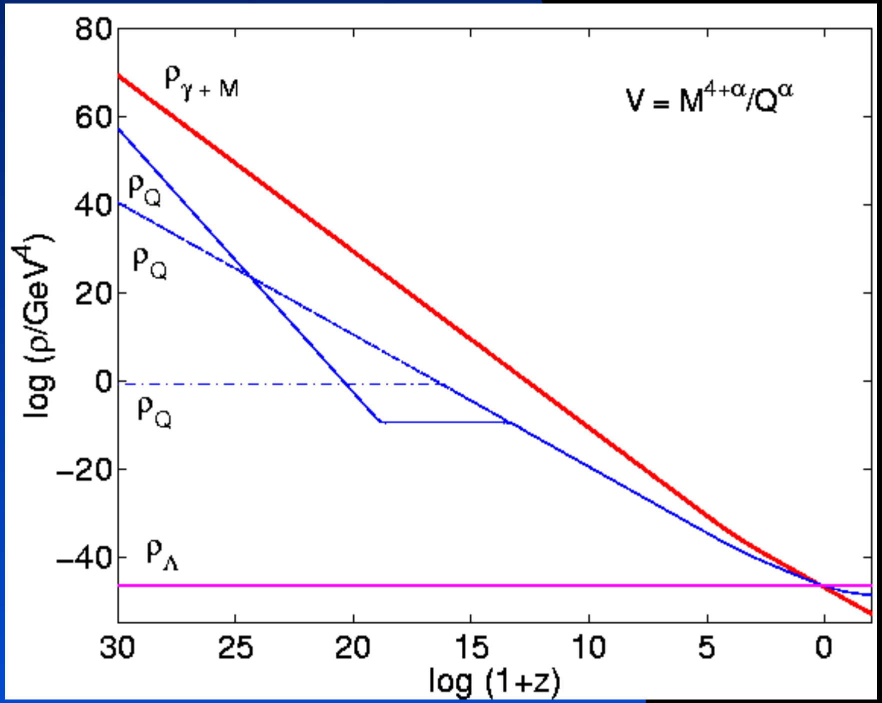
Zlatev, Wang and Steinhardt

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

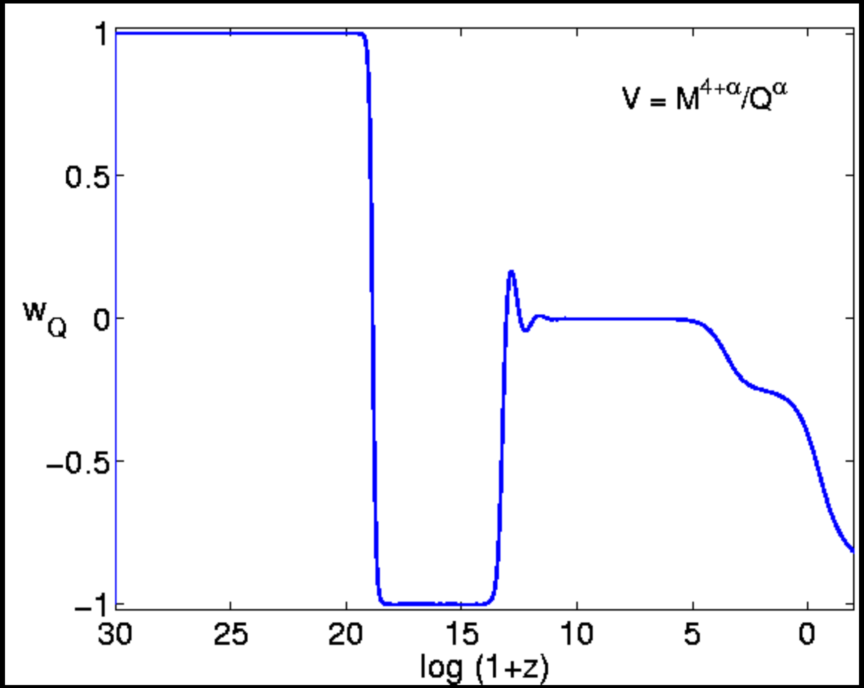
$$\lambda = \frac{-\alpha}{\kappa\phi}$$

$$\Gamma - 1 \equiv \frac{1}{\alpha}$$

Find: $$\phi = \phi_i \left(\frac{a}{a_i} \right)^{3(1+w_B)/(2+\alpha)}$$ and $$w_\phi = \frac{\alpha w_B - 2}{2+\alpha}$$



$$\alpha = 6$$



Fine Tuning in Quintessence

Need to match energy density in Quintessence field to current critical energy density.

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

$$\rho_{\Lambda} \leq \frac{H_0^2}{\kappa^2} \approx 10^{-47}\ \text{GeV}^4$$

Find: $$y_c^2 = \frac{\kappa^2 V}{3H^2} \propto \kappa^2 \phi^2$$

so: $$H^2 = \frac{V}{\phi^2} \propto \kappa^2 \rho_{\phi} \Rightarrow \phi_0 \approx M_{pl}$$

Hence: $$M = \left[\rho_{\phi}^{0}\, M_{pl}^{\alpha}\right]^{1/(4+\alpha)} \Rightarrow \alpha = 2; M = 1\, GeV$$

A few models

1. Inverse polynomial – found in SUSY QCD - Binetruy

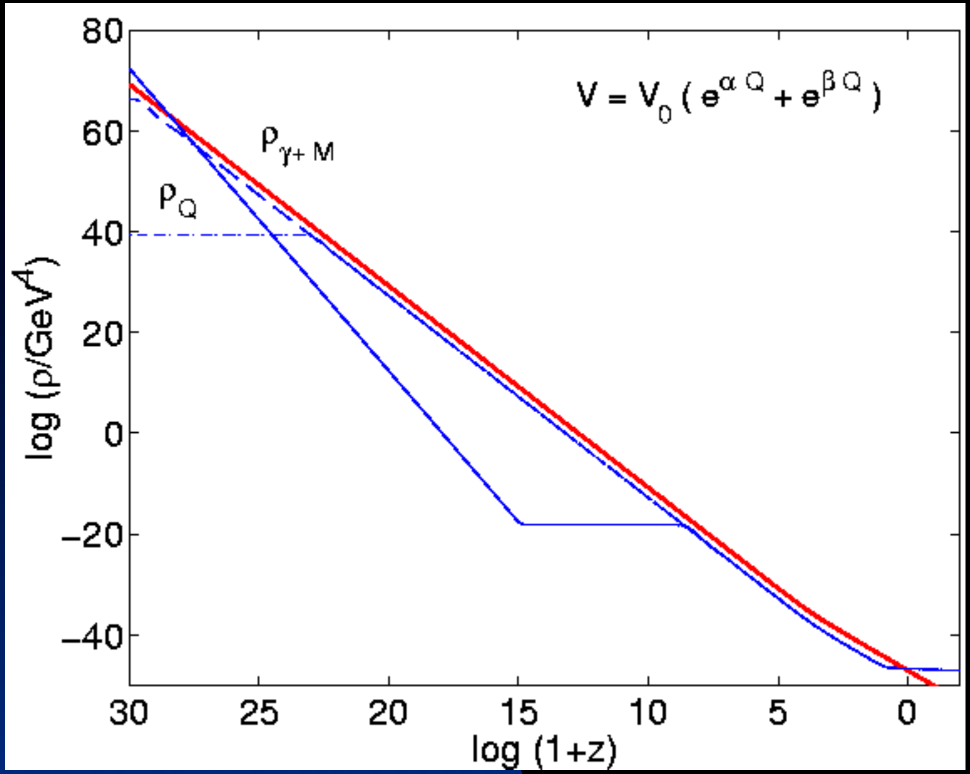
2. Multiple exponential potentials – SUGR and String compactification.

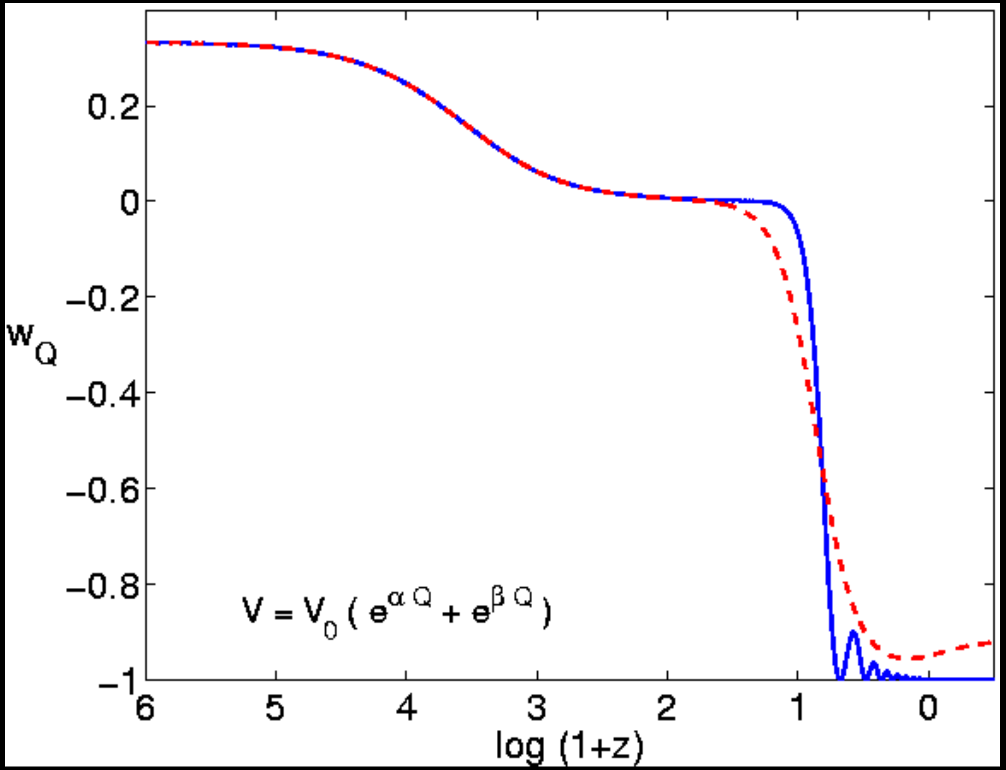
Barreiro, EC, Nunes

$$V(\phi) = V_1 + V_2$$
$$= V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi}$$

Enters two scaling regimes depends on lambda, one tracking radiation and matter, second one dominating at end. Must ensure do not violate nucleosynthesis constraints.

$\alpha = 20; \beta = 0.5$





Scaling for wide range of i.c.

Fine tuning: $$V_0 \approx \rho_\phi \approx 10^{-47}\ \text{GeV}^4 \approx (10^{-3}\ \text{eV})^4$$

Mass: $$m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \text{eV}$$

Fifth force !

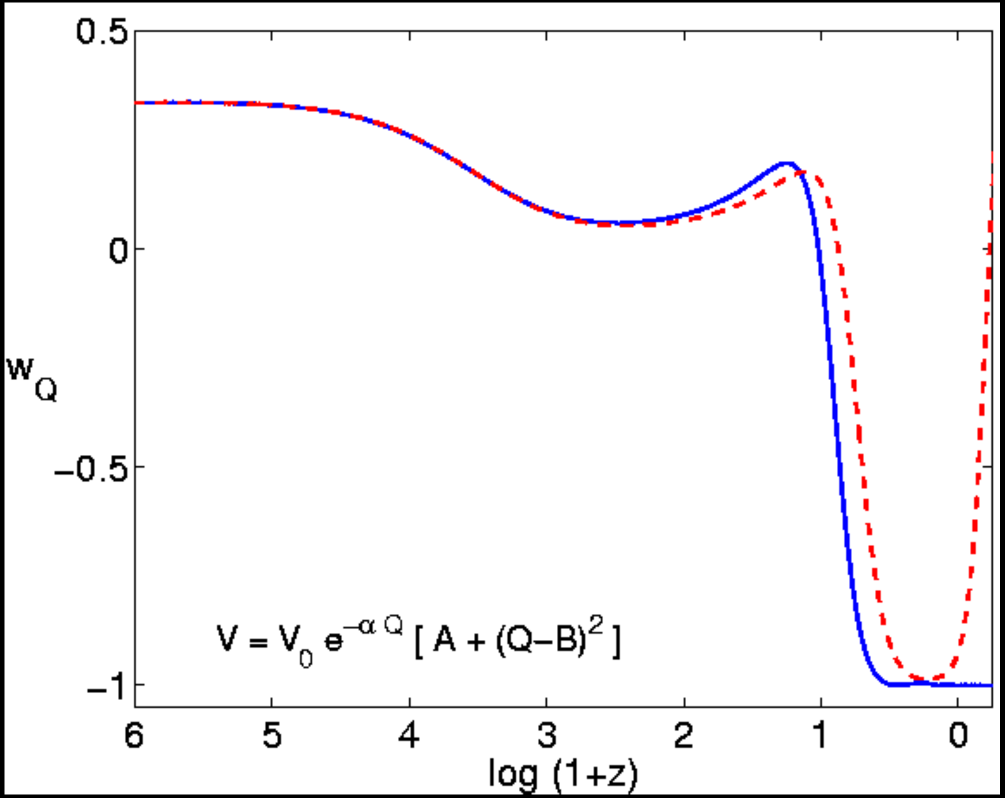
3. Albrecht-Skordis model – Albrecht and Skordis

$$V(\phi) = V_0\, e^{-\alpha\kappa\phi}\left[A + (\kappa\phi - B)^2\right]$$

-- Brane models

Early times: exp dominates and scales as rad or matter.

Field gets trapped in local minima and univ accelerates



Fine tuned as in previous cases.

K-essence v Quintessence

K-essence -- scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model? -- Armendariz-Picon, Mukhanov, Steinhardt

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form.

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{16\pi G}R + K(\phi)\tilde{p}(X)\right]$$

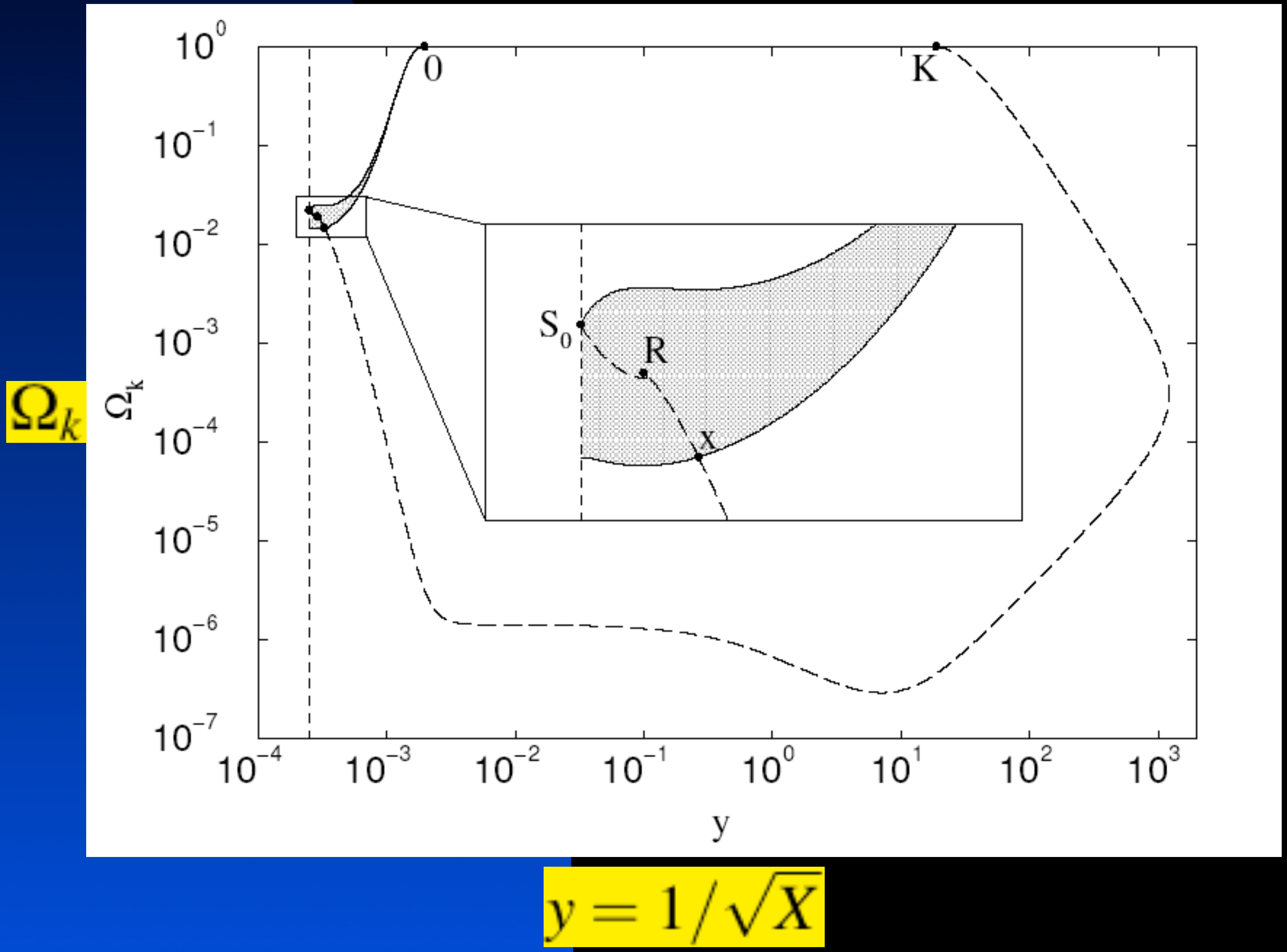
$$K(\phi) > 0,\ X = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$

Eqn of state $w_k = \frac{\tilde{p}(X)}{\tilde{\varepsilon}(X)} = \frac{\tilde{p}(X)}{2X\tilde{p}'(X) - \tilde{p}(X)}$ can be < -1

However also requires similar level of find tuning as in Quintessence

Fine tuning in K-essence as well: -- Malquarti, EJC, Liddle

Not so clear that K-essence solves the coincidence problem. The basin of attraction into the regime of tracker solutions is small compared to those where it immediately goes into K-essence domination.



Shaded region is basin of attraction for stable tracker solution at point R. All other trajectories go to K-essence dom at point K.

Based on K-essence model astro-ph/0004134, Armendariz-Picon et al.

Modified gravity as an alternative -- f(G) Dark Energy

Zhou, EJC and Saffin

Consider modified gravity:
$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + f(G) + \mathcal{L}_r + \mathcal{L}_m\right)$$

with Gauss-Bonnet combination:
$$G = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$

Einstein eqns complicated :
$$\begin{aligned} & R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \\ &+8[R_{\mu\rho\nu\sigma} + R_{\rho\nu}g_{\sigma\mu} - R_{\rho\sigma}g_{\nu\mu} - R_{\mu\nu}g_{\sigma\rho} + R_{\mu\sigma}g_{\nu\rho} \\ &+\frac{1}{2}R(g_{\mu\nu}g_{\sigma\rho} - g_{\mu\sigma}g_{\nu\rho})]\nabla^\rho\nabla^\sigma f_G + (Gf_G - f)g_{\mu\nu} = T_{\mu\nu} , \end{aligned}$$

Intro :
$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$$

Have :
$$R = 6(\dot{H} + 2H^2) , \qquad G = 24H^2(\dot{H} + H^2) ,$$

Following Amendola for f(R) consider writing as dynamical phase plane system to obtain fixed points:

Intro dimensionless variables :

$$
\begin{aligned}
x_1 &= \frac{G f_G}{3H^2}\,, \\
x_2 &= -\frac{f}{3H^2}\,, \\
x_3 &= -8H\dot{f}_G\,, \\
x_4 &= \Omega_r = \frac{\rho_r}{3H^2}\,, \\
x_5 &= \frac{G}{24H^4} = \frac{\dot{H}}{H^2} + 1\,.
\end{aligned}
$$

$$f_G = \mathrm{d}f/\mathrm{d}G$$

leads to :

$$
\begin{aligned}
\frac{\mathrm{d}x_1}{\mathrm{d}N} &= -\frac{x_3 x_5}{m} - x_3 x_5 - 2x_1 x_5 + 2x_1\,, \\
\frac{\mathrm{d}x_2}{\mathrm{d}N} &= \frac{x_3 x_5}{m} - 2x_2 x_5 + 2x_2\,, \\
\frac{\mathrm{d}x_3}{\mathrm{d}N} &= -x_3 + 2x_5 - x_3 x_5 + 1 - 3x_1 - 3x_2 + x_4\,, \\
\frac{\mathrm{d}x_4}{\mathrm{d}N} &= -2x_4 - 2x_4 x_5\,, \\
\frac{\mathrm{d}x_5}{\mathrm{d}N} &= -\frac{x_3 x_5^2}{x_1 m} - 4x_5^2 + 4x_5\,,
\end{aligned}
$$

$$N = \ln(a/a_i)$$

$$
\begin{aligned}
m &= \frac{G f_{GG}}{f_G}\,, \\
r &= -\frac{G f_G}{f} = \frac{x_1}{x_2}\,.
\end{aligned}
$$

$$\Omega_m = 1 - x_1 - x_2 - x_3 - x_4$$

$$w_{DE} = \frac{p_{DE}}{\rho_{DE}} = \frac{16H^3 \dot{f}_G + 16H\dot{H}\dot{f}_G + 8H^2 \ddot{f}_G - Gf_G + f}{Gf_G - f - 24H^3 \dot{f}_G}$$

$$w_{eff} = -1 - \frac{2\dot{H}}{3H^2},$$

$$w_{DE} = \frac{-2x_5 - x_4 - 1}{3(x_1 + x_2 + x_3)},$$
$$w_{eff} = -\frac{1}{3}(2x_5 + 1).$$

Critical points and critical lines :

$$L_1 : \{x_1 = 1 - x_2, x_2 = x_2, x_3 = 0, x_4 = 0, x_5 = 1\},$$
$$\Omega_m = 0, \quad \Omega_r = 0, \quad \Omega_{DE} = 1, \quad w_{DE} = -1, \quad w_{eff} = -1,$$
$$L_2 : \{x_1 = \frac{1}{6}x_3, x_2 = -\frac{1}{3}x_3, x_3 = x_3, x_4 = 0, x_5 = -\frac{1}{2}, m = -\frac{1}{2}\},$$
$$\Omega_m = 1 - \frac{5}{6}x_3, \quad \Omega_r = 0, \quad \Omega_{DE} = \frac{5}{6}x_3, \quad w_{DE} = 0, \quad w_{eff} = 0,$$
$$L_3 : \{x_1 = \frac{x_5}{x_5 - 2}, x_2 = -\frac{2x_5}{x_5 - 2}, x_3 = \frac{2(x_5 - 1)}{x_5 - 2}, x_4 = 0, x_5 = x_5, m = -\frac{1}{2}\},$$
$$\Omega_m = 0, \quad \Omega_r = 0, \quad \Omega_{DE} = 1, \quad w_{DE} = -\frac{2}{3}x_5 - \frac{1}{3}, \quad w_{eff} = -\frac{2}{3}x_5 - \frac{1}{3},$$
$$L_4 : \{x_1 = \frac{1}{4}x_3, x_2 = -\frac{1}{2}x_3, x_3 = x_3, x_4 = 1 - \frac{3}{4}x_3, x_5 = -1, m = -\frac{1}{2}\},$$
$$\Omega_m = 0, \quad \Omega_r = 1 - \frac{3}{4}x_3, \quad \Omega_{DE} = \frac{3}{4}x_3, \quad w_{DE} = \frac{1}{3}, \quad w_{eff} = \frac{1}{3}.$$

Critical points:

For general $f(G)$ models, there are 4 continuous lines of critical points:

$$\mathcal{L}_1 \;:\; (1-x_{20}, x_{20}, 0, 0, 1)$$

de Sitter

$$\mathcal{L}_2 \;:\; (\frac{1}{6}x_{30}, -\frac{1}{3}x_{30}, x_{30}, 0, -\frac{1}{2})\,,\ \mathbf{m}(-\frac{1}{2}) = -\frac{1}{2}$$

scaling with matter$(\Omega_{DE}/\Omega_m = 5x_{30}/(6-5x_{30}))\,,\ w_{DE} = 0$

$$\mathcal{L}_3 \;:\; (\frac{x_{50}}{x_{50}-2}, -\frac{2x_{50}}{x_{50}-2}, \frac{2(x_{50}-1)}{x_{50}-2}, 0, x_{50})\,,\ \mathbf{m}(-\frac{1}{2}) = -\frac{1}{2}$$

dark energy dominated, $w_{DE} = -2/3x_{50} - 1/3$

$$\mathcal{L}_4 \;:\; (\frac{1}{4}x_{30}, -\frac{1}{2}x_{30}, x_{30}, 1-\frac{3}{4}x_{30}, -1)\,,\ \mathbf{m}(-\frac{1}{2}) = -\frac{1}{2}$$

scaling with radiation$(\Omega_{DE}/\Omega_r = 3x_{30}/(4-3x_{30}))\,,\ w_{DE} = 1/3$

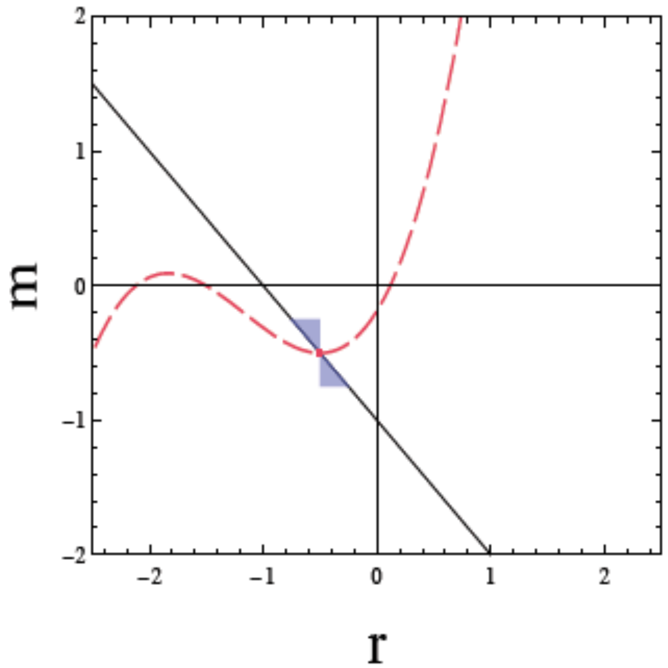
$\mathcal{L}_2$: Standard Matter Point

Eigenvalues:

$$\mathbf{0}\,, \quad -1\,, \quad 3(m'(-\tfrac{1}{2})+1)\,, \quad -\frac{3}{4}\pm\frac{1}{4}\sqrt{\frac{96-71x_{30}}{x_{30}}}$$

Standard Matter Point

- $x_{30} = -8H\dot{f}_G \to 0^-$
- $m(-\frac{1}{2}) = -\frac{1}{2}$
- $m'(-\frac{1}{2}) > -1$



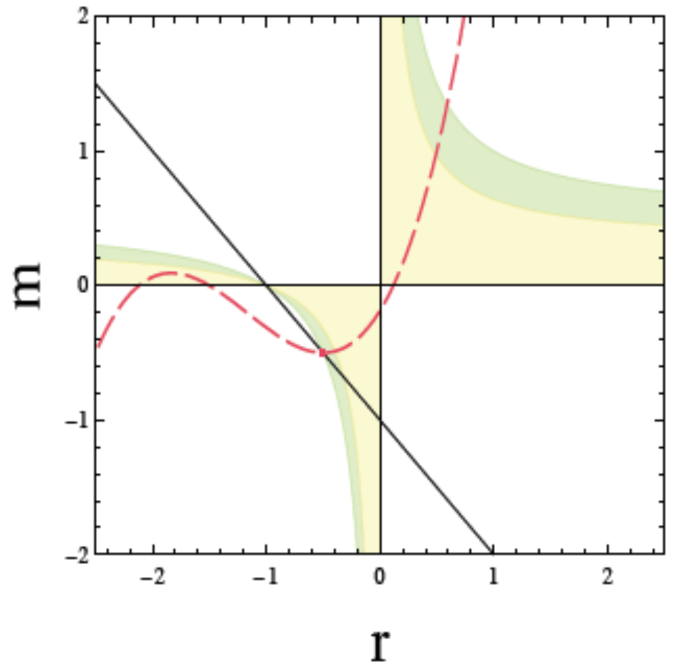
$\mathcal{L}_1$: de Sitter Point

Eigenvalues:

$$\mathbf{0}\,, \quad -4\,, \quad -3\,, \quad -\frac{3}{2} \pm \frac{1}{2}\sqrt{25 - \frac{8(r_1+1)}{r_1 m_1}}$$

Stable de Sitter Point

- Stable Spiral:
  Area enclosed by
  r axis and
  $m_{dS*}(r) = 8(r+1)/25r$
- Stable Node:
  Area enclosed by
  $m_{dS*}(r) = 8(r+1)/25r$
  and $m_{dS}(r) = (r+1)/2r$



$\mathcal{L}_3$: Dark Energy Dominated Point

Eigenvalues:

$$\mathbf{0}\,, \quad -2x_{50}-1\,, \quad -x_{50}-2\,, \quad -2x_{50}-2\,, \quad -2(x_{50}-1)(m'(-\frac{1}{2})+1)$$

Stable Quint.-like Point

- $-\frac{1}{2} < x_{50} < 1$
- $m(-\frac{1}{2}) = -\frac{1}{2}$
- $m'(-\frac{1}{2}) < -1$

Stable Phantom-like Point

- $x_{50} > 1$
- $m(-\frac{1}{2}) = -\frac{1}{2}$
- $m'(-\frac{1}{2}) > -1$

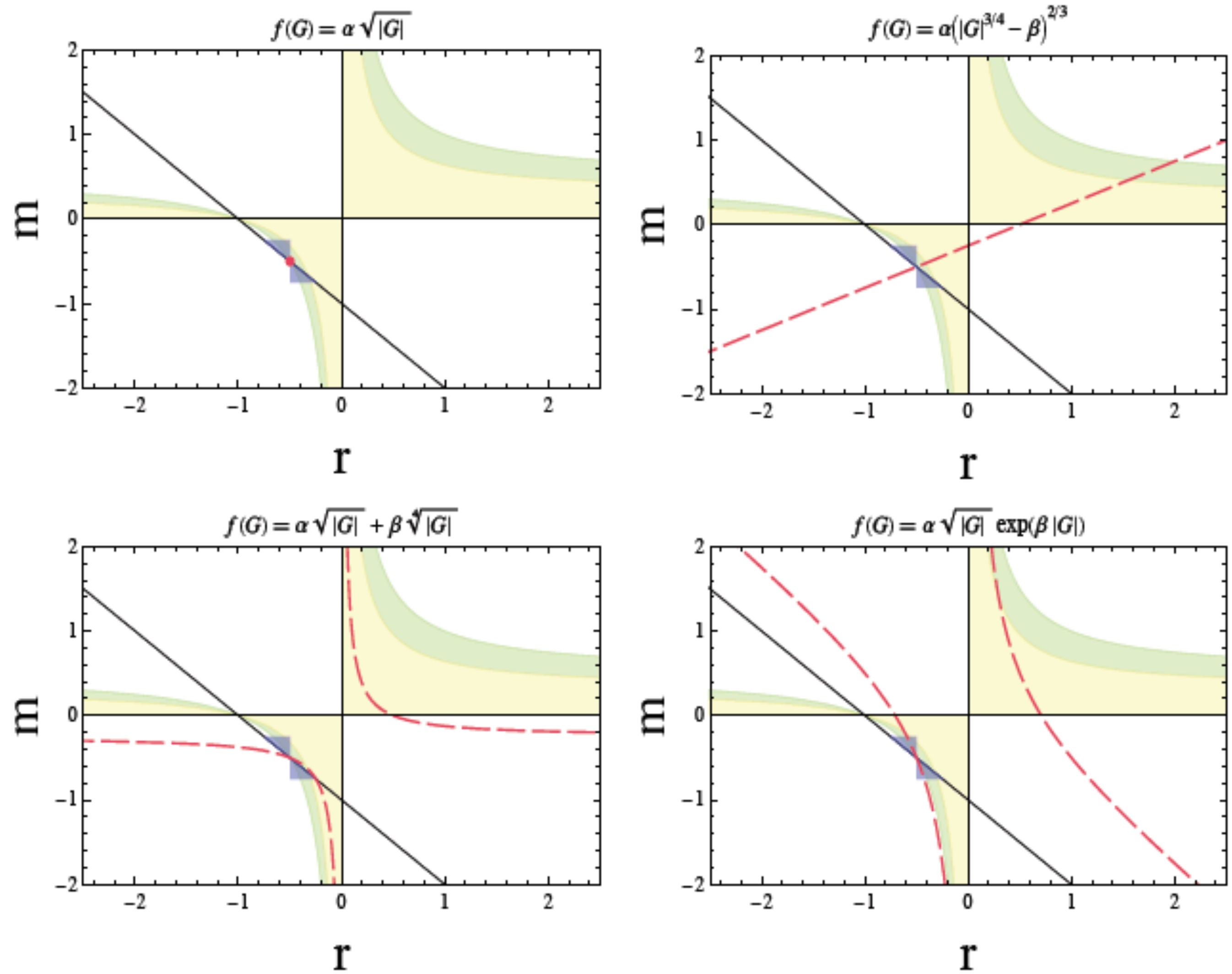
Toy Models

Cosmologically Viable Trajectory

Radiation dominated point $\mathcal{L}_4 \to$ Standard matter point $\mathcal{L}_2 \to$ Stable de Sitter point $\mathcal{L}_1$ or Stable phantom-like point $\mathcal{L}_3$

Toy models for which $m(r)$ can be analytically obtained:

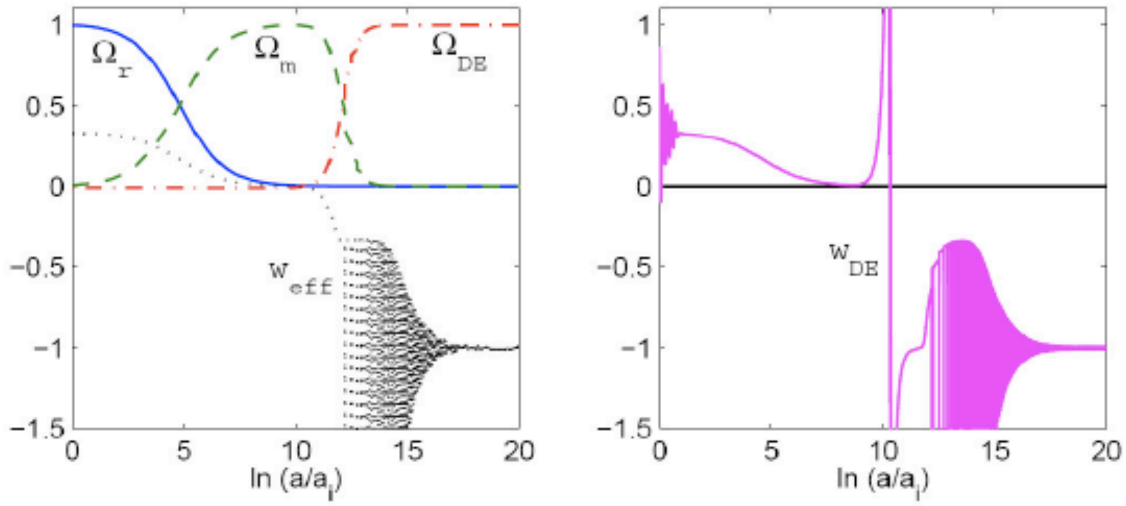
$$
\begin{aligned}
\mathbf{f(G)} &: \quad \mathbf{m(r)} \\
\alpha G^n &: \quad n-1 \\
\alpha(G^p-\beta)^q &: \quad (1-q)/qr+p-1 \\
\alpha G^p+\beta G^q &: \quad p+q-1+pq/r \\
\alpha G^p \exp(\beta G) &: \quad -r+p/r \\
\alpha G^p \exp(\beta/G) &: \quad -r-p/r-2 \\
\alpha G^p\,[\ln(\beta G)]^q &: \quad A(r,p,q)/qr
\end{aligned}
$$



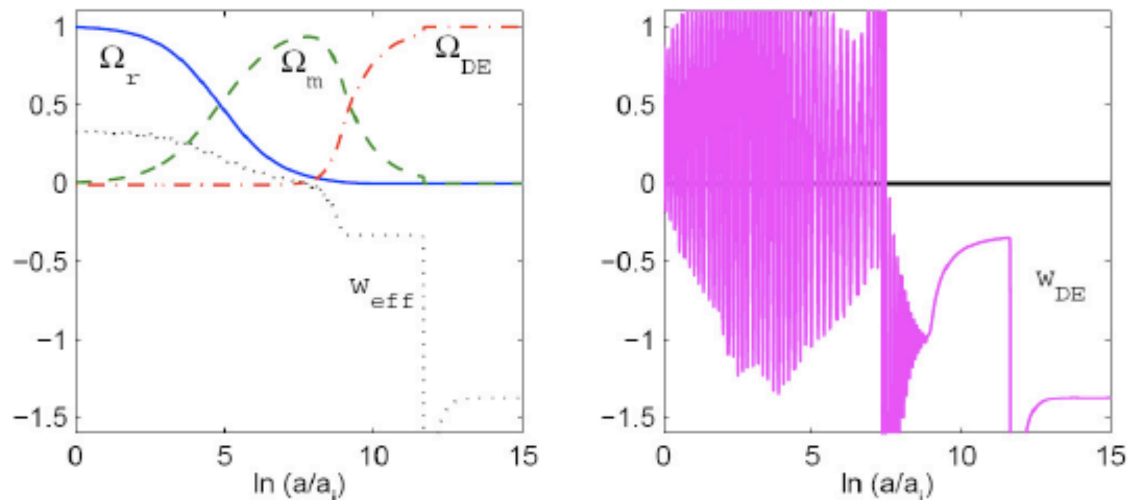
$f(G) = \alpha\sqrt{|G|}$
$f(G) = \alpha\left(|G|^{3/4} - \beta\right)^{2/3}$
$f(G) = \alpha\sqrt{|G|} + \beta\sqrt[4]{|G|}$
$f(G) = \alpha\sqrt{|G|}\,\exp(\beta\,|G|)$
m
r

Successful Trajectories: $f(G) = \alpha(G^{3/4} - \beta)^{2/3}$

de Sitter DE:



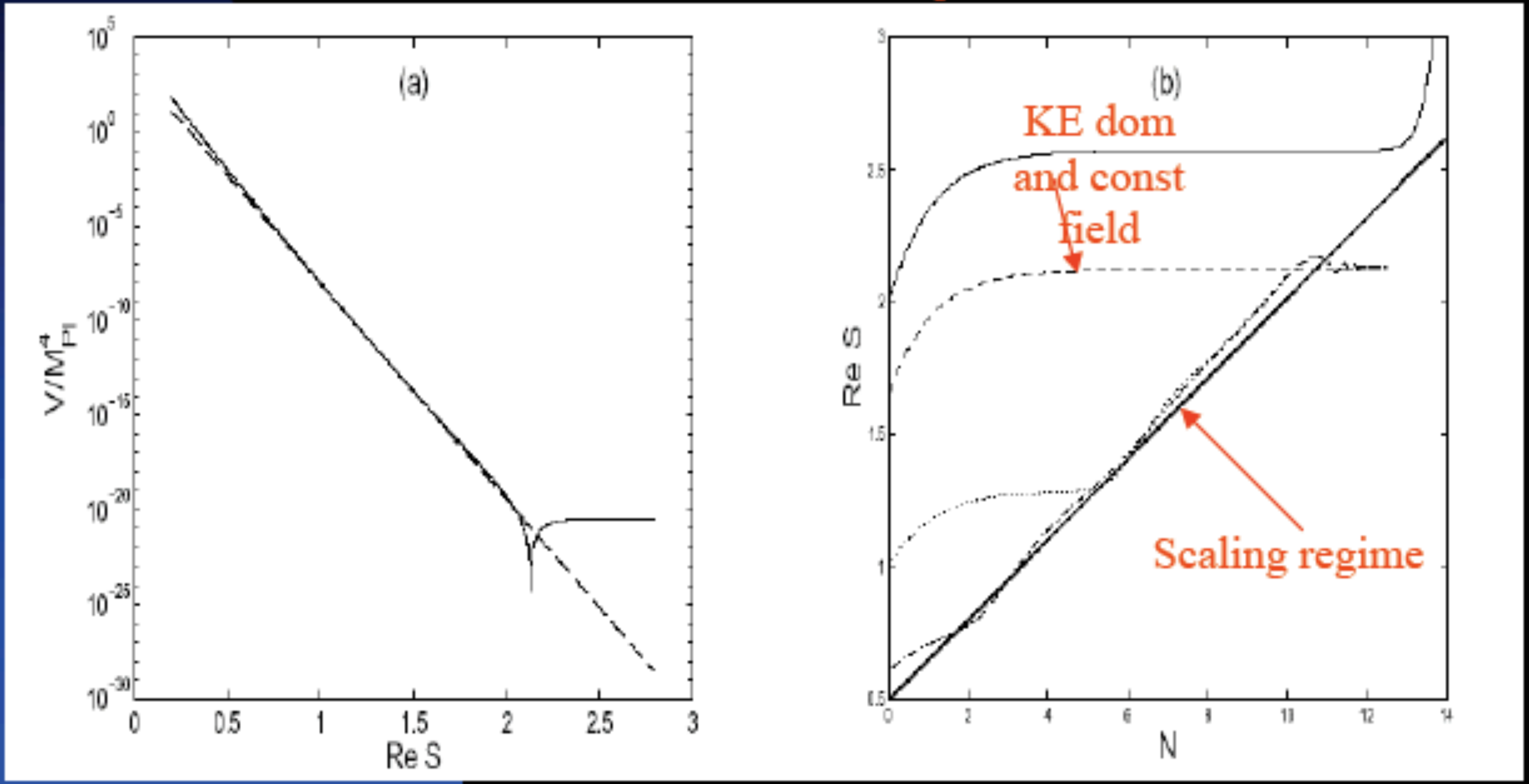
Phantom-like DE:



Useful way of stabilising moduli in string cosmology. Sources provide extra friction when potentials steep.

Barreiro, de Carlos and EC : hep/th-9805005

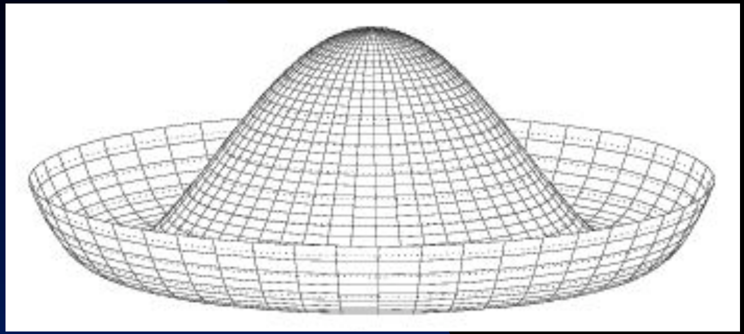
Brustein, Alwis and Martins : hep-th/0408160



Two condensate model with $V \sim e^{-a\,\mathrm{Re}S}$ as approach minima

Barreiro et al : hep-th/0506045

3. Original cosmic strings, in gauge theory :



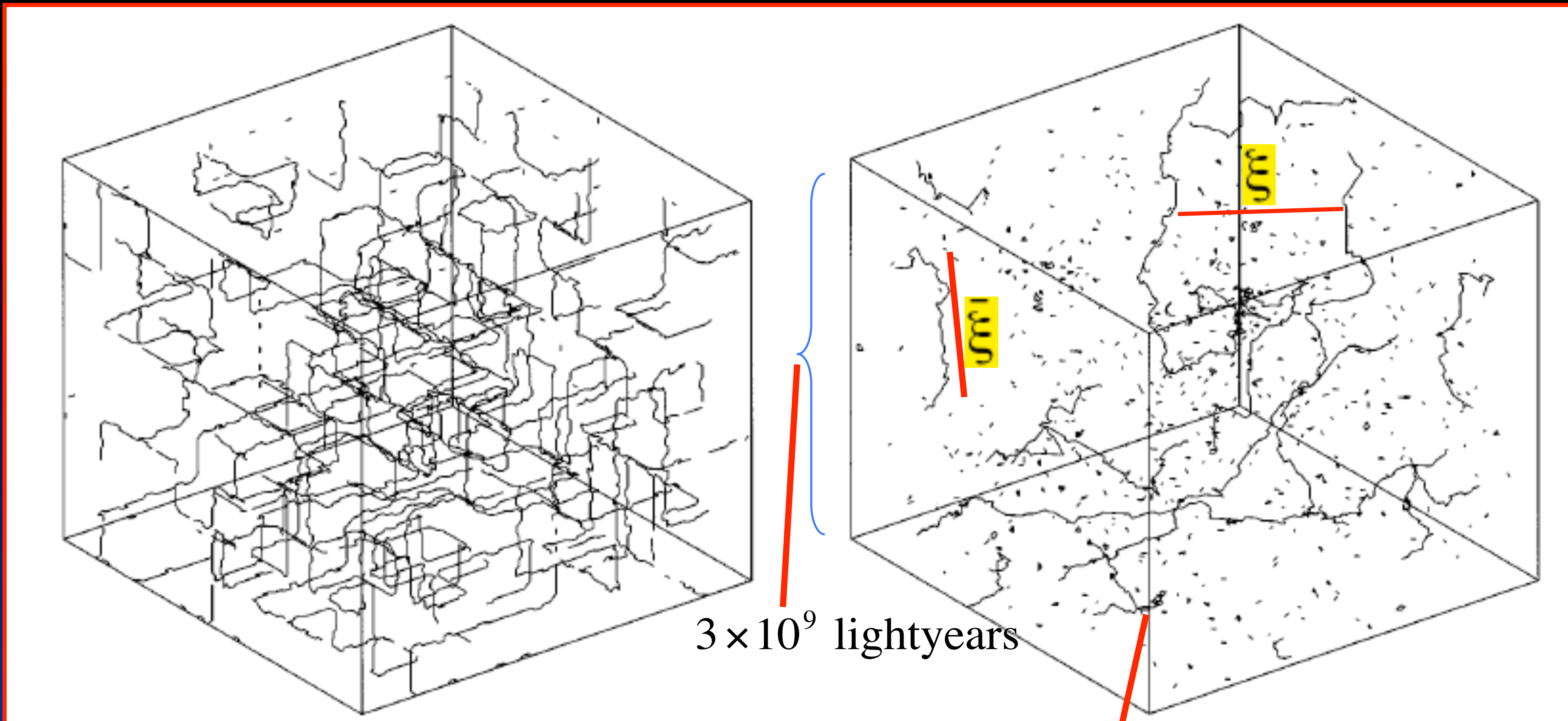
Spontaneously broken $U(1)$ symmetry, has magnetic flux tube solutions (Nielsen-Oleson vortices).

Network would form in early universe phase transitions where $U(1)$ symmetry *becomes* broken. Higgs field roles down the potential in different directions in different regions (Kibble 76).

String tension : μ Dimensionless coupling to gravity : G μ
GUT scale strings : G $\mu \sim 10^{-6}$ -- size of string induced metric perturbations.

Length scales on networks

[Vincent et al]



Initial

Scaling

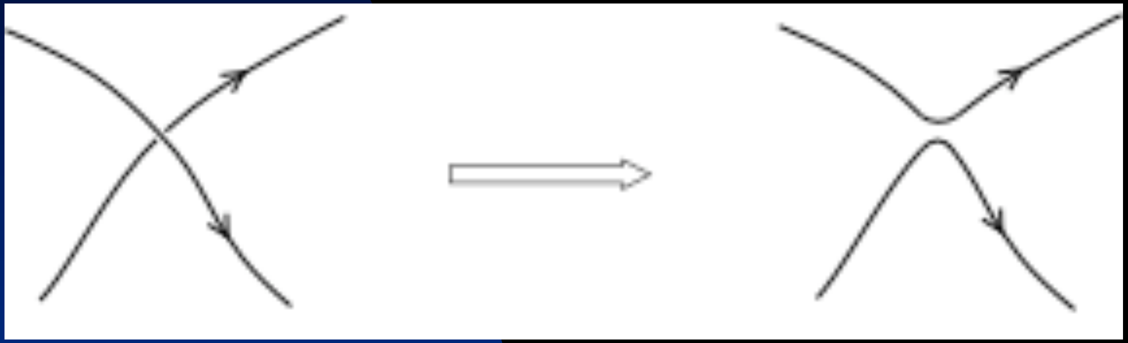
$\bar{\xi}$ - persistence length of string

ξ - interstring distance

ζ - small scale structure on network

Observational consequences : 1980's and 90's

Single string networks evolve with Nambu-Goto action, decaying primarily by forming loops through intercommutation and emitting gravitational radiation and possibly particles.



For gauge strings, reconnection probability P~1

Scaling solutions are reached where energy density in strings reaches constant fraction of background energy density:

$$\rho_{string}/\rho_{rad} \sim 400G\mu$$

$$\rho_{string}/\rho_{mat} \sim 60G\mu$$

[Albrecht &Turok; Bennett & Bouchet; Allen & Shellard]

Density increases as P decreases because takes longer for network to lose energy to loops. Recent re-analysis of loop production mechanisms suggest two distributions of long and small loops.

Single one-scale model: (Kibble + many...)

Infinite string density $\rho = \frac{\mu}{L^2}$

$$\dot{\rho} = -2\frac{\dot{a}}{a}\rho - \frac{\rho}{L}$$

Expansion   Loss to loops

Correlation length $L(t) = \xi(t)t, \quad a(t) \sim t^{\beta}$ Scale factor

$$\frac{\dot{\xi}}{\xi} = \frac{1}{2t}\left(2(\beta - 1) + \frac{1}{\xi}\right)$$

Scaling solution $\xi = [2(1-\beta)]^{-1}.$

Need this to understand the behaviour with the CMB.

Velocity dependent model: (Shellard and Martin)

$$\dot{\rho} = -2\frac{\dot{a}}{a}(1+v^2)\rho - \frac{\tilde{c}\,v\,\rho}{L},$$

RMS vel of segments

$$\dot{v} = (1-v^2)\left(\frac{k}{L} - 2\frac{\dot{a}}{a}v\right)$$

Curvature type term encoding small scale structure

$$k = \frac{2\sqrt{2}}{\pi}\left(\frac{1-8v^6}{1+8v^6}\right)$$

$$\xi^2 = \frac{k(k+\tilde{c})}{4\beta(1-\beta)}, \qquad v^2 = \frac{k(1-\beta)}{\beta(k+\tilde{c})}$$

Both correlation length and velocity scale

Multi tension string network: (Avgoustidis & Shellard 08, Avgoustidis & EJC 10)

$$\dot{\rho}_i = -2\frac{\dot{a}}{a}(1+v_i^2)\rho_i - \frac{c_i v_i \rho_i}{L_i} - \sum_{a,k} \frac{d_{ia}^k \bar{v}_{ia} \mu_i \ell_{ia}^k(t)}{L_a^2 L_i^2} + \sum_{b,\, a\leq b} \frac{d_{ab}^i \bar{v}_{ab} \mu_i \ell_{ab}^i(t)}{L_a^2 L_b^2}$$

Expansion

Loop of \`i' string

Segment of \`i' collides with \`a' to form segment \`k' -- removes energy

Segment of \`i' forms from collision of \`a' and \`b' -- adds energy

$$\dot{v}_i = (1-v_i^2)\left[\frac{k_i}{L_i} - 2\frac{\dot{a}}{a}v_i + \sum_{b,\, a\leq b} b_{ab}^i \frac{\bar{v}_{ab}}{v_i} \frac{(\mu_a + \mu_b - \mu_i)}{\mu_i} \frac{\ell_{ab}^i(t) L_i^2}{L_a^2 L_b^2}\right]$$

$$v_{ab} = \sqrt{v_a^2 + v_b^2} \qquad \mu_i \equiv \mu_{(p_i,q_i)} = \frac{\mu_F}{g_s}\sqrt{p_i^2 g_s^2 + q_i^2} \qquad \rho_i = \frac{\mu_i}{L_i^2}$$

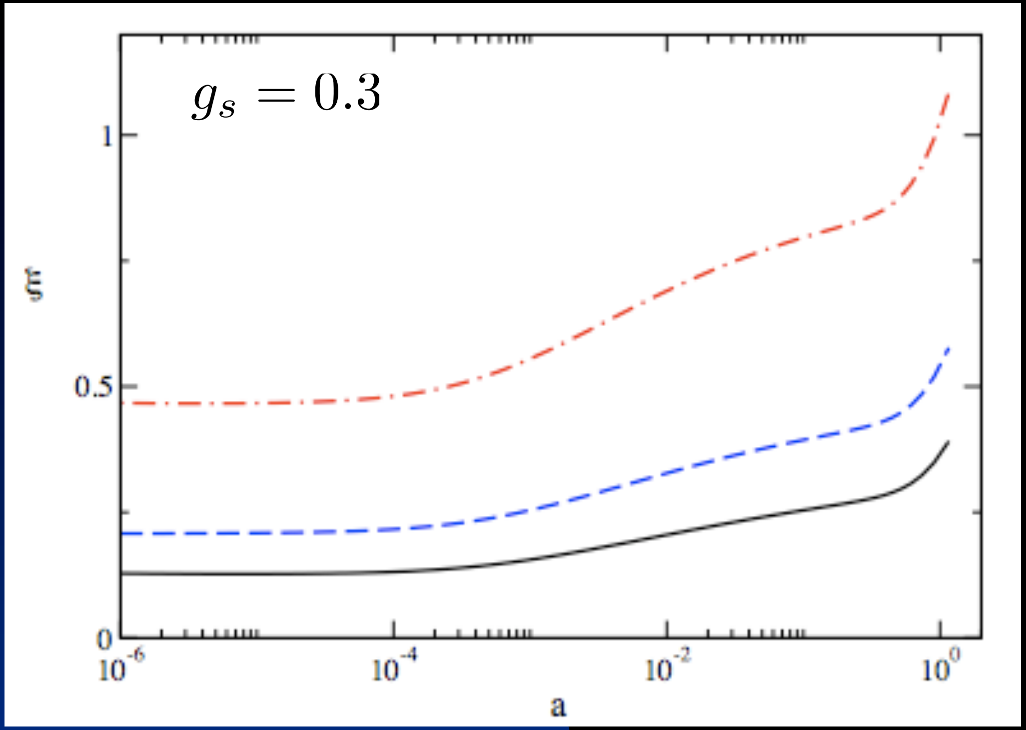
\`k' segment length

$$\ell_{ij}^k = \frac{L_i L_j}{L_i + L_j}$$

d_{ia}^k incorporate the probabilities of intercommuting and the kinetic constraints. They have a strong dependence on the string coupling g_s and we are still getting to the bottom of that dependence -- not easy !

$$\{(p,q)_i\} = \{(1,0),(0,1),(1,1),(1,2),(2,1),(1,3),(3,1)\}\ , \quad (i=1,...,7)$$

Pourtsidou, Avgoustidis, EJC, Pogosian and Steer



Example - 7 types of (p,q) string. Only first three lightest shown - scaling rapidly reached in rad and matter.

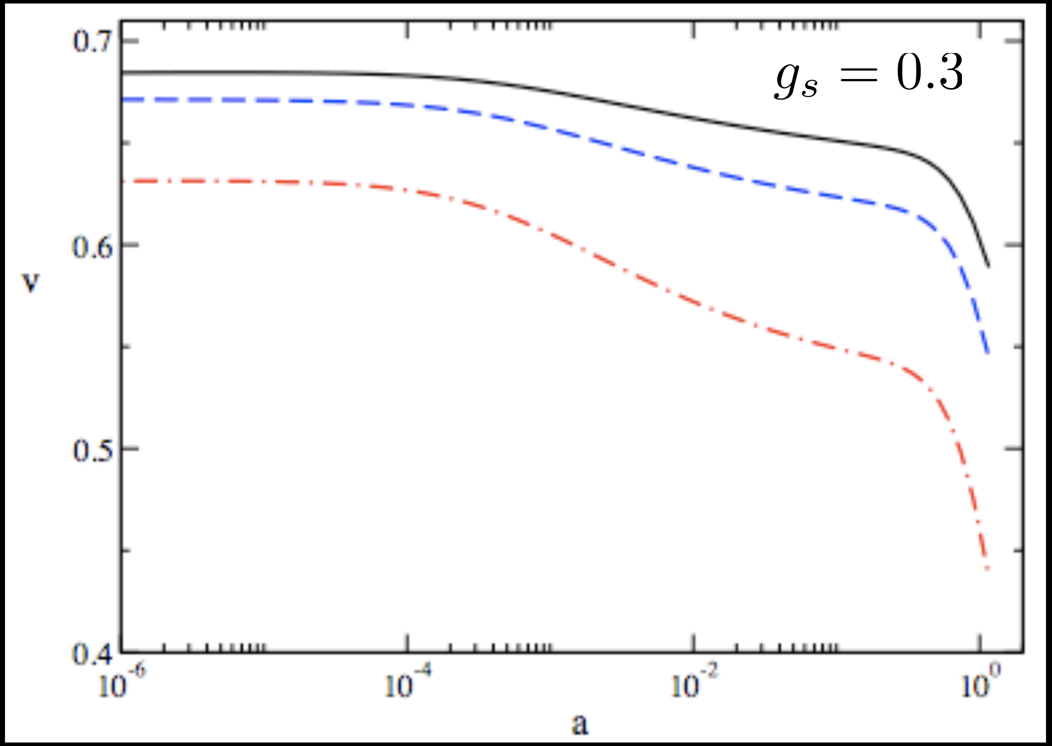
Densities of rest suppressed.

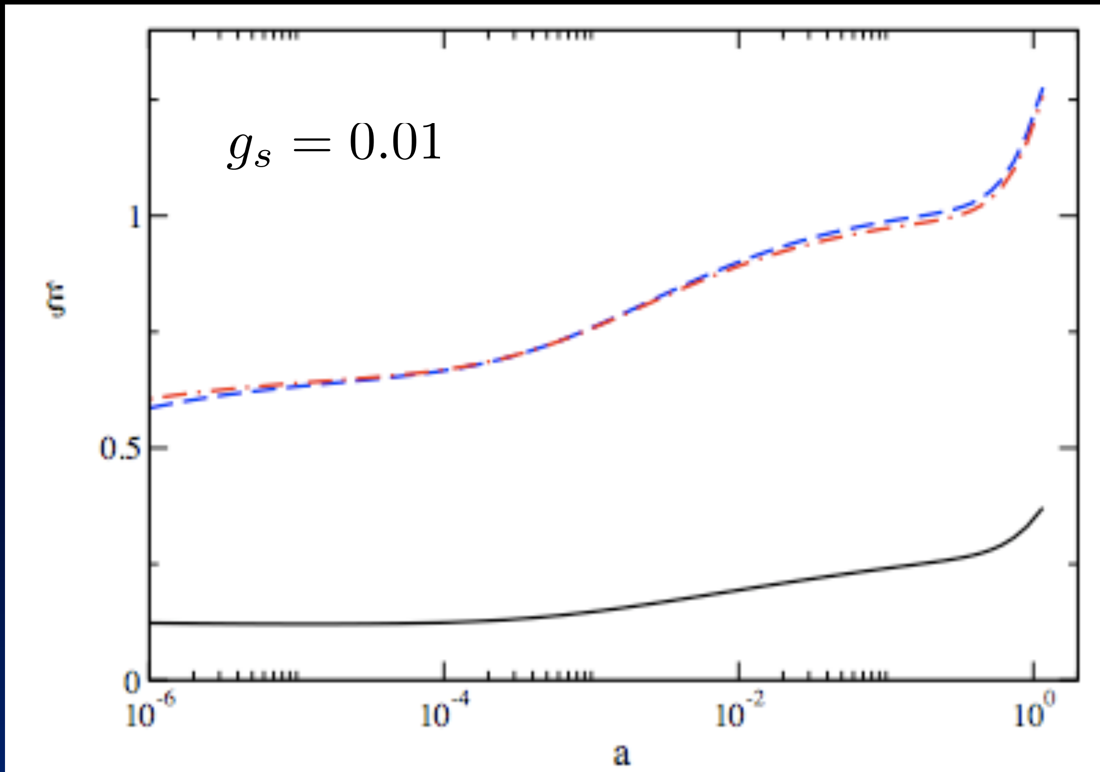
Black -- (1,0) -- Most populous
Blue dash -- (0,1)
Red dot dash -- (1,1)

Deviation from scaling at end as move into Λ domination.

Velocities of first three most populous strings:

F and D strings dominate both the number density and the energy density for larger values of g_s=0.3 - 1





As before for correlation lengths but now with g_s=0.01

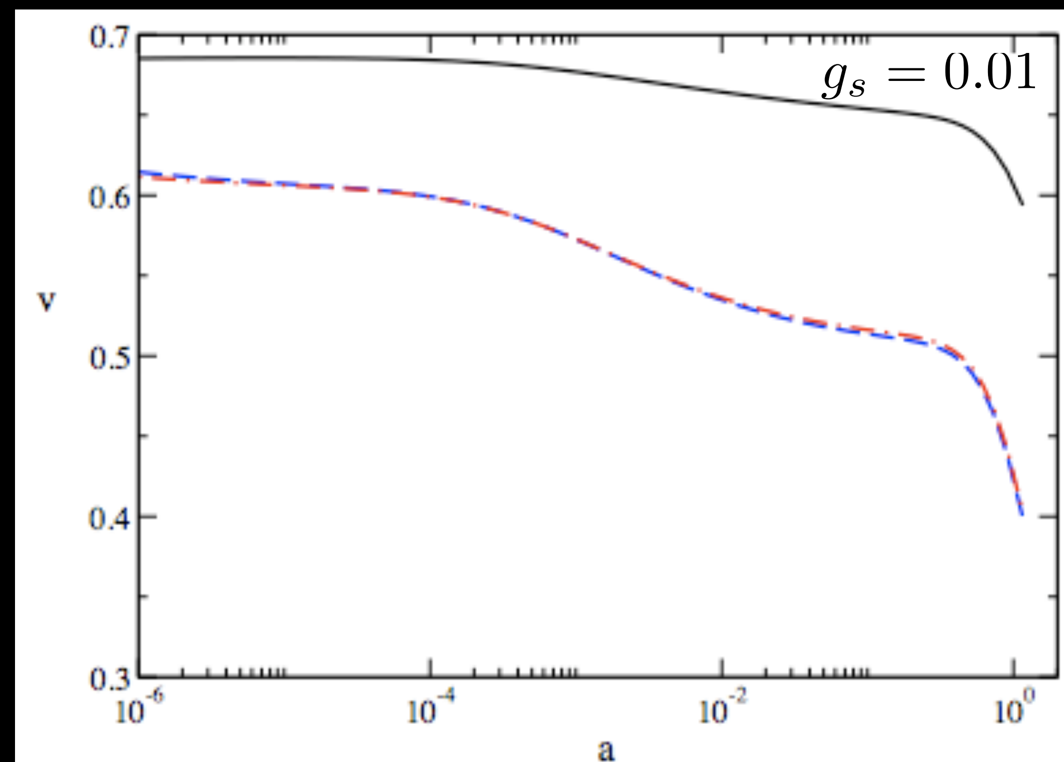
Black -- (1,0) -- Most populous
Blue dash -- (0,1)
Red dot dash -- (1,1)

Note (0,1) and (1,1) almost identical because tensions so similar. Note also F string has much larger number density, where as heavier D string (100 times here) is less common. Same is true for (F,D) string, so now have two heavy and one light string.

As before for velocities but now with g_s=0.01

Now have situation where energy density of network is dominated by the heavier and rare D and (F,D) strings even though the light F string is more populous. This is in contrast to previous case.

This impacts on position of B-mode peak in CMB.



These scaling solutions feed into the CMB codes to obtain the power spectrum of strings.

Conclusions

1. Scaling behaviour can be found in many systems in cosmology as well as many other areas of science.

2. This opens up the possibility of a phase space description of the system of interest.

3. It allows us to analyse the system by looking for the fixed points and discovering their stability even though we may not have the full analytic solutions for the systems.

4. In doing so it allows us to determine analytically the late time behaviour, the attractor solutions, which is often what we are after.

And so where are we today?

- Exciting time in cosmology -- Big Bang huge success.
- String - theory suggests we can consistently include gravity into particle physics.
- What started the big bang ?
- How did inflation emerge – if at all ?
- How did the spacetime dimensions split up?
- Where did the particle masses come from?
- Why are there just three families of particles?
- Why is the Universe accelerating today?
- What is the dark matter
- Where is all the anti-matter?

Thank you for listening and good luck to you all with your research.