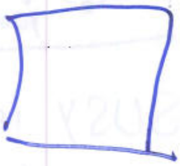
String Theory (A. Hanany) lec. 1 15/01/08 ①

Books: GSW I, II   Zwiebach
Polchinski I, II B^2S

3 perspectives

1. worldsheet



$\Sigma \rightarrow$ spacetime
↑ Field theories on world sheet
↑ conformal

2. spacetime
26 (bosonic)
10 (super)
11 (M-theory)
anything (non-critical)

Effective actions of SUGRA in 10, 11, 26 d

only massless modes considered, neglect the tower of massive modes

Classical sol$^{\underline{n}}$s
Black holes
branes (extended objects in various dim)

massless modes control the dynamic of the system

3. Brane
↑ QFTs in various dim. (1+3 dim - Poincaré inv$^{\underline{t}}$)
with no gravity

Supersymmetry

no tachyons ← instability of the theory

Know much more how to do:

non-renormalisation

-ve 2^{nd} derivative
⇒ negative mass
⇒ tachyon

Maximal SUSY

32 supercharges ⟷ $\mathcal{N}=8$ SUSY in 4d.
(maximal SUSY)

total # that spinors have.

$\{Q_a, Q_b\} = E$ closed, finitely generated.

rep. supermultiplets

16

8

→ 4 ...... ⟷ $\mathcal{N}=1$ SUSY in 4d.

2 ⟷ $\mathcal{N}=1$ SUSY in 3d

1

(Super)string theories?

5 superstring theories in 10d

1 in 11d (no strings) M-theory.

32 supercharges M-theory

massless fields graviton $g_{\mu\nu}$ (traceless symmetric 2^{nd} rank tensor)

spin-2 meaningful only 4d (misnomer)

How many propagating d.o.f.?

in 4d 2 physical d.o.f.

$$0 = m^2 = p^2 = p_0^2 - \vec{p}^2$$

$$p = (p_0, p_1, \dots, p_{d-1}) \to (|p|, 0 \dots 0, p)$$

sig. $+, -, -, \dots -$

Little gp: subgp. of Lorentz gp. that preserves momentum

$SO(d-2)$

irreps of little group counts # d.o.f.

String Theory (lec. 2) 16/01/08

Next week: Tue 4-5, no Wed
No Tue 23, W 30
T Feb 5 4-5

1.3 perspectives

1. world sheet 2d CFT
2. spacetime, effective low energy actions (contain gravity)
3. brane QFT, effective (light modes) no gravity

SUSY

1. no tachyons
2. hierarchy (Hanany doesn't take this pt. to be a motivation)
3. We know how to calculate things

1+5 theories

11d (no string) — world sheet observers can't get this theory

10d — can be done on worldsheet

Let us start with M-theory:

M-theory

- no microscopic description
- not many computations one can do

supersymmetric 32 supercharges

polarisation, physical d.o.f., propagating d.o.f.

$g_{\mu\nu}$ Little gp.

Weinberg: QFT I, II, III

count # pol. according to irrep. of $SO(d-2)$.

$d=5$: littlegp $\rightarrow SO(3)$

$\frac{1}{2}(3)(4)-1=5$

Vector rep. 3 $\qquad 3\otimes 3 = \underset{\text{tr}}{1} \oplus 3 \oplus \underset{\text{trless symmetric}}{\boxed{5}}$

any d: $\dfrac{(d-2)(d-1)}{2}-1$

$= \frac{1}{2}d(d-3)$.

$d=11$: $\frac{11\times 8}{2}=44$

$d=3$: 0 (in 3 dim, when we have gravity, gravitons doesn't propagate
- just static background
- Topological QFT).

in $d<3$ -ve d.o.f.

$\Rightarrow$ ~~take so~~ gauge invts take same d.o.fs from matter.

gauge field 1-form

# of d.o.f. is in vector rep of $SO(d-2)$, which is $d-2$.

gauge theory in $d=2$ has no propagation.

In $d=1$, $-$ve d.o.f. $\underbrace{\text{n scalar + 1 gauge field}}_{\text{n-1 d.o.f's.}}$

In $d=3$, # d.o.f. gauge field = # d.o.f. scalar field } they describe the same physics

scalar $\xleftrightarrow{\text{dual}}$ gauge field.

in 4 dim we have just these.

But for $d>4$, we have other gauge fields

p-forms p-th rank antisym. tensors

2-form, 3-form

B C

in 11d $C^{(3)}$ 3-form

pol. $= \frac{(d-2)(d-3)(d-4)}{6}$

p-th rank antisymm. tensor of $SO(n)$ has dim. $\binom{n}{p}$.

3-form: $d=11$: $\frac{9\times8\times7}{6} = 84$

$Q^2=0$

$Q|0\rangle$, $|0\rangle$

$\boxed{\text{In } d=11}$ graviton 44 + 3-form 84 $= 128 = \boxed{2^7} \to$ supercharge can only present or not

That's how SUSY selects the no. of dimensions, and which fields to work with.

In conclusion, we have

$$\boxed{g_{\mu\nu} \, , \; C^{(3)}}$$

spinor ψ_μ – gravitino (Rarita-Schwinger in 4d) $\Big\}$ 11d: 128.

11d sugra is the low energy limit of M-theory, only massless fields

3-form has $\dfrac{(d-2)(d-3)(d-4)}{6}$ d.o.f.
graviton has

11d sugra has only 3 fields: $\boxed{g_{\mu\nu}, C^{(3)}, \psi_\mu}$

Keep lowest orders of derivatives in the action

5 superstrings

	IIA	IIB	I	Heterotic $\left< \begin{matrix} SO(32) \\ E_8 \times E_8 \end{matrix} \right\}$ choice of gauge gps.
super charges:	32	32	16	16 ~~xxx~~

(32 is minimal amount in 11d)

$\boxed{IIA}$ **bosonic**

ϕ, A, $C^{(3)}$, B, g

dilaton, 1-form, 3-form, 2-form, graviton

$II = 2 \times 16$ SUSY Q's

We expect sum 128 dofs, know we have 32 supercharges

$$\left.\begin{aligned} \# g &: \frac{d(d-3)}{2} = 35 \\ \# C^{(3)} &: \frac{8\cdot7\cdot6}{6} = 56 \\ \# B &: \frac{8\cdot7}{2} = 28 \\ \# A &: 8 \\ \# \phi &= 1 \end{aligned}\right\} \text{sum } 128.$$

This content looks like 1 comp from 11 d

Look at: 11d $g_{\mu\nu}$

Let $a = 1, \dots, 10$

g_{ab} identify as graviton in 10d	$g_{a,11}$ 1 form in 10d	$g_{11,11}$ scalar.

So, IIA g, A, and ϕ can combine into the 11d graviton; the # of d.o.f. match.

now the 3-form, decompose the 11d 3-form:

$$C^{11d}_{\mu\nu\lambda} : \quad \underset{\displaystyle C^{(3)}_{IIA}}{\underset{\|}{C_{abc}}}, \quad \underset{\displaystyle B_{(IIA)}}{\underset{\|}{C_{ab,11}}}$$

again Dof's match.

One can do a similar analysis for fermions

<u>fermions</u> ψ_c, ψ_s dim 8 (one spin index normal spinor in 10d)

$\psi_{\mu c}, \psi_{\mu s}$ dim 56 (gravitino)

We characterise the fields according to reps of $SO(8)$, the little gp. of 10d.
Of course, $\exists$ natural 8d rep (vector), but also 2 spinor reps;

vector 8_v, spinor 8_s and complex conj. spinor 8_c.

Great. So, $\bar{\psi}_s = \psi_c$, now its time to talk about χrality - this only exists in <u>even</u> dim (otherwise $\gamma_5 = \pm 1$).

4d. γ_μ $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ (or $g_{\mu\nu}$)
For us, we will take Minkowski η
sig. $+ - - -$
The of course $\exists\ \gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$
γ_5 measures χrality of spinors.
Weyl spinors are e.spinors of γ_5: $\gamma_5\psi_\pm = \pm\psi_\pm$.

Now to any dimensions:
Γ_μ: $\{\Gamma_\mu, \Gamma_\nu\} = 2g_{\mu\nu}$
$\Gamma = \Gamma_0 \ldots \Gamma_{d-1}$ which is nontrivial for even d.
2 χralities in 10d, ψ_c & ψ_s
They are Weyl, but also Majorana
$\exists$ a convention where one would have
$\Gamma\psi_c = \psi_c$ or $\Gamma\psi_s = -\psi_s$
(or vice-versa)

Type IIA superstring is non-χral (no distinction between χralities)

[all interactns are sym. wrt. χrality exchange.]

8

Count # SUSY's by counting # gravitinos in supermultiplets

→ we have 1 left moving gravitino
& 1 right " "

So IIA is described as
(1,1) SUSY in 10d

[32 is max in 10 d, 16 is min in 10 d]

$\boxed{\text{IIB}}$ $\underbrace{\phi, B, g}_{\text{worldsheet talk "NS sector" (same for IIA)}}$, $\underbrace{C^{(0)}, C^{(2)}, C^{(4)+}}_{\text{"RR sector"}}$ only the self-dual parts

dofs: $\underbrace{1, 28, 35, 1, 28, 35}_{128}$

When you have a 'middle dimensionful' object, say $C^{(4)}$ in 8d

$*F_5 = *dC^{(4)} = G_5$

$C^{(4)}$ separates into self dual
antiself dual

each with $\frac{8 \cdot 7 \cdot 6 \cdot 5}{24 \cdot 2} = 35$

(so have to pick 1 for the 128)

String Theory (lec. 3) 22/01/08 ①

Lectures on week of Feb 4			
T	5	4-5	5-6?
W	6	10-12	
Th	7		10-11? 12-1?
F	8		4-5?

Cancelled:
W 23 10-12
3 of week of Jan 27

1+5 supersymmetric theories

11d sugra (low energy limit of M theory)

10d theories Type IIA, IIB 32 SUSY's
Type I, Heterotic SO(32), $E_8 \times E_8$

11 d sugra: minimal SUSY in 11d
↳ smallest spinor rep which has 32 dims.

Sugra multiplet (massless) $g_{\mu\nu}$, ψ_μ, $C^{(3)}$
44 128 84

Effective action: Poincaré inv.
low # derivatives
supersymmetric.
} give a unique effective action.

(observer in spacetime describe the system using this effective action) all particles appear in LH and RH (2)

Type IIA: non-χral

χrality operator $\Gamma = \Gamma_0 \dots \Gamma_{11}$ $\quad \Gamma\psi_+ = +\psi_+$ $\quad \Gamma\psi_- = -\psi_-$

$\{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu}$ $\quad \eta_{\mu\nu} = (1, -1, \dots, -1)$

minimal SUSY in 10d is $N=1$ (16 supercharges)

Type IIB is χral (2,0) SUSY in 10d.

Type IIA

massless sugra multiplet

bosonic: $\underbrace{g,\ B,\ \phi}_{\text{NS sector}}$ $\quad$ $\underbrace{C^{(1)},\ C^{(3)}}_{\text{RR}}$

35 28 1 $\quad$ 8 56

fermionic: ψ_+ ψ_- ψ_μ^+ ψ_μ^-

8 8 56 56

Type IIB

bosonic: g, B, ϕ $\quad$ $C^{(0)}$, $C^{(2)}$, $C^{(4)}$

35 28 1 $\quad$ 1 28 35

can rotate between these two-forms

fermionic: $2\psi_+$ $\quad$ $2\times\psi_\mu^-$

2×8 $\quad$ 2×56

Type I sugra multiplet ($m=0$)

$$\begin{array}{ccc} g & B & \phi \\ 35 & 28 & 1 \end{array} \qquad \begin{array}{cc} \psi_+ & \psi_\mu^- \\ 8 & 56 \end{array}$$

Vector multiplet ($m=0$)

$$\begin{array}{cc} A_\mu, & \psi \\ 8 & 8 \end{array}$$ (Majorana-Weyl fermion)

in 2D we can have Majorana & Weyl.

U(1) gauge group in 10d

SYM theories with gauge gp. G.

A_μ^a, ψ^a $\quad a=1,\dots,\dim G$.

Type 1 has 1 gravity multiplet (G-plet)

& 1 Vector multiple in adj. rep. of SO(32).

$\dim SO(32) = 16\cdot 31 = 496$.

Same massless sector of Heterotic SO(32)

Heterotic $E_8 \times E_8$ has $2\times \underbrace{248}_{\text{dim. of } E_8} = 496$.

*Anomalies (gravitational anomalies) force # vector multiplets to be 496. in 10d.

SUSY + $\exists$ of gravitons $\Rightarrow$ 1+5 theories.

<u>Branes</u>: solutions of e.o.m.

carry changes w.r.t. to generalised gauge fields.

in 4d: A, $F = dA$

$$dF = \delta^{(3)} \quad \text{mag. mon.}$$
$$d{*F} = \delta^{(3)} \quad \text{elec. object}$$

$${*F} = \tilde{F}_{\mu\nu}\, dx^\mu \wedge dx^\nu, \quad \tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\lambda\sigma} F^{\lambda\sigma}$$

F has degree n

${*F}$ has degree $D-n$

$dF = \delta^{(3)}$ $\to$ has degree 3

$$\delta^{(n)} = \delta(x)\, dx_1 \ldots dx_n.$$

10 dim = 1 time + 9 space

IIA

$C^{(1)}$ in 10d: $F^{(2)} = dC^{(1)}$, $dF^{(2)} = \delta^{(3)}$ magnetic $\to$ $\boxed{\text{6-brane}}$

$dF^{(2)} = \delta^{(9)}$ electric $\to$ $\boxed{\text{0-brane}}$

$C^{(3)}$ in 10d: $F^{(4)} = dC^{(3)}$, $dF^{(4)} = \delta^{(5)}$ magnetic $\to$ $\boxed{\text{4-brane}}$

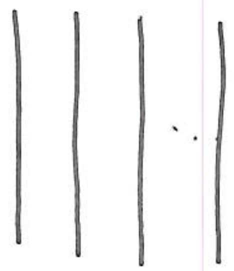
$d{*F^{(4)}} = \delta^{(7)}$ electric $\to$ $\boxed{\text{2-brane}}$

B: $H = dB$ $\quad dH = \delta^{(4)}$ 5-brane.

$d{*H} = \delta^{(8)}$ string (fundamental).

fundmental string = string coupled to electric field

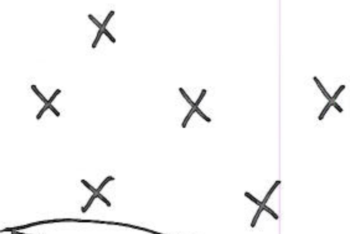
String Theory (4/02/08) ①



Moduli space of vacuum config.

Space of vacuum states
$\langle \phi \rangle$ — modulus
dimension $(9-p)N$,
denote $\vec{\phi}_i \in (9-p)$-vector

N Dp-branes



Energy of system indep. of position
→ configuration space of vacuum states all of same energy

$U(1)^N \quad |\vec{\phi}_i - \vec{\phi}_j| \neq 0 \qquad i = 1, \ldots, N$
generic point on moduli space $U(1)^N$

$\vec{\phi}_i = \vec{\phi}_j, \quad U(2) \times U(1)^{N-2}$ rank N

$U(2) \supset U(1)^2$
$U(N) \supset U(1)^N$
maximal commuting subgroup

rank is # of commuting element

$\vec{\phi}_1 = \vec{\phi}_2 = \vec{\phi}_3 \qquad U(3) \times U(1)^{N-3}$ rank N

$$\mathbb{R}^{N(9-p)}/S_N = \mathbb{R}^{9-p} \times \frac{\mathbb{R}^{(N-1)(9-p)}}{S_N} = \mathcal{M}$$

in singular pts of moduli space: gauge enhancement
fixed point of the symmetric gp. S_N
— when branes collide, the symmetric gp. fixes permutation.

When all branes collide

$\vec{\phi}_i = \vec{\phi}$, $U(N)$

enhancement: $U(1)^N \longrightarrow U(N)$

$V = m^2\phi^2$

If the scalar field is massive, then the mass forces $\langle\phi\rangle = 0$.

String theory: Type IIA, ϕ dilaton

$\mathcal{M}$ parametrised by e^{ϕ}

later, identify with string coupling g_s

moduli space $= \mathbb{R}^+$

Type IIB: Two scalars $\phi, C^{(0)}$

$$\tau = \frac{i}{e^{\phi}} + C^{(0)}$$

What's the moduli space of type IIB string?

Answer: $SO(2,1)/SO(2)$

gp. m'fold parametrised by 3 scalars

subject to 1 relation

with no brane

M-Theory: no scalars ← gravity multiplet contains no scalar.

(strongly coupled)

→ no perturbation theory

We mean M-theory in flat space - no other object

If we add branes, there are a lot of scalars.

10d theories 16 SUSY's

1 scalar $\mathbb{R}^+$.

M-theory: M2, M5

Type IIA: D0, D2, D4, D6, F1, NS 5

g_{MN}, $C^{(3)}$, M $\quad M,N=0,\dots,10$

$g_{\mu\nu}$, A_μ, ϕ, $C^{(3)}$, $B^{(2)}$ IIA $\quad \mu,\nu=0,\dots,9$

$g_{\mu\nu}$, $g_{\mu 10}$, $g_{10,10}$

$g_{\mu 10} \leftrightarrow A_\mu$, $\quad g_{10,10} \leftrightarrow \phi$

M-theory on $\mathbb{R}^{1,9} \times S^1_R$

$R \leftrightarrow g_s$

C_{MNP}

$C_{\mu\nu\rho}$, $C_{\mu\nu 10}$

$C^{(3)}$ $\quad B_{\mu\nu}$

String Theory 5/02/08

1

Week of Feb 4: T 4-5, 5-6
W 10-11, 11-12
Th 10-11, 12-1
F 4-5

Feb 11: M 12-1
T 4-5, 5-6
W 10-11, 11-12
Th 10-11, 12-1
F 4-5

Reminder:

5 superstrings in 10d: $B^{(2)}$, $C^{(0)}$, $C^{(2)}$, $C^{(4)+}$ IIB

1 M-theory in 11d: $C^{(3)}$ | $C^{(1)}$, $C^{(3)}$ IIA

Branes: electron is a source for E-field

$$d * F = \delta^{(3)} \quad \text{in } (3+1)d$$

$$dF = \delta^{(3)} \quad \text{in } (3+1) \text{ magnetic source.}$$

$C^{(4)}$; $F^{(5)}$ field strength

$*F^{(5)}$

$B^{(2)}$, $F^{(3)} = dB$ $\quad \underline{d * F^{(3)} = \delta^{(8)}}$ ← object localised in 8 dim

string is an electric source for $B^{(2)}$ in 10d.

$dF^{(3)} = \delta^{(4)}$ 5-brane.

<u>M-theory</u>: $C^{(3)} \to$ 5-brane M-object
$\to$ 2-brane E-object.

Any 10d theory $B \to$ 5-brane M-object
$B \to$ 1-brane E-object (string)

$\downarrow$

all of theory in 10d have B-field

$\Downarrow$

all of them have string $\to$ fundamental string

Observer on world sheet of string when quantised must get the spectrum it couples to (i.e. B-field) name of 2-form that couples electrically to string

If we're able to quantise 2-brane, we must have $C^{(3)}$.

<u>Type IIA</u>: $C^{(3)}, C^{(1)}$

$C^{(3)} \to$ 4-brane, $\to$ 2-brane

$C^{(1)} \to$ 6-brane, $\to$ 0-brane

In type IIA, we have even dim. branes

Type IIB : add dim. branes

$C^{(2)}$ → 5-brane, → string

$C^{(4)}$ → 3-brane, → 3-brane } 1 object: dyonic → source of electric & mag. field.

$C^{(0)}$ → 7-brane, → −1-brane − instanton (happen once & disappears)

$dC^{(0)} = F^{(1)}$

$d * F^{(1)} = \delta^{(10)}$ localised in space-time

$C^{(k)}$ in d dim.

$\underbrace{d * F^{(k+1)}}_{d-(k+1)+1} = \delta^{(d-k)} \longrightarrow$ brane is $d-(d-k)-1 = k-1$ (elec.)

$dF^{(k+1)} = \delta^{(k-2)} \longrightarrow (d-1)-(k+2) = d-k-3$ brane (mag.)

9-2-3

Heterotic string; Type I

$\underbrace{F^a_{\mu\nu}}$ $SO(32)$, $E_8 \times E_8$

non-abelian gauge fields

$\mathrm{Tr}(F \wedge F) = \delta^{(4)}$ (instanton in 3+1 dim)

an instanton in higher dim is an object localised in $\underline{\underline{4d}}$ (abuse of name)

↳ very different from the electric source of $C^{(0)}$ i.e. -1-brane.

10d; such an object is a 5-brane.

A brane can be localised in time (Euclidean brane)
← a bit like "instanton" - happen once in time, and disappears.

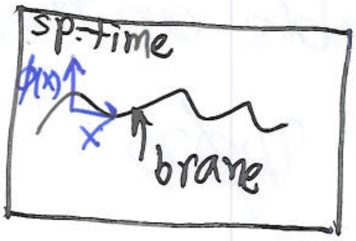
Euclidean 5-brane in M-theory 1...6 fills
localised; 0, 7, 8, 9, 10

Why not 2 times?

2 times-generators ⇒ losing unitarity

Spectrum of branes

common feature:
localised in space, embedded into space

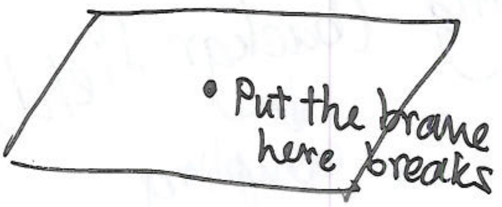


$\phi_i(x)$ — i runs over transverse sp.
μ runs over world vol. coords of brane.

$\boxed{X^\mu}$ brane — n scalars in $p+1$ dimensions.
p is $\underbrace{\text{dim.}}_{\text{space}}$ of brane.
$n = \#$ of localised directions.

Poincaré invce in $p+1$ dim.
$\Rightarrow$ QFT in $p+1$ dim on the brane.

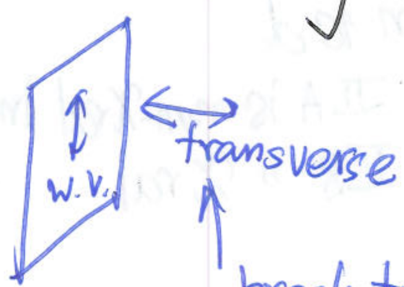
massless scalars



breaks translational invt.

Goldstone theorem:

n-massless scalars due to spontaneous breaking of translation invce in n direction.



break translation sym. in this dir.

scalar fields admit VEV

$\langle\phi\rangle$ position of brane in spacetime

So, all branes have scalar fields on them.

$$\sum_{i=1}^{n}\int d^{p+1}x\, \partial_\mu\phi_i\,\partial^\mu\phi_i + \psi\not{\phi}\psi$$

If SUSY broken,

massless fermions.

gauge fields (not all branes have fermions or gauge fields but all branes have scalar).

spacetime fields still live on the brane but are slowly moving (backgr. field)

$\downarrow$ coupling

Fundamental string in any of 10d theories

8 scalar fields in 1+1 dim.

Type IIA, IIB } 8 fermions | in 10d

IIA is non-χral in 1+1 χral

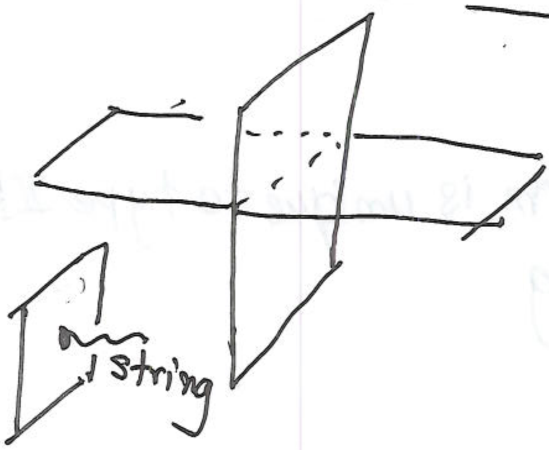
IIB " χral non-χral

Supersymmetry – understand supermultiplets in various dim.

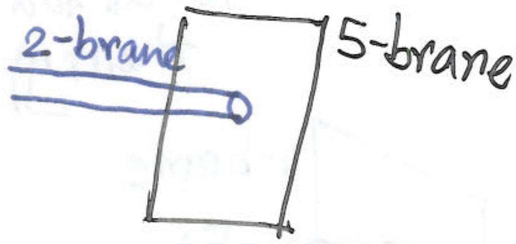
① massless supermultiplet with n-scalars in p+1 dims.

② Gauss law

Branes can end on branes.



M-theory 5-brane
2-brane



observer on brane see a point-like object as the string move.

Charged particle

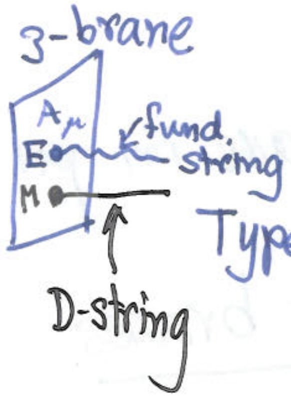
$$\int_{S^2} \vec{E}\cdot d\vec{A} = Q$$

$\boxed{\text{When a brane ends on another brane, gauss law implies the existence of gauge field}}$

string ends on brane → 1-form

$\underbrace{\text{2-brane}}_{\text{end of 2-brane is a string}}$ ends on 5-brane → 2-form on world-volume of 5-brane.

EM duality : should be magnetically charged object.



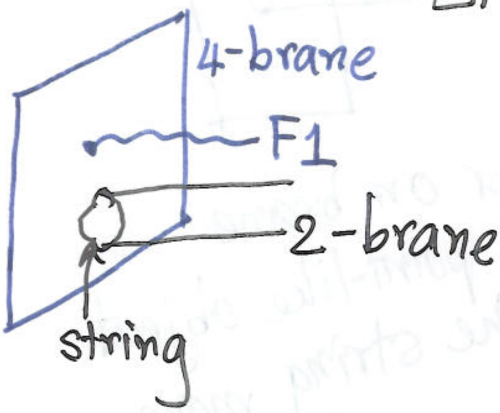
1-form dual to 1-form in 3+1 dim

Type IIB : add dim branes

B , $C^{(2)}$

B: Fund. string (universal for all 10d theory)

$C^{(2)}$: This 2-form is unique to type IIB D-string



$C^{(3)}$

Ex. derive all possible branes ending on branes.

String theory 6/02/08 1

Fundamental String B, $H=dB$

$$d*H=\delta^{(8)}Q\underbrace{\theta(x)}_{\text{Heaviside fn.}} \qquad d\theta=\delta \quad d\delta=0$$

Take differential of both side:

$$0=\delta^{(9)}Q.$$

String cannot end in spacetime unless its charge is zero; it must end on some thing!

$$d*_{10}H=\delta^{(8)}Q\,\theta(x)+\boxed{\overbrace{*_6F\wedge\delta^{(4)}}^{\text{to make it localise in the worldvol.}}\ \ *_6F \text{ is 4-form}} \rightarrow \text{interaction with the brane}$$

$$\Rightarrow\quad 0=\delta^{(9)}Q-d*_6F\,\delta^{(4)}$$

$$\Rightarrow\quad d*_6F=Q\delta^{(5)}$$

Q ← charge of the string

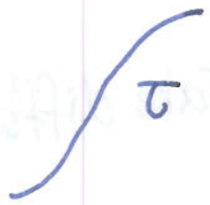
A source of electric field on the worldvol.

Necessary condition for a brane to end on another brane is existence of gauge field on the bigger brane that satisfies the gauss law

EM: gauge field A couples to a charge Q

$$Q\int_{W.V.} A = Q\int_{W.L.} A_\mu \frac{\partial x^\mu}{\partial \tau} d\tau = A_\mu J^\mu$$

W.V. is world line of the particle



$$J^\mu = Q\underbrace{\int_{W.L.} \frac{\partial x^\mu}{\partial \tau} d\tau}_{\substack{\text{velocity of the particle} \\ \text{along the worldline}}}$$

2-form couples to a string with charge Q

$$Q\int_{\substack{W.V. \\ \text{of string}}} B = Q\int B_{\mu\nu} \frac{dx^\mu}{d\sigma^1}\frac{dx^\nu}{d\sigma^2} d^2\sigma = B_{\mu\nu} J^{\mu\nu},$$

$J^{\mu\nu} = Q\int \frac{dx^\mu}{d\sigma^1}\frac{dx^\nu}{d\sigma^2} d^2\sigma$ is the current of the string.

In spacetime, this interaction looks like

$$Q\int d^{10}x \, B \, \delta^{(8)}$$

Kinetic term for H is $\int d^{10}x \, \underbrace{H \wedge * H}_{\text{generalisation of } F\wedge *F}$.

$B \wedge *_6 F \, \delta^{(4)}$ postulate an interaction between spacetime field B, w.v. field F

$$\int d^{10}x \Big(\underbrace{H \wedge *_{10} H}_{\text{kinetic term}} + Q B \delta^{(8)} - \underbrace{B \wedge *_6 F \delta^{(4)}}_{\text{interaction}} \Big)$$

When brane ends on another brane, there is an interaction term

$$\int C \wedge * G,$$

where C is a gauge field couples electrically to the small brane, G is a field strength on w.v. of larger brane.

M-theory M2 ends on M5 brane

expect $\int_{M5} d^6x \, C^{(3)} \wedge * G^{(3)}$.

$C^{(3)}$: M-theory 3-form.

$G^{(3)}$: field strength living on M5 brane

String dyonic in 6d.

D-brane is a brane over which a fundamental string ends (D = Dirichlet).

An observer living on the fundmental string will see 2 types: open, closed.

Open string: 2 b.c. N, D

N: end is free
D: end is fixed

N along w.v. directions
D " transverse directions.

Dp-brane has p+1 N b.c.
9-p D b.c.

Type I is an open string with D9 branes

Dp branes carry charge wrt. gauge fields (p+1)-forms.

Type IIA: $C^{(1)}$, $C^{(3)}$

E, E, M, M

D0, D2, D4, D6, D8

No object to coupling with but from the point of view of the observer it should exist

massive IIA

Type IIB: $C^{(0)}$, $C^{(2)}$, $C^{(4)}$ †

E, E, M, E/M, M

D(-1), D1, D3, D5, D7

dyonic

Every D-brane will carry a 1-form gauge field in their W.V. due to F-string ending on them

Dp brane breaks half of SUSY: 32 SUSY Q's

$$\downarrow$$

16 SUSY Q's

16 SUSY Q's preserved on WV of the brane

$\Rightarrow$ SUSY-gauge theory in p+1 dims with 16 supercharges.

So we need to come up with 8 fermionic dof's. These will be the gauginos living in the vector multiplet.

$\Rightarrow$ 8 gaugino d.o.f's in Fermions living in p+1 dimensions

There is a simple way of doing this in groups. Rep. theory of orthogonal gps $SO(9,1)$.

Symmetry: $\underset{\substack{\uparrow \\ \text{Lorentz on} \\ \text{spacetime}}}{SO(9,1)} \supset \underset{\substack{\uparrow \\ \text{Lorentz on} \\ \text{brane WV.}}}{SO(p,1)} \times \underset{\substack{\uparrow \\ \text{rotations in} \\ \text{remaining dims} \\ \text{transverse to brane}}}{SO(9-p)}$

brane breaks this rot$^{\underline{n}}$

How do our reps decompose?

$16 \longrightarrow$

$\overline{16} \longrightarrow$

an example ...

E.g., if $p=5$,

$$SO(9,1) \supset SO(5,1) \times SO(4)$$

$$16 \longrightarrow (4,4)$$

$$\underline{16} \longrightarrow (\underline{4},4)$$

[spinor rep $\quad SO(4) = SU(2) \times SU(2)$

$\quad 4 \longrightarrow (2,1) \oplus (1,2)$]

So really $\quad 16 \longrightarrow (4,2,1) \oplus (4,1,2)$

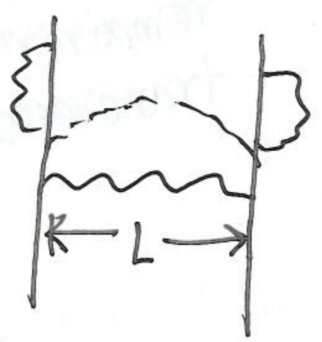
$$\underline{16} \longrightarrow (\underline{4},2,1) \oplus (\underline{4},1,2)$$

In IIB, there are 2 copies of 16 since it's chiral.

Surviving spinors are $(4,2,1) \oplus (4,1,2)$.

Now we know the field contents, what can we do with this?

- Consider 2 Dp-branes



Dp $\qquad$ Dp

When both ends are on the same Dp-brane, then we get the massless fields $A_\mu^{(a)}$ and $\phi_i^{(a)}$ $9-p$ of them $a=1,2$.

- The 9-p scalars are expected by Goldstone theorem.

Now we have a new situation where the string goes between two branes. Quantising this string - has a lowest mode of course - the frequency of the zero mode will be determined by the rep.

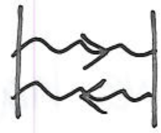
The lowest lying mode is a particle of mass M proportional to the distance between branes.
↳ need to talk about scales

Fundamental string has a tension (mass per unit length)
$= \alpha' = \# \, l_s^{-2}$
↳ π's etc.

so l_s is our fundamental unit of length
$M = l_s^{-2} L$ when $L = 0$ we get massless objects.

SUSY, only one type of multiplet with gauge fields In 16 SUSY's massless gauge fields
⟶ non abelian gauge sym. at $L = 0$
U(2) gauge theory.

dim $U(2)=4$, 4 massless gauge fields.

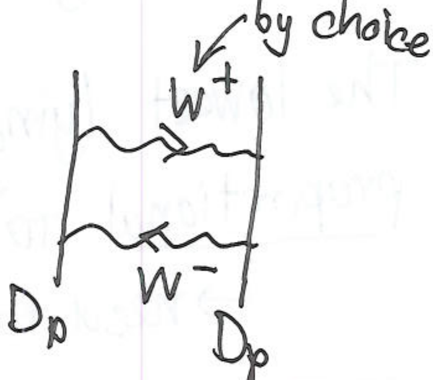
This is consistent with our picture because we neglected to mention the orientation 

This is a brane realisation of Higgs mechanism.

When $L\neq 0$ gauge thy is $U(1)^2$ with 2 massive W-bosons, mass $\sim L$

↳ by analogy to standard model.

$$\underset{L\neq 0}{U(1)^2} \longrightarrow \underset{L=0}{U(2)}$$



We know it's $U(2)$ by counting. In $(p+1)$-dims, we have our vector multiplet $9-p$ scalars
$p-1$ polarisations

Non-abelian gauge field carries 2-indices spacetime & internal (adj.)

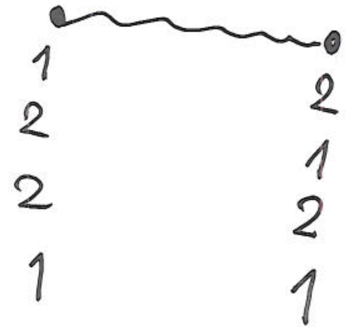
- Refresh the Higgs Mechanism for an $SU(2)$ gauge thy with adj. matter, specifically, adj. scalar ϕ^a_i.

Why adjoint? Well, the gauge thy is in adjoint and the scalars are in the same SUSY multiplet and [susy trans, gauge trans.] $=0$, so scalars are in adjoint too.

Or we can look from WV perspective.

On Worldsheet of fundamental strings: 4 types of open strings

$\Rightarrow$



Need to quantise each of them, one by one, and compute the spectrum. The result confirms what we are saying about the low lying mode gauge gp. and behaviour with L.

So, we got two things:

1) How non-abelian gauge theories arise in string theory? (stack of branes on top of each other)
2) Higgs mechanism

Such things cannot be found on fund. strings

$\rightarrow$ there can be no gauge fields on the string.

Ex. What is the world volume theory on N coincident M5 branes?

Reminder: 5+1 → massless spectrum → branes

fundamental string open / closed | scalars - Goldstone th'm

fundamental string: 8 scalars (8 transverse dir.)
string in D dimensions has D-2 transverse directions
→ D-2 massless scalars
does not match with Lorentz inv. in (D-1)+1 dim

$$\mathcal{L} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} g^{\alpha\beta}$$

metric in spacetime

metric on the worldsheet (2d)

$X(\sigma,\tau)$

There are D of them, not D-2! discrepancy!

→ fix gauge (reparametrisation) light cone gauge.

→ Lose manifestly Lorentz inv., but see explicit physical d.o.f.

Massless excitation on brane:
fermions
gauge fields → particle phenomenology!
but not gravitons - they live in space time, but can be induced to the brane.

Dp-branes – ends of fundamental strings.
from the p.o.v. of observer on string
$\rightarrow$ open string.

Dp-brane $p = -1, \dots, 9$ $\quad$ $9-p$ D b.c.
$\quad$ $p+1$ N b.c.

p odd lives in type IIB
p even " in type IIA

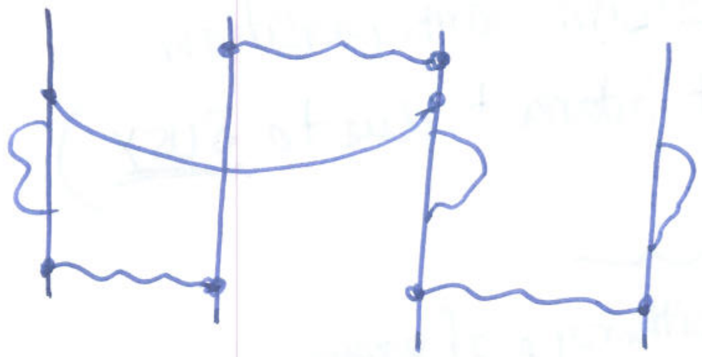
QFT's in p+1 dim gauge theory with 16 supercharges.

<u>Higgs mechanism</u>

parallel Dp-branes
4 massless gauge fields (L=0)
(V. multiplets).

V-plet	$1\ A^a_\mu$,	$1\lambda^a$	
10d	8	8	dofs
d	$\mu = 0, \dots 9$		
d=9	$A_\mu, \phi = A_9$	$\lambda \xrightarrow{\text{decompose}}$	
d=p+1	A_μ, 9-p scalars	fermions.	

$L \neq 0$ $\quad U(1)^2$ $\quad$ 2 massive, W-bosons



N parallel Dp-branes

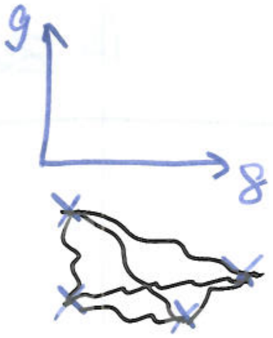
$U(1)^N$ for generic case

$N \cdot (9-p)$ sclars

2 Dp branes collide
more massless field
N^2 vector multiplet
If all branes coincide,
then all N^2 are massless

$p=7$ $\quad 0 \; 1 \; \dots \; 7$

$\langle \phi_{8,9} \rangle$



$$U(N) \text{ gauge theory} = \underbrace{U(1)}_{\substack{\text{centre} \\ \text{of} \\ \text{mass} \\ \text{of} \\ \text{the branes}}} \times \underbrace{SU(N)}_{N^2-1}$$

Take $N=2$, $\quad U(2) \xrightarrow[L\neq 0]{} U(1)^2$

$$\cancel{U(1)} \times SU(2) \longrightarrow \cancel{U(1)} \times U(1)$$

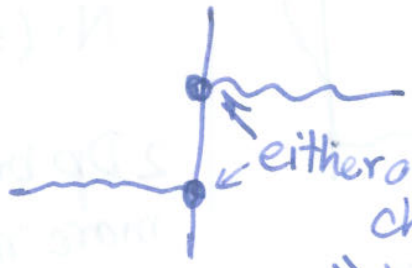
Centre of mass $\quad$ spon. sym. breaking

Think of those $U(1)$'s as sum & difference between positions of the branes.

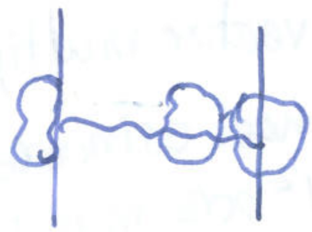
Spontaneous breaking of gauge sym.

⇓

moduli space of vacuum configuration
(branes do not interact due to SUSY)

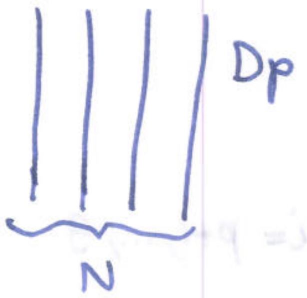


either opp. of same charge depending "in" or "out" directions.



When taking all contributions of exchanging all strings, the force betwen 2 branes is zero.

String Theory (8/02/2008)

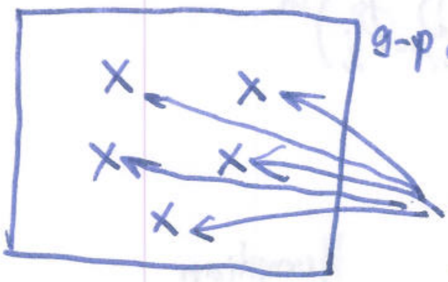


Moduli space of vacua

$$\mathcal{M} = \mathbb{R}^{(9-p)N} / S_N$$

$$= \underbrace{\mathbb{R}^{9-p}}_{\text{CoM motion}} \times \frac{\mathbb{R}^{(9-p)(N-1)}}{S_N}.$$

Spacetime perspective:



9-p dim. space

these pts can move any where. identical ← this is why we divide by S_N.

Brane perspective: gauge fields

16 SUSY's vector multiplet in Adj. rep. of $U(N)$

An observer on the brane will write down an effective action:

$$\int d^{p+1}x \left(F^a_{\mu\nu} F^{\mu\nu}_a \right)$$

calculation is easy:

$$SO(9,1) \supset SO(p,1) \times SO(9-p)_R$$

* scalars are in v-rep. of $SO(9-p)$,
* fermions are spinors of $SO(9-p)$,
* gauge fields are singlets.

$\underbrace{\text{R-symmetry}}_{\text{global sym. of Lagrangian}}$ acts differently on bosons & fermions

Start with $(9+1)d$: A_μ, λ ②

$$\int d^{10}x \, F^a_{MN} F^{MN}_a \qquad M: \mu, i$$

$$\mu = 0, \dots, p, \quad i = p+1, \dots, 9$$

$$A_M \rightarrow A_\mu, \phi_i$$

$$\int d^{p+1}x \left(F^a_{\mu\nu} F^{\mu\nu}_a \right) + \left(F^a_{\mu i} F^{\mu i}_a \right)$$

just forget about $9-p$ dimension

$F^a_{\mu i} F^{\mu i}_a \rightarrow (D_\mu \phi_i)^a$

$$F = dA + A \wedge A$$

$$F_{\mu\nu} = \partial_\mu A_\nu + i \underbrace{[A_\mu, A_\nu]}$$

in $9-p$ dim: $\mathrm{Tr}([\phi_i, \phi_j])^2$ ϕ_i are Hermitian $N \times N$ matrices.

$$\boxed{\text{The potential energy in } p+1 \text{ dimension} \quad \Big\} \quad V = \sum_{i<j} \mathrm{Tr}([\phi_i, \phi_j])^2.}$$

Look for the solution of $V = 0$ (vacuum configuration)

$$\Rightarrow [\phi_i, \phi_j] = 0$$

$$\Rightarrow \phi_i = \begin{pmatrix} \lambda_i^{(1)} & & \\ & \ddots & \\ & & \lambda_i^{(N)} \end{pmatrix}.$$

There are $N(9-p)$ parameter subject to S_N permutation.

① ∃ extra dim. or not
② they are compact or not ⇐ study of branes

Relationship between branes and algebra

$\vec{\phi}_i$ 9-p vector

$$(M_W)_{ij} = |\vec{\phi}_i - \vec{\phi}_j|$$

U(1) decouples, because we do not consider CM motion; we just take CM to be at the origin of 9-p dim.

root system of SU(N):

$$\sum_{i=1}^{N} \vec{E}_i = 0, \quad \vec{E}_i \cdot \vec{E}_j = \delta_{ij}$$

$\vec{E}_i$ = vector in N dim

roots in Adj. rep:

$\pm(E_i - E_j)$ — W boson

$(N-1)N$ roots of adjoint rep. of SU(N)

N−1 more vector multiplets

Assign position of brane with roots of SU(N) alg.

$E_8 \times E_8$, SO(32), U(N)

E_6 SO(N)

E_7 Sp(N)

$\boxed{\text{Next show: M: 12-1}}$

<u>Stringtheory</u> 11/02/08

Two basic parameters in string theory

l_s in 10d – stringlength = basic scale in the theory

g_s string coupling → not the scale

$g_s = e^{\langle\phi\rangle}$ ← dilaton

All theories in 10d have it.

VEV of scalar field

Only one scale, the rest are VEVs of scalar fields.

l_s, is the fundamental unit of length, appears in the action on the world sheet of F-string

$$S = T\int\sqrt{h}\,h^{\alpha\beta}\,\partial_\alpha X^\mu \partial_\beta X_\mu\, d^2\sigma$$

↑ tension of the string

$$T = \frac{\text{mass}}{\text{length}} = \# \, l_s^{-2}$$

↑ some no. of order 1, e.g. π.

X^μ are coords in spacetime but scalar fields on w.s.

Expansion in coupling of w.s. l_s^{-2}

quantum: $\int DX\, e^{iS/\hbar}$

quantum correction are power in $\alpha' = l_s^2$.

Spacetime Perspective:

l_s - basic unit of length

derivative expansion - power of length

all dimension full operators measured in unit of l_s.

α' expansion - low energy expansion (since higher α' powers correspond to higher l_s^2 power and so to higher derivative terms)

Suppose we consider string in curved spacetime

$$S = T\int \sqrt{h}\, h^{\alpha\beta}\partial_\alpha X_\mu \partial_\beta X_\nu \, d^2\sigma \, \underbrace{G^{\mu\nu}(X)}$$

can think of the collection of couplings
not free field theory any more - hard to solve in most cases!

g_s | $\int d^2\sigma \, \underbrace{\phi R}$ ← Ricci scalar w.r.t. metric on W.S

dilaton   scalar field (dilaton) couples to a Ricci scalar

Def^n any thing coupling to a Ricci scalar is a dilaton.

$\phi(X^\mu(\sigma))$ quantum field

$\langle \phi \rangle \neq 0$

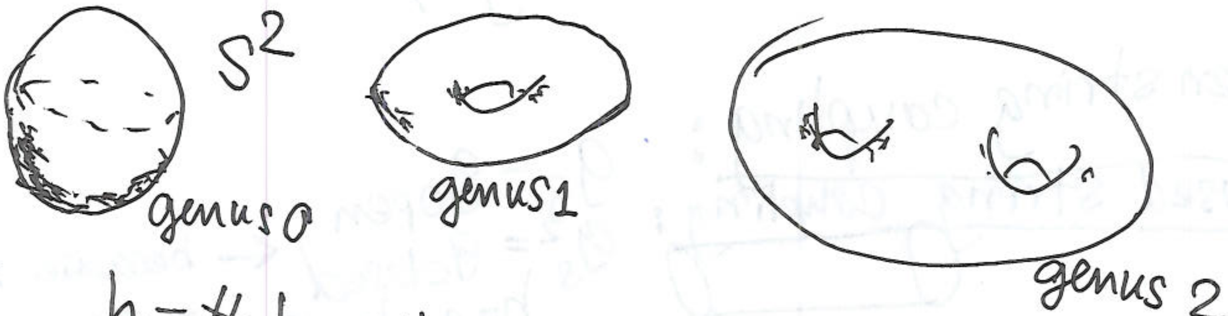
ϕ_0 no σ dependence.

zero mode

$$\int d^2\sigma\, \phi R \xrightarrow{\text{take zero mode}} \phi_0 \underbrace{\int d^2\sigma\, R}_{\substack{\text{topological invt,}\\ \text{i.e. Euler charater } \chi \\ \text{of the worldsheet}}} = \phi_0 \chi$$

<u>Classification of W.S.</u>

S^2



$h = \#$ handles
$= $ genus

$b = \#$ of boundaries

$$\boxed{\chi = 2 - 2h - b}$$

$$\phi_0 \chi = \phi_0 (2 - 2h - b)$$

$$e^{-S} = e^{-\phi_0 (2-2h-b)}$$

Set $g_s = e^{\phi_0}$ string coupling.

# of genus = loops in string pert$^{\underline{b n}}$ theory

$\Rightarrow$ string loop expansion.

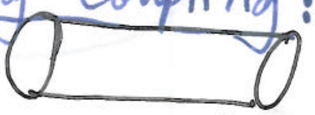
l_s - small   low energies

g_s small   tree level string theory.

<u>Klein bottle</u> $h=0$, $b=0$, unoriented
interesting when we deal with
unoriented string (type I simplest example
of unoriented strings).

Open string coupling: $g_s = g_{open}$

<u>Closed string coupling</u>: $g_s^2 = g_{closed}$ ← because $2h$ ← handle



cylinder

$h=0$, $b=2 \Rightarrow \chi = 0$.

↳ can be thought of

- closed string tree level propagator
- open string one loop amplitude

M-theory: one scale (Planck length)
no VEV's

no small parameter (l_p)

INTRODUCE SCALE:

Curvature scale: $[R] = l_p^{-2}$

If curvature is small can use small l_p limit (low energy)

$$\frac{1}{l_p^9}\int R\, d^{11}X$$

Because $l_p \ll$ every scale of curvature, we can use GR. to predict other things without quantum correction.

Braves:

Branes: tension $= \dfrac{\text{mass}}{\text{Volume}}$ $\dfrac{1}{(l_{s,p})^{p+1}}$

M2: $T_2 \sim \dfrac{1}{l_p^3}$

M5: $T_5 \sim \dfrac{1}{l_p^6}$ precise factor of 2π are determined by imposing Dirac quantisation condition.

Taking into account that charge & tension are equal $\Leftarrow\!\Rightarrow$ SUSY brane.

BPS (SUPERSYMMETRIC)

$$T_{F1} = \frac{1}{\ell_s^2}, \quad T_{Dp} = \frac{1}{g_s \ell_s^{p+1}}$$

When $g_s \ll 1$, the brane is a very heavy object

$$T_{Dp} \gg T_{F1}$$

at weak coupling

$$T_{NS5} = \frac{1}{g_s^2 \ell_s^6}.$$

String Theory (12/02/2008)

W 10-11, 11-12
Th 10-11, 12-1
F 4-5, 10-11?

Next M 12-1
Long break
Next Lecture Mar 3 12-1

Last time g_s, ℓ_s $\quad \ell_p \leftarrow$ only scale in M-theory

When the curvature length $\gg \ell_p$,
low energy limit 11d sugra!

	Tension	
M2	ℓ_p^{-3}	indep. of string coupling
M5	ℓ_p^{-6}	indep. of string coupling
F1	ℓ_s^{-2}	fund
Dp	$\frac{1}{g_s \ell_s^{p+1}}$	D-brane
NS5	$\frac{1}{g_s^2 \ell_s^6}$	Solitonic $\leftarrow$ can't compute tension etc., since non-pertbtive.

- much heavier than Dp
- Also, a solitonic object
- has topology of sphere

String theories are pertbtive in nature.
So, from F-string p.o.v., g_s is small.

If g_s is large, we have to include all topologies - hard calculation. At $g_s \sim 1$, need infinitely many topologies.

From spacetime observer p.o.v., effective actions for massless fields. In this p.o.v., the calculation for NS5 is possible - it is a magnetic source of 2 form field.

Duality: reduces # of field theories. One theory at a range of parameters the same as another theory at a different range.

$\boxed{\text{M-theory on } M_{10} \times S^1 \leftrightarrow \text{Type IIA on } M_{10}}$

l_p, R ← radius of circle $\qquad l_s, g_s$

↕ expectation value of a scalar $g_{10,10}$

spectrum are the same
interaction are the same
for any 10-dim m'fold

M5 ⟷ NS5

M2 ⟷ D2

M5 on S^1 ⟷ D4

M2 on S^1 ⟷ F1

? ⟷ D0

? ⟷ D6

$$M5 \leftarrow \frac{1}{\ell_p^6} = \frac{1}{g_s^2 \ell_s^6} \rightarrow NS5 \quad 1$$

$$M2 \leftarrow \frac{1}{\ell_p^3} = \frac{1}{g_s \ell_s^3} \longrightarrow D2 \quad 2$$

$$\Rightarrow \boxed{g_s \ell_s^3 = \ell_p^3}$$

$g_s \gg 1$

M-theory side

$g_s \ll 1 \leftarrow$ string perbtn

$$M5 \text{ on } S^1 \leftarrow \frac{R}{\ell_p^6} = \frac{1}{g_s \ell_s^4} \qquad D4 \quad 3$$

$$M2 \text{ on } S^1 \leftarrow \frac{R}{\ell_p^3} = \frac{1}{\ell_s^2} \qquad F1 \quad 4$$

$$\Rightarrow \boxed{g_s \ell_s = R} \quad 2, 4$$

$g_s \to \infty \qquad R \to \infty$

$\qquad\qquad\qquad\;\; \parallel$

$\qquad\qquad$ 11 dim. M-theory

- S-duality: How IIA theory behaves when $g_s \to \infty$?

(No duality like this in many field theories...)

M-theory on $M_{10} \times S^1$ $\Longleftrightarrow$ IIA on M_{10} 4

$\boxed{?} \Leftarrow \frac{1}{R} = \frac{1}{g_s l_s} \leftarrow$ D0

looks like KK mode

Ans: gravitational momentum mode

R small = large mass

$\boxed{?} \Leftarrow \frac{R^2}{l_p^9} = \frac{1}{g_s l_s^7} \leftarrow$ D6

Ans: K.K. monopole.

$$\underset{\text{9 spatial dim.}}{SO(9)} \supset \underset{8}{SO(8)}$$

$$\underset{\text{3 form}}{84} \longrightarrow \underset{\text{3 form}}{56} \oplus \underset{\text{2 form}}{28}$$

$$\underset{\text{graviton}}{44} \rightarrow 35 \oplus 8 \oplus \underset{\text{scalar}}{1}$$

$g_{10,10}$

By introduce a circle, we introduce a scalar

VEV of this scalar ~~this~~ is the radius R

$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ $\mu,\nu = 0, \dots, 10$

in 11d

$g_{10,10} (dx^{10})^2$

natural interpretation is the radius of that dim.

A Few ways of getting scalars:

1. compactification → scalars live in spacetime
2. Putting branes ↘ scalars localise in $(p+1)$-dim only.

Standard model: 1 Higgs field ← one scalar

the rest are VEV:

α fine structure const.
should be thought of a VEV of a scalar field in string theory

Yukawa & Gauge coupling are also VEVs of same scalar fields.

Type IIB: ⟷ Type IIB

$\boxed{\tau = \frac{i}{g_s} + \frac{C^{(0)}}{2\pi}}$ $g_s, l_s, C^{(0)}$

$\underbrace{g_s', l_s', C^{(0)'}}$ $\tau = \frac{i}{g_s'} + \frac{C^{(0)}}{2\pi}$

$e^{\langle\phi\rangle}$ ← dilaton

axion couples to D-1 E
D7 M

B

$C^{(2)}$

$C^{(4)+}$

F1 ⟷ D1

NS5 ⟷

D1

D5

(b)

$$\text{Type IIB} \xleftrightarrow[\tau = \frac{i}{g_s} + \frac{C^{(0)}}{2\pi}]{\text{S-duality}} \text{Type IIB}$$

$g_s, \ell_s, C^{(0)}$ — axion couples to D1 E, D7 M

$\underbrace{g_s', \ell_s', C^{(0)\prime}}$ $\quad \tau' = \frac{i}{g_s'} + \frac{C^{(0)}}{2\pi}$

$B \longrightarrow C^{(2)}$

$C^{(2)} \longleftrightarrow B$

$C^{(4)+} \longleftrightarrow C^{(4)+}$

F1 $\longleftrightarrow$ D1 $\quad \frac{1}{\ell_s^2} = \frac{1}{g_s'\ell_s'^2}$

NS5 $\longleftrightarrow$ D5

D1 $\longleftrightarrow$ F1 $\quad \frac{1}{g_s\ell_s^2} = \frac{1}{\ell_s'^2}$

D5 $\longleftrightarrow$ NS5

D3 $\longleftrightarrow$ D3 $\quad \frac{1}{g_s\ell_s^4} = \frac{1}{g_s'\ell_s'^4} = \frac{g_s}{\ell_s'^4}$.

$$\boxed{g_s' = \frac{1}{g_s}}$$

Compactification

$\boxed{\text{KK}}$ spectrum of massless states is very rich.

Compact manifold $\rightarrow$ scalars
gauge fields
fermions
gravitinos

brane wrapping in 9 or 10 dim.

2 types of string

Toroidal compactification: preserves SUSY (32 supercharges)

M-theory on S^1	IIA	M on T^2
		$SO(9) \supset SO(7) \times SO(2)$
84 $C^{(3)}$	$56+28$	$35+2\times 21+7$
44 $g_{\mu\nu}$	$35+8$	$27+2\times 7+3\times 1$
128 ψ_μ	$8_s+56_s+8_c+56_c$	

$C_{\mu\nu\lambda}$ $C_{\mu\nu 9}$ $C_{\mu\nu 10}$ $C_{\mu 9 10}$

2 string charges
3 1-form charges

$\dfrac{SL(2,\mathbb{R})}{SO(2)} \times \mathbb{R}^+$

complex scalar

scalar

$g_{\mu\nu}$

2 vectors
$g_{\mu,10}$
$g_{\mu,9}$

2 gauge fields (graviphoton)

3 scalars
$g_{10,10} \equiv R_{10}$
$g_{10,9}$
$g_{9,9} \equiv R_9$

Moduli space
= space of values that scalar can admit

$R_{10}, R_9 \gg l_p \rightarrow 11d$

One small, one large $\rightarrow$ IIA

both small $\rightarrow$ M on T^2

How many theories with 32 SUSY's in 9d? $\boxed{\text{One}}$

Type IIB on S^1

$g_{\mu\nu}$, $C^{(2)}$, B, $C^{(4)+}$ ϕ, $C^{(0)}$

35 28 each 35 2×1.

$35 \longrightarrow 27 + 7 + 1$

grav. in 9 dim gauge scalar

$2\times$ $28 \longrightarrow 21 + 7$

$35 \longrightarrow 35$ $C^{(4)+}$ $C_{\mu\nu\lambda\sigma}$

$2\times$ $1 \longrightarrow 1$ $dC^{(4)} = * dC^{(4)}$.

Type IIB scalar manifold $\dfrac{SL(2,\mathbb{R})}{SO(2)}$

$\boxed{\text{Type IIA on } S^1 \text{ is dual to Type IIB on } S^1.}$

How 4 parameters match

IIB IIA

$\tau, l_s, R_B \longleftrightarrow l_s, g_A, R_A$

$\boxed{\text{So, the theory with N coincidence M5 branes has no scalar}}$

String Theory (13/02/08) 1

One theory with 32 SUSY's in 9d

Scalar manifold is $\frac{SL(2,\mathbb{R})}{SO(2)} \times \mathbb{R}^+$

$SO(2,1) = SL(2,\mathbb{R})$ in the literatures

different corner on maduli space.

M on T^2 ; Type IIA on S^1 ; Type IIB on S^1

Can take the rest 9 dim to be anything.

1 gravity multiplet 128 boson
128 ferm.

Bosonic sector

		(mag, elec)
1 graviton	27	
1 $C^{(3)}$	35	(2,3)
2 $C^{(2)}$	2×21	2(1,4)
3 $C^{(1)}$	3×7	3(0,5)
3 scalar: $\underbrace{\phi_1, \phi_2}_{\text{non-compact}}$, $C^{(0)}$	3×1	(−1,6)

$C^{(0)}$: compact scalar = 0-form gauge field
- excitations are electric & mag. object

(−1,6): Only for compact scalar.

Comment on gauge inv.

EM: A_μ, $\delta A_\mu = \partial_\mu \lambda$

A, $\delta A = d\lambda$

any form: $C^{(n)}$, $\delta C^{(n)} = d\chi^{(n-1)}$

After g.transf.: $C^{(0)} \simeq C^{(0)} + R$ — identify with

electric object under $C^{(0)} \Rightarrow$ instanton (D-1 brane).

magnetic object under $C^{(0)} \Rightarrow$ vortex ← Codimension 2 object

similar to electrostatic line of charge ← logarithmic divergence in P.E.

$C^{(0)}$ gauge field; inst. dim $G^{(1)} = dC^{(0)} \xleftrightarrow{\text{mag. dual}} G^{(3)} = d\,\underbrace{C^{(2)}}_{}$

give rise to electrical object called string!
- cosmic string
- vortices

Type IIA: D0, D6

M-theory: $g_{\mu 10}$ → elec: graviton momentum mode: mass $\sim \frac{1}{R}$ (0-brane); → mag: KK monopole. (6-brane)

$D(-1)$ = D-instanton

			M on T^2	IIA on S^1	IIB on S^1
1	$C^{(3)}$	$(2;3)$	(M2, M5 on T^2)	(D2, D4 on S^1)	(D3 on S^1, D3)
2	$C^{(2)}$ ↳ transf. in spinor rep. of $SL(2,\mathbb{R})$	$2(\cancel{1};4)$	(2×M2 on S^1, 2×M5 on S^1)	(D2 on S^1 or F1; D4, or NS5 on S^1)	(F1 or D1; D5 on S^1 or NS5 on S^1)
3	$C^{(1)}$	$3(0;5)$	(M2 on T^2, 2 momentum modes; M5, 2KK-monopole on S^1)	(D0, F1 on S^1, mom-mode in IIA; NS5, D6 on S^1, KK-mon. on S^1)	(F1 on S^1, D1 on S^1, mom-mode; D5, NS5, KK on S^1)
	$C^{(6)}$	$(-1,6)$	(m-mode on S^1; ?)	(D0 on S^1; D6)	(D(-1); D7 on S^1)

have to check! ← can be KK monopole (?!) ↳ a bit like D8.

$\underline{M \text{ on } T^2}$; $\underline{\text{Type IIA on } S^1}$; $\underline{\text{IIB on } S^1}$

$\ell_p, R_1, R_2, g_{10,9}$ $\quad \ell_A, g_A, R_A, C_g$ $\quad \ell_B, g_B, R_B, C^{(0)}$

Tension formulae:

1-form in type IIA $C^{(1)}$, C_μ ← one of this can be g and this wraps on S^1 ← scalar

(2;3) ↑ $T_2 = \frac{1}{\ell_p^3} = \boxed{\frac{1}{g_A \ell_A^3} = \frac{R_B}{g_B \ell_B^4}}$

$T_3 = \frac{R_A R_B}{\ell_p^6} = \boxed{\frac{R_A}{g_A \ell_A^5} = \frac{1}{g_B \ell_B^4}}$

$\boxed{\frac{\ell_A^2}{R_A} = R_B}$ (*)

R_B compactified on a large radius $\xleftrightarrow{\text{dual}}$ R_A compactified on a small radius

T-duality: small-large radius duality

strings (1;4) ↑

$\frac{R_1}{\ell_p^3} = \frac{R_A}{g_A \ell_A^3} = \frac{1}{g_B \ell_B^2}$

$\frac{R_2}{\ell_p^3} = \frac{1}{\ell_A^2} \cancel{\overset{(*)}{=} \frac{1}{R_A R_B}} = \frac{1}{\ell_B^2}$ $\quad \left[\overset{(*)\Rightarrow}{\frac{1}{\ell_A^2}} = \frac{1}{R_A R_B} \overset{?}{=} \frac{1}{\ell_B^2} \right]$

KK monopole: $\frac{R_1^2 R_2}{\ell_p^9}$ $\frac{R_2^2 R_1}{\ell_p^9}$ (#) $\boxed{\frac{R_1 R_2}{\ell_p^6} = T_6 \text{ in } 9d}$

$(-1, 6)$

How many theories with 32 SUSY's in 8d?

One theory but with many faces!

Ex. work out the matter content in the massless gravity multiplet in dimension $d = 3, \ldots, 8$?

9d: $SO(9) \supset SO(7)$

M on T^2 8d: $SO(9) \supset SO(6) \times SO(3)$ maximal

$84 \longrightarrow$

$44 \longrightarrow$

7d $SO(9) \supset SO(5) \times SO(4)$

---

dynkin diagram of $\mathbb{E}_{10}$

$\dim SO(n) = \frac{1}{2}n(n-1)$

M-theory

compactify on S^1

$\mathbb{R}^+$ IIA

IIB $\qquad \frac{SL(2,\mathbb{R})}{SO(2)}$

9d $\qquad \frac{SL(2,\mathbb{R})}{SO(2)} \times \mathbb{R}^+$

7 scalars 8d $\qquad \frac{\overset{=8}{SL(3,\mathbb{R})}}{\underset{\text{dim } 3}{SO(3)}} \times \frac{\overset{=3}{SL(2,\mathbb{R})}}{\underset{\text{dim } 1}{SO(2)}}$

14 scalars 7d $\qquad \frac{SL(5)=24}{SO(5)=10}$

25 scalars 6 $\qquad \frac{SO(5,5)}{SO(5)\times SO(5)}$

42 scalars 5 $\qquad \underset{78}{\mathbb{E}_{6(6)}}/Sp(4) = 36$ max. subgp. $Sp(4)$

70 scalars 4 $\qquad \mathbb{E}_{7(7)}/SU(8)$??

128 scalars 3 $\qquad \underset{248.}{\mathbb{E}_{8(8)}}/\underset{120}{SO(16)}.$

10 :

9d : max. form $C^{(4)+}$

8d : $C^{(3)}$

7d : $C^{(2)+}$

⋮ $C^{(2)}$

3d : $C^{(0)}$

all fields in 3-d are scalars; gravitons have no propagating d.o.f.

U-duality of M on T^n is given by $\mathbb{E}_{n(n)}/\text{max. compact subgp.}$

$E_5 = D_5 \qquad D_5 = SO(10)$

$E_4 = A_4 \qquad A_n = SU(n+1)$

$E_3 = A_2 \times A_1$

$E_2 = A_1 \times U(1)$

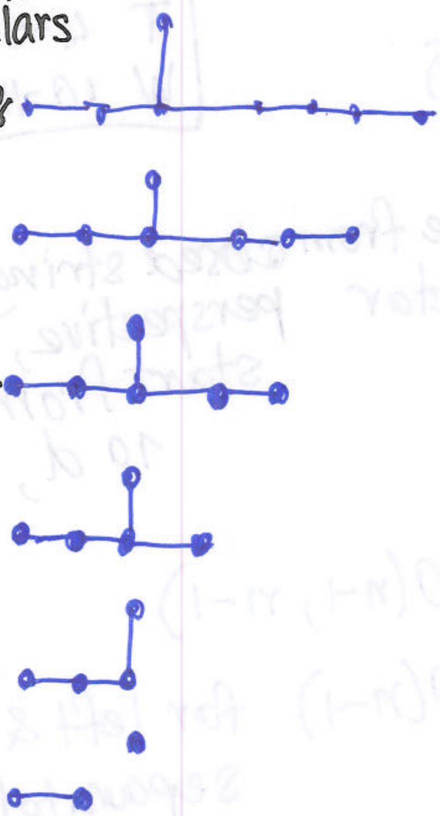
$E_1 = A_1 \quad \tilde{E}_1 = U(1)$

$E_0 = \emptyset.$

String theory 14/2/08

Symmetry pattern U-duality gp.

(non-compact) $\mathbb{E}_n$ $\quad n$ # compact directions starting from 11-dim.

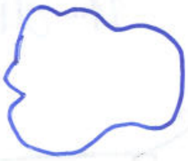


dimension	scalars				
3	128		$E8$		$\frac{E_{8(8)}}{SO(16)}$
4	70		$E7$		$\frac{E_{7(7)}}{SU(8)}$
5	42		$E6$		$\frac{E_{6(6)}}{Sp(4)}$
6	25		$E_5 = D_5$	$SO(10)$	$\frac{SO(5,5)}{SO(5)\times SO(5)}$
7	14		$E_4 = A_4$	$SU(5)$	$\frac{SL(5)}{SO(5)}$
8	7		$E_3 = A_2 \times A_1$	$SU(3)\times SU(2)$	$\frac{SL(3,\mathbb{R})}{SO(3)} \times \frac{SL(2,\mathbb{R})}{SO(2)}$
9	2		$E_2 = A_1 \times U(1)$	$SU(2)\times U(1)$	$\frac{SL(2,\mathbb{R})}{SO(2)} \times \mathbb{R}^+$
10	2	IIB	$E_1 = A_1$	$SU(2)$	$\frac{SL(2,\mathbb{R})}{SO(2)}$
	1	IIA	$\tilde{E}_1 = U(1)$		
11	0.		$E_0 = \phi$		

$\frac{G}{H}$ $\quad G$ non-compact, H maximal compact subgp.

$$U \to \begin{cases} S \text{ duality} & \text{strong } g_s \\ \underbrace{T \text{ duality}}_{\frac{SO(n-1,\,n-1)}{SO(n-1)\times SO(n-1)}} & \text{weak } g_s \end{cases}$$

F 10-11, 4-5
M 12-1
Long break
M Mar 5, 12-1
T 4-5, 5-6
W 10-11, 11-12



closed string

$\vec{P}_R$, $\vec{P}_L$ $(n-1)$-dim vector → because from closed string perspective, start from 10 d, not 11d!

right moving, left moving

$(\vec{P}_L)^2-(\vec{P}_R)^2$ invariant under $SO(n-1,n-1)$

and $SO(n-1)$ for left & right separately.

$$E_{n+1} \supset \underbrace{D_n \times U(1)}_{\text{maximal subgp.}}$$

Take $n=4$ (6 non-cpt dim.)

$$E_5 = D_5 \supset D_4 \times U(1)$$

$$SO(10) \supset SO(8)\times SO(2), \quad SO(5,5) \supset \underbrace{SO(4,4)}_{\text{T duality gen.}} \times \underbrace{SO(1,1)}_{\text{S duality gen.}}$$

$N = 1, 2, 4, 8$ $\quad d=4$

\# of superchanges $\quad$ 4 8 16 32

$\boxed{\text{N=8 sugra in (3+1)-d} \uparrow \text{only theory in 4-d with 32 supercharges}}$

Moduli problem: no scalars have been observed
→ some mechanisms that these scalars gain mass at least 100 GeV.

All theories (except IIB) are non-Xral
But we can put branes into them to get Xrality

Observed fermions are Xral

Manifold $\quad \mathbb{R}^{d,1} \times T^n \quad d+1+n = 11$

IIB $\quad \mathbb{R}^{3,1}, \ AdS_5 \times S^5 \quad$ 32 SUSY's.

Theories with 16 SUSY's

As we go down with SUSY, there are more & more options – more complicated.
(We have to ask whether each theory is a consistent one or not)

$$10d \begin{cases} \text{max. SUSY } 32 \\ \text{min. SUSY } 16 \end{cases}$$

closed string theories

$10\,d$ Type I , Het $E_8 \times E_8$, Het $SO(32)$

$SO(32)$ Open strings

(Heterotic = L moving & R moving are different objects)

1 sugra multp.

g, B, ϕ

35 28 1 = 64 bosonic d.o.f.

$\psi_\mu \quad \lambda$

56 8

1 Vector multiplet

A_μ, λ

$G = E_8 \times E_8$ Instanton

$SO(32)$

Type I $\longleftrightarrow$ Het $SO(32)$

Moduli space $\mathbb{R}^+$ $g_s = e^{\phi}$

g_I , g_H

Branes:

Comments T-duality

Type IIA on S^1 $\longleftrightarrow$ Type IIB on S^1

F1 $\longleftrightarrow$ F1 $\qquad \dfrac{1}{l_A^2} = \dfrac{1}{l_B^2} = \dfrac{1}{l_S^2}$

D2 on S^1 $\longleftrightarrow$ D1 $\qquad \dfrac{R_A}{g_A l_S^3} = \dfrac{1}{g_B l_S^2} \rightarrow \dfrac{1}{g_B} = \dfrac{R_A}{g_A l_S}$

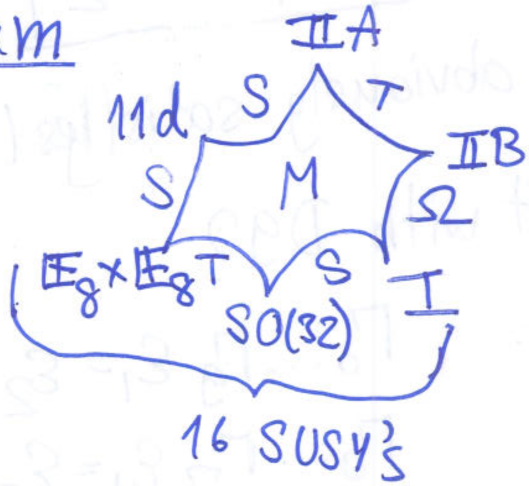
$$\boxed{\frac{R_B}{g_B^2} = \frac{R_A}{g_A^2}}.$$

M on T^2, IIA on S^1, IIB on S^1

Ex. M on $T^2 \longleftrightarrow$ IIB on S^1

We have done M on $T^2 \longleftrightarrow$ IIA on S^1.

Duality diagram



massless spectrum

Massless spectrum

Het $SO(32)$	Type I
1 SUGRA m-plet $\longleftarrow$	strings, 5-brane
1 v-plet $G = SO(32)$

$$T_{F_1} = \frac{1}{\ell_H^2}, \quad T_{NS5} = \frac{1}{g_H^2 \ell_H^6}.$$

D9-brane all over spacetime

~~D7 coexists with D9~~

D5

~~D3~~

D1

Type II Dp-brane has 16 SUSY's

Q_1, Q_2

$\{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu}.$

IIA: $\Gamma_0 \ldots \Gamma_9 Q_1 = Q_1$

$\Gamma_0 \ldots \Gamma_9 Q_2 = -Q_2$

IIB: $\Gamma_0 \ldots \Gamma_9 Q_1 = Q_1$

$\Gamma_0 \ldots \Gamma_9 Q_2 = Q_2$

$\varepsilon_1 Q_1 + \varepsilon_2 Q_2$ $\quad \boxed{\Gamma_0 \ldots \Gamma_p \varepsilon_1 = \varepsilon_2} \quad (*)$

D9-brane abviously satisfies $(*)$.

Can D7 coexist with D9?

Suppose so: $\Gamma_0 \ldots \Gamma_9 \varepsilon_1 = \varepsilon_2$

$\Gamma_0 \ldots \Gamma_7 \varepsilon_1 = \varepsilon_2$

Consider $(\Gamma_0 \ldots \Gamma_7)(\Gamma_0 \ldots \Gamma_9)\varepsilon_1 = \varepsilon_2$

$\boxed{\text{Dp, Dp+4 can be mutually SUSY}}$

$0,\dots,p$ $\quad 0\cdots p+4$

$\boxed{\text{Dp, } D_{p+2} \text{ are not mutually supersym.}}$

$\boxed{\text{Dp, } D_{p+6} \text{ are not}}$

$\boxed{D_p, D_{p+8} \text{ Supersymmetric}}$

I $\qquad\qquad\qquad$ Heterotic

D9-brane

D5 $\quad \dfrac{1}{g_I \ell_I^6} = \dfrac{1}{g_H^2 \ell_H^6} \quad$ NS S

D1 $\quad \longleftrightarrow \quad$ F1

$$\frac{1}{g_I \ell_I^2} = \frac{1}{\ell_H^2}$$

$\boxed{\text{Dp-brane tension in type I} = \dfrac{1}{g_I \ell_I^{p+1}}}$

Solution: $\boxed{\begin{aligned} \frac{1}{g_I} &= g_H \\ g_I \ell_I^2 &= \ell_H^2 \end{aligned}}$

S-duality

Weak in type I

$\longleftrightarrow$ strong in type II

(vice-versa)

If $g_I \sim \mathcal{O}(1)$, then both fails.

Compactification

(toroidal compactification preserves no. of supercharges) 16

massless multiplets

$\mathbb{R}^{1,8} \times S^1$

1 g-plet — 10d

g	B	ϕ	ψ_μ	λ
35	28	1	56	8

1 V-plet

10d

A_μ	λ
8	8

→ No scalar

9d	A_μ	A_9	λ
	7	1	8

→ there is a scalar

Moduli space of vacua

$\overset{10}{g} \to \overset{9}{g} + A_\mu + \phi$

$35 \to 27 + 7 + 1$

2 gauge fields

$B \to B + A_\mu$

$28 \to 21 + 7$

2 scalars

$\psi_\mu \to \psi_\mu + \lambda$

$56 \to 48 + 8$

$\phi \to \phi$

$\lambda \to \lambda$

$\psi_\mu : (d-3)2^{\left[\frac{d-3}{2}\right]}$ degrees of freedom in d dimension

g	B	$2A$	2ϕ
27	21	27	21

9-plet

g, B, A, ϕ

ψ_μ, λ

gravity multiplet in 10 d

$$G_{10} \longrightarrow G_9 + V_9$$
$$V_{10} \longrightarrow V_9$$

String Theory (15/02/08)

M	March 3 12-1
T	4-5, 5-6
W	10-11, 11-12
Th	10-11, 12-1
F	4-5

T-duality: $SO(n,n)$

$$V = (\vec{V}_L, \vec{V}_R), \quad V^2 = \vec{V}_L^{\,2} - \vec{V}_R^{\,2}$$

$\dfrac{SO(n,m)}{SO(n)\times SO(m)}$ Walf spaces

$$\frac{SU(n,m)}{SU(n)\times SU(m)}$$

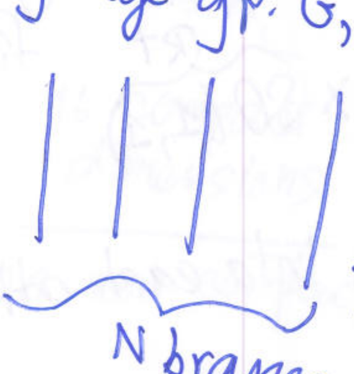
$$\frac{SL(n)}{SO(n)}$$

9d theories 16 SUSY's

Het $SO(32)$ on S^1, Type I on S^1, Het $E_8\times E_8$ on S^1.

- In 10d, there is 1 scalar (dilaton) from gravity multiplet
  $\Rightarrow$ dim. of moduli sp. $= 1$
- In 9d G-plet has 1 scalar, $U(1)$ V-plet 1 scalar

gauge gp. G, 1 scalar in adj



$$G = U(N)$$

N branes

9-p scalars in adj. rep.

Dp in Type II

$$U(N) \longrightarrow U(1)^N, \quad \prod_{i=1}^{k} U(n_i) \quad \sum_{i=1}^{k} \underset{\text{rank of gauge gp.}}{n_i} = N$$

④

Compactify on T^n $\longrightarrow$ $\dfrac{SO(16+n,n)}{SO(16+n)\times SO(n)} \times \mathbb{R}^+$

Non-compact gp.

- $SO(3,1)$ 6 generators $\begin{cases} 3 \text{ compact gen. (rotations)} \\ 3 \text{ non-compact (boosts)} \end{cases}$
- $SO(n,m)$ Subgps $SO(n)$, $SO(m)$ compact
  $\frac{n(n-1)}{2} + \frac{m(m-1)}{2}$
  $\Rightarrow$ Non-compact nm.
- $E_{8(n)}$ $n = \#$ non-comp. gens $- \#$ comp. gens
  maximal non-compact when $n =$ rank
  M on T^n has $E_{n(n)}$ U-duality symmetry.

$\frac{G}{H}$ $G \to$ maximal non-compact
$H \to$ maximal compact subgp.

$\dim G$, $\underbrace{\dim H}_{\#\text{ compact}}$

$\dim G - \dim H = \#$ non-compact

$\dim G - 2\dim H = n$

$E_{8(8)}$ 128

248 120

$\frac{E_{8(8)}}{SO(16)}$ $SO(16)$

$E_{7(7)}$

133

$\frac{E_{7(7)}}{SU(8)}$ $SU(8)$ ($N=8$ SUGRA in 4d).

70 63

$$E_{6(6)} \qquad \frac{E_{6(6)}}{Sp(4)} \qquad Sp(4)$$
$$78 \qquad 42 \qquad 36$$

$$E_{5(5)} = SO(5,5) \qquad \frac{SO(5,5)}{SO(5)\times SO(5)} \qquad SO(5)\times SO(5)$$
$$45 \qquad 25 \qquad 20$$

$$E_{4(4)} = SL(5) \qquad \frac{SL(5)}{SO(5)} \qquad SO(5)$$
$$24 \qquad 14 \qquad 10$$

$$E_{3(3)} = SL(3)\times SU(2) \qquad \frac{SL(3)\times SL(2)}{SO(3)\times SO(2)} \qquad SO(3)\times SO(2)$$
$$8+3 \qquad 7 \qquad 3+1=4$$

9d theories with 16 SUSY's

1 G-plet, 17 V-plets

generic pt in moduli space

moduli space of V-plets is $\frac{SO(17,1)}{SO(17)}$.

At singularities of this moduli space, additional massless states $U(1)^{17}\times U(1)$

$\hookrightarrow$ must come from V-plets (we have only G-plet & V-plets)

Non-abelian gauge gps, any of
$SU(n_i)$, $SO(2n_i)$, E_{n_i} such that $\sum n_i = 17$
Consider forexample $SO(34)$:
At generic points, $SO(17)$
At singularity, enhancing of sym. $SO(34)$.

<u>8d theories with 16 supercharges</u>

$G_8 \equiv$ G-plet in 8d
$V_8 \equiv$ V-plet in 8d

$G_9 \rightarrow G_8 + V_8$ 18 V-plets (17 V-plets in 9d + 1 from this)

V_8 has A_μ, A_8, A_9
V-plet moduli space is of dim 36

~~Scalar manifold~~ $\dfrac{SO(18,2)}{SO(18)\times SO(2)}$

G_8 has 1 scalar (dilaton) $\mathbb{R}^+$

The 18 V-plets transform in 18 of $SO(18)$ ⑧
commutes with supersymmetry.

$SO(2)$ acts differently on cpts of V-plets

scalars have charge ± 2
fermions " " ± 1 (2 spinors $4_1, \bar{4}_{-1}$)

- R-symmetry acts differently on different cpts on the supermultiplets (does not commute with SUSY)
$SO(2)_R$

Generic pt. in moduli space gauge gp. is $U(1)^{18}$.
$\times U(1)^2$

graviphoton coming from 2 G-plets

any of A, D, E s.t. $\sum_i n_i = 18$

$SO(36)$ $SU(19)$.

This continues up to $d=5$

Type I on $T^5 \times \mathbb{R}^{4,1}$
or Het $SO(32)$ T^5
or Het $E_8 \times E_8$ T^5

They are T-dual to each other

$T^n \times \mathbb{R}^{9-n,1}$

V-plet moduli space $\dfrac{SO(16+n,n)}{SO(16+n)\times SO(n)}$

G-plet " " $\mathbb{R}^+$

$16+n$ V-plets, each with n scalars: $n(16+n)$

Total rank $16+n$ $SO(n)_R$

and n graviphotons in vector rep. of $SO(n)_R$, $U(1)^n$.

<u>What happen at $d=4$?</u>

In $d=3+1$, V-plet mod. space $\dfrac{SO(22,6)}{SO(22)\times SO(6)}$, $U(1)^{28}$

G_n in n dimensions has 1 2-form.

4d: scalar dual to 2-form

axion in G-plet $\dfrac{SL(2,\mathbb{R})}{SO(2)}$, $\tau = ie^{\phi} + C^{(0)}$

(it must be counted in 4d, but not higher dimensions).

Sometimes people don't distinguish between 22 (V-plet) and 6 (G-plet)

3d. gravity as no d.o.f.
all gauge fields are scalars.

$$\frac{SO(24,8)}{SO(24)\times SO(8)}.$$

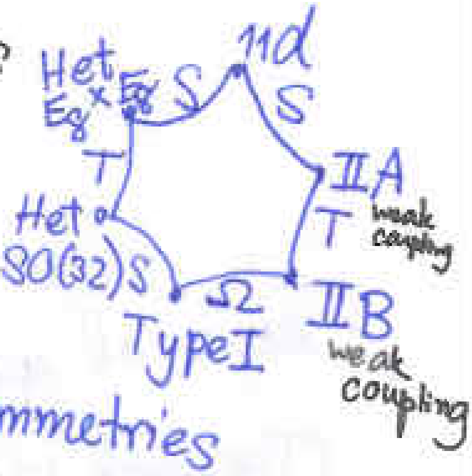
~~[illegible]~~
3 theories in 9d & 16 SUSY's:

\# V-plets 17
9
1.

String Theory 18/02/08 ①

Summary: 5 string theories in 10d ($\to 2+3$)

1 in 11d 32 SUSYs



Look at massless supermultiplets

32 SUSYs 1 G-plet

Moduli space of vacua – E_n symmetries for M-theory on $\mathbb{R}^{1,10-n} \times T^n$.

possibly other background with 32 susy's

$AdS_5 \times S^5$ (IIB)

$AdS_7 \times S^4$ (M)

$AdS_4 \times S^7$

16 SUSYs G-plet, V-plet ($\to$ SO(32) adj. rep.)

in 6d there is another T-plet.

Generalised EM $\to$ branes, branes end on branes.

|||| N coincident D_p brane Higgs mech.

Brane realisation to Higgs mechanism $U(N)$ with adj. matter.

Classical gps

A, B, C, D $\leftarrow$ SO(N) N even

SU(N) SO(N) Sp(N)

Theories with 16 SUSYs

moduli space $\dfrac{SO(16+n,n)}{SO(16+n)\times SO(n)}$ $16+n$ V-plets, n scalars

Type I on $\mathbb{R}^{10-n}\times T^n$.

dimension $n(16+n)$.

rank of gauge gp. in V-plet is always $16+n$.
(The rank is always const., no matter where we put the branes – problematic for phenomenology.
SM: $\underbrace{SU(3)\times SU(2)\times U(1)}_{\text{rank } 4}$.)

Want a mechanism for rank reduction
lowering SUSY
Xral matter.

rank reduction

9 dim 16 SUSYs, Type I, Het with $E_8\times E_8$,
on S^1_R Het $SO(32)$.

T-duality
$R_E R_O = \ell_s^2$ remains the same

$$\frac{R_E}{g_E^2}=\frac{R_O}{g_O^2}.$$

Type II with Dp-branes

Under T duality:

IIA on S^1 $\longleftrightarrow$ IIB on S^1

g_A, R_A $\qquad$ g_B, R_B

$$R_A R_B = l_s^2$$

$$\frac{R_A}{g_A^2} = \frac{R_B}{g_B^2}$$

$$\frac{1}{g_A} = \frac{R_B}{g_B l_s}.$$

Dp in IIA (can either wrap or not wrap the S^1)

Dp wraps S^1 in IIA

$$\frac{1}{g_s l_s^{p+1}} \cdot R = \frac{R_A}{g_A l_s^{p+1}} = \frac{R_A R_B}{l_s^{p+1} g_B l_s} = \frac{1}{g_B l_s^p}$$

$\downarrow$ tension of $D_{(p-1)}$ brane in IIB does not wrap S^1

$$\boxed{\text{Dp on } S^1 \xleftrightarrow{T} D(p-1) \text{ unwrapped}}$$

On Dp: gauge theory in p+1 dim; one direction is compact $\mathbb{R}^{1,p-1} \times S^1$

Gauge theory in (p+1)-dim - one dimension is compact with radius R	$\overset{T}{\longleftrightarrow}$	Gauge theory in p-dim with $\underbrace{\text{scalar (compact)}}_{\text{lives on a circle with radius } \frac{\ell_s^2}{R}}$

[On Dp brane (9-p) scalars]

Very important to phenomenology.

Rank reduction

<u>Orientifold plane</u>

Op-plane in Type II on $\underbrace{\mathbb{R}^{1,p}}_{0,\dots,p} \times \mathbb{R}^{9-p} / \mathbb{Z}_2$

p: same p as Dp-brane.

$\mathbb{R}^{9-p}$: Space transverse to this Op-plane; $p+1, \dots 9$

p even in IIA

p odd in IIB

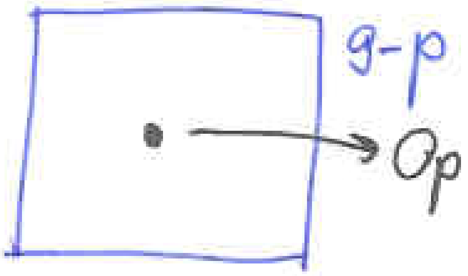
divide by a $\mathbb{Z}_2$ symmetry (mirror)

Coordinates X_i in 9-p dim. $X_i \to -X_i$

reversing orientation on string $\quad i = p+1, \dots, 9$

local coord. on fund. string z $\quad z \to \bar{z}$

Op is the $\underbrace{\text{fixed point}}_{\text{origin in (9-p) space}}$ of this action



preserves 16 SUSYs, carries $\underbrace{\text{charge same}}_{\text{not equal value of charge, but the same gauge field.}}$ as Dp-brane, $\pm 2^{5-p}$.

String Theory 3/03/08

T	4-5, 5-6	
W	10-11, 11-12	
Th	10-11	
F	4-5	

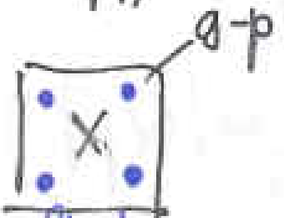
- Supermultiplets in various dim 32, 16, 8, 4, 2
- Branes ending on branes
- tension formulae & duality
- Higgs mechanism $\leftrightarrow$ branes

Orientifolds fundamental string; spacetime; Branes.

Fund. string: $X^\mu(\sigma)$ $z = \sigma_1 + i\sigma_2$ $z \to \bar{z}$

orientifold: reflection of space-time

" on worldsheet.



$$X^\mu(z) \to -X^\mu(\bar{z}) \quad \mu = p+1, \dots, 9$$

$$X^\mu(z) \to X^\mu(\bar{z}) \quad \mu = 0, \dots, p$$

reflections of D branes (Op) in a mirror

Op-plane — fixed plane of this action

origin in 9-p dim

Brane perspective: root system (dynkin diagram)

B, C, D systems

Spacetime perspective:

orientifold is a charge object under same forms that couple to Dp-branes. In units that Dp carries +1, Op carries $\pm 2^{p-5}$.

Tension is equal to the charge (x the unit of length)

Q_p charge of Op-plane then

$$T_p = Q_p \underbrace{\frac{1}{g_s \ell_s^{p+1}}}_{\text{tension of Dp-plane.}}$$

BPS objects for Dp-brane $\frac{T_p}{(g_s \ell_s^{p+1})^{-1}} = |Q_p|$

Anti-Dp brane $\overline{D_p}$ carries charge -1

carries tension $+1$.

Positive Op-plane has +ve tension

Negative charge Op-plane has −ve tension

③

Orientifold is not a dynamical object (it does not propagate). It is also an object with gravitational attraction (+ve) / rephlsion (−ve).

Presence of all other fields cancel the gravitational repulsion of O_p with −ve charge.

If spacetime has O_p^- plane, then we can put Dp brane on it $\Rightarrow$ increase tension & reduce the magnitude of the charge. We can put as many Dp as we'd like.

$$\begin{cases} \text{Total charge} = \underbrace{Q(O_p)}_{-2^{p-5}} + \underbrace{Q(nD_p)}_{+\,n} \\ \text{Total tension } \underbrace{-2^{p-5} + n}_{\text{reduce the repulsion}} \end{cases}$$

What happens if this is zero.

If the spacetime non-compact, then we have only 1 op-plane, But if it is compact, you have more, e.g.

S^1 2 op-planes

S^2 4 op-planes

⋮

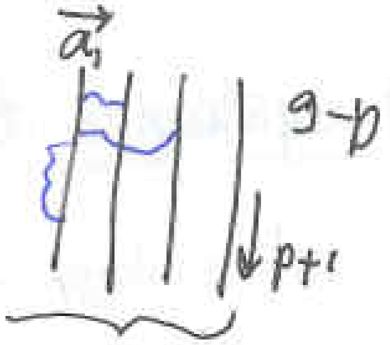
Supersymmetry ⇒ energy or energy density to be zero.

In SUSY backgds, want total tension to be zero.

If Op^+ planes present, then problem for SUSY. Can get away if space is non-compact

Root systems:

A_n root system $\longleftrightarrow$



n+1 parallel Dp-brane.

$(n+1)^2$ possible boundary conditions for fund. string. $\longrightarrow$ Chan-Paton factors

M_{ij} $\quad i, j = 1, \dots, n+1$

i: where string begins

j: where string ends.

Set of masses: Set of all distances between branes:

$\vec{a}_i \longrightarrow (p+1, \dots, 9)$

$i = 1, \dots, n+1$

$|\vec{a}_i - \vec{a}_j|$

- $(n+1)^2$ masses.
- $(n+1)$ massless corresponding to $U(1)^{n+1}$ gauge sym. (never broken for any $\vec{a}_i$).

$\downarrow$

It can be broken if we put in transver brane

Higgs mech.

n^2+n non-zero masses (W-bosons)

root systems describe W-bosons

A_n root system for $SU(n+1)$

define $(n+1)$ vectors in $\mathbb{R}^{n+1}$;

basis $e_i = (0, \dots, 0, \underset{\uparrow \, i^{th}}{1}, 0, \dots, 0)$

$E_i = e_i - e_{i+1}$ simple roots n

---

whenever write e_i, pick one of the branes

$\Rightarrow$ E_i is the distance between two adjacent branes.

---

positive roots $e_i - e_j \quad i<j$

positive linear combination of simple roots.

$e_1 - e_3 = (e_1 - e_2) + (e_2 - e_3)$

$= E_1 + E_2$

---

negative roots $e_i - e_j \quad i>j$

roots (+ve & -ve) $n^2 + n$.

$\hookrightarrow$ W-bosons.

$n=1$: e_1, e_2 $\quad e_1=(1,0)$ $\quad e_1-e_2$ positive

$\qquad\qquad\qquad e_2=(0,1)$ $\quad e_2-e_1$ negative.

$\alpha = \{\text{set of all roots}\}$

$$m_\alpha = \left|\sum_{i=1}^{n+1} a_i \cdot \alpha\right|.$$

String theory 4/03/08

Orientifolds

①

- SO, Sp gauge theories
- rank reduction
- perspectives: $\boxed{\text{w.s.}}$ $\mathbb{Z}_2$ symmetry

$\partial X^\mu \bar\partial X_\mu$.

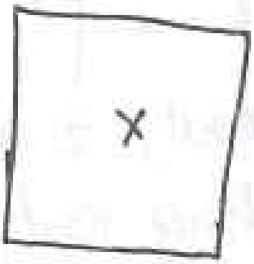
$z \to \bar z$

$X_\mu \to -X_\mu$.

Op-plane for each know Dp-brane

$\boxed{\text{space-time}}$ - charge $\pm 2^{p-5}$

- tension (can be $-ve$ $\Downarrow$ gravitational repulsive)



compact: end of the world $\downarrow$ space not time

$\boxed{\text{brane}}$ - root lattices

$\vec{a}_1$ $\vec{a}_{n+1}$

$n+1$

A_n root lattice, $\mathbb{R}^{n+1}$ $e_i = (0,\dots,0,\overset{i}{1},0,\dots)$

orthogonal to $(1,\dots,1)$ $E_i = e_i - e_{i+1}$ basis (simple roots) $\Downarrow$ $\boxed{SU(n+1) \text{ not } U(n+1)}$

positive roots $e_i - e_j$ $i<j$

negative roots $e_i - e_j$ $i>j$ $\Big\}$ n^2+n

$SU(n+1) \times U(1) = U(n+1)$

Orthogonality to $(1,1,\dots,1)$ $\Rightarrow$ c.M is fixed

The roots of A_n are 1-1 correspondence with W-bosons.

The length of the root is $\sqrt{2}$

$$\underset{\text{dim Adj } U(n+1).}{\overset{\shortparallel}{(n+1)^2}} \text{ possible strings}$$

$$e_i - e_j = \sum_{k=i}^{j} E_k$$

$$\text{tr } E_i = 0$$

$$m_{ij} = |\vec{a}_i - \vec{a}_j|$$

$\alpha \in \{\text{set of roots}\}$

$$m_\alpha = \left| \sum_{i=1}^{n+1} \vec{a}_i \alpha_i \right|$$

For every root, $\exists$ a corresponding root.

generalize to all possible root systems

	dim Adj.
$A_n - SU(n+1)$	n^2+2n
$B_n - SO(2n+1)$	$n(2n+1)$
$C_n - Sp(n)$	$n(2n+1)$
$D_n - SO(2n)$	$n(2n-1)$

$E_6, 7, 8$

G_2

F_4

$B_n : \mathbb{R}^n \quad \underbrace{E_i = e_i - e_{i+1}}_{n-1}$

$$E_n = e_n$$

This system has <u>an origin</u> whereas A_n system has no origin reference.

B_n: $(\pm 1, \cdots, 0)$ $2n$ — length 1

$(\underset{i}{\pm 1}, 0, \cdots, 0, \underset{j}{\pm 1}, 0, \cdots, 0)$ $2(n^2-n)$ — length $\sqrt{2}$

$\Big\} 2n^2$

length α^2

$\boxed{\text{Def}^{\underline{n}} \text{ If all roots have the same length, it is a simply laced algebra}}$

C_n: $\mathbb{R}^n$, $\underbrace{E_i = e_i - e_{i+1}}_{n-1}$ $E_n = 2e_n$

root system
$(0, \underset{i}{\pm 1}, 0, \cdots, 0, \underset{j}{\pm 1}, 0, \cdots, 0)$ $2(n^2-n)$ — length $\sqrt{2}$

$(0, \cdots, 0, \underset{i}{\pm 2}, 0, \cdots, 0)$ $2n$ — 2

$\Big\} 2n^2$

D_n: $\mathbb{R}^n$, $E_i = e_i - e_{i+1}$, $E_n = e_{n-1} + e_n$ simple roots

$(0, \cdots, 0, \pm 1, 0, \cdots, 0, \pm 1, 0, \cdots, 0)$ $2(n^2-n)$.

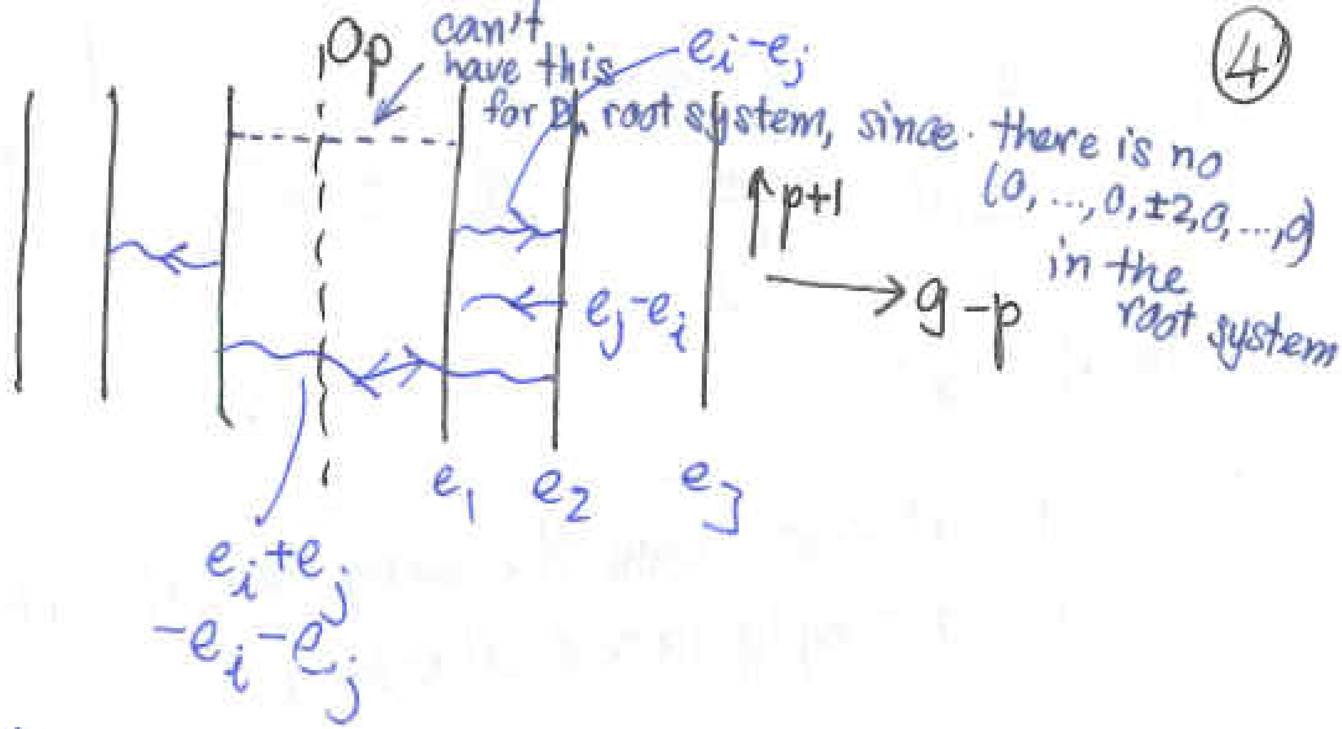
dim Adj = rank + # roots

A_n: $n^2+2n = n + (n^2+n)$

B_n: $n(2n+1) = n + 2n^2$

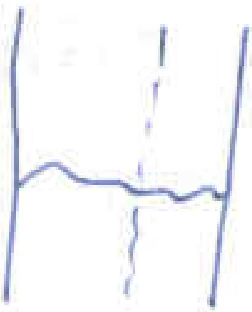
C_n: " " "

D_n: $n(2n-1) = n + 2(n^2-n)$.



<u>Rule</u>: No strings between branes & their images.

- brane realisation of D_n-algebra



$\mathbb{Z}_2$ projection removes the zero mode of this string
$\Rightarrow$ even if the brane coincide with its image (i.e. with orientifold), there is no massless spectrum corresponding to it.

⑤

Spectrum

$$m_\alpha = \left| \sum_{i=1}^{n} \vec{a}_i \alpha_i \right| , \quad m_{\pm i, \pm j} = \left| \pm \vec{a}_i \pm \vec{a}_j \right|$$

moduli sp. of vacua.

= modulispace of n points in $9-p$ dimensions

dimension is $n(9-p)$

A_n alg $\quad \mathcal{M} = \dfrac{(\mathbb{R}^{9-p})^{(n+1)}}{S_{n+1}}$

$$\mathcal{M}_{D_n} = \frac{(\mathbb{R}^{9-p})^n}{\underbrace{S_n \ltimes (\mathbb{Z}_2)^{n-1}}_{\text{Weyl gp. of } D_n}} = \frac{(\mathbb{R}^{9-p})^{\text{rank}}}{W}$$

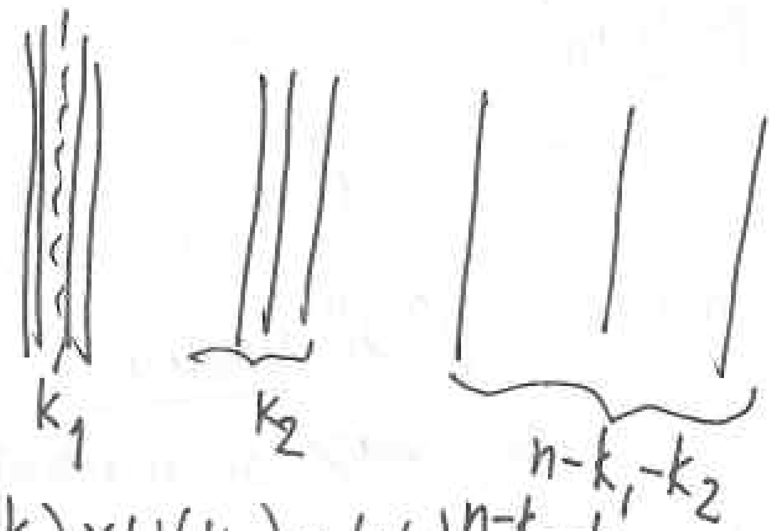
In general $\quad \boxed{\mathcal{M} = \dfrac{(\mathbb{R}^{9-p})^{\text{rank}}}{W}}$

D_n system

$U(k) \times U(1)^{n-k}$

total rank n.

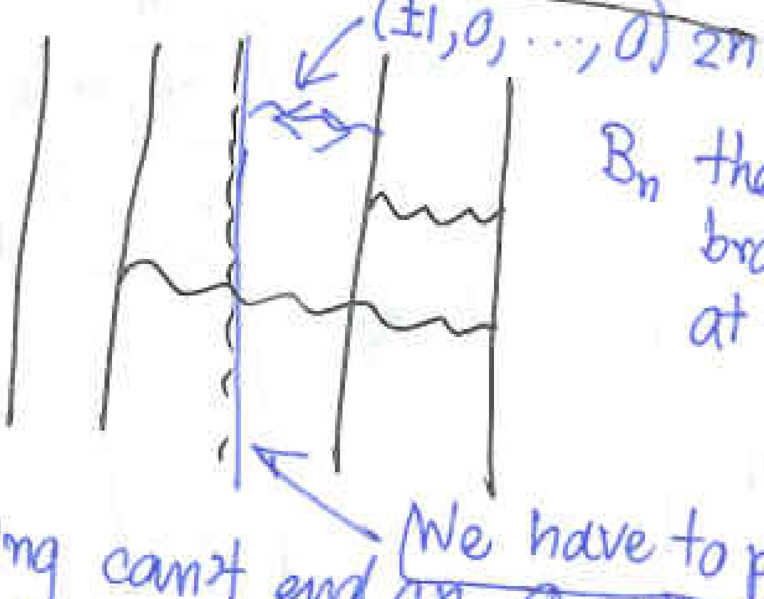
$SO(2k) \times U(1)^{n-k}$



$$SO(2k_1)\times U(k_2)\times U(1)^{n-k_1-k_2}.$$

$SO(2)$ dim 1
$U(1)$ dim 1
} no extra massless particle.

B$_n$ system

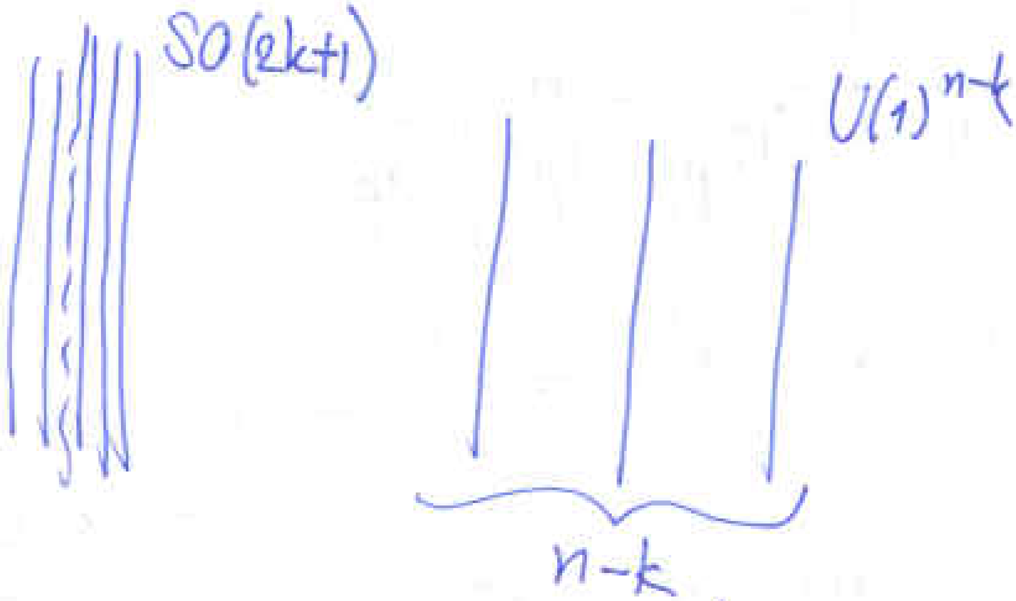


B_n there is a brane stuck at Op-plane.

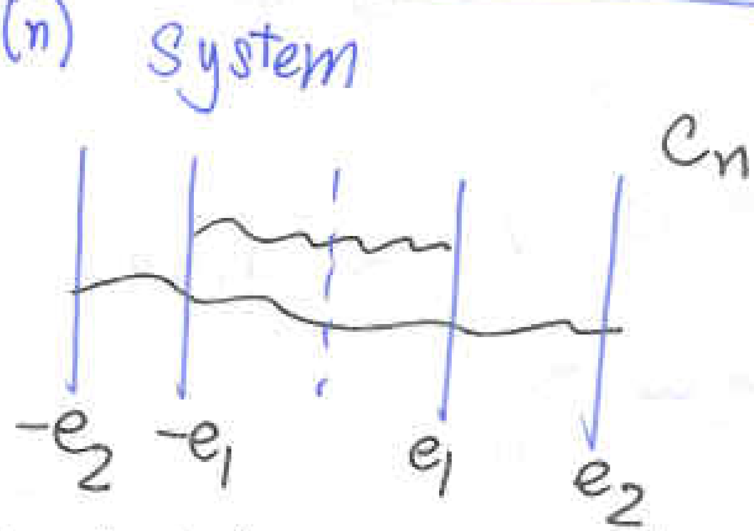
We have to propose existence of a D-brane on Op-plane

(string can't end on Op-plane because Op-plane is not dynamical there can't be a gauge field on Op-plane $\xrightarrow[\text{Gauss}]{}$ string can't end on Op-plane)

$$M = \frac{(\mathbb{R}^{9-p})^n}{S_n \times \mathbb{Z}_2^n}.$$ ← the brane at orientifold can't move



$C_n = Sp(n)$ system



No stuck brane on Op-plane, but a string ending on a brane & its image does have zero mode.

A "D_n-string" has no zero mode → string stretch between a brane & its image.

A "C_n-string" has a zero mode.

↓ Turn out to be just a choice of b.c.

Now we know how to characterise A_n, B_n, C_n, D_n

A_n,	B_n,	C_n,	D_n
No O_p	$O_p^- + D_p$	O_p^+	O_p^-

$\underline{p=9}$: $2^{p-5} = 2^{9-5} = 16 = \text{rank } SO(32)$ → 496

$= \text{rank } Sp(16)$ → not 496

charge of $O9^\pm + nD9$ branes

$$Q = n \pm 16$$

One situation $Q = 0 \Rightarrow n = 16$

$$O9^- \Longleftrightarrow D_n$$

Type I.

In 10d, A_{10} is the 10 form which couples electrically to D9 brane.

$$Q \int_{\text{space time}} A_{10} = 0$$ by gauss law

(charge must be absorbed by something otherwise it has no where to go $\therefore$ charge $= 0$).

Cosmological constant: proportional to Q

To keep it 0, must have $Q=0$.

otherwise, break SUSY.

<u>String Theory</u> (5/03/08) ①

Orientifold planes

$Op \quad p=1,\dots,9$

3 types $\longrightarrow B_n \longrightarrow O_p^- + \frac{1}{2}D_p \quad nD_p \quad SO(2n+1)$

stuck Dp-brane; physical Dp-brane

$C_n \longrightarrow O_p^+ \quad nD_p \quad Sp(n)$

can't be there ∵ it breaks SUSY and lead to inconsistency.

$D_n \longrightarrow O_p^- \quad nD_p \quad SO(2n)$

$p=9$: $n=16$ (Orientifold reduces SUSY charges by $\frac{1}{2}$)

$O9^- + 16\,D_9$ branes $\longleftrightarrow$ Type I string.

All strings have Neumann B.C, no Dirichlet.

+

Gravity multiplet

+ Vector multiplet

$\uparrow$

Type IIB/Ω

Ω: orientation reversal.

$z \to \bar{z}$

Actually, we can calculate 1-pt fn. (tadpole) of the 10-form A_{10}. QFT: only 1-pt fn. of a scalar field can be non-zero, but A_{10} is not a scalar, so 1-~~po~~point fn. of A_{10} must be zero. Since it $\propto$ charge Q, Q=0

① Gravitational <u>Anomaly</u>:

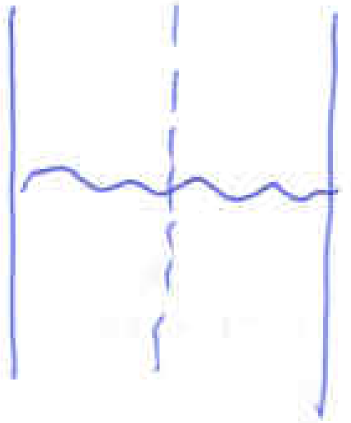
\# v. plets = 496

② tadpole (Gauss-law)

③ <u>Scale anomaly</u>:

total charge Q=0

$\Rightarrow$ total tension = 0.

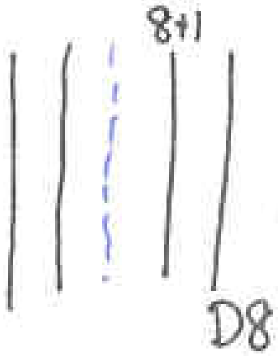


C_n has a zero mode
D_n has no zero mode

C_n violates: grav. anomaly, tadpole (gauss-law) & scale anomaly (+ve tension)

$p=8$: O8 planes
$0 \cdots 8$, x^9

9 direction



SO(2n) gauge theory in 8+1 dim

↳ codim 1 in 9-space

Since D8-brane is charges, by knowledge of electrostatic, electric field jump across the D8-brane

codimension 1 object (Domain wall) $\boxed{\text{a plate in 3-space}}$

Example: electron in 1+1d is a domain wall

$E_+ - E_- = Q$

$\int_{S^2} \vec{E} \cdot d\vec{A} = Q$

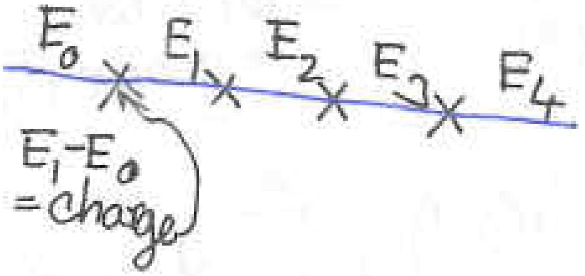
D8-brane couples electrically to A_9 with a field strength $F_{10} = *F_0$

Kinetic term $\sim \int F_{10} \wedge *F_{10}\, d^{10}x$

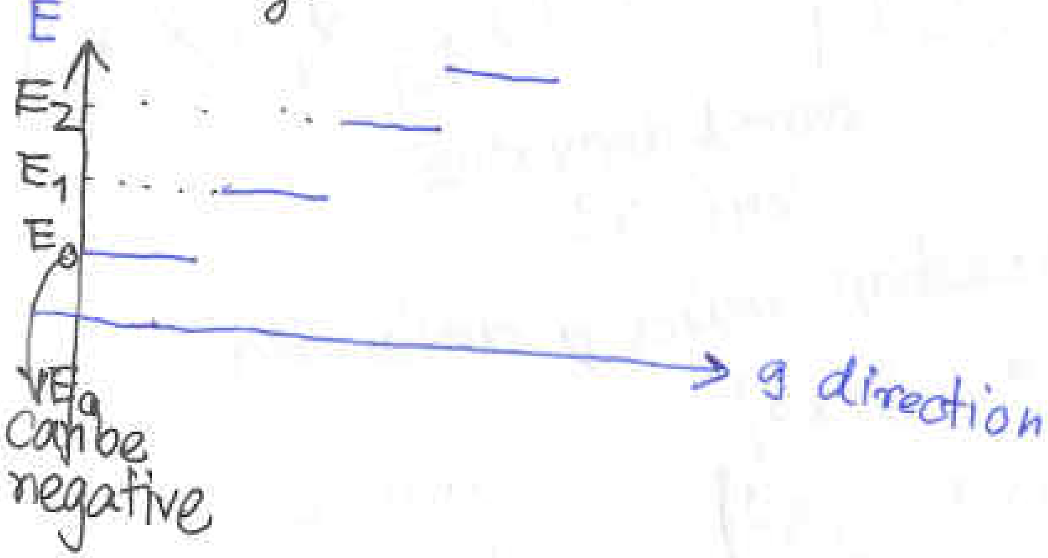
$$= \int d^{10}x\,(F_0)^2.$$ contribution to a cosmological constant in 10d

If $F_{10} \neq 0$, we have problem with SUSY.

F_0 is just the electric field E: $F_0 = E$

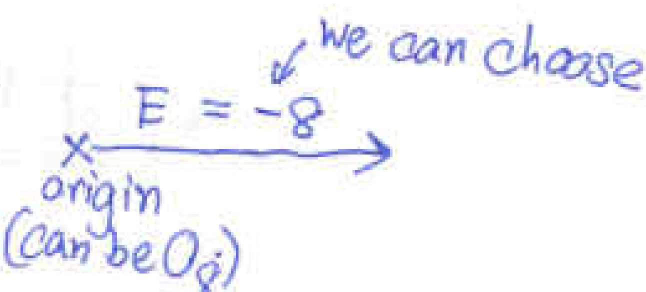


Electric field is piecewise const.



$\mathbb{R}^{1,8} \times \mathbb{R}/\mathbb{Z}_2$

$\mathbb{R}^{1,8} \times \mathbb{R}^+$



$\frac{1}{g_s}$ satisfies eq$^{\underline{n}}$ of motion:

g_s ← member of V-plet.

Type I $9+1$ g_s is in grav. multiplet,
but in 9 dim, g_s lives in vec-multiplet
$\mathbb{R}^{1,8} \times \mathbb{R}^+$

D8 is a source of elec. field
it must also be a source
of vec-multiplet

$$dF_0 = \delta(x9)$$
$$d{*}F_{10} = \delta(x9).$$

} second deriv. in gauge field

$$\Rightarrow \left(\frac{1}{g_s}\right)'' = \delta(x9), \quad \sum_{i=1}^{n} Q_i \cdot \delta(x^9 - x_i).$$

second derivative wrt. x9

Sign of δ function makes a difference

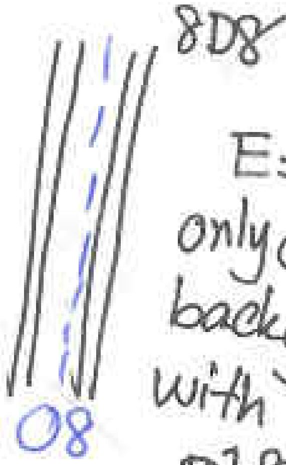
$$f''(x9) = \delta(x9)$$
$$f(x9) = \frac{1}{2}|x9|$$

convention

$\frac{1}{g_s}$ is piecewise linear. At some point $\frac{1}{g_s}$ will hit 0.

$$\frac{1}{g_s}(x^9)=0$$

Space has to be compact, unless the field is zero identically



E=0 only constant background with $\mathbb{R}^{1,8}\times\mathbb{R}^{+}$ fixed dilaton $\left(\frac{1}{g_s}\right)$

<u>Weyl group</u>

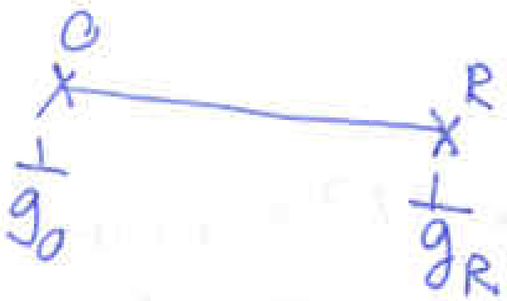
$W_{D_n}=S_n\ltimes\mathbb{Z}_2^{n-1}$

$W_{\substack{B_n\\C_n}}=S_n\ltimes\mathbb{Z}_2^{n}$

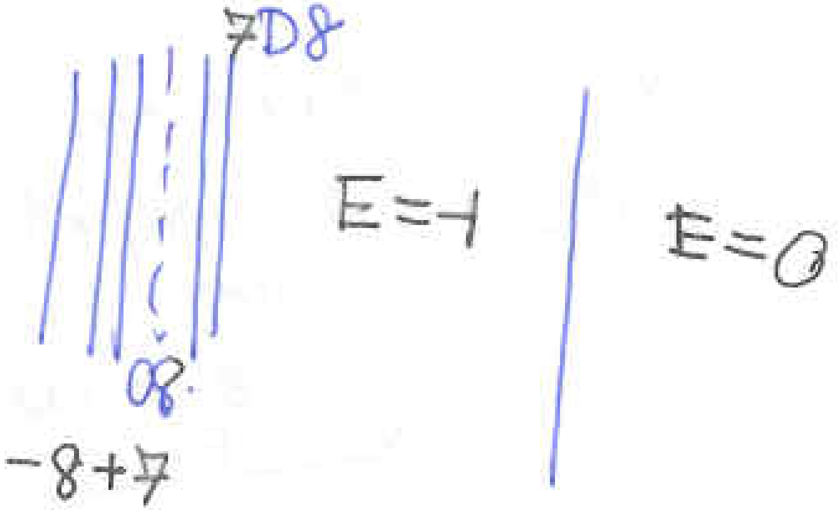
$E_n=e_{n-1}+e_n$

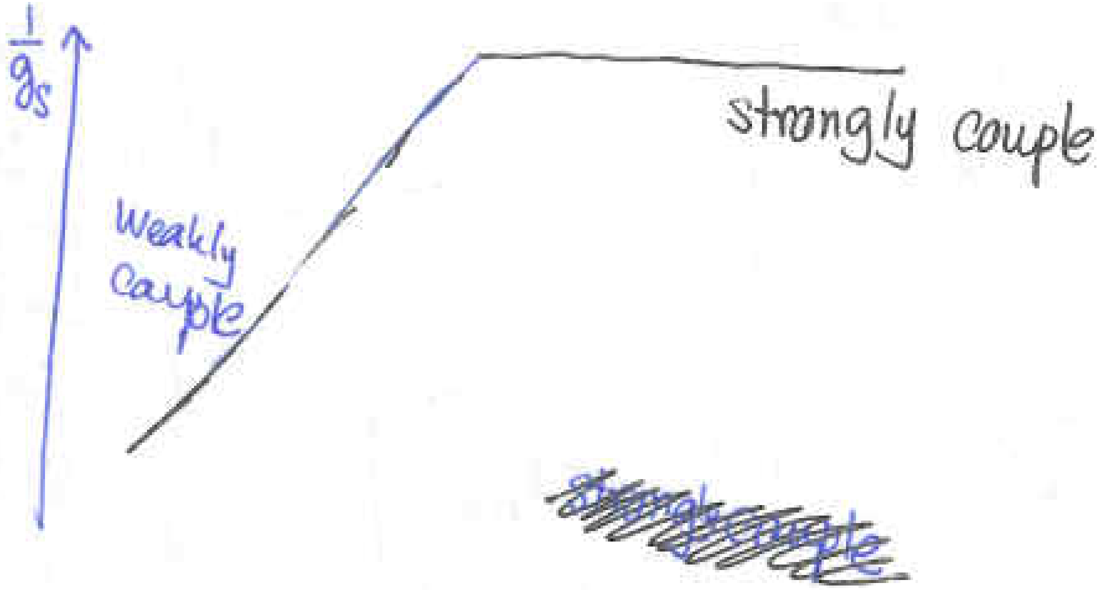
$\left.\begin{array}{l} e_{n-1}\to e_{n-1} \\ e_n\to -e_n \end{array}\right\}$ reflection on E_{n-1} permutation $n,n-1$

$$\frac{1}{g_s}(x^9)=\frac{1}{2}\sum_i Q_i\,|x^9-x_i|$$
$$+\frac{1}{2}\left(\frac{1}{g_0}+\frac{1}{g_R}\right)$$

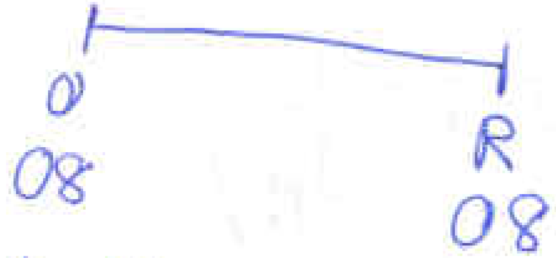


compact manifold.





$$\mathbb{R}^{1,8} \times S^1/\mathbb{Z}_2$$



T-duality takes D9-brane to D8-brane

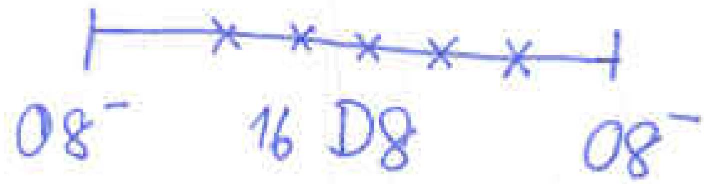
wrap $\frac{1}{R}$. wrap R

O9$^-$ splits to two O8$^-$ planes at the ends of the interval. such that the sum of charges match:

$$2Q_{O8^-} = Q_{O9^-} = -16$$

$$\Rightarrow \quad Q_{O8^-} = -8.$$

T-duality of type I leads to 16 D8 brane and 2 $O8^-$-planes



$$\frac{1}{g_s}(x^9) = \frac{1}{2}\sum_i Q_i |x^9 - x_i| + \frac{1}{2}\left(\frac{1}{g_0} + \frac{1}{g_R}\right)$$

charges from orientifolds & 16 D8-branes

This is called type I′ in literature

$$\mathbb{R}^{8,1} \times \underbrace{I}_{\parallel\, S^1/\mathbb{Z}_2} \leftarrow \text{interval}$$

$\mathbb{Z}_2$ is a combination of space reflection & orientation reversal.

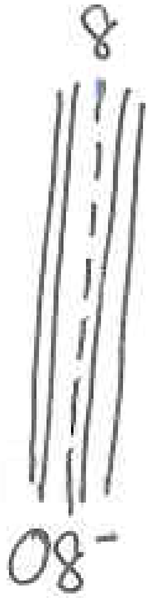
How many scalars? Each D8-brane 1 scalar.

vec multiple in 9 dim

$\frac{1}{g_s}, R$ → gravity multip

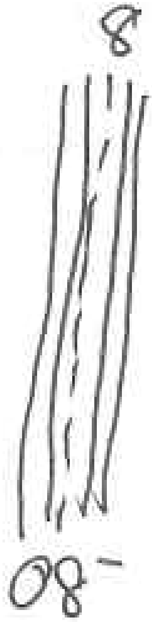
17 scalars from V-plet
1 scalar from G-plet

$$\boxed{M = \frac{SO(17,1)}{SO(17)} \times \mathbb{R}^+}$$



$E = 0$

$\frac{1}{g_s} = \text{const.}$



$SO(16) \times SO(16)$

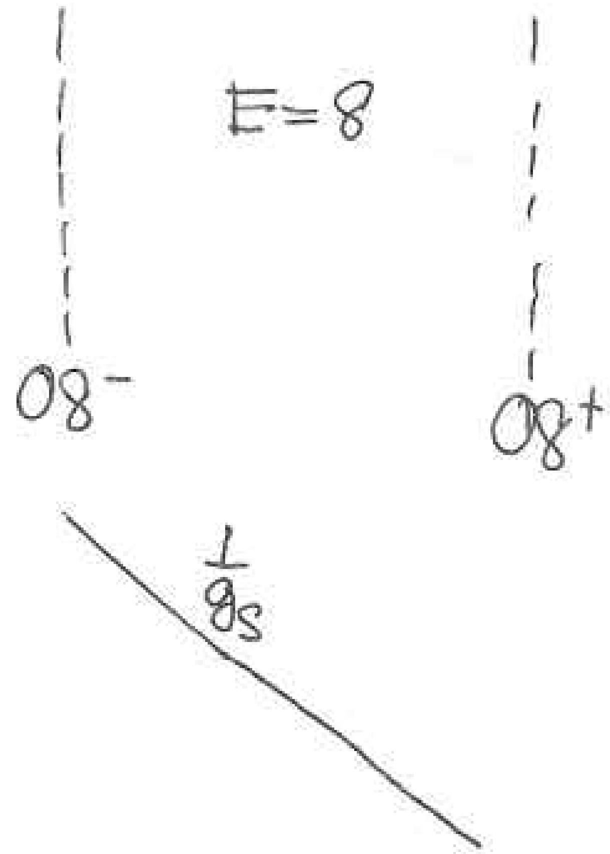
We can tune g_s to be very small and have perturbative background.

A-type gauge enhancement
D- " " "
E- " " "

$\boxed{\text{All simply laced gauge gps with total rank 17}}$

"
length of roots $= \sqrt{2}$

rank reduction



no D8 branes
1 G-plet
1 V-plet
$\mathcal{M} = \mathbb{R}^+ \times \mathbb{R}^+$

Vacua in 9 dim (16 susy) are characterised by # V-plets.

17 · V-plets: Het $SO(32)$ on S^1
9 - V-plets: Het $E_8 \times E_8$ on S^1
1 V-plet: Type I on S^1
Type I'

String theory (6/03/08)

① F 4-5
M 12-1
T 4-5, 5-6

Orientifold planes

Background: Minkowski $\mathbb{R}^{1,d}$, T^n,

preserve supersymmetry

$O9^-$
16 D9 $\longleftrightarrow$ Type I

Op-planes ($\frac{1}{2}$ SUSY)

$O8^-$ $O8^+$

16 D8

$O8^-$ —— $O8^-$

$\frac{SO(17,1)}{SO(17)} \times \mathbb{R}^+$, all simply laced gauge groups of rank 17 can appear on mod. sp.
$E_8 \times E_8 \times SU(2)$, $SO(34)$, $SU(18)$.

Even the Op-planes & branes backreact on backgd. the groups of the mod. space are still the same, but the metric on mod. sp. changes

$O8^-$ 8 D8 —— $O8^-$ 8 D8

string coupling is constant

How to get $E_8 \times E_8 \times SU(2)$, $SO(34)$, $SU(18)$?

Heterotic on S^1 $\left\{ SO(32), E_8 \times E_8 \right.$

Narain moduli space

E$_8$ root system in $\mathbb{R}^8$

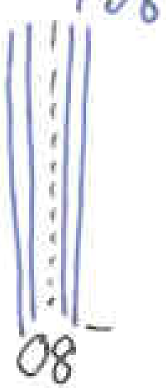
8 simple roots: based on D_7

$E_n \supset D_{n-1} \times U(1)$.

$E_1 = e_1 - e_2, \dots, E_6 = e_6 - e_7, E_7 = e_6 + e_7$

$E_8 = \frac{1}{2} \sum_{i=1}^{8} e_i$ → can be associated with $U(1)$

240 roots

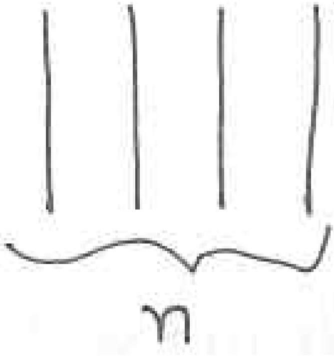


enhanced symmetry

$= SO(14)$ symmetry

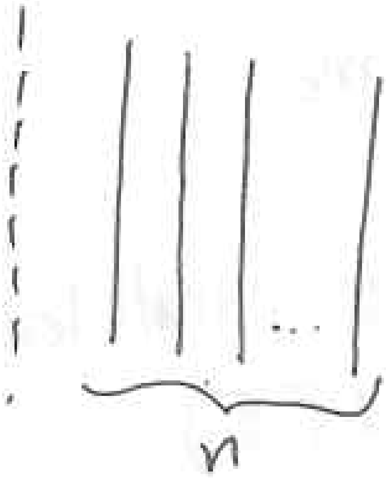
$E_8 \supset SO(16) \supset SO(14) \times U(1)$.

have to tune 7 parameters.



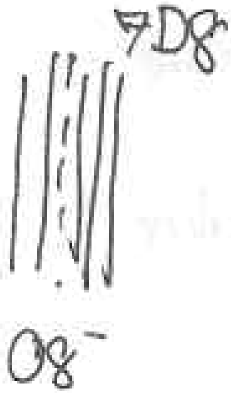
have to tune $n-1$ pararam. to get $U(n)$ enhanced sym.

$n-1=$ rank of $SU(n)$



Have to tune n pararams. to get enhanced $SO(2n)$.

n params $=$ rank of $SO(2n)$.



tune 7 params.

8th parameter is $\frac{1}{g_s l_s}$

in V-plet, gauge field has dimension 1.

A scalar field has dimension $\frac{d}{2}-1$.

$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2$

$[\mathcal{L}]=d$

$[\partial]=1$

A gauge coupling has dimension $2-\frac{d}{2}$.

$\mathcal{L}=\frac{1}{g^2}\mathrm{tr}F_{\mu\nu}^2$

$[F]=2$ $[\frac{1}{g^2}]=d-4$

$[g]=2-d/2$

$[g\phi] = 1.$

This is a natural object to be in the V-plet.

$\frac{1}{g_s l_s}$ = tension of D0-brane.

If $\frac{1}{g_s l_s} = 0$, new massless particles
(D0 and $\overline{D0}$)
charge +1 −1

O8$^-$ with g_s → the symmetry is $U(1)$.

g_s is a real scalar in V-plet

Type IIA on $\mathbb{R}^{1,8} \times S^1/\mathbb{Z}_2$

$\frac{1}{g_s l_s} = 0$ two new massles particles
(because they can't live in gravity multiplet).

2 new V-plet.

3 V-plets with charges 0, +1, −1.

by Higgs mechanism $U(1) \to SU(2)$ gauge symmetry

3 V-plets

$\uparrow$

dim of Adj $SU(2) = 3$.

Exercise For n D8-branes on $O8^-$ & $\frac{1}{g_s l_s} = 0$ the gauge sym. is E_{n+1}.

String Theory (7/03/08) ① M 12-1 T 4-6

$O8^-$ plane, associated param. $\frac{1}{g_s}$

vev order parameter

gauge symmetry $U(1) \longrightarrow SU(2)$

Adjoint Higgs mechanism, strong coupling effect (when $g_s \to \infty$).

D0-brane becomes massless when $g_s \to \infty$.

- tension (mass) formula for D_p-brane is protected from quantum corrections (exact for all orders of pertbn)
- Type I' $\longleftrightarrow$ Type IIA on $\mathbb{R}^{1,8} \times S^1 / \mathbb{Z}_2$

16 D8

$O8^-$ —— $O8^+$

All simply laced gauge gps of total rank 17 are allowed. e.g. $SO(34)$, $SU(18)$

$$\left[\mathcal{L} = \underbrace{\frac{1}{\alpha'} \partial X \cdot \partial X}_{\text{kinetic terms}} \quad \text{stringy corrections are determined by } g_s \right]$$

E_n : $n-1$ D8 branes on $O8^-$ plane with $\frac{1}{g_s} \to 0$.

$E_n \supset U(1) \times D_{n-1}$

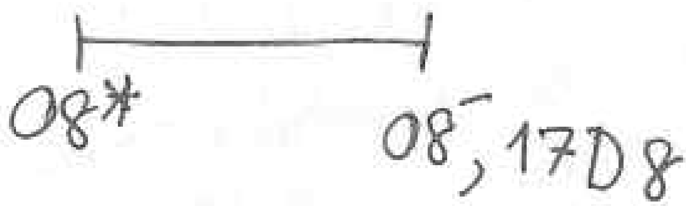
7 D8-branes

$$\underbrace{SO(14) \times U(1)}_{64,\,\overline{64},\,120} \longrightarrow E_8$$

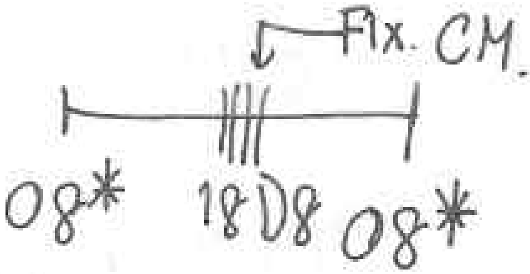
spinor rep. of rotation gp.

Adj of SO(14)

$E_1 \longrightarrow O8^* + $ D8-brane.



Tune 17 params.
(No string coupling left)



8 para + 8 para + 1 para = 17 para.

$SO(34)$, $SU(18)$, $E_8 \times E_8 \times \underline{SU(2)}$

tune 7 D8-branes.

7 D8-brane

can put the other two on top of each other.

O9, O8 ③

$$\overset{16\,D8}{\vdash\!\!-\!\!-\!\!-\!\!\dashv}$$

$O8^-$ $O8^-$

$\vdash\!\!-\!\!-\!\!-\!\!\dashv$ 1 v-plet

$O8^-$ $O8^+$ $\mathcal{M} = \mathbb{R}^+ \times \mathbb{R}^+$

cantune the g_s to be ∞ at this size. v-plet G-plet

$U(1)$, $SU(2)$.

What is the behaviour of type I′ at $g_s \to \infty$

Type I′ = Type IIA on $\mathbb{R}^{8,1} \times S^1/\mathbb{Z}_2$

8D8 $E=0$ 8D8

$O8^-$ $O8^-$ g_s anything

$SO(16) \times SO(16)$.

M-theory on $S^1_R \overset{S}{\longleftrightarrow}$ Type IIA g_s, ℓ_s

ℓ_p, R

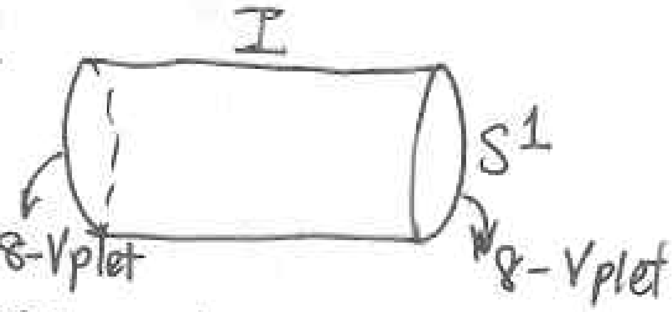
$$\left.\begin{aligned} g_s \ell_s^3 &= \ell_p^3 . \\ g_s \ell_s &= R . \end{aligned}\right\} \text{Any manifold not just flat}$$

Type I′ on $\mathbb{R}^{8,1} \times I \longleftrightarrow$ M-theory on $\mathbb{R}^{8\times1} \times I \times S^1$

M-theory on cylinder $= S^1 \times I$

Type I' on $R^{8,1} \times I$

$\leftrightarrow$ M theory on cylinder



is a theory with 16 Supercharges, 17 V-plets

Therefore, another point in moduli space of 9 dimension theories with 17 V-plets.

17+1 real moduli Type I on S^1 $\frac{1}{g_s}$ is in G-plet

17+1 real moduli Type I' $\frac{1}{g_s}$ V-plet, R in G-plet

Het SO(32) on S^1

$E_8 \times E_8$ on S^1.

} 17 V-plets.

Recall: duality between type I & Het SO(32) in 10d. $g_I = \frac{1}{g_H}$

- No scalar in V-plet in 10 dim:

$\Rightarrow$ No v-plet moduli space

- 1 scalar in G-plet = dilaton.

$\mathcal{M} = \mathbb{R}^+$

String theory backgrounds are typically flat.
Take backgd. with general metric
→ can do worldsheet calculations where string propagate in that backgd.

$$\mathcal{L} = \underbrace{G^{\mu\nu}(X)}_{\text{coupling}} \partial X_\mu \bar{\partial} X_\nu .$$

~~gauge~~ coupling depends on scalar fields
= field dependent ~~gauge~~ couplings.

Compute β-function eqns. to each of these couplings

$\beta_{\mu\nu} = R_{\mu\nu}(G) = 0$

Insist that the ~~code~~ theory to be scale independant

≡ CFT background.

General coord. invariant on worldsheet
⇒ conformal invariant.

$\underbrace{\text{Cylinder}}_{\substack{17\ V\text{-plet} \\ 2\ \text{boundaries}}},\ \underbrace{T^2, \mathbb{R}^2}_{32\ \text{SUSY's}},\ \underbrace{\text{Möbius}}_{9V\text{-plet}},\ \underbrace{\text{Klien bottle}}_{O8^+, O8^-}$ are flat

$AdS_a \times S^b$

$a+b = 10$
$ 11$

$\left.\begin{array}{l} R_{AdS_a} < 0 \\ R_{S^b} > 0 \end{array}\right\}$ total space is a good solution for CFTs.

Choose AdS backgd, must be compensated by +ve curvature backgd. S.

String Theory (10/03/08)

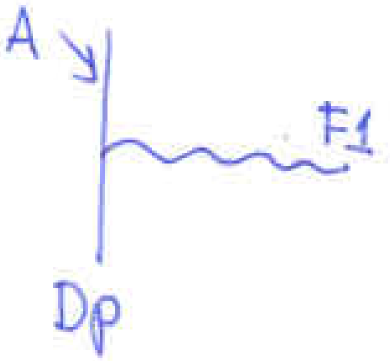
Branes ending on branes

Can D1 end on D5-brane?

need gauge field which absorb the flux lines satisfy Gauss' law.

F1 ending Dp, using T&S dualities

If there is a gauge field on the brane $\langle {E \atop M}$



Terminology: codim 1 domain wall $\to \phi''(x) = \delta(x^9)$ D8 in IIA

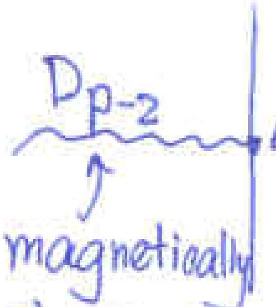
" 2 Vortex $\to \Delta\phi = \delta^2(x)$ D7 in IIB

" 3 monopole KK monopole in M-theory on S^1

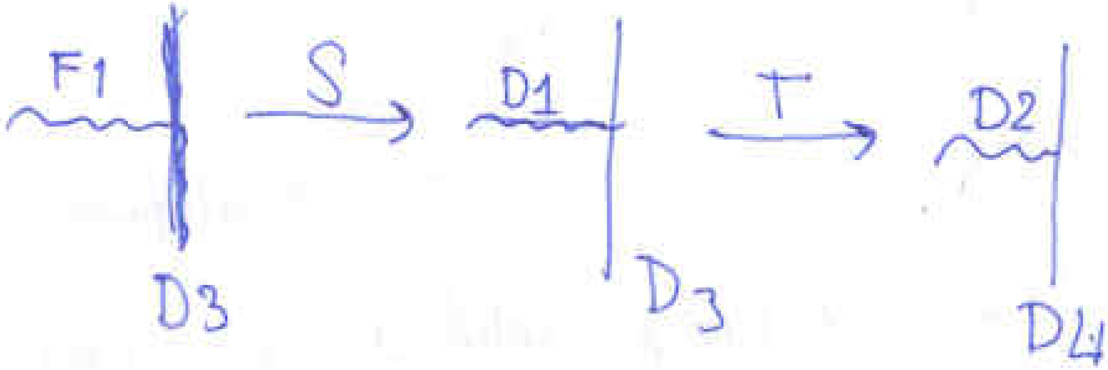
$$\int_{S^2} F = Q_M$$

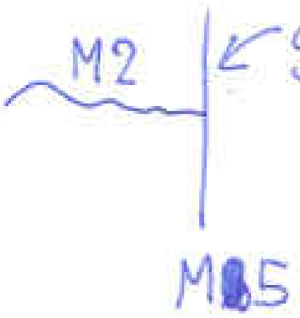
" 4 instanton $\to F\wedge F = \delta^4$ D1 inside D5 brane.

Monopole inside Dp-brane is a (p-3)-brane.



the boundary is (p-3) dim.





gauge field lives on M5 is 2-form

∵ end of M2 is string

2-form in 6d has 6 d.o.f. $\left(\begin{array}{c}\text{self dual}\\ + \\ \text{anti self-dual}\end{array}\right)$

↳ 3 scalars correspoing to self dual.

5 scalars in transverse directions.

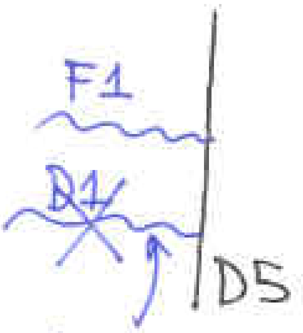
On Dp brane have bosonic d.o.f: A_{p-1} + scalars_{9-p}

Using S-duality

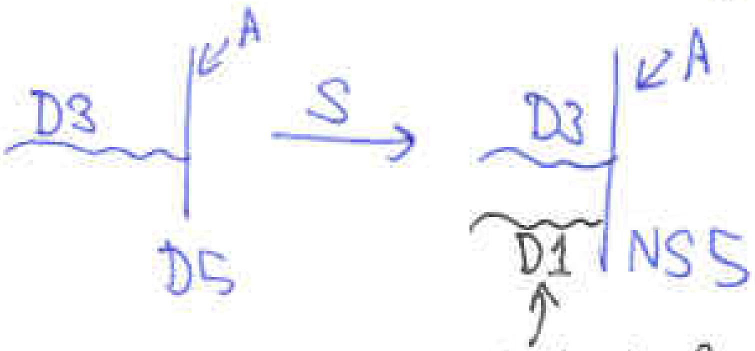
$$D4 \xrightarrow{S} M5, \quad B^{(2)} \ \& \ 5 \text{ scalars.}$$

Scalar in D4 $\xrightarrow[\text{promoted to}]{}$ two-form in M5

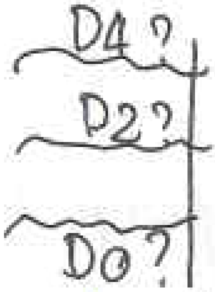
$\boxed{\text{String is a dyonic object in 6d}}$



This would give extra gauge field that we don't want $\Rightarrow$ can't fit into multiplet.



S dual of F1.



NS5 in Type IIA

even-brane

Euclidean D0 (can choose 1 direction to be space not time).

4 scalars on NS5-brane in IIA
+
1 compact $g_s l_s$.

M5 on S^4 transverse to M5

$0,\dots,5 \quad \underbrace{6\,7\,8\,9}_{\mathbb{R}^4} \quad 10$

compact

$\phi(x) \simeq \phi(x) + g_s l_s$

$0,\dots,5$

T-duality: Dp-brane end on NS5-brane.

even for IIA
odd for IIA

The end of D6 is 1+5 dim.

What's massless muliplet on NS5 in IIA?

$B^{(2)-}$, D2 ending on NS5 is again self-dual or anti self-dual.

Compact scalar is a 0-form gauge field that has E & M excitations.

$\swarrow$ $\searrow$

-1-brane. 3-brane.

$\downarrow$ $\uparrow$

D0 ending on NS5

bdry of D4-brane on NS5.

String Theory (11/03/08)

32 SUSYs, Moduli spaces

Branes (break $\frac{1}{2}$ SUSY)

locally will be the matter content of 8 SUSY & χral.

4 SUSY

non-chiral as a whole

Three intersecting branes

rank reduction, Op-planes Op^+, Op^-: 9dim: $O8^+$ $O8^-$

preserve $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of SUSY

When tilted, can have more configuration.

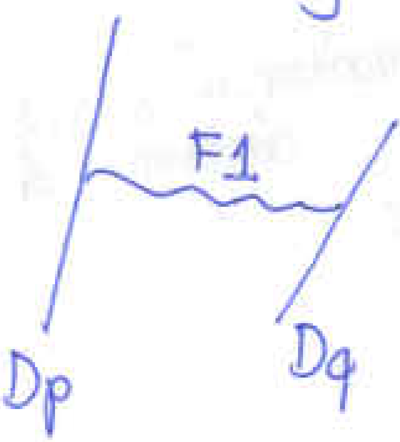
$$\int_{M5} C^{(3)} \wedge H^{(3)}$$

live on the world vol. of the brane.

Brane creation

A system Dp, Dq

What happens when they are mutually supersymmetric?



0 1 2 ... 9

DD
NN
ND
DN

total # of ND (DN) directions = 0 mod 4 are mutually SUSY.

$D5$ & $D9$ $\xrightarrow{\text{T dualise}}$ Dp & D_{p+4}

0 ... 5 $\quad \underbrace{0 \dots 5}_{N}$

$p=2$ $\quad$ D2 012

D6 $\underbrace{012}_{NN}$ $\underbrace{3456}_{ND}$ $\underbrace{789}_{DD}$ → mutually SUSY

D3 & D5

D3	012	3 N	D	D	D
D5	012	D	4 N	5 N	6 N
	3NN	4ND		3DD	

different orientation.

T dual.

→ Dp & D_{p+2}

p NN
4 ND
6−p DD → # of scalar in V-plet of 8 SUSY.

Cases with ND b.c. equal to 8

D1 & D9

D0 & D8

D(-1) & D7

$\boxed{\text{\# of DD directions is 1}}$

↑ criteria for brane creation

Dp D(8-p)

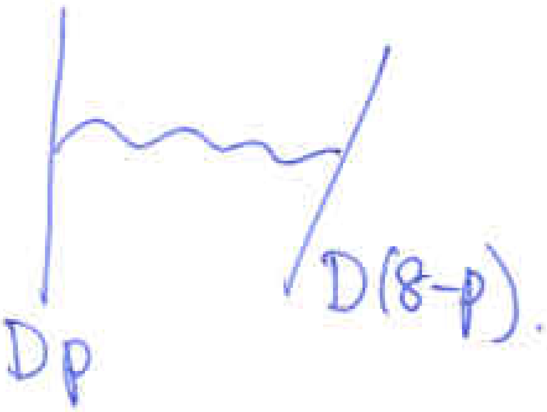
$$\begin{array}{ll} Dp & 0\,1\ldots p \\ D(8-p) & \underbrace{\ldots\quad p+1\ldots}_{ND}\ \underset{\substack{\uparrow \\ DD}}{8} \end{array}$$

In x^9-direction

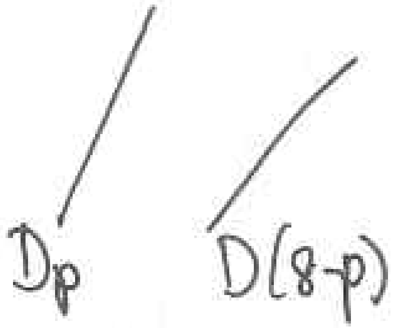
x^9: 

The prob. of collision is almost 1.
Such objects are said to <u>linked</u>.

# of DD directions is 1 there is brane creation.



1 Xral fermionic 0-mode
↓
orientation of branes.

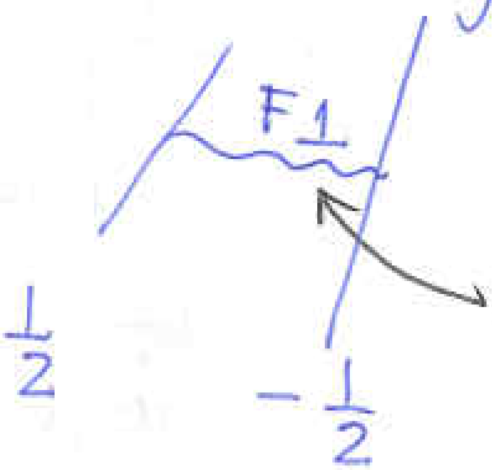


Dp induces a charge on D(8-p) brane which depends on position in X^9.

violate charge conservation!

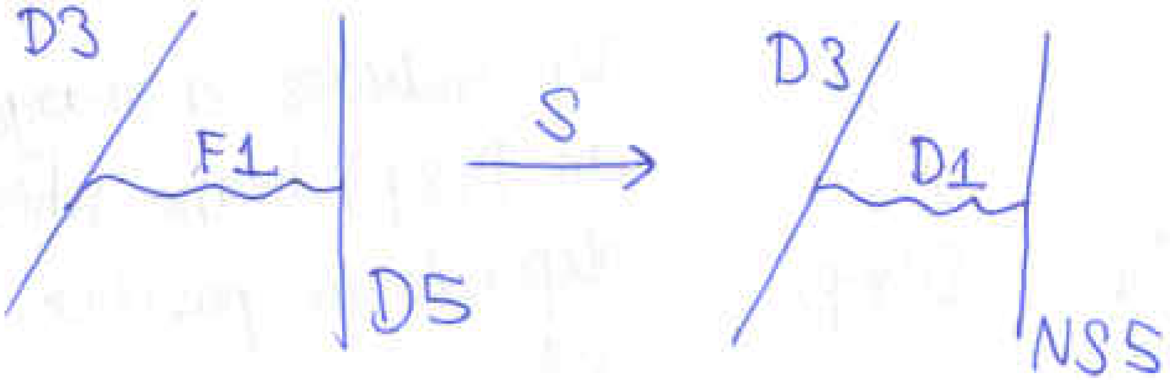
$\left(\frac{1}{2} \leftrightarrow -\frac{1}{2}\right)$

In order to preserve charge conservation, introduce a string: charged preserve.

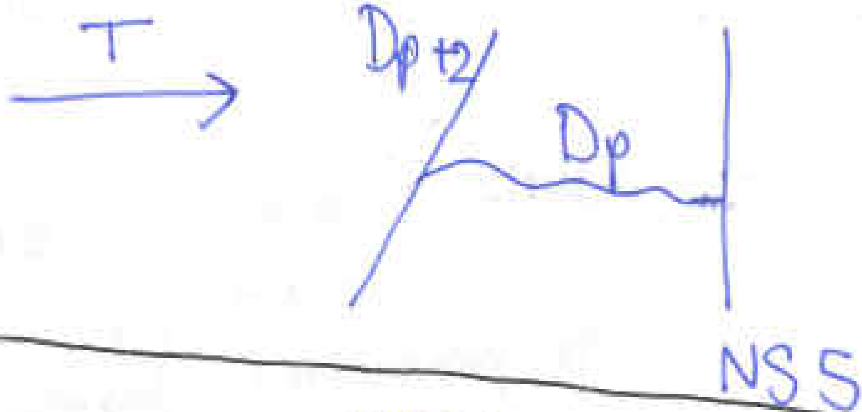


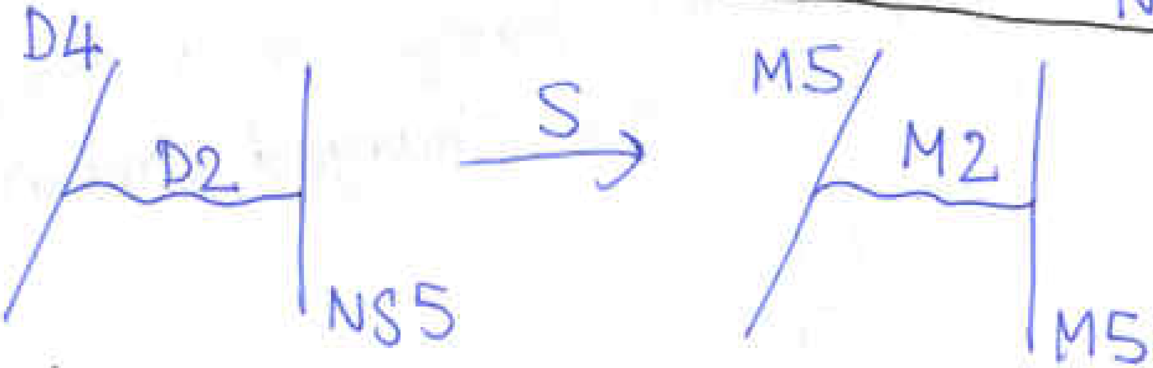
string creation!

The presence of fermionic mode gives rise to spinors in D0-D8 branes.



When D3 crosses NS5, D1 created



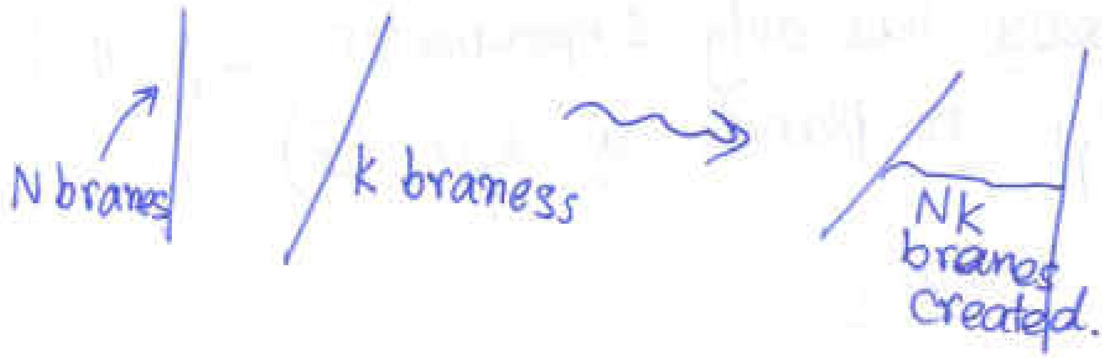


M_5 has energy density in 6 dim, whereas M2 has energy density in 3 dim. M_5 is more massive than M_2.
Can look at M_2 as a small pertbn to M_5:

M5 0 1 2 ... 5
M5 0 1 6 7 8 9

When these brane cross along X^{10} direction M2 is created along 01, and an interval along X^{10} between two M5-branes.

⑥



Type IIB moduli sp. $\frac{SL(2,\mathbb{R})}{SO(2)}$ $\tau = \frac{i}{g_s} + \frac{C^{(0)}}{2\pi}$.

$\downarrow$

2 dim

Type IIA $\mathbb{R}^+$

codim 2 (vortex)

$\rightarrow$ axion

The logarithmic potential

in condensed matter = vortices.

$\Delta C^{(0)} = \delta^{(2)}$.

$C^{(0)} \sim \log|X|$.

2π-rotation has to be across a branch cut.

D7 mag. charged wrt. $C^{(0)}$

$\Rightarrow$ vortex on the axion $\xleftarrow{\text{massless}}$ excit^{ts}.

axial field

In 4d, $\int \underbrace{B^{(2)}}_{J} \wedge F$ whatever. field strength.

"coupling" associated with charge.

(axion charge = string charge = vortex charge).

<u>M-theory</u> has only 1 dimensionfull parameter (1 scale)
M_{pl} (or planck length ℓ_p or G)

<u>no scalars</u>

When we say ℓ_p small or large, we have to compare to typical curvature: E.

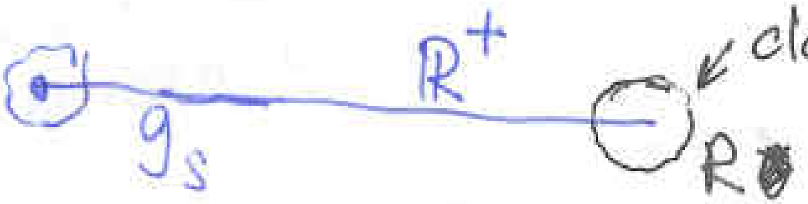
$$E\ell_p \ll 1 \rightarrow \text{11d SUGRA.}$$
$$\gg 1 \; ?$$
$$\sim 1 \; ?$$

<u>Type II theory</u> has ℓ_s, g_s

there is moduli space of vacua $\left\langle \begin{array}{l} \text{1 dim IIA} \\ \text{2 dim IIB} \end{array} \right.$

can take. $g_s \ll 1$

Have full control $\rightarrow$ just free field theory.



classical supergravity

compactification of M-theory.

By duality, $g_s \ell_s = R$

$R \gg \ell_p$.

We can write down Actions of branes but none of them is solvable.

Not much meaning to make G_N run (because G_N is dimfull)

$E^{d-2} G_N \ll 1$

$\gg 1$

~ 1

---

(1) How to predict the value of cosmological const. in string theory?

$\int d^D x \sqrt{G}\, \Lambda \qquad [\Lambda] = D$

natural grav. scale $\Lambda \sim c\, l_p^{-D}$

can be

- SM spont. breaking scal.
- SUSY breaking scale
- confinement scale
- χral sym. breaking scale.

No scale gives c of order 1.

small numbers in physics

$\frac{F_G}{F_E} \sim 10^{-40} \qquad \alpha \sim 10^{-2}$.

(2) Find a microscopic of M-theory.

(3) Multitude of vacua. – is there a "right" vacuum? match to SM

How many consistent non-SUSY background?

(4) How to do string cosmology? ↔ particle physics

5. strings $\longleftrightarrow$ Gauge fields
QCD

6. Dynamical SUSY breaking

String Theory Practice Exam

Amihay Hanany*

March 5, 2008

1. Write down the equations for branes ending on branes for the case of a Dp-brane ending on a NS5-brane. Please include:

   (a) The source equation.

   (b) The term in the action which indicates the interaction between the bulk gauge field and the field strength which is localized on the brane.

   (c) The corresponding Gauss Law, either in differential form or in integral form.

2. **11-dimensional supergravity.**

   (a) Write down the massless field content for this theory.

   (b) Count the number of degrees of freedom for each massless field and write them as irreducible representations of the little group.

   (c) Verify that the number of bosonic degrees of freedom is equal to the number of fermionic degrees of freedom.

3. Using tension formulas write down the S-duality relations in Type IIB. Write down all possible branes and their corresponding transformation laws under S-duality.

4. (a) Compute the matter content of the supergravity multiplet in 8 dimensions and with 32 supercharges. Specify the different fields and their multiplicity. What is the dimension of the moduli space of vacua?

*a.hanany@imperial.ac.uk

(b) Find the coset space, G/H which is the scalar manifold of the theory, with G the maximally non-compact version of the E_n algebra and H is it maximal compact subgroup.

(c) How many different 2-branes are there?

5. **SYM actions in various dimensions.**

   (a) Write down the classical, 16 supercharges, SYM action for a vector multiplet in 10 dimensions for an arbitrary simple gauge group G. Please include the kinetic term for both the gauge field and the gaugino.

   (b) By applying dimensional reduction to this theory compute the action for a vector multiplet in 9 dimensions. Find the moduli space of vacua for this theory.

   (c) Compute the mass of a W-boson for the special case $G = SU(2)$. Generalize this computation to W-bosons for a gauge group $G = SU(n)$.

   (d) Apply again dimensional reduction and compute the action in 8 dimensions. What is the moduli space of vacua in this case?

   (e) Write down the bosonic part of the action for any dimension $d < 10$.