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The Paradox of New Members: Strategic Foundations and Experimental Evidence

by

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Abstract

Power indices suggest that adding new members to a voting body may affect the balance of power between the original members even if their number of votes and the decision rule remain constant. Some of the original members may actually gain, a phenomenon known as the *paradox of new members*. We show that the paradox can occur as an equilibrium of a non-cooperative bargaining game based on the Baron-Ferejohn (1989) model of legislative bargaining. We implement this game in the laboratory and find empirical support for the paradox.

Keywords: voting, non-cooperative bargaining, power indices, experiments, paradox of new members

JEL classification: C70; C92

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1. **Introduction**

The paradox of new members was identified by Brams and Affuso (1976), and refers to the possibility that the addition of a new member to a voting body may increase the voting power of some original members, even if the voting body is deciding over the allocation of a fixed pie, and the relative voting weights of the original members and the decision rule remain constant. Brams and Affuso based their analysis on the application of Shapley and Banzhaf power indices to cooperative games. In this paper we show that their paradox emerges as an equilibrium outcome of a noncooperative legislative bargaining game. We also implement this game in the laboratory, and observe the paradox in our experiment.

The bargaining game we study uses the same bargaining procedure as the seminal Baron and Ferejohn (1989) model. Players are bargaining over a fixed budget. A player is randomly selected to submit a proposal, which is then voted on. If the proposal passes the proposal is implemented (closed rule), otherwise the game goes to the next period and the procedure is repeated. Whereas the Baron-Ferejohn model uses a one-person one-vote and majority voting rule, we examine weighted voting rules. As we show, introducing weighted voting into the Baron-Ferejohn model allows the paradox of new members to arise in the sense that equilibrium payoffs of an existing member may increase after the addition of a new member. Furthermore, the paradox arises for the original voting games studied by Brams and Affuso.

In our experiment participants bargain over a fixed budget, and we measure power by the average earnings of each voter type. We study three treatments, corresponding to the examples in Brams and Affuso. In our VETO treatment there are three voters, one of whom is a ‘strong’ player with veto power. Our ENLARGED treatment adds a fourth voter, so that the strong player loses his veto power. Theoretically the paradox should occur in the comparison of these two treatments, because the addition of a new member empowers the non-veto players. In our SYMMETRIC treatment the three voters have the same nominal voting weights as in VETO, but the quota is modified so that any two voters can form a winning coalition. Compared to SYMMETRIC, the addition of a fourth voter in our ENLARGED treatment increases the voting power of the strong player. Thus the paradox is predicted for this comparison as well.

There have been several previous experiments investigating the Baron-Ferejohn procedure (McKelvey, 1991, Diermeier and Morton, 2005, Fréchette et al., 2005a, 2005b, 2005c, and Kagel et al., 2007), but as far as we are aware none have studied the paradox of new members.[2] Fréchette et al. (2005a, 2005b) study several treatments similar to our SYMMETRIC treatment, while Kagel et al. (2007) study treatments similar to our SYMMETRIC and VETO treatments. As we report later, our results are remarkably consistent with theirs given the wide range of procedural differences between the experiments. The ENLARGED treatment has not been previously studied, and of course it is the comparison of this treatment with the others that allows us to test whether the paradox occurs. We find that the addition of a new member does indeed cause average earnings to change in the direction implied by the paradox.

The remainder of the paper is organized as follows. In the next section we describe our theoretical framework and apply this to three bargaining games. In section 3 we describe how we implement these games experimentally, and in section 4 we present our results. Section 5 discusses the results and offers concluding comments.

2. Strategic Foundations of the Paradox

2.1 Brams and Affuso's Examples

Brams and Affuso use three examples to illustrate the paradox of new members. The examples are given in Table 1.

Table 1. Brams and Affuso's Examples

	Votes controlled by player 1	Votes controlled by player 2	Votes controlled by player 3	Votes controlled by player 4	Votes needed to pass a proposal
VETO	3	2	2	–	5
SYMMETRIC	3	2	2	–	4
ENLARGED	3	2	2	1	5

[2] Montero et al. (2008) investigate the paradox using a relatively unstructured bargaining procedure whereby any player can put or amend a proposal on the table at any time. In their experiment the paradox is observed but, as we report later, there are some important differences between their results and those reported here.

In all examples player 1, who we will refer to as the 'strong' player, has three votes while players 2 and 3 (the 'weak' players) have two votes each. In the first example these three players comprise the voting body, and five votes are needed to pass a proposal. Here, the strong player has veto power, since no proposal can be passed without her votes, thus we refer to this as the VETO example. The second example is identical except that only four votes are needed to pass a proposal. As a consequence no player has a strategic advantage over the others, since any two members have enough votes to pass a proposal, and so we refer to this as the SYMMETRIC example. In the third example, ENLARGED, five votes are needed to pass a proposal, and there is an additional member with a single vote.

Clearly, the voting weights of strong and weak players are the same in all examples. Relative to VETO, ENLARGED holds the total number of votes required to pass a proposal constant. Brams and Affuso show that if voting power is represented by the Shapley value or Banzhaf index, the weak players have greater voting power in ENLARGED. Relative to SYMMETRIC, ENLARGED holds constant a simple majority decision rule. Brams and Affuso show that the power indices give the strong player greater voting power in ENLARGED.

2.2 A Non-cooperative Bargaining Procedure

Here we study a non-cooperative game based on extending the Baron-Ferejohn model to weighted voting games. Let there be a voting body with n voters, who are assumed to be selfish and risk neutral. There is a budget of 120 points to be divided between the voters by weighted majority rule. The bargaining procedure starts with a voter being randomly selected to make a proposal, with each voter having an equal chance of being selected. The proposer proposes a division of the budget such that each voter receives a nonnegative whole number of points, and the whole budget is distributed. The voters vote on the proposal simultaneously. If the total number of votes in favor is sufficient to pass the proposal, the proposal is implemented and each voter receives the amount of points specified in the proposal. Otherwise, a new proposer is selected, again each voter with the same probability. As in our experiment, we consider a finite horizon of 20 rounds and no explicit discounting. If no agreement is reached by the end of the 20 rounds all voters get zero.[3]

[3] We show in Appendix A that the paradox is also an equilibrium phenomenon in the standard model with a perfectly divisible budget, an infinite horizon and discounting provided the discount factor is sufficiently large.

There is a multiplicity of subgame perfect equilibria (cf. Norman, 2002). Our goal is not to analyze them all, but to show that the paradox *may* arise as an equilibrium phenomenon. In order to do this, we focus on equilibria in which strategies do not depend on past offers or voting behavior. We also assume that all voters vote as if their vote is pivotal in order to eliminate equilibria in which a proposal is accepted or rejected by all voters regardless of whether the voters actually prefer the proposal to pass, because no one of the voters can unilaterally change the outcome. We also assume that a voter accepts an offer if he is indifferent between accepting and rejecting. Furthermore, we assume that strategies are symmetric in the sense that voters of the same type play the same strategy and strategies also treat voters of the same type symmetrically.

Denote voter i's equilibrium payoff at the beginning of round t by y_i^t. Given our assumptions, this value does not depend on past offers and counteroffers. At time $t-1$, it is optimal for a voter to accept an offer that gives him at least y_i^t. The proposer will then look for the cheapest group of voters that control enough votes to achieve a majority, and offer them exactly these values (rounded up to the nearest integer), keeping the residual. Following common practice, we will say that the proposer "proposes to" the voters that are offered at least y_i^t, and, if the offer is accepted, we refer to these players as the "coalition partners".

2.3 The VETO Treatment

This game is analyzed by Winter (1996) with a perfectly divisible budget. Since there is a finite horizon, the game can be solved by backward induction.

Recall voters are assumed to care only about their own material payoffs. This implies that, in a subgame perfect equilibrium, a voter must accept any offer giving him a positive number of points at round 20, and he is indifferent between accepting and rejecting offers giving him 0 points. Given our assumption that a voter must accept if he is indifferent, the proposer allocates 120 to himself and 0 to everybody else in round 20, and $y_i^{20} = 40$ for all *i*.

In round 19, suppose the strong player is the proposer. He has a choice between making the best acceptable offer (i.e., offering 40 to one of the weak players) and an offer that will be rejected. Since 120 – 40 > 40, the strong player strictly prefers to make an acceptable proposal, and is indifferent between proposing to any of the weak players. Our symmetry assumption implies that he will propose to each of them with probability ½. As for the weak players, they strictly prefer to offer 40 to the strong player and keep 80 rather than make an unacceptable proposal and expect 40 on average in round 20. The expected payoff for the strong player at the beginning of round 19 is given by $y_1^{19} = \frac{1}{3}\left[120 - y_2^{20}\right] + \frac{2}{3} y_2^{20}$. Since $y_1^{20} = y_2^{20} = 40$, $y_1^{19} = \frac{160}{3}$. For a weak player, $y_2^{19} = \frac{1}{3}\left[120 - y_1^{20}\right] + \frac{1}{6} y_2^{20}$, or $y_2^{19} = \frac{100}{3}$.

In round 18, the same reasoning applies, except that since proposals must be integers player 1 must be offered at least 54, and players 2 and 3 must be offered at least 34. Denoting rounded up expected payoffs by $\overline{y}_i^{\,t}$, expected payoffs can be obtained recursively using

$y_1^t = \frac{1}{3}\left[120 - \overline{y}_2^{\,t+1}\right] + \frac{2}{3}\overline{y}_1^{\,t+1}$ and $y^t{}_2 = \frac{1}{3}\left[120 - \overline{y}_1^{\,t+1}\right] + \frac{1}{6}\overline{y}_2^{\,t+1}$, where $t = 1, \ldots, 19$ and $\overline{y}_1^{\,20} = \overline{y}_2^{\,20} = 40$.

Iterating this equation we obtain $\overline{y}_1^{\,2} = 118, \overline{y}_2^{\,2} = 2$. Thus, in round 1, the strong player offers 2 to one of the weak players if he is the proposer, and a weak player offers 118 to the strong player if he is the proposer. These strategies imply expected payoffs $y_1^1 = 118, y_2^1 = 1$.

2.4 The SYMMETRIC Treatment

This is the original game analyzed by Baron and Ferejohn. Under our assumption of symmetry of equilibrium strategies, the equilibrium is very simple. In round 20 the proposer will take all 120 points, and since all players are equally likely to be proposers $y_i^{20} = 40$ for all i. In the previous round, the proposer offers 40 to another player and keeps 80; under symmetry each player is equally likely to propose to the other two and we obtain $y_i^{19} = 40$ for all i, and indeed $y_i^t = 40$ for all i and t.

Before the game starts, the expected payoff of each player is 40. However, once the proposer for round one is selected, this player will propose that he gets 80 and another, randomly selected, player gets 40. This proposal will be passed.

2.5 The ENLARGED Treatment

If the strong player is selected to be proposer, he will seek the favorable vote of one weak player (the new member is never of use to the strong player). As for a weak player, he will seek the favorable vote of either the strong player or the other two players depending on which votes are cheaper to obtain. The new member will seek the favorable vote of two others.

Expected payoffs in round 20 are $y_i^{20} = 30$ for all i. In round 19, the strong player will offer 30 to one weak player (the symmetry assumption implies that he will be equally likely to propose to any of the weak players). As for the weak players, they strictly prefer to offer 30 to the strong player. The situation facing the new member is more complicated. He could offer 30 to any two others. Assuming that he chooses to propose to the two weak players, expected payoffs in round 19 are $y_1^{19} = 37.5, y_2^{19} = 33.75, y_4^{19} = 15$ and iterating this reasoning, allowing only integer offers, we obtain the minimal acceptable offers for each type in round one: $\bar{y}_1^2 = 45, \bar{y}_2^2 = 31, \bar{y}_4^2 = 15$. It turns out that if the new member breaks ties differently, then expected payoffs in round 19 will be different, but by the end of the iterative process the minimal acceptable offers in round one are unaffected.

Given these minimal acceptable offers it is clear that when the strong player is proposer he will randomize between (89,31,0,0) and (89,0,31,0), when player 2 is the proposer he will propose (45,75,0,0), when player 3 is the proposer he will propose (45,0,75,0), and when the new member is the proposer he will propose (0,31,31,58).

2.6 The Paradox of New Members

If proposers are chosen randomly with equal probability, then it is straightforward to compute the expected payoff of each type of player from the equilibria identified above. These are presented in Table 2.

Table 2. Expected Payoffs

	Strong Player	Weak Player	New Player
VETO	118	1	NA
SYMMETRIC	40	40	NA
ENLARGED	44.75	30.375	14.5

The paradox is displayed in two cases. First, beginning from the VETO treatment, the addition of the new player clearly benefits the weak player. Second, beginning from SYMMETRIC the addition of the new player benefits the strong player. Thus Brams and Affuso's paradoxes are also evident in this non-cooperative equilibrium.

Note, however, the assumptions lying behind this result. First our result is based on the analysis of a game of complete information. We assumed players are expected utility maximizers, that utility depends only on own point earnings, that players are risk neutral, and that all of this is common knowledge. Second, equilibrium analysis of this game provides many subgame perfect equilibria, and we adopt several refinement criteria in order to select one. Thus, it is not clear how empirically relevant the paradox may be.

3. Experimental Design and Procedures

The experiment was conducted at the University of Nottingham using subjects recruited from a university-wide pool of undergraduate students.[4] Four sessions were conducted with each treatment, with either 12 subjects (VETO and SYMMETRIC treatments) or 16 subjects (ENLARGED treatment) per session. Thus, 160 subjects participated in total.

All sessions used an identical protocol. Upon arrival, subjects were given a written set of instructions that the experimenter read aloud.[5] Subjects were then allowed to ask questions by raising their hands and speaking to the experimenter in private. Subjects were not allowed to communicate with one another throughout the session, except via the decisions they entered on their terminals.

The decision-making phase of the session then consisted of 10 periods. At the beginning of each period subjects were assigned to groups of either three or four (depending on treatment). Subjects were informed that they would not know who of the other people in the room were in their group, that group compositions would change from period to period, and that the same set of subjects would never be matched together twice. At the beginning of each period subjects were assigned an ID that changed from period to period. They were also assigned roles determining how many votes they controlled, and roles also varied across

[4] We recruited subjects using the ORSEE software (Greiner, 2004). The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007).

[5] A copy of the instructions for the VETO treatment can be found in Appendix B. The instructions for the other treatments paralleled these.

periods. We adopted these procedures (anonymous decision making together with changing roles and group compositions) in order to make it difficult for a subject to build up a reputation across periods.

In each period a group played a multi-round bargaining game over the division of 120 points. At the beginning of round one all group members submitted proposals over how to divide the 120 points. When all group members had submitted a proposal, one was selected at random and revealed to all group members. Group members then voted for or against the proposal. If the total number of votes for the proposal met the quota of 5 votes (VETO and ENLARGED) or 4 votes (SYMMETRIC) the proposal was passed and each group member received the proposed number of points. Otherwise bargaining proceeded to round two. All rounds had the same structure up to round 20. If no agreement was reached by the end of round 20, the period would end with each group member earning zero points.[6]

An important aspect of this design is that in each round all players submit proposals, then one is randomly selected, and then all players observe and vote on the randomly selected proposal. From a theoretical point of view this is equivalent to the more common description of the game in the theoretical literature, where a player is randomly selected to be the proposer and then only the randomly selected player makes a proposal. The advantage of our version is that it allows us to observe proposals from all players in every round. As we shall see in the next section, this information is useful not only for analyzing the determinants of proposals, but also for the analysis of voting patterns.

At the end of the experiment subjects were privately paid according to their accumulated point earnings from all 10 periods, using an exchange rate of 3p per point (VETO and SYMMETRIC treatments) or 4p per point (ENLARGED treatment). Earnings averaged £12 per subject and ranged from a minimum of £3.90 to a maximum of £20.20 (at the time of the experiment £1 = $1.92). Sessions lasted, on average, 50 minutes, with no session taking longer than 70 minutes.

[6] Subjects had 90 seconds to submit a proposal and 30 seconds to cast their votes. If they failed to make a decision within this time constraint, the computer made a default decision. The default proposal was that each group member received zero, and the default voting decision was to reject. In fact this time constraint was rarely binding: across all sessions only 7 of 2824 proposals were made by the computer, and only 8 of 2824 voting decisions were made by the computer.

A final noteworthy aspect of our design was that prior to the first period of a session we divided the subjects into two equally-sized matching groups, and then in the rest of the session formed groups from within these matching groups, with no information passing across the two matching groups.[7] This feature of our design was chosen for statistical reasons. Since subjects in one matching group cannot influence or be influenced by the decisions of subjects in the other matching group, our design generates two independent observations for each session and allows exact tests of our primary hypotheses. These primary hypotheses refer to the paradox of new members. We test the hypotheses that average earnings for the weak player are higher in ENLARGED than in VETO, and that average earnings for the strong player are higher in ENLARGED than in SYMMETRIC.[8]

4. Results

4.1 Overview

We observed rapid agreements in the experiment. Figure 1 shows the distribution of rounds in which agreement was reached. Out of the 480 games, 62% resulted in an agreement in the first round, and more than 90% ended within 3 rounds. Very few games got close to the 20-round deadline, and none actually reached the deadline.

In all three treatments a proposal can be implemented without unanimous support, and so within a group a subset of the players can pass a proposal that gives zero to outsiders. Thus, the reason for fast agreements in SYMMETRIC and ENLARGED may be that, though there are 20 rounds available for bargaining and there is no discounting, players have a strong incentive to accept a positive offer as otherwise they may end up with nothing. In VETO weak players similarly have a strong incentive to accept positive offers, but the strong player can afford to be patient since his vote is required for an agreement. Interestingly, only in VETO did we observe

[7] To ensure comparability between matching groups, we randomly formed groups, assigned roles, and selected proposers for one matching group prior to the first session. The same random draws were then used for all matching groups.

[8] Note that these hypotheses are derived from the expected payoffs in the equilibria identified in Section 2, which assumed players were equally likely to be the proposer in any round. We can also identify expected payoffs on the basis of the experimental realizations of who would be proposer in round one. This distinction is irrelevant for VETO, where the average earnings of a player type are independent of the proposer frequencies. In contrast, in SYMMETRIC and ENLARGED the actual frequencies matter, but the paradox still holds. The weak player's average earnings would change from 1 in VETO to 29.3 (rather that 30.375) in ENLARGED, and the strong player's earnings would change from 41 (rather than 40) in SYMMETRIC to 47 (rather than 44.75) in ENLARGED.

any trend in the duration of bargaining: we found that as the session progressed the duration of bargaining increased. Figure 2 shows how the average round of agreement changed across periods for each treatment. For formal statistical tests we use the matching group as the unit of observation. In VETO the Spearman rank correlation coefficient between period and average round of agreement is positive for all eight matching groups indicating a significant trend ($p = 0.008$); for SYMMETRIC and ENLARGED the corresponding p-values are both 0.727.

Figure 1. Distribution of Rounds in which Groups reached an Agreement

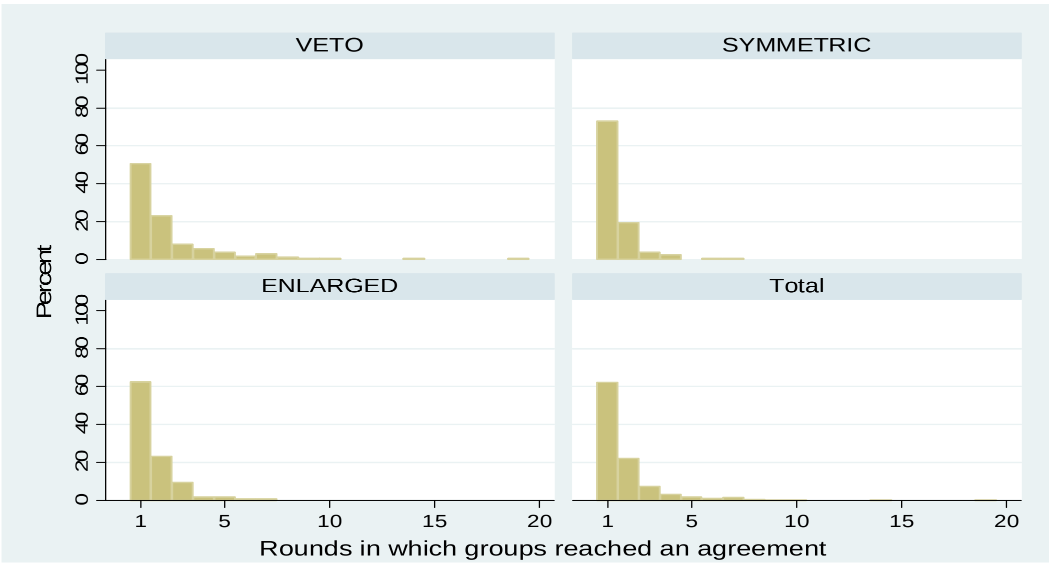
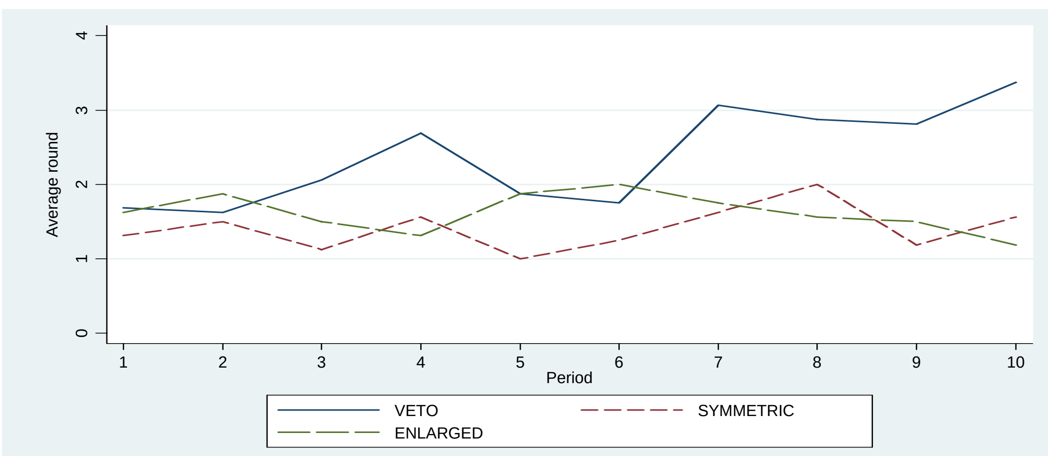
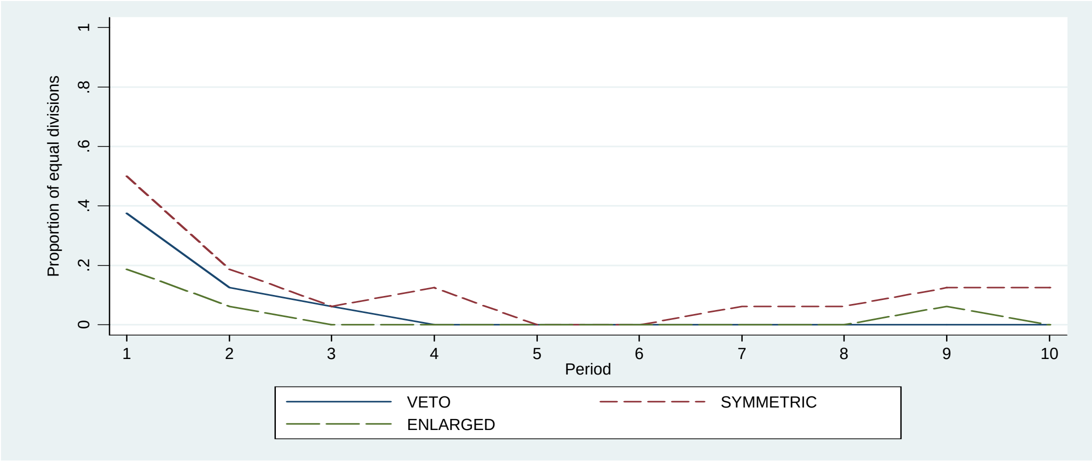


Figure 2. Average Duration of Bargaining



In the first period agreements to divide the 120 points equally among all members were quite frequent, occurring in 17 of the 48 (≈ 35%) groups. For many subjects this must have seemed a natural and acceptable outcome. However, as shown in Figure 3, equal divisions were observed less frequently in later periods, and in the last period only 2 groups (≈ 4%) agreed upon an equal division. Equal divisions are most commonly observed in the SYMMETRIC treatment, where theoretically all players have, ex ante, equal bargaining power; even here, an equal division of the pie is quite rare, occurring in only 20/160 (≈ 13%) games.

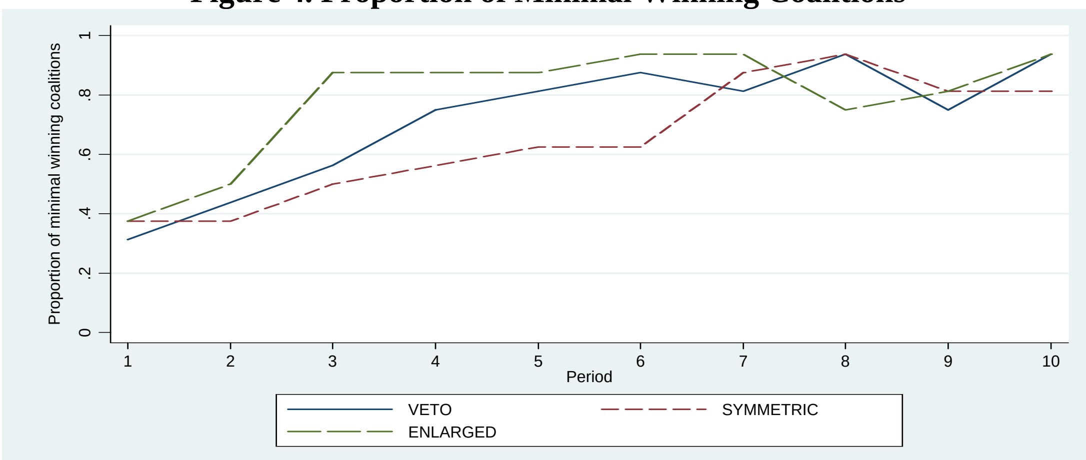
Figure 3. Proportion of Equal Divisions



Agreements to split the pie equally tended to be replaced by agreements that gave some players zero. Thus, while in two-person ultimatum game bargaining experiments it is well-known that subjects resist small offers, here as in other experiments on multi-person bargaining subjects are willing to propose distributions that give zero to another subject, and others are willing to vote for such a proposal (see, e.g., Güth and van Damme, 1998, Okada and Riedl, 2005). A winning coalition in which the votes of all coalition members are essential to pass a proposal and the coalition maximizes its point earnings by allocating zero points to outsiders is called a *minimal winning coalition*. In the equilibria discussed in Section 2, all winning coalitions are minimal.

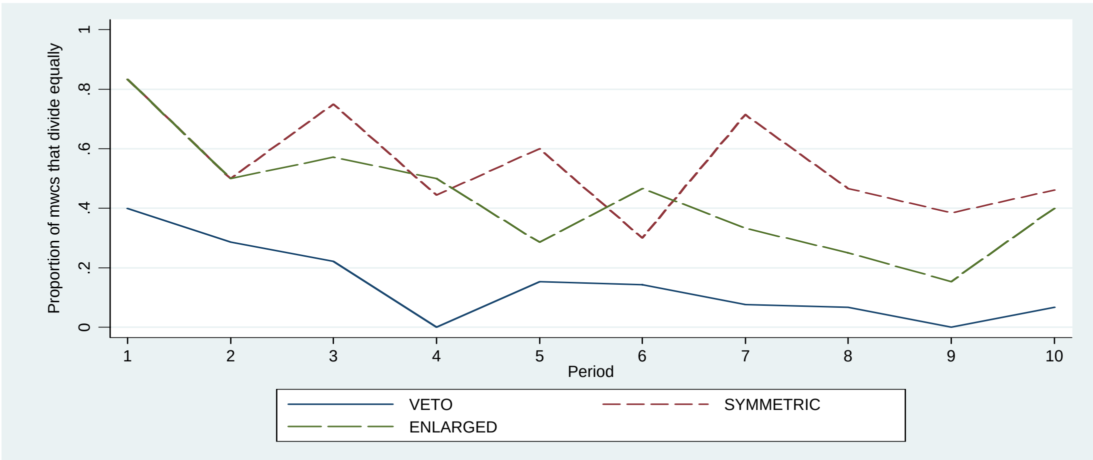
The frequency of minimal winning coalitions in our data is shown in Figure 4. In all treatments the frequency of minimal winning coalitions increases across periods. Taking all three treatments together, minimal winning coalitions formed in 17/48 groups (≈ 35%) in the first period, compared with 43/48 (≈ 90%) in the last period. In VETO, minimal winning coalitions, which must include the strong player, formed in 115/160 (≈ 72%) games overall. In SYMMETRIC, minimal winning coalitions, which can be comprised of any two players, formed in 104/160 (≈ 65%) games overall. In ENLARGED there can be two different types of minimal winning coalition: the strong player with one of the weak players, or the two weak players with the new member. In this treatment, the first type of coalition was much more common. Minimal winning coalitions excluding the strong player formed in only 19/160 (12%) games, while those including the strong player occurred in 107/160 (≈ 67%) games.

Figure 4. Proportion of Minimal Winning Coalitions



We interpret the increase in the frequency of minimal winning coalitions as reflecting a tendency toward more strategic behavior. On top of this, within minimal winning coalitions we see a tendency for the division of the pie to change over the course of the session. Figure 5 shows that in the first period the pie was often split equally among members of the coalition, but that by the last period this kind of outcome was much less frequent. This pattern is particularly clear in VETO, where strong players quickly realized that they only needed the vote of one of the weak players, and that weak players were willing to accept proposals that excluded the other weak player. In the final period minimal winning coalitions formed in 15 of 16 VETO groups, but only in one of these did the members of that coalition share the pie equally. The pattern is less pronounced in the other treatments. In the last four periods of the ENLARGED treatment about a third of the minimal winning coalitions divided the pie equally among its members, and in the last four periods of the SYMMETRIC treatment about a half of the minimal winning coalitions divided the pie equally among its members.

Figure 5. Proportion of Minimal Winning Coalitions that Divide Equally



The trend in Figure 5 seems to reflect a process where players learn from experience about their bargaining power and how to exploit it. At the same time, examining the proposals agreed by minimal winning coalitions suggests that some players were unable to fully extract their equilibrium shares. In VETO the strong player is predicted to get 118 points, almost the whole pie. In contrast, even restricting attention to minimal winning coalitions, the strong player gets only 77 points. In SYMMETRIC ex post equilibrium divisions give 80 points to the proposer and 40 points to one of the other players, and so the randomly selected proposer has a considerable theoretical advantage. In fact, even within minimal winning coalitions the average allocation gives only 62 points to the proposer, much more equitable (within the winning coalition) than predicted. In ENLARGED players can derive a theoretical bargaining advantage either from having more votes than other players or from being randomly selected to be proposer. The strong player is predicted to get 89 points as a proposer, though averaging across all minimal winning coalitions in which she was the proposer she only obtained 69 points. On the other hand a weak proposer is predicted to demand 75 points and offer the strong player 45 points, whereas when a weak player was successful in proposing this coalition the split was 63-57-0-0 in favor of the strong player. Thus, where voting weights or the position of proposer confer bargaining advantages, players are not fully able to exploit these.

It is interesting to compare the patterns from our SYMMETRIC and VETO treatments with those observed in previous experiments. Our SYMMETRIC treatment is similar to some of the treatments reported in Fréchette et al. (2005a, 2005b) and Kagel et al. (2007) in that, while voting weights vary across voters, any two of the three voters can form a winning coalition. We focus on the (inexperienced, undiscounted) treatment of Fréchette et al. (2005b) as this is procedurally closest to ours. In both experiments agreements happen quickly: on average their games lasted 1.6 rounds (our games lasted 1.4 rounds), and immediate agreements occurred in 68% of their games (73% of our games). Also in both experiments egalitarian outcomes are uncommon: 7% of their games resulted in egalitarian divisions (12% of our games). In both experiments most games resulted in minimal coalitions: 69% of their games resulted in a minimal winning coalition (66% of our games). These patterns broadly support the Baron-Ferejohn model predictions. At the same time, there are some notable deviations from the model predictions. The model predicts ex post divisions that give the proposer 2/3 of the pie and one of the other players 1/3 of the pie. In their data the average allocation gives the proposer just 51% of the pie (in our data 47% of the pie), and even focussing on minimal winning coalitions, their proposers only get 55% of the pie (ours get 52% of the pie).

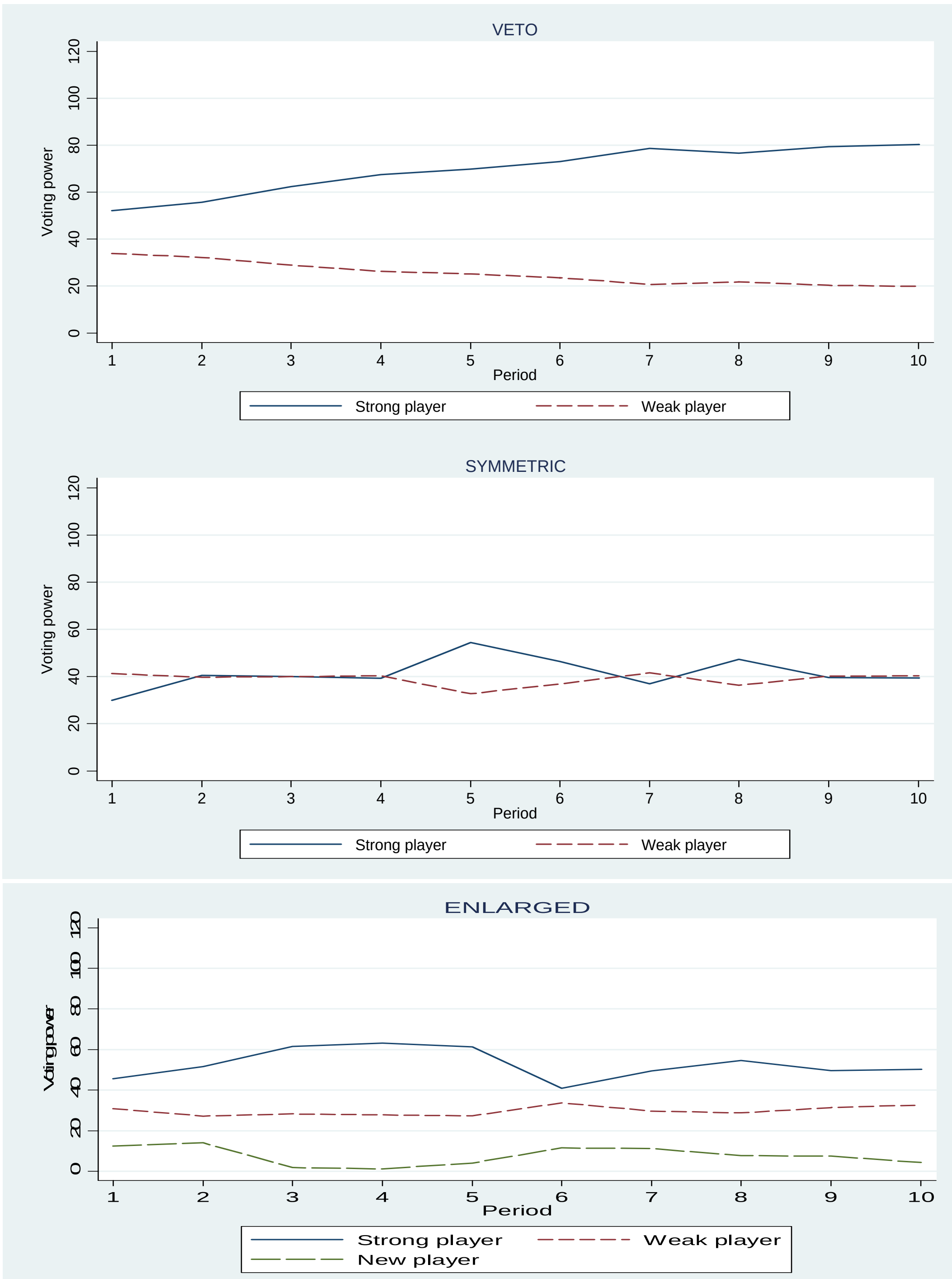
The results from our VETO treatment are also qualitatively similar to results from the (inexperienced, low delay cost) treatment of Kagel et al.'s (2007) Veto Game.[9] In their Veto Games agreements happened quickly (although not as quickly as in a control treatment somewhat similar to our SYMMETRIC treatment): 54% of their games (and 51% of our games) ended in round one and on average their games lasted 1.95 rounds (our games lasted 2.38 rounds). As in our VETO treatment, they observe an increasing tendency toward minimal winning coalitions over the course of the session, and overall 59% of their games (72% of our games) result in minimal winning coalitions. Just like in our experiment, however, the veto player's earnings are considerably less than predicted. She is predicted to get 92% as the proposer and 80% as the recipient of a proposal. Even within minimal winning coalitions she only gets 62% as proposer and 52% as recipient. In our VETO treatment the strong player is predicted to get 98% either as proposer or recipient, and even within minimal winning coalitions she only gets 67% as proposer and 61% as recipient.

[9] Although we emphasize that on top of the many procedural differences between experiments, their use of discounting makes us hesitant to read too much into quantitative differences between our results and theirs.

4.2 Formal Analysis of Voting Power

In this section we look at the implication of these developments in coalition formation, and in payoffs within coalitions, for players' earnings, which we use to measure voting power. Figure 6 shows how this develops across periods for each treatment and player-role.

Figure 6. Voting Power



In the SYMMETRIC and ENLARGED treatments the voting power of each player-type appears to be stable while in the VETO treatment it appears that there are trends in voting power. In VETO the Spearman rank correlation coefficients between the strong player's earnings and period are positive for each matching group, so we can reject the null hypothesis that earnings are equally likely to increase or decrease with experience (p-value = 0.008). The significant increase in the strong player's earnings is due to changes in earlier periods: there is no evidence of a relationship between earnings and period in the last four periods (the p-value based on Spearman rank correlation coefficients is 0.727). A similar analysis reveals no significant trends in SYMMETRIC or ENLARGED. [10]

Average earnings for each player type are presented in Table 3. In this table and in subsequent analysis we present results based on all periods and last four periods. For SYMMETRIC and ENLARGED this does not make much difference. Interestingly, for these treatments average earnings are quite close to the equilibrium expected payoffs given in Table 2. On the other hand in VETO, the strong player’s earnings are substantially below the equilibrium expected payoff given in Table 2, but the discrepancy is smaller in the last four periods.

Table 3. Measures of voting power

	VETO		SYMMETRIC		ENLARGED	
	All periods	Last 4 periods	All periods	Last 4 periods	All periods	Last 4 periods
Strong player	69.51	78.70	41.39	40.81	52.81	50.98
Weak player	25.24	20.64	38.92	39.59	29.78	30.64
New player	–	–	–	–	7.61	7.71

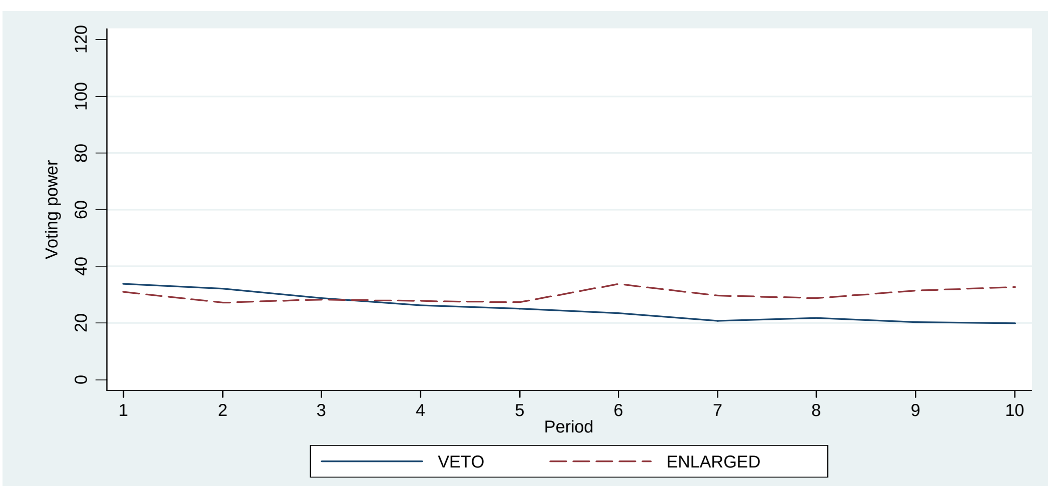
[10] For SYMMETRIC and ENLARGED the corresponding p-values based on all periods are both 0.727; based on the last four periods they are 1.000 (SYMMETRIC) and 0.727 (ENLARGED).

There is strong evidence that voting weights affect voting power in the VETO and ENLARGED treatments. In both treatments the strong player earns more than a weak player in every single matching group, whether we focus on all periods or just the last four periods, and thus the strong player earns significantly more than the weak player (one-sided sign-test p-value = 0.004). Similarly, the weak player earns significantly more than the new player in ENLARGED (one-sided sign-test p-value = 0.004). In the SYMMETRIC treatment we find that the earnings of the strong player are not significantly different from the earnings of the weak players (all periods: two-sided sign-test p-value = 1.000; last four periods: two-sided sign-test p-value = 0.727).

The main question motivating the design of our experiment was whether adding a new player to a weighted voting game, while retaining the relative voting weights of original players, could increase the voting power of an original player. This is the paradox of new members, as identified by Brams and Affuso (1976) based on application of Shapley and Banzhaf power indices. This is also an equilibrium of the bargaining model discussed in Section 2.

Our first test of whether the paradox occurs in our experiment is based on the weak player's earnings in VETO and ENLARGED, displayed in Figure 7.

Figure 7. Weak Player’s Voting Power and the Paradox of New Members

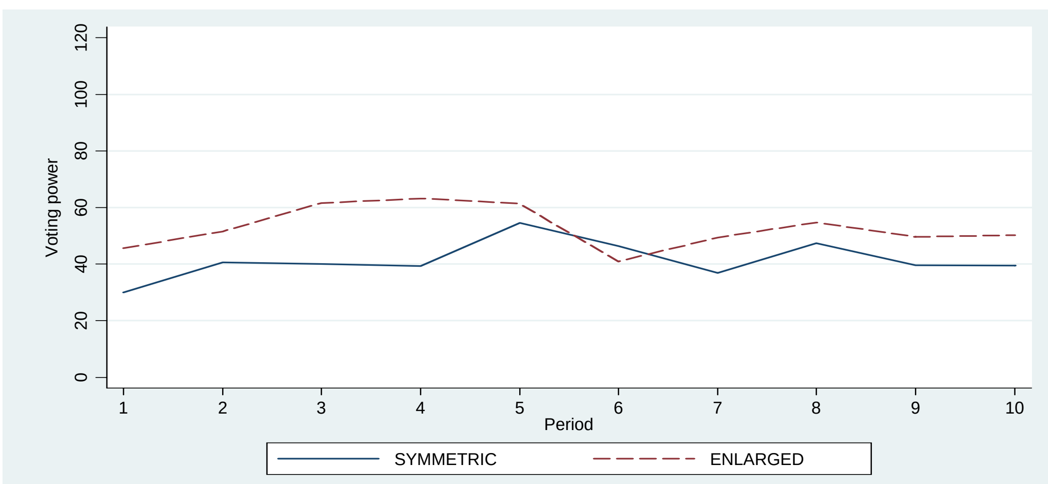


We test the null hypothesis that the weak player's earnings are the same in VETO and ENLARGED against the alternative hypothesis that the weak player's earnings are greater in

ENLARGED.[11] We reject the null hypothesis: the weak player's earnings are significantly higher in ENLARGED (based on all periods the one-sided Wilcoxon test p-value is 0.023; based on just the last four periods the one sided Wilcoxon test p-value is 0.001). The source of the paradox is that the addition of a new member provides the weak players with an alternative minimal winning coalition. This possibility appears to enhance a weak player's ability to extract larger shares of the pie from the strong player, despite the fact that the alternative coalition very seldom forms. For example, looking at winning coalitions from the last four periods consisting of the strong player and one of the weak players we see that in VETO the strong player gets about 77 points, compared with about 66 points in ENLARGED.

Our experimental design provides a second opportunity to test for the paradox of new members. Starting from the SYMMETRIC voting game, adding a new player with one vote and increasing the quota from 4 votes (out of 7) to 5 votes (out of 8) produces the ENLARGED voting game, and can in equilibrium increase the strong player's voting power. Figure 8 displays the strong player's voting power in these two treatments.

Figure 8. Strong Player's Voting Power and the Paradox of New Members



In our experiment the strong player earns significantly more in ENLARGED than SYMMETRIC (based on all periods the one-sided Wilcoxon test p-value is 0.001; based on just

[11] Thus, the null hypothesis is that there is no treatment effect, and the alternative hypothesis is that there is a treatment effect in the theoretically predicted direction.

the last four periods the one-sided Wilcoxon test p-value is 0.029). Thus, in our experiment, the paradox of new members is also observed in the second Brams and Affuso example.

Here, the source of the paradox is rather different. The strong player's bargaining power within a minimal winning coalition does not appear to be very different between SYMMETRIC and ENLARGED treatments. In a minimal winning coalition the strong player gets 64 points in SYMMETRIC (63 if we focus on the last four periods) and 66 in ENLARGED (68 if we focus on last four periods). Instead, the main reason why the strong player earns more in ENLARGED than SYMMETRIC is that he is included more often in minimal winning coalitions. He is included in 85% of minimal winning coalitions in ENLARGED (78% if we focus on last four periods), compared with 63% in SYMMETRIC (64% if we focus on last four periods). This is qualitatively consistent with the equilibrium discussed in Section 2. There, ex ante, the strong player is expected to be included in 75% of minimal winning coalitions in ENLARGED compared with 67% in SYMMETRIC, while when included in a minimal winning coalition the strong player expects to get 60 points in both ENLARGED and SYMMETRIC.

4.3 Analysis of Voting Patterns

The primary determinant of whether a given player type votes in favor of a proposal is, perhaps unsurprisingly, how much that player is offered. A player is more likely to accept higher offers. For example, the strong player accepted 10/22 (45%) offers of 40 points in ENLARGED, compared with 37/47 (79%) offers of 60 points. However, the amount that must be offered to secure an acceptance varies considerably across player types. In ENLARGED the weak player accepted 39/52 (75%) of offers giving him 40 points, and the new player accepted 5/5 (100%) offers giving him 40 points. Also, the propensity for a given type to accept a given proposal varies across treatments. Looking again at offers of 40 points we see that the strong player accepted 55% of these in SYMMETRIC and 26% in VETO, while the weak player accepted 57% in SYMMETRIC and 67% in VETO. [12]

While these patterns are very stable in ENLARGED, in SYMMETRIC and VETO we observe a decrease in the propensity to accept offers of 40 points. Based on the last four periods

[12] We focus on offers of 40 points since, pooling over all treatments, this is the most common offer made (other than offers of 0 points).

both the strong and weak players accept 33% of such offers in SYMMETRIC, while the strong player never accepted and the weak player accepted 42% of such offers in VETO.

Figure 9. Voting Patterns

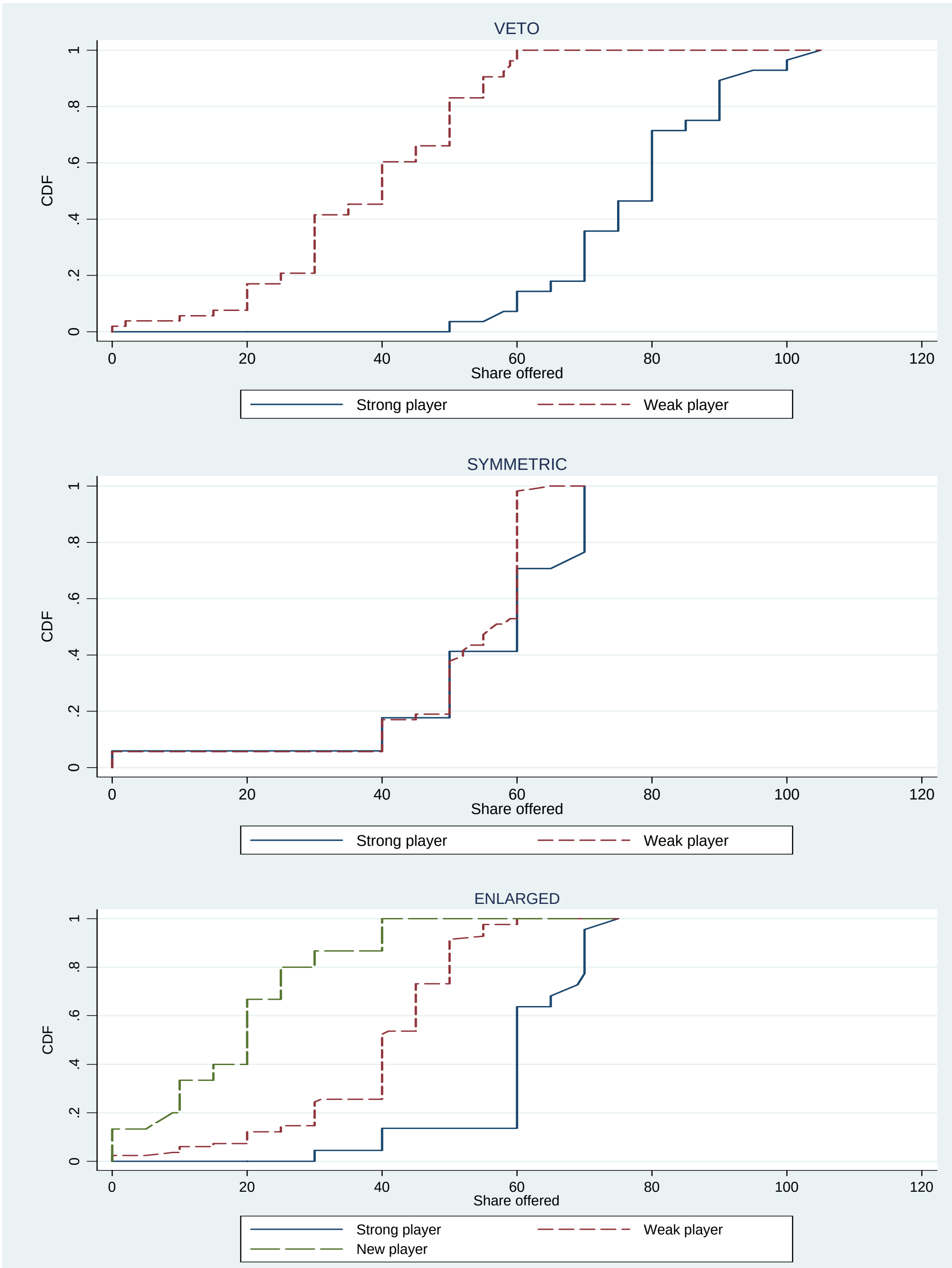


Figure 9 displays the empirical cumulative distribution functions of accepted offers. Votes of proposers are excluded from the analysis, and we focus on the last four periods.[13] In VETO it is clear that accepted offers tend to be lower for weak players than for strong players. Half of the acceptances by weak players were for offers less than or equal to 40, whereas for the strong player half of acceptances were for offers less than or equal to 80. Likewise in ENLARGED there is a natural ordering whereby accepted offers by strong players tend to be higher than the accepted offers of weak players, which in turn tend to be higher than the accepted offers of new players. In SYMMETRIC we present separate functions for strong and weak players. However, the functions are not very different and as we report below differences in the acceptance behavior of the different types are insignificant. Thus, in SYMMETRIC the strong player with three votes behaves no differently from a weak player with two votes in terms of voting behavior.

For a formal statistical analysis of voting behavior we estimated probit models where the dependent variable is whether or not a player voted for a proposal. Again, we excluded proposers from the analysis and focus on the last four periods.[14] As explanatory variables we included the number of points offered to the player and the number of points the player demanded in her own proposal. Since observations are independent across, but not within, matching groups, we cluster on independent matching groups to obtain robust standard errors. The results are presented in Table 4.

As expected, the share offered to a player is a strong explanatory factor for whether a player will accept an offer. For all player types in all treatments, the more a player is offered the more likely she is to accept. Offering one additional point increases the probability of acceptance by between 0.9 percentage points (Strong player in VETO treatment) and 8.8 percentage points (New player in ENLARGED). We also included the share that a player demanded in their own proposal as an explanatory variable – recall that all subjects, not just the randomly selected proposer, completed a proposal in every round. We expected that the share

[13] The functions are quite stable across periods in ENLARGED, and are stable across the last four periods in SYMMETRIC and VETO. The functions in SYMMETRIC tend to be further to the left in earlier periods, and those in VETO tend to be closer together in earlier periods.

[14] We also conducted probit estimations on all periods including period dummies, and found significant period effects in the VETO and SYMMETRIC treatments. For this reason we restrict attention to the last four periods. For the last four periods we also conducted estimations including period dummies, and found these to be jointly insignificant in all cases.

demanded would reveal an aspiration level of a subject, and so a given offer would more likely be accepted if the share demanded is lower. As can be seen in Table 4, the marginal effect of this variable is indeed negative in all cases, although it is not significant for the weak player in the VETO treatment, or for either player type in the SYMMETRIC treatment.

Table 4. Probit Analysis of Voting Behavior

Independent Variables	*Dependent Variable: Accept=1, Reject=0*						
	VETO		SYMMETRIC		ENLARGED		
	Strong Player	Weak Player	Strong Player	Weak Player	Strong Player	Weak Player	New Player
Share offered	0.009***	0.011***	0.012***	0.015***	0.015***	0.030***	0.088***
	(0.003)	(0.001)	(0.003)	(0.002)	(0.008)	(0.004)	(0.027)
Share demanded	–0.008**	–0.000	–0.005	–0.004	–0.025**	–0.016**	–0.011***
	(0.003)	(0.003)	(0.002)	(0.003)	(0.023)	(0.006)	(0.005)
Obs.	115	273	58	146	61	154	73
Pseudo R^2	0.3624	0.4707	0.4557	0.3985	0.6445	0.6845	0.7261

Notes: Based on data from last four periods. Table lists marginal effects, evaluated at the means of the independent variables, and standard errors, clustered on independent matching groups, in parentheses. * denotes significance at the 10-percent level, ** at the 5-percent level, and *** at the 1-percent level.

We also examined whether the voting behavior of different types within a treatment differ. Based on likelihood ratio tests the behavior of strong and weak players in the VETO treatment differ significantly (p = 0.000), as does behavior of strong, weak, and new players in ENLARGED (p = 0.000). However, in SYMMETRIC there is no significant difference between the voting behavior of strong and weak players (p = 0.975). Finally, we examined whether the voting behavior of a given player type varied across treatments. Again using likelihood ratio tests, we find voting behavior differs significantly across treatments for both strong players (p = 0.042) and weak players (p = 0.000).

5. Discussion and Conclusion

There are two approaches in the theoretical literature for solving games: the axiomatic approach, which is based on specifying some desirable properties for the solution and the strategic approach, which is based on the equilibrium of an explicit bargaining game. The paradox of new members was introduced into the literature in terms of power indices grounded in the axiomatic approach. In this paper we follow the strategic approach and show that the addition of a new member to a voting body can increase the equilibrium payoffs of original members in the context of a specific bargaining procedure. Thus the paradox of new members can arise as an equilibrium phenomenon in this bargaining procedure.

The bargaining procedure we use was introduced by Baron and Ferejohn and is widely used in political science. Other theoretical analyses of the Baron-Ferejohn procedure with weighted voting include Banks and Duggan (2000), Diermeier et al. (2003), Snyder et al. (2005), Montero (2006), Kalandrakis (2006) and Eraslan and McLennan (2006). There are other bargaining procedures we could have chosen. In particular, since it is possible to devise a bargaining procedure to make equilibrium payoffs coincide with the Shapley value, and the Shapley value displays the paradox, the possibility of the paradox in a non-cooperative setting was already implicitly established. However, while the Baron-Ferejohn procedure seems a natural abstraction of bargaining under majority rule, these other bargaining procedures seem more contrived. In particular, these other procedures require unanimity rather than majority in order to pass a proposal: Hart and Mas-Colell (1996), Pérez-Castrillo and Wettstein (2001) and Laruelle and Valenciano (2005) require the unanimous consent of all players; in Vidal-Puga (2007) a coalition is formed gradually, with players having the option to join it or leave the game in a random order, and the final allocation must be accepted unanimously by all members of the coalition. Other implementations of the Shapley value like Gul (1989) apply only to particular classes of games that do not include majority games. Our theoretical contribution is to demonstrate that Brams and Affuso's theoretical paradox holds in the leading model of legislative bargaining.

The theoretical possibility that the paradox may emerge in the non-cooperative Baron-Ferejohn procedure motivated us to ask whether it actually is observed in the laboratory. Our answer is affirmative. In all of our treatments we find that as sessions progress subjects become less and less willing to propose and vote for equal divisions of the pie and more and more willing to propose and vote for proposals that give zero to other subjects. Although these trends lead to minimal winning coalitions, as predicted in equilibrium, we see some substantial deviations from equilibrium, even in later periods. In particular, players are generally unable to exploit either proposer power or advantageous voting weights to the full extent predicted by equilibrium. Despite these deviations we find that average earnings of each player-type are quite stable in our SYMMETRIC and ENLARGED treatments, and not far from equilibrium expected payoffs. Consequently, when comparing these treatments the theoretical 'paradox' is observed. In our VETO treatment, although the weak players' earnings move closer to equilibrium with experience, even by the end of the experiment the weak player's earnings are far above equilibrium. This makes it harder to observe the paradox in the comparison of the VETO and ENLARGED treatments, but we do observe the paradox in this case as well: the weak players' earnings are higher in the enlarged voting body.

Previously, Montero et al. (2008) studied the paradox in a less structured environment where any player could place (or amend) a proposal on the table at any time, any player could vote for or against any proposal on the table at any time, and the first proposal on the table to achieve the quota was implemented. They found that voting powers are quite close to Shapley Values and so the paradox is also observed in their experiment. This environment seemed natural for testing hypotheses based on cooperative game theory, but of course it is difficult to compare behavior with equilibria. Nevertheless, it is interesting to note that the voting powers associated with a given player role differ across the experiments. For example, in the VETO treatment the strong player earns more in the less structured procedure than in the Baron-Ferejohn procedure[15]. Similarly, the strong player in the ENLARGED treatment earns more in the less structured environment. A similar result, albeit in a different context, is observed in de Groot Ruiz et al. (2007). They compare a non-cooperative voting game with a more natural, but less structured, voting game and find that a median voter (the 'strong player' in their context) is able to secure a higher payoff in a less structured voting game. Together these results suggest that voting powers depend not only on voting weights and quotas, but also on specific features of the agenda.

[15] This is despite the fact that the strong player gets almost the whole pie in the equilibrium of the Baron-Ferejohn procedure, while in the less structured environment the Shapley Value assigns 2/3rd of the pie to the strong player.

We conclude by commenting on whether there have been any 'real-world' manifestations of the paradox, and if so whether these are recognized by the players. It turns out that application of the Baron-Ferejohn model to the first two enlargements of the EU Council of Ministers suggests instances of the paradox. Table 5 displays the voting weights and the stationary expected equilibrium payoffs of the various countries, where the latter are based on the infinite-horizon Baron-Ferejohn model with no discounting and where all countries have equal recognition probabilities.[16]

Table 5. Voting Weights and Voting Power on the EU Council of Ministers

	1958 voting weights (% voting power)	1973 voting weights (% voting power)	1981 voting weights (% voting power)
W. Germany	4 (23.8)	10 (15.9)	10 (16)
France	4 (23.8)	10 (15.9)	10 (16)
Italy	4 (23.8)	10 (15.9)	10 (16)
United Kingdom		10 (15.9)	10 (16)
Belgium	2 (11.9)	5 (7.9)	5 (8)
Netherlands	2 (11.9)	5 (7.9)	5 (8)
Greece			5 (8)
Denmark		3 (7.1)	3 (4)
Ireland		3 (7.1)	3 (4)
Luxembourg	1 (4.8)	2 (6.3)	2 (4)
Quota	12	41	45

It is well known that the first enlargement exhibits the paradox according to cooperative power indices, because it enhanced Luxembourg's voting power (despite the fact that

[16] For detailed derivations see Montero (2007).

Luxembourg's relative voting weight fell). Before the enlargement Luxembourg could never feature in a minimal winning coalition, while after the enlargement it could, for example, form a minimal winning coalition with the four largest countries. Thus the first enlargement transformed Luxembourg from a 'dummy player' into a player with positive bargaining power. The Baron-Ferejohn model yields a qualitatively similar result, although note that even before the enlargement Luxembourg's bargaining power was positive, since it could obtain a positive share as a proposer.

More intriguing is the second enlargement. Previous analysis based on cooperative power indices suggested a further increase in Luxembourg's voting power (see Brams and Affuso, 1985). The Baron-Ferejohn model suggests a different manifestation of the paradox: the second enlargement increased the voting power of the six largest countries.[17] Whether the cooperative power index or the noncooperative equilibrium prediction provides the better measure of theoretical power is open to question. Moreover, which (if any) country's voting power, measured in terms of its realized share of whatever is being bargained over, really did increase as a result of the enlargement is difficult to determine empirically. The advantage of the experimental approach is that it can systematically examine the effects of voting weights and voting rules, including features of the extensive form governing the way in which proposals are made and votes cast, in order to enhance our understanding of voting power.

[17] This seems to us to be even more paradoxical, as not only do existing members get more voting power from an enlargement, but a proposal to enlarge would obtain enough votes to be passed. In this respect this manifestation of the paradox differs from any others of which we are aware.

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Appendix A: The Paradox in the Infinite Horizon Game with Discounting

General model and results

Let there be a voting body with n voters, who are assumed to be selfish and risk neutral. Denote the set of voters by $N = \{1,2,...,n\}$, the vector of weights by $\omega = (\omega_1,...,\omega_n)$, where ω_i is the number of votes controlled by voter i, and the number of votes required to pass a proposal by q. There is a budget of size 1 to be divided between the voters by weighted majority rule. The bargaining procedure starts with a voter being randomly selected to make a proposal (with voter i being selected with probability θ_i, where $\theta_i \geq 0$ and $\sum_{i \in N} \theta_i = 1$). The proposer proposes a division of the budget such that, if x_i is the amount allocated to voter i, $x_i \geq 0$ for all i, and $\sum_i x_i = 1$. The voters vote on the proposal simultaneously. If the total number of votes in favor is at least q, the proposal is implemented and each voter i receives x_i. Otherwise, a new proposer is selected (again each voter is selected with probability θ_i). All players discount future payoffs by $\delta \in [0,1]$. We denote the noncooperative game by $G(\omega, q, \theta, \delta)$.

Because responders vote on the proposal simultaneously, there may be subgame perfect equilibria in which a proposal is accepted or rejected by all players regardless of whether the players actually prefer the proposal to pass, because none of the responders can unilaterally change the outcome. For the theoretical analysis that follows we eliminate weakly dominated strategies, that is, each responder is assumed to vote as if he is pivotal. Equivalently we may assume that voters vote sequentially.

We look for stationary subgame perfect equilibria (SSPE), that is, subgame perfect equilibria in which players use stationary strategies. In this context, a stationary strategy is a strategy that does not condition on past offers, on responses to past offers or on t. For a proposer, a stationary strategy specifies a (time-invariant) probability distribution over the set of possible payoff divisions. For a responder, it assigns a (time-invariant) probability of acceptance to each possible proposal. The probability of acceptance may depend on the entire proposed vector as well as on history elements of the current bargaining round like the identity of the proposer and the responses of the other players to the current proposal. However, any stationary equilibrium is payoff equivalent to another one in which players use "cutoff" strategies, accepting any offer that gives them at least their continuation value (see Yan, 2002). Because equilibrium strategies are stationary, a player's continuation value (that is, a player's expected payoff after a proposal is rejected) is time invariant. Subgame perfection implies that a player must accept any offer above the continuation value.[18]

Two general results are known for $\delta < 1$. First, there is immediate agreement (Okada, 1996, theorem 1); second, interchangeable players must have the same expected payoffs (Montero, 2002, lemma 2). For $\delta = 1$, the two results still hold if there are no veto players. If there are veto players, some qualifications are needed. Delay is possible, though only when it is immaterial (i.e. when it does not affect expected payoffs), and all interchangeable *nonveto* players must have the same expected payoffs.

The two results are restated below allowing for $\delta = 1$.

Let y_i^t be player i's equilibrium expected payoff at the beginning of round t, before the proposer is selected. Stationarity implies that y_i^t is independent of t, so the superscript can be dropped. The continuation value if a proposal is rejected is δy_i.

The following lemma shows that agreement must occur with probability 1, and must be immediate in the absence of veto players. Any agreement is such that the proposer chooses the "cheapest" group of players that has enough votes to complete a winning coalition, and offers them exactly their continuation values.

Lemma 1. (cf. Okada, 1996) In any SSPE of the game $G(\omega, q, \theta, \delta)$:

1.1 $\sum_{i \in N} y_i = 1$.

1.2 If player i proposes a payoff vector x, and this payoff vector is accepted with positive probability, then there is a coalition S such that $x_j = \delta y_j$ for all $j \in S \setminus \{i\}$, $x_j = 0$ for all $j \in N \setminus S$, and S solves the maximization problem

$$\min_{T: i \in T, \sum_{j \in T} \omega_j \geq q.} \sum_{j \in T \setminus \{i\}} y_j .$$

1.3 If there are no veto players (i.e., if $\sum_{j \in N \setminus \{i\}} \omega_j \geq q$ for all i), all proposals are accepted.

1.4 If $\delta < 1$ or if there are no veto players, $y_i > 0$ for all i.

[18] A player may accept or reject an offer of exactly the continuation value. Yan's discussion implies that little is lost by assuming that players always break ties in favor of acceptance; we will however not make any assumptions on how ties are broken in the proofs that follow.

Proof. 1.1. Suppose $\sum_{i\in N} y_i < 1$. Consider the situation of player i as a proposer. By stationarity, making a proposal that is rejected yields δy_i. Subgame perfection together with sequential voting (or elimination of weakly dominated strategies) implies that any proposal that allocates $y_j + \varepsilon$ ($\varepsilon > 0$) to each $j \in N \setminus \{i\}$ (and $1 - \sum_{j\in N\setminus\{i\}} y_j - (n-1)\varepsilon$ to player i) must be accepted, and, if ε is sufficiently small, is preferred by i to making a proposal that is not accepted. Since this reasoning is applicable to any proposer, agreement is always immediate and no payoffs are wasted, thus $\sum_{i\in N} y_i = 1$, a contradiction.

1.2. If player i's proposal is accepted with positive probability, the set of players that are offered at least their continuation value (including of course i) must control at least q votes (otherwise, sequential voting or elimination of weakly dominated strategies would ensure that the proposal is rejected). Call this set S. If any player in $S \setminus \{i\}$ is getting more than δy_j, i would not be playing a best response since it could reduce x_j slightly. If S were not the solution of the optimization problem above, let S' be a solution. Then i could offer $\delta y_j + \varepsilon$ to each $j \in S' \setminus \{i\}$ and be better-off.

1.3. Finally, if there are no veto players, all players must strictly prefer to make acceptable proposals. Each player i can obtain a positive payoff by offering $y_j + \varepsilon$ to each j in $N \setminus \{i,k\}$, where k is such that $y_k > 0$. The only case in which this would not be possible is if $y_i = 1$, but then i would be excluded by all other players, contradicting $y_i = 1$.

1.4. Finally, if $\delta < 1$ or if there are no veto players, all players must have a positive payoff. Since all proposals must allocate a nonnegative share to all players, a player can never have a negative payoff. As a proposer, a player can have a positive payoff by offering $\delta y_j + \varepsilon$ to each other player if $\delta < 1$, or, if there are no veto players, by offering $\delta y_j + \varepsilon$ to each j in $N \setminus \{j,k\}$, where k is such that $y_k > 0$.

Let i be the proposer, and S the coalition of lemma 1.2. We will often abuse terminology and refer to S as the *proposed coalition*. The players $j \in S \setminus \{i\}$ are the *coalition partners*. The next corollary is well known and follows from lemma 1.2 and 1.3.

Corollary 1. (cf Harrington, 1990) If $\delta < 1$ or if there are no veto players, player i's payoff as a proposer exceeds player i's payoff as a coalition partner.

With veto players and $\delta = 1$, a player earns exactly the same as proposer and as coalition partner. The proposer's advantage stems from the fact that the proposer can exclude some players with a positive continuation value.

Two players i and j are of the same type if they are interchangeable, that is, if for any coalition S that does not contain i or j, $\sum_{k \in S} \omega_k + \omega_i \geq q$ if and only if $\sum_{k \in S} \omega_k + \omega_j \geq q$. Two players with the same weight must be of the same type, but the converse is not necessarily true. Montero (2002, lemma 2) shows that any two players of the same type must have the same continuation value in an SSPE if they have the same probability of being proposer. This lemma assumes $\delta < 1$. For $\delta = 1$, any two *nonveto* players of the same type must have the same continuation value.

Lemma 2. (cf. Montero, 2002) Let i and j be two players of the same type, and assume the bargaining procedure is such that i and j have the same probability of being proposers ($\theta_i = \theta_j$). In any SSPE of the game $G(\omega, q, \theta, \delta)$, $y_i = y_j$ if $\delta < 1$ or if i and j are not veto players.

Proof. For $\delta = 0$, the proposer is playing the equivalent of an ultimatum game, and the result is trivial. In what follows, assume $\delta > 0$.

Suppose there is an SSPE with $y_i \neq y_j$. Without loss of generality let $y_i > y_j$. Expected equilibrium payoffs satisfy the following equations

$$y_i = \theta_i y_i^i + r_i \delta y_i \qquad (1)$$

$$y_j = \theta_j y_j^j + r_j \delta y_j \qquad (2)$$

where θ_i is the probability that i is selected to be proposer (by assumption, $\theta_i = \theta_j$), y_i^i is the expected equilibrium payoff for player i conditional on being proposer, and r_i is the probability that i receives a proposal in the equilibrium.

We now show $y_i > y_j$ leads to a contradiction.

First, notice that $r_i \le r_j$. Players other than i and j will never include i and exclude j in a proposal. As for i and j themselves, if j includes i in the coalition with positive probability there must be a coalition S containing i and j that is optimal for player j as a proposer, that is, $y_i + \sum_{k \in S \setminus \{i,j\}} y_k \le \sum_{k \in T} y_k$ for all T such that $\sum_{k \in T} \omega_k + \omega_j \ge q, T \subseteq N \setminus \{i,j\}$. This implies that $y_j + \sum_{k \in S \setminus \{i,j\}} y_k < \sum_{k \in T} y_k$. Thus, i strictly prefers to propose coalition S rather than any of the coalitions that do not contain j, and must propose to j with probability 1.

Second, since i has better alternatives than j, it is clear that $y_i^i \ge y_j^j$.

Third, $y_i^i - y_j^j \le y_i - y_j$. If i never proposes to j, it cannot be optimal for j to propose to i, thus $y_i^i - y_j^j = 0 < y_i - y_j$. If i proposes to j, j always has the option to propose the same coalition, thus $y_i^i - y_j^j \le \delta\left(y_i - y_j\right)$.

Let $y_i^i - y_j^j = y_i - y_j - \varepsilon$ ($\varepsilon \ge 0$). Subtracting (2) from (1) and taking into account that $\theta_i = \theta_j$, we obtain

$$(y_i - y_j)(1 - \theta_i - r_i\delta) = -\varepsilon\theta_i - (r_j - r_i)\delta y_j \tag{3}$$

The RHS of expression (3) is negative or 0, whereas the LHS is positive or zero. The only case in which they can be equal is if they are both 0. This is not possible for $\delta < 1$. If $\delta = 1$, it requires $r_i = 1 - \theta_i$. From the RHS, $r_i = r_j$ and thus $r_j = 1 - \theta_j$. From equations (1) and (2), it follows that there is no advantage of being proposer for either player ($y_i^i = \delta y_i$), and any players outside the proposed coalition must have a continuation value of 0. If i is not a veto player, j could allocate 0 to i and offer all other players a sufficiently small $\varepsilon > 0$. The proposal would pass and j would be better-off. Thus, the only case that does not lead to a contradiction is $\delta = 1$ with i and j being veto players.

Lemma 2 refers to expected payoffs, not to strategies. Strategies need not always be symmetric, but we will focus on symmetric strategies.

We now calculate the SSPE of the three voting games assuming that all players have the same probability of being proposer.

VETO

Veto players must get everything when $\delta = 1$ (Winter, 1996). This is because veto players must be in the final coalition with probability 1, and (from equation (1)) must receive the same payoff as proposers and as coalition partners ($y_i^i = y_i$). Assuming that a nonveto player j has a positive expected payoff would lead to a contradiction, since the veto player could obtain more than y_i as a proposer by excluding j . Because $\sum_{i \in N} y_i = 1$, agreement must occur with probability 1. This does not exclude delay in equilibrium, but (because of stationarity) it implies that in each round of bargaining players must agree with positive probability.

For $\delta < 1$, we have immediate agreement (lemma 1.3), $y_2 = y_3$ (lemma 2), and $y_2 > 0$ (lemma 1.4). This implies that the veto player must propose to each of the other players with the same probability in equilibrium. The following equation holds: $y_1 = \frac{1}{3}[1 - \delta y_2] + \frac{2}{3}\delta y_1$, or $y_1 = \frac{2-\delta}{6-5\delta}$, which implies $y_2 = \frac{2(1-\delta)}{6-5\delta}$.

SYMMETRIC

This is a particular case of Baron-Ferejohn (1989) with $n = 3$. Agreement is immediate in equilibrium and $y_i = \frac{1}{3}$ for all i. Equilibrium strategies are not completely determined; adding the requirement of symmetry they are such that each player is equally likely to propose to any of the other two.

ENLARGED

Since there are no veto players, agreement is immediate in equilibrium. Moreover, symmetry implies $y_2 = y_3$. Also, all players have a positive continuation value. Taking this into account, strategies are as follows. Because $y_4 > 0$, player 1 will always propose to player 2 or player 3. Player 2 and 3's behavior depends on how y_1 compares with $y_2 + y_4$. Player 4 will propose to players 2 and 3, unless $y_1 \le y_2$. However, it is easy to see that $y_1 \le y_2$ would lead to a contradiction.[19]

Three possibilities can be considered in turn, depending on players 2 and 3's strategy. The only equilibrium corresponds to 2 and 3 proposing to 1. In order for $y_2 = y_3$, 1 must propose to each of them with the same probability. Expected payoffs are the solution to the following system of equations

$$y_1 = \frac{1}{4}[1-\delta y_2] + \frac{2}{4}\delta y_1$$
$$y_2 = \frac{1}{4}[1-\delta y_1] + \frac{3}{8}\delta y_2$$
$$y_3 = \frac{1}{4}[1-2\delta y_2]$$

The solution to this system is $y_1 = \frac{8-5\delta}{4(\delta^2-7\delta+8)}$, $y_2 = \frac{4-3\delta}{2(\delta^2-7\delta+8)}$, $y_3 = \frac{4\delta^2-11\delta+8}{4(\delta^2-7\delta+8)}$. For $\delta = 1, y_1 = \frac{3}{8}, y_2 = \frac{1}{4}, y_3 = \frac{1}{8}$. Notice that $y_1 = y_2 + y_4$ for $\delta = 1$, so players 2 and 3 do not have a unique optimal coalition, but the only equilibrium is that they propose to player 1.

Comparing expected payoffs from different games, we observe that the paradox occurs for sufficiently high values of δ. The approximate thresholds are 0.61 for the comparison between VETO and ENLARGED, and 0.82 for the comparison between SYMMETRIC and ENLARGED.

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[19] Since $y_4 > 0$, $y_1 \le y_2$ implies that players 2 and 3 propose to 1. Then $y_1 \ge \frac{1}{4}(1-\delta y_2) + \frac{1}{2}\delta y_1$ and $y_2 \le \frac{1}{4}(1-\delta y_1) + \frac{3}{8}\delta y_2$. It follows that $y_1 \ge \frac{8-5\delta}{4(\delta^2-7\delta+8)}$ and $y_2 \le \frac{4-3\delta}{2(\delta^2-7\delta+8)}$, and $y_1 > y_2$ for all $\delta > 0$, a contradiction.

Appendix B: Instructions for VETO treatment

Introduction

This is an experiment about group decision-making. There are other people in this room who are also participating in this experiment. You must not talk to them or communicate with them in any way during the experiment. The experiment will take no more than 90 minutes, and at the end you will be paid in private and in cash. The amount of money you earn will depend on the decisions that you and the other participants make.

In this experiment you will participate in ten periods. In each period you will be in a group with two other people, but you will not know which of the other people in this room are in your group. The people in your group will change from period to period, and in particular you will never be matched with the same set of two other people twice. The decisions made by you and the other people in your group will determine how many points you earn in that period. At the end of the experiment you will be paid according to your total point earnings from all ten periods. You will be paid 3p per point.

Description of a period

At the beginning of each period, you will be randomly allocated a subject identification number, either 1, 2, or 3. (Thus, your identification number may change from period to period.)

Within each group of three people **one person will control three votes**, **one person will control two votes,** and **one person will control two votes**. The assignments of votes to subject identification numbers will vary from period to period.

In each period **you and the other people in your group have 120 points to divide**. You and the other people in your group can make proposals about how these points are to be divided among the group members. You and the other people in your group then cast votes for or against proposals. **The first proposal to receive *five* or more votes will be enforced**. When a proposal is enforced the period ends and each person earns the number of points specified in that proposal.

How you make and vote on proposals

A period can consist of up to 20 rounds of making and voting on proposals.

At the beginning of each period your computer screen will look like the one shown in Figure 1. You must propose the number of points that each person in your group will receive. For each person you can type in any whole number between 0 and 120, but the total number of points received by the group members must add up to 120. Note that the row where you propose how much you yourself will receive is in boldface.

When you have completed a proposal you click on the "submit" button to submit it. You will have ninety seconds to make a proposal. If you do not submit a proposal in ninety seconds the computer will submit a proposal for you, and this proposal will propose that all group members get zero.

After all three group members have submitted proposals the computer will randomly choose one of them and the three group members will vote on this proposal. A sample screen is shown in Figure 2 (except that the entries marked XXX will be the numbers one of the group members entered in their proposal). Note that the row proposing how much you will receive is in boldface. You must then cast your votes either for or against the proposal by clicking accept or reject. If you accept, all the votes you control are cast in favour of the proposal, while if you reject, all the votes you control are cast against the proposal.

You will have thirty seconds to cast your votes. If you do not cast your votes in thirty seconds the computer will vote for you, and will vote against the proposal.

If the proposal receives five or more votes in favour it will be enforced. This means that the period will end and this proposal will determine the number of points you earn in that period.

If the proposal receives less than five votes, the process will be repeated: once again all group members will make proposals, one of them will be randomly chosen, and all group members will get to vote on this proposal. This process will continue until a proposal is enforced, or until 20 rounds of making and voting on proposals have passed. If no proposal has been enforced after the twentieth vote the period will end and all three group members will receive zero.

Ending the session

At the end of period ten your total points from all periods will be converted to cash at a rate of 3p per point and you will be paid this amount in private and in cash. The computer will keep track of your point earnings from period to period, but if you want to keep a record for yourself you can use the attached Record Sheet.

If you have any questions raise your hand and a monitor will come to your desk to answer them. Now, please begin period 1.

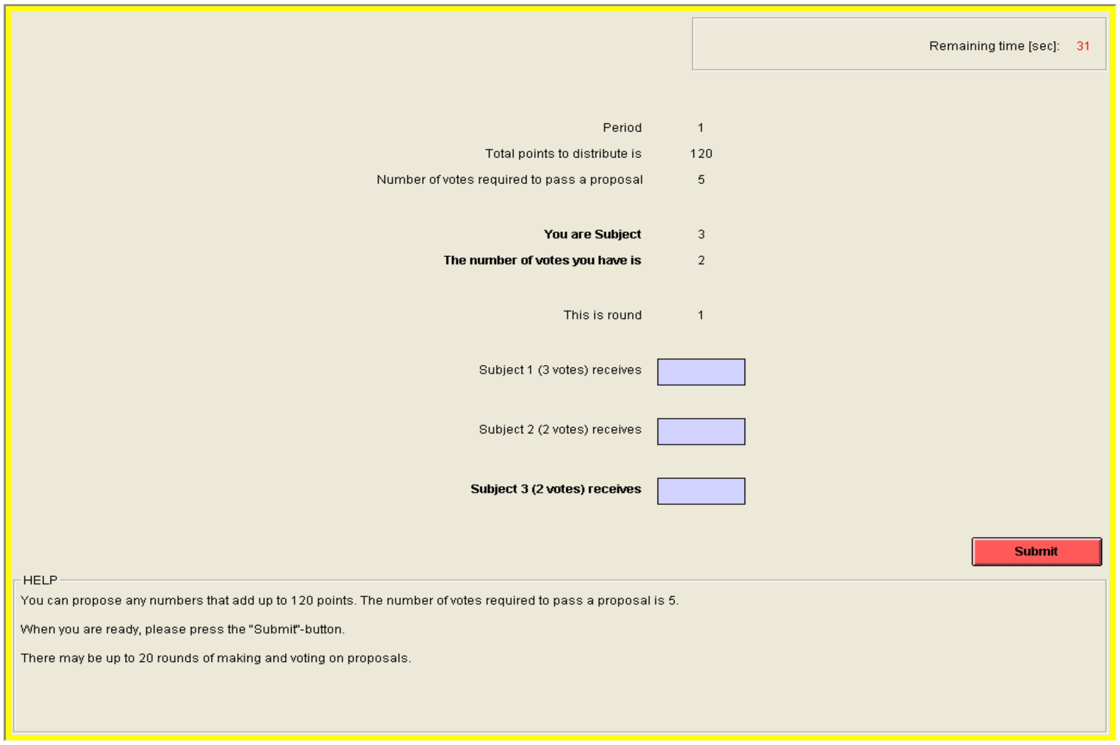

FIGURE 1. SCREEN FOR MAKING PROPOSALS

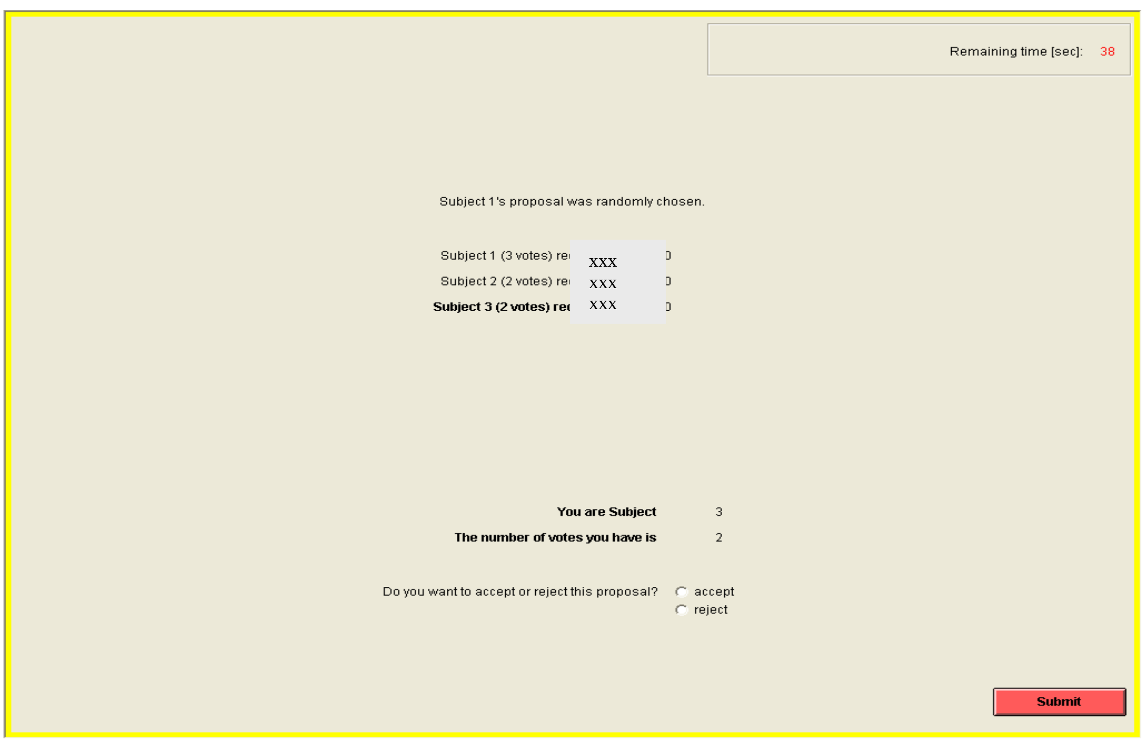

FIGURE 2: SCREEN FOR VOTING ON PROPOSALS