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Dual Sourcing with Price Discovery*

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Abstract

Reverse auctions are considered a fast and inexpensive price discovery tool to award procurement contracts and it is often desirable to award contracts to more than one supplier. We propose a new procurement procedure that is based on a reverse auction. Shares are allocated endogenously, depending on the suppliers' bids. The procedure obtains dual sourcing by assigning positive shares to the two most competitive bids and uses discarded bids to endogeneize the reserve price. In equilibrium the two most competitive suppliers are awarded contracts. Surprisingly, when discarded suppliers are competitive enough, the procedure not only allows taking advantage of dual sourcing but also generates lower procurement expenditures than a standard auction for sole sourcing. We also show that providers reveal their costs truthfully and that the procurement procedure can be used in different scenarios concerning what providers know about each others' costs, provided the assumption of private values holds.

Keywords: Dual Sourcing, Procurement Auctions, Contests, Price Discovery
Journal of Economic Literature Classification Numbers: **D44**, **D47**.

1 Introduction

Procurement is an important part of economic activity. Procurement of government contracts represents 19.96% for OECD countries and 14.48% for non-OECD countries, while the value of procurement transactions in the private sector is estimated to be even larger than in the public sector.[1] In this paper we propose a new procurement procedure that builds on reverse auction formats used in real-life procurement markets and has the potential to reconcile the conflicting aims of expenditure minimization and dual sourcing.

It is uncontroversial that procurement procedures should minimize costs. In some procurement markets, however, it is not only important to minimize procurement costs but also to avoid dependence on a single provider. Having only one supplier risks that the buyer is 'locked in' with one provider and experiences shortage in the case that this supplier cannot fulfil his obligations. Currently, for example, the state of Texas buys influenza vaccines from Novartis and Sanofi Pasteur, while meningococcal vaccines are provided by Sanofi Pasteur and Glaxosmithkline.[2] Similarly, in the private sector, Nokia and Toyota follow a dual sourcing strategy in order to reduce supply chain risk.[3] Since having several providers might require to forgo economies of scale or to buy from providers with different efficiency levels, conventional wisdom holds that there is a trade-off between expenditure minimization and a dual (or even multiple) sourcing strategy. The main contribution of this paper is to propose a procurement procedure that results in dual sourcing but has the potential to avoid this trade-off.[4]

Reverse auctions are considered a fast and inexpensive price discovery tool to award procurement contracts. A commonly employed dual sourcing strategy is as follows.[5] The buyer announces (i) what kind of objects or services are intended to be bought; (ii) the budget constraint (or reserve price, or bid ceiling); and (iii) an ex-ante specified proportion, say 70% and 30%, in which the two winning providers share the total amount to be bought, with the supplier proposing the lower price receiving the larger proportion. Providers make their bids and the two suppliers proposing the lowest prices are chosen. Tunca and Wu (2009) observe that this procedure results in the Vickrey outcome in which the two lowest-cost

[1] See OECD (2002) and Dimitri et al. (2006).

[2] See http://www.txsmartbuy.com/contracts/view/1741, accessed on 09/12/2015, and http://www.window.state.tx.us/procurement/contracts/all_term/, accessed on 09/12/2015, respectively.

[3] See http://www.scdigest.com/assets/newsViews/07-08-15-1.php?cid=1178, accessed on 09/12/2015, and http://www.scdigest.com/ontarget/12-03-07-2.php?cid=5576, accessed on 09/12/2015, respectively.

[4] A similar trade-off appears in procurement when there are affirmative action considerations and our procedure might be of interest in this context. For further motivation of a share auction in the context of affirmative action and a discussion of the trade-off between expenditure minimization and minority representation see Alcalde and Dahm (2013).

[5] Bilateral negotiations between the buyer and each supplier are the traditional price discovery tool and the main alternative to reverse auctions. For further details on the following procedure see Tunca and Wu (2009) and the clearing-price auction in Cramton et al. (2015).

providers win a share at the price offered by the third lowest cost provider. The procurement procedure that we propose modifies this format slightly, because rather than using an exogenous sharing rule, we postulate that suppliers share the total amount to be bought *depending on the bids submitted*. Shares are based on the difference between the buyer's budget (or reserve price) and the price proposed by each supplier. In what follows we refer to this difference as a supplier's bid. Shares are assigned depending on the relative difference of the bids of suppliers as a percentage of the largest bid (submitted by the supplier proposing the lowest price).

In this paper we investigate different procurement games in different informational environments but all these games have the following feature in common. In equilibrium the most efficient provider's price is strictly lower than the one of the second lowest cost provider, which in turn is weakly lower than the third lowest cost level. This implies that total procurement costs are always strictly lower than the third lowest cost, the outcome in the aforementioned Vickrey auction for dual sourcing. Our procedure results in very competitive procurement, because the winning suppliers are not only concerned with outbidding their rivals and obtaining a positive share, but also compete to increase the relative size of their shares. Given that the procurement procedures considered always outperform the standard Vickrey auction for dual sourcing, we employ a much stronger benchmark in this paper. We compare our procedure to a standard Vickrey auction for sole sourcing, in which procurement costs are equal to the second lowest cost. In other words, we ask when a buyer, who is not interested in dual sourcing, prefers to use our procedure. She will do so when procurement costs are lower with our procedure than with sole sourcing, in which case the trade-off between expenditure minimization and dual sourcing disappears.

We start our analysis with a simultaneous bidding game in which the two providers who propose the lowest prices are assigned procurement shares.[6] Discarded prices, however, are used for price discovery. The lowest discarded price is used to replace the initial budget (or reserve price). As a consequence the initial budget does not affect the equilibrium outcome and the buyer can save costly resources when determining it. In addition, the procedure avoids setting the initial reserve price too low and accidentally deterring participation of suppliers in the auction. Given the revised budget, the previously explained assignment rule for procurement shares is used to allocate shares to the two winners. We show that there is a multiplicity of undominated Nash equilibria in each of which the two lowest cost providers are assigned shares. Moreover, if discarded suppliers are competitive enough (as measured by the cost difference between the second and the third lowest cost providers), then we can guarantee that in all these equilibria procurement costs are lower than in a Vickrey auction for sole sourcing.

In the simultaneous mechanism the multiplicity of equilibria comes from the fact that

[6]Notice that even though only two suppliers win shares, any provider who wishes to participate can do so. In this sense there is free entry and equal treatment of providers, which is normatively appealing.

in equilibrium the second lowest price might lie anywhere between two extremes. In one extreme, the second lowest price just outbids the third lowest cost provider (by proposing a price equal to the third lowest cost) and the third lowest cost provider submits a relatively uncompetitive price. In the other extreme, the third lowest cost provider is much more competitive proposing a price just above his cost. As a result, the second lowest price is much lower. From the buyer's point of view the latter situation is much more attractive, because prices are more competitive and procurement costs are lower. In order to obtain this configuration of prices as a unique equilibrium prediction, we propose in Section 4 a two stage procedure.[7] In the first stage, the price discovery stage, providers propose prices. Based on these prices two suppliers are chosen to compete in the second stage. In this contest stage the two providers can adjust their initial price proposals within some bounds. We show that there is a unique Subgame Perfect Nash equilibrium in which suppliers do not use a weakly dominated price report in the first stage. Interestingly, providers reveal their costs truthfully at this price discovery stage, so that we can guarantee that the two lowest cost providers are assigned procurement shares. Moreover, compared to the simultaneous game, we derive a less demanding condition implying that in equilibrium procurement costs are lower than in a Vickrey auction for sole sourcing.

Up to this point our analysis considers the polar case in which suppliers are completely informed about each others' characteristics.[8] In Section 5, we consider the other polar case in which each provider only knows his own cost structure. We propose a variant of a reverse English (or Japanese) auction in which the buyer decreases the price continuously over time.[9] In our auction, the price decreases until all suppliers have dropped out. As before the third lowest price is used in order to revise the initial budget. In equilibrium only the two lowest cost providers obtain positive shares and shares depend on the drop out decisions. Although providers initially do not have information about each other, we show that during the course of the auction all the relevant information is revealed so that at the unique equilibrium the providers' prices and shares coincide with those in the unique equilibrium of the sequential procedure under complete information. Moreover, the equilibrium is in weakly dominant strategies. This implies that the procurement procedure can be used in different scenarios concerning what providers know about each others' costs, provided the assumption of private values holds.

[7]As observed by Tunca and Wu (2009) different forms of two-stage processes are frequently used in real life procurement situations. One form has a second round of bidding among the winners of the first stage.

[8]The complete information setting is considered appropriate when providers know each other well (Moldovanu and Sela, 2003). Examples are construction contracting (Bernheim and Whinston, 1986) or when technology can be considered to be stable (Anton and Yao, 1992, p. 691).

[9]Our auction is a reverse auction variant of the English auction analysed in Milgrom and Weber (1982). As observed by Tunca and Wu (2009) English reverse auctions are frequently used in real life procurement situations and advances in communication and information technologies make it possible to employ new approaches, like online bidding events.

Our analysis builds on our earlier paper, Alcalde and Dahm (2013), which considered the case of two suppliers and proposed to allocate procurement shares depending on the bids of suppliers. The assignment rule that we use to allocate shares to the two winning suppliers is a special case (more precisely the so-called case of unit elasticity) of their *Contested Procurement Auction* (CPA henceforth).[10] The present paper complements our earlier analysis in two important ways. First, the focus on two suppliers did not allow for a comparison to the Vickrey auction for dual sourcing, and so could not show that the latter is *always* outperformed. Second, our earlier paper took the budget constraint as exogenously given and therefore did not take advantage of price discovery to revise it and strengthen competition. Price discovery allows us to derive a condition under which equilibrium procurement costs are lower than in a Vickrey auction for sole sourcing. This condition is much less demanding than a similar condition in our earlier paper, which is based on the initial budget constraint.

Our paper also relates and contributes to several strands of literature. First, there is a literature on split-award auctions (Wilson, 1979; Bernheim and Whinston, 1986; Anton and Yao, 1987, 1989, 1990, 1992; Perry and Sákovics, 2003; Kremer and Nyborg, 2004; Inderst, 2008; Tunca and Wu, 2009; Anton et al., 2010; Gong et al., 2012; Bag and Li, 2014). A major difference is that with our procedure the split is endogenous rather than exogenous, allowing to induce strong competition and to improve upon sole sourcing. Second, given that we look at procurement from a design point of view, our paper also relates to the literature on optimal design of procurement auctions (Myerson, 1981; Dasgupta and Spulber, 1990; Maskin and Riley, 2000). As observed by Tunca and Wu (2009), p. 763, the theoretically optimal mechanisms are so complex that "they are almost never employed in procurement auctions as implemented today." In contrast, all procurement procedures that we propose are small variations of mechanisms used in real life procurement markets. In addition, we analyse the properties of a given procedure, rather than following a mechanism design approach, because we wish to exploit the strategic properties (related to dominance) that our procedure possesses. Thus, as Ewerhart and Fieseler (2003), who compare the unit-price-contract mechanism for procurement to a standard auction, we compare our procedure to a Vickrey auction for sole sourcing. Third, in our model we assign procurement shares based on bids of suppliers. This links our paper to the literature on contests, in which a so-called contest success function assigns win probabilities (or shares of a resource) depending on efforts of contestants. Most of this literature assumes that effort is exerted, even if a contestant does not win (the so-called all-pay contests; for recent surveys see Corchón, 2007; Konrad, 2009). Our study complements a small literature in which effort is only exerted if a contestant wins (the so-called winner-pay contests; see Skaperdas and Gan, 1995; Wärneryd,

[10]The name comes from a related solution for bankruptcy situations, the so-called *Contested Garment Principle*, see Dagan (1996).

2000; Corchón and Dahm, 2011; Yates, 2011).[11]

This paper is organized as follows. The next section introduces the procurement problem and the rule used to assign shares. Section 3 analyses the simultaneous bidding procedure, while Section 4 investigates the two stage bidding game. Both Sections 3 and 4 assume complete information. Section 5 supposes private information and considers the dynamic English reverse auction. Lastly, Section 6 offers some concluding remarks.

2 The Procurement Problem and a Solution

A buyer wishes to buy a certain quantity of a perfectly divisible good. The total amount to be acquired is normalized to 1. The buyer has an exogenously determined budget b. This budget specifies the maximum amount she can spend and will be interpreted as a reserve price. There are $n > 2$ potential suppliers. Each provider has a constant average cost $c_i \geq 0$ and sufficient capacity to supply the total amount to be acquired. We assume that[12]

$$c_1 < c_2 < \ldots < c_n < b,$$

and we start our analysis assuming that this is common knowledge among suppliers.[13]

The buyer's only objective is to minimize procurement costs. In particular, she does not value the advantages of dual sourcing over sole sourcing. This benchmark assumption makes it more difficult that the buyer prefers dual sourcing to sole sourcing, and hence makes our results more surprising.

For the case of two suppliers Alcalde and Dahm (2013) propose a solution to this problem. In the *Contested Procurement Auction* (CPA) providers simultaneously propose prices at which they are willing to provide the good. The CPA allocates to each supplier a share of the total amount to be acquired. Each supplier's share increases in the difference between the buyer's reserve price and the price proposed by this provider. This difference can be interpreted as the bid of this supplier. Given the so defined bids, shares are assigned depending on the relative difference of the suppliers' bids.[14] Alcalde and Dahm (2013) show that the

[11]Our analysis of the two-stage procedure also relates to elimination tournaments in which only winners proceed to later stages (Fullerton and McAfee, 1999; Fu and Lu, 2012), and the buyer's decision between dual and sole sourcing relates to the choice between a lottery contests and an all-pay auction (Fang, 2002; Epstein et al., 2013; Franke et al., 2014; Matros and Possajennikov, 2015).

[12]We exclude equalities for simplicity. This is the conservative assumption to make. Assuming that the most efficient providers have the same cost, it is much less surprising that bidding results in very competitive prices and overall procurement costs are low.

[13]We relax this assumption in Section 5. Similar to Bernheim and Whinston (1986) or Anton and Yao (1989), we assume that the buyer does not know the providers' costs.

[14]To be fully precise, Alcalde and Dahm (2013) consider a family of assignment rules that differ in the sensitivity of a supplier's procurement share with regard to his price. Suppose that suppliers 1 and 2 propose prices p_1 and p_2, respectively, with $p_1 \leq p_2$. In the simplest case of unit elasticity, the CPA assigns to providers

CPA has interesting strategic properties and potentially generates low procurement costs. The strategic properties are that the equilibrium is unique (Theorem 1), that the game is dominance solvable (Theorem 2), and that bids have a simple algebraic expression (Corollary 1). Consequently, the identification of equilibrium prices is straightforward. Moreover, when firms are heterogeneous enough (i.e., their costs are different enough, see expression (1) below), then equilibrium procurement costs are lower than in a Vickrey auction for sole sourcing. For later reference, Alcalde and Dahm's Corollary 2 states that the buyer's expenditures are lower than c_2 if

$$\frac{c_2 - c_1}{b - c_2} > \left(\frac{13}{8} + \frac{5}{8}\sqrt{17}\right) \approx 4.20. \tag{1}$$

A natural extension of the CPA to the case of n providers is as follows. Suppose that, given the suppliers' costs $C = (c_1, \ldots, c_i, \ldots, c_n)$ and the reserve price b, suppliers simultaneously choose prices $P = (p_1, \ldots, p_i, \ldots, p_n)$ at which they are willing to provide the good. In order to introduce the *Generalized Contested Procurement Auction* (GCPA), assume also that prices are increasingly ordered, i.e. for each $i < n-1$, $p_i \leq p_{i+1}$.[15] The GCPA allocates procurement shares to providers in the following way:

(a) if $p_1 < b$, then each supplier receives

$$\varphi_i^{GCPA}(P|\, b) = \sum_{j=i}^{n} \frac{\min\left\{p_{j+1}, b\right\} - \min\left\{p_j, b\right\}}{j\,(b - p_1)}, \tag{2}$$

with $p_{n+1} = b$;

(b) if $p_1 = b$, then all suppliers reporting the lowest price share the total amount equally at that price; and

(c) if $p_1 > b$, then there is no provision and each provider's share is zero.

Notice that the recursive allocation rule in (2) assigns equal shares in case of ties. It also incorporates a feasibility condition, because if a provider's price is higher than the buyer's budget constraint, his share is zero (except when part (b) applies).[16]

2 and 1 the shares $\varphi_2^{CPA} = (b - p_2)/[2(b - p_1)]$ and $\varphi_1^{CPA} = 1 - \varphi_2^{CPA}$, respectively. From a normative point of view, the CPA can be motivated through a connection to the framework of bargaining with claims, because it coincides with the relative claim-egalitarian solution (see Corchón and Dahm, 2010).

[15] If necessary relabel the set of bidders.

[16] In addition, the assignment rule reduces to the CPA for $n = 2$ and preserves the desirable mathematical properties of the rule for two agents. Specifically, it is anonymous so that shares are independent of providers' labels and depend only on prices. It is also continuous (everywhere but when all the providers select the whole budget) and the shares are monotonic in bids. Lastly, the assignment is homogeneous, that is, it is independent of the numéraire employed.

Given this allocation of procurement shares, consider the following normal form game Γ^{GCPA}. The n suppliers constitute the set of players. Each agent's strategy space is $[0, b]$. For each $P = (p_1, \ldots, p_i, \ldots, p_n)$, supplier i receives the procurement share

$$s_i(P) = \varphi_i^{GCPA}(P|\, b)$$

and his profit is

$$\Pi_i(P) = s_i(P)(p_i - c_i).$$

Although Γ^{GCPA} is a simple generalization of Alcalde and Dahm (2013), it turns out that it might not preserve some of the strategic properties of the CPA. In particular, there might be more than one Nash equilibrium and at some of the equilibria the ordering of prices might differ from the ordering of costs. The following Example 1 sheds further light on these issues. In addition, the example shows how the buyer can use the information revealed by the less efficient providers to reduce her procurement cost.

Example 1 There are three suppliers with costs $C = (50, 100, 103)$ and the buyer's budget constraint is $b = 150$. Simple computation identifies two Nash equilibria, $\widehat{P} = (\widehat{p}_1, \widehat{p}_2, \widehat{p}_3)$ and $\widetilde{P} = (\widetilde{p}_1, \widetilde{p}_2, \widetilde{p}_3)$, which are described in the following table.

	Prices			Shares			Cost
	p_1	p_2	p_3	$s_1(P)$	$s_2(P)$	$s_3(P)$	$\sum p_i s_i(P)$
$\widehat{P}$	107.13	121.08	126.50	0.57	0.25	0.18	114.10
$\widetilde{P}$	107.57	125.00	122.33	0.58	0.20	0.23	114.36

Note that at equilibrium $\widetilde{P}$ we have that $\widetilde{p}_3 < \widetilde{p}_2$ holds, even though it is assumed that $c_2 < c_3$.

Now, assume that the buyer learns that supplier 3's cost is $c_3 = 103$. Such a discovery about feasible procurement prices has not only an informational value but can also be used to strengthen competition between the two most efficient providers in the following way. The buyer can refine her (estimate of the reasonable) reserve price and set $b' = 103$. With this reserve price the equilibrium prices of providers 1 and 2 are 96.70 and 101.50, respectively. Procurement shares are $s_1 = 0.88$ and $s_2 = 0.12$. Moreover, overall procurement costs decline to $97.27 < c_2$, which –in contrast to the costs associated to Γ^{GCPA} with the initial reserve price– is lower than the expenditure associated to a Vickrey auction for sole sourcing.

Example 1 suggests that the existence of a competitive pool of potential suppliers might be beneficial for the buyer. Competition is strengthened, because price discovery results in a more realistic reserve price, and this in turn strengthens competition. The remainder of the paper formalizes this intuition by endogeneizing the reserve price in different ways.

3 Simultaneous Bidding with Endogenous Reserve Price

As observed in the Introduction, a commonly employed procedure to follow a dual sourcing strategy is as follows. The buyer stipulates a reserve price and an ex-ante specified proportion, say 70% and 30%, in which the two winning providers share the total amount to be acquired. Providers make their bids, the two suppliers choosing the lowest prices receive positive shares, and the supplier proposing the lowest price receives the larger proportion.

In this section we endogeneize the buyer's input into this procedure, reducing considerably the information required in practice. As in the standard procedure, the two suppliers choosing the lowest prices receive positive shares, but the size of shares is endogenous, as shares depend on the relative difference of the suppliers' bids. Moreover, the prices of discarded suppliers are used for price discovery in order to determine the reserve price.

Consider the following variation of the game Γ^{GCPA} introduced in Section 2. Providers simultaneously choose prices $P = (p_1, \dots, p_i, \dots, p_n)$, with $p_i \in [0, b]$ for each supplier i. Given the suppliers' prices, the third lowest price determines the 'endogenous' budget constraint $\widehat{t}(P)$. Formally, $\widehat{t}(P) \in \{p_1, \dots, p_i, \dots, p_n\}$ is such that

(a) $\{i : p_i \leq \widehat{t}(P)\}$ has at least three agents; and

(b) $\{h : p_h < \widehat{t}(P)\}$ has at most two agents.

To illustrate this definition notice that when $p_1 \leq p_2 \leq p_3 \leq \dots \leq p_n$ holds, we have that $\widehat{t}(P) = p_3$ even if $p_2 = p_3$ or $p_1 = p_3$. For each $P = (p_1, \dots, p_i, \dots, p_n)$, supplier i receives procurement share

$$s_i^{\mathrm{R}}(P) = \varphi_i^{GCPA}\left(P \mid \widehat{t}(P)\right),$$

and his profit is

$$\Pi_i^{\mathrm{R}}(P) = s_i^{\mathrm{R}}(P)(p_i - c_i). \tag{3}$$

Note that, since $s_i^{\mathrm{R}}(P)$ is based on the endogenous reserve price $\widehat{t}(P)$ rather than the initial budget b, in practice this game results in dual sourcing.[17] This implies that providers who select prices higher or equal than the third highest price receive zero profits. Denote by Γ^{R} the game previously described; i.e. the n suppliers constitute the set of players, each agent's strategy space is $[0, b]$, and for each P player i's profit follows $\Pi_i^{\mathrm{R}}(P)$ as described in equation (3).

In the remainder of this section we provide a positive analysis of Γ^{R}. We will see that this game has a Nash equilibrium and that each such Nash equilibrium, say P^*, has the following properties:

[17] As it is described, the set of active providers might also consist of more than two suppliers (when more than two providers select the lowest price) or be a singleton. Nevertheless, this will not be the case when agents act strategically.

(1) Prices: $p_1^* < p_2^* < p_i^*$ for each $i \geq 3$, with $c_2 < p_2^* \leq c_3$.

(2) Profits: $\Pi_i^{\mathrm{R}}(P^*) > 0$ for $i \leq 2$ and $\Pi_i^{\mathrm{R}}(P^*) = 0$ for $i \geq 3$.

(3) Procurement costs: if supplier 3 is competitive enough, then the buyer's overall expenditures are lower than c_2.

In equilibrium the two most competitive suppliers are awarded contracts. These suppliers obtain positive profits and have thus a strong incentive to participate in the procedure. Moreover, when competition is strong enough, the procurement expenditures generated are lower than in a Vickrey auction for sole sourcing. The following Lemma 1 characterizes the set of Nash equilibria for Γ^{R}. Theorem 1 offers a refinement of these equilibria as well as results on profits and overall procurement costs that are true in all Nash equilibria.

Lemma 1 $P^* = \left(p_1^*, \ldots, p_i^*, \ldots, p_n^*\right)$ is a Nash equilibrium for Γ^{R} if, and only if, it satisfies the following conditions:

(a) for each $i \geq 3$,

$$p_i^* \geq \widehat{t}(P^*);$$

(b) provider 2's strategy satisfies

$$c_2 < p_2^* = \frac{\widehat{t}(P^*) + c_2}{2} \leq c_3; \text{ and}$$

(c) provider 1's strategy is described by

$$c_1 < p_1^* = \widehat{t}(P^*) - \frac{\left(\widehat{t}(P^*) - c_1\right)^{\frac{1}{2}} \left(\widehat{t}(P^*) - c_2\right)^{\frac{1}{2}}}{2}.$$

Proof. We prove first that any P^* fulfilling conditions (a), (b) and (c) is a Nash equilibrium for Γ^{R}. Observe that condition (a) establishes that

$$\widehat{t}(P^*) = \min_{i \geq 3} \left\{p_i^*\right\}.$$

This implies that, for each $i \geq 3$,

$$\Pi_i^{\mathrm{R}}(p^*) = \left(p_i^* - c_i\right) s_i^{\mathrm{R}}(P^*) = 0. \tag{4}$$

This is because $s_i^{\mathrm{R}}(P^*) = \varphi_i^{GCPA}\left(P^* | \widehat{t}(P^*)\right) = 0$. Moreover, since for provider $i \leq 2$, $p_i^* > c_i$ and $s_i^{\mathrm{R}}(P^*) > 0$, the two providers get positive profit, $\Pi_i^{\mathrm{R}}(p^*) > 0$.

Now, assume that P^* is not an equilibrium. Then, there should be a provider, say j, and a strategy for him, say p_j such that

$$\Pi_j^{\mathrm{R}}\left(p_j, P_{-j}^*\right) > \Pi_j^{\mathrm{R}}\left(P^*\right). \tag{5}$$

Assume that $j \notin \{1,2\}$. Therefore, by (4),

$$s_j^{\mathrm{R}}\left(p_j, P_{-j}^*\right) > 0, \tag{6}$$

which implies, by condition (b), that $p_j < p_2^* \leq c_j$. Note that this contradicts condition (5), because

$$\Pi_j^{\mathrm{R}}\left(p_j, P_{-j}^*\right) = \left(p_j - c_j\right) s_j^{\mathrm{R}}\left(p_j, P_{-j}^*\right) < 0 = \Pi_j^{\mathrm{R}}\left(P^*\right).$$

Therefore $j \in \{1,2\}$. Consider the real valued function g_j defined as

$$g_j(x) = \Pi_j^{\mathrm{R}}\left(x, P_{-j}^*\right). \tag{7}$$

Note that $g_j(x) > 0$ if, and only if, $x \in \left(c_j, \widehat{t}(P^*)\right)$. Moreover, g_j is strictly concave and continuously differentiable in this interval. Then, by optimizing g_j in $\left(c_j, \widehat{t}(P^*)\right)$ we obtain condition (b) for $j=2$, or condition (c) when $j=1$.

We now turn to the necessary condition. Let $\widetilde{P} = (\tilde{p}_1, \ldots, \tilde{p}_i, \ldots, \tilde{p}_n)$ be a Nash equilibrium; we will prove that it has to fulfil conditions (a), (b) and (c).

Notice that there should be some i such that $\tilde{p}_i < b$. Otherwise, any seller can ensure to be the sole provider by declaring $p_i' = \frac{b+c_i}{2}$. By selecting this strategy her profit will be $\frac{b-c_i}{2}$, which is larger than her profit when each of the providers reports b, namely $\frac{b-c_i}{n}$. Therefore, $\widetilde{P}$ must satisfy that, for each provider i

$$\tilde{p}_i \geq c_i \text{ whenever } s_i^{\mathrm{R}}\left(\widetilde{P}\right) > 0. \tag{8}$$

Without loss of generality we can assume that $\tilde{p}_1 \leq \tilde{p}_2$. Observe that if $\tilde{p}_2 < \tilde{p}_h$ for each $h \geq 3$, then $\tilde{p}_2 \leq c_3$. Otherwise, provider 3 might deviate by proposing

$$p_3' = \frac{\tilde{p}_2 + c_3}{2},$$

which satisfies $c_3 < p_3' < \tilde{p}_2$ and, therefore

$$\Pi_3^{\mathrm{R}}\left(p_3', \widetilde{P}_{-3}\right) = \left(p_3' - c_3\right) s_3^{\mathrm{R}}\left(p_3', \widetilde{P}_{-3}\right) > 0 =$$

$$= \left(\tilde{p}_3 - c_3\right) s_3^{\mathrm{R}}\left(\widetilde{P}\right) = \Pi_3^{\mathrm{R}}\left(\widetilde{P}\right),$$

contradicting that $\widetilde{P}$ is a Nash equilibrium.

In what follows we assume, without loss of generality, that $\tilde{p}_3 \leq \tilde{p}_h$ for each $h \geq 3$. Suppose that $\tilde{p}_3 \leq \tilde{p}_2$. This implies that $s_3^{\mathrm{R}}\left(\widetilde{P}\right) > 0$. By equation (8) we have that $\tilde{p}_3 \geq c_3 > c_2$.

Consider the following three cases, which exhaust all the possibilities:

(1) $\tilde{p}_2 > \max\{\tilde{p}_1, \tilde{p}_3\}$.

In such a case, $s_2^{\mathrm{R}}\left(\tilde{P}\right) = 0$. Therefore, provider 2 might improve her profit by selecting

$$p_2' = \frac{c_2 + \tilde{p}_3}{2},$$

because

$$s_2^{\mathrm{R}}\left(p_2', \widetilde{P}_{-2}\right) > 0,$$

and thus

$$\Pi_2^{\mathrm{R}}\left(p_2', \widetilde{P}_{-2}\right) = \left(p_2' - c_2\right) s_2^{\mathrm{R}}\left(p_2', \widetilde{P}_{-2}\right) > 0.$$

(2) $\tilde{p}_2 = \max\{\tilde{p}_1, \tilde{p}_3\} > \min\{\tilde{p}_1, \tilde{p}_3\}$.

Then, $\widehat{t}\left(\widetilde{P}\right) = \tilde{p}_2$. Moreover, $\Pi_2^{\mathrm{R}}\left(\widetilde{P}\right) = 0$. By (8) we have that $c_2 < c_3 \leq \tilde{p}_3 \leq \tilde{p}_2$ and hence there exists $p_2' \in (c_2, \tilde{p}_2)$ such that $c_2 < p_2' < \widehat{t}\left(p_2', \tilde{P}_{-2}\right)$. This implies that $s_2^{\mathrm{R}}\left(p_2', \widetilde{P}_{-2}\right) > 0$ and, consequently

$$\Pi_2^{\mathrm{R}}\left(p_2', \widetilde{P}_{-2}\right) > 0 = \Pi_2^{\mathrm{R}}\left(\widetilde{P}\right).$$

(3) $\tilde{p}_2 = \tilde{p}_1 = \tilde{p}_3$.

In such a case, by (8) we have that $c_2 < c_3 \leq \tilde{p}_3 = \tilde{p}_2 \leq \tilde{p}_h$ for each $h > 3$. Therefore,

$$0 < \Pi_2^{\mathrm{R}}\left(\widetilde{P}\right) \leq \frac{1}{3}\left(\tilde{p}_3 - c_2\right).$$

Note that, by declaring $p_2' = \frac{c_2 + \tilde{p}_3}{2}$ seller 2 becomes the sole provider, getting a profit

$$\Pi_2^{\mathrm{R}}\left(p_2', \widetilde{P}_{-2}\right) = \frac{1}{2}\left(\tilde{p}_3 - c_2\right) > \frac{1}{3}\left(\tilde{p}_3 - c_2\right) = \Pi_2^{\mathrm{R}}\left(\widetilde{P}\right).$$

Therefore, we conclude that $\tilde{p}_2 < \tilde{p}_h$ for all $h \geq 3$. A similar argument can be used to see that $\tilde{p}_1 < \tilde{p}_h$ for all $h \geq 3$. Hence,

$$\widehat{t}\left(\widetilde{P}\right) = \min_{h \geq 3} \tilde{p}_h > \max\{\tilde{p}_1, \tilde{p}_2\},$$

which implies that $\max\{\tilde{p}_1, \tilde{p}_2\} \leq c_3$. To conclude our proof, we mention that the expressions in equations (b) and (c) are derived in Alcalde and Dahm (2013), Corollary 1. ■

The multiplicity of equilibria in the previous result comes in part from the fact that in equilibrium provider 3 might set his price very close to c_2. While such a competitive bid benefits the buyer, it might not be a good prediction of supplier 3's behaviour, because such a price is strictly dominated. More precisely, for all suppliers i the strategy $p_i \leq c_i$ is strictly dominated by any $p_i' \in (c_i, b)$. This is so, because selecting p_i provider i is certain to get non-positive profits, while choosing p_i' he is certain not to make losses and, depending on his rivals' strategies, his profits might be strictly positive. Assuming that providers restrict to undominated strategies, we have that $\widehat{t}(P^*) > c_3$.

Our next result reports that Γ^{R} has undominated Nash equilibria. Moreover, with regard to equilibrium profits and procurement costs, the restriction to undominated strategies is not important. Lastly, Theorem 1 provides a condition on the providers' heterogeneity, assuring that the buyer's expenditures are lower than in a Vickrey auction for sole sourcing.

Theorem 1 Γ^{R} has at least one undominated Nash equilibrium, P^*. For each Nash equilibrium P^*, and agent i, $\Pi_i^{\mathrm{R}}(P^*) > 0$ if, and only if, $c_i < c_3$. Moreover, if

$$\frac{c_2 - c_1}{c_3 - c_2} > \left(\frac{13}{4} + \frac{5}{4}\sqrt{17}\right) \approx 8.40, \tag{9}$$

then, for each Nash equilibrium P^*,

$$\sum_{i=1}^{n} p_i^* s_i^{\mathrm{R}}(P^*) < c_2.$$

Proof. First, consider $\widehat{P}$ such that,

(a) $p_n = \min\left\{\frac{b+c_n}{2}, 2c_n - c_{n-1}\right\}$;

(b) for each $n > i \geq 3$, $\hat{p}_i = \min\left\{\frac{c_{i+1}+c_i}{2}, 2c_i - c_{i-1}\right\}$;

(c) $\hat{p}_2 = \frac{c_2+\hat{p}_3}{2}$; and

(d) $\hat{p}_1 = \hat{p}_3 - \frac{\sqrt{(\hat{p}_3-c_1)\,(c_2-c_1)}}{2}$.

It is easy to check that $\widehat{P}$ fulfils the conditions established in Lemma 1 and thus is a Nash equilibrium for Γ^{R}. Moreover each provider selects an undominated $p_i \in (c_i, b)$.

Second, let $P^* = \left(p_1^*, \ldots, p_i^*, \ldots, p_n^*\right)$ be a Nash equilibrium for Γ^{R}. By Lemma 1 we have that

$$p_2^* = \frac{\widehat{t}(P^*) + c_2}{2} \leq c_3, \text{ and} \tag{10}$$

$$p_1^* = \widehat{t}(P^*) - \frac{\left(\widehat{t}(P^*) - c_1\right)^{\frac{1}{2}} \left(\widehat{t}(P^*) - c_2\right)^{\frac{1}{2}}}{2}. \tag{11}$$

Notice that $c_i < p_i^* < \widehat{t}(P^*)$ for $i = 1, 2$ hold. Consequently, both suppliers 1 and 2 make a strictly positive profit; all other providers are not awarded contracts and receive zero profit.

Lastly, notice that

$$\frac{\widehat{t}(P^*)-c_1}{\widehat{t}(P^*)-c_2}=1+\frac{c_2-c_1}{\widehat{t}(P^*)-c_2}\geq 1+\frac{c_2-c_1}{2(c_3-c_2)},$$

where the last inequality comes from the fact that (10) implies that $\widehat{t}(P^*)\leq 2c_3-c_2$. Using Corollary 2 in Alcalde and Dahm (2013), we have that

$$\frac{c_2-c_1}{2(c_3-c_2)}>\left(\frac{13}{8}+\frac{5}{8}\sqrt{17}\right)\approx 4.20,$$

which is equivalent to the condition in the statement and assures that at any Nash equilibrium procurement costs are strictly lower than c_2. ■

We have seen that in all equilibria of Γ^{R} the two most efficient suppliers are assigned positive procurement shares. We conclude this section by providing bounds on the (undominated) equilibrium prices of these providers. As previously explained, in each equilibrium P^* in which suppliers restrict to undominated strategies we have $\widehat{t}(P^*)>c_3$. Moreover, part (b) of Lemma 1 implies that $\widehat{t}(P^*)\leq 2c_3-c_2$. Notice that the interval $(c_3,2c_3-c_2]$ is the smaller, and thus we can think of the multiplicity of equilibria as being smaller, the closer c_2 and c_3 are. Applying part (b) of Lemma 1 again implies at any equilibrium P^* that

$$p_2^*\in\left(\frac{c_2+c_3}{2},c_3\right]. \tag{12}$$

Moreover, for $\widehat{t}(P^*)$ given, provider 1's price follows expression (11) above. Note that for c_1 and c_2 given, p_1^* is a convex function of $\widehat{t}(P^*)$, it decreases for $\widehat{t}(P^*)$ close to c_2 and reaches a minimum at

$$\widehat{t}(P^*)=\frac{c_1+c_2}{2}+\frac{\sqrt{3}}{3}(c_2-c_1). \tag{13}$$

Taking all together, we can ensure that p_1^* is bounded from below and from above. The lower bound is such that

$$p_1^*\geq \min\left\{\frac{1}{2}(c_1+c_2)+\frac{\sqrt{3}}{4}(c_2-c_1),2c_3-c_2-\left(\frac{(2c_3-c_2-c_1)(c_3-c_2)}{2}\right)^{\frac{1}{2}}\right\},$$

whereas the upper bound is such that

(a) if $105c_3<27c_1+78c_2+28\sqrt{6}(c_2-c_1)$, then[18]

$$p_1^*<c_3-\frac{\sqrt{(c_3-c_1)(c_3-c_2)}}{2}, \text{ and}$$

[18] This inequality follows from comparing the right hand side of expression (11) for $\widehat{t}(P^*)=c_3$ and for $\widehat{t}(P^*)=2c_3-c_2$.

(b) if $105c_3 \geq 27c_1 + 78c_2 + 28\sqrt{6}(c_2 - c_1)$, then

$$p_1^* \leq 2c_3 - c_2 - \frac{\sqrt{(2c_3 - c_2 - c_1)(c_3 - c_2)}}{2}.$$

The following example summarizes the results of this section.

Example 2 There are three suppliers with costs $C = (50, 100, 110)$ and the buyer's budget constraint is $b = 150$. In this section we have shown that in all equilibria the two most efficient providers receive procurement shares. The assumption of undominated prices and part (b) of Lemma 1 imply that supplier 3's price belongs to the interval $(110, 120]$. Therefore, there are two extreme cases:

(a) when $p_3 \to 110 = c_3$, then $p_1 \to 97.75$ and $p_2 \to 105$. Procurement costs are 99.23, which is lower than c_2.

(b) when $p_3 = 120 = 2c_3 - c_2$, then $p_1 = 101.29$ and $p_2 = 110$. Procurement costs are 103.62, which is strictly higher than c_2.

We conclude that Γ^R has equilibria in which the procurement procedure does not perform better than a Vickrey auction for sole sourcing. On the other hand, from the derivation of (9) it also follows that there exists an undominated Nash equilibrium with strictly lower procurement costs than a standard auction if the left hand side of (9) is strictly lower than 4.20. This improves upon (1), the condition for the two-provider case, as $c_3 < b$. In the next section we propose a closely related sequential procedure and argue that it leads to an equilibrium with this property.

4 Truthful Reserve Price Discovery

As observed in the Introduction, some commonly employed procurement procedures include two-stage processes that have a second round of bidding among the winners of the first stage. In this section we propose such a mechanism in order to improve upon the simultaneous procedure analysed in the previous section. In the first stage, the price discovery stage, suppliers are asked to reveal their true marginal cost. The two most efficient providers are selected to compete in the second stage. In this second stage, the contest stage, the two providers compete in the *Contested Procurement Auction* CPA (Alcalde and Dahm, 2013), with the buyer's reserve price endogenously determined by the third lowest report in the price discovery stage.

We show that this procedure induces truthful reporting at the price discovery stage. The intuition for this is as follows. First, it is not beneficial to exaggerate costs, as it harms the prospect to progress to the second stage. Second, the reason why it is not beneficial to understate costs follows the logic of King Solomon's Dilemma, as formalized by Glazer and Ma (1989): if a supplier claims to be more efficient than he really is, he risks to compete in the contest stage under the conditions of his report. The contest stage depends on his report through the endogenous reserve price, implying that reporting false low prices might induce negative profits.

We formalize this two-stage game as follows. We start with some notation. For a given vector of prices $P = (p_1, \ldots, p_i, \ldots, p_n)$, define $A(P) = \left\{i \colon p_i < \widehat{t}(P)\right\}$, and $B(P) = \left\{i \colon p_i = \widehat{t}(P)\right\}$. Lastly, similar to Glazer and Ma (1989, p. 225) we introduce $\epsilon > 0$ such that $\epsilon < \left|c_i - c_j\right|$ for any pair of suppliers $i \neq j$. The role of this fixed parameter will become clear shortly, so at this point we only remark that ϵ is determined by the buyer as part of the procurement procedure and that for interpretative purposes we think of this number as being very small.

Definition 1 The CPA with Entry game Γ^{E} is a two-stage game. The n suppliers constitute the set of players. At the price discovery stage, each provider reports a price $p_i \in [0, b]$. Given the reported prices, $P = (p_1, \ldots, p_i \ldots, p_n)$, two suppliers advance to the contest stage, according to the rules below. At the contest stage, both providers revise their price selecting

$$r_i \in \left[0, \widehat{t}(P)\right]. \tag{14}$$

Based on these revised prices procurement shares follow

$$s_i^{\mathrm{E}}\left(P^{\mathrm{E}}\right) = \varphi_i^{GCPA}\left(P^{\mathrm{E}} \middle| \widehat{t}(P) + \epsilon\right),$$

where P^{E} is the vector in which the $h-th$ component is r_h if h is one of the suppliers participating in the contest stage, and p_h otherwise.[19]

To conclude the description of Γ^{E} we describe now how the competitors in the contest stage are selected.

(a) If $A(P)$ has two elements, then the competitors in the contest stage are the suppliers in $A(P)$.

(b) If $A(P)$ is a singleton, then one of the competitors in the contest stage is the supplier in $A(P)$. The other provider is selected with equal probability from the suppliers in $B(P)$.

(c) If $A(P)$ is empty, then the two competitors in the contest stage are selected with equal probability from the suppliers in $B(P)$.

[19]Notice that the revised reserve price exceeds the upper bound of the prices at the contest stage by $\epsilon > 0$. This is a technical condition needed to apply Theorem 2 in Alcalde and Dahm (2013) that allows us to conclude that the contest stage is dominance solvable. It also guarantees that the shares in the contest stage are strictly positive. Nevertheless, given that ϵ is assumed to be very small, for interpretative purposes we will think of the revised reserve price as coinciding with the third most efficient price report of the price discovery stage.

In our analysis of Γ^{E} we focus on Subgame Perfect Nash Equilibria (SPNE henceforth), applying a backward induction argument. Similarly to, for example, Baron and Kalai (1993) or Austen-Smith and Banks (2005, Section 4.1), we apply a notion of stage-undominated strategies to the price discovery stage. In order to define such a notion for the first stage of Γ^{E}, however, it is not necessary to assume that providers anticipate equilibrium play in the second stage. More precisely, the following observations are sufficient. Given a price vector P from the price discovery stage and conditional on competing at the contest stage, a provider i anticipates the following about the contest stage, independently with whom he competes and what price his rival sets:

(a) If $c_i > \widehat{t}(P)$, then any $r_i \leq \widehat{t}(P)$ implies that the provider receives a strictly positive share at a price strictly below cost, and obtains a strictly negative profit.

(b) If $c_i = \widehat{t}(P)$, then any $r_i < \widehat{t}(P)$ again implies losses, and the best the supplier can do is setting $r_i = \widehat{t}(P)$, which guarantees zero profits.

(c) If $c_i < \widehat{t}(P)$, then setting $r_i = (c_i + \widehat{t}(P))/2$ yields a strictly positive share at a price exceeding costs, and thus strictly positive profits (even though this might not the equilibrium price in the anticipated subgame).

Given that providers anticipate (a)-(c), we apply the usual notion of weak dominance to the first stage. That is, a price report p_i is weakly dominated at the price discovery stage if there exists another price report p'_i such that for all price report vectors $P_{-i} = \left(p_j\right)_{j \neq i}$ of his rivals $\pi_i\left(P_{-i}, p'_i\right) \geq \pi_i\left(P_{-i}, p_i\right)$ holds, with strict inequality for some P_{-i}. A price report p'_i is a weakly dominant report for supplier i at the price discovery stage if it weakly dominates every other report. In order to show that for each supplier i the truthful price report $p_i = c_i$ is weakly dominant we proceed in two steps.

Lemma 2 For each supplier i any price report $p_i < c_i$ is weakly dominated by $p'_i = c_i$.

Proof. Consider a given supplier, say i, a price report vector $P_{-i} = \left(p_j\right)_{j \neq i}$ of his rivals, and let $p_{(1)}$ and $p_{(2)}$ be the lowest competing prices. More precisely, let $p_{(1)} \leq p_{(2)} \leq p_j$ for each $j \notin \{(1), (2), i\}$. We assume $0 \leq p_i < p'_i = c_i$ and consider the following cases, which exhaust all the possibilities.

(a) $p'_i < p_{(2)}$. Then $i \in A(P)$ and $i \in A(P')$, so that with both reports i proceeds to the second stage. Moreover, $\widehat{t}(P) = \widehat{t}(P')$, and hence $\pi_i\left(P_{-i}, p_i\right) = \pi_i\left(P_{-i}, p'_i\right)$.

(b) $p_{(2)} < p_i$. Then with both reports provider i does not proceed to the second stage and obtains zero profits.

(c) $p_i \leq p_{(2)} < p'_i$. In this case provider i proceeds with some positive probability to the second stage when reporting p_i, but does not proceed when reporting p'_i. In the latter case his profits are zero. In the former case i's profit is strictly negative, as $c_i > \widehat{t}(P)$.

(d) $p_{(2)} = p_i'$. Then $i \in A(P)$ and $i \in B(P')$, so that reporting p_i supplier i is certain to proceed to the second stage, while reporting p_i' supplier i only proceeds with a positive probability to the second stage. Since however $\widehat{t}(P') = \widehat{t}(P) = c_i$, profits are zero when proceeding to the second stage and hence $\pi_i(P_{-i}, p_i) = \pi_i\left(P_{-i}, p_i'\right)$.

Note that in all cases it is weakly better to report p_i' rather than p_i, with a strict preference in case (c). Hence, any price report $p_i < c_i$ is weakly dominated by $p_i' = c_i$. ■

Lemma 3 For each supplier i any price report $p_i > c_i$ is weakly dominated by $p_i' = c_i$.

Proof. Consider a given supplier, say i, and a price report vector $P_{-i} = \left(p_j\right)_{j \neq i}$ of his rivals, and let $p_{(1)}$ and $p_{(2)}$ be the lowest competing prices. More precisely, let $p_{(1)} \leq p_{(2)} \leq p_j$ for each $j \notin \{(1), (2), i\}$. We assume $c_i = p_i' < p_i$ and consider the following cases, which exhaust all the possibilities.

(a) $p_i < p_{(2)}$. Then $i \in A(P)$ and $i \in A(P')$, so that with both reports i proceeds to the second stage. Moreover, $\widehat{t}(P) = \widehat{t}(P')$, and hence $\pi_i(P_{-i}, p_i) = \pi_i\left(P_{-i}, p_i'\right)$.

(b) $p_i' < p_{(2)} < p_i$. In this case reporting p_i' supplier i proceeds to the second stage and obtains a strictly positive profit, as $\widehat{t}(P') > c_i$. On the other hand, reporting p_i his profits are zero, because he does not proceed to the second stage.

(c) $p_i' < p_{(2)} = p_i$. Then $c_i < \widehat{t}(P') = \widehat{t}(P)$, and provider i strictly prefers to report p_i', as reporting p_i he does not always proceed to the second stage.

(d) $p_{(2)} < p_i'$. Then with both reports provider i does not proceed to the second stage and obtains zero profits.

(e) $p_{(2)} = p_i'$. Then, when reporting p_i' supplier i has a positive probability of proceeding to the second stage in which case his profit is zero, as $\widehat{t}(P') = c_i$. Reporting p_i his profits are also zero, because he does not proceed to the second stage.

Note that in all cases it is weakly better to report p_i' rather than p_i, with a strict preference in cases (b) and (c). Hence, any price report $p_i > c_i$ is weakly dominated by $p_i' = c_i$. ■

Lemmata 2 and 3 imply that for each supplier i the truthful price report $p_i = c_i$ weakly dominates every other report at the price discovery stage. We summarize with the following result.

Theorem 2 Γ^{E} has a unique SPNE in which providers do not use a weakly dominated price report at the price discovery stage. In this SPNE suppliers report their marginal costs truthfully at the first stage and the contest stage is dominance solvable. Moreover, if

$$\frac{c_2 - c_1}{c_3 + \epsilon - c_2} > \left(\frac{13}{8} + \frac{5}{8}\sqrt{17}\right) \approx 4.20, \tag{15}$$

then the buyer's procurement expenditures in this SPNE are lower than c_2.

Proof. Following Lemmata 2 and 3, we have that reporting marginal costs truthfully at the first stage is a weakly dominant report for each supplier. Thus, providers 1 and 2 compete in the contest stage. Theorems 1 and 2 in Alcalde and Dahm (2013) imply that this subgame has a unique equilibrium and is dominance solvable. As a consequence, condition (15) can be established by replacing b with $c_3 + \epsilon$ in condition (1). ■

Notice that condition (15) improves upon condition (1), as $c_3 + \epsilon < b$. This shows that the existence of a competitive pool of potential suppliers and endogeneizing the reserve price through the two-stage procedure Γ^E is beneficial for the buyer, compared to the *Contested Procurement Auction* with only two suppliers.

We conclude this section by pointing out that Γ^E also admits equilibria in which providers use a weakly dominated price report at the price discovery stage. This, however, is not surprising. It is well known that standard Vickrey auctions also admit equilibria in which suppliers employ a weakly dominated strategy.

Example 3 Reconsider Example 2 in which there are three suppliers with costs $C = (50, 100, 110)$ and the buyer's budget constraint is $b = 150$.

First, consider a standard sealed-bid second-price auction for sole sourcing among the two most efficient suppliers. Notice that the price pair $(100, 50)$ is a Nash equilibrium in which both providers use a weakly dominated strategy and the more efficient supplier is not selected.

Second, consider a sealed-bid Vickrey auction for dual sourcing among all three providers, in which the two winning suppliers provide their shares at the highest price. Suppose winners obtain equal shares. Notice that the price pair $(100, 110, 100)$ is a Nash equilibrium in which all providers use a weakly dominated strategy and the second most efficient supplier is not selected.

Third, consider Γ^E. On one hand, notice that the first stage price report triple $(100, x, 100)$ with $x \in [110, b]$ is part of an equilibrium in which all providers use a weakly dominated price report at the price discovery stage and the second most efficient supplier does not proceed to the contest stage.[20] On the other hand, the focus on weakly undominated price reports yields—as in Vickrey auctions—a unique prediction. The reserve price is revised to $\widehat{t}(P) = 110$, providers 1 and 2 proceed to the contest stage, and the final provision prices (as $\epsilon \to 0$) are $r_1^* = 97.75$ and $r_2^* = 105.00$. Procurement shares are 80% for provider 1 and 20% for supplier 2, while expenditures are $99.23 < c_2$.

[20]Notice that the construction of such an equilibrium requires in addition to the use of weakly dominated strategies the following two features that we find unreasonable. First, supplier 3 must 'preempt' provider 2 by reporting a price of at most c_2, as otherwise provider 2 could profitably deviate and lower his report sufficiently to proceed to the contest stage. Second, supplier 2 must make this 'preemption' profitable for provider 3 by fixing the reserve price high enough. In other words, supplier 2 can avoid the coordination on such an equilibrium by managing the beliefs of his rivals in such a way that they are certain that he reports a price strictly below c_3.

Fourth, consider Γ^{R} as analysed in Example 2. We see that we can think of the two-stage procedure Γ^{E} as choosing among the multiple equilibrium outcomes of Γ^{R} the outcome based on the most competitive reserve price. As a result condition (15) improves upon condition (9).

5 Private Information and Cost Discovery

The analysis so far considered the polar case in which each supplier is completely informed about the costs of each of his rivals. As argued in the Introduction, this might be realistic in some situations. In other instances, however, this assumption is less appealing. In this section we relax the assumption of complete information and consider the other polar case in which each supplier only has (private) information about his own costs, but does not know the costs of his rivals. We analyse a simple continuous-time version of our procedure that is easily implementable through the increased ability to communicate in real time via the Internet. During the course of the auction, information about rivals is not needed, because all the relevant information is revealed. In particular, discarded suppliers reveal their costs truthfully. As a result, at the unique equilibrium the procurement shares and profits of providers coincide with those in the complete information environment of Theorem 2.

Since the procedure is based on an *electronic reverse auction*, we refer to it as 'ε-Procurement' mechanism. We describe the corresponding game $\Gamma^{\varepsilon\text{-P}}$ as follows.

Definition 2 The 'ε-Procurement' game $\Gamma^{\varepsilon\text{-P}}$ is a continuous-time mechanism. The n suppliers constitute the set of players. At each moment $m \in [0,1]$ all providers simultaneously select their messages (or actions) $a_{im} \in \{0,1\}$, where 0 is interpreted as 'continuing' and 1 as 'stopping'. We denote by m_i the first moment at which i selects $a_{im_i} = 1$, and thus $a_{im} = 0$ for each $m < m_i$.

Given the sequence of actions of providers, supplier i's price is given by

$$p_i = (1 - m_i)\, b, \tag{16}$$

where b is the initial (exogenous) reserve price. At $m = 1$ the auction closes and we interpret this as all suppliers choosing at this moment message 1.[21] At the conclusion of the auction all prices are hence determined and each supplier's procurement share follows

$$s_i^{\varepsilon\text{-P}}(P) = \varphi_i^{GCPA}\left(P \mid \widehat{t}(P)\right). \tag{17}$$

His profit is thus given by

$$\Pi_i^{\varepsilon\text{-P}}(P) = s_i^{\varepsilon\text{-P}}(P)\,(p_i - c_i)\,. \tag{18}$$

[21] That is, we use the convention that $p_i = 0$ when $a_{im} = 0$ for each $m \in [0,1]$. Notice also that once supplier i selects $a_{im} = 1$ his price is determined. Therefore, his actions at any $m > m_i$ are inconsequential.

Given the definition of the game $\Gamma^{\varepsilon\text{-P}}$, we formalize now the assumptions on observability and the informational structure, which are common knowledge. Initially, at moment $m = 0$, each provider i knows only his own costs c_i and the initial (exogenous) reserve price b. We suppose that suppliers have no information about their rivals' costs. During the course of the auction, however, additional information about providers is revealed, because the message of each supplier at each moment in time is observable by all other providers. Hence, at each moment $m \in [0,1]$ each provider i knows the messages selected by all suppliers at any $m' < m$. We represent supplier i's relevant information at any $m \in [0,1]$ by

$$I_i(m) = \left(c_i, A(m), \left(m_j\right)_{j \notin A(m)}\right), \tag{19}$$

where $A(m)$ denotes the set of providers whose prices are still undetermined; i.e. $j \in A(m)$ if for each $m' < m$, $a_{jm'} = 0$. The last entry in (19), the collection of m_j for $j \notin A(m)$, are the drop out decisions of the remaining providers, which are observable and determine the prices of these suppliers.

A strategy for provider i prescribes for each $\widetilde{m} \in [0,1)$ an action $a_{i\widetilde{m}}$ that depends both on $\widetilde{m}$, which determines the current price, and on the provider's current information, which is collected in $I_i(\widetilde{m})$. Consider the strategy described by

$$a^* = \begin{cases} 0 & \text{if } \alpha(I_i(\widetilde{m})) < (1-\widetilde{m})\, b \\ 1 & \text{otherwise} \end{cases}, \tag{20}$$

where

$$\alpha(I_i(\widetilde{m})) = \begin{cases} c_i & \text{if } |A(\widetilde{m})| \geq 3 \\ \frac{\widehat{t}(P)+c_i}{2} & \text{if } |A(\widetilde{m})| = 2 \\ \widehat{t}(P) - \sqrt{\frac{\left(\widehat{t}(P)-c_i\right)\left(\widehat{t}(P)-\delta\right)}{2}} & \text{otherwise} \end{cases}, \tag{21}$$

and δ is the second-lowest price; i.e. $\delta = (1-\overline{m})\, b$, where $\overline{m}$ is such that $A(\overline{m})$ has two providers and, for any m with $\overline{m} < m \leq \widetilde{m}$, $A(m)$ is a singleton. The strategy a^* requires the provider to remain active until the price drops below a certain threshold. This threshold depends on the number of suppliers remaining in the auction. If more than two suppliers remain active, then the provider waits until his marginal cost is reached. If all but one other supplier have dropped out, then the provider shades his bid by dropping out earlier. These drop out decisions mimic the provision prices in part (b) and (c) of Lemma 1. We have the following result.

Proposition 1 In game $\Gamma^{\varepsilon\text{-P}}$, it is a weakly dominant strategy to follow a^*.

Proof. We show that deviating from a^* at any $\widetilde{m} \in [0,1)$ a provider can never increase his profits but in some circumstances may decrease it. Consider a provider, say i, and any

$\widetilde{m} \in [0,1)$. Notice that if $m_i < \widetilde{m}$ holds, then i's action at $\widetilde{m}$ does not affect his profits. Hence assume that i's action at each $m < \widetilde{m}$ was $a_{im} = 0$. Consider the following cases, which exhaust all possibilities.

(a) $A(\widetilde{m})$ has at least three providers. If $a_{i\widetilde{m}} = 1$, then $p_i = (1-\widetilde{m})\,b \geq \widehat{t}(P)$, which implies that $s_i^{\text{ε-P}}(P) = 0$. Notice that this is true even when there is another providers who also drops out in this moment. Suppose provider i compares dropping out at $\widetilde{m}$ to dropping out at $m' > \widetilde{m}$ and denote by $m'' \geq \widetilde{m}$ the next moment in which a bidder $j \neq i$ with $j \in A(\widetilde{m})$ drops out of the auction. There are two possibilities.

  (i) $c_i \geq (1-\widetilde{m})\,b$. Then m'' must be such that $c_i \geq (1-m'')\,b$. There are again two possibilities. If $m' \leq m''$, then provider i's share and profits are zero. If $m' > m''$, then provider i's share is strictly positive and his profits are strictly negative, because $m' > \widetilde{m}$ implies that $c_i > (1-m')\,b$. Therefore, as prescribed by (20) and (21), it is optimal to select $a_{i\widetilde{m}} = 1$, which guarantees zero profits.

  (ii) $c_i < (1-\widetilde{m})\,b$. There are again two possibilities. If m'' is such that $c_i < (1-m'')\,b$, then provider i can choose m' as prescribed by (20) and (21) and obtain strictly positive profits. If m'' is such that $c_i \geq (1-m'')\,b$, then provider i can again follow (20) and (21) and choose $m' = 1 - c_i/b$, guaranteeing himself zero profits.

  This implies that when $A(\widetilde{m})$ has at least three providers it is profitable to remain active until the price reaches marginal costs, as described in (20) and (21).

(b) $A(\widetilde{m})$ has two providers. This implies that when the auction closes and all prices are determined, $p_i \leq (1-\widetilde{m})\,b < \widehat{t}(P)$. Moreover, at moment $\widetilde{m}$ the value of $\widehat{t}(P)$ is publicly known. Therefore, by Theorem 4 in Alcalde and Dahm (2013) we have that i's optimal decision is as described in (20) and (21).

(c) i is the only provider in $A(\widetilde{m})$. This implies that i is the supplier proposing the lowest price. Again, by Theorem 4 in Alcalde and Dahm (2013), the unique optimal decision for i is as described in (20) and (21).

This concludes the proof that a^* is a weakly dominant strategy. ■

The intuition why the strategy a^* is weakly dominant is simple. First, as in a standard English auction, it cannot be optimal to continue when the price falls below marginal costs, as this can only result in a loss. Also, dropping out before the price reaches marginal costs risks forgoing potential gains and cannot be optimal either. Second, once only two providers are left, the key observation is the following. The supplier dropping out first is certain to submit the second lowest price, and the optimal second lowest price does not depend on the lowest price. Third, once only one supplier is left, he observes the drop out decision and thus prices of all other bidders. He shades his bid compared to what would be his optimal second lowest price, resolving optimally the trade-off between procurement share and mark up.

The fact that providers have a weakly dominant strategy implies that the procurement procedure can be used under different assumptions of what providers know about each others' marginal costs, provided the assumption of private values holds. If, however, there is interdependence of providers' costs, the drop out decisions of rivals might reveal information about a supplier's cost and it is not weakly dominant to follow the strategy a^*.

When all providers follow the weakly dominant strategy a^* each supplier $i > 2$ reveals his costs truthfully by choosing $p_i = c_i$. This implies that the revised reserve price coincides with c_3. Thus, the games $\Gamma^{\varepsilon\text{-P}}$ and Γ^{E} generate very similar equilibrium outcomes. We record this with the following result. The proof is immediate taking into account Proposition 1 and Theorem 2.

Theorem 3 The weakly dominant strategies of providers in $\Gamma^{\varepsilon\text{-P}}$ induce the following prices

$$p_i = \begin{cases} c_i & \text{if } i \geq 3 \\ \frac{c_3+c_2}{2} & \text{if } i = 2 \\ c_3 - \sqrt{\frac{(c_3-c_2)(c_3-c_1)}{4}} & \text{if } i = 1 \end{cases} \tag{22}$$

Moreover, if

$$\frac{c_2 - c_1}{c_3 - c_2} > \left(\frac{13}{8} + \frac{5}{8}\sqrt{17}\right) \approx 4.20,$$

then the buyer's procurement expenditures are lower than c_2.

6 Concluding Remarks and Extensions

This paper proposed a new procurement procedure for dual sourcing. Commonly employed dual sourcing strategies fix procurement shares and the reserve price exogenously. In contrast, our procedure uses the bids of suppliers in order to endogeneize both the allocation of shares and the reserve price. We have shown that in equilibrium providers reveal their costs truthfully and that the two most competitive suppliers are awarded contracts. Moreover, the procedure can be used under different assumptions of what providers know about each others' costs, provided the assumption of private values holds.

Our procedure always outperforms a Vickrey auction for dual sourcing. For this reason we make the extreme benchmark assumption that the buyer does not value the advantages of dual sourcing at all and compare our procedure to a Vickrey auction for sole sourcing. We have shown that the existence of a competitive pool of potential suppliers benefits the buyer, because when discarded suppliers are competitive enough (as measured by the cost difference between the second and the third lowest cost providers), then we can guarantee that procurement costs are even lower than in a Vickrey auction for sole sourcing.

There are several interesting ways in which our analysis might be extended. In particular, our assignment rule for shares assumes the elasticity of a supplier's procurement share with respect to his price to be one. Following Alcalde and Dahm (2013) this could be generalized to other values of the elasticity. The results in our earlier paper suggest that under such a generalization truthful revelation of marginal costs of discarded bids still occurs in equilibrium. Theorem 3 in Alcalde and Dahm (2013) implies then that the buyer can choose the elasticity in such a way that procurement expenditures are lower than in a Vickrey auction for sole sourcing, even when in the setting of the present paper (with unit elasticity) this is not possible. This shows that different assignment rules for shares might yield further interesting results.

In addition there are two other assumptions that might be starting points for interesting generalizations. First, we assumed the providers' marginal costs to be constant. Introducing economies of scale poses a challenge, as it should make it more difficult to reconcile the aims of expenditure minimization and having more than one provider. But the latter can still be desirable. Scherer (2007), for instance, analyses for influenza vaccines the trade-off between economies of scale and protection against stochastic shortage risk through having more than one provider. He concludes that for plausible scenarios sole sourcing is not optimal. Second, we have not modelled how the buyer values dual sourcing. Instead, we showed that the procurement procedure proposed outperforms a Vickrey auction for sole sourcing, even though the buyer does not put any value on dual sourcing. In reality, however,–like in Scherer (2007)–the buyer will be willing to trade-off some savings for dual sourcing. In principle one might model the value of dual sourcing by postulating a function that trades off the conflicting aims, maybe through a generalization to a CES utility function. Further work modelling the interplay of both economies of scale and the value of dual sourcing involves challenging questions for future research.

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