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The conflating effects of education and financial competition in an OLG growth model with Nelson-Phelps human capital

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Abstract

The Nelson-Phelps (N-P) concept of human capital, which determines the speed at which a new technology may be implemented, is considered within an AK, overlapping-generations model to produce a generalized dynamic form. Finance firms are assumed to act as local monopolies in the market for loans to production firms but as monopsonistic price-takers in the deposit market for households. Households also vote for taxes that are earmarked to pay for public education, which determines the subsequent level of N-P human capital. The main result is that a concentrated financial market structure, although *directly* lowering economic growth, may *indirectly* raise it through provoking a political economy response of voting for higher taxes to pay for a greater future, level of N-P human capital.

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1. Introduction

Two factors are often regarded separately in the analysis of economic growth, the level of human capital and the role of financial markets. This paper contends that it may be necessary to consider these two concerns jointly, because the effects of a particular form of human capital with varying degrees of financial sector competition may be conflated in such a way that the empirical analysis of one in isolation may lead to misleading conclusions. In particular, we suggest that the effect of financial monopoly on deposit interest rates may be so detrimental to household utility as to provoke a political economy response of voting for higher taxes to fund a greater public investment in education. In the model developed below, we find that education tends to be more important than finance for economic growth, while financial competition remains more beneficial than financial monopoly. Thus omitting human capital from an empirical analysis of financial effects could lead to the false inference that economic growth is higher because of rather than in spite of the presence of financial monopoly. Similarly, we suggest that full capital mobility in a global model reduces the incentive to invest in human capital with negative consequences for economic growth.

The empirical results for human capital in growth models in general have not been unequivocal. Benhabib and Spiegel (1994) and Pritchett (2001) find that human capital is not significant in empirical models of economic growth, but responses to this point are given by Temple (1999) in terms of econometric issues and by Engelbrecht (2003) with reference to model specification. However, measurements of human capital substantially improve the performance of the original neoclassical growth model of Solow (1956) model in terms of its convergence properties and goodness of fit. The additional variables indicative of human capital included by Barro (1991) and Mankiw, Romer and Weil (1992) have generated the new and preferred *augmented* form of the Solow model. The modelling of human capital has also launched a range of endogenous growth model, discussed in Romer (1986) but made explicit in Lucas (1988). Human capital becomes an engine of long-run economic growth if there are non-decreasing returns in a broader measure of the capital stock.

Apart from differing assumption concerning returns-to-scale, another key distinction in the literature from Nelson and Phelps (1966) is between human capital as a factor input synonymous with *knowledge* and as *cognitive ability* that speeds up the process in applying a new and technically demanding production technology. The purpose of this present paper is to explore the implications of marrying non-decreasing returns in a broad measure of capital according to Romer (1986) and Lucas (1988) with the Nelson and Phelps' concept of human capital in order to re-examine the nexus between finance and economic growth.

A consensual view is that a strong association exists between financial development and economic growth, although assessments of the underlying causality may vary. Levine (1997) is relatively sanguine about the initiating role of finance, while other reviewers, for instance, Pagano (1993) and Driffill (2003), have expressed more reservation. Moreover, in spite of the fact that this literature has defined financial development mainly in terms of extensions to the range of instruments, we suggest that the extent of competition within markets for given instruments may be at least as important for at least two reasons. First, the economic benefits of any particular financial instrument will depend on a sufficient degree of market competition among its providers to ensure its efficient pricing and allocation. Secondly, the emergence of financial initiatives providing additional instruments that are both novel and substitutable with existing ones may be roughly equivalent toh an increase in market competition over the provision of existing ones. Thus, an analysis of financial market structure from an aerial macroeconomic perspective may roughly approximate one of financial development.

In an earlier review of this literature, Pagano (1993) makes the basic point that imperfect financial competition may reduce capital accumulation both by raising the costs of borrowing and by reducing rates of returns to lenders. More recent attention has turned to the effects of market structure within an environment of uncertainty because of asymmetric information between borrowers and lenders. A review this particular strand of the theoretical literature in Cetorelli (2001) finds what amounts to an embarrassment of riches, because of assumption-sensitive results: financial monopoly may either worsen or improve economic outcomes under asymmetric information.

The approach of this present paper may be regarded as something of a compromise with assumptions of local monopoly in lending markets but with allowance for varying degrees of monoposonistic competition in the deposit market. Petersen and Rajan (1995) argue that monopoly finance serves to alleviate inherent asymmetric information problems, and they also find supporting evidence to this effect. Thus assuming monopoly in the loan market may pre-empt some fundamental informational problems, and thus provide a rationale for the present assumption of perfect information. Combining these two financial market assumptions with another two, one of an AK production technology and another of Nelson-Phelps human capital, generates some interesting results.

The Nelson-Phelps version of human capital has at least three major implications. First, within an *AK* model, it will theoretically affect the level of long-run growth and not just the speed of catch-up. In dealing with some empirical issues, Engelbrecht (2003) correctly states that Nelson and Phelps human capital, in the original paper, pertains only to *diffusion*, while the separate mechanism of *innovation* has been subsequently attributed to them has led to some confusion with the separate debate over *differences versus levels*. However, in any model with non-decreasing returns, any parameter, including one for diffusion, will necessarily have an impact upon the long-run growth rate through affecting the scale of production, since income is the base for savings.

Secondly, it will potentially dominate the other determinants of economic growth. In the context of the present generalized model, the standard version of the AK model may be interpreted as the case of a full complement of Nelson-Phelps human capital where knowledge spill-overs can be implemented instantaneously. Allowing for the factor of delay will, like any other parameter, affect the steady-state of the model under the assumption of non-decreasing returns to scale. The other polar case where human capital is entirely absent delivers a static model for output. Thus, within this generalization, while any other factor, including one for financial market competition, may affect the long-run economic growth outcome, it may not be a necessary as a precursor in the way that some degree of Nelson-Phelps human capital is.

Thirdly and of special significance to this present paper, Nelson-Phelps human capital also determines the interest elasticity of the aggregate demand for capital. In any endogenous growth model founded on constant returns with respect to a broad measure of the capital stock, where production firms' loan demands are also being satisfied, the solution for the borrowing interest rate, namely, the marginal product of capital, is parametric (with A as a solution at A for its most basic form where $y_t = Ak_t$). Then, loan market structure - in the absence of rationing - whether in terms of the degree of concentration or in the imposition of interest rate ceilings - cannot matter for economic growth. However, within the generalized form of the present model, loan market structure is relevant, because of the result that the level of Nelson-Phelps human capital determines the interest elasticity of the aggregate capital stock. Furthermore, as the analysis proceeds to show, increasing the level of Nelson-Phelps human capital is equivalent in effect to making loan markets more competitive. Thus, human capital may affect economic growth through an additional mechanism, which, to our knowledge, has not been previously considered in the literature.

An implication of this third point is that with an intermediate degree of Nelson-Phelps human capital there will be externalities in the setting of loan interest rates. Taking on board Petersen and Rajan (1995), we assume that finance firms - through as many branches as necessary - act as interest-setting local monopolists with regard production firms, but that they also act as Cournot competitors in the deposit market. With respect to the demand for loans, their decisions will not only directly affect those of their own customers but also indirectly those of third parties, other production firms, through the working of an investment externality.

The two sides to their operations of financial intermediaries implies a trade-off with respect to their degree of concentration. Having fewer finance firms is more efficient for internalizing the interest setting externality, but is also more costly in terms of increased monopsonistic power on the deposit taking side of their operations. The net effect will depend, of course, on relative parameter values, and interest elasticity of saving is a key parameter at least for the closed-economy example under consideration.

The analysis also has some interesting implications from a political economy perspective. Naturally, the economic returns to increasing levels of public education are in raising future levels of human capital, but its costs current because immediate tax funding is required. We find that the returns to public education *in terms of household utility* is greater where financial markets are less competitive because deposit interest rates are then lower. Another finding is that that the returns to education *in terms of economic growth* generally dominate those of financial market competition. The implication of combining these two findings is that monopolistic finance may lead to higher economic growth by provoking the political economy response of voting for more education as a corrective measure. Thus, in any empirical analysis that either omitted appropriate variables for human capital or included them within an incorrect specification, the empirical relationship between economic performance and financial structure would be wrongly attributed.

The organization of the paper is as follows. *Sections 2 and 3* lay out the model, consisting of production and household sectors, followed by the financial sector. *Section 4* looks at a simple political economy aspect: how the financial structure might affect voting for taxes to pay for education and future human capital. *Section 5* considers some issues that are secondary to the main analysis and *Section 6* provides a summary.

2. Production-firms and households

2.1 Production firms

There is a continuum of production function of measure unity, each with a CRS, Cobb-Douglas production function, indexed z and expressed in per capita terms as

$$y_t(z) = Ak_t(z)^{\alpha} b_t^{1-\alpha} \ , \qquad (1)$$

where $k_t(z)$ is z's capital stock and b_t is an *applied* technology that is common to all firms within the (same) economy. The model turns on the distinction between *applied* technology, b_t, and *actual* technology, T_t, as first suggested by Nelson and Phelps (1966),

$$b_t = T_t^{\theta_t} b_{t-1}^{\ 1-\theta_t} \qquad \theta_t = \theta(\varepsilon_{t-1}) \in (0,1) \ . \qquad (2)$$

Human capital determines the speed, θ_t, at which the former converges to the latter. The speed of learning new techniques depends on the level to which the young workforce was educated, ε_{t-1}, in a previous period of schooling; and an exponential process is specified in line with a Cobb-Douglas consideration. [2]

[2] An exponential process is specified here in accordance with the original paper by Nelson and Phelps (1966), which also simplifies our results. Benhabib and Siegel (2003) find some empirical support for a logistic process, which, with an additional human capital effect on innovations, implies that countries below an educational threshold may never catch up with the technology leading country. We suggest that the existence of educational thresholds may also be also be tested within an exponential specification.

The benefit of education is in providing basic cognitive skills necessary for the application of a given technology. In the absence of any cognitive ability at all, $\theta_t = 0$, current workers merely apply the previous period's technology, $b_t = b_{t-1}$, which, in a generational setting, suggests the transference of traditional techniques, possibly, through apprenticeship learning. The other polar case, $\theta_t = 1$, implies $b_t = T_t$, where full cognitive ability allows the state of the art technology to be immediately implemented. Nelson-Phelps human capital is usually regarded as determining the speed at which a country catches up with another, technology-leading country. Here we are specifying catch-up to be of an applied to an objective technology, which may also pertain within a single-country setting

The other key feature of the model is that, as suggested in Romer (1986), actual technology may arise as an investment externality, which is reflected in the general capital stock,

$$T_t = k_t \tag{3}$$

Clearly, an assumption of learning-by-doing does not square with the notion of Nelson-Phelps human capital. Nor would regarding equation (3) as a reduced-form that arising from a separate *R and D* sector that is funded either publically through income taxes or privately from profits be consistent with the model: this would be tarry with equation (1) for the general case of $\theta_t < 1$ for equation (2).[3] We merely assume only that the level of the aggregate capital stock, k_t, is commensurate with the degree of sophistication in production, which is an inspiration for technological progress.

Equations (2 and (3) give

$$b_t = k_t^{\theta_t} b_{t-1}^{1-\theta_t}, \tag{4}$$

which into (1) implies

$$y_t(z) = Ak_t(z)^{\alpha} k_t^{(1-\alpha)\theta_t} b_{t-1}^{(1-\alpha)(1-\theta_t)} = Ak_t(z)^{\alpha} \left(\Pi_{j=0}^{\infty} k_{t-j}^{\theta(1-\theta)^j} \right)^{1-\alpha} \tag{5}$$

$$y_t(z) = Ab_0^{1-\alpha} k_t(z)^{\alpha}$$

[3] Aghion and Howitt (1999) discuss the likelihood of some interesting complementaries between R and D and basic education where they are each public funded.

The model nests the Romer case where $\theta = 1$, so that $y_t(z) = Ak_t(z)^{\alpha} k_t^{1-\alpha}$; Solow where $\theta = 0$, so that $y_t(z) = Ab_0^{1-\alpha} k_t(z)^{\alpha}$; and a generalization of Romer where $0 < \theta < 1$, where although knowledge spill-overs may be exploited, it takes time, while a better educated workforce takes less time than others. Henceforth, we focus on the Romer cases, $0 < \theta \le 1$.

Next, each production firm z has the following profit function

$$\pi_t(z) = Ak_t(z)^{\alpha} b_t^{1-\alpha} - R_t^F(z, j)k_t(z) - w_t(z)$$

where $R_t^F(z, j)$ is production firm z's loan interest factor set by finance firm j, the marginal cost of funds , comprising the interest *rate* plus an assumed depreciation rate of unity, and where $w_t(z)$ is the wage. Profit is maximized where the marginal factor costs equal the marginal products,

$$R_t^F(z, j) = \alpha Ak_t(z)^{\alpha-1} k_t^{(1-\alpha)\theta_t} b_{t-1}^{(1-\alpha)(1-\theta_t)} \quad (6)$$

$$w_t = (1-\alpha)Ak_t(z)^{\alpha} k_t^{(1-\alpha)\theta_t} b_{t-1}^{(1-\alpha)(1-\theta_t)} \quad (7)$$

In symmetric equilibrium, $k_t(z) = k_t$, $\forall z$, $R_t(z, j) = R_t$, $\forall z, j$, factor prices are

$$R_t^F = \alpha Ak_t^{\alpha+(1-\alpha)\theta_t - 1} b_{t-1}^{(1-\alpha)(1-\theta_t)} \quad (8)$$

$$w_t = (1-\alpha)Ak_t^{\alpha+(1-\alpha)\theta_t} b_{t-1}^{(1-\alpha)(1-\theta_t)} \quad (9)$$

In addition, within a steady-state growth equilibrium, $G = k_{t+i}/k_{t+i-1}$, $\forall i$, equation (4) implies the following relationship between the applied technology and the capital stock

$$\begin{array}{lcl} b_t = G^{1-1/\theta} k_t > 1 & \text{for} & 0 < \theta < 1 \\ b_t = b_{t-\infty} & & \theta = 0 \end{array} \quad (10)$$

The implication for factor prices in equations (8) and (9) where $0 < \theta < 1$ is that

$$R^F = \alpha AG^{-(1-\alpha)(1-\theta)/\theta} \quad (11)$$

$$w_t = (1-\alpha)AG^{-(1-\alpha)(1-\theta)/\theta} k_t \quad (12)$$

Note also that the output-capital ratio in symmetric equilibrium is $y_t/k_t = AG^{(1-\alpha)(1-1/\theta)}$, which is a negative function of the economic growth factor where $\theta < 1$ and a positive function of human capital where $G > 1$.

2.2 The relationship between economic growth and firms' borrowing interest rates

Equation (11) constitutes the first main building block of the model, and has the following properties.

Result 1: *The interest factor on loans, R^F, is (i) negatively related to economic growth ($G > 1$) in the general case where $0 < \theta < 1$, and is (ii) a positive function of human capital, θ, where $G > 1$.*

There are two corollaries to this result. First, inverting the equation to give $G = \left(\alpha A / R^F\right)^{\theta/(1-\theta)(1-\alpha)}$ provides a very succinct partial equilibrium model of economic growth, if R^F and θ may safely be regarded as parameters. Economic growth is inversely related to firms' borrowing rates of interest and positively depends on the speed parameter, θ, reflecting human capital. Thus, Nelson Phelps human capital determines long-run growth as well as transition within an Ak model.

Secondly, in the standard form of this model, implicitly where $\theta = 1$ - and also where production firms' are on their demand curves, conditions in the loan market do not matter for loan interest rates, because there is then a unique solution at $R^F = \alpha A$ that holds for configurations of all variables. Thus, the growing literature investigating the effects of loan market effects on long-run economic growth is misplaced, unless either loans are rationed or else there is less than full catch-up, $\theta < 1$, as considered here.

Furthermore, the second part the *Result* states that the interest factor is positive related to the Nelson-Phelps measure of human capital. This also applies to the alternative Lucasian]specification where human capital enters as a factor input, h_t, in the production process. With a Cobb-Douglas production function, $y_t = Ak_t^{\alpha} h_t^{1-\alpha}$, the interest factor is solved as $R_t^F = \alpha A(h_t/k_t)^{1-\alpha}$ and, similarly, is positively related to the share of human capital.

Having determined a relationship between economic growth and firms' interest factors, the next stage of the model is to specify a relationship between the mark-up of the lending interest factor over the deposit interest factor and economic growth. Finally, the model is closed with a third stage in the following *Section* where this mark-up is determined.

2.3 The relationship between economic growth and the loan-deposit interest mark-up factor

The basic form of the Diamond overlapping-generations model is applied where non-altruistic households are assumed to live for two periods. They have the following utility function,

$$U_t = (1-\mu)\ln c_t^Y + \mu \ln c_{t+1}^O \qquad (13)$$

from consumption, c, both when young and old, where μ, is the relative weight placed on the latter.

A minor departure is the assumption that households supply labour both when young and old in the respective proportions, $1-\eta$ and η. This is made to ensure a savings function of some elasticity from a discounting effect of the rate of interest where $\eta > 0$. If the first period covers the individual's lifespan from ages 20 to 50 and the second from ages 50 to 80, and if households supply the same of hours each year until full retirement at 65, then $\eta = 1/3$. Labour is supplied inelastically in each period and its life-time supply is normalised at unity.

How profits are returned to households within this framework is not without unimportance. Roberts (2009) considers three cases possibilities: that they are transferred to the young or to the old as transfers or as dividend payments on financial sector equity. We leave aside the first two options in order to avoid an analysis of redistributive fiscal policy, which motivated this earlier paper but would constitute a distraction here. [4] We simplify by assuming that the equity of the financial sector is not traded and that its profits are either wasted, consumed by the financial sector itself or else expatriated.

Each household saves by holding deposits d_t which yield the rate of return R_{t+1}^D. The household budget constraints for each period are

$$c_t^Y = (1-\eta)(1-\tau_t)w_t - d_t, \qquad c_{t+1}^O = \eta(1-\tau_{t+1})w_{t+1} + (1-\tau_{t+1})R_{t+1}^D d_t$$

All incomes are taxed at the fixed rate τ_t, and tax revenue is used to finance the education of the young, which determines the future level of human capital, θ_{t+1}, plus other government expenditures, which take the share, $\hat{\tau}_t$,

$$\tau_t = \hat{\tau}_t + \lambda\theta_{t+1} \tag{14}$$

The parameter λ is important, because its inverse defines the rate at increased spending on education translates into higher future human capital.

The maximization of utility in (13) subject to these constraints gives a solution for total household savings,

$$d_t = \mu(1-\eta)(1-\tau_t)w_t - (1-\mu)\eta R_{t+1}^{D\ -1} w_{t+1} \tag{15}$$

The second-period tax drops out, because it reduces both the second-period income and the factor by which it is discounted. Using the mark-up definition,

$$R^F \equiv MR^D, \qquad M \geq 1, \tag{16}$$

and exploiting the constant factor shares property implies the saving elasticity is solved as

$$\varepsilon_{t+1} = \frac{(1-\mu)\eta\left(\dfrac{1-\alpha}{\alpha}\right)M_{t+1}k_{t+1}}{\mu(1-\eta)(1-\tau_t)w_t - (1-\mu)\eta\left(\dfrac{1-\alpha}{\alpha}\right)M_{t+1}k_{t+1}} \tag{17}$$

Deposits provide the funds to determine the following period's capital stock through theprocess of financial intermediation,

$$k_{t+1} = d_t, \tag{18}$$

[4] Transferring financial profits to the young has a beneficial effect on economic growth through consumption smoothing, and this may dominate the adverse interest rate effect of imperfect financial competition where interest elasticities are low [Roberts (2009)].

Equations (15), (16) and (18) again with property of constant factor income shares give

$$k_{t+1} = \frac{\mu(1-\eta)(1-\tau_t)w_t}{1+(1-\mu)\eta\big((1-\alpha)/\alpha\big)M_{t+1}} \tag{19}$$

Equation (19) into (17) gives

$$\varepsilon_{t+1} = (1-\mu)\eta\big((1-\alpha)/\alpha\big)M_{t+1} \tag{20}$$

Note that neither the tax nor the wage is part of the solution for the elasticity. Economic growth in the steady-state is also solved as

$$G = \left(\frac{\mu(1-\eta)(1-\hat{\tau}-\lambda\theta)(1-\alpha)A}{1+(1-\mu)\eta(\alpha^{-1}-1)M}\right)^{\frac{\theta}{1-\alpha(1-\theta)}}, \tag{21}$$

which implies the following.

Result 2: *Economic growth is a decreasing function of the loan-deposit interest mark-up.*

A higher mark-up factor reduces economic growth, because it implies a lower deposit interest rate. It is also clear that some level of human capital, $\theta > 0$, is necessary for economic growth, $G > 1$. The model will be completed after a solution for the mark-up factor M is found.

3. The financial sector

We assume that there are $1 \leq N$ financial firms and consider the general case of imperfect competition where $N < \infty$. The number N may be considered as an implicit solution from exogenous distributions of entry and/or operating costs in conjunction with a non-negative net profit condition governing entry and exit. Making this explicit would not only be trivial for the analysis but also make it less transparent, since the *number of firms*, unlike their incentives to exist represented by assigned relative parameter values, has more tangible meaning. The only requirement might be a “stability condition” that the entry of new finance firms should drive profits down and not up. However, even this would be unnecessary where entry is controlled through regulation.

3.1 The loan interest rate setting externality

The first point to note is that the combination of Nelson-Phelps human capital with the AK production feature implies an interest setting externality. This is shown as follows. First, inverting equation (6) gives production firm z's demand for capital as

$$k_{t+1}(z) = \left(\alpha A / R^F_{t+1}(z)\right)^{1/(1-\alpha)} k_{t+1}{}^{\theta_t} b_t^{1-\theta_t}$$

As production firms are of measure unity, $k_{t+1} = \int_0^1 k_{t+1}(h)dh$, the aggregate demand for capital is similarly given as

$$k_{t+1} = \int_0^1 \left(\left(\alpha A / R^F_{t+1}(h)\right)^{1/(1-\alpha)} dh \right) k_{t+1}{}^{\theta_t} b_t^{1-\theta_t}$$

or, after rearranging, by

$$k_{t+1} = \left(\int_0^1 \left(\alpha A / R^F_{t+1}(h)\right)^{1/(1-\alpha)} dh \right)^{1/1-\theta_t} b_t$$

Substituting this back into the original equation gives the individual firm's demand for capital,

$$k_{t+1}(z) = \left(\alpha A / R^F_{t+1}(z)\right)^{1/(1-\alpha)} \left(\int_0^1 \left(\alpha A / R^F_{t+1}(h)\right)^{1/(1-\alpha)} dh \right)^{\theta_t / 1-\theta_t} b_t$$

Clearly, its demand depends not only on its own borrowing cost, $R^F_{t+1}(z)$, but on all those of other firms through $\left(\int_0^1 R^F_{t+1}(h)^{-1/(1-\alpha)} \right)^{\theta_t/(1-\theta_t)}$ where $\theta_t > 0$. There would be a tendency under non-cooperative behaviour for finance firms to set loan interest rates at too high a rate by failing to account for the combined and external effects of their lending on the aggregate capital stock.

However, this externality may be internalized with a finite number of financial firms, N; and the fewer present, the more effectively so. Consider a single finance firm i that lends to an average share, $1/N$, of all production firms, which are indexed p,i and of measure unity. Its rivals $j \neq i$ lend to the remaining production firms of weight $1-1/N$ and indexed $q, j \neq p,i$. Finance firm i will in total lend

$$k_{t+1}(i) = \int_0^{1/N} \left(\alpha A / R_{t+1}^F(p,i)\right)^{1/(1-\alpha)} dp, i \times$$

$$\left(\int_0^{1/N} \left(\alpha A / R_{t+1}^F(p,i)\right)^{1/(1-\alpha)} dp, i + \int_{1/N}^{1} \left(\alpha A / R_{t+1}^F(q,j)\right)^{1/(1-\alpha)} dq, j \right)^{\theta_t/1-\theta_t} b_t \tag{22}$$

With symmetry across the two dimensions of production and of finance firms, the effective interest elasticity of the demand for its loans is given by

$$\frac{R_{t+1}^F(i)}{k_{t+1}(i)} \frac{\partial k_{t+1}(i)}{\partial R_{t+1}^F(i)} = -\left(\frac{1}{1-\alpha}\right)\left(1 + \left(\frac{\theta}{1-\theta}\right)\frac{1}{N}\right)$$

As $\theta \to 1$, this elasticity term tends to infinmity, but for the general case of the model where $0 < \theta < 1$, the absolute value of this expression is decreasing in N: other things being equal, a more concentrated market-structure leads to a higher loan interest elasticity. For the single finance firm or the cartel case ($N = 1$) this is $(1-\theta)^{-1}$ times the elasticity for the competitive case ($N \to \infty$). The scale of this relative magnitude is clearly a positive and convex function of the level of human capital, θ.

3.2 Financial profit maximization

The profit function of each finance firm, indexed i, is given by

$$\pi_{t+1}(i) = \int R_{t+1}^F(p,i) k_{t+1}(p,i) dp - R_{t+1}^D d_t(i) \tag{23}$$

Finance firms, i ,set individual interest rates, $R_{t+1}^F(i, p)$ when dealing with each production firm, p, on the lending side of their operations, but engage in Cournot competition on the deposit side, taking the deposit interest factor, R_{t+1}^D, as given. This is determined as an aggregate market outcome that equates the aggregate demand for loans with the total supply of deposits. Thus, aggregate lending in response to an array of individually administered loan interest rates ultimately determines an aggregate deposit interest rate through a market clearing condition.

Nevertheless, in setting loan interest rates, $R_{t+1}^F(i)$, finance firms also account for the indirect effects of their individual choices on $R_{t+1}^F(i)$ on the aggregate outcome R_{t+1}^D through anticipating the effects of their own contributions to lending, $k_{t+1}(i)$, on total lending, which determines the aggregate capital stock, k_{t+1}. In terms of the solution, each maximizes its own profit in equation (21), both accounting for the loan interest setting externality evident in equation (20) and the effect on the market deposit interest factor through an inversion of (14) subject to equation (15). [Equation (9) also enters the solution.] In the a steady-state, symmetric equilibrium, the first-order condition for financial profit maximization is

$$R^F - \frac{R^F - R^D}{1-\alpha} - \frac{1}{(1-\alpha)(1-\theta)N}\left(\theta(R^F - R^D) - \varepsilon^{-1}R^D\right) = 0 \text{ where}$$

$$\varepsilon \equiv (\partial d / \partial R^D)(R^D / d) \tag{24}$$

where ε is the interest elasticity of deposits.

3.3 Constant savings elasticity

There is some benefit in initially abstracting from the precise form of the saving function in order to consider a constant deposit interest elasticity, $\bar{\varepsilon}$. This is in reducing equation (24) to a linear solution for the mark-up factor, M,

$$R^F_{t+1} \equiv MR^D_{t+1}, \qquad M \equiv \frac{\theta + (1-\theta)N + \bar{\varepsilon}^{-1}}{\theta + \alpha(1-\theta)N},$$

$$\frac{\partial M}{\partial N} \equiv \frac{(1-\theta)\left((1-\alpha)\theta - \alpha\bar{\varepsilon}^{-1}\right)}{\left(\theta + \alpha(1-\theta)N\right)^2}, \quad \frac{\partial M}{\partial \theta} \equiv \frac{-\left(\bar{\varepsilon}^{-1} + \left(1 - \alpha(1+\bar{\varepsilon}^{-1})\right)N\right)}{\left(\theta + \alpha(1-\theta)N\right)^2} \tag{25}$$

The following four *Results* are worth noting.

Result 3: *The loan-deposit interest mark-up factor is decreasing (increasing) in the number of finance firms if both the level of human capital and the interest elasticity of deposits are relatively low (high):* $\varepsilon\theta < (>) \alpha/(1-\alpha)$, $\theta < 1$

From: $$\frac{\partial M}{\partial N} \equiv \frac{(1-\theta)\left((1-\alpha)\theta - \alpha\bar{\varepsilon}^{-1}\right)}{\left(\theta + \alpha(1-\theta)N\right)^2},$$

In essence, there is trade-off with regard to market structure where there are intermediate levels of N-P human capital, $0 < \theta < 1$. With fewer finance firms, there is more scope to internalise the loan interest setting externality but there is also greater monopsonistic power in the deposit market.[5] Gylfason (1993) lists a number of relevant studies and averaging them finds an interest elasticity of 0.3. Taking this value and assigning the Cobb-Douglas capital share parameter, α, its stylized value of one-third, we find that, externality for all values of θ, the factor of market power in lowering deposit interest rates dominates the effect of internalizing the lending. For the opposite outcome, there would need to be both a sufficiently high value for θ and an interest elasticity well in excess of 0.5, which may be feasible and is not inconsistent with some of the single country elasticities that are reported by Gylfason (1993).

Result 4: *The loan-deposit interest mark-up factor is decreasing in the level of human capital, either if the elasticity is relatively high,* $\varepsilon \geq \alpha/(1-\alpha)$ *or if both the elasticity and the number of finance firms are low,* $\bar{\varepsilon} < \alpha/(1-\alpha)$ *and* $N < 1/\left(\alpha-(1-\alpha)\bar{\varepsilon}\right)$.

From: $$\frac{\partial M}{\partial \theta} \equiv \frac{-\left(\bar{\varepsilon}^{-1}+\left(1-\alpha(1+\bar{\varepsilon}^{-1})\right)N\right)}{\left(\theta+\alpha(1-\theta)N\right)^2}$$

If the interest elasticity is very low, monopsony power becomes a dominating factor, and the interest setting externality may exploit this. This is more likely to occur where N is high. However, in this case, the magnitude of the response will be rather low, because the denominator would also be very large as a squared term that is also increasing in N.

Result 5: *The loan-deposit mark-up is unaffected by the number of finance firms concentration with a full measure of Nelson-Phelps human capital (*$\theta = 1$*).*

If $\theta = 1,\ \partial M/\partial N = 0$.

Result 6: *The loan-deposit mark-up is less affected by the level of human capital, the more competitive is the deposit market.*

It is evident that $\partial\left(\partial M/\partial \theta\right)/\partial N < 0$ and that as $N \to \infty, \partial M/\partial \theta \to 0$.

Result 6 is restatement of the earlier discussion that conditions in the loan market are important only outside the usual representation of the *AK* model. *Result* 6 is important both

[5] This is analogous to the result of Calmfors and Driffill (1988) with respect to labour union power and macroeconomic performance.

for a main conclusion that emerges later in the analysis and also because it guarantees the stability condition that the entry of new finance firms would depress financial profits.

3.4 The actual solution

Now departing from the constant interest elasticity of deposits assumption, equations (20) and (24) give

$$M=\frac{1}{2}\left(\frac{\theta+(1-\theta)N}{\theta+\alpha(1-\theta)N}\right)+\sqrt{\frac{1}{4}\left(\frac{\theta+(1-\theta)N}{\theta+\alpha(1-\theta)N}\right)^2+\frac{\alpha}{(1-\mu)\eta(1-\alpha)\left(\theta+\alpha(1-\theta)N\right)}} \tag{26}$$

3.5 The parameterisation

We parameterise $(1-\mu)\eta$ on the basis (i) that, according to Gylfason (2003), *on average*, $\varepsilon=0.3$, (ii) that the capital income share takes its standard value, $\alpha=1/3$ and (iii) and by making the judgements that, on average, $\theta=0.75$ and $N=3$. Equation (24) for the elasticity is satisfied for these values if $(1-\mu)\eta=0.09/2.9=0.03103$, which in turn implies a mark-up of 4.833. [Over 40 years this is roughly equivalent to an interest spread of 4% per annum.] Then with respect to the parameter composite $(1-\mu)\eta$ we generate unconstrained values for the mark-up, M, for ranges of values for θ and for N. From this basis, it is then possible to generate values for growth, G, from equation (21), thence firms' borrowing interest factor from equation (11) and finally the implied values for households' deposit interest factor from the first and third results. The remaining parameterisations are based on the assumption that with a full complement of Nelson-Phelps human capital $(\theta=1)$, the annual growth rates and the lending interest rates are respective at 3% and 7.5%. Given $\alpha=1/3$, the latter requires that $A=54.133$. Then calculating the corresponding mark-up value ($M=4.545$), we deduce that $\mu(1-\eta)=0.1159$. The results are presented in the following *Table*.

Table : The mark-up, economic growth, firms borrowing interest and households' deposit interest factors *where* $\alpha = 1/3,\ A = 54.133,\ (1-\mu)\eta = 0.09/2.9 = 0.03103,\quad \mu(1-\eta) = 0.1159$									
		$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$		$N\to\infty$
$\theta=0$	M	8.612	6.640	5.785	5.286	4.952	4.710		3
	G	1	1	1	1	1	1		1
	R^F	6.619	6.091	5.862	5.728	5.639	5.575		5.117
	R^D	0.769	0.917	1.013	1.084	1.139	1.183		1.706
$\theta=0.25$	M	6.764	5.964	5.404	5.118	4.871	4.681		3
	G	1.434	1.451	1.463	1.470	1.475	1.480		1.522
	R^F	8.780	8.574	8.428	8.353	8.289	8.238		7.788
	R^D	1.298	1.438	1.560	1.632	1.702	1.760		2.596
$\theta=0.5$	M	5.723	5.388	5.137	4.939	4.779	4.645		3
	G	1.967	1.985	1.999	2.010	2.019	2.027		2.130
	R^F	11.496	11.425	11.371	11.329	11.294	11.265		10.899
	R^D	2.009	2.120	2.214	2.294	2.363	2.425		3.633
$\theta=0.75$	M	5.038	4.929	4.833	4.748	4.671	4.601		3
	G	2.581	2.592	2.602	2.611	2.618	2.626		2.804
	R^F	14.619	14.602	14.590	14.579	14.569	14.560		14.349
	R^D	2.901	2.962	3.019	3.071	3.119	3.165		4.783
$\theta\to1$	M	4.545	4.545	4.545	4.545	4.545	4.545		4.545,3
	G	3.262	3.262	3.262	3.262	3.262	3.262		3.262
	R^F	18.044	18.044	18.044	18.044	18.044	18.044		18.044
	R^D	3.970	3.970	3.970	3.970	3.970	3.970		3.970

The effects of financial competition are found to be trivial economic growth, and even non-existent in the limits of human capital where $\theta = 0,1$, but have small effects in reducing firms' borrowing interest rates and in increasing households' deposit interest rates. Human capital, however, has a large effect in increasing both of these interest rates, but, most of all, a very powerful effect on economic growth. [6]

4. 4. Voting for education

4.4.1 An open-form solution

We now consider the level of human capital from a political economy perspective where the young household is also deemed to be the median voter. Strictly speaking with zero population growth and a universal two-period life-span, the identity of the median voter cannot be determined. However, with some arbitrarily low level of either population growth or of early death, this assumption may be sustained.

The utility response for a young household of a rise in human capital is

$$\frac{\partial V_t}{\partial \theta_{t+1}} = -\left(\frac{1-\mu}{c_t^Y}\right)\left(\lambda(1-\eta)w_t - \frac{\partial d_t}{\partial \theta_{t+1}}\right) +$$
$$\frac{\mu}{c_{t+1}^O}\left(-\lambda\left(\eta w_{t+1} + d_t R_{t+1}^D\right) + (1-\hat{\tau}-\theta_{t+1}\lambda)\left(\eta \frac{\partial w_{t+1}}{\partial \theta_{t+1}} + R_{t+1}^D \frac{\partial d_t}{\partial \theta_{t+1}} + d_t \frac{\partial R_{t+1}^D}{\partial \theta_{t+1}}\right)\right) = 0$$

The full details are relegated to the *Appendix*, but considering the first two stages of the solution gives some insight. First, the application of the Euler equation,

$$\frac{\partial V_t}{\partial d_t} = -\left(\frac{1-\mu}{c_t^Y}\right) + \frac{\mu}{c_{t+1}^O}(1-\hat{\tau}-\theta_{t+1}\lambda)R_{t+1} = 0,$$

implies

$$\frac{\partial F}{\partial \theta} = -\lambda\left((1-\eta)w_t + \frac{\eta(w_{t+1}/R_{t+1}^D) + d_t}{1-\hat{\tau}-\lambda\theta_{t+1}}\right) + \frac{\eta}{R_{t+1}^D}\frac{\partial w_{t+1}}{\partial \theta_{t+1}} + \frac{d_t}{R_{t+1}^D}\frac{\partial R_{t+1}^D}{\partial \theta_{t+1}}$$

---

[6] We note that if $\theta = 0$ and $N \le 2$, $R^D < 1$, that is , the deposit interest rate in real terms is negative. Disallowing negative nominal interest rates, this is admissible only in the presence of inflation, which one might also associate with economies with such low parameter values. In the worse case, $\theta = 0$ $N = 2$, the annual inflation rate would not need to exceed the low two-thirds of a percentage point for periods of forty years. In the absence of inflation, a non-negativity condition for nominal interest rates implies $R^D = 1$, which would only have second-order effects both on the gains from human capital investment and on output.

Then, substituting in the definition of the interest factor mark-up renders this

$$\frac{\partial F}{\partial \theta} = -\lambda\left((1-\eta)w_t + \frac{\eta\left(w_{t+1} / R^D_{t+1}\right) + d_t}{1-\hat{\tau}-\lambda\theta_{t+1}} \right) + \frac{\eta}{R^D_{t+1}}\frac{\partial w_{t+1}}{\partial \theta_{t+1}} + d_t\left(\frac{1}{R^F_{t+1}}\frac{\partial R^F_{t+1}}{\partial \theta_{t+1}} - \frac{1}{M_{t+1}}\frac{\partial M_{t+1}}{\partial \theta_{t+1}} \right) \quad (27)$$

The first term is the negative effect of higher taxes on household utility. The remaining three terms are each positive and represent the effects of higher future human capital (i) on the future wage, (ii) on the future deposit interest factor, *holding constant the mark-up factor*) and (iii) on the same *through a fall in the mark-up factor*. This last term drops out where the finance market is competitive, but is the very powerful in the monopolistic case.

Proposition: *A competitive financial structure reduces the incentive to invest in Nelson-Phelps human capital.*

The intuition follows from *Result 6*: the more competitive the financial sector, the less scope there is for reducing the mark-up, so that the returns to education in part (iii) are necessarily smaller, other things being equal.

As a consequence, it is also possible to show by way of numerical example that a more concentrated financial sector may lead to higher economic growth through this political economy mechanism in equation (27).

4.4.2 A numerical example

We parameterise the model in order determine the relative growth rates under a competitive financial market structure and a competitive one and a monopolistic one. We assign a value for the annual growth rate of 2% in the competitive case from which the value of the technology A as a free parameter may be deduced and then applied exogenously in the solution for monopoly growth rate.

Also, we work backwards by assuming a value of human capital in the competitive case from which a value λ may be inferred. For the other choices of parameter values chosen, we find that if households vote for $\theta = 0.7$ in the competitive case, then $\lambda = 0.0991$. This implies that about 7% of income is taxed for the funding of education, but that it would need to be about 10% for $\theta = 1$. These values are consistent with the OECD finding that expenditure as a share of income for a number of countries ranges between 5% and 8%. We obtain the following result.

Numerical example: *There is a political economy equilibrium which is consistent with a steady-state annual growth rate of 2% in the competitive case (* $N \to \infty$ *) where* $\theta = 0.7$ *. In the monopoly case (* $N = 1$ *), the same parameter values imply* $\theta = 1$ *and a steady-state annual growth rate of 2.25%.* [This is proved in the *Appendix*.]

It is noteworthy that in the competitive case, where the value of the interest mark-up value is fixed, a higher θ raises the deposit interest factor through raising the marginal product of capital and firms' lending rates, since the financing effect of higher taxes on wages reduce the rate of capital accumulation. Conversely, in the monopolistic case, an equilibrium where $\theta = 1$ is supported, because an increase in human capital has a powerful effect in raising the deposit interest factor via a reduction in the mark-up factor. Thus, economic growth may be higher with a monopolistic financial sector, because from a political economy perspective this raises the incentive to vote for higher taxes and more education, leading to greater future levels of Nelson-Phelps human capital.

5. Further points

A particular implication of the analysis is the relationship between democracy and long-run economic growth. In general and outside this model, democracy may be beneficial for growth by providing a stable mechanism for political change. Conversely, it may reduce growth if voters prefer redistribution to efficiency or present consumption to the provision of public infrastructure. Barro (1996) finds that if some of the associated benefits of democracy are controlled for, it has a weakly negative effect on economic growth. This present model with N-P human capital suggest that the economic growth benefits of majority voting are stronger where financial markets less highly competitive, where there is less incentive to raise educational infrastructure.

This model may be extended to an open economy that consists of a number of countries. It would be of particular interest, if one country were designated as a technology leader with its internal economic factors driving the global growth rate. N-P human capital would then, more conventionally, determine the speed at which the follower countries catch up. It would also be of insightful to consider global interactions with various possibilities for international capital mobility. The effects of increased financial competition from abroad would be ambiguous in a way that would closely resemble concentration effects within a single country setting. On the one hand, the international mobilization of deposit savings could counteract the effects of deposit market monopsony at home, while, on the other, a second stage of mobility, which allows foreign firms also to enter into loan market competition with existing domestic ones, would have a similar crowding effect that would make it difficult for the loan market externality to be internalized. The effects of these two forms of capital mobility on both global growth and on the international income distribution - at least with a single leader and an archetypal follower country - warrant further analysis.

6. Concluding comments

The paper started off by generalising the *AK* model by including the Nelson-Phelps concept of human capital that determines the speed of catch-up of the applied to an objective technology. A full complement of human capital was interpreted to describe the standard form of this particular model, where market concentration in the market for loans to firms is irrelevant to loan interest rate determination and to economic growth. It was shown that Nelson-Phelps human capital would not just determine the rate of technology catch-up in a global setting but also the long-run economic growth. Another effect of increasing Nelson-Phelps human capital is in raising the interest elasticity of the aggregate demand for loans. This opens up an additional mechanism through which education may affect economic growth where financial markets are imperfectly competitive.

Another result was of a trade-off with respect to financial market concentration. From the point of view of the investment externality alone, it is always more efficient to have fewer firms, but this also is consistent with greater monopsonistic exploitation in the deposit market. A relative interest inelastic supply of savings, as suggested by some empirical findings, suggests net benefits from increased financial market competition. However, if the financial sector is more competitive, there is less incentive to invest in human capital. Thus, financial monopoly might benefit economic growth not in and of itself but in making it more worthwhile to incur the extra tax costs of increasing the public provisionof education.

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Appendix: Proof of the Example

To restate equation (27),

$$\frac{\partial F}{\partial \theta} = -\lambda\left((1-\eta)w_t + \frac{\eta(w_{t+1}/R^D_{t+1}) + d_t}{1-\hat{\tau}-\lambda\theta_{t+1}}\right) + \frac{\eta}{R^D_{t+1}}\frac{\partial w_{t+1}}{\partial \theta_{t+1}} + d_t\left(\frac{1}{R^F_{t+1}}\frac{\partial R^F_{t+1}}{\partial \theta_{t+1}} - \frac{1}{M_{t+1}}\frac{\partial M_{t+1}}{\partial \theta_{t+1}}\right)$$

Equations (8) and (9) imply

$$\frac{1}{R^F_{t+1}}\frac{\partial R^F_{t+1}}{\partial \theta_{t+1}} = (1-\alpha)\left(\ln\left(\frac{k_{t+1}}{b_t}\right) - (1-\theta_{t+1})\frac{1}{k_{t+1}}\frac{\partial k_{t+1}}{\partial \theta_{t+1}}\right) \tag{A1}$$

$$\frac{1}{w_{t+1}}\frac{\partial w_{t+1}}{\partial \theta_{t+1}} = (1-\alpha)\ln\left(\frac{k_{t+1}}{b_t}\right) + \left(\alpha + (1-\alpha)\theta_{t+1}\right)\frac{1}{k_{t+1}}\frac{\partial k_{t+1}}{\theta_{t+1}} \tag{A2}$$

Equation (10) also implies that in the steady-state

$$\ln\left(k_{t+1}/b_t\right) = \theta^{-1}\ln G, \tag{A3}$$

so that the steady-state form of (A1) is

$$\frac{1}{R^F_{t+1}}\frac{\partial R^F_{t+1}}{\partial \theta_{t+1}} = (1-\alpha)\left(\frac{1}{\theta}\ln G - (1-\theta_{t+1})\frac{1}{k_{t+1}}\frac{\partial k_{t+1}}{\partial \theta_{t+1}}\right), \tag{A4}$$

The constant factor income shares property of the Cobb-Douglas production function with identity (25) implies

$$\frac{w_{t+1}}{R_{t+1}^{D}} = \left(\frac{1-\alpha}{\alpha}\right) k_{t+1} M_{t+1} \tag{A5}$$

This with equations (A2) and (A3) gives

$$\frac{1}{R_{t+1}^{D}} \frac{\partial w_{t+1}}{\partial \theta_{t+1}} = \left(\frac{(1-\alpha)^2}{\alpha \theta} \ln G k_{t+1} + \frac{(1-\alpha)\left(\alpha + (1-\alpha)\theta_{t+1}\right)}{\alpha} \frac{\partial k_{t+1}}{\theta_{t+1}} \right) M_{t+1} \tag{A6}$$

Equations (14) and (19) give

$$k_{t+1} = \frac{\mu(1-\eta)(1-\hat{\tau}_t - \lambda \theta_{t+1}) w_t}{1 + (1-\mu)\eta \left(\frac{1-\alpha}{\alpha}\right) M_{t+1}} \tag{A7}$$

The derivative

$$\frac{\partial k_{t+1}}{\partial \theta_{t+1}} = \frac{-\lambda \mu (1-\eta) w_t - (1-\mu)\eta \left(\frac{1-\alpha}{\alpha}\right) k_{t+1} \frac{\partial M_{t+1}}{\partial \theta_{t+1}}}{1 + (1-\mu)\eta \left(\frac{1-\alpha}{\alpha}\right) M_{t+1}}$$

may also be expressed as

$$\frac{\partial k_{t+1}}{\partial \theta_{t+1}} = -\left(\frac{\lambda}{1-\hat{\tau} - \lambda \theta_{t+1}} + \frac{(1-\mu)\eta \left(\frac{1-\alpha}{\alpha}\right) \frac{\partial M_{t+1}}{\partial \theta_{t+1}}}{1 + (1-\mu)\eta \left(\frac{1-\alpha}{\alpha}\right) M_{t+1}} \right) k_{t+1} \tag{A8}$$

Equation (A7) may also be written in terms of the wage

$$(1-\eta) w_t = \left(\frac{1 + (1-\mu)\eta \left(\frac{1-\alpha}{\alpha}\right) M_{t+1}}{\mu\left(1-\hat{\tau} - \lambda \theta_{t+1}\right)} \right) k_{t+1} \tag{A9}$$

Equation (A8) into (A4) and (A6) and these two with (A5) and (A9) into (27), noting $d_t = k_{t+1}$.

$$\frac{\partial F}{\partial \theta} = -\lambda\left(\frac{1+\mu+\eta\left(\frac{1-\alpha}{\alpha}\right)M_{t+1}}{\mu(1-\hat{\tau}-\lambda\theta_{t+1})}\right)k_{t+1}$$

$$+\eta(1-\alpha)\left(\frac{1-\alpha}{\alpha\theta}\ln G+\left(\frac{\alpha+(1-\alpha)\theta}{\alpha}\right)\frac{1}{k_{t+1}}\frac{\partial k_{t+1}}{\partial \theta_{t+1}}\right)M_{t+1}k_{t+1} \quad \text{(A10)}$$

$$+(1-\alpha)\left(\frac{1}{\theta}\ln G-(1-\theta)\frac{1}{k_{t+1}}\frac{\partial k_{t+1}}{\partial \theta_{t+1}}\right)k_{t+1}-\frac{k_{t+1}}{M_{t+1}}\frac{\partial M_{t+1}}{\partial \theta_{t+1}}$$

In the competitive case, $N \to \infty$, so that $\partial M/\partial \theta = 0$ and

$$\frac{\partial F}{\partial \theta} \to -\lambda\left(\frac{1+\mu+\eta\left(\frac{1-\alpha}{\alpha}\right)M_{t+1}}{\mu(1-\hat{\tau}-\lambda\theta_{t+1})}\right)k_{t+1}$$

$$+\eta(1-\alpha)\left(\frac{1-\alpha}{\alpha\theta}\ln G+\left(\frac{\alpha+(1-\alpha)\theta}{\alpha}\right)\frac{1}{k_{t+1}}\frac{\partial k_{t+1}}{\partial \theta_{t+1}}\right)M_{t+1}k_{t+1} \quad \text{(A10C)}$$

$$+(1-\alpha)\left(\frac{1}{\theta}\ln G-(1-\theta)\frac{1}{k_{t+1}}\frac{\partial k_{t+1}}{\partial \theta_{t+1}}\right)k_{t+1}$$

where

$$\frac{\partial k_{t+1}}{\partial \theta_{t+1}} \to -\left(\frac{\lambda}{1-\hat{\tau}-\lambda\theta_{t+1}}\right)k_{t+1} \quad \text{(A8C)}$$

We consider an interior solution, $\partial F/\partial \theta \to 0$. (A10C) - containing (A8C) - may then be presented as

$$\frac{\theta\lambda}{1-\hat{\tau}-\lambda\theta} = \frac{(1-\alpha)\left(1+\eta\left(\frac{1-\alpha}{\alpha}\right)M_{t+1}\right)\ln G}{\frac{1+\mu+\eta\left(\frac{1-\alpha}{\alpha}\right)M_{t+1}}{\mu}+\eta(1-\alpha)\left(\frac{\alpha+(1-\alpha)\theta}{\alpha}\right)M_{t+1}-(1-\alpha)(1-\theta_{t+1})}$$

After taking out common terms, this reduces to

$$\frac{\theta\lambda}{1-\hat{\tau}-\lambda\theta}=\frac{(1-\alpha)\ln G}{\mu^{-1}+\left(\alpha+(1-\alpha)\theta\right)} \tag{A11C}$$

Some parameter value choices are made: $\ln G(\infty)=0.7921$ (ie. 2% per annum over 40 years), $\alpha=0.33$, $\hat{\tau}=0.3$, $\mu=0.25$ and $\theta=0.7$. Equation (A11C) then holds where $\lambda=0.0991$.

Using these same values, including the solved value $\lambda=0.0991$, implies that the growth equation,

$$G=\left(\frac{\mu(1-\eta)(1-\hat{\tau}-\lambda\theta)(1-\alpha)A}{1+(1-\mu)\eta(\alpha^{-1}-1)M}\right)^{\frac{\theta}{1-\alpha(1-\theta)}}, \tag{21}$$

is satisfied where $A=32.596$.

Finally, we first, we conjecture that in the monopolistic case where $N=1$, $\theta=1$. Applying all the above values, $\alpha=0.33$, $\hat{\tau}=0.3$ and $\mu=0.25$, including the two solved values $\lambda=0.0991$ and $A=32.596$, to equation (21), where $\theta=1$ and $M=4.5451$, gives $\ln G(1)=0.8923\left(>\ln G(\infty)=0.7921\right)$, which over 40 years horizon implies a higher, annual growth rate of 2.25%.

Secondly, this conjecture in proved by showing that for all these values, $\alpha=0.33$, $\hat{\tau}=0.3$ and $\mu=0.25$, $\lambda=0.0991$, $A=32.596$, $\theta=1$, $M=4.5451$ plus $\ln G(1)=0.8923$, there equation (A10) becomes

$$\frac{\partial F}{\partial\theta}\geq 0 \tag{A10M}$$

In fact, we find $\partial F/\partial\theta>0$, which suggests a mathematical optimum where $\theta>1$, but, in economically, θ, is bounded at unity.