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*Working Paper 12/13*

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Produced By:

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# Endogenous Participation in Imperfect Labor and Capital Markets

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## Abstract

We introduce endogenous participation in an economy with labor and financial market frictions. Agents can choose to be workers or entrepreneurs or not to participate in any market. We examine how the transition rates between these three options are affected by productivity shocks (business cycle conditions) and by changes in the level of market frictions (cross-country institutional quality variations).

**Keywords:** endogenous participation; labor market frictions; financial market frictions.

**JEL:** G32, J64

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## 1. Introduction

Each year many workers decide to become self-employed while many self-employed become employees and the levels of these transitions depend on macroeconomic conditions. For example, panel (a) of Figure 1 suggests that each year in Great Britain between 1.5% and 2% of those in employment move into self-employment. There is some suggestion that this rate may be pro-cyclical. Meanwhile, about 2% to 3% of those out of employment enter self-employment each year. Panel (b) shows that each year about 10% of the self-employed become employed while about 6% of the self-employed exit into non-employment.<sup>1</sup>

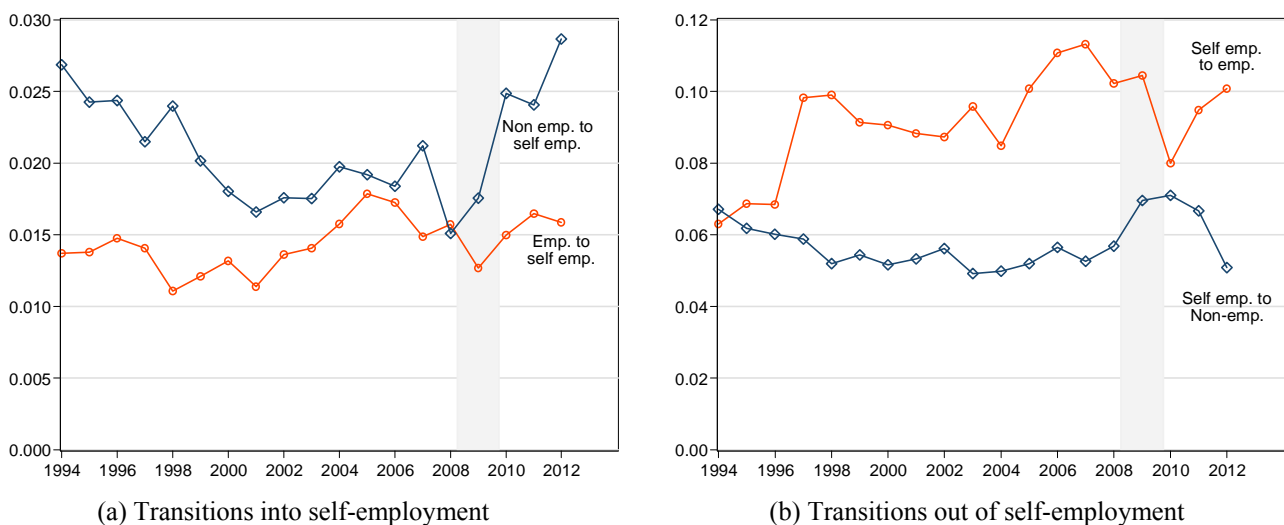


Figure 1. Annual transition rates into and out of self-employment. Shaded area indicates consecutive quarters of negative GDP growth. Sample includes all individuals aged 16-60. Weighted to UK population. Source: Quarterly Labour Force Survey 5-quarter longitudinal panel.

Clearly, the choice between self-employment and employment is influenced not only by the state of the economy but also by conditions in the financial and labor markets. The level of labor market frictions affects availability of jobs while the level of financial market frictions affects availability of funding for prospective entrepreneurs.

We analyze a simple model with labor and financial market frictions where agents can stay away from the market, or enter the labor market, or become entrepreneurs.<sup>2</sup> We do so, by introducing a labor market in the Holmström and Tirole (1997) fixed investment model. Then we examine how the transition rates between these three options are affected by productivity

<sup>1</sup> The fact that transition rates out of self-employment are so much higher is related to the fact that the stock of self-employed is much smaller than the stocks of employed and non-employed (unemployed plus those people between 16 and 60 who do not participate in the labor market).

<sup>2</sup> There is an extensive literature that investigates the interactions between financial and labor market frictions (see, for example, Acemoglu, 2011; Arnold, 2002; Farmer, 1985; Greenwald and Stiglitz, 1993; Hall, 2011; Wasmer and Weil, 2004). However, in all these papers agents can be either workers or self-employed without being allowed to move between the two forms of employment.

shocks (business cycle conditions) and by changes in the level of market frictions (cross-country institutional quality variations).

## 2. The Model

The one-period single-good economy is populated by a continuum of risk-neutral agents of measure 1. Agents have three options. They can become entrepreneurs and run a project or they can enter the labor market or they can abstain from any economic activity. Entering the labor market entails a utility cost  $\gamma$  distributed across the population according to the function  $F$  (with density  $f$ ) on the interval  $[\underline{\gamma}, \bar{\gamma}] \in R^+$ . The utility of an agent is equal to  $C - \gamma I$  where  $C$  denotes consumption and  $I \in \{0,1\}$  equals 1 if the agent enters the labor market and 0 otherwise. Agents are endowed with one indivisible unit of labor and  $z$  units of the good that is distributed across the population according to the function  $G$  (with density  $g$ ) on the interval  $[\underline{z}, \bar{z}] \in R^+$ . The distributions  $F$  and  $G$  are independent.<sup>3</sup>

There is a risky technology that requires an entrepreneur's labor to manage it, an investment of  $K$  units of the good and one unit of labor. With probability  $p^H \in \{p^L, p^H\}$ , ( $0 < p^L < p^H < 1$ ) the technology yields  $X$  units of the single good and with probability  $1-p^H$  yields nothing. Success depends on the entrepreneur's behavior. Working hard increases the likelihood of success while shirking offers a private benefit  $B$ .

**Assumption 1:**  $K > \bar{z}$

The inequality implies that no agent can self-finance a project. We restrict our attention to cases where projects are socially efficient only when entrepreneurs exert effort.

**Assumption 2:**  $p^H(X - w) - rK \geq 0 > p^L(X - w) - rK + B$

$w$  and  $r$  denote the equilibrium wage (paid at the completion of the project) and the gross interest rate.

### 2.1. The Financial Market

Agents who do not become entrepreneurs either invest their endowments in the competitive financial market or they store them. They choose the former option if: (a) entrepreneurs, who are protected by limited liability, exert effort, and (b) the interest rate is not less than 1 (the return to storage). Entrepreneurs exert effort if the incentive compatibility constraint

$$p^H(X - R - w) \geq p^L(X - R - w) + B$$

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<sup>3</sup>This assumption is relaxed in the last section.

is satisfied, where  $R$  denotes the loan repayments. Rearranging yields

$$X - w - \varphi \geq R$$

where  $\varphi \equiv \frac{B}{p^H - p^L}$  measures agency costs. The constraint sets an upper bound on the repayment. The lenders' zero-profit condition implies that the repayment of an entrepreneur with endowment equal to  $z$  must satisfy

$$p^H R = (K - z)r$$

Combining the above two conditions we find that only those agents with endowments greater than

$$z^* = K - \frac{p^H}{r}(X - w - \varphi) \tag{1}$$

obtain external funds. We refer to the rest of the agents as 'financially constrained'.<sup>4</sup>

## 2.2. The Labor Market

The division of surplus between entrepreneurs, who have secured a loan, and workers is determined by a generalized Nash bargaining rule. When wages are set above the market clearing wage some of those agents who enter the labor market are not matched with firms and become involuntary unemployed; a possibility that agents anticipate when they decide whether or not to enter the labor market. Let  $\pi$  denote the employment rate, i.e. the proportion of agents that enters the labor market and are matched.

Consider the wage determination process. Let  $z^I$  measure internal finance. The entrepreneur's expected payoff conditional on a successful bargaining outcome equals  $p^H(X - w) - (K - z^I)r + (z - z^I)r$  while when bargaining fails her payoff equals  $(z - z^I)r$  given that she is protected by limited liability. The worker's expected payoff conditional on a successful bargaining outcome equals  $p^H w + zr - \gamma$  while when bargaining fails her payoff equals  $zr - \gamma$ . Let  $\alpha$  measure the bargaining power of workers. Then,

$$w = \operatorname{argmax}(p^H w)^\alpha (p^H(X - w) - (K - z^I)r)^{1-\alpha}$$

**Lemma 1:** *The optimal level of internal finance is independent of the endowment level.*

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<sup>4</sup> Giannetti (2011) provides evidence showing that liquidity constraints affect negatively the probability of being self-employed.

**Proof:** Setting the first-order-condition of the optimization problem equal to zero and solving for the wage yields  $w = \frac{\alpha(p^H X - (K - z^I)r)}{p^H}$ . The proposition follows from the observation that the wage is increasing in the level of internal finance,  $z^I$ . ■

The lemma implies that entrepreneurs contribute the lowest possible amount of internal finance. Thus we have  $z^I = z^*$ .

**Lemma 2:** *The common wage is given by*

$$w = \frac{\alpha}{1-\alpha} \varphi \equiv \theta \varphi$$

**Proof:** Setting  $z^I = z^*$ , implies that  $w = \frac{\alpha(p^H X - (K - z^*)r)}{p^H}$ . By substituting (1) for  $z^*$  and solving for the wage rate we complete the proof. ■

The equilibrium wage depends on the degree of imperfections in both markets.<sup>5</sup> The higher the bargaining power of workers (higher  $\alpha$ ) the higher the wage is. The wage also increases with  $\varphi$  a measure of financial market agency costs. As the level of internal finance increases the entrepreneur's obligation to her creditors decreases thus increasing the surplus whose division is negotiated between the two parties.

Above, we assumed that the allocation of bargaining power is independent of the employment rate. In what follows, we consider the more realistic case where the bargaining power of workers is increasing in the employment rate. As the employment rate increases the influence of the outsiders, i.e. the involuntary unemployed, declines. Thus, we consider the new wage function

$$w = \frac{\alpha(\pi, \delta)}{1-\alpha(\pi, \delta)} \varphi = \delta \theta(\pi) \varphi; \theta'(\pi) > 0; \delta > 0 \quad (2)$$

$\delta$  is a shift parameter.

## 2.2. Occupational Decisions

The expected utility derived from entering the labor market is

$$U^L = \pi p^H w + rz - \gamma$$

The first term denotes expected labor income and the second term denotes gross financial income. The expected utility of entrepreneurs is

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<sup>5</sup> This dependence was originally shown by Peroti and Spier (1993) while its macroeconomic implications have also been analyzed by Wasmer and Weil (2004).

$$U^E = p^H(X - w) - rK + rz$$

When the agent decides neither to become an entrepreneur nor to participate in the labor market her utility  $U^N$  is given by her financial income

$$U^N = rz$$

Assumption 2 implies that  $U^E - U^N > 0$ . All agents with  $z \geq z^*$  either become entrepreneurs or enter the labor market. Next, comparing  $U^L$  and  $U^N$  we find that among the agents with endowments less than  $z^*$ , those with  $\gamma > \pi p^H w$  do not enter the labor market.

Let

$$\gamma^H = \pi p^H w \tag{3}$$

Comparing  $U^E$  and  $U^L$  we find that among those agents with endowments greater than  $z^*$ , those with  $\gamma > \pi p^H w - (p^H(X - w) - rK)$  do not enter the labor market. Let

$$\gamma^L = \pi p^H w - (p^H(X - w) - rK) \geq 0 \tag{4}$$

Assumption 2 implies that  $\gamma^H > \gamma^L$ . Clearly if  $\gamma^L = 0$  then all agents with  $z \geq z^*$  will become entrepreneurs.

### 2.3. Equilibrium

The availability of storage implies that there are two types of equilibria. There is one equilibrium where the financial market clears at a gross interest rate greater than unity and nobody stores. The mass of financially unconstrained agents is sufficiently high to allow the market to clear. Small perturbations only change the interest rate without affecting output and employment.

We focus on the other equilibrium where the gross interest rate is equal to one (return to storage) and some endowments are stored. Labor market clearing implies

$$\pi\{F(\gamma^H)G(z^*) + F(\gamma^L)(1 - G(z^*))\} = (1 - F(\gamma^L))(1 - G(z^*)) \tag{5}$$

The left-hand side equals the supply of labor. The first term in the brackets equals the mass of financially constrained agents who enter the labor market while the second term equals the corresponding mass of unconstrained agents. The right hand-side equals the demand for labor (unconstrained agents who become entrepreneurs).

Equations (1) and (5) solve for the parameters  $z^*$ ,  $\gamma^H$ ,  $\gamma^L$ ,  $w$  and  $\pi$ . The amount invested in storage  $V$  is

$$V = \hat{z} - K(1 - F(\gamma^L))(1 - G(z^*)) \quad (5)$$

The second term of the right-hand side equals aggregate investment. The following proposition summarizes the comparative static results of the model when  $\gamma^L = 0$ .

**Proposition 1:** (*Equilibrium with Storage*)

$$(a) \frac{d\pi}{d\phi} < 0, (b) \frac{dV}{d\phi} > 0, (c) \frac{d\pi}{d\delta} < 0, (d) \frac{dV}{d\delta} > 0, (e) \frac{d\pi}{dX} > 0, \text{ and } (f) \frac{dV}{dX} < 0$$

After an increase in either workers' bargaining power or in agency costs the wage rate increases causing drops in output (increase in storage) and employment and thus on the employment rate. In contrast, a positive productivity shock has the opposite effects. The results related to the employment rate still hold when  $\gamma^L > 0$ . A sufficient condition, but by no means necessary, for the results related to storage to be still valid is that the direct effects on output dominate the indirect employment effects on the wage rate.

### 3. Macroeconomic Implications

We first analyze the impact of productivity shocks on employment and participation rates to clarify the implications of our model for the behavior of workers over the business cycle. Then we introduce variations in market frictions to assess how cross-country institutional differences affect cross-country variations in participation and employment rates.

#### 3.1. Productivity Shocks

Consider an increase in  $X$ . Proposition 1 implies that the employment rate will rise. When  $\gamma^L = 0$ , the participation rate is equal to  $\frac{(1-G(z^*))F(\gamma^H)}{(1-G(z^*))} = F(\gamma^H)$ . Differentiating with respect to  $X$  yields

$$f(\gamma^H)p^H \frac{d\pi}{dX} \delta (1 + \theta'(\pi)) \theta > 0$$

The participation rate increases after a positive productivity shock. When  $\gamma^L > 0$  there is an additional effect. The improvement in productivity boosts both profits and wages. If the former effect dominates then  $\gamma^L$  declines having a negative effect on the participation rate.

#### 3.2. Institutional Variations

A higher value of  $\delta$  signifies a stronger union and thus a less flexible labor market. Similarly, a higher  $\phi$  captures a less efficient financial market. Proposition 1 suggests that an increase in any of these parameters has a negative effect on the employment rate. What happens to the



participation rate is more complicated and for simplicity we restrict our attention to the case when  $\gamma^L = 0$ .

Differentiating the participation rate with respect to  $\varphi$  yields

$$f(\gamma^H)p^H\delta\left(\frac{d\pi}{d\varphi}\varphi(\theta(\pi)+\pi\theta'(\pi))+\pi\theta(\pi)\right)\geq 0$$

The term in the brackets is positive which implies if  $\left|\frac{\frac{d\pi}{d\varphi}\varphi(\theta(\pi)+\pi\theta'(\pi))}{\pi\theta(\pi)}\right| < 1$  the whole expression will be positive. The expression equals the elasticity of the expected wage function  $\pi w$  (see (2)) with respect to the level of financial frictions. When it is less than 1 (the effects of an increase in frictions on the wage dominate its effects on the employment rate) the rise in frictions has a positive effect on the participation rate. This, for example, will be the case for countries with, *ceteris paribus*, higher employment rates.

We draw two implications about the relationship between cross-country institutional variations and corresponding variations in labor market outcomes. The symmetry of the wage function with respect to  $\delta$  and  $\varphi$  implies that the two variations have the same qualitative effects. Furthermore, more flexible markets imply higher employment rates. However, cross-country variations in participation rates depend on the relative responses of wages and employment rates to institutional variations. Among countries with high frictions those with higher employment rates have higher participation rates.

#### 4. Extensions

We have assumed that the distributions of endowments and entry costs are independent. We consider the more plausible case of negative correlation. For simplicity, suppose that  $\gamma$  and  $z$  are perfectly negatively correlated. Let  $z = \hat{\gamma} - \lambda\gamma$ ,  $\hat{\gamma} > 0$ ,  $\lambda > 0$  and define  $\gamma^* = \frac{\hat{\gamma}-z^*}{\lambda}$  as the cutoff level of labor market entry costs such that agents with a lower cost have access to external funds. (2), (3) and (4) are still the same. In any equilibrium with  $\gamma^L > 0$  the inequalities  $\gamma^L < \gamma^* < \gamma^H$  must hold. Agents with  $\gamma < \gamma^L$  enter the labor market, agents with  $\gamma^L \leq \gamma < \gamma^*$  become entrepreneurs, agents with  $\gamma^* \leq \gamma < \gamma^H$  enter the labor market, and agents with  $\gamma \geq \gamma^H$  do not participate. For simplicity consider the case when  $\gamma^L = 0$ .

Equilibrium in the labor market implies  $\pi(F(\gamma^H) - F(\gamma^*)) = F(\gamma^*)$  while the participation rate equals  $\frac{F(\gamma^H)-F(\gamma^*)}{1-F(\gamma^*)}$ . The denominator equals the mass of agents that are financially

constrained and the numerator equals the mass of labor market entrants. Higher frictions have a negative effect on the employment rate and an ambiguous effect on the participation rate. These results are qualitatively the same as those derived above.

## **References**

- Acemoglu, D., 2001. Credit market imperfections and persistent unemployment. *European Economic Review* 45, 665-679.
- Holmström, B., and Tirole, J., 1997. Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics* 112, 663-692.
- Arnold, L., 2002. Financial market imperfections, labour market imperfections and business cycles. *Scandinavian Journal of Economics* 104, 105-124.
- Farmer, R., 1985. Implicit contracts with asymmetric information and bankruptcy: The effect of interest rates on layoffs. *Review of Economic Studies* 52, 427-442.
- Giannetti, M., 2011. Liquidity constraints and occupational choice. *Finance Research Letters* 8, 37-44.
- Greenwald, B., Stiglitz, J., 1993. Financial market imperfections and business cycles. *Quarterly Journal of Economics* 108, 77-114.
- Hall, R., 2011. The long slump. *American Economic Review* 101, 431-469.
- Peroti, E., Spier, K., 1993. Capital structure as a bargaining tool: The role of leverage in contract renegotiation. *American Economic Review* 83, 1131-1141.
- Wasmer, E., Weil, P., 2004. The Macroeconomics of labor and credit market imperfections. *American Economic Review* 94, 944-963.

## Appendix

### Proof of Proposition 1:

Setting  $r = 1$  and totally differentiating (4') we get

$$\begin{aligned}
& \left\{ \begin{array}{l} F(\gamma^H)G(z^*) + (1 - G(z^*))F(\gamma^L) + \\ \pi(p^H \delta\theta(\pi)\varphi + \pi p^H \delta\theta'(\pi)\varphi)f(\gamma^H)G(z^*) + \\ (1 + \pi)f(\gamma^L)(1 - G(z^*))(p^H \delta\theta(\pi)\varphi + \pi p^H \delta\theta'(\pi)\varphi + p^H \delta\theta'(\pi)\varphi) + \\ (\pi(F(\gamma^H) - F(\gamma^L)) + (1 - F(\gamma^L)))g(z^*)p^H \delta\theta'(\pi) \end{array} \right\} d\pi \\
& + \left\{ \begin{array}{l} \pi^2 f(\gamma^H)p^H G(z^*)\delta\theta(\pi) + \\ (\pi(F(\gamma^H) - F(\gamma^L)) + (1 - F(\gamma^L)))(1 + \delta\theta(\pi))g(z^*)p^H + \\ (1 + \pi)f(\gamma^L)(1 - G(z^*))(\pi p^H \delta\theta(\pi) + p^H \delta\theta(\pi)) \end{array} \right\} d\varphi \\
& + \left\{ \begin{array}{l} \pi^2 f(\gamma^H)G(z^*)p^H \theta(\pi)\varphi + (\pi(F(\gamma^H) - F(\gamma^L)) + (1 - F(\gamma^L)))g(z^*)p^H \theta(\pi)\varphi + \\ (1 + \pi)f(\gamma^L)(1 - G(z^*))(\pi p^H \theta(\pi)\varphi + p^H \theta(\pi)\varphi) \end{array} \right\} d\delta \\
& - \left\{ (\pi(F(\gamma^H) - F(\gamma^L)) + (1 - F(\gamma^L)))g(z^*)p^H + (1 + \pi)(1 - G(z^*))f(\gamma^L)p^H \right\} dX = 0
\end{aligned}$$

From which the proofs of parts (a), (c) and (e) directly follow.

Totally differentiating (5) we get

$$\begin{aligned}
\frac{dV}{d\varphi} &= \frac{\partial V}{\partial \pi} \frac{d\pi}{d\varphi} + \frac{\partial V}{\partial \varphi} \\
&= \left\{ \begin{array}{l} f(\gamma^L)(1 - G(z^*))K((p^H \delta\theta(\pi)\varphi + \pi p^H \delta\theta'(\pi)\varphi + p^H \delta\theta'(\pi)\varphi)) + \\ K(1 - F(\gamma^L))g(z^*)p^H \delta\theta'(\pi)\varphi \end{array} \right\} \frac{d\pi}{d\varphi} \\
&+ f(\gamma^L)(1 - G(z^*))K(\pi p^H \delta\theta(\pi) + p^H \delta\theta(\pi)) \\
&+ K(1 - F(\gamma^L))g(z^*)(1 + \delta\theta(\pi))p^H
\end{aligned}$$

$$\begin{aligned}
\frac{dV}{d\delta} &= \frac{\partial V}{\partial \pi} \frac{d\pi}{d\delta} + \frac{\partial V}{\partial \delta} \\
&= \left\{ \begin{array}{l} f(\gamma^L)(1 - G(z^*))K(p^H \delta\theta(\pi)\varphi + \pi p^H \delta\theta'(\pi)\varphi + p^H \delta\theta'(\pi)\varphi) + \\ K(1 - F(\gamma^L))g(z^*)p^H \delta\theta'(\pi)\varphi \end{array} \right\} \frac{d\pi}{d\delta} \\
&+ [Kf(\gamma^L)(1 - G(z^*))p^H \theta(\pi)\varphi + K(1 - F(\gamma^L))g(z^*)p^H \theta(\pi)\varphi]
\end{aligned}$$

$$\begin{aligned} \frac{dV}{dX} &= \frac{\partial V}{\partial \pi} \frac{d\pi}{dX} + \frac{\partial V}{\partial X} \\ &= \left\{ \begin{array}{l} f(\gamma^L)(1 - G(z^*))K(p^H \delta\theta(\pi)\varphi + \pi p^H \delta\theta'(\pi)\varphi + p^H \delta\theta'(\pi)\varphi) + \\ K(1 - F(\gamma^L))g(z^*)p^H \delta\theta'(\pi)\varphi \end{array} \right\} \frac{d\pi}{dX} \\ &\quad - [Kf(\gamma^L)(1 - G(z^*))p^H + K(1 - F(\gamma^L))g(z^*)p^H] \end{aligned}$$

Parts (b), (d) and (f) follow directly from parts (a), (b) and (c) for the case when  $\gamma^L = 0$ . For  $\gamma^L > 0$ , we need the additional assumption that the direct effects on output dominate the indirect employment effects on the wage rate and thus on the participation rate. ■

### Negative Correlation between $\gamma$ and $z$

We show that when  $z$  and  $\gamma$  are perfectly negatively an increase in the level of financial frictions will have a negative impact on the employment rate. Totally differentiating the labor market equilibrium condition  $\pi(F(\gamma^H) - F(\gamma^*)) = F(\gamma^*)$  we get

$$\begin{aligned} &\left\{ F(\gamma^H) - F(\gamma^*) + \pi^2 f(\gamma^H) p^H \delta\theta'(\pi)\varphi + \pi f(\gamma^H) p^H \delta\theta(\pi)\varphi \right. \\ &\quad \left. + (1 + \pi) f(\gamma^*) \frac{1}{\lambda} p^H \delta\theta'(\pi)\varphi \right\} d\pi \\ &\quad + \left\{ \pi^2 f(\gamma^H) p^H \delta\theta(\pi) + (1 + \pi) f(\gamma^*) \frac{1}{\lambda} p^H (1 + \delta\theta(\pi)) \right\} d\varphi = 0 \end{aligned}$$

The result follows from the observation that all terms are positive.

Differentiating the participation rate  $\frac{F(\gamma^H) - F(\gamma^*)}{1 - F(\gamma^*)}$  with respect to  $\varphi$  we find that its sign is the same as the sign of the following expression

$$\begin{aligned} &f(\gamma^H) p^H \delta \left( \frac{d\pi}{d\varphi} \varphi (\theta(\pi) + \pi\theta'(\pi)) + \pi\theta(\pi) \right) (1 - F(\gamma^*)) \\ &\quad + f(\gamma^*) \frac{1}{\lambda} p^H \left( \delta \left( \theta'(\pi)\varphi \frac{d\pi}{d\varphi} + \theta(\pi) \right) + 1 \right) \geq 0 \end{aligned}$$