Optimal Organization of Financial Intermediaries

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Abstract

This paper provides a unified framework for endogenizing two distinct organizational structures for financial intermediation. In one structure, called Bank, the intermediary is financed by issuing debt contracts to investors, and thus resembles commercial banks . In the other structure, called Fund, the intermediary is financed by issuing equity contracts to investors, thus resembling private-equity funds. The paper considers the advantage of the Bank structure relative to the Fund structure. It finds that in the former incentives can be provided in a less costly way, but the latter is more robust to negative shocks on the asset side. Our model predicts that relative to banks, private equity funds are more involved in the running of the firms that they finance, contribute more to the success of these firms, and provide funds to higher-risk, higher-return firms.

JEL Classification: D86, G00

1 Introduction

In economic environments where transaction costs, informational asymmetries and incomplete markets inhibit direct relationships between borrowers and lenders, financial intermediaries provide indirect ways for bringing the two parties together. Up to a very large extent this intermediation role is performed by *banks*. A defining characteristic of banks is that on their liability side they raise funds mainly by offering fix obligations to investors (depositors). In the last twenty years, we have seen a rapid growth of an alternative class of financial intermediaries, namely, *private equity funds*, that, unlike banks, raise

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funds by offering equity claims to their investors who are known as limited partners (see Mertrick and Yasuda, 2010). Some types of *private equity funds* finance, like banks, a variety of new investments for firms unable to access directly the capital markets. For example, *venture capital* specializes in financing young, innovative firms, *growth capital* finances expansion activities of relatively mature firms, and *mezzanine capital* offers investors preferred equity to finance activities of small firms that are unable to raise sufficient funds in the capital market.¹ The volume of capital managed by private equity funds has risen from \$5 billion in 1980 to \$100 billion in 1994 to about \$1 trillion in 2012.²

The co-existence of two distinct organization structures for financial intermediation raises the following questions. What are the relative advantages of each structure? Which are the types of firms more likely to seek funding from each structure? We address these questions, in a unified framework where depending on the values of parameters, the optimal equilibrium contractual arrangement in equilibrium corresponds to one of these two structures.

In our model, an intermediary that bridges entrepreneurs and many small investors provides a service that can potentially increase the probability of success of the projects that it finances. Examples of this type of services include consultation, marketing, and controlling entrepreneurial moral hazard. The provision of this service, however, is unobservable to the investors. As a result, the intermediary is liable to a moral hazard problem. So far, our model is similar to Holmström and Tirole (1997). The innovation of our paper is that, after the funds have been invested, the projects are subject to a shock, oserved only by the intermediary, that divides projects into two types: A type h project can benefit from the intermediary's service while a type l project cannot. Using a mechanism design approach we solve for the optimal contracts on both sides of the intermediary's balance sheet. We find that the equilibrium organization structure of financial intermediation can take one of only two types, depending on the nature of securities they issue to investors. It is either debt in which case we will refer to the organization structure as 'Bank', or it is equity in which case we will refer to the organization structure as 'Fund'.

The trade-off between Bank and Fund is that while Bank has the advantage of providing incentives $^{-1}$ We are mainly concerned with intermediaries that finance new projects so we will ignore private equity funds specializing in leverage buyouts that is in the acquisition of established firms and other types of intermediaries that invest in financial assets such as hedge funds and mutual funds.

² The first couple of figures were taken from Fenn, Liang and Prowse (1995) while the last figure is reported in Metrick and Yasuda (2012). To put these figures in perspective, the total loans and leases granted to businesses and households by U.S. commercial banks form 1/10/2012 till 30/9/2012 according to FDIC was approximately \$7 trillion.

to the intermediary at a lower cost, Fund is more robust to negative realizations (type l) of the shock. In order to understand this trade-off, consider the case where the intermediary considers financing two projects that are known to be type h. Then, as Laux (2001) has demonstrated, in equilibrium the intermediary makes qualitative asset transformation. The contract that the intermediary agrees with the investors is debt, leaving the intermediary nothing when only one project succeeds. Next, consider what happens when we introduce the shock which impacts the projects after the investments have been made. Suppose that one project is type l and the other is type h. Further, suppose that the probability of success of either type l projects or type h projects that are not monitored is very close to zero. Under the debt contract the intermediary gets nothing even if it increases the success probability of the type hproject through monitoring. Therefore, the intermediary has no incentives to monitor that project. Put differently, bad news about one project ruins any incentives to monitor any of the projects. In contrast, suppose that the intermediary is financed with equity contracts. Whenever one project succeeds, the intermediary receives a share of the revenues from the successful project, which offers it incentives to monitor the type h project thus increasing its success probability even if the other project is destined to fail. In summary, the Bank structure provides incentives at a lower cost, but the Fund structure is more robust to bad news. We show that this trade-off between Bank and Fund holds for a wide set of parameters and that the two organization structures are the only ones occurring in equilibrium.

Our model makes the following predictions: (a) equity-financed intermediaries are more intensively involved than banks in monitoring the firms that they finance; (b) the bigger the difference that the intermediary's input makes, the more likely it is that the intermediary is organized as a Fund, and (c) the likelihood of Bank financing relative to Fund financing is positively correlated with the cost of monitoring and negatively correlated with the risk of the projects. The above predictions are consistent with the evidence reported by Metrick and Yasuda (2010) for private equity funds. In particular, prediction (c) implies that private equity funds are more likely than banks to finance projects with small probability of success and huge returns conditional on success which is consistent with the evidence provided by Sahlman (1990) and Kerr, Nanda and Rhodes-Kropf (2014) for venture capital.

Our work is related to various strands of the financial economics literature. For single-project financing, Innes (1990) is the first to demonstrate the optimality of debt for providing incentives under moral hazard. Laux (2001) has demonstrated that with multiple projects cross-pledging can further enhance incentives; see also Tirole (ch.4, 2006). More generally, the optimality of debt contracts in providing incentives related to information problems has been repeatedly demonstrated in the literature; see among others, Diamond (1984), Gale and Hellwig (1985), and Gorton and Pennacchi (1990). In contrast, in this paper, by introducing uncertainty about project types into a setting similar to Holmström and Tirole (1997), we show that the optimal security that the intermediary issues to investors can be either debt or equity.

In our model intermediaries provide a second service, namely, they gather information about their clients. That particular role has also been addressed by Ramakrishnan and Thakor (1984) and Millon and Thakor (1985), however, in these papers intermediaries do not transfer money from investors to entrepreneurs. In contrast, in our paper intermediaries learn project types after the funding has taken place.

Our paper follows a well-established literature that views intermediation as a solution to the problem of delegated monitoring. For example, Diamond (1984) finds a role for financial intermediation in Townsend's (1990) costly-state verification framework by showing that sufficient diversification reduces delegations costs to zero.³ In Calomiris and Kahn (1991) the incentives to monitor are provided by the ability of depositors to withdraw their deposits at will. The monitoring service that intermediaries provide in our model is similar to that in Holmström and Tirole (1997). The aim of all these papers has been to identify the advantages of bank loans over direct finance while our main concern is to compare the solutions to the delegated monitoring problem provided by alternative organization structures for financial intermediaries.

Lastly, our paper is also related to the fast growing theoretical literature on private equity that mainly specializes on the organizational structure of venture capital.⁴ Although our model is too abstract to account for the many complex arrangements associated with these methods of finance (such as stage financing and the decision to go public that demand a dynamic framework; see Gompers, 1995), it provides a unified framework that sheds lights on private equity funds in a broad perspective by letting them compete on a level playing field against another main form of financial intermediation, namely, banks. Further, our model suggests that some features of venture capital, like the funding of

³Since Diamod's (1984) many other authors have analyzed the delegating monitoring problem with the costly-state verification framework (e.g. see Williamson, 1986; Krasa and Villamil, 1992; Winton, 1995; Cerasi and Daltung, 2000; Hellwig, 2000).

⁴As we indicated above leverage buy-outs are not directly related to this study given that they are concerned with the re-organization of firms. See Cuny and Talmor (2007) for a review of the private equity theoretical literature.

high-risk and high-return firms, may be partly accounted for by considerations realted to agency costs.

We organize the paper as follows. In section 2, we describe the model and in Section 3 we solve it and present the main results. In addition, to the derivation of equilibrium intermediation mechanisms we also compare them with alternative direct mechanisms. In section 4, we consider the robustness of our results to (a) an increase in the number of projects thus introducing the possibility of diversification, and (b) a more general contracting environment, and we also discuss some empirical predictions of our model. In Section 5 we offer some concluding comments. All proofs can be found in the Appendix.

2 The Model

There are four dates: 0, 1, 2 and 3. There is a single good that can be stored or invested or consumed. There are two types of risk neutral agents: entrepreneurs and investors. There are two entrepreneurs (E1 and E2) each endowed with a project that requires an investment of 1 unit at date 0. There is a large number of investors each having a very small endowment of the good. Their aggregate endowment is larger than 2. The competitive net interest rate is equal to 0, the net return to storage.

Each project can either succeed or fail. At date 3, when a project succeeds, it returns R, while when it fails, it returns nothing. The probability of success of a project depends on (a) a binary shock, and (b) the input of a service, which we will refer to as 'monitoring', and captures any help in managing, marketing, and identifying potential consumers. Any of the investors can potentially provide the monitoring service. But as in Diamond (1984), to avoid cost replication, monitoring will be delegated to one single investor, whom we refer to as the monitor (hereafter M). The binary shock is realized at date 1. At date 0, it is common knowledge that the shock is identically and independently distributed across projects. With probability p a project is of type h while with probability 1 - p its type is l. After observing the type of a project, M chooses whether or not to monitor it. Monitoring does not affect the probability of success of a type l project which is equal to \underline{q} . In contrast, for a type h project M can, by monitoring, increase its probability of success to q. If M decides not to monitor the probability of success is still equal to \underline{q} . M incurs a fixed cost c when monitoring a project. Investors cannot observe neither project types nor M's monitoring choice. Let $p_s \equiv pq + (1-p)\underline{q}$. We assume that ex ante projects are efficient but those projects that are not monitored destroy value.

Condition 1 $p_s R > 1 + ps$ and 1 > qR

Entrepreneurs need to obtain funds from investors to finance their projects. There are large numbers of investors and monitors and therefore they are price-takers leaving entrepreneurs as the residual claim holders. In this section, we assume M to be a financial intermediary bridging investors and the entrepreneurs, as Diamond (1984) does. Therefore, entrepreneurs issue contracts to M, who in turn issues contracts to investors. However, as Wang (2012) has argued, it might be feasible for M to provide the service only and entrepreneurs to obtain funds directly from investors. Later we will consider the advantages and disadvantages of financial intermediation relative to direct finance. All parties are protected by limited liability.

The timing of the model is as follows. At date 0, E1 and E2 sign contracts with the same M. Then, M issues securities and sells them to investors thus raising funds to finance the two projects. At date 1, M learns the types of both projects. This information is private and not verifiable, however, each entrepreneur learns the type of her own project. At date 2, M decides whether or not to monitor each project. Lastly, at date 3, each project either succeeds or fails and payments are made according to the terms of the contracts signed.

2.1 Organization Structures of Financial Intermediation

For the moment, we assume that a contract signed between one of the entrepreneurs and M can depend only on the outcome of that entrepreneur's projects. Later, we will consider how our results are affected when we relax this restriction by allowing for multilateral contracting between the two entrepreneurs and M. Then, given that there are only two possible outcomes, the only type of contract that each entrepreneur can agree with M is one that specifies what M will receive when the project succeeds. We are going to restrict attention to symmetric equilibria where the two entrepreneurs choose cooperatively the identical contracts that they offer to M.⁵ Thus, on the asset side of M, contracts are represented by a positive number, m, denoting the payment to M from an entrepreneur whose project has been successful. Next, we turn our attention on the liability side of M's balance sheet. Given that project returns are independent there are four possible states of the world. Thus, on the liability side the contract is a profile $\{r_{ij}\}$, where i, j = 1, 0 represents the success (1) or failure (0) of each project. Limited liability

⁵The symmetric Nash equilibrium contract that each entrepreneur offers to M is identical to the contract that corresponds to our co-operative solution. Clearly, there exists a continuum of other non-symmetric Nash equilibria that we ignore where one entrepreneur, conditional on success, pays M less and the other pays M more.

and symmetry imply that $r_{00} = 0$, $r_{01} = r_{10} \equiv r_1 \leq m \leq R$ and $r_{11} = r_2 \leq 2m$.⁶ Following Innes (1990) we will pay special attention to contracts that satisfy the following payoff monotonicity condition (MC):

Condition 2 *MC*: $r_2 \ge r_1$.

Innes (1990) motivates the introduction of this constraint by the possibility that borrowers (in our case M) can pad their revenues and thus avoiding the higher repayments. Of particular interest are organization structures whose liabilities take the form of either equity or debt.

Definition 1 The financial intermediary is organized as a Fund (F) if its liability contract is equity: $r_2 = 2r_1$.

The equity of the fund is sold at price $\frac{2m}{r}$ per share. Investors offer two units of their endowments for $\frac{r}{m}$ shares of the fund and M holds the rest of shares.

Definition 2 The financial intermediary is organized as a Bank (B) if its liability contract is debt: either $r_1 = m$ and $m \le r_2 < 2m$ or $r_1 = r_2 \le m$

The arrangement is a standard debt contract with face value r_2 . When the intermediary is organized as a Bank it makes *qualitative asset transformation* given that the assets held by investors cannot be issued directly by a single entrepreneur.

We will demonstrate that in any equilibrium that satisfies MC only these two arrangements are possible.

3 Equilibrium Organization of Financial Intermediation

The objective of entrepreneurs, as residual claimers, is to minimize the cost of external finance. The first decision that the two entrepreneurs need to take is whether to offer incentives to M to monitor only when the both projects are type h or to ensure that M monitors a project whenever its type is h regardless the other project's type. After comparing the two cases, the entrepreneurs decide what contract to offer to M.

⁶Limited liability of the monitor implies that $r_1 \leq m$ and $r_2 \leq 2m$, while limited liability for the entrepreneur implies that $m \leq R$.

3.1 Case 1: M Monitors Only When Both Projects Are Type h

Suppose that the two entrepreneurs would like M to monitor only when both projects are type h. Then, each project's ex ante probability of success is equal to $p^2q + (1-p^2)\underline{q} \equiv p_z$. The probability that both projects are type h is equal to p^2 and the probability of success of all projects that are not monitored is equal to \underline{q} . The two entrepreneurs choose m so that (a) M has an incentive to monitor only when both projects are good, and (b) the investors' participation constraint is satisfied. Thus, the problem that two entrepreneurs solve is given by

Problem 1 min m subject to:

$$IC1: 2q(1-q)(m-r_1) + q^2(2m-r_2) - 2c$$

$$\geq \underline{q}(1-q)(m-r_1) + q(1-\underline{q})(m-r_1) + q\underline{q}(2m-r_2) - c$$

$$IC2: 2q(1-q)(m-r_1) + q^2(2m-r_2) - 2c$$

$$\geq 2\underline{q}(1-\underline{q})(m-r_1) + \underline{q}^2(2m-r_2)$$

$$IC3: 2\underline{q}(1-\underline{q})(m-r_1) + \underline{q}^2(2m-r_2)$$

$$\geq \underline{q}(1-q)(m-r_1) + q(1-\underline{q})(m-r_1) + q\underline{q}(2m-r_2) - c$$

$$PC1: [p^2q(1-q) + (1-p^2)\underline{q}(1-\underline{q})]2r_1 + [p^2q^2 + (1-p^2)\underline{q}^2]r_2 \geq 2$$

$$r_1 \leq m \leq R \text{ and } r_2 \leq 2m$$

IC1 and IC2 are the incentive compatibility constraints that ensure that when both projects are type h M has an incentive to monitor both of them. On the left-hand side of the weak inequalities in each of these two constraints we have M's expected payoff from monitoring both projects when their type is h. In that case, each project succeeds with probability q. Thus, with probability q^2 both projects succeed, M gets 2m from the two entrepreneurs, and returns r_2 to investors, which implies a payoff for M of $2m - r_2$. Furthermore, with probability q(1 - q), only one project succeeds and then M's payoff equals $m - r_1$. The right-hand side of IC1 is M's expected payoff when she only monitors one project. Then one of the projects succeeds with probability \underline{q} while the other project succeeds with probability q. Similarly, the right-hand side of IC2 is equal to M's payoff when she does not monitor any of the projects IC3 is the incentive compatibility constraint that ensures that when only one project is type hM prefers not to monitor at all. On the left-hand side of the weak inequality we have M's payoff when she does not monitor the type h project in which case the probability of success of each project equals \underline{q} . On the right hand-side of the weak inequality we have the same expression as those on the right-hand side of the weak inequality in IC1, showing M's net expected payoff when she monitors the type h project. Lastly, the solution must also satisfy the participation constraint of the investors. At date 0, with probability p^2 both projects are type h, M monitors both of them, and each project succeeds with probability q; with probability $1 - p^2$ at least one project is type h, M does not monitor any project, and each project succeeds with probability \underline{q} . Thus, the probability that only one project succeeds is equal to $2[p^2q(1-q) + (1-p^2)\underline{q}(1-\underline{q})]$, while the probability that both projects succeed is equal to $\tau \equiv p^2q^2 + (1-p^2)\underline{q}^2$. Then, the right-hand side of PC1 shows the expected payoff of investors when M monitors only when both projects are type h, in which case she monitors both projects.

Proposition 1 Suppose that the two entrepreneurs would like M to monitor only when both projects are of type h and let $c_{\Delta} \equiv \frac{c}{q-q}$. Then,

(i) Finance is feasible if and only if $c_{\Delta} \leq \min\left(\frac{q+\underline{q}}{\tau}(p_z R-1), R-\frac{1-(q+\underline{q})}{p_z-\tau}\right)$.

(ii) If finance is feasible then

(a) if $c_{\Delta} \leq \frac{q+q}{p_z-\tau}$ the optimal contract is given by

$$r_1 = m^Z = \frac{1}{p_z} + \frac{\tau}{p_z(q+\underline{q})}c_{\Delta};$$

$$r_2 = \frac{2}{p_z} - \frac{2c_{\Delta}}{q+\underline{q}}\frac{p_z - \tau}{p_z} \ge r_1,$$

(b) if $c_{\Delta} > \frac{q+q}{p_z-\tau}$ the optimal contract is given by

$$r_1 = \frac{1}{p_z - \tau}; r_2 = 0;$$

 $m^Z = \frac{1 - q - q}{p_z - \tau} + c_\Delta.$

Proof. See the Appendix.

Given that the two entrepreneurs would like M to monitor only when both projects are type h optimality requires that M's payoff is minimized when only one project succeeds and concentrated on that state when both projects succeed. Given m^Z , the asset asside of M's balance sheet, this leads to a liability contract that takes away M's revenue in states when only one project succeeds, namely $r_1 = m^Z$. Then, r_2 is pinned down by the investors' participation constraint. This type of contract is called 'live or die' and was originally derived by Innes (1990).⁷ In case (a) the payoff to investors is constrained by the limited liability of M. The term $\frac{\tau}{p_z(q+q)} < 1$ captures the cross-pledging effect

⁷See also Tirole (p.133, 1996).

(see Tirole, 2006, p.159). For p = 1, the term reduces to $\frac{q}{q+q}$, which is the same as in Laux (2001) and Tirole (2006). The complexity here is due to the uncertainty at date 0 about the type of each of the two projects. Introducing this uncertainty is important for our work since, as we will show in the following section, it allows for multiple financial intermediation organization structures. When $c_{\Delta} \leq \frac{q+q}{2p_z-\tau} < \frac{q+q}{p_z-\tau}$ the liability contract is debt with face value r_2 and the intermediary defaults when at least one project fails, otherwise, MC is violated. This is because when c_{Δ} is sufficiently high, which in turn implies that r_1 is high, the investors' participation constraint requires that we set a value for r_2 that is lower than r_1 , in which case MC is not satisfied. In case (b) the contract is constrained by the condition that the payoff of investors cannot be negative and the contract always violates MC. The following proposition characterizes the optimal contract for the case when MC must be satisfied.

Proposition 2 Suppose that the two entrepreneurs would like M to monitor only when both projects are type h and let $c_{\Delta} \equiv \frac{c}{q-q}$. In addition, suppose that contacts must satisfy MC. Then,

- (i) Finance is feasible if and only if $c_{\Delta} \leq \min\left(\frac{q+q}{\tau}(p_z R-1), R-\frac{2-(q+q)}{2p_z-\tau}\right)$.
- (ii) If finance is feasible then

(a) if $c_{\Delta} \leq \frac{q+q}{2p_z-\tau}$ the optimal contract is given by

$$r_1 = m^Z = \frac{1}{p_z} + \frac{\tau}{p_z(q+\underline{q})}c_\Delta;$$

$$r_2 = \frac{2}{p_z} - \frac{2c_\Delta}{q+\underline{q}}\frac{p_z-\tau}{p_z} \ge r_1,$$

(b) if $c_{\Delta} > \frac{q+q}{2p_z-\tau}$ the optimal contract is given by

As the liability contract of M is debt with face value r_2 , M is organized as a Bank.

Proof. See the Appendix. \blacksquare

When $c_{\Delta} > \frac{q+q}{2p_z-\tau}$ MC is binding and the liability contract now changes to debt. The intermediary defaults only when both projects fail.

Figure 1 shows the optimal contract in the (c_{Δ}, R) plane, where $\alpha_B \equiv \frac{2 - (q + \underline{q})}{2p_z - \tau}$,



Figure 1: The optimal contract if M monitors only when both projects are type h

To the left of the kinked bold line the combinations of low project returns R and high agency costs c_{Δ} imply that finance is not feasible. Above the kink, that is for sufficiently high agency costs, the contract design is further restricted by MC. Ignoring the MC constraint, the optimal contract requires that $r_2 < r_1$ which violates monotonicity. Notice that even if monitoring costs are equal to zero unless $R \ge \frac{1}{p_z}$ investors cannot break even.

3.2 Case 2: M Always Monitors When A Project Is type h

Now, consider the case when the two entrepreneurs would like M to monitor a project whenever it is type h. In this case, each project's *ex ante* success possibility is equal to p_s . Now, they solve the following problem:

Problem 2 min m subject to:

IC1, IC2,
IC4:
$$\underline{q}(1-q)(m-r_1) + q(1-\underline{q})(m-r_1) + q\underline{q}(2m-r_2) - c$$

 $\geq 2\underline{q}(1-\underline{q})(m-r_1) + \underline{q}^2(2m-r_2)$
PC2: $p_s(1-p_s)2r_1 + p_s^2r_2 \geq 2$
 $r_1 \leq m \leq R \text{ and } r_2 \leq 2m$

The incentive compatibility constraints IC1 and IC2 are common to both problems, given that once more the two entrepreneurs would like M to monitor both projects when both are type h. However, in this new problem the two entrepreneurs would like M to monitor whenever a project is type h, independently of the type of the other project, and, therefore, IC4 is obtained from IC3 by reversing the direction of the weak inequality. The participation constraint is also similar to that of Problem 1; the only difference is that a project's *ex ante* probability of success is now higher $(p_s > p_z)$.

Proposition 3 Suppose that M monitors every type h project and let $c_{\Delta} \equiv \frac{c}{q-q}$. Then,

(i) If $\frac{q}{q} < \frac{1+p}{p}$ then finance is feasible if and only if $c_{\Delta} \leq \min\left(\frac{2q}{p_s}\left(R - \frac{1}{p_s}\right), R - \frac{1-2q}{p_s(1-p_s)}\right)$. When finance is feasible

(a) if $c_{\Delta} \leq \frac{2q}{p_s(1-p_s)}$ the optimal contract is given by

$$r_1 = m^S = \frac{1}{p_s} + \frac{p_s}{2\underline{q}}c_{\Delta};$$

$$r_2 = \frac{2}{p_s} - \frac{1 - p_s}{\underline{q}}c_{\Delta}.$$

(b) if $c_{\Delta} > \frac{2q}{p_s(1-p_s)}$ the optimal contract is given by

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$$r_1 = \frac{1}{p_s(1-p_s)}; r_2 = 0;$$

$$n^s = c_\Delta + \frac{1-2\underline{q}}{p_s(1-p_s)}.$$

(ii) If $\frac{q}{\underline{q}} \ge \frac{1+p}{p}$ then finance is feasible if and only if $c_{\Delta} \le R - \frac{1}{p_s}$. When finance is feasible the optimal contract is given by

$$r_1 = \frac{r_2}{2} = \frac{1}{p_s};$$
$$m^S = c_\Delta + \frac{1}{p_s}.$$

As the liability contract of M is equity, M is organized as a Fund.

Proof. See the Appendix.

In part (i) of Proposition 3, MC is not necessarily satisfied. More specifically, for part (i-a) when $c_{\Delta} \leq \frac{2q}{2p_s - p_s^2} < \frac{2q}{p_s(1-p_s)}$ the liability contract is debt with face value r_2 and the intermediary defaults when at least one project fails, otherwise, MC is violated. For part (i-b) the contract is constrained by the condition that the payoff of investors cannot be negative and the contract always violates MC. The following proposition characterizes the optimal contract for the case when MC must be satisfied.

Proposition 4 Suppose that M monitors every type h project and let $c_{\Delta} \equiv \frac{c}{q-q}$. In addition, suppose that contacts must satisfy MC. Then,

(i) If $\frac{q}{q} < \frac{1+p}{p}$ then finance is feasible if and only if $c_{\Delta} \leq \min\left(\frac{2q}{p_s}(R-\frac{1}{p_s}), R-\frac{2(1-q)}{2p_s-p_s^2}\right)$. When finance is feasible

(a) if $c_{\Delta} \leq \frac{2q}{2p_s - p_s^2}$ the optimal contract is given by

$$r_1 = m^S = \frac{1}{p_s} + \frac{p_s}{2\underline{q}}c_{\Delta};$$

$$r_2 = \frac{2}{p_s} - \frac{1 - p_s}{\underline{q}}c_{\Delta}.$$

(b) if $c_{\Delta} > \frac{2q}{2p_s - p_s^2}$ the optimal contract is given by

$$r_1 = r_2 = \frac{2}{2p_s - p_s^2};$$

$$m^s = c_\Delta + \frac{2(1-q)}{2p_s - p_s^2}.$$

As the liability contract of M is debt, M is organized as a Bank.

(ii) If $\frac{q}{\underline{q}} \ge \frac{1+p}{p}$ then finance is feasible if and only if $c_{\Delta} \le R - \frac{1}{p_s}$. When finance is feasible the optimal contract is given by

$$r_1 = \frac{r_2}{2} = \frac{1}{p_s};$$

 $m^S = c_\Delta + \frac{1}{p_s}.$

As the liability contract of M is equity, M is organized as a Fund.

Proof. See the Appendix.

The intuition behind the switch in the liability contract stated in Propositions 3 and 4 is as follows. First, on the asset side, M's revenue, conditional on the number of successful projects, is given by (0, m, 2m). The payment to M is equal to the difference between this revenue and his liability obligation, $(0, r_1, r_2)$, which is given by $(0, m - r_1, 2m - r_2)$. Therefore, the design of the liability contract matters for M's incentives to monitor. Second, in the present case the entrepreneurs want M to monitor an h-type project even if the other project is a l-type, which succeeds with probability \underline{q} . This requires that M is paid even if only one prject is successful. The lower is \underline{q} , the higher the payment to M must be when only one project succeeds. For example, if $\underline{q} = 0$, namely, if the l-type project never has a chance to succeed, M has an incentive to monitor the other h-type project only if she receives a apyment in those states when only one project succeeds (i.e. $m - r_1 > 0$). This consideration has a negative effect on r_1 and a positive effect on r_2 (in order to make the investors break even), and thus we get the equity contract. Therefore, the lower is \underline{q} , the more likely is that the the equity contract is optimal. This explains why the switch between debt and equity contracts for the present case depends on the ratio q/q. This also explains why in the previous case, where the entrepreneurs want M to monitor an h-type project only if the other one is also an h-type, this switch is not there: the other project succeeds with a relatively greater probability, q, and thus the concern that drives the equity contract does not arise.

3.3 Equilibrium Organization Structure

The equilibrium structure of financial intermediation is decided by the two entrepreneurs since they have all the bargaining power. For $\frac{q}{q} \leq \frac{1+p}{p}$ the optimal intermediation structure, if it exists, is always Bank. For $\frac{q}{q} > \frac{1+p}{p}$ and when both intermediation structures are feasible, we need to compare Proposition 2 with Proposition 4. In case 1, with probability p^2 both projects are type h and thus M monitors both of them, which implies expected profits $q(R - m^Z)$ for each entrepreneur given that each project succeeds with probability q. With probability $(1 - p^2)$ at least one project is bad and M monitors does not monitor at all. Then, each project succeeds with probability \underline{q} , which implies that each entrepreneur's utility is given by $\underline{q}(R - m^Z)$. Therefore, in case 1 each entrepreneur's expected utility is equal to $V^Z = p^2 q(R - m^Z) + (1 - p^2)(\underline{q}(R - m^Z)) = p_z(R - m^Z)$. Similarly, in case 2, each entrepreneur's expected utility is equal to $V^S = pq(R - m^S) + (1 - p)(\underline{q}(R - m^S)) = p_s(R - m^S)$. By comparing the expressions for the two expected utilities and restricting attention to parameter configurations such that at least one organization structure is feasible we arrive at the following result:

Theorem 1 Equilibrium Organization Structure:

- (1) Suppose that $\frac{q}{q} \leq \frac{1+p}{p}$. The equilibrium structure is Bank.
- (2) Suppose that $\frac{q}{\underline{q}} > \frac{1+p}{p}$. Then,
 - (a) if $qq > (p_s p_z) (2 (q + q))$ then
 - (i) Fund is the only feasible organization structure if

$$c_{\Delta} \leq R - \frac{1}{p_s}$$
 and $\frac{(p_s - p_z)(q + \underline{q}) + q\underline{q}}{p_s q\underline{q}} > R;$

(ii) Bank is the only feasible organization structure if

$$c_{\Delta} \leq \min\left(R - \frac{2 - (q + \underline{q})}{2p_z - \tau}, \frac{q + \underline{q}}{\tau}(p_z R - 1)\right) and \frac{(p_s - p_z)(q + \underline{q}) + q\underline{q}}{p_s q\underline{q}} \leqslant R;$$

(iii) when both Bank and Fund are feasible Bank dominates Fund if

$$\min\left(\frac{(p_s - p_z)(q + \underline{q}) + qq}{(p_s - p_z)(q + \underline{q})}c_{\Delta}, c_{\Delta} + \frac{q\underline{q}}{(p_s - p_z)(2p_z - \tau)}\right) > R$$

otherwise, Fund dominates Bank.

(b) If $q\underline{q} \leq (p_s - p_z) \left(2 - (q + \underline{q})\right)$ then the equilibrium structure is Fund; either it is the only feasible organization structure or it dominates Bank.

Proof. See the Appendix.

Figure 2 shows for the case $q\underline{q} > (p_s - p_z)[2 - (q + \underline{q})]$ and for each pair (c_{Δ}, R) whether finance is feasible and, if so, the equilibrium financial intermediation structure.



Figure 2: The equilibrium structure for Theorem (2a)

We can better understand the trade-off between the two organization structures by comparing M's corresponding expected payoffs. We concentrate our discussion on the case $\frac{q}{q} > \frac{1+p}{p}$ where a trade-off between the two structures exists. We know that in both cases investors break even. M's expected payoffs are given by $U^B = \frac{2(p^2q^2+(1-p^2)q^2)}{q+q}\frac{c}{q-q}$ under Bank and $U^F = \frac{2p_sc}{q-q}$ under Fund. It is straightforward to show that $U^B < U^F$.⁸ The intuition behind this inequality is that the cost of providing incentives to M is lower under Bank. The optimality of debt contracts in the presence of moral hazard is well known in the literature (e.g. Innes,1990; Laux, 2001; Tirole, 2006). This is a consequence of the 'maximum incentive principle', which says that the agent shall receive a positive payment only when all the informative signals display the values indicating she has chosen high effort.⁹ Therefore, M

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$$\begin{split} sign\left\{U^F - U^B\right\} &= sign\left\{pq + (1-p)\underline{q} - \frac{2[p^2q^2 + (1-p^2)\underline{q}^2]}{q+\underline{q}}\right\} = \\ sign\left\{(1-p)p\left(q^2 - \underline{q}^2\right) + q\underline{q}\right\} \end{split}$$

which is clearly positive.

⁹See Laffont and Martimont (2003) and Bolton and Dwatripont (2005).

should receive a payoff only when both projects succeed. By introducing uncertainty about the project's type this paper finds a disadvantage of debt contracts. The very feature that enables them to provide incentives at a lower cost makes them less robust to bad news. To see this, consider the extreme case where $\underline{q} = 0$, where according to the theorem the equilibrium structure is Fund. Suppose that only one project is type h. Under the Bank structure, as there is at most one successful project, M's payoff will be 0 if she monitors the type h project and thus she does not monitor at all. One piece of bad news is sufficient to destroy M's incentives to monitor. In contrast, under the Fund structure M monitors the only type h project and receives a share of the output. We conclude that while it is cheaper to provide incentives by opting for the Bank structure, the alternative Fund structure is more robust to the ralization of negative shocks.

3.4 Intermediate versus Direct Finance

Thus far, we have assumed that M is a financial intermediary that provides both the monitoring service and transfers funds from investors to entrepreneurs. But is it possible that an arrangement of direct finance, where M only provides the monitoring service, can do better? Below we show that this is never the case and moreover if contracts are restricted to be bilateral as in the case of financial intermediation, direct finance does worse.

Proposition 5 Suppose that the contract between an entrepreneur and M can only be conditioned on that entrepreneur's project's outcome. Then M monitors a project whenever it is type h. This allocation can be implemented by financial intermediation with a Fund arrangement.

Proof. See the Appendix.

With bilateral contracts M's payoff is equal to 0 when both projects fail, m when only one project succeeds and 2m when both projects succeed. This compensation structure generates incentives to monitor whenever a project is of type h. Thus, under the direct mechanism each entrepreneur hires an M (not necessarily the same) as a director at a wage $\frac{c}{q-q}$ and issues equity with return rate $\frac{1}{p_s}$ directly to investors. Clearly, the Fund is indistinguishable from direct finance. There are two complications with the direct mechanism. Firstly, investors would be vulnerable to the possibility that the entrepreneur terminates the contract with M. In contrast, under the Fund structure by bundling the monitoring with the financing activities this possibility does not arise. This is because while a firm can find an excuse to fire a director, it cannot fire a creditor before it clears its debt obligations to him. Secondly, the Fund has the advantage of a more efficient governance structure. Irrespectively, of the organization structure, investors will not commit their funds unless they are convinced that entrepreneurs will be monitored. Under direct finance this can only be achieved if investors oversee all entrepreneurs separately to ensure that their activities are monitored. Put differently, they depend on the governance structure of each entrepreneur's firm. In contrast, under the Fund structure investors can rely on the governance structure of only one firm, namely, the Fund.

The following result is a direct consequence of the above proposition.

Corollary 1 Consider parameter values such that, according to Theorem 1, Bank dominates Fund. Then, financial intermediation dominates direct mechanism.

The advantage of the Bank structure is its ability to perform asset transformation. The only way to implement the Bank solution through a direct mechanism is by using multilateral mechanisms. that allow for contracts between one entrepreneur and M that are contingent on the outcome of the other entrepreneur's project. This imposes a stronger requirement on the information of the relevant parties. What happens with multi-lateral contracting is discussed below.

4 Discussion

In this section we (a) examine the implications of diversification for the choice of intermediation structure, (b) consider a more general contracting environment in order to consider the robustness of our solutions, and (c) discuss the empirical relevance of our model.

4.1 Diversification and Internal Control

Up to this point, we have assumed that there are only two entrepreneurs. What happens if the financial intermediary M can fully diversify its assets as the number of the entrepreneurs goes to infinity? Full diversification offers the Bank structure some advantages because (a) under the Bank structure M's profits are zero while under the Fund structure M earns positive profits, and (b) there is no aggregate uncertainty about the portfolio return which cancels the advantage that Fund has. Does that consideration of full diversification destroy the trade-off between the two structures considered above?

The answer is negative if internal control problems are associated with large enterprises. In our analysis above, the problem is assumed away given that M is able to monitor the two projects by herself. When there are large number of projects, M would need to delegate the monitoring activities to others. Given that monitoring is not observable by third parties, M faces the problem of monitoring these monitors. Define as "internal control cost" the cost incurred by M to ensure that a monitor incurs the cost c by monitoring the assigned project. The following proposition makes clear that the Bank structure suffers more from internal control problems than the Fund structure does.

Proposition 6 If each monitor can only monitor one project and the internal control cost is larger than c, then the only equilibrium structure is Fund.

Proof. See the Appendix.

The reason that Bank suffers more from internal control problems is rooted in the very feature that enables it to save on incentive costs. Under the Bank structure, each monitor imposes some negative externality, "cross pledging", upon other monitors given that failure to monitor on his part reduces their expected income. It is exactly because of this externality that Bank saves on incentive costs relative to Fund. Thus, if each monitor's behavior is not controlled to internalize the externality, cross pledging would not work and the Bank structure would collapse. In contrast, the Fund does not incur internal control costs since each monitor's contract can be arranged independently with each monitor obtaining a positive share of the output from the project she monitors.

Thus, our main conclusions about the trade-off between the two main financial intermediation structures is robust to the consideration of full diversification since internal control problems limit the Bank's capacity for diversification.

4.2 Generalizing the Contractual Environment

One restriction that we have imposed on contract design is that each entrepreneur can condition the terms of his agreement with M only on the outcome of his own project. This seems to be reasonable given the potential high costs of information gathering associated with conditioning the terms of each contract on the outcomes of other projects. In any case, the following proposition demonstrates that the equilibrium structures derived above are also solutions to the contracting problem where no such restriction is imposed as long as monitoring costs are not too high. **Proposition 7** As longs as $c_{\Delta} \leq \min\left(\frac{q+q}{2p_z-\tau}, \frac{2q}{p_s(2-p_s)}\right)$, alloing from ultilateral contracting does not change the equilibrium allocation.

Proof. See the Appendix.

What gives rise to the constraint in the statement of Proposition 7 is the requirement that the optimal mechanism satisfies MC. However, from Proposition 5 we know that any mechanism, direct or intermediated, will be similarly restricted if (a) MC must be satisfied, and (b) the contracts between each entrepreneur and M cannot be conditioned on the outcomes of both projects.

4.3 Empirical Predictions

Our model yields a number of empirical predictions about (a) the ability of each organization structure to raise pledgeable income, (b) the relationship between firm-specific characteristics and the two sources of funds, and (c) the operations of the two organization structures.

Prediction 1 Keeping profitability constant Bank is more likely to dominate when monitoring costs are high and the payoff to monitoring is low.

The payoff to monitoring is directly related to the difference $q - \underline{q}$ which, as in Holmström and Tirole (1997), is a measure of agency costs. By monitoring good quality projects the onitor increases their probability of success from q to \underline{q} . Clearly, as the difference between these two probabilities declines the payoff to monitoring declines too. Then the prediction follows directly from Theorem 1 by noticing that c_{Δ} is increasing in the size of the monitoring cost c and decreasing in q - q.

Prediction 2 Keeping monitoring costs and the payoff to monitoring constant Fund is more likely when profitability is high.

Our next prediction follows directly from Predictions 1 and 2.

Prediction 3 The Fund structure is more attractive for firms in high-tech innovative sectors and startup firms.

Our model predicts that firms with high q, low $q - \underline{q}$ and low R, which, according to evidence provided by Dunne, Roberts and Samuelson (1988), these are the characteristics of mature firms, are

Measuring profitability by R the prediction follows directly from Figure 2.

more likely to be financed by banks. In contrast, according to Sahlman (1990) the profitability of young firms in high-risk innovative sectors is very strong conditional on survival, and also these are the firms more likely to be financed by venture capitalists.

Prediction 4 Fund structured intermediaries monitor more intensively that Bank structured intermediaries.

This prediction is an immediate consequence of Propositions 1 and 2. When it is optimal to monitor only when the quality of both projects are type h we find that the optimal organization structure is Bank. In contrast, as long as the incentives for monitoring are not too low, when it is always optimal to monitor a type h project the optimal organization structure is Fund. In our model monitoring takes the form of active participation in decision-making which is consistent with the activities of equity-based funs and, as Gompers (1995) and Sahlman (1990) observe, especially those of venture capitalists. In contrast, bank monitoring is more related to screening and information gathering which are outside the scope of this paper.

5 Conclusion

We have used a mechanism design approach to solve a delegated monitoring problem and we have found that the organization structure of financial intermediation can take one of two forms, namely, Bank or Fund. For each structure we have derived both their asset and liability contracts and then by comparing the solutions we have identify their advantages and disadvantages. The trade-off between the two structures is that it is cheaper to offer incentives a monitor using the Bank option but the Fund alternative is more robust to the arrival of bad news. The model is quite simple, but is still rich enough to explain many empirical regularities. For example, we find that Fund is more likely than Bank in financing start-up firms and firms in innovative industries, and Fund, on average, monitors more intensively its clients than Bank does.

The simplicity of the model triggers the question about the theory's robustness and relevance. We have found that our results are robust to generalizing the contracting environment and to increasing the number of projects. We have demonstrated that the Bank organization structure cannot be replicated by bilateral relationships and that the same is true for Fund as long as multilateral contracting is costly.

Financial intermediaries channel trillions of funds from investors to entrepreneurs providing various

services to their customers. There is a lot of progress made in understanding their advantages over direct finance. However, there are still questions about them that are not very well understood. Do the providers of the monitoring services considered in this paper need to be organized as intermediaries? Given that these providers are organized as intermediaries what determines their organization structure? Put differently, why private equity funds and commercial banks coexist? This paper is an attempt to further understand these interesting problems.

6 Appendix

6.1 Proof of Proposition 1

Let m^Z denote the optimal symmetric solution for m. Constraint IC1 can be written as:

$$(m^{Z} - r_{1}) (2q (1 - q) - \underline{q} (1 - q) - q (1 - \underline{q})) + (2m^{Z} - r_{2}) (q^{2} - q\underline{q}) \ge c \iff$$

$$(m^{Z} - r_{1}) (q - \underline{q}) (1 - 2q) + (2m^{Z} - r_{2}) q (q - \underline{q}) \ge c \iff$$

$$m^{Z} - ((1 - 2q) r_{1} + qr_{2}) \ge c_{\Delta}$$
(A1)

IC2 can be written as:

$$2(m^{Z} - r_{1})(q(1 - q) - \underline{q}(1 - \underline{q})) + (2m^{Z} - r_{2})(q^{2} - \underline{q}^{2}) \ge 2c \iff$$

$$2(m^{Z} - r_{1})(1 - q - \underline{q})(q - \underline{q}) + (2m^{Z} - r_{2})(q + \underline{q})(q - \underline{q}) \ge 2c \iff$$

$$m^{Z} - \left((1 - q - \underline{q})r_{1} + \frac{q + \underline{q}}{2}r_{2}\right) \ge c_{\Delta} \qquad (A2)$$

PC1 can be written as:

$$[p^2q(1-q) + (1-p^2)\underline{q}(1-\underline{q})]2r_1 + [p^2q^2 + (1-p^2)\underline{q}^2]r_2 \ge 2$$
(A3)

We can then write Problem 1 as:

$$\min_{m^{Z}, r_{1}, r_{2}} m^{Z}, \text{ s.t.}(A1), (A2), \text{ IC3 and } (A3); \text{ and} \\ 0 \le r_{1} \le m^{Z} \le R, \ 0 \le r_{2} \le 2m^{Z}.$$

We prove the following results:

Lemma 1 If IC2 (A2) is binding then IC3 is not binding.

Proof. The lemma follows by setting equal the two sides of (A2) and then substitutiong the right-hand side for the left-hand side of (A1). QED \blacksquare

Lemma 2 (A2) is binding.

Proof. First, notice that minimization of m^Z implies that either (A1) or (A2) must be binding. Second, notice that (A2) implies (A1) if and only if

$$(1-q-\underline{q})r_1 + \frac{q+\underline{q}}{2}r_2 \ge (1-2q)r_1 + qr_2 \iff (q-\underline{q})r_1 \ge \frac{q-\underline{q}}{2}r_2 \iff r_1 \ge \frac{r_2}{2}.$$

And vice versa. Thus, if (A1) is binding, contrary to the lemma, then $r_1 \leq \frac{r_2}{2}$. In this case, $m^Z = ((1-2q)r_1 + qr_2) + c_{\Delta}$ and the problem of the two entrepreneurs is equivalent to:

min
$$H^1$$
 = $(1-2q)r_1 + qr_2$ subject to (A3) and
 $0 \leq r_1 \leq m^Z \leq R, 0 \leq r_2 \leq 2m^Z$.

Certainly (A3) is binding and it follows that $dr_2 = -\frac{2[p^2q(1-q)+(1-p^2)\underline{q}(1-\underline{q})]}{p^2q^2+(1-p^2)\underline{q}^2}dr_1$. Then,

$$dH^{1} = \left[(1-2q) - q \frac{2[p^{2}q(1-q) + (1-p^{2})\underline{q}(1-\underline{q})]}{p^{2}q^{2} + (1-p^{2})\underline{q}^{2}} \right] dr_{1}.$$

Consider the expression in the square brackets

$$\begin{split} 1 - 2q &< q \frac{2[p^2 q (1 - q) + (1 - p^2) \underline{q} (1 - \underline{q})]}{p^2 q^2 + (1 - p^2) \underline{q}^2} \iff \\ p^2 q^2 + (1 - p^2) \underline{q}^2 &< 2q \left(p^2 q^2 + (1 - p^2) \underline{q}^2 + p^2 q (1 - q) + (1 - p^2) \underline{q} (1 - \underline{q}) \right) \iff \\ p^2 q^2 + (1 - p^2) \underline{q}^2 &< 2q \left(p^2 q + (1 - p^2) \underline{q} \right) \end{split}$$

where given that $q > \underline{q}$ the last inequality clearly holds. Thus $\frac{dH^1}{dr_1} < 0$ and the solution for this case is to set $r_1 = \frac{r_2}{2}$, which implies that the overall optimal solution for Problem 1 is in the region $r_1 \ge \frac{r_2}{2}$, that is, (A2) is binding. QED

With (A2) binding, $m^Z = (1 - q - \underline{q})r_1 + \frac{q+q}{2}r_2 + c_{\Delta}$ and the problem of the two entrepreneurs is equivalent to:

$$\min H^2 = (1 - q - \underline{q})r_1 + \frac{q + q}{2}r_2 \text{ subject to (A3) and}$$
$$0 \leq r_1 \leq m^Z \leq R, 0 \leq r_2 \leq 2m^Z.$$

Notice that the limited liability conditions imply:

$$0 \leq r_1 \leq (1 - q - \underline{q})r_1 + \frac{q + \underline{q}}{2}r_2 + c_\Delta \tag{A4}$$

$$0 \leq r_2 \leq 2(1 - q - \underline{q})r_1 + (q + \underline{q})r_2 + 2c_\Delta$$
 (A5)

Lemma 3 (a) If $c_{\Delta} \leq \frac{q+q}{p_z-\tau}$ then $r_1 = \frac{1}{p_z} + \frac{\tau}{p_z(q+q)}c_{\Delta}$, $r_2 = \frac{2}{p_z}(1 - \frac{p_z-\tau}{q+q}c_{\Delta})$, $m^Z = r_1$, and (b) if $c_{\Delta} \geq \frac{q+q}{p_z-\tau}$ then $r_1 = \frac{1}{p_z-\tau}$, $r_2 = 0$, $m^Z = \frac{1-q-q}{p_z-\tau} + c_{\Delta}$.

Proof. Once more the binding (A3) implies that $dr_2 = -\frac{2[p^2q(1-q)+(1-p^2)\underline{q}(1-\underline{q})]}{p^2q^2+(1-p^2)\underline{q}^2}dr_1$. Then, $dH^2 = (1-q-\underline{q})dr_1 + \frac{q+q}{2}dr_2 = \left[(1-q-\underline{q}) - \frac{q+q}{2}\frac{2[p^2q(1-q)+(1-p^2)\underline{q}(1-\underline{q})]}{p^2q^2+(1-p^2)\underline{q}^2}\right]dr_1$. Next, we show that the expression in the square brackets is negative.

$$\begin{split} 1 - q - \underline{q} &< \frac{q + \underline{q}}{2} \frac{2[p^2 q (1 - q) + (1 - p^2) \underline{q} (1 - \underline{q})]}{p^2 q^2 + (1 - p^2) \underline{q}^2} \iff \\ p^2 q^2 + (1 - p^2) \underline{q}^2 - (q + \underline{q}) [p^2 q^2 + (1 - p^2) \underline{q}^2] < (q + \underline{q}) [p^2 q (1 - q) + (1 - p^2) \underline{q} (1 - \underline{q})] \iff \\ p^2 q^2 + (1 - p^2) \underline{q}^2 < (q + \underline{q}) (p^2 q + (1 - p^2) \underline{q}) \iff -q \underline{q} < 0 \end{split}$$

where the last inequality clearly holds. Thus $\frac{dH^2}{dr_1} < 0$, which implies the entrepreneurs must set r_1 as high as possible. Therefore, the second inequality of (A4) is binding, unless the first inequality of (A5) $0 \le r_2$ is binding, given that from the binding (A3) we know that r_1 and r_2 are negatively related. Part (a) of the lemma follows when the first inequality of (A5) is not binding. Then, from the binding second inequality of (A4) and the binding (A3) we get the values of r_1 and r_2 which is the solution obtained when $r_2 \ge 0$, which will be the case if $c_{\Delta} \le \frac{q+q}{p_z-\tau}$. Part (b) of the lemma follows when the first inequality of (A5) is binding and thus $r_2 = 0$. Then it follows from the binding (A3) that $r_1 = \frac{1}{p_z-\tau}$. QED

Therefore, as long as finance is feasible, that is the limited liability condition of the two entrepreneurs is satisfied, the solution given in Lemma 3 is the solution to Problem 1. Now we are left to check the feasibility of finance, namely, that the constraint $m^Z \leq R$ is satisfied. Let $T_1 \equiv \frac{q+q}{p_z-\tau}$, $T_2 \equiv \frac{q+q}{\tau}(p_z R-1)$ and $T_3 \equiv R - \frac{1-q-q}{p_z-\tau}$. Then we have the following result:

Lemma 4 Finance is feasible if and only if $c_{\Delta} \leq \min(T_2, T_3)$.

Proof. If $c_{\Delta} \leq T_1$, then from Lemma 2 we have $m^Z = \frac{1}{p_z} + \frac{\tau}{p_z(q+q)}c_{\Delta}$. Therefore, $m^Z \leq R$ if and only if $c_{\Delta} \leq T_2$. If $c_{\Delta} \geq T_1$, then from Lemma 2 we have $m^Z = \frac{1-q-q}{p_z-\tau} + c_{\Delta}$. Therefore, $m^z \leq R$ if and only if $c_{\Delta} \leq T_3$. To complete the proof we consider the following two cases:

(a) If $R \leq \frac{1}{p_z - \tau}$ then $T_1 \geq T_3 \geq T_2$ which in turn implies that $T_1 > \max(T_2, T_3)$. In this case, if $c_{\Delta} \geq T_1$, then $c_{\Delta} > T_3$ and thus finace is not feasible. If $c_{\Delta} < T_1$, finance is feasible if $c_{\Delta} \leq T_2$. Therefore, if $R \leq \frac{1}{p_z - \tau}$ finance is feasible if and only if $c_{\Delta} \leq T_2$.

(b) If $R \ge \frac{1}{p_z - \tau}$ then $T_1 \le T_3 \le T_2$ which in turn implies that $T_1 \le \min(T_2, T_3)$. In this case, if $c_{\Delta} < T_1$, then $c_{\Delta} < T_2$. Thus finance is feasible. If $c_{\Delta} \ge T_1$, then finance is feasible if and only if $c_{\Delta} \le T_3$. Therefore, in this case, finance is feasible if and only if $c_{\Delta} \le T_3$.

Then the lemma follows from $T_3 \ge T_2$ if and only if $R \le \frac{1}{p_z - \tau}$. QED

Proposition 1 follows directly from Lemma 3 and Lemma 4. QED

6.2 Proof of Proposition 2

We begin with the proof of part (ii). In the solution obtained in part (ii-a) of Proposition 1, MC (that is $r_2 \ge r_1$) is satisfied as long as $c_{\Delta} \le \frac{q+q}{2p_z-\tau}$ and this completes the proof of part (ii-a). To obtain the solution for part (ii-b) set $r_2 = r_1$ and then use (A3).

Next, we to check the feasibility of finance, namely, that the constraint $m^Z \leq R$ is satisfied. Let $T_1^* \equiv \frac{q+q}{2p_z-\tau}, T_2 \equiv \frac{q+q}{\tau}(p_z R - 1)$ and $T_3^* \equiv R - \frac{2-(q+q)}{2p_z-\tau}$. Then we have the following result:

Lemma 5 Finance is feasible if and only if $c_{\Delta} \leq \min(T_2, T_3^*)$.

Proof. Suppose that $c_{\Delta} \leq T_1^*$, that is MC is satisfied. Then as in Proposition 1 $m^Z \leq R$ requires that $c_{\Delta} \leq T_2$. If $c_{\Delta} \geq T_1^*$, then $r_2 = r_1$ and we have $m^Z = c_{\Delta} + \frac{2-(q+q)}{2p_z - \tau}$. It follows that $m^z \leq R$ if and only if $c_{\Delta} \leq T_3^*$. To complete the proof we observe that:

(a) If $R \ge \frac{2}{2p_z - \tau}$ then $T_1^* \le T_3^* \le T_2$ and (b) if $R \le \frac{2}{2p_z - \tau}$ then $T_1^* \ge T_3^* \ge T_2$. The proof is completed by following exactly the same steps as those used for the proof of Lemma 4. QED

This completes the proof of Proposition 2. QED

6.3 Proof of Proposition 3

Let m^S denote the optimal symmetric solution for m. Constraints (A1) and (A2) must also be satisfied by the solution of Problem 2. We also need to add IC4 which can be written as

$$\left(\underline{q}\left(\underline{q}-q\right)+\left(1-\underline{q}\right)\left(q-\underline{q}\right)\right)\left(m^{S}-r_{1}\right)+\underline{q}\left(q-\underline{q}\right)\left(2m^{S}-r_{2}\right) \geq c \iff \left(1-2\underline{q}\right)\left(m^{S}-r_{1}\right)+\underline{q}\left(2m^{S}-r_{2}\right) \geq c_{\Delta} \iff m^{S}-\left(1-2\underline{q}\right)r_{1}+\underline{q}r_{2} \geq c_{\Delta}.$$
(A6)

And PC2 which can be written as:

$$2p_s(1-p_s)r_1 + p_s^2 r_2 \ge 2 \tag{A7}$$

The problem of the two entrepreneurs is to

$$\min_{m^{Z}, r_{1}, r_{2}} m^{S}, \text{ s.t.}(A1), (A2), (A6) \text{ and } (A7); \text{ and} 0 \le r_{1} \le m^{S} \le R, \ 0 \le r_{2} \le 2m^{S}.$$

Lemma 6 (A6) is binding.

Proof. Notice that (A6) implies (A2) if and only if

$$(1 - 2\underline{q}) (m^{S} - r_{1}) + \underline{q}(2m^{S} - r_{2}) \le (1 - q - \underline{q})(m^{S} - r_{1}) + \frac{q + \underline{q}}{2}(2m^{S} - r_{2}) \iff (q - \underline{q}) (m^{S} - r_{1}) \le \frac{q - \underline{q}}{2}(2m^{S} - r_{2}) \iff r_{1} \ge \frac{r_{2}}{2}.$$

Thus, if $r_1 \ge \frac{r_2}{2}$, then (A6) implies (A2) which, in turn, implies (A1) and vice versa. We first consider the case $r_1 \le \frac{r_2}{2}$. Following a similar argument as the one used for the proof of Proposition 1, we find that the problem of the two entrepreneurs is equivalent to

$$\min H^1 = (1 - 2q)r_1 + qr_2$$
 subject to (A7)

From the binding (A7) it follows that $dr_2 = -\frac{2(1-p_s)}{p_s}dr_1$. Then $dH^2 = \left[(1-2q) - q\frac{2(1-p_s)}{p_s}\right]dr_1$. Next, we show that the expression in the square brackets is negative.

$$1 - 2q < q \frac{2(1 - p_s)}{p_s} \iff p_s < 2q$$

which clearly holds. Thus $\frac{dH^2}{dr_1} < 0$ for $r_1 \le \frac{r_2}{2}$ and the solution for this case is $r_1 = \frac{r_2}{2}$. Therefore, the optimal solution for Problem 2 must lie in the region $r_1 \ge \frac{r_2}{2}$. This implies (A6) is binding. QED Therefore

Therefore,

$$m^{S} = (1 - 2q)r_1 + qr_2 + c_{\Delta}$$

and the problem of the two entrepreneurs is to

$$\min H^3 = (1 - 2q)r_1 + qr_2$$
 subject to (A7)

Lemma 7 (a) If $\frac{q}{\underline{q}} > \frac{1+p}{p}$ then $r_1 = \frac{1}{p_s}$, $r_2 = \frac{2}{p_s}$, $m^S = c_{\Delta} + \frac{1}{p_s}$, and

(b) if
$$\frac{q}{\underline{q}} \leq \frac{1+p}{p}$$
 then
(i) if $c_{\Delta} \leq \frac{2\underline{q}}{p_s(1-p_s)}$ then $r_1 = m^S = \frac{1}{p_s} + \frac{p_s}{2\underline{q}}c_{\Delta}$, $r_2 = \frac{2}{p_s} - \frac{1-p_s}{\underline{q}}c_{\Delta}$, and
(ii) if $c_{\Delta} > \frac{2\underline{q}}{p_s(1-p_s)}$ then $r_1 = \frac{1}{p_s(1-p_s)}$, $r_2 = 0$, $m^s = c_{\Delta} + \frac{1-2\underline{q}}{p_s(1-p_s)}$.

Proof. From the binding (A7) we get $dH^3 = \left[(1-2\underline{q}) - \underline{q}\frac{2(1-p_s)}{p_s}\right]dr_1$. The expression in the square brackets has the same sign as:

$$p_s - 2\underline{q} = pq + (1-p)\underline{q} - 2\underline{q} = \frac{q}{\underline{q}} - \frac{1+p}{p} \ge 0$$

Once more, we need to consider two cases:

(a) Suppose that $\frac{q}{q} > \frac{1+p}{p}$ which implies that $\frac{dH^3}{dr_1} > 0$. Now it is optimal to set r_1 as small as possible, which implies that $r_1 = \frac{r_2}{2}$. All constraints are equivalent and binding. Using (A7) we get $r_1 = \frac{1}{p_s}$. Then $r_2 = \frac{2}{p_s}$ which, in turn, implies that $m^S = c_{\Delta} + \frac{1}{p_s}$.

(b) Suppose that $\frac{q}{\underline{q}} \leq \frac{1+p}{p}$. Then $\frac{dH^3}{dr_1} < 0$ in which case it is optimal to set r_1 as large as possible, however, because of limited liability, we get $r_1 = m^S$. However, this is only feasible if the constraint $0 \leq r_2$ is not binding, that is $c_\Delta \leq \frac{2q}{p_s(1-p_s)}$. From the binding (A6) and (A7), we obtain the solution to part (b-i) of the lemma. If the constraint $0 \leq r_2$ is binding, that is $c_\Delta > \frac{2q}{p_s(1-p_s)}$, part (b-ii) of the lemma, then we set $r_2 = 0$ and from the binding (A7) we get $r_1 = \frac{1}{p_s(1-p_s)}$ and by substituting these solutions into $m^S = (1 - 2\underline{q})r_1 + \underline{q}r_2 + c_\Delta$ we get $m^S = c_\Delta + \frac{1-2q}{p_s(1-p_s)}$.

This completes the proof of the lemma. QED \blacksquare

Therefore, as long as finance is feasible, that is the limited liability condition of the two entrepreneurs is satisfied, the solution given in Lemma 7 is the solution to Problem 2. Now we are left to check the feasibility of finance, namely, that the constraint $m^S \leq R$ is satisfied. Let $T_4 \equiv \frac{2q}{p_s(1-p_s)}, T_5 \equiv \frac{2q}{p_s}(R-\frac{1}{p_s})$ and $T_6 \equiv R - \frac{1-2q}{p_s(1-p_s)}$. Then we have the following result:

Lemma 8 (a) If $\frac{q}{q} > \frac{1+p}{p}$ then finance is feasible if and only if $c_{\Delta} \leq R - \frac{1}{p_s}$, and (b) if $\frac{q}{q} \leq \frac{1+p}{p}$ then finance is feasible if and only if $c_{\Delta} \leq \min(T_5, T_6)$.

Proof. (a) Given that $m^S = c_{\Delta} + \frac{1}{p_s} \leq R$, finance is feasible if and only if $c_{\Delta} \leq R - \frac{1}{p_s}$.

(b) If $c_{\Delta} \leqslant T_4$, in which case $m^S = \frac{1}{p_s} + \frac{p_s}{2q}c_{\Delta}$, finance is feasible if and only if $c_{\Delta} \le T_5$. If $c_{\Delta} > T_4$, in which case $m^S = c_{\Delta} + \frac{1-2q}{p_s(1-p_s)}$, finance is feasible if and only if $c_{\Delta} \leq T_6$. It is straightforward to check that if $R \leq \frac{1}{p_s(1-p_s)}$ then $T_4 \geq T_6 \geq T_5$ and if $R \leq \frac{1}{p_s(1-p_s)}$ then $T_4 \leq T_6 \leq T_5$. By following exactly the same steps as those used for the proof of Lemma 4 we can show that $c_{\Delta} \leq \min(T_5, T_6)$.

This completes true proof. QED $\hfill\blacksquare$

Proposition 3 follows directly from Lemma 7 and Lemma 8. QED

6.4 **Proof of Proposition 4**

Part (ii) of the proposition is identical to part (ii) of Proposition 3 given that MC is not binding. In the solution obtained in part (i-a) of Proposition 3, MC (that is $r_2 \ge r_1$) is satisfied as long as $c_{\Delta} \le \frac{2q}{2p_s - p_s^2}$ and this completes the proof of part (i-a). To obtain the solution for part (i-b) set $r_2 = r_1$ and then use (A7).

Next, we to check the feasibility of finance, namely, that the constraint $m^S \leq R$ is satisfied. Let $T_4^* \equiv \frac{2q}{2p_s - p_s^2}, T_5 \equiv \frac{2q}{p_s} (R - \frac{1}{p_s})$ and $T_6^* \equiv R - \frac{2(1-q)}{2p_s - p_s^2}$. Then we have the following result:

Lemma 9 Finance is feasible if and only if $c_{\Delta} \leq \min(T_5, T_6^*)$.

Proof. Suppose that $c_{\Delta} \leq T_4^*$, that is MC is satisfied. Then as in Proposition 3 $m^S \leq R$ requires that $c_{\Delta} \leq T_5$. If $c_{\Delta} \geq T_4^*$, then $r_2 = r_1$ and we have $m^S = c_{\Delta} + \frac{2(1-q)}{2p_s - p_s^2}$. It follows that $m^z \leq R$ if and only if $c_{\Delta} \leq T_6^*$. To complete the proof we observe that:

(a) If $\frac{2}{2p_s - p_s^2} \leq R$ then $T_5 \geq T_6^* \geq T_4^*$, and (b) if $\frac{2}{2p_s - p_s^2} > R$ then $T_5 < T_6^* < T_4^*$. The proof is completed by following exactly the same steps as those used for the proof of Lemma 4. QED

This completes the proof of Proposition 4. QED

Proof of Theorem 1 6.5

Part (1) follows directly from Propositions 2 and 4.

For the proof of part (2) we will use the following two results:

Lemma 10 $\frac{1}{p_s} < \frac{2 - (q + \underline{q})}{2p_z - \tau} \iff q \underline{q} < (p_s - p_z)(2 - (q + \underline{q}))$

Proof.

$$\frac{1}{p_s} < \frac{2 - (q + \underline{q})}{2p_z - \tau} \iff$$

$$2p_z - \tau < 2p_s - p_s(q + \underline{q}) \iff$$

$$p_s(q + \underline{q}) - \tau < 2(p_s - p_z) \iff$$

$$p_s(q + \underline{q}) - p_z(q + \underline{q}) + p_z(q + \underline{q}) - \tau < 2(p_s - p_z) \iff$$

$$(p_s - p_z)(q + \underline{q}) + q\underline{q} < 2(p_s - p_z) \iff$$

$$q\underline{q} < (p_s - p_z)(2 - (q + \underline{q}))$$

Q.E.D. ■

Lemma 11 $\frac{2}{2p_z - \tau} < \frac{(p_s - p_z)(q + \underline{q}) + q\underline{q}}{p_s q\underline{q}} \iff q\underline{q} < (p_s - p_z)(2 - (q + \underline{q}))$

Proof.

$$\begin{array}{lll} \displaystyle \frac{2}{2p_z - \tau} &< \displaystyle \frac{(p_s - p_z)(q + \underline{q}) + q\underline{q}}{p_s q\underline{q}} \Leftrightarrow \\ \\ \displaystyle 2p_s q\underline{q} &< \displaystyle (2p_z - \tau)[(p_s - p_z)(q + \underline{q}) + q\underline{q}] \Leftrightarrow \\ \\ \displaystyle 2(p_s - p_z)q\underline{q} &< \displaystyle (2p_z - \tau)(p_s - p_z)(q + \underline{q}) - \tau q\underline{q} \Leftrightarrow \\ \\ \displaystyle \tau q\underline{q} &< \displaystyle (p_s - p_z)((2p_z - \tau)(q + \underline{q}) - 2q\underline{q}) \Leftrightarrow \\ \\ \displaystyle \tau q\underline{q} &< \displaystyle (p_s - p_z)[2(p_z(q + \underline{q}) - q\underline{q}) - \tau(q + \underline{q})] \Leftrightarrow \\ \\ \displaystyle \tau q\underline{q} &< \displaystyle (p_s - p_z)[2\tau - \tau(q + \underline{q})] \Leftrightarrow \\ \\ \displaystyle q\underline{q} &< \displaystyle (p_s - p_z)(2 - (q + \underline{q})) \end{array}$$

Q.E.D. ■

From Propositions 2 and 4, Fund is the only feasible structure if $\min\left(\frac{q+q}{\tau}\left(p_zR-1\right), R-\frac{2-(q+q)}{2p_z-\tau}\right) < c_{\Delta} \leq R-\frac{1}{p_s}$ and Bank is the only feasible structure if $\min\left(\frac{q+q}{\tau}\left(p_zR-1\right), R-\frac{2-(q+q)}{2p_z-\tau}\right) \geq c_{\Delta} > R-\frac{1}{p_s}$. We can now prove the following result:

Lemma 12 Suppose that $q\underline{q} > (p_s - p_z)(2 - (q + \underline{q}))$. Then $\min\left(\frac{q+q}{\tau}(p_zR - 1), R - \frac{2-(q+\underline{q})}{2p_z - \tau}\right) > R - \frac{1}{p_s} \iff \frac{(p_s - p_z)(q+\underline{q}) + q\underline{q}}{p_s q\underline{q}} < R$

Proof. Since

$$\min\left(\frac{q+q}{\tau}\left(p_{z}R-1\right), R-\frac{2-(q+q)}{2p_{z}-\tau}\right) = \begin{cases} \frac{q+q}{\tau}\left(p_{z}R-1\right) \text{ if } R < \frac{2}{2p_{z}-\tau} \\ R-\frac{2-(q+q)}{2p_{z}-\tau} \text{ if } R \ge \frac{2}{2p_{z}-\tau} \end{cases}$$

we have

$$\min\left(\frac{q+\underline{q}}{\tau}\left(p_z R-1\right), R-\frac{2-(q+\underline{q})}{2p_z-\tau}\right) > R-\frac{1}{p_s}$$
(A8)

is equivalent to

$$\begin{cases} \frac{q+q}{\tau} \left(p_z R - 1 \right) > R - \frac{1}{p_s} \text{ if } R < \frac{2}{2p_z - \tau} \\ R - \frac{2 - (q+q)}{2p_z - \tau} > R - \frac{1}{p_s} \text{ if } R \ge \frac{2}{2p_z - \tau} \end{cases}$$
(A9)

Note that if $\frac{2-(q+\underline{q})}{2p_z-\tau} < \frac{1}{p_s}$ (or from Lemma 10 $q\underline{q} > (p_s - p_z)(2 - (q + \underline{q})))$, the lower branch of (A9) holds. Therefore, if (A8) holds we must have either

$$R \ge \frac{2}{2p_z - \tau}$$

or

$$R < \frac{2}{2p_z - \tau}$$
 and $\frac{q + \underline{q}}{\tau} (p_z R - 1) > R - \frac{1}{p_s}$

Given that $\frac{q+q}{\tau}(p_z R - 1) > R - \frac{1}{p_s} \iff \frac{p_z(q+q)-\tau}{\tau}R > \frac{(q+q)p_s-\tau}{\tau p_s} \iff R > \frac{(p_s-p_z)(q+q)+qq}{p_s qq}$, the last expression is equivalent to

$$\frac{(p_s - p_z)(q + \underline{q}) + q\underline{q}}{p_s q\underline{q}} < R < \frac{2}{2p_z - \tau}$$

Therefore, using Lemma 11 we find that (A8) is equivalent to $\frac{(p_s - p_z)(q + \underline{q}) + q\underline{q}}{p_s q\underline{q}} < R.$ QED

Then, when $q\underline{q} > (p_s - p_z)(2 - (q + \underline{q}))$, Bank is the only feasible structure if $\frac{(p_s - p_z)(q + \underline{q}) + qq}{p_s q\underline{q}} < R$ and $\min\left(R - \frac{2 - (q + \underline{q})}{2p_z - \tau}, \frac{q + q}{\tau}(p_z R - 1)\right) \ge c_\Delta$, and Fund is the only feasible structure if $\frac{(p_s - p_z)(q + \underline{q}) + qq}{p_s q\underline{q}} > R$ and $R - \frac{1}{p_s} \ge c_\Delta$. This completes the proofs of parts (2-a-i) and (2-a-ii).

Next, we prove part (2-a-iii). When both organization structures are feasible Bank dominates Fund if $V^Z > V^S$, that is

$$p_{z}(R - m^{Z}) > p_{s}(R - m^{S}) \iff$$

$$p_{z}\left(R - \max\left(\frac{1}{p_{z}} + \frac{\tau}{p_{z}(q + \underline{q})}c_{\Delta}, c_{\Delta} + \frac{2 - (q + \underline{q})}{2p_{z} - \tau}\right)\right) > p_{s}\left(R - (c_{\Delta} + \frac{1}{p_{s}})\right) \iff$$

$$p_{z}\left(R - \max\left(\frac{1}{p_{z}} + \frac{\tau}{p_{z}(q + \underline{q})}c_{\Delta}, c_{\Delta} + \frac{2 - (q + \underline{q})}{2p_{z} - \tau}\right)\right) > p_{s}\left(R - (c_{\Delta} + \frac{1}{p_{s}})\right) \iff$$

$$p_{s}c_{\Delta} + 1 - \max\left(1 + \frac{\tau}{(q + \underline{q})}c_{\Delta}, p_{z}c_{\Delta} + \frac{p_{z}\left(2 - (q + \underline{q})\right)}{2p_{z} - \tau}\right) > (p_{s} - p_{z})R \iff$$

$$\min\left(p_{s}c_{\Delta} + 1 - \left(1 + \frac{\tau}{(q + \underline{q})}c_{\Delta}\right), p_{s}c_{\Delta} + 1 - \left(p_{z}c_{\Delta} + \frac{p_{z}\left(2 - (q + \underline{q})\right)}{2p_{z} - \tau}\right)\right) > (p_{s} - p_{z})R \iff$$

$$\min\left(\frac{(p_{s} - p_{z})(q + \underline{q}) + q\underline{q}}{(p_{s} - p_{z})(q + \underline{q})}c_{\Delta}, c_{\Delta} + \frac{q\underline{q}}{(p_{s} - p_{z})(2p_{z} - \tau)}\right) > R.$$

Finally, we prove part (2-b). Lemma 10 implies that $\frac{1}{p_s} < \frac{2-(q+\underline{q})}{2p_z-\tau}$ which in turn implies that $\min\left(R - \frac{2-(q+\underline{q})}{2p_z-\tau}, \frac{q+\underline{q}}{\tau}(p_z R - 1)\right) < R - \frac{1}{p_s}$. Therefore, Bank can never be the only feasible structure. Next, we show that if $q\underline{q} \leq (p_s - p_z)\left(2 - (q + \underline{q})\right), V^S \geq V^Z$.

We have

$$V^{S} = p_{s}(R - m^{S}) = p_{s}\left(R - \left(c_{\Delta} + \frac{1}{p_{s}}\right)\right)$$

and

$$V^{Z} = p_{z}(R - m^{Z}) = \left\{ \begin{array}{l} p_{z}\left(R - \left(\frac{1}{p_{z}} + \frac{\tau}{p_{z}(q+q)}c_{\Delta}\right)\right) \text{ if } c_{\Delta} \leq \frac{q+q}{2p_{z}-\tau} \\ p_{z}\left(R - \left(c_{\Delta} + \frac{2 - (q+q)}{2p_{z}-\tau}\right)\right) \text{ if } c_{\Delta} > \frac{q+q}{2p_{z}-\tau} \end{array} \right\}.$$

For $c_{\Delta} \leq \frac{q+q}{2p_z-\tau}$ we have

$$\begin{split} V^S &\geq V^Z \Leftrightarrow \\ p_s \left(R - \left(c_\Delta + \frac{1}{p_s} \right) \right) &\geq p_z \left(R - \left(\frac{1}{p_z} + \frac{\tau}{p_z(q + \underline{q})} c_\Delta \right) \right) \Leftrightarrow \\ (p_s - p_z) R &\geq \left(p_s - \frac{\tau}{q + \underline{q}} \right) c_\Delta |_{\left(R \geq c_\Delta + \frac{2 - (q + \underline{q})}{2p_z - \tau}\right)} \Leftrightarrow \\ (p_s - p_z) \left(c_\Delta + \frac{2 - (q + \underline{q})}{2p_z - \tau} \right) &\geq \left(p_s - \frac{\tau}{q + \underline{q}} \right) c_\Delta \Leftrightarrow \\ (p_s - p_z) \frac{2 - (q + \underline{q})}{2p_z - \tau} &\geq \left(p_z - \frac{\tau}{q + \underline{q}} \right) c_\Delta |_{\left(p_z(q + \underline{q}) - \tau = q\underline{q}\right)} \Leftrightarrow \\ \frac{(p_s - p_z)(q + \underline{q}) \frac{2 - (q + \underline{q})}{2p_z - \tau}}{q\underline{q}} &\geq c_\Delta |_{\left(c_\Delta \leq \frac{q + \underline{q}}{2p_z - \tau}\right)} \Leftrightarrow \\ \frac{(p_s - p_z)(q + \underline{q}) \frac{2 - (q + \underline{q})}{2p_z - \tau}}{q\underline{q}} &\geq \frac{q + \underline{q}}{2p_z - \tau} \Leftrightarrow \\ (p_s - p_z)(2 - (q + \underline{q})) &\geq q\underline{q} \end{split}$$

which is true. For $c_{\Delta} \geq \frac{q+q}{2p_z - \tau}$ we have

$$V^{S} \geq V^{Z} \iff$$

$$p_{s}\left(R - (c_{\Delta} + \frac{1}{p_{s}})\right) \geq p_{z}\left(R - \left(c_{\Delta} + \frac{2 - (q + \underline{q})}{2p_{z} - \tau}\right)\right) \iff$$

$$(p_{s} - p_{z})R \geq 1 + (p_{s} - p_{z})c_{\Delta} - \frac{2 - (q + \underline{q})}{2p_{z} - \tau}p_{z}|_{\left(R \geq c_{\Delta} + \frac{2 - (q + \underline{q})}{2p_{z} - \tau}\right)} \iff$$

$$(p_{s} - p_{z})\left(c_{\Delta} + \frac{2 - (q + \underline{q})}{2p_{z} - \tau}\right) \geq 1 + (p_{s} - p_{z})c_{\Delta} - \frac{2 - (q + \underline{q})}{2p_{z} - \tau}p_{z} \iff$$

$$(p_{s} - p_{z})\frac{2 - (q + \underline{q})}{2p_{z} - \tau} \geq 1 - \frac{2 - (q + \underline{q})}{2p_{z} - \tau}p_{z}|_{\left(p_{z}(q + \underline{q}) - \tau = q\underline{q}\right)} \iff$$

$$(p_{s} - p_{z})\frac{2 - (q + \underline{q})}{2p_{z} - \tau} \geq \frac{q\underline{q}}{2p_{z} - \tau} \iff$$

$$(p_{s} - p_{z})(2 - (q + \underline{q})) \geq q\underline{q},$$

which is true. QED

6.6 Proof of Proposition 5

As in the case with intermediated finance, the entrepreneurs may prefer M to monitor only when both projects are type h, or they may prefer him to monitor any project as long as it is type h. For each case, we first figure out the optimal direct mechanism and then compare it with the optimal financial intermediation arrangement. Let m^d (the superscript d denotes that 'direct mechanism') denote the payment to M from an entrepreneur whose project succeeds.

Lemma 13 If the entrepreneurs would like M to monitor only when both projects are type h, then the optimal (and only feasible) contract is $m^d = c_{\Delta}$.

Proof. The problem for each entrepreneur is to choose the lowest value of m^d that satisfies the following constrainnts:

$$2q^{2}m^{d} + 2q(1-q)m^{d} - 2c \geq 2q\underline{q}m^{d} + [q(1-\underline{q}) + \underline{q}(1-q)]m^{d} - c \iff$$

$$m^{d} \geq c_{\Delta}$$
(A10)

$$2q^{2}m^{d} + 2q(1-q)m^{d} - 2c \geq 2\underline{q}^{2}m^{d} + 2\underline{q}(1-\underline{q})m^{d} \iff$$

$$m^{d} \geq c_{\Delta}$$
(A11)

and

$$2\underline{q}(1-\underline{q})m^{d} + \underline{q}^{2}2m^{d} \geq [\underline{q}(1-q) + q(1-\underline{q})]m^{d} + q\underline{q}2m^{d} - c \iff$$

$$2\underline{q}m^{d} \geq (\underline{q}+q)m^{d} - c \iff m^{d} \leq c_{\Delta}.$$
(A12)

where (A10) requires that when both projects are type h M prefers to monitor both of them rather than only one project, (A11) requires that when both projects are type h M prefers to monitor both of them rather than none of them and (A12) requires that when only one project is type h, M prefers not to monitor. The above three inequalities imply that the only feasible contract is $m^d = c_{\Delta}$. QED

Lemma 14 If the entrepreneurs would like M to monitor a project whenever it is good, then the optimal contract is $m^d = c_{\Delta}$.

Proof. The problem for each entrepreneur is to choose the lowest value of m^d that satisfies (A10), (A11) and the constraint

$$[\underline{q}(1-q) + q(1-\underline{q})]m^d + q\underline{q}2m^d - c \geq 2\underline{q}(1-\underline{q})m^d + \underline{q}^22m^d \iff (\underline{q}+q)m^d - c \geq 2\underline{q}m^d \iff m^d \geq c_{\Delta}.$$
(A13)

where (A13) requires that when only one project is type h, M prefers not to monitor it. The entrepreneurs' problem becomes

$$\min m^d$$
, s.t. $m^d \ge c_\Delta$.

The solution is $m^d = c_\Delta$. QED

The two lemmas imply that the entrepreneurs prefer M to monitor every type h project. QED

6.7 Proof of Proposition 6

If the internal control cost is bigger than c, then the monitors will not be monitored but we still need two monitors, call them M1 and M2 to monitor the two entrepreneurs. Thus, the only way to induce them to monitor is to offer them sufficient incentives. Consider problem 1. Suppose the incentive scheme for Mk (k = 1, 2) is m_{ij}^k where i, j = 1, 0 denote the success or failure of each of the two projects. Given that M2 monitors, M1 has an incentive to monitor if and only if $q(1-q)m_{10}^1 + q(1-q)m_{01}^1 + q^2m_{11}^1 - c \ge$ $\underline{q}(1-q)m_{10}^1 + (1-\underline{q})qm_{01}^1 + q\underline{q}m_{11}^1$. The last expression is equivalent to $(1-q)m_{10}^1 - qm_{01}^1 + qm_{11}^1 \ge \frac{c}{q-\underline{q}}$. The optimal incentive scheme requires $m_{01}^1 = 0$. Then $(1-q)m_{10}^1 + qm_{11}^1 \ge \frac{c}{q-\underline{q}}$. That means that if project 1 succeeds M1 expects at least $\frac{c}{q-\underline{q}}$, which is exactly what M1 will get under the Fund structure. QED

6.8 Proof of Proposition 7

We are going to prove the Proposition 7 in two steps. We will first demonstrate that as long as both intermediaterd and direct financial contracts can be conditioned on the outcomes of both projects that financial intermediation and direct finance implement the same set of allocations. Then we will prove that as long as $c_{\Delta} \leq \min\left(\frac{q+q}{2p_z-\tau}, \frac{2q}{p_s(2-p_s)}\right)$ the intermediated finance contracts between each entrepreneur and M that do not condition the apyoff to M on the outcoes of both projects implement the same allocations with the direct mechanism that allows for such conditioning. **Lemma 15** If intermediated and direct financial contracts can be conditioned on the outcomes of both projects then financial intermediation and direct finance will implement the same set of allocations.

Proof. Consider the direct mechanism whereby each entrepreneur signs contracts separately with M and investors. Denote the contracts agreed between each entrepreneur (E1 and E2) and M by m_{ij}^{dk} , where k = 1, 2 denotes the entrepreneur and i, j = 0, 1 denote the outcomes of the two projects. Thus, for example, m_{10}^{d1} is the payment from E1 to M when project 1 succeeds but project 2 fails. Treating symmetrically the two entrepreneurs and taking into account their limited liability implies that the general contract can be written as (m_1^d, m_2^d) , where m_1^d is the payment to M when only one project succeeds (received from the entrepreneur with the successful project) and m_2^d is the payment when both projects succeed from each entrepreneur). Similarly, a general contract to investors can be denoted by (r_1^d, r_2^d) , where r_1^d is the payment to the investors when only one project succeeds received from the successful project and r_2^d is the payment from each entrepreneur to investors when both projects succeeds.

Consider the case under financial intermediation in which the contract on the asset side (namely the contract between an entrepreneur and the intermediary) can be contingent on the outcomes of both projects. An asset side contract is (m_1, m_2) , where m_1 denotes the payment from an entrepreneur with a successful project to M when the other project fails and m_2 denotes the corresponding payment when both projects succeed. The contract on the liability side is still given by (r_1, r_2) .

Then, any arrangement under financial intermediation, $(m_1, m_2; r_1, r_2)$, can be implemented by an arrangement under the direct mechanism where $m_1^d = m_1 - r_1$ and $m_2^d = m_2 - r_2/2$; $r_1^d = r_1$ and $r_2^d = r_2/2$. Moreover, any arrangement under the direct mechanism, $(m_1^d, m_2^d; r_1^d, r_2^d)$, can be implemented by an arrangement under financial intermediation where $m_1 = r_1^d + m_1^d$ and $m_2 = r_2^d + m_2^d; r_1 = r_1^d$ and $r_2 = 2r_2^d$. QED

Lemma 16 Suppose that $c_{\Delta} \leq \min(\frac{q+q}{2p_z-\tau}, \frac{2q}{p_s(2-p_s)})$. Then, Bank and Fund implement the same allocations with the direct mechanism where the contracts between each entrepreneur and M can be conditioned on the outcomes of both projects.

Proof. The following observation is useful when considering the optimal direct mechanism. In the direct mechanism, the contract to investors, (r_1^d, r_2^d) , has no effect on the incentives of the monitor. Therefore, any contract such that the participation constraint of investors is satisfied as an equality is

optimal from the point of view of entrepreneurs. once more the two entrepreneurs might prefer M to monitor only when both projects are type h or to monitor a project so long as it is type h. For each case, we derive the optimal direct mechanism and then consider when it can be implemented by Bank or Fund. \blacksquare

Result 1 If the entrepreneurs would like M to monitor only when both projects are type h, then the optimal arrangement is $m_1^d = 0, m_2^d = \frac{c_{\Delta}}{q+q}$.

Proof. A feasible direct mechanism must satisfy the limited liability constraint, $m_1^d \ge 0, m_2^d \ge 0$ and the IC constraints that require that M has incentives to monitor if and only if both projects are good. The following incentive constraint states that, when both projects are good, M prefers to monitor both rather than only one of them:

$$2q^{2}m_{2}^{d} + 2q(1-q)m_{1}^{d} - 2c \geq 2q\underline{q}m_{2}^{d} + [q(1-\underline{q}) + \underline{q}(1-q)]m_{1}^{d} - c \iff (1-2q)m_{1}^{d} + 2qm_{2}^{d} \geq c_{\Delta}$$
(A14)

The following incentive constraint requires that, when both projects are good, M prefers to monitor both of them rather than none:

$$2q^2m_2^d + 2q(1-q)m_1^d - 2c \geq 2\underline{q}^2m_2^d + 2\underline{q}(1-\underline{q})m_1^d \iff (1-q-\underline{q})m_1^d + (q+\underline{q})md_2^d \geq c_\Delta$$
(A15)

There are two additional constraints ensuring that when only one project is good M prefers not to monitor at all which, following the same argument as the one used for the proof of Proposition 1, we can show that in equilibrium do not bind. At date 0, the expected payment to M from each entrepreneur is equal to

$$p^{2} \left(q^{2} m_{2}^{d} + q(1-q)m_{1}^{d}\right) + \left(1-p^{2}\right) \left(\underline{q}^{2} m_{2}^{d} + \underline{q}(1-\underline{q})m_{1}^{d}\right) = \gamma m_{1}^{d} + \tau m_{2}^{d}$$

where $\tau = p^2 q^2 + (1 - p^2) \underline{q}^2$ and $\gamma = p^2 q (1 - q) + (1 - p^2) \underline{q} (1 - \underline{q})$. Thus the optimal mechanism solves the problem

$$\min H^4 = \gamma m_1^d + \tau m_2^d \text{ subject to (A14) and (A15)}$$

(A15) implies (A14) if and only if $m_1^d \le m_2^d$. First, consider the case $m_1^d \le m_2^d$, which implies that (A15) is binding and thus $dm_2^d = -\frac{(1-q-q)}{q+q}dm_1^d$. Then $dH^4 = \gamma dm_1^d + \tau dm_2^d = \left[\gamma - \tau \frac{(1-q-q)}{q+q}\right]dm_1^d$. In the proof of Proposition 1, we have shown that $1 - q - \underline{q} < (q + \underline{q})\frac{\gamma}{\tau}$, which implies $\gamma - \tau \frac{(1-q-q)}{q+\underline{q}} > 0$. Thus

 $\frac{dH^4}{dm_1^d} > 0$, which, in turn, implies that it is optimal to set $m_1^d = 0$. Next, consider the case $m_1^d \ge m_2^d$, which implies that (A14) is binding and thus $dm_2^d = -\frac{1-2q}{2q}dm_1^d$. Then $dH^4 = \gamma dm_1^d + \tau dm_2^d = \left[\gamma - \tau \frac{1-2q}{2q}\right]dm_1^d$. In the proof of Proposition 1, we have shown that the expression in the brackets is positive and thus $\frac{dH^4}{dm_1^d} > 0$. Thus the solution for this case is to set $m_1^d = m_2^d$ which implies that the overall optimal solution is in the region $m_1^d \le m_2^d$ and, therefore, $m_1^d = 0$ and given that (A15) is binding, $m_2^d = \frac{c_A}{q+q}$. QED

Result 2 The optimal direct mechanism given by Result 1, $(m_1^d = 0, m_2^d = \frac{c_{\Delta}}{q+q})$, can be implemented by Bank if and only if $c_{\Delta} \leq \frac{q+q}{2p_z-\tau}$.

Proof. An arrangement under financial intermediation $(m; r_1, r_2)$ implements a direct mechanism (m_1^d, m_2^d) only if

$$m - r_1 = m_1^d \tag{A16}$$

$$2m - r_2 = 2m_2^d (A17)$$

and the PC of investors

$$[p^2q(1-q) + (1-p^2)\underline{q}(1-\underline{q})]2r_1 + [p^2q^2 + (1-p^2)\underline{q}^2]r_2 = 2.$$

When $m_1^d = 0$; $m_2^d = \frac{c_{\Delta}}{q+q}$, these three equations give $m = \frac{1}{p_z} + \frac{\tau}{p_z(q+q)}c_{\Delta} = r_1$ and $r_2 = \frac{2}{p_z}(1 - \frac{p_z - \tau}{q+q}c_{\Delta})$. This is the Bank solution. But the liability contract satisfies the MC, namely, $r_2 \ge r_1$ if and only if $c_{\Delta} \le \frac{q+q}{2p_z - \tau}$. QED

Result 3 Suppose the entrepreneurs would like M to monitor a project whenever it is type h. Then if $\frac{q}{\underline{q}} \leq \frac{1+p}{p}$, the optimal direct arrangement is $(m_1^d = 0, m_2^d = c_\Delta/(2\underline{q}))$, otherwise it is given by $m_1^d = m_2^d = c_\Delta$.

Proof. In addition to constraints (A14) and (A15) the following constraint that states that when only one project is good M prefers to monitor it, must also be satisfied:

$$\left(\underline{q}(1-q)+q(1-\underline{q})\right)m_{1}^{d}+q\underline{q}2m_{2}^{d}-c \geq \underline{q}(1-\underline{q})2m_{1}^{d}+\underline{q}^{2}2m_{2}^{d}\iff \left(\underline{q}\left(\underline{q}-q\right)+(1-\underline{q})\left(q-\underline{q}\right)\right)m_{1}^{d}+\underline{q}\left(q-\underline{q}\right)2m_{2}^{d}\geq c\iff \left(1-2\underline{q}\right)m_{1}^{d}+\underline{q}2m_{2}^{d}\geq c_{\Delta}$$
(A18)

Thus the optimal mechanism solves the problem

min
$$H^4 = \gamma m_1^d + \tau m_2^d$$
 subject to (A14), (A15) and (A18)

Notice that (A18) implies (A15) which in turn implies (A14) if and only if $m_1^d \leq m_2^d$ and vice versa. We first consider the case $m_1^d \geq m_2^d$ which implies that (A14) is binding and thus $dm_2^d = -\frac{1-2q}{2q}dm_1^d$. Following the same steps as in Result 1 we find that it is optimal to set $m_1^d = m_2^d$. Therefore, the optimal solution for the above problem must lie in the region $m_1^d \leq m_2^d$ which implies that (A18) is binding and thus $dm_2^d = -\frac{1-2q}{2q}dm_1^d$. Then $dH^4 = \gamma dm_1^d + \tau dm_2^d = \left[\gamma - \tau \frac{1-2q}{2q}\right]dm_1^d$. The expression in the brackets has the same sign as $\frac{1+p}{p} - \frac{q}{q} \geq 0$ and, thus, we need to consider two cases.

(a) Suppose that $\frac{q}{\underline{q}} \leq \frac{1+p}{p}$. Then $\frac{dH^4}{dm_1^d} \geq 0$ in which case it is optimal to set m_1^d as small as possible, that is, $m_1^d = 0$. It follows from binding (A18) that $m_2^d = c_\Delta/(2\underline{q})$.

(b) Suppose that $\frac{q}{q} > \frac{1+p}{p}$ which implies that $\frac{dH^4}{dm_1^4} < 0$. Now it is optimal to set m_1^d as large as possible, which implies that $m_1^d = m_2^d$ given that $m_1^d \le m_2^d$. By substituting $m_1^d = m_2^d$ into the binding (A18), we find that $m_1^d = m_2^d = c_{\Delta}$.

QED

Result 4 If $\frac{q}{\underline{q}} < \frac{1+p}{p}$, the optimal direct mechanism can be implemented by Bank if and only if $c_{\Delta} \leq \frac{2q}{p_s(2-p_s)}$. If $\frac{q}{\underline{q}} \geq \frac{1+p}{p}$, the optimal direct mechanism can be implemented by Fund.

Proof. Similarly to the proof of Result 2, to implement a direct mechanism (m_1^d, m_2^d) , the arrangement under financial intermediation, $(m; r_1, r_2)$, shall satisfy (A16) and (A17). However, in this case PC of investors is different, because as the probability of one project succeeding is p_s , (r_1, r_2) now satisfies:

$$2p_s(1-p_s)r_1 + p_s^2r_2 = 2.$$

(a) If $\frac{q}{q} < \frac{1+p}{p}$, the optimal direct mechanism is $(m_1^d = 0, m_2^d = c_\Delta/(2\underline{q}))$. The solutions of the three equations are given by $r_1 = m = \frac{1}{p_s} + \frac{p_s}{2\underline{q}}c_\Delta$, $r_2 = \frac{2}{p_s} - \frac{1-p_s}{\underline{q}}c_\Delta$, which corresponds to the solution for Bank. The liability contract satisfies the MC, namely, $r_2 \ge r_1$ if and only if $c_\Delta \le \frac{2\underline{q}}{p_s(2-p_s)}$.

(b) If $\frac{q}{\underline{q}} \geq \frac{1+p}{p}$, the optimal direct mechanism is $m_1^d = m_2^d = c_\Delta$. Now the solutions of the three equations are given by $r_1 = r_2/2 = \frac{1}{p_s}$ and $m = \frac{1}{p_s} + c_\Delta$, which corresponds to the solution for Fund.

QED

Lemma 16 follows from Results 2 and 4. QED

Proposition 7 follows from Lemma 15 and Lemma 16. QED.

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