Quantity Measurement, Balanced Growth, and Welfare in Multi–Sector Growth Models*

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Abstract

Multi-sector models use numeraires to aggregate whereas the NIPA use the Fisher index. Since the resulting GDP statistics differ considerably, we must choose one aggregation method to compare model and data GDP. For a model of structural change, we show that the numeraire investment offers the least restrictive way of constructing a balanced growth path (BGP), but the Fisher index is often a measure of welfare changes and captures the growth slowdown due to Baumol's Costs Disease. We advocate to construct the BGP with the numeraire investment but to connect model GDP calculated with the Fisher index to the data.

Keywords: Balanced Growth; Baumol's Cost Disease; Fisher Index; Multi-Sector Growth Models; Structural Change.

JEL classification: O41; O47; O51.

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1 Introduction

Multi-sector growth models are ubiquitous in modern macroeconomics. Analyzing them requires the aggregation of sectoral value added to economy-wide GDP. The theoretical literature aggregates with numeraires whereas the National Income and Product Accounts (NIPA) of most of industrialized countries now aggregate with the Fisher quantity index. We ask whether the aggregation method matters for anything important and if one of them is preferable. We study these questions in a three-sector model of structural change, which has an investment sector and two consumption sectors that produce goods and services. Our model contains the most important challenges to aggregation: relative prices change; the sectoral composition changes because relative prices and income change.

We show that the aggregation method matters theoretically for the existence and the properties of a balanced growth path (BGP). Constructing a BGP involves the least restrictions with the numeraire investment and the most restrictions with the Fisher index. Moreover, if the BGP constructed with a numeraire exhibits structural change from goods to services, then GDP growth measured with the Fisher index exhibits a slow down, although GDP growth measured with the numeraire is constant. In other words, the Fisher index detects the implications of Baumol's (1967) Cost Disease on GDP growth whereas the numeraire does not.

We document that the aggregation method also matters empirically. Compared to the Fisher index, average annual GDP growth in the postwar U.S. is 0.27 percentage points lower if it is measured with the numeraire consumption and 1.03 percentage points higher if it is measured with the numeraire investment. Moreover, GDP growth is approximately constant if it is measured with the numeraire investment but slows down if it is measured with the numeraire consumption or the Fisher index. Since over long horizons, the resulting cumulative differences in GDP levels are sizeable, we conclude that GDP must be constructed with the same aggregation method in both model and data. This leaves the question whether it is preferable to construct model and data GDP with a numeraire or with the Fisher index.

We show that using the Fisher index has two conceptual advantages: it is independent of which numeraire is used for the construction of the BGP; if utility is homothetic, the Fisher index approximates a measure of welfare changes. This welfare result is a discrete-time version of the one derived by Durán and Licandro (2017) for continuous time. We differ from Durán and Licandro (2017) in that we use a new method of proof that provides separate first-order approximations for the Laspeyres and the Paasche indexes.

We conclude that while it is most tractable to construct a BGP with the numeraire investment, using the Fisher index is preferable for connecting model GDP to data GDP. We therefore advocate to proceed in three steps: (i) construct a BGP in the model by using a numeraire (preferably investment); (ii) construct model GDP by applying the Fisher index to the sectoral value added from the BGP; (iii) connect this measure of model GDP to data GDP.

The organization of the rest of the paper follows the two-step procedure of Herrendorf et al. (2014). We first study a two-sector growth model with investment and consumption where aggregation issues arise from changes in relative prices. We then study a three-sector growth model with structural change from consumption goods to services where additional aggregation issues arise from changes in the sectoral composition.

2 Two-sector growth model

2.1 Environment

The household is endowed with initial capital $K_0 > 0$ and one unit of time in each period. Capital K_t accumulates according to

$$K_{t+1} = (1-\delta)K_t + X_t,$$

where $\delta \in [0,1]$ and X_t is investment.

The utility function is

$$\sum_{t=0}^{\infty} \beta^t \log(C_t),$$

where $\beta \in (0,1)$ and C_t is consumption.

The sectoral production functions for consumption and investment are:

$$C_t = K_{ct}^{\theta}\Big(A_{ct}L_{ct}\Big)^{1-\theta}, \tag{1a}$$

$$X_t = K_{xt}^{\theta}\Big(A_{xt}L_{xt}\Big)^{1-\theta}, \tag{1b}$$

where $\theta \in (0,1)$ is the capital-share parameter; K_{it} and L_{it} are sectoral capital and labor; A_{it} captures exogenous, sector-specific, labor-augmenting technological progress.[1]

[1] Having the same θ across sectors has the advantage that the production side aggregates. Herrendorf et al. (2015) established

Capital and labor are freely mobile between the sectors and the usual feasibility constraints apply:

$$K_{ct} + K_{xt} \leq K_t,$$
$$L_{ct} + L_{xt} \leq L_t = 1.$$

2.2 Competitive equilibrium

A competitive equilibrium is a sequence of prices and an allocation such that: given prices, the allocation solves the household's problem and the firms' problems in each sector; markets clear. Since the two-sector model is well known, we state the standard equilibrium properties without deriving them. Since we want to study different numeraires, it is convenient to initially denominate all prices in current dollars.[2]

Profit maximization in each sector implies that the rental prices for capital and labor, r_t and w_t, equal the marginal revenue products. Denoting the prices of the sectoral outputs by p_{xt} and p_{ct}, this gives for $i \in \{g, s\}$:

$$r_t = p_{it}\theta\left(\frac{K_{it}}{L_{it}}\right)^{\theta-1} A_{it}^{1-\theta},$$
$$w_t = p_{it}(1-\theta)\left(\frac{K_{it}}{L_{it}}\right)^{\theta} A_{it}^{1-\theta}.$$

Combining the first-order conditions gives the usual result that the capital–labor ratios are equalized:

$$\frac{K_{xt}}{L_{xt}} = \frac{K_{ct}}{L_{ct}} = \frac{K_t}{L_t} = K_t, \tag{2}$$

where the last equality follow from the fact that $L_t = 1$. The relative price is inversely related to relative sector TFPs:

$$\frac{p_{ct}}{p_{xt}} = \left(\frac{A_{xt}}{A_{ct}}\right)^{1-\theta}. \tag{3}$$

Figure 1 shows that the empirically relevant case is $\widehat{A}_{xt} > \widehat{A}_{ct}$, where a "hat" denotes a growth factor. We will focus on this case from now on.

that Cobb-Douglas production functions with equal capita-share parameters nonetheless captures the key features of labor reallocation in the postwar U.S.

[2] Greenwood et al. (1997) and Oulton (2007) developed versions of the two-sector model. Herrendorf et al. (2014) solved a similar version as is used here.

Combining (2)–(3), equations (1) become:

$$C_t = K_t^{\theta} A_{ct}^{1-\theta} L_{ct}, \tag{4a}$$

$$X_t = K_t^{\theta} A_{xt}^{1-\theta} L_{xt}. \tag{4b}$$

(3) and (4) imply that the expenditure ratio equals the labor ratio::

$$\frac{p_{ct}C_t}{p_{xt}X_t} = \frac{L_{ct}}{L_{xt}}.$$

Hence, we can restrict our attention to analyzing the properties of the expenditure ratio.

The household maximizes its utility subject to the budget constraint and the feasibility constraints. The first-order conditions imply the usual consumption-Euler equation and transversality condition:

$$\frac{p_{ct+1}C_{t+1}}{p_{ct}C_t} = \beta \frac{p_{xt+1}}{p_{xt}} \left[1 - \delta + \frac{r_{t+1}}{p_{xt+1}} \right],$$
$$0 = \lim_{t\to\infty} \beta^t \frac{p_{xt}K_{t+1}}{p_{ct}C_t}.$$

2.3 Aggregation and balanced growth

We now study the existence and the properties of a balanced growth path (BGP) equilibrium. Standard definitions of BG require the growth rates of *all* variables to be constant (including zero). This is too restrictive in multi-sector models of structural change in which relative prices and the sectoral composition change.

There are less stringent alternatives in the literature. Kongsamut et al. (2001) introduced *Generalized Balanced Growth Path* (GBGP): "*A GBGP is a trajectory along which the real interest rate is constant*". Although often applied, GBG is too loose in our context because it does not require constant growth of all aggregate variables. Ngai and Pissarides (2007) introduced *Aggregate Balanced Growth Path* (ABGP): "*We define an aggregate balanced growth path such that aggregate output, consumption, and capital grow at the same rate.*" This concept ABGP works well for our purpose if we make two minor modifications: (i) aggregate quantities are expressed in the same units;[3] (ii) the growth rates of aggregate quantities are not necessarily the same.[4] We therefore use the following modified concept of ABG:

[3] For example, we will require that $p_{ct}C_t$ grows at a constant rate. The distinction between $p_{ct}C_t$ and C_t is relevant because in general C_t does not grow at a constant rate in models of structural change.

[4] We will encounter an example in Proposition 1 below where, with numeraire consumption, the growth rates of capital and

Definition 1 *An ABGP is an equilibrium path along which aggregate quantities expressed in the same units grow at constant rates (including zero).*

Note that changes in the sectoral composition are permitted to take place underneath the ABGP. In contrast, BG rules out such changes because it requires all variables to grow at constant rates.

We now aggregate sectoral outputs to GDP using the numeraires consumption and investment. We do not consider the numeraire labor, because it fits with the income approach instead of the product approach that we pursue here. Using the abbreviations

$$P_{ct} \equiv \frac{p_{ct}}{p_{xt}}, \qquad P_{xt} \equiv \frac{p_{xt}}{p_{ct}},$$

and superscripts to denote the numeraire, GDP in units of a numeraire is defined as:

$$Y_t^X \equiv P_{ct}C_t + X_t = K_t^{\theta} A_{xt}^{1-\theta}, \tag{5a}$$

$$Y_t^C \equiv C_t + P_{xt}X_t = K_t^{\theta} A_{ct}^{1-\theta}, \tag{5b}$$

where the equalities follow from (3)–(4).

Proposition 1

(i) Let X be the numeraire:

an ABGP exists if and only if $\widehat{A}_x$ *is constant; along the ABGP,* $\widehat{Y}_t^X = \widehat{A}_x$ *and* $\widehat{C}_t = \widehat{A}_x^{\theta}\widehat{A}_{ct}^{1-\theta}$.

(ii) Let C be the numeraire:

an ABGP exists if $\widehat{A}_x$ *and* $\widehat{A}_c$ *are constant; along the ABGP,* $\widehat{Y}_t^C = \widehat{C}_t = \widehat{A}_x^{\theta}\widehat{A}_c^{1-\theta}$.

GDP are constant yet different along the ABGP.

Proof. We begin by eliminating prices and consolidating the equilibrium conditions so that the only unknowns are equilibrium quantities:

$$Y_t^X = K_t^\theta A_{xt}^{1-\theta}, \qquad Y_t^C = K_t^\theta A_{ct}^{1-\theta} \tag{6a}$$

$$1 = \frac{C_t}{K_t^\theta A_{ct}^{1-\theta}} + \frac{X_t}{K_t^\theta A_{xt}^{1-\theta}}, \tag{6b}$$

$$\widehat{K}_{t+1} = \frac{X_t}{K_t} + 1 - \delta, \tag{6c}$$

$$\left(\frac{\widehat{A}_{xt+1}}{\widehat{A}_{ct+1}}\right)^{1-\theta} \widehat{C}_{t+1} = \beta\left[1 - \delta + \theta\left(\frac{K_{t+1}}{A_{xt+1}}\right)^{\theta-1}\right], \tag{6d}$$

$$0 = \lim_{t\to\infty} \beta^t \left(\frac{A_{ct}}{A_{xt}}\right)^{1-\theta} \frac{K_{t+1}}{C_t}. \tag{6e}$$

Depending on the numeraire, the first or second equality in (6a) applies.

Proof of Part (i). Necessity: We need to show that the existence of an ABGP implies that A_{xt} grows at a constant rate. Since the growth of Y_t^X and K_t is constant along the ABGP, this follows from the first equality of (6a).

Sufficiency: We need to show that if $\widehat{A}_x$ is constant, then an ABGP exists. We do so by constructing a path $\{Y_t^X, K_t, X_t, C_t\}_{t=0}^\infty$ such that Y_t^X and K_t grow at constant factors, $L_t = 1$ is constant, and the first equation of (6a) and (6b)–(6e) are satisfied.

We first construct $\{Y_t^X, K_t, X_t, C_t\}_{t=1}^\infty$. Set $\widehat{K}_t = \widehat{A}_x$, which is constant, and define $\{\widehat{Y}_t^X\}_{t=1}^\infty$ such that the first equation of (6a) is satisfied for all $t > 0$ if it is satisfied at $t = 0$. In particular,

$$\widehat{Y}_t^X = \widehat{K}_t^\theta \widehat{A}_{xt}^{1-\theta} = \widehat{A}_x.$$

We define $\{\widehat{X}_t\}_{t=1}^\infty$ such that equation (6c) is satisfied for all $t > 0$ if it is satisfied at $t = 0$. Since

$$\frac{X_t}{K_t} = \widehat{K}_{t+1} - (1-\delta) = \widehat{A}_x - (1-\delta),$$

this implies X_t/K_t must be constant. Thus, we set $\widehat{X}_t = \widehat{A}_x$. We define $\{\widehat{C}_t\}_{t=1}^\infty$ such that (6b) is satisfied for all $t > 0$ if it is satisfied at $t = 0$. Since $\widehat{K}_t = \widehat{X}_t = \widehat{A}_x$, (6b) implies that $C_t/(K_t^\theta A_{ct}^{1-\theta})$ must be constant. Hence, we set $\widehat{C}_t = \widehat{A}_x^\theta \widehat{A}_{ct}^{1-\theta}$.

Next, we set (Y_0^X, K_0, X_0, C_0) such that (6a)–(6c) hold at $t = 0$ and the Euler equation (6d) holds for all $t \geq 0$. Together with the previous growth factors, this uniquely determines $\{Y_t^X, K_t, X_t, C_t\}_{t=0}^{\infty}$. Using consumption growth and that $K_{t+1}/A_{xt+1} = K_0/A_{x0}$, (6d) becomes:

$$\widehat{A}_x = \beta\left(1 - \delta + \theta\left(\frac{K_0}{A_{x0}}\right)^{\theta-1}\right).$$

We choose the unique solution $K_0 > 0$ given $A_{x0} > 0$. Given K_0, we then set $Y_0^X \equiv K_0^{\theta} A_{x0}^{1-\theta}$ and $X_0 \equiv [\widehat{A}_x - (1-\delta)]K_0$ to satisfy (6a)–(6b) at $t = 0$. Given X_0 and K_0, we choose C_0 to satisfy (6b) at $t = 0$:

$$C_0 = \left(1 - \frac{X_0}{K_0^{\theta} A_{x0}^{1-\theta}}\right) K_0^{\theta} A_{c0}^{1-\theta}.$$

To show that the transversality condition (6e) holds, we substitute the growth factors for K_{t+1} and C_t into the right-hand side:

$$\beta^t \left(\frac{A_{ct}}{A_{xt}}\right)^{1-\theta} \frac{K_{t+1}}{C_t} = \beta^t \frac{K_0}{C_0} \widehat{A}_x.$$

Since this converges to zero as $t \to \infty$, we have constructed an ABGP.

Proof of Part (ii). Sufficiency: The proof is exactly the same as for Part (i), except now the second equation of (6a) applies and

$$\widehat{Y}_t^C = \widehat{K}_t^{\theta} \widehat{A}_{ct}^{1-\theta} = \widehat{A}_x^{\theta} \widehat{A}_c^{1-\theta}.$$

QED

Proposition 1 shows that constructing an ABGP with numeraire X is possible under less restrictive conditions than with numeraire C: whereas $\widehat{A}_{ct}$ may change with numeraire X, we are able to establish the existence of an ABGP with numeraire C only if both $\widehat{A}_x$ and $\widehat{A}_c$ are constant. Given (3), this implies that only with numeraire X can the ABGP match the fact that in the postwar U.S. the average annual growth rate of p_{ct}/p_{xt} varied widely; it was 0.63% during 1955–1975, 1.46% during 1975–1995, and 2.33% during 1995–2015.

Proposition 1 also shows only if GDP growth is measured with the numeraire consumption does it equal consumption growth. This is noteworthy because aggregate output measured in units of consumption is often viewed as an indicator of well being. This notion goes back to Weitzman (1976) who showed for a continuous-time, two-sector growth model without technological progress that the present discounted sum of future consumption equals the present value of receiving ad infinitum today's Net Domestic Product (NDP) measured in units of consumption. Dasgupta and Mäler (2000) clarified that Weitzman's result has a welfare interpretation only if utility is linear. Asheim and Weitzman (2001) replied by showing that if utility is concave and real NDP is constructed with a Divisia consumption price index, then welfare increases if and only if real NDP increases. We emphasize that this is a *qualitative* result that says that welfare and real NDP move together. Below, we will provide a *quantitative* result that says that, if the utility function is homothetic, then the change in GDP measured with the Fisher index approximates the change in a measure of welfare based on compensating expenditures.

Lastly, Proposition 1 shows that GDP growth depends on the choice of the numeraire, because $\widehat{Y}_t^X = \widehat{A}_x$ whereas $\widehat{Y}_t^C = \widehat{A}_x^{\theta}\widehat{A}_{ct}^{1-\theta}$. This immediately implies the following result:

Proposition 2 *If $\widehat{A}_{ct} < \widehat{A}_{xt}$, then $\widehat{Y}_t^C < \widehat{Y}_t^X$.*

The result of the proposition is intimately linked to the behavior of the relative price. To see this, recall that (5) defined the growth rates of GDP in units of a numeraire as:

$$Y_t^X \equiv P_{ct}C_t + X_t, \qquad Y_t^C \equiv C_t + P_{xt}X_t. \tag{7}$$

As mentioned before, Equation (3) and Figure 1 imply that the empirically relevant case is $\widehat{A}_x > \widehat{A}_{ct}$ and $\widehat{P}_{xt} < \widehat{P}_{ct}$. Since the relative price is the only difference in the definitions of Y_t^X and Y_t^C, it is obvious that $\widehat{Y}_t^C < \widehat{Y}_t^X$.

That GDP growth depends on the choice of numeraire is undesirable. We will see next that using the Fisher index results in a measure of GDP growth that is independent of the choice of numeraire. In this context, it will be important that along the ABGP $P_{ct}C_t/X_t$ is constant, which follows the proof of Proposition 1 implies that $P_{ct}C_t$ and X_t grow at the same factor along the ABGP.

2.4 Aggregation with the Fisher index

For any two adjacent periods, the Fisher quantity index is defined as the geometric average of the Laspeyres and Paasche quantity indexes:[5]

$$\widehat{Y}_t^F \equiv \sqrt{\widehat{Y}_t^L \cdot \widehat{Y}_t^P} \equiv \sqrt{\frac{p_{ct-1}C_t + p_{xt-1}X_t}{p_{ct-1}C_{t-1} + p_{xt-1}X_{t-1}} \cdot \frac{p_{ct}C_t + p_{xt}X_t}{p_{ct}C_{t-1} + p_{xt}X_{t-1}}}. \tag{8}$$

[5] Whelan (2002) offers a more detailed discussion of the Fisher index.

Proposition 3 *GDP growth with the Fisher (quantity) index is independent of the numeraire.*

Proof. The claim follows by pulling out p_{ct-1} and p_{ct} or p_{xt-1} and p_{xt} from the numerators and denominators of equation (8). **QED**

GDP levels with the Fisher index are obtained by choosing a reference year and chaining the growth rates. For example, choosing year 0 as the reference year and denoting the nominal GDP of period 0 by Y_0,

$$Y_t^F = \widehat{Y}_t^F \cdot ... \cdot \widehat{Y}_1^F \cdot Y_0.$$

It is straightforward to show that:

$$Y_t^F = \sqrt{\frac{Y_t^C}{Y_0^C} \cdot \frac{C_t + P_{xt-1}X_t}{C_{t-1} + P_{xt}X_{t-1}} ... \frac{C_1 + P_{x0}X_1}{C_0 + P_{x1}X_0}} \cdot Y_0 = \sqrt{\frac{Y_t^X}{Y_0^X} \cdot \frac{P_{ct-1}C_t + X_t}{P_{ct}C_{t-1} + X_{t-1}} ... \frac{P_{c0}C_1 + X_1}{P_{c1}C_0 + X_0}} \cdot Y_0.$$

Since the behavior of the terms under the square root is hard to characterize analytically, Y_t^F is generally not suited for obtaining analytical results. In contrast, chaining GDP growth rates calculated with the numeraires C and X gives back the GDP levels Y_t^C and Y_t^X defined above and remains tractable.

The simplicity of the two–sector model implies that we can analytically characterize how the different measures of GDP growth are related to each other along an ABGP. Rearranging the terms in (8) while using that $\widehat{Y}_t^C = \widehat{C}_t$ and $\widehat{Y}_t^X = \widehat{X}_t$ gives:

$$\widehat{Y}_t^F = \widehat{Y}_t^C \sqrt{\frac{1 + \frac{P_{xt}X_t}{C_t}\frac{P_{xt-1}}{P_{xt}}}{1 + \frac{P_{xt-1}X_{t-1}}{C_{t-1}}\frac{P_{xt}}{P_{xt-1}}}} = \widehat{Y}_t^X \sqrt{\frac{1 + \frac{P_{ct}C_t}{X_t}\frac{P_{ct-1}}{P_{ct}}}{1 + \frac{P_{ct-1}C_{t-1}}{X_{t-1}}\frac{P_{ct}}{P_{ct-1}}}}.$$

Recalling (3), that $\widehat{A}_x$ must be constant for an ABGP to exist, and that $P_{xt}X_t)/C_t$ and $P_{ct}C_t)/X_t$ are constant along an ABGP, we get:

$$\widehat{Y}_t^F = \widehat{Y}_t^C \sqrt{\frac{1 + \frac{P_xX}{C}\frac{\widehat{A}_x}{\widehat{A}_{ct}}}{1 + \frac{P_xX}{C}\frac{\widehat{A}_{ct}}{\widehat{A}_x}}} = \widehat{Y}_t^X \sqrt{\frac{1 + \frac{P_cC}{X}\frac{\widehat{A}_{ct}}{\widehat{A}_x}}{1 + \frac{P_cC}{X}\frac{\widehat{A}_x}{\widehat{A}_{ct}}}}. \tag{9}$$

Hence, our maintained assumption that $\widehat{A}_x > \widehat{A}_{ct}$ implies that $\widehat{Y}_t^C < \widehat{Y}_t^F < \widehat{Y}_t^X$. Moreover, $\widehat{Y}_t^F$ is constant iff $\widehat{A}_c$ is constant.

Proposition 4

(i) $\widehat{Y_t^C} < \widehat{Y_t^F} < \widehat{Y_t^X}$ *along any ABGP with numeraire C or X.*

(ii) $\widehat{Y_t^F}$ *and* $\widehat{Y_t^X}$ *are constant along any ABGP with numeraire C.*

(iii) $\widehat{Y_t^C}$ *and* $\widehat{Y_t^F}$ *are constant along any ABGP with numeraire X iff* $\widehat{A_{ct}}$ *is constant.*

Proposition 4 implies that GDP growth with the Fisher index lies between GDP growth with the numeraires. This raises the question of how large the differences between $\widehat{Y_t^C}$, $\widehat{Y_t^X}$ and $\widehat{Y_t^F}$ are in the data. Figure 2 and Table 1 show that they are sizeable. Since the differences among the measures of GDP growth are too large to ignore, we must measure GDP growth in the same way in the model and in the data.[6]

Figure 2 also shows that while GDP growth measured in the numeraire investment has a constant long-run trend, GDP growth measured with the Fisher index or with the numeraire consumption slows down.[7] This raises the question under which conditions there can be a growth slowdown in the two-sector model. Proposition 4 said that if we choose the numeraire investment, then GDP growth in the Fisher index and in the numeraire consumption may not grow at constant rates along the ABGP, so there is room for a GDP growth slowdown. The next proposition specifies under what condition there actually is a growth slowdown:

Proposition 5 *If* $\widehat{A_x}$ *is constant while* $\widehat{A_{ct}}$ *decreases, then along any ABGP with numeraire investment: the growth rate of GDP measured with numeraire consumption slows down; the first-order approximation of the growth rate of GDP measured with the Fisher index slows down.*

Proof. To show the claim that $\widehat{Y_t^C}$ decreases, recall (5):

$$Y_t^C = K_t^\theta A_{ct}^{1-\theta}.$$

Proposition 1 implies that along the ABGP with numeraire X_t, K_t grows at factor $\widehat{A_x}$, which is constant. Hence, the assumption that $\widehat{A_{ct}}$ decreases implies that $\widehat{Y_t^C}$ decreases.

To show that a first-order approximation of $\widehat{Y_t^F}$ slows down, we take the log of the first equation of

[6] Whelan (2003) is one of the few authors who appreciated this. He calibrated a two-sector growth model measuring quantities with the Fisher index. We add to his analysis a comparison among different measures of GDP growth, paying particular attention to the welfare properties of the Fisher index and to Baumol's Cost Disease.

[7] In the figure, C is private nondurable consumption and X is private investment in fixed assets and consumer durables. Note that Y_t^C is initially above Y_t^X because, as Figure 1 shows, P_{xt} initially increases.

(9):

$$\widehat{Y_t^F} \approx \Delta \log(Y_t^F) \approx \Delta \log(Y_t^X) + \frac{1}{2}\frac{P_c C}{X}\left(\frac{\widehat{A_{ct}}}{\widehat{A_x}} - \frac{\widehat{A_x}}{\widehat{A_{ct}}}\right). \tag{10}$$

The right-hand side slows down because: $P_c C/X$ and $\widehat{A_x}$ are constant; $\Delta \log(Y_t^X) \approx \widehat{Y_t^X}$ is constant along the ABGP with numeraire investment; $\widehat{A_{ct}}$ slows down by assumption. **QED.**

The growth slowdown of GDP measured with the Fisher index that we described in the previous proposition is intimately linked to the behavior of the relative price of investment. To see this in the model, replace the TFP growth rates in equation (10) by relative prices:

$$\widehat{Y_t^F} \approx \Delta \log(Y_t^F) \approx \Delta \log(Y_t^X) + \frac{1}{2}\frac{P_c C}{X}\left(\frac{P_{ct-1}}{P_{ct}} - \frac{P_{ct}}{P_{ct-1}}\right).$$

GDP growth slows down along the ABGP with numeraire investment iff the relative price of consumption increases. Figures 1 and 2 show that this is borne out by the data too: the three measures of GDP growth started to diverge when the relative price of investment started to decrease around 1960; before 1960, the three measures of GDP growth remained close to each other.

Given the results of this section, we advocate the following strategy for using the two-sector model in quantitative work: (i) construct an ABGP in the model using investment as the numeraire; (ii) construct model GDP by applying the Fisher index to the ABGP; (iii) connect model GDP to data GDP. Following this strategy has two advantages: (i) using the numeraire X for the construction of the ABGP does not restrict the growth rates of consumption and the relative price to be constant; (ii) using the Fisher index results in a measure of GDP growth that is independent of the numeraire with which the ABGP is constructed and that detects the GDP growth slowdown.

2.5 Welfare changes

We start by defining an indirect utility function and an expenditure function that are needed to construct compensating expenditure. The household's problem gives rise to the standard value function V:[8]

$$V(K_t, A_{xt}, A_{ct}) \equiv \max_{C_t, X_t}\left\{\log(C_t) + \beta V\left(X_t + (1-\delta)K_t, \widehat{A}_x A_{xt}, \widehat{A}_{ct} A_{ct}\right) : \frac{A_{ct}}{A_{xt}} C_t + X_t \leq Y_t^X = K_t^\theta A_{xt}^{1-\theta}\right\},$$

[8]We could have written the value function also with C as the numeraire.

where we used that $A_{it+1} = \widehat{A}_{it} A_{it}$. It is convenient to summarize the state variables by $S_t \equiv (K_t, A_{xt}, A_{ct})$ and write $V(S_t) = V(K_t, A_{xt}, A_{ct})$.

Following Durán and Licandro (2017), we define an indirect utility function as:[9]

$$v(P_{ct}, Y_t^X; S_t) \equiv \max_{C_t, X_t} \left\{ \log(C_t) + \beta V(S_{t+1}) : P_{ct} C_t + X_t \leq Y_t^X \right\}.$$

where $S_{t+1} = (X_t + (1-\delta)K_t, \widehat{A}_x A_{xt}, \widehat{A}_{ct} A_{ct})$. The definition of the indirect utility function drops the constraints $Y_t^X = K_t^\theta A_{xt}^{1-\theta}$ and $P_{ct} = A_{ct}/A_{xt}$ for period t, but leaves them in place for all subsequent periods. Hence, it gives the value of the program also for realizations of income and relative prices that are not consistent with equilibrium in period t. Similarly, the minimum-expenditure function is defined as:

$$e(P_{ct}, v; S_t) = \min_{C_t, X_t} \left\{ P_{ct} C_t + X_t : \log(C_t) + \beta V\left(X_t + (1-\delta)K_t, \widehat{A}_x A_{xt}, \widehat{A}_{ct} A_{ct}\right) \geq v \right\}.$$

We now develop a measure of welfare changes that is based on compensating expenditure differences. The basic idea goes back to Fisher and Shell (1972), who generalized the index of Könus (1939) to situations in which preferences evolve over time. They emphasized that since utility is an ordinal concept, one must not compare the utility levels from periods $t-1$ and t. Instead, they calculated compensating expenditure levels by imposing indifference in terms of the *same* indirect utility function.[10] Building on the ideas of Weitzman (2000) and Licandro et al. (2002), Durán and Licandro (2017) showed how to apply the true quantity index of Fisher and Shell (1972) to the two-sector growth model with general recursive preferences. The basic insight is that it does not matter whether the time dependence of $u_t(\cdot)$ and $e_t(\cdot)$ arises from evolving preferences, as in Fisher and Shell's model, or from evolving state variables, as in the growth model.[11] While Durán and Licandro (2017) used *continuous* time, we develop a true quantity index for *discrete* time. Using discrete time is both more natural for connecting

[9]In dynamic contexts like ours there are two indirect utility functions: a period one and a present-value one. Our indirect utility function is a recursive formulation of the present-value indirect utility function, that is, the present value of the current and all future utilities that result under optimal behavior. In recursive formulation, that present value is a function of current income, current prices, and the current realizations of the state variables. To avoid confusion with the language used in Durán and Licandro (2017), we call the indirect value function an indirect utility function.

[10]Although the original index of Fisher and Shell is a true cost-of-living index, it is straightforward to apply the underlying principles to the construction of the corresponding true quantity index.

[11]Fisher and Shell dismissed the forward-looking perspective because yesterday's tastes are no longer relevant today. In contrast, the forward-looking perspective is meaningful when yesterday's indirect utility function represents past realizations of the state variables.

model data to NIPA data and also is more cumbersome because it requires a careful distinction between different reference periods. A novelty of our work is that this leads to two perspectives: the backward-looking (forward-looking) perspective uses prices and realizations of the state variables from "today" ("yesterday").

The backward-looking perspective compares today's observed expenditure, Y_t^X, with the compensating expenditure that make the household indifferent between having them at today's prices and having yesterday's expenditure at yesterday's prices. Imposing indifference in terms of today's utility, this gives $e\Big(P_{ct}, v(P_{ct-1}, Y_{t-1}^X; S_t); S_t\Big)$. The backward-looking true quantity index is:

$$\widehat{FS}_{t,t-1} \equiv \frac{Y_t^X}{e\Big(P_{ct}, v(P_{ct-1}, Y_{t-1}^X; S_t); S_t\Big)}.$$

The forward-looking perspective compares yesterday's observed expenditure, Y_{t-1}^X, with the compensating expenditure that make the household indifferent between having them at yesterday's prices and having today's expenditure at today's prices. Imposing indifference in terms of yesterday's utility, this gives $e\Big(P_{ct-1}, v(P_{ct}, Y_t^X; S_{t-1}); S_{t-1}\Big)$. The following forward-looking true quantity index is:

$$\widehat{FS}_{t-1,t} \equiv \frac{e\Big(P_{ct-1}, v(P_{ct}, Y_t^X; S_{t-1}); S_{t-1}\Big)}{Y_{t-1}^X}.$$

The Fisher-Shell true quantity index is the geometric average of the forward- and backward-looking indexes:

$$\widehat{FS}_t \equiv \sqrt{\widehat{FS}_{t-1,t} \cdot \widehat{FS}_{t,t-1}}.$$

The next proposition states one of our main results that the Fisher quantity index first-order approximates the Fisher-Shell true quantity index. While this result is a discrete-time version of the one of Durán and Licandro (2017), we use a more direct method of proof that provides additional first-order approximations for the Laspeyres and the Paasche indexes.

Proposition 6 *In the two-sector growth model,*

$$\widehat{FS}_{t-1,t} \approx \widehat{Y}_t^L,$$

$$\widehat{FS}_{t,t-1} \approx \widehat{Y}_t^P,$$

$$\widehat{FS}_t \approx \widehat{Y}_t^F.$$

Proof. We prove the claims by establishing that the Laspeyres and Paasche quantity indexes are first-order approximations to the forward-looking and backward-looking Fisher-Shell true quantity indexes:

$$\widehat{FS}_{t-1,t} \approx \frac{X_t + P_{ct-1}C_t}{Y_{t-1}^X}, \qquad \widehat{FS}_{t,t-1} \approx \frac{Y_t^X}{X_{t-1} + P_{ct}C_{t-1}}.$$

Two identities are helpful:

$$\frac{\partial e\Big(P_{ct}, v(P_{ct}, Y_t^X; S_t); S_t\Big)}{\partial v} \frac{\partial v(P_{ct}, Y_t^X; S_t)}{\partial Y_t^X} = 1, \tag{11}$$

$$\frac{\partial e\Big(P_{ct}, v(P_{ct}, Y_t^X; S_t); S_t\Big)}{\partial v} \frac{\partial v(P_{ct}, Y_t^X; S_t)}{\partial P_{ct}} = -C_t. \tag{12}$$

(11) follows by taking the derivative of $e_t(\cdot)$ with respect to Y_t^X and rearranging. (12) follows from Roy's identity,

$$\left[\frac{\partial v(P_{ct}, Y_t^X; S_t)}{\partial Y_t^X}\right]^{-1} \frac{\partial v(P_{ct}, Y_t^X; S_t)}{\partial P_{ct}} = -C_t,$$

and (11).

We establish that $\widehat{FS}_{t-1,t} \approx \widehat{Y}_t^L$ by showing that $e\Big(P_{ct-1}, v(P_{ct}, Y_t^X; S_{t-1}); S_{t-1}\Big) \approx X_t + P_{ct-1}C_t$. Interpreting $e\Big(P_{ct-1}, v(P_{ct}, Y_t^X; S_{t-1}); S_{t-1}\Big)$ as a function of (P_{ct}, Y_t^X) and linearizing around (P_{ct-1}, Y_{t-1}^X) gives:

$$\begin{aligned} e\Big(P_{ct-1}, v(P_{ct}, Y_t^X; S_{t-1}); S_{t-1}\Big) \approx\ & e\Big(P_{ct-1}, v(P_{ct-1}, Y_{t-1}^X; S_{t-1}); S_{t-1}\Big) \\ & + \frac{\partial e\Big(P_{ct-1}, v(P_{ct-1}, Y_{t-1}^X; S_{t-1}); S_{t-1}\Big)}{\partial v} \frac{\partial v(P_{ct-1}, Y_{t-1}^X; S_{t-1})}{\partial Y_{t-1}^X}(Y_t^X - Y_{t-1}^X) \\ & + \frac{\partial e\Big(P_{ct-1}, v(P_{ct-1}, Y_{t-1}^X; S_{t-1}); S_{t-1}\Big)}{\partial v} \frac{\partial v(P_{ct-1}, Y_{t-1}^X; S_{t-1})}{\partial P_{ct-1}}(P_{ct} - P_{ct-1}). \end{aligned}$$

Using (11)–(12) and that $e\Big(P_{ct-1}, v(P_{ct-1}, Y^X_{t-1}; S_{t-1}); S_{t-1}\Big) = Y^X_{t-1}$ gives:

$$
\begin{aligned}
e\Big(P_{ct-1}, v(P_{ct}, Y^X_t; S_{t-1}); S_{t-1}\Big) &\approx e\Big(P_{ct-1}, v(P_{ct-1}, Y^X_{t-1}; S_{t-1}); S_{t-1}\Big) + (Y^X_t - Y^X_{t-1}) - C_{t-1}(P_{ct} - P_{ct-1}) \\
&= Y^X_t - C_{t-1}(P_{ct} - P_{ct-1}) \\
&= X_t + P_{ct-1}C_t + (C_t - C_{t-1})(P_{ct} - P_{ct-1}) \\
&\approx X_t + P_{ct-1}C_t,
\end{aligned}
$$

where the last step leaves out the second-order terms.

We establish that $\widehat{FS}_{t,t-1} \approx \widehat{Y}^P_t$ by showing that $e\Big(P_{ct}, v(P_{ct-1}, Y^X_{t-1}; S_t); S_t\Big) \approx X_{t-1} + P_{ct}C_{t-1}$. The proof follows by interpreting $e\Big(P_{ct}, v(P_{ct-1}, Y^X_{t-1}; S_t); S_t\Big)$ as a function of (P_{ct-1}, Y^X_{t-1}), linearizing it around (P_{ct}, Y^X_t), and following the same steps as before:

$$
\begin{aligned}
e\Big(P_{ct}, v(P_{ct-1}, Y^X_{t-1}; S_t); S_t\Big) &\approx e\Big(P_{ct}, v(P_{ct}, Y^X_t; S_t); S_t\Big) \\
&\quad + \frac{\partial e\Big(P_{ct}, v(P_{ct}, Y^X_t; S_t); S_t\Big)}{\partial v} \frac{\partial v(P_{ct}, Y^X_t; S_t)}{\partial Y^X_t}(Y^X_{t-1} - Y^X_t) \\
&\quad + \frac{\partial e\Big(P_{ct}, v(P_{ct}, Y^X_t; S_t); S_t\Big)}{\partial v} \frac{\partial v(P_{ct}, Y^X_t; S_t)}{\partial P_{ct}}(P_{ct-1} - P_{ct}) \\
&= Y^X_{t-1} - C_t(P_{ct-1} - P_{ct}) \\
&\approx X_{t-1} + P_{ct}C_{t-1}.
\end{aligned}
$$

QED

We end this section by pointing out that the Fisher-Shell true quantity index abstracts from several relevant features of reality that affect welfare, including inequality, leisure, and life expectancy. Jones and Klenow (2016) proposed a broader welfare measure that takes these features into account and implemented it for a set of countries.

3 Three-sector Growth Model

We now disaggregate consumption into goods and services and study structural change from the goods to the services sector. Additional aggregation issues then arise from changes in the composition of the consumption expenditures. We will see that these composition changes importantly affect the behavior of

GDP growth measured with the Fisher quantity index. For simplicity, we keep the investment sector as it was in the two-sector model. We note that the results that follow would continue to hold in two alternative specifications: a simpler two-sector growth model that does not have capital; a more elaborate four-sector model in which structural change takes place in both consumption and investment; see Herrendorf et al. (2018) for an analysis of the latter.

We omit the full description of the three-sector model and highlight only the parts that are different from the two-sector model. There are now production functions for consumption goods, C_{gt}, and services, C_{st}:

$$C_{gt} = K_{gt}^{\theta}\left(A_{gt}L_{gt}\right)^{1-\theta},$$
$$C_{st} = K_{st}^{\theta}\left(A_{st}L_{st}\right)^{1-\theta}.$$

The period utility now equals:

$$C_t = u(C_{gt}, C_{st}), \tag{13}$$

where u satisfies the standard regularity conditions.

Similar results to (3), (4), and (5) hold for $i \in \{g, s\}$:

$$P_{it} \equiv \frac{p_{it}}{p_{xt}} = \left(\frac{A_{xt}}{A_{it}}\right)^{1-\theta},$$
$$C_{it} = K_t^{\theta} A_{it}^{1-\theta} L_{it},$$
$$Y_t^X \equiv P_{gt}C_{gt} + P_{st}C_{st} + X_t = K_t^{\theta} A_{xt}^{1-\theta}.$$

Figure 3 shows that the empirically relevant case is $\widehat{A}_{xt} > \widehat{A}_{gt} > \widehat{A}_{st}$.

There are two known classes of period-utility functions (13) for which an ABGP with structural change from goods to services exists: the homothetic CES utility functions with C_{gt} and C_{st} being complements studied by Ngai and Pissarides (2007); non-Gorman utility functions studied by Boppart (2014) and Alder et al. (2017).[12] In what follows, we will study the behavior of GDP growth with these utility functions. Given the results from the two-sector model, we consider only the numeraire X. To be

[12]Since the last two papers started from indirect utility functions, the statement should be interpreted as referring to utility functions that gives rise to their demand system.

able to obtain sharp, analytical results, we also assume that all $\widehat{A}_i$ are constant.

Proposition 7 *Suppose that X is the numeraire, $\widehat{A}_i$ is constant for $i \in \{x, g, s\}$, and $\widehat{A}_g > \widehat{A}_s$. If an ABGP with structural change from goods to services exists, then GDP growth measured by $\widehat{Y}_t^F$ slows down along the ABGP.*

Proof. The first step is to recognize that L_x is constant along a ABGP:

$$L_{xt} = \frac{K_t^\theta A_x^{1-\theta} L_{xt}}{K_t^\theta A_{xt}^{1-\theta}} = \frac{X_t}{Y_t^X} = \frac{1}{\frac{P_{ct}C_t}{X_t} + 1},$$

which is constant along ABGP.

We show the claim by showing that both the Laspeyres and the Paasche index decline along ABGP. In the three-sector model, the Laspeyres index is:

$$\begin{aligned}
\widehat{Y}_t^L &\equiv \frac{P_{gt-1}C_{gt} + P_{st-1}C_{st} + X_t}{P_{gt-1}C_{gt-1} + P_{st-1}C_{st-1} + X_{t-1}} = \frac{\frac{P_{gt-1}}{P_{gt}}P_{gt}C_{gt} + \frac{P_{st-1}}{P_{st}}P_{st}C_{st} + X_t}{P_{gt-1}C_{gt-1} + P_{st-1}C_{st-1} + X_{t-1}} \\
&= \frac{\left(\frac{\widehat{A}_{gt}}{\widehat{A}_{xt}}\frac{A_{xt}}{A_{gt}}\right)^{1-\theta} C_{gt} + \left(\frac{\widehat{A}_{st}}{\widehat{A}_{xt}}\frac{A_{xt}}{A_{st}}\right)^{1-\theta} C_{st} + X_t}{K_{t-1}^\theta A_{xt-1}^{1-\theta}} \\
&= \frac{\left(\frac{\widehat{A}_g}{\widehat{A}_x}\right)^{1-\theta} K_t^\theta A_{xt}^{1-\theta} L_{gt} + \left(\frac{\widehat{A}_s}{\widehat{A}_x}\right)^{1-\theta} K_t^\theta A_{xt}^{1-\theta} L_{st} + K_t^\theta A_{xt}^{1-\theta} L_{xt}}{K_{t-1}^\theta A_{xt-1}^{1-\theta}} \\
&= \widehat{A}_x \left\{ \left[\left(\frac{\widehat{A}_g}{\widehat{A}_x}\right)^{1-\theta} - \left(\frac{\widehat{A}_s}{\widehat{A}_x}\right)^{1-\theta} \right] L_{gt} + \left(\frac{\widehat{A}_s}{\widehat{A}_x}\right)^{1-\theta} (1 - L_x) + L_x \right\},
\end{aligned}$$

where we used that $L_{gt} + L_{st} + L_x = 1$ and $\widehat{K}_t = \widehat{A}_x$ along ABGP. Since $\widehat{A}_g > \widehat{A}_s$ and L_{gt} declines along the ABGP with structural change, $\widehat{Y}_t^L$ declines.

Using the same steps gives:

$$\begin{aligned}
\widehat{Y}_t^P &= \frac{P_{gt}C_{gt} + P_{st}C_{st} + X_t}{\frac{P_{gt}}{P_{gt-1}}P_{gt-1}C_{gt-1} + \frac{P_{st}}{P_{st-1}}P_{st-1}C_{st-1} + X_{t-1}} \\
&= \frac{K_t^\theta A_{xt}^{1-\theta}}{K_{t-1}^\theta A_{xt-1}^{1-\theta} \left[\left(\frac{\widehat{A}_x}{\widehat{A}_g}\right)^{1-\theta} L_{gt-1} + \left(\frac{\widehat{A}_x}{\widehat{A}_s}\right)^{1-\theta} L_{st-1} + L_{xt-1} \right]} \\
&= \frac{\widehat{A}_x}{\left[\left(\frac{\widehat{A}_x}{\widehat{A}_g}\right)^{1-\theta} - \left(\frac{\widehat{A}_x}{\widehat{A}_s}\right)^{1-\theta} \right] L_{gt-1} + \left(\frac{\widehat{A}_x}{\widehat{A}_s}\right)^{1-\theta} (1 - L_x) + L_x}.
\end{aligned}$$

Since $\widehat{A}_g > \widehat{A}_s$ and L_{gt} declines along the ABGP with structural change, $\widehat{Y}_t^P$ declines. **QED**

Proposition 7 implies that GDP growth measured with the Fisher index slows down along any ABGP. This occurs because the Fisher index picks up the effects of Baumol's Cost Disease resulting from the reallocation from the goods sector with high productivity growth to the services sector with low productivity growth. Figure 2 showed that the overall growth slowdown has been large in the postwar U.S. Several papers showed that the contribution of Baumol's cost disease to the overall growth slowdown is sizeable; see for example Duernecker et al. (2017).

If the period utility is homothetic, then we can go further than Proposition 7:

Proposition 8 *Suppose that X is the numeraire, $\widehat{A}_i$ is constant for $i \in \{x, g, s\}$, and $\widehat{A}_g > \widehat{A}_s$. If $u(C_{gt}, C_{st})$ is homothetic, then: (i) the growth rate of the price of aggregate consumption, $\widehat{P}_{ct}$, increases over time; (ii) the growth rate of welfare measured by $\widehat{FS}_t$ slows down along the ABGP.*

Proof. We start with the proof of claim (i). Let $C_t \equiv u(C_{gt}, C_{st})$ be aggregate (composite) consumption. The expenditure function is the minimum cost of buying (C_{gt}, C_{st}) to achieve consumption level C_t:

$$E_t \equiv E(P_{gt}, P_{st}, C_t) \equiv \min_{C_{gt}, C_{st}} \left\{ P_{gt}C_{gt} + P_{st}C_{st} : u(C_{gt}, C_{st}) \geq C_t, P_{gt}, P_{st} \geq 0 \right\}.$$

If preferences are homothetic, then the expenditure function can be written as

$$E(P_{gt}, P_{st}, C_t) = P_{ct}C_t \equiv P_c(P_{gt}, P_{st})C_t.$$

See Shephard (1953), Chapter 4 for a proof. Shephard also proves that $P_{ct} \equiv P_c(P_{gt}, P_{st})$ is homothetic.

Next, we show that if preferences are homothetic, then:

$$\frac{\partial P_c(P_{gt}, P_{st})}{\partial P_{it}} \frac{P_{it}}{P_{ct}} = \frac{P_{it}C_{it}}{P_{ct}C_t}. \tag{14}$$

This follows from Sheppard's lemma which states that

$$C_{it} = \frac{\partial E(P_{gt}, P_{st}, C_t)}{\partial P_{it}} = \frac{\partial P_c(P_{gt}, P_{st})}{\partial P_{it}} C_t,$$

implying

$$\frac{\partial P_c(P_{gt}, P_{st})}{\partial P_{it}} = \frac{C_{it}}{C_t}.$$

Multiplying both sides with P_{it}/P_{ct} proves the claim.

To study the dynamics of P_{ct} over time, we linearise $P_{ct+1} = P_c(P_{gt+1}, P_{st+1})$ around (P_{gt}, P_{st}):

$$P_{ct+1} \approx P_c(P_{gt}, P_{st}) + \frac{\partial P_c(P_{gt}, P_{st})}{\partial P_{gt}}(P_{gt+1} - P_{gt}) + \frac{\partial P_c(P_{gt}, P_{st})}{\partial P_{st}}(P_{st+1} - P_{st}),$$

implying

$$\frac{P_{ct+1} - P_{ct}}{P_{ct}} \approx \frac{\partial P_c(P_{gt}, P_{st})}{\partial P_{gt}} \frac{P_{gt}}{P_{ct}} \frac{P_{gt+1} - P_{gt}}{P_{gt}} + \frac{\partial P_c(P_{gt}, P_{st})}{\partial P_{st}} \frac{P_{st}}{P_{ct}} \frac{P_{st+1} - P_{st}}{P_{st}}.$$

Using (14), this becomes:

$$\frac{P_{ct+1} - P_{ct}}{P_{ct}} \approx \frac{P_{st+1} - P_{st}}{P_{st}} + \frac{P_{gt}C_{gt}}{P_{ct}C_t}\left(\frac{P_{gt+1} - P_{gt}}{P_{gt}} - \frac{P_{st+1} - P_{st}}{P_{st}}\right)$$

By assumption, the growth rates of the prices of goods and services relative to investment are constant. Moreover, $(P_{gt}C_{gt})/(P_{ct}C_t)$ and the term in brackets decrease over time. Hence the growth rate of P_{ct} increases over time.

The proof of claim (ii) follows by going through the exact same steps as in the two-sector model. Therefore, we omit it. **QED**

The proposition shows that homothetic utility allows for two results in addition to the more general result of a GDP growth slowdown. First, structural change implies that the price of aggregate consumption relative to investment increases, which, of course, is the condition for the GDP growth slowdown from the two-sector model. Remarkably, this condition is satisfied although we assume in this section that that all sectoral TFPs grow at constant rates. Second, as in the two-sector model, the GDP growth slowdown translates into a welfare growth slowdown.[13]

Three papers are closely related to the last two propositions. Ngai and Pissarides (2004) mentioned that Baumol's Cost Disease can lead to a GDP growth slowdown when GDP growth is calculated with constant relative prices. However, they did not pursue the growth slowdown further but framed their

[13] It is unknown whether the welfare result extends to non-homothetic utility functions; see Diewert (1976) and Diewert and Mizobuchi (2009) for more discussion. The challenge with non-homothetic utility functions is that the price index depends also on the growing level of consumption, implying that the first-order Taylor approximations contains additional terms that are unrelated to the Fisher index.

entire analysis in terms of a balanced growth path and constant GDP growth measured in a current numeraire. Moro (2015) provided an interesting model in which Baumol's Cost Disease reduces GDP measured with the Fisher index. His analysis differs from our analysis because he focused on the role of differences in the sectoral intermediate-input shares in a cross section of middle- and high-income countries. In independent work, Leon-Ledesma and Moro (2017) asked to what extent structural change may lead to violations of the Kaldor (1961) growth facts. In their simulation results, based on the model of Boppart (2014), structural change leads to a growth slowdown of GDP measured with the Fisher index.

Although in some aspects our work is similar to these three papers, two features set what we have done apart: we have analytically characterized the behavior of GDP growth measured with a numeraire and with the Fisher index along an aggregate balanced growth path with structural change; we have proven that with homothetic utility the slowdown affects not only GDP growth measured with the Fisher index but also welfare growth.

Duernecker et al. (2017) study the natural follow up question whether GDP growth will slow further in the coming years. A particular worry is that the slowest-growing services industries could take over the economy. They find that substitutability within the service sector prevents that from happening.

4 Conclusion

Which aggregation method is preferable to analyze multi-sector growth models with structural change and connect them to the data from the NIPA? We have shown that the numeraire investment offers the least restrictive way of constructing an ABGP, but that the Fisher index has the advantage that the implied GDP growth is independent of the choice of numeraire, captures the GDP growth slowdown resulting from Baumol's Cost Disease, and is a measure of welfare changes if utility is homothetic. We have advocated to proceed in three steps: (i) construct the model's BGP with the numeraire investment; (ii) calculate model GDP with the Fisher index; (iii) connect this measure of model GDP to the NIPA.

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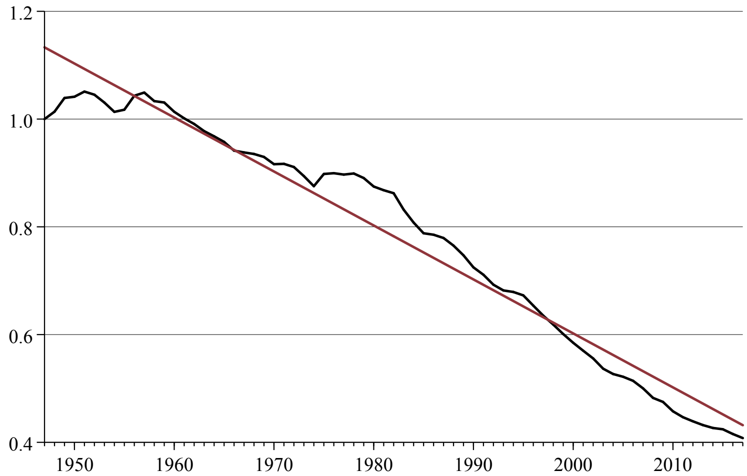
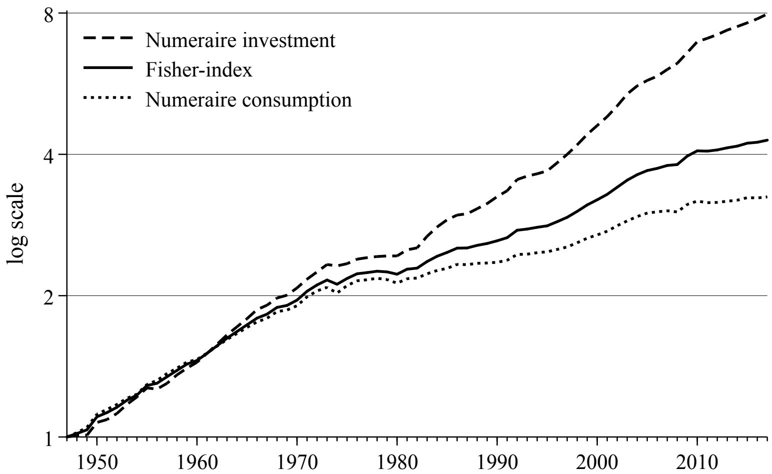
Figures and Tables

Figure 1: The Price of Investment Relative to Consumption in the U.S. (1947=1)



Source: NIPA, Bureau of Economic Analysis, own calculations. Investment: Private fixed investment and consumer durables, Consumption: Private nondurable goods and services consumption.

Figure 2: U.S. GDP per hour with different aggregation methods

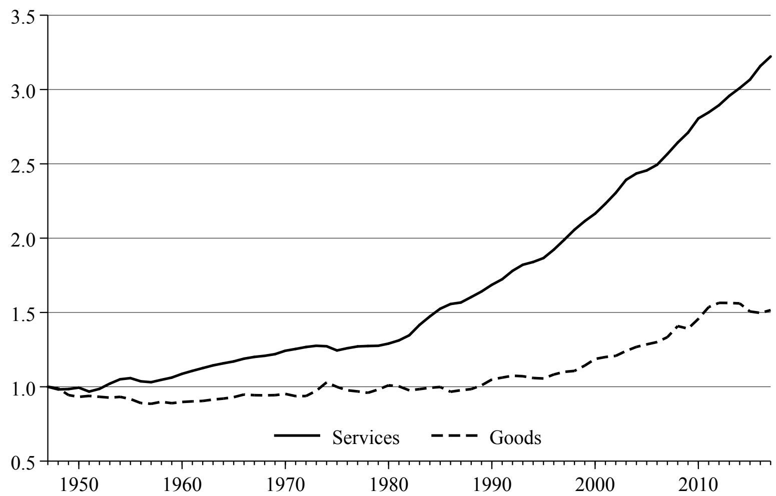


Source: NIPA, Bureau of Economic Analysis, "Hours Worked in Total U.S. Economy and Subsectors"; BLS; own calculations. Investment: Private fixed investment and consumer durables. Consumption: Private nondurable goods and services consumption. GDP deflator: Fisher-index of private fixed investment, consumer durables, private nondurable goods, and services consumption.

Table 1: U.S. GDP per hour 1947–2017

Units	Average annual growth rate	Level after 70 years
C	1.67	0.83
F	1.94	1.00
X	2.97	2.02

Figure 3: The Price of Goods and Services relative to Investment in the U.S. (1947=1)



Source: NIPA, Bureau of Economic Analysis, own calculations. Goods: Private nondurable consumption, Services: Private services consumption, Investment: Private fixed investment and consumer durables.