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Codes JEL: I32 (Measurement and Analysis of Poverty), O15 (Income Distribution), D31 (Personal Income Distribution)

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Abstract

Living standard indicators are complex nonlinear statistics based on fundamental components (income or consumption data, prices, household characteristics and environment). The statistical distributions of these components, which are often incompletely obseved, are central to study utilitarian social evaluation functions (USEF) and associated inequality measures. One important case on which we focus is when the living standard indicator can be represented as a ratio of two weakly statistically associated components. Our typical example is that of the effect of the distribution of price indices on USEFs or on inequality measures.

First, we provide theoretical decompositions into the effect of change in price index dispersion and the effect of change in aggregate level of prices, with known directions of components. Second, new notions describing aversion and sensitivity of USEFs to price dispersion are derived. Third, Atkinson's inequality measures are shown to belong to an interval whose bounds are inequality measures without price index dispersion. These bounds can be estimated without disposing of a sample of price indices. Fourth, a higher dispersion of prices at constant arithmetic (respectively harmonic) mean price level is shown to increase (respectively decrease) social welfare and reduce (respectively augment) inequality. Finally, we illustrate the theoretical results by using data from Rwanda.

Résumé

Les indicateurs de niveau de vie sont des statistiques nonlinéaires reposant sur plusieurs composantes plus fondamentales (revenus ou consommation, prix, caractéristiques et les situations des ménages). Les distributions statistiques de ces composantes, qui sont souvent incomplètement observé, sont centrales pour la compréhension du niveau des fonctions utilitaristes de bien-être social (FUBES) et des mesures associées d'inégalité. Nous nous intéressons dans ce travail au cas où l'indicateur de niveau de vie peuvent être représenté comme un ratio de deux composantes indépendantes.

Notre example typique est celui de l'effet de la distribution des indices de prix sur le neveau de FUBES ou de measures d'inégalité. D'abord, nous fournissons une décomposition théorique en effet du changement de la dispersion des indices de prix et effet du changement du niveau aggrégé des prix, avec signe connu des composantes. Puis, nous proposons de nouvelles notions décrivant l'aversion ou la sensibilité des FUBES á la dispersion des prix. Nous montrons également que les mesures d'inégalité d'Atkinson appartiennent à un intervalle dont les bornes sont des mesures d'inégalité sans dispersion des indices de prix. Ensuite, nous montrons que pour de nombreuses FUBES usuelles une plus forte dispersion des prix pour une moyenne arithmétique (resp. harmonique) constante des indices de prix peut augmenter (resp. diminuer) le bien-être social et rédure (resp. augmenter) l'inégalité. Finalement, nous illustrons les résultats théoriques à partir de données du Rwanda.

1 Introduction

In social welfare studies, living standard indicators incorporated in a social evaluation function are generally taken as given. When living standards have been decomposed, it has been done by using additive specifications, mostly to distinguish several sources of income, or risk[1]. However, living standard indicators can be better seen as a nonlinear combination of fundamental components: on the one hand income or consumption data, and on the other hand variables describing prices, household characteristics and environment.Unfortunately, many of these elements are unobserved and vary across households. Social researchers are therefore constrained to work under severe assumptions of incomplete information. What can be said about the impact of the distributions of unobserved variables on social welfare? Can we provide intervals with observable bounds in which the true social welfare indicators would be included?

In this paper we investigate a new direction of analysis for social welfare indicators. Our approach is based on the simultaneous consideration of: 1) the functional form of the composite living standard function; 2) the statistical links between the different components of the living standard variable; 3) the variational inequalities implied by generalised concavity properties of the kernel of the social evaluation function. Naturally, this work programme is too general to produce immediate results. Therefore, we start the analysis with the most obvious question. Linear problems have already been well studied and we shall not concentrate on them. In contrast, the ratio function, which plays a fundamental role in the definition of the living standard variable, for example with the incorporation of price indices or equivalence scales, has not been studied. We focus on this case first. Second, we study situations in which the numerator and denominator of the living standard variable are weakly statistically linked for the utilitarian social evaluation function (USEF). We postpone the investigation of more specific types of statistical dependence. Third, we investigate kernels of the USEF that are

[1] e.g. Khilstrom et al. (1981), Shorrocks (1982), Chantreuil and Trannoy (1999).

concave in the real living standard, which correspond to desirable transfer axioms and are convex in the variable at the denominator of the living standard variable, which we shall show is satisfied for the usual functional forms.

We study in this paper the consequences of the ratio functional form for living standard variables in USEFs and in inequality measures, by simultaneously taking into account the distributions of the numerator and the denominator variables. USEFs can be related to Pareto optimal situations, are easy to manipulate and enjoy separability properties. It has been shown that the USEFs are the only social evaluation functions satisfying various sets of attractive axioms[2].

Because the price index is present at the denominator of the living standard, if the price deflation, for prices that distinct households face in separate situations, is inaccurate, then apparent welfare differences between households may derive from non-deflated price differences[3]. In LDCs, because of substantial transport costs and transaction costs as well as deficient information, prices may be very variable across regions. In industrial countries, spatial price dispersion may also occur. Housing rents, or goods subject to local taxes are commodities associated with high spatial price dispersion. The proximity of production sites is another source of geographical dispersion in prices.

The influence of spatial price deflation on social welfare has not attracted

[2]See for example Harsanyi (1955), d'Aspremont and Gevers (1977), Sen (1986), Fleurbaey (1986), Chakravarty (1990), Cowell (1993), Blackorby, Bossert and Donaldson (1999b), Bossert and Weymark (2000). Thus, any welfarist ex-ante social evaluation functional satisfying anonymity and the weak Pareto principle is Utilitarian.

[3]In this paper, we examine the price dispersion such that it appears through the dispersion of price indices. Indeed, price indices are sufficient statistics for the calculation of real living standards when nominal living standards are known. Changes in price dispersion across products are not treated, even if they contribute to changes in dispersion of price indices. Moreover, we do not deal in this paper with the already studied effect on individual welfare of the instability in individual prices (Turnovsky, Shalit and Schmitz (1980), Helms (1985), Ebert (1994)). We emphasize that, although these papers deal with a similar topic, their results rest on different mathematical bases and are not directly related.

much attention in the theoretical literature[4]. Applied welfare economists[5] generally choose to deflate living standards using Laspeyres or Paasche price indices (more rarely using true price indices[6]) to account for geographical price dispersion. Price indices have been extensively studied in the theoretical literature[7].

The results that we present are useful on five grounds. Firstly, they help understand the impact of the distribution of price indices on USEFs and on inequality measures. Notably, we derive decompositions in terms of an aggregate level effect and a dispersion effect of the price indices. Secondly, they reveal in which conditions the price index dispersion is socially advantageous, which is a new result. Thirdly, they allow the analysis of specific USEFs in terms of their sensitivity to the price index distribution. For small dispersion and a given USEF, price index distributions can be ordered for their impact on social welfare by using only simple central tendency and dispersion statistics for price indices. Fourthly, they exhibit the special roles of harmonic and arithmetic means of price indices in welfare analysis. Publishing such spatial price statistics would be useful to welfare analysis. Fifthly, they provide lower and upper bounds for inequality measures under price index dispersion. These bounds can be expressed as inequality measures with deflation based on an aggregate mean of price indices. Therefore, they do not necessitate the observation of a national sample of price indices.

We define the general framework describing the effects of the price index distribution on USEFs in section 2. We analyse these theoretical effects in the case of weak statistical association of price indices and nominal living standards in section 3. We derive notions of aversion and sensitivity of USEFs to the price dispersion in section 4. In section 5, we study the consequences of these results for inequality indices. We present an empirical illustration using data from Rwanda in 1983 in section 6. Finally, we conclude in section 7. All proofs are in the appendix.

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[4]Roberts (1980), Slivinski (1983) and Blackorby, Bossert and Donaldson (1999a) examine when welfare prescriptions can be independent from the price configuration in the economy. They all find that this is impossible, except for unsatisfactory welfare indicators.

[5]e.g. Muellbauer (1974), Atkinson (1975), Glewwe (1990), Slesnick (1993).

[6]A true price index is a ratio of cost functions that accounts for household responses to prices. We do not deal in this paper with substitution effects in price indices.

[7]e.g. Fisher and Shell (1972), Pollak (1978), Diewert (1981), Foss, Manser and Young (1982), Baye (1985), Diewert (1990), Selvanathan and Rao (1995).

2 The General Framework

2.1 Generalities

In welfare analysis, accounting for price differences across households and periods implies that the living standard indicator for household i at period t is better defined as

$$y_{it} = \frac{x_{it}}{I_{it}} \tag{1}$$

where I_{it} is the price index associated with household i and period t. The USEFs are defined by

$$W = \int_0^{+\infty} u(y)\, d\mu(y) \tag{2}$$

where μ is the probability distribution of real living standards y and u is a kernel function (u(y) represents the welfare of a household with real living standard y). To study the role of the price index distribution, we consider the joint distribution of x and I, F(x, I). In all this paper we assume that all the considered integrals are finite, which is satisfied with actual data and usual functional forms for u.

We first compare the USEF without price deflation (assuming I = 1 for all households), with the USEF taking into account the effect of the price index distribution (using deflated living standard indicators, y = x/I). To distinguish the contributions of the distributions of x and I, we condition on x.

Definition 1 *The variation in the USEF caused by the price deflation is*

$$\Delta W = \int_0^{+\infty} \int_0^{+\infty} u(\frac{x}{I})\ dF_2(I \mid x)\, dF_1(x) - \int_0^{z} u(x)\ dF_1(x) \tag{3}$$

with F_1 the c.d.f. of the marginal distribution of x and F_2 the c.d.f. of the distribution of I conditionally on x.

One can also consider that eq. 3 describes the situation where x is a living standard indicator for which a crude price index has been used, while y corresponds to a more accurate deflation. Another interpretation is that eq. 3 describes the difference between USEFs with and without price index uncertainty (although without accounting for household expectations or responses to this uncertainty). Alternatively, under the assumption that the responses of nominal living standards to price changes can be neglected, as it is plausible in the very short term, ΔW can be considered as the change in social welfare implied by a modification of the price index distribution. In this interpretation, x is the real living standards before price changes and x/I is the real living standards after price changes. To simplify, we emphasise in this paper the first interpretation of mere deflation effect[8].

2.2 The general case

a) Majoration results

The important factors for this article are the concavity properties of the kernel function of the USEF, which are related to transfer axioms[9]. In particular, the regressive transfer axiom ensures that the kernel function of USEFs is concave. To derive results relatively to the usual level of price stabilisation that is the arithmetic mean, we need to consider the following function.

Definition 2 *$K_x(I) \equiv u(\frac{x}{I})$ defines the kernel function of the USEF considered as a function of I conditionally on the value of x.*

Then, we can derive the following majoring results:

Proposition 1 *If a social evaluation function W can be written in the following form*

$$W = \int_0^{+\infty} u(y)\, d\mu(y) \tag{4}$$

where μ is the distribution of living standard y and u is the kernel function, then,

a) if u is concave, we have

$$W \leq \int_0^{+\infty} u \left[\int_0^{+\infty} \frac{x}{I} dF_2(I \mid x) \right] dF_1(x) \tag{5}$$

$$\leq u \left[\int_0^{+\infty} \int_0^{+\infty} \frac{x}{I} dF_2(I \mid x) dF_1(x) \right] = u(\bar{y}) \tag{6}$$

[8]Note that the problem under study is different from the consideration of multidimensional welfare analysis in which the utility function would admit (x, I) as argument. Indeed, there is no direct ethical property of u attached to variable I. It is truly a problem of decomposition of the living standard variable.

[9]e.g. Donaldson and Weymark (1986), Chakravarty (1990).

b) if u is convex, we have the opposite inequalities.

c) if K_x is convex, we have

$$W \geq \int_0^{+\infty} u \left[\frac{x}{\int_0^{+\infty} I \, dF_2(I \mid x)} \right] dF_1(x) \tag{7}$$

d) if K_x is concave, we have the opposite inequality.

Ineq. 5 (respectively 7) shows that the actual USEF is majored (respectively minored) by a USEF calculated by using deflated living standards where the deflation is made using a price index specific to each household. Ineq. 6 shows that the majoring can be pushed one step further with W smaller than the utility of the representative consumer with living standard the arithmetic mean of living standards. This well known result is useful to discuss distortions brought in welfare analysis by the hypothesis of the representative agent, and here also the position of this hypothesis with respect to the hypothesis of no price index dispersion. Alternatively, $W < u(\bar{y})$ is an analogue of the comparison between utility outcomes with/without uncertainty.

b) The lognormal case

We discuss in appendix 2 the reasons why the hypothesis of independence of nominal living standards (n.l.s.) and price indices (p.i.) is credible as well as the reasons why p.i. and n.l.s. could be statistically linked. All in all, we believe that in many cases it is reasonable to assume some weak statistical association, when there is no compelling evidence of association between n.l.s. and p.i. However, we first devote some attention to the case of dependence. This case is not likely to lead to unambiguous results as we now show by using a lognormal distributional assumption[10] as an approximation of the distribution of (x, I).

Proposition 2 *Let us assume that (x, I) follows a lognormal distribution*

[10] Other distributional assumptions yield similar outcomes.

$$LN\left[\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right].$$

Then,

$$\Delta W = \Delta W^0$$
$$+\rho.[\int_0^{+\infty}\int_0^{+\infty} u(\tfrac{x}{I})\frac{1}{2\pi\sigma_1\sigma_2 I x}$$
$$\exp\left\{-\frac{(\ln(x)-m_1)^2}{2\sigma_1^2} - \frac{(\ln(I)-m_2)^2}{2\sigma_2^2}\right\}\frac{(\ln(x)-m_1).(\ln(I)-m_2)}{\sigma_1\sigma_2}\, dI\, dx] + O(\rho^2)$$

where ΔW^0 is the bias in the USEF when $\rho = 0$, and ρ is the coefficient of correlation of the logarithms of x and I.

When the values of distribution parameters (m_1, m_2, σ_1, σ_2) can be estimated, the first-order expansion can be used to study the quality of the approximation ΔW^0. The value of the integrals in the second term of the Taylor expansion directly depends on the kernel function of the USEF as well as on the values of parameters m_1, m_2, σ_1, σ_2, and implicitly on all parameters through the presence of x and I under the integral. Then, the coefficient of ρ is not of determined sign, and little unambiguous information seems to be deducible from such formula. Similar difficulties arise when the price correction is implemented using a constant aggregate price index. In all cases, the derived formula makes ΔW^0 appear as a polar case worth consideration when the statistical link between I and x is small. We study this case without distribution assumption in section 3.

3 The Case of Weak Statistical Association for the USEF

We now concentrate on the case of 'weak statistical association for the USEF' of nominal living standards and price indices. We show in that situation that under reasonable generalised concavity conditions, we can explicit several signed contributions of the price index distribution to the USEF. The discussion is related to analyses of risk in additive indicators of social welfare[11]. We first need to determine what we mean by 'weak statistical association

[11]e.g. Atkinson (1970), Rotschild-Stiglitz (1970), Ravallion (1988), Lambert (1993), Foster and Sen (1997).

of the USEF'. This is the object of the next definition and of the following proposition.

Definition 3 *a)* ***Condition C1:*** *x and I are said to be* ***weakly statistically associated at the numerator for the USEF*** *if*

$\int_0^{+\infty} u\left[x\int_0^{+\infty}\frac{1}{I}dF_2(I\mid x)\right]dF_1(x) = \int_0^{+\infty} u\left[x\int_0^{+\infty}\frac{1}{I}dF_2(I)\right]dF_1(x)$,

where $F_2(.|x)$ denotes the distribution of I conditionally on x, while $F_2(.)$ denotes the marginal distribution of I.

b) Condition C2: *x and I are said to be* ***weakly statistically associated at the denominator for the USEF*** *if*

$\int_0^{+\infty} u\left[\frac{x}{\int_0^{+\infty} I dF_2(I\mid x)}\right]dF_1(x) = \int_0^{+\infty} u\left[\frac{x}{\int_0^{+\infty} I dF_2(I)}\right]dF_1(x)$.

c) Condition C3: *x and I are said to be* ***weakly statistically associated for the USEF*** *if they satisfy C1 and C2.*

Proposition 3 *A sufficient condition for C3 is that x and I are independent.*

Of course, conditions C1, C2, C3 can be considerably weaker than the independence. However, for the clarity of economic argument we shall often express our results as being obtained under independence. As mentioned above, the credibility of this assumption is discussed in the appendix.

The price index intervening in C1 at the right-hand-side of the equation is the harmonic mean of all price indices, H. The following Proposition 3.3. implies that under C1 (respectively C2) and with u concave (respectively K_x convex) the deflated USEF can be majored (respectively minored) by a USEF calculated without price dispersion as soon as the aggregate level of prices is adequately defined.

Proposition 4 *Under C1:*

a) if u is concave, then

$$W \leq \int_0^{+\infty} u\left(\frac{x}{H}\right) dF_1(x) \leq u\left(\bar{y}\right) \tag{8}$$

b) if u is convex, then the inequalities have the other direction.
Under C2:
c) if K_x is convex, then

$$W \geq \int_0^{+\infty} u\left(\frac{x}{\bar{I}}\right) \, dF_1(x) \tag{9}$$

d) if K_x is concave, then the inequality has the other direction.

We can use the results established in Proposition 3.3 to decompose the deviation of the deflated USEF from the USEF measured with a fixed level of prices in two components: the effect of price dispersion at a constant aggregate level of prices, plus the effect of a change in the aggregate level of prices. Because of stochastic dominance reasons, choosing the fixed aggregate level of prices smaller (respectively larger) than the harmonic (respectively arithmetic) mean of price indices yields components with explicit signs when u (respectively K_x) is concave or convex.

Proposition 5 *If C1 is satisfied:*

a) If u is concave and if the aggregate level of prices is defined by J inferior or equal to H, the harmonic mean of price indices, then the variation in the measured social welfare, caused by the deflation, is such that
$\Delta W = \Delta W_1 + \Delta W_2$ *, where*

$$\Delta W_1 = W - \int_0^{+\infty} u\left(\frac{x}{J}\right) \, dF_1(x) \tag{10}$$

$$\Delta W_2 = \int_0^{+\infty} u\left(\frac{x}{J}\right) \, dF_1(x) - \int_0^{+\infty} u(x) \, dF_1(x) \tag{11}$$

$\Delta W_1 \leq 0$. $\Delta W_2 \geq 0$ *if* $J \leq 1$, *and* $\Delta W_2 \leq 0$ *if* $J \geq 1$.

b)If instead u is convex, then $\Delta W_1 \geq 0$ *for all* $J \geq H$, *while the sign of* ΔW_2 *remains the same.*

Proposition 6 *If C2 is satisfied:*

a) If K_x is concave and if the aggregate level of prices is J inferior or equal to the arithmetic mean of price indices, $\bar{I}$, then the variation in measured social welfare, caused by the deflation, is such that
$\Delta W = \Delta W_3 + \Delta W_4$ *, where*

$$\Delta W_3 = W - \int_0^{+\infty} u\left(\frac{x}{J}\right) dF_1(x) \tag{12}$$

and

$$\Delta W_4 = \int_0^{+\infty} u\left(\frac{x}{J}\right) dF_1(x) - \int_0^{+\infty} u(x)\ dF_1(x). \tag{13}$$

$\Delta W_3 \leq 0$. $\Delta W_4 \geq 0$ if $J \leq 1$, and $\Delta W_4 \leq 0$ if $J \geq 1$.

b) If instead K_x is convex, then $\Delta W_3 \geq 0$ for $J \geq \bar{I}$, while the sign of ΔW_4 remains the same.

When the dispersion of price indices is fixed, the effect on social welfare of a change in the mean price index is opposite to the variation in the mean price index. When the arithmetic mean of price indices, $\bar{\text{I}}$, is fixed, then with K_x convex, which we shall show is satisfied for the usual functional forms of u, the dispersion of price indices increases the level of the USEF. Since $\bar{\text{I}}$ is the usually considered stabilisation level of prices, this implies that any policy changing the geographical distribution of prices should be monitored not only for its effects on the general level of prices, but also for its effect on the dispersion of price indices, even in a static framework. This somewhat surprising result stems from the location of the price index as a divisor of the nominal living standard. Because of the asymmetric shape of the inverse function, the impact of a larger spread of price indices is stronger for a fall in prices than for an augmentation in prices.

An intuition of this is clear in the following example. Consider a population composed of two people who, in situation A, have living standards respectively of levels 1 and 1 (for example, represented by their fixed wages) and facing price indices equal respectively to 2 and 2. Suppose now that after further observation we discover that prices must be corrected in such a way that in situation B the people now face price indices respectively equal to: 2 - 1 = 1 and 2 + 1 = 3. Clearly, the arithmetic average of price indices has not changed. The real living standards of the poor people in situation B are now respectively equal to 1 and 1/3. Although it depends on the risk aversion that one may want to consider, many observers would agree that the situation of the first person has improved much more than the situation of the second person has deteriorated.

In proposition 3.4, the magnitude of ΔW_1 depends on the extent of the concavity of u and in proposition 3.5, the magnitude of ΔW_3 depends on the extent of the convexity of K_x. This invites us to develop measures of the sensitivity of the USEF to price dispersion that would depend on the curvatures of u and K_x .

4 USEF Sensitivity to Price Dispersion

We now derive measures of the sensitivity of a specific USEF to the price index distribution. Although, they have no normative content that would be based on ethical feelings about the price distribution, we shall show that these measures are partially related to Arrow-Pratt risk aversion coefficients. Each sensitivity measure induces a partial ordering on price index distributions for social welfare analysis. It enables one to rank small differences in price configurations using only central tendencies and variances of an error term. For a given price configuration, it can also be used to show that one USEF is more or differently sensitive to price dispersion than another. In that sense, these sensitivity measures also generate a partial ordering on USEFs. The sensitivity measures are based on a distance between USEFs measured with and without price dispersion. This distance can first be described in terms of equivalent constant-price-index and measures of inequality in prices.

4.1 Equivalent constant-price-index and inequality in prices

We first define the *equivalent constant-price-index* (ECPI) that conserves the USEF level while discarding price index dispersion, and *measures of inequality in prices* that describe the price dispersion in terms of deviation of the ECPI from the reference levels H and $\bar{I}$.

Definition 4 *I_e, the* ***equivalent constant-price-index*** *(ECPI) is the real number defined as the solution to*

$$\int_0^{+\infty}\int_0^{+\infty} u\left(\frac{x}{I}\right)\, dF(x,I) = \int_0^{+\infty} u\left(\frac{x}{I_e}\right)\, dF_1(x) \qquad (14)$$

Two measures of inequality in prices are defined as: $AIP = I_e$-H and $RIP = (I_e - H)/H$.

Other measures of inequality in prices are: $AI\tilde{P} = \bar{I} - I_e$ and $RI\tilde{P} =$

$(\bar{I} - I_e)/\bar{I}$.

The right-hand-side term in eq. 14 is generally strictly increasing in I_e. In that case, the value of I_e is unique. $AIP, \tilde{AIP}, RIP$ and $\tilde{RIP}$ are analogue to inequality indices defined from income distribution[12]. Similarly to Blackorby and Donaldson (1980a), it is possible to consider AIP and $\tilde{AIP}$ as absolute inequality measures in prices if the USEF is translatable in prices, and RIP and $\tilde{RIP}$ as relative inequality measures in prices if the USEF is homothetic in prices. RIP (respectively $\tilde{RIP}$) is symmetric, bounded between 0 and 1, and sensitive to changes in prices that preserve spread in the sense of H (respectively $\bar{I}$). RIP and $\tilde{RIP}$ are cardinally significant and remain invariant under affine transformation of u. They can be interpreted as the proportional welfare loss generated by the existence of price index dispersion. When the price index dispersion around the reference level H or $\bar{I}$ is small, we now show how to derive indicators of the USEF sensitivity to price index dispersion that do not depend on the price index distribution.

4.2 Aversion and sensitivity indicators

Under C1 and if u is concave we have from proposition 3.3 by redefining I as $H + \varepsilon$:

$$W = \int_0^{+\infty} \int_0^{+\infty} u\left(\frac{x}{H + \varepsilon}\right) \, d\mu(\varepsilon) \, dF_1(x) \leq \int_0^{+\infty} u\left(\frac{x}{H}\right) \, dF_1(x) \qquad (15)$$

where μ is the c.d.f. of ε.

Under C2 and if K_x is convex, we have from proposition 3.4 by defining I as $\bar{I}+\zeta$:

$$W = \int_0^{+\infty} \int_0^{+\infty} u\left(\frac{x}{\bar{I} + \zeta}\right) d\nu(\zeta) dF_1(x) \geq \int_0^{+\infty} u\left(x/\bar{I}\right) dF_1(x) \qquad (16)$$

where ν is the c.d.f. of ζ.

Ineq. 15 suggests that it is possible to define a positive indicator of the '*aversion to price index dispersion at H*' since the absence of price index dispersion at H would be preferred. We proceed by defining small deviations

[12]Atkinson (1970), Blackorby and Donaldson (1980b).

of I (denoted ρ_I) or x (denoted ρ_x) from the distribution of x/H, that bring back to the USEF level measured by using real living standards.

By contrast, ineq. 16 implies that we can define a positive indicator of the '*sensitivity to price dispersion at $\bar{I}$*' since the situation with price dispersion at $\bar{\mathrm{I}}$ would be preferred. As above, we define small deviations of I (denoted $\tilde{\rho}_I$) or x (denoted $\tilde{\rho}_x$) from the distribution of x/$\bar{\mathrm{I}}$, that bring back to the USEF level measured by using real living standards. $\rho_I, \rho_x, \tilde{\rho}_I, \tilde{\rho}_x$ are analogue to the risk premium for small risks in consumer theory (Pratt (1964), Kihlstrom et al. (1981))[13].

Definition 5 *a) ρ_I is the solution of*

$$\int_0^{+\infty} \int_0^{+\infty} u(\frac{x}{I})\, dF_2(I)\, dF_1(x) = \int_0^{+\infty} u\left(\frac{x}{H+\rho_I}\right) dF_1(x) \qquad (17)$$

b) ρ_x is the solution of

$$\int_0^{+\infty} \int_0^{+\infty} u(\frac{x}{I})\, dF_2(I)\, dF_1(x) = \int_0^{+\infty} u\left(\frac{x-\rho_x}{H}\right) dF_1(x) \qquad (18)$$

c) $\tilde{\rho}_I$ is the solution of

$$\int_0^{+\infty} \int_0^{+\infty} u(\frac{x}{I})\, dF_2(I)\, dF_1(x) = \int_0^{+\infty} u\left(\frac{x}{\bar{I}-\tilde{\rho}_I}\right) dF_1(x) \qquad (19)$$

d) $\tilde{\rho}_x$ is the solution of

$$\int_0^{+\infty} \int_0^{+\infty} u(\frac{x}{I})\, dF_2(I)\, dF_1(x) = \int_0^{+\infty} u\left(\frac{x+\tilde{\rho}_x}{\bar{I}}\right) dF_1(x) \qquad (20)$$

When x is an income level, ρ_x can be interpreted as the monetary loss owing to price dispersion at aggregate constant-price-index H, i.e. the risk premium associated with the price index dispersion for price stabilisation at H. It measures the aggregate amount of income transfer that would be

[13]There have been numerous attempts to generalise various aspects of the Pratt-Arrow risk aversion parameters to the case of multivariate risk in an additive income framework or in a multiple goods or assets context (e.g. Karni (1979), Ross (1981)). The case of the ratio decomposition of living standard and the case of the risk on price indices for USEFs have never been studied.

necessary to compensate the price index dispersion while maintaining the aggregate price index level at H. ρ_I is the price index increase equivalent to the price index dispersion at aggregate constant-price-index H. It is a shadow price index of this price index dispersion. $\tilde{\rho}_x$ can be interpreted as the monetary gain of the price dispersion at aggregate constant-price-index $\bar{\text{I}}$, i.e. minus the risk premium associated with the price index dispersion for price stabilisation at $\bar{\text{I}}$. It measures the aggregate amount of income transfer that would be necessary to compensate an elimination of the price index dispersion while maintaining the aggregate price index level at $\bar{\text{I}}$[14]. $\tilde{\rho}_I$ is the price index reduction equivalent to the price index dispersion at aggregate constant-price-index $\bar{\text{I}}$. It is a shadow price index of the elimination of price index dispersion at $\bar{\text{I}}$. Herein, since u is concave, ρ_x and ρ_I are positive, and since K_x is usually convex, $\tilde{\rho}_x$ and $\tilde{\rho}_I$ are positive. In other situations all these parameters may be negative.

Because for any distribution F_1, we have
$\int_0^{+\infty} u\left(\frac{x}{H+\rho_I}\right) dF_1(x) = W = \int_0^{+\infty} u\left(\frac{x}{I_e}\right) dF_1(x)$,
then if u is strictly increasing in y and if the density of x is strictly positive, we have $\rho_I = I_e - H = AIP$. Similarly, $\tilde{\rho}_I = \bar{I}$ - $I_e = A\tilde{IP}$. Therefore, ρ_I and $\tilde{\rho}_I$ (and therefore ρ_x and $\tilde{\rho}_x$ in another sense) are measures of the inequality in price indices.

We now derive explicit expressions of ρ_I, ρ_x, $\tilde{\rho}_I$ and $\tilde{\rho}_x$ for small disturbances of prices and incomes. For this purpose, we assume that the kernel function u is twice differentiable and strictly increasing. Let ε be a random term independent from x, with a small variance σ_ε^2 and with its centered moments of higher order negligible. ε is added to the constant-price-index H, then $E\varepsilon = \bar{I} - H$. We need moreover to assume that $\bar{I}$ and H are close, which is generally the case, so as to validate the expansion approach. The calculus is based on Taylor expansions of:

a) $\int_0^{+\infty} u\left(\frac{x}{H+\rho_I}\right) dF_1(x)$ for small ρ_I ;

b) $\int_0^{+\infty} u\left(\frac{x-\rho_x}{H}\right) dF_1(x)$ for small ρ_x ;

c) $\int_0^{+\infty} u\left(\frac{x}{H+\varepsilon}\right) dF_1(x)$ for small ε.

The comparison of the results obtained for a) (respectively b)) and the expectation of the expression in c) provides an approximate expression of ρ_I

[14] Using analogues to Pratts's denominations, we could as well denote $-\rho_x$ as 'the risk premium for I', ρ_x as the 'asking price for I', $-\rho_x$ as the 'insurance premium for I', except that for ρ_x we consider K_x convexity instead of u concavity.

(respectively ρ_x). For $\tilde{\rho}_I$ and $\tilde{\rho}_x$ a similar approach is used by considering ζ be a random term independent from x, with a small variance σ_ζ^2 and with its centered moments of higher order negligible. ζ is added to the constant-price-index $\bar{I}$, then $E\zeta = 0$.

Proposition 7 *Under the above assumptions,*

$$\rho_I \simeq -\left[\sigma_\varepsilon^2 + (\bar{I} - H)^2\right] \frac{\left[\int_0^{+\infty} \frac{x^2}{H^2}\, u''(\frac{x}{H}) + 2\frac{x}{H}\, u'(\frac{x}{H})\, dF_1(x)\right]}{2\, \left[\int_0^{+\infty} xu'(\frac{x}{H})\, dF_1(x)\right]} - (H - \bar{I}) \quad (21)$$

$= \frac{\left[\sigma_\varepsilon^2 + (\bar{I} - H)^2\right]}{2} SDA - (H - \bar{I})$,
which defines SDA, the ***social denominator-aversion (at H).***

$$\begin{aligned}\rho_x \quad &\simeq \quad -\left[\sigma_\varepsilon^2 + (\bar{I} - H)^2\right] \frac{\left[\int_0^{+\infty} \frac{x^2}{H^3} u''(\frac{x}{H}) + 2\frac{x}{H^2}\, u'(\frac{x}{H})\, dF_1(x)\right]}{2\left[\int_0^{+\infty} u'(\frac{x}{H})\, dF_1(x)\right]} \\ &\quad -(H - \bar{I}) \frac{\int_0^{+\infty} \frac{x}{H}\, u'(\frac{x}{H})\, dF_1(x)}{\int_0^{+\infty} u'(\frac{x}{H})\, dF_1(x)} \qquad (22)\end{aligned}$$

$= \frac{\left[\sigma_\varepsilon^2 + (\bar{I} - H)^2\right]}{2} SNA_1 - (H - \bar{I})\ SNA_2$,
which defines SNA_1 and SNA_2, the ***social numerator-aversions (at H)****, respectively of type I and type II.*

$$\tilde{\rho}_I \simeq \sigma_\zeta^2 \frac{\int_0^{+\infty} \frac{x^2}{\bar{I}^2}\, u''(x/\bar{I}) + 2\frac{x}{\bar{I}}\, u'(x/\bar{I})\, dF_1(x)}{2\ \int_0^{+\infty} xu'(x/\bar{I})\, dF_1(x)} \quad (23)$$

$= \frac{\sigma_\zeta^2}{2} SDS$,
which defines SDS, the ***social denominator-sensitivity (at $\bar{I}$).***

$$\tilde{\rho}_x \simeq \sigma_\zeta^2 \frac{\int_0^{+\infty} \frac{x^2}{\bar{I}^3} u''(x/\bar{I}) + 2\frac{x}{\bar{I}^2}\, u'(x/\bar{I})\, dF_1(x)}{2\ \int_0^{+\infty} u'(x/\bar{I})\, dF_1(x)} \quad (24)$$

$= \frac{\sigma_\zeta^2}{2} SNS$,

which defines SNS, the ***social numerator-sensitivity (at $\bar{I}$).***

Naturally, both parameters ρ_x and ρ_I (and similarly for $\tilde{\rho}_x$ and $\tilde{\rho}_I$) could be simultaneously used for the compensation. This would yield an infinity of solution couples (ρ_x, ρ_I), which can be shown to be composed of formulae 21 and 22 multiplied respectively by some coefficients α and $1-\alpha$. *SDA* and *SNA₁* are parameters describing the sensitivity of the USEF to price index dispersion as measured by $\left[\sigma_\varepsilon^2 + (\bar{I} - H)^2\right]$. SNA_2 is a parameter describing the sensitivity of the USEF to the deviation $(H - \bar{I})$ at constant $\mathrm{E}\varepsilon^2$. The deviation $(H - \bar{I})$ can be interpreted as a measure of the asymmetry of the distribution. Note that the formula for ρ_I also accounts for $H - \bar{I}$. *SDS* and *SNS* are parameters describing the USEF sensitivity to price index dispersion as measured by σ_ζ^2. As the infixes 'numerator' and 'denominator' suggest, aversion and sensitivity parameters can be similarly defined for other variables at the denominator of the living standard. Moreover, notions of relative sensitivity and relative aversion can be derived by considering multiplicative rather than additive random terms. To save space, we do not give their formulae here. There exist correspondences between positivity of aversion (respectively sensitivity) parameters to price dispersion, positivity of parameters ρ_x and ρ_I (respectively $\tilde{\rho}_x$ and $\tilde{\rho}_I$), and the concavity of u (respectively the convexity of K_x).

One can also consider the individual welfare status of a specific individual. Thus, a simple application of proposition 4.3 to a population of one individual gives:

Definition 6 ***Individual aversion measures.***

$$IDA = -\frac{\frac{x^2}{H^2}\,u''(\frac{x}{H}) + 2\frac{x}{H}\,u'(\frac{x}{H})}{x\,u'(\frac{x}{H})} = -\frac{x}{H^2}\frac{u''}{u'} - \frac{2}{H} = -\frac{1}{H}\left[e(u', x/H) + 2\right] \tag{25}$$

$$INA_1 = -\frac{\frac{x^2}{H^3}\,u''(\frac{x}{H}) + 2\frac{x}{H^2}\,u'(\frac{x}{H})}{u'(\frac{x}{H})} = -\frac{x^2}{H^3}\frac{u''}{u'} - 2\frac{x}{H^2} = -\frac{x}{H^2}\left[e(u', x/H) + 2\right] \tag{26}$$

$$INA_2 = \frac{x}{H} \tag{27}$$

where $e(u', \frac{x}{H})$ is the elasticity of u' at the point $\frac{x}{H}$, i.e. the opposite of the Arrow-Pratt relative risk aversion coefficient.

Individual sensitivity measures.

$$IDS = \frac{\frac{x^2}{\bar{I}^2}\, u''(x/\bar{I}\,) + 2\frac{x}{\bar{I}}\, u'(x/\bar{I}\,)}{x\, u'(x/\bar{I})} = \frac{x}{\bar{I}^2}\frac{u''}{u'} + \frac{2}{\bar{I}} = \frac{1}{\bar{I}}\left[e(u', x/\bar{I}) + 2\right] \quad (28)$$

$$INS = \frac{\frac{x^2}{\bar{I}^3}\, u''(x/\bar{I}\,) + 2\frac{x}{\bar{I}^2}\, u'(x/\bar{I}\,)}{u'(x/\bar{I}\,)} = \frac{x^2}{\bar{I}^3}\frac{u''}{u'} + 2\frac{x}{\bar{I}^2} = \frac{x}{\bar{I}^2}\left[e(u', x/\bar{I}) + 2\right] \quad (29)$$

All these measures are homogenous of degree 0 with respect to function u. Moreover, $(x/H).IDA = INA_1$ and $(x/\bar{I}).IDS = INS$. This leads one to consider in that case INA_1 and INS as 'absolute aversion' and 'absolute sensitivity' and IDA and IDS as 'relative aversion' and 'relative sensitivity', with respect to the real living standard. IDA, INA_1, IDS and INS can be directly related to the relative risk aversion in consumer theory. This is not the case for SNA_1, SDA, SNS and SDS because the presence of integrals prevents simplifications. Owing to the link with the Arrow-Pratt risk aversion, the utility function can be expressed as a primitive function based on IDA, INA_1, IDS or INS. For example, we have: $u \sim \int e^{-\int(\frac{IDA.H+2}{x})}$. In practice, only a few functional forms have been used for USEF. We consider them in the next section by first studying the concavity properties of u and K_x.

5 Various Functional Forms

Proposition 8 ***Concavity properties of u and K_x.***

a) $u(y) = y^\alpha, 0 < \alpha \leq 1$.
u is convex for $\alpha \geq 1$ and concave for $\alpha \leq 1$. K_x is convex for all $\alpha > 0$.

b) $u(y) = ln(y)$.
u is concave and K_x is convex.

c) $u(y) = ay^2 + by + c$.
u is convex for $a \geq 0$ and concave for $a \leq 0$. K_x is convex for $x/I \geq$

$-b/3a$ *(always true when a and b have the same sign) and concave for* $x/I \leq -b/3a$.

d) $u(y) = K_1 - K_2\, e^{-\alpha y}, \alpha > 0$, K_1 and K_2 are positive constants.

u is concave. K_x *is convex for* $I \geq \alpha/3$ *(always true for usual values of* α*) and concave for* $I \leq \alpha/3$.

These results are deduced from the signs of u$''$ and K''_x. It is easy to see that in usual cases of economic interest, K_x is convex. Indeed, a), b) and d) are the most usual functional forms, for example used in cost-efficiency analysis (see Dreze and Stern (1987)). For d), α is often of the same magnitude than $1/\bar{y}$, which makes the condition $I \geq \alpha/3$ always satisfied for usual price indices. u is generally chosen concave as a result of transfer axioms. The case c) can be considered as a second-order approximation of the kernel function around a fixed value of y.

We now present the aversions and sensitivities to price dispersion for these usual functional forms of USEFs, substituting everywhere x/H with y. Because we always have $y.IDA = INA_1$, $INA_2 = x/H$, $y.IDS = INS$ and $IDS = -(H/\bar{I}).IDA$, only IDA is shown in the following proposition.

Proposition 9 *IDA for usual functional forms.*

$USEF$	$u(y)$	$e = y\frac{u''}{u'}$	IDA
W_a	$y^\alpha, 0 < \alpha \leq 1$	$\alpha - 1$	$-\frac{1}{H}.(\alpha+1)$
W_b	$ay^2 + by + c$	$\frac{2ay}{2ay+b}$	$-\frac{2}{H}\frac{3ay+b}{2ay+b}$
W_c	$\ln y$	-1	$-\frac{1}{H}$
W_d	$K - e^{-\alpha y}, \alpha > 0$	$-\alpha y$	$\frac{y\alpha-2}{H}$

The formulae of the $IDAs$ reveal that each USEF is specifically sensitive to the price dispersion. Different types of dependency of the *IDA* on living standard correspond to different functional forms. When *H* is given, W_a and W_c, which are the most frequently used in practice, have fixed *IDA*, ranging from *1/H* to *2/H*. By contrast, the forms W_b and W_d are characterised by *IDA* varying with the real living standard, linearly for W_d and homographically for W_b. W_d corresponds to *IDA* increasing with y, while W_b can be associated with increasing (when ab $>$ 0) or decreasing (when ab $<$ 0) *IDA* with y. If one wants to avoid (at the individual level) an impact of the price index distribution that would depend on living standards, one may favour the forms W_a and W_c, or one may select values of the USEF parameters that minimise the *IDA*.

Table 0 shows the values of these usual USEFs for the example of section 3. The price dispersion at $\bar{I}$ increases the USEF level in all these cases. We now apply the results obtained for USEFs to inequality measures.

6 Consequences for Inequality Measures

We must first remember the definition of Atkinson's inequality measures (Atkinson (1970). Similar inequality measures can be defined by using $\bar{I}$ instead of H under C3.

Definition 7 *An Atkinson's inequality measure associated with the USEF of kernel u is:*

$AI = 1 - y_e/\bar{y}$ *where* $u(y_e) = \int_0^{+\infty} u(y)dF(y)$ *and* F *is the c.d.f. of* y.

When the utility function u is strictly increasing and the density of y is strictly positive, this inequality measure can be written $AI = 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(y)dF(y)\right)}{\bar{y}}$.

We also define another similar inequality measure

$$\tilde{AI} = 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(y)dF(y)\right)}{\bar{x}/\bar{I}}.$$

Under C3, by using proposition 3.4 and 3.5 when u is concave and K_x is convex, with u^{-1} increasing and $\bar{y} = \bar{x}/H$, we can derive upper and lower bounds of these inequality measures.

Proposition 10 *If C3 is satisfied:*

$$a)\ 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(x/\bar{I})dF_1(x)\right)}{\bar{x}/H} \geq AI \geq 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(x/H)dF_1(x)\right)}{\bar{x}/H}.$$

$$b) 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(x/H)dF_1(x)\right)}{\bar{x}/\bar{I}} \leq \tilde{AI} \leq 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(x/\bar{I})dF_1(x)\right)}{\bar{x}/\bar{I}}.$$

Then, as soon as H and $\bar{I}$ are available, it is possible to propose an interval in which the inequality measure with price dispersion is included, and such that the upper and lower bounds are inequality measures without price dispersion that can be estimated without observing a whole sample of price indices. Using these results, it is possible to decompose the effect of the price index distribution on inequality measures. We first define the following inequality measures under C3.

Definition 8

$$AI(y) = 1 - \frac{u^{-1}\left(\int_0^{+\infty}\int_0^{+\infty} u(x/I)\, dF_2(I)\, dF_1(x)\right)}{\bar{x}/H}.$$

$$AI(x/H) = 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(x/H)\, dF_1(x)\right)}{\bar{x}/H}.$$

$$AI(x) = 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(x)\, dF_1(x)\right)}{\bar{x}}.$$

$$A\tilde{I}(y) = 1 - \frac{u^{-1}\left(\int_0^{+\infty}\int_0^{+\infty} u(x/I)\, dF_2(I)\, dF_1(x)\right)}{\bar{x}/\bar{I}}.$$

$$A\tilde{I}(x/\bar{I}) = 1 - \frac{u^{-1}\left(\int_0^{+\infty} u(x/\bar{I})\, dF_1(x)\right)}{\bar{x}/\bar{I}}.$$

Then, we obtain the following results.

Proposition 11 ***a) with H as a reference price index****, the variation of AI owing to the deflation is:*

$\Delta AI = \Delta AI_1 + \Delta AI_2$

where $\Delta AI_1 = AI(y) - AI(x/H)$ *and* $\Delta AI_2 = AI(x/H) - AI(x)$.

If u is concave, then $\Delta AI_1 \geq 0$. *If u is convex, then* $\Delta AI_1 \leq 0$. *Any constant price index* $J \leq H$ *if u concave (or* $J \geq H$ *if u convex) could be used instead of H in these formulae.*

b) with $\bar{\mathbf{I}}$ as a reference price index, *the variation of* $A\tilde{I}$ *owing to the deflation is:*

$\Delta A\tilde{I} = \Delta A\tilde{I}_1 + \Delta A\tilde{I}_2$

where $\Delta A\tilde{I}_1 = A\tilde{I}(y) - A\tilde{I}(x/\bar{I})$ *and* $\Delta A\tilde{I}_2 = A\tilde{I}(x/\bar{I}) - AI(x)$.

If K_x *is convex, then* $\Delta A\tilde{I}_1 \leq 0$. *If* K_x *is concave, then* $\Delta A\tilde{I}_1 \geq 0$.

Any constant price index $J \geq \bar{I}$ if K_x convex (or $J \leq \bar{I}$ if K_x concave) could be used instead of $\bar{I}$ in these formulae.

Note that when K_x is convex, as it is expected, *the inequality at $\bar{I}$ fixed is a decreasing function of the price index dispersion.* In contrast, the signs of ΔAI_2 and $\Delta A\tilde{I}_2$ are less obvious. For example, ΔAI_2 is of the sign of

$\delta = u^{-1}\left(\int_0^{+\infty} u(x)\; dF_1(x)\right) - H.u^{-1}\left(\int_0^{+\infty} u(x/H)\; dF_1(x)\right)$.

For W_a and W_c, $\delta = 0$ and there is no effect of the price level on inequality. Let us now consider the case of W_d with $u(t) = 1 - e^{-\alpha t}$, $\alpha > 0$. Then, δ is of the sign of

$\delta_1 = -\ln\left[1 - \int_0^{+\infty} 1 - e^{-\alpha x} dF_1(x)\right] + H.\ln\left[1 - \int_0^{+\infty} 1 - e^{-\alpha x/H} dF_1(x)\right]$.

which is negative for $H > 1$ and positive for $H < 1$, by applying Minkowski inequality. However, in general the sign of ΔAI_2 is unknown.

7 Empirical Illustration

The illustration that we now present has several purposes. First, we want to investigate not only the directions, but also the magnitude of the components of USEF changes. Second, the theoretical results are based on Assumption C3. Using empirical estimates, this condition is not strictly imposed even when the independence of x and I is statistically rejected. Thus, we check that the theoretical results are robust in an empirical context. Third, we want to assess the empirical usefulness of the theoretical bounds for inequality indices. Finally, we want to estimate the sensitivity to spatial price dispersion for usual functional forms. We use data from Rwanda in 1983. Since we present a mere illustration, we shall be brief on the presentation of the data and the indicators.

7.1 The data

Rwanda is one of the poorest countries in the world, with 1983 per capita GNP of US$ 270 per annum. Data for the estimation is taken from the Rwandan national budget-consumption survey, conducted by the government of Rwanda and the French Cooperation and Development Ministry, in the rural part of the country from November 1982 to December 1983 [15]. 270

[15]See Ministère du Plan (1986a). The main part of the collection was designed with the help of INSEE (French national statistical institute). The author was

households grouped in 90 clusters were surveyed about their consumption. The collection was organised in four rounds: A, B, C, D[16], which allows us to separate the seasonality of prices from their geographical dispersion. The consumption indicators and the price indices are discussed in Muller (1999). The sample used in the estimation corresponds to a mean real consumption of 51 176 Frw (Rwandan Francs[17]) or 10613 Frw per capita. The average household has 5.22 members.

The price indices calculated with a quarterly national basis for each cluster and each of the four quarters are presented in Table 1. Harmonic means of the Laspeyres price indices across households, H, are smaller but close to arithmetic means, $\bar{I}$. $\bar{I}$ and H are higher in quarter D and lower in quarter A.

7.2 Tests and Estimates

The living standard indicator is the per capita consumption. The P-values of Khi-square tests of independence between nominal living standards and Laspeyres price indices are respectively equal for quarters A, B, C, D to 0.340, 0.701, 0.304, 0.287. The hypothesis of independence is therefore never rejected at usual statistical levels.

Table 2 shows the variations of USEFs caused by the price index distribution: ΔW_1 and ΔW_2 for the four quarters and four kernel functions (u(y) = y^2, $\sqrt{y}$, 1-$e^{-x/\bar{x}}$, ln(y)). The empirical results are consistent with the theoretical results. At all quarters and for all USEFs, the effect of price index dispersion at H, ΔW_1, is negative when u is concave, and it is positive for u convex. The effect of price index dispersion at $\bar{I}$, ΔW_3, is always positive, which is consistent with K_x always convex. The signs of the effects of the aggregate levels of price indices, ΔW_2 and ΔW_4 correspond to the predicted signs for all quarters and all USEFs. Indeed, the aggregate harmonic means of price indices is always greater than 1. In this dataset, the relative changes in magnitude of the USEF range from very small (about 1 per mil for u(y)

himself involved in this project as a technical advisor from the French Ministry of Cooperation and Development. More than 95% of the population live in rural areas (Bureau National du Recensement (1984)).

[16]Round A: 01/11/1982 upto 16/01/1983. Round B: 29/01/1983 upto 01/05/1983. Round C: 08/05/1983 upto 07/08/1983. Round D: 14/08/1983 upto 13/11/1983.

[17]In 1983, the average exchange rate was 100.17 Frw for 1 US $ (source: IMF, International Finance Statistics).

$= \ln(y)$) to notable (upto 13 % with $u(y) = y^2$). However, the meaning of such absolute numbers for a USEF is unclear.

Table 3 shows the variations of Atkinson inequality measures owing to price index distribution. Note first that because $AI(x/I) < 0$ when u is convex, the signs of $\Delta AI_1/AI(x/\bar{I})$ are different from the signs of ΔAI_1. Again, the empirical results reflect the theoretical results. When u is concave and K_x convex, the price index dispersion at H increases the inequality at H, but diminishes it at $\bar{I}$. As expected from the theory, the signs of ΔAI_2 and ΔAI_4 are ambiguous and the values are quasi-null. Here, most of the relative variation of inequality indices comes from the pure price index dispersion and not from the changes in absolute level of price indices. The relative variations $\Delta AI_1/AI(x/H)$ are sometimes substantial. They vary for u concave from 2.4% to 13.9%. Moreover, important differences occur across quarters with a much larger impact of price dispersion at H in quarter C. There are also large differences for different functional forms, which shows the importance of the specification of the inequality measure. Similar remarks are valid for $\Delta AI_3/AI(x/\bar{I})$ that varies for u concave from -2.5% down to -32.14 %.

We have calculated the same type of statistics (not shown here) using the household size instead of the price index. They reveal that the signs of ΔW_1, $\Delta AI_1$and ΔAI_3 are generally ambiguous, which is consistent with the household size not satisfying the condition of weak statistical link with nominal living standard. Moreover, ΔAI_2or ΔAI_4 are in that case non negligible because the central tendencies of the variable household size are very different from one. To study the effect of the distribution of the household size (or of another equivalence scale) at the denominator of the living standard variable, one can apply the approach of this paper only after an estimation of a model where the household size is regressed as $f(x) + \varepsilon$, where f is a function to determine (for example using nonparametric estimation methods) and ε is an error term that must be weakly statistically associated with x. However, in that case the functions u and K_x to consider must incorporate the influence of function f and their concavity properties are therefore more delicate to characterise.

Table 4 shows Atkinson indices AI and their two bounds not depending on price index dispersion. The interval between the bounds is generally narrow enough for these bounds to provide a good assessment of inequality. Similar results are found with inequality measures $\tilde{AI}$ and associated lower and upper bounds.

Finally, Table 5 presents estimates of aversion and sensitivity parameters $H.SDA$, $\bar{I}.SDS$, SNA_1 and SNS. To simplify $\bar{I}.SDS$ and $H.SDA$ are shown, while SNA_1 and SNS are presented to illustrate the orders of magnitude of these sensitivity and aversion parameters. Table 6 shows the explicit expressions for parameters SNS and SDS for the used functional forms. Formulae for parameters SNA_1 and SDA can be obtained by substitution of Ī by H. It is remarkable that for several chosen functional forms, simplifications occur even with the presence of integrals in the theoretical formulae. In contrast, numerical estimation is necessary for $u(y) = 1 - e^{-y/\bar{y}}$. Sensitivity parameters SDS are always smaller for $u(y) = ln(y)$ than for $u(y) = \sqrt{y}$, themselves smaller than for $u(y) = y^2$. As a matter of fact, SDS for $u(y) = y^\alpha$ is an increasing function of α. The estimation shows with the present dataset that SDS for $u(y) = 1 - e^{-y/\bar{y}}$ is inferior to SDS for $u(y) = \sqrt{y}$, but can be below or above the SDS for $u(y) = \ln(y)$. Similar orderings occur with SNA_1 and SNS parameters.

8 Conclusion

Social welfare measures are based on living standard variables that are generally not decomposed in elementary components, or only in a restrictive additive fashion. However, other functional decompositions are necessary if we want to understand the contributions to social welfare of the distributions of economically fundamental components. A crucial functional form to study is the ratio of the two variables, so as to deal with the role of the distribution of price indices, or that of equivalence scales.

Using likely conditions of weak statistical link between price indices and nominal living standards and of generalised concavity properties of the utilitarian social evaluation functions (USEF), we decompose the measurement bias in USEFs and Atkinson's inequality measures, caused by neglecting the price index distribution. Two signed components are exhibited: the effect of the aggregate level of price indices, and the effect of the dispersion of price indices at constant aggregate price level. If the constant aggregate price level is superior or equal to the arithmetic mean of price indices, then the dispersion of price indices increases the USEF and diminishes the inequality. In contrast, if the stabilisation level is inferior or equal to the harmonic mean, the price index dispersion decreases the USEF and increases the inequality. Since observed arithmetic and harmonic means are generally close, the domain of price stabilisations for which there remains ambiguous results is likely to be narrow. Moreover, we derive measures of aversion and sensitivity of the USEF to the price index dispersion, which help rank price index distributions and USEFs. An empirical illustration using data from Rwanda confirms our theoretical results and shows that the impact of price index dispersion on welfare can be considerable.

The results are useful for several purposes. First, they help understand the impact on USEFs and inequality measures of the distribution of price indices. Second, they allow the analysis of the sensitivities of specific USEFs to the price index distribution. Third, they reveal that spatial price dispersion can be socially advantageous. Four, they exhibit the special roles of arithmetic and harmonic means of price indices in welfare analysis. Finally, they provide lower and upper bounds for inequality measures with price index dispersion. These bounds are inequality measures with deflation based on arithmetic and harmonic means of price indices. Therefore, they do not require the observation of a national sample of price indices.

We are investigating several extensions of the theoretical results of this paper. First, the propositions can be extended to other variables than price indices at the denominator of the living standard indicator. Second, similar results can be obtained with poverty measures expressed in the form of expectations of kernel functions. More generally, other functional forms and statistical conditions may be useful for welfare analyses involving the disaggregation of the living standard variable.

Appendix 1: Proofs.

Proof of Proposition 2.3: Consequences of Jensen's inequality applied to the internal integral after conditioning on x, then to the external integral. QED.

Proof of proposition 2.4:

The log-joint-density of a couple (x, I) is:

$LL = -\ln(2\pi) - \ln(\sigma_1) - \ln(\sigma_2) - \frac{1}{2}\ln(1-\rho^2) - \ln(x) - \ln(I)$
$-\frac{1}{2(1-\rho^2)}\left[\frac{(\ln(x)-m_1)^2}{\sigma_1^2} + \frac{(\ln(I)-m_2)^2}{\sigma_2^2} - 2\rho\frac{(\ln(x)-m_1).(\ln(I)-m_2)}{\sigma_1\sigma_2}\right].$

The marginal log-density of x is:

$\ln(f_1) = -\frac{1}{2}\ln(2\pi) - \ln(\sigma_1) - \ln(x) - \frac{(\ln(x)-m_1)^2}{2\sigma_1^2}.$

By difference, the log-density of the conditional law of I with respect to x is:

$\ln(f_{2|1}) = -\frac{1}{2}\ln(2\pi) - \ln(\sigma_2) - \frac{1}{2}\ln(1-\rho^2) - \ln(I)$
$-\frac{1}{2(1-\rho^2)}\left[-\rho^2\frac{(\ln(x)-m_1)^2}{\sigma_1^2} + \frac{(\ln(I)-m_2)^2}{\sigma_2^2} - 2\rho\frac{(\ln(x)-m_1).(\ln(I)-m_2)}{\sigma_1\sigma_2}\right].$

Then, using these densities in the calculation of the Taylor expansion of ΔW with respect to ρ gives the result. QED.

Proof of Proposition 3.2: Obvious. QED.

Proof of Proposition 3.3: Consequence of Proposition 2.3, and of the equalities in C1. QED.

Proof of proposition 3.4:

With u concave and under C1, we obtain from Proposition 3.3
$W \leq \int_0^{+\infty} u\left(x\int_0^{+\infty}\frac{1}{I}\,dF_2(I)\right)\,dF_1(x)$
$= \int_0^{+\infty} u\left(\frac{x}{H}\right)\,dF_1(x),$
where H is the harmonic mean of price indices.
For any constant price index J inferior to H we have
$W \leq \int_0^{+\infty} u\left(\frac{x}{J}\right)\,dF_1(x)$ because of stochastic dominance, which gives $\Delta W_1 \leq 0$. The sign of ΔW_2 can also be deduced from stochastic dominance. The proof is similar for u convex. QED.

Proof of Proposition 3.5: Similar to the proof of proposition 3.4. QED.

Proof of proposition 4.3:

On the one hand, we have by using a Taylor expansion and taking the expectation with respect to the law of ε.

$W = \int_0^{+\infty} u\left(\frac{x}{H+\varepsilon}\right) dF_1(x)\, d\mu(\varepsilon) = \int_0^{+\infty} u(\frac{x}{H})\, dF_1(x)$
$+E\,\varepsilon \left(\int_0^{+\infty} -u'(\frac{x}{H})\frac{x}{H^2}\, dF_1(x)\right)$
$+\frac{1}{2}\, E\,\varepsilon^2 \left[\int_0^{+\infty} u''(\frac{x}{H})\frac{x^2}{H^4} + 2u'(\frac{x}{H})\frac{x}{H^3}\, dF_1(x)\right] + o(\sigma_\varepsilon^2)$,

which can be simplified because $\mathrm{E}\,\varepsilon = \bar{I} - H$, into:

$$W \simeq \int_0^{+\infty} u(\frac{x}{H})\, dF_1(x) + (\bar{I} - H)\left(\int_0^{+\infty} -u'(\frac{x}{H})\frac{x}{H^2}\, dF_1(x)\right) + \frac{(\bar{I}-H)^2 + \sigma_\varepsilon^2}{2}\left[\int_0^{+\infty} u''(\frac{x}{H})\frac{x^2}{H^4} + 2\frac{x}{H^3}u'(\frac{x}{H})\, dF_1(x)\right].$$

On the other hand, we can derive the expansions for the expressions of W with ρ_I and ρ_x, when they are small relatively to H and x.

$W = \int_0^{+\infty} u(\frac{x}{H+\rho_I})\, dF_1(x) = \int_0^{+\infty} u(\frac{x}{H})\, dF_1(x) - \rho_I . \left[\int_0^{+\infty} u'(\frac{x}{H})\frac{x}{H^2}\, dF_1(x)\right] + o(\rho_I)$,which gives the result.

A similar approach with ρ_x yields

$W = \int_0^{+\infty} u(\frac{x-\rho_x}{H})\, dF_1(x) = \int_0^{+\infty} u(\frac{x}{H})\, dF_1(x) -\ \rho_x . \left[\int_0^{+\infty} u'(\frac{x}{H})\frac{1}{H}\, dF_1(x)\right] + o(\rho_x)$,which gives the result.

The calculus is similar for $\tilde{\rho}_I$ and $\tilde{\rho}_x$. QED.

Proof of Proposition 5.2: Direct calculus. QED.

Proof of Proposition 6.4: Because of Proposition 3.4, we have if u is concave:

$AI(y) \geq AI(x/H)$, therefore $\Delta AI_1 \geq 0$. Similarly, because of proposition 3.5, if K_x is convex, $\Delta A\tilde{I}_1 \leq 0$. QED.

Appendix 2: Discussion of the dependence between price indices and nominal living standards.

There are several reasons why prices and nominal living standards could be statistically linked. First, the lack of market integration would generate local supply and demand effects. Then, local prices, and the structure of relative prices, would be different in poor areas from in rich areas. Rich households may have a specific composition of their consumption and output, and live in a better economic environment than poor households. However, the evidence for lack of market integration is generally unclear. Second, a significant correlation between price indices and nominal living standards may arise from using true price indices because of the curvature of an exact metric of utility. However, the common practice in applied work is to use Laspeyres or Paasche indices, and we want to focus on actually calculated indices. Third, using unit-values instead of observed market prices may entail spurious correlation between consumption levels and unit-values (Deaton (1988, 1990)). However, market prices should be used when possible.

On the other hand, there also exist good reasons to believe in the independence of price indices and nominal living standards. First, in social welfare literature, the possible statistical links between nominal living standards and price indices are often neglected. Second, we mention in section 7, independence tests for rural Rwanda, whose results do not reject the hypothesis of independence. This implies that this hypothesis is not unrealistic in some contexts. Third, even when at the local level there are significant statistical links between on the one hand, prices or the structure of relative prices and levels of consumption or income on the other hand, this does not imply that the link between nominal living standards and price indices is significant. Indeed, the living standard indicators frequently incorporate demographic information that blurs the relation between prices and living standards. Moreover, price indices may average out the correlation of relative prices (or of specific prices) with the living standard level.

Four, even when the link between x and I is statistically significant, we do not expect it to be strong. To this extent, the study of the polar case of weak statistical link for the USEF provides a useful insight as an approximation of the actual situation. Five, in theoretical welfare or risk analyses, independence restrictions for several sources of risk are not uncommon (e.g. Kihlstrom et al. (1981)). Six, if prices can be considered as locally homogeneous, it is possible to stratify the studied regions in smaller areas where the independence assumption will be valid. Then, one can aggregate the USEFs obtained for these areas. Seven, and finally, weaker conditions than the independence (see section 3) are sufficient to obtain the results of this paper.

Appendix 3: Tables

Table 0: Values of some USEFs for the example.

u(y)	Situation (1/2,1/2)	Situation (1,1/3)
y^α	$2^{1-\alpha}$	$1+3^{-\alpha}$
ay^2+by+c	$2(a/4+b/2+c)$	$(a+b+c)+(a/9+b/3+c)$
$ln(y)$	$-2\ln 2 \simeq -1.38$	$-\ln 3 \simeq -1.09$
$K-e^{-\alpha y}$	$2.(K-e^{\alpha/2})$	$2K-e^{\alpha}-e^{\alpha/3}$
$\sqrt{y}$	$\sqrt{2} \simeq 1.41$	$1+1/\sqrt{3} \simeq 1.57$
$-e^{-y}$	$-2/\sqrt{e} \simeq -1.21$	$-1/e-e^{-1/3} \simeq -1.08$
y^2+y	$3/2=1.5$	$2+1/9+1/3 \simeq 2.44$

Table 1: Price index statistics weighted by the sampling scheme

		A	B	C	D
Laspeyres price indices					
	simple mean	1.028	1.058	1.051	1.075
	weighted mean	1.030	1.058	1.058	1.070
	standard deviation	0.115	0.119	0.131	0.099
	coefficient of variation	0.111	0.112	0.124	0.092
Harmonic mean					
	simple mean	1.015	1.043	1.034	1.065
	weighted mean	1.016	1.044	1.040	1.059

256 household observations in 90 clusters. The means of the national price index means over the four quarters is 1.053. The coefficient of variation of the national price index means over the four quarters is 0.018.

Table 2: Variations of USEF estimates owing to the price index distribution

$u(y)$	*Quarter*	$\frac{\Delta W_1}{W(x/H)}$	$\frac{\Delta W_2}{W(x)}$	$\frac{\Delta W_3}{W(x/I)}$	$\frac{\Delta W_4}{W(x)}$
y^2	A	0.0162	-0.0331	0.0432	-0.0581
y^2	B	0.0242	-0.0834	0.0522	-0.107
y^2	C	0.0875	-0.0754	0.126	-0.107
y^2	D	0.0447	-0.109	0.0660	-0.127
$\sqrt{y}$	A	-0.000117	-0.00838	0.00645	-0.0148
$\sqrt{y}$	B	-0.00113	-0.0215	0.00563	-0.0281
$\sqrt{y}$	C	-0.000202	-0.0194	0.00861	-0.0280
$\sqrt{y}$	D	-0.000510	-0.0285	0.00454	-0.0334
$1-e^{-y/\bar{y}}$	A	0.00408	0.0140	-0.00659	0.0249
$1-e^{-y/\bar{y}}$	B	0.00476	0.0324	-0.00490	0.0425
$1-e^{-y/\bar{y}}$	C	0.0103	0.0314	-0.00322	0.0454
$1-e^{-y/\bar{y}}$	D	0.00444	0.0401	-0.00222	0.0471
$\ln y$	A	-0.000845	-0.00214	0.000826	-0.00381
$\ln y$	B	-0.000907	-0.00565	0.000855	-0.00740
$\ln y$	C	-0.00117	-0.00502	0.00108	-0.00727
$\ln y$	D	-0.000677	-0.00761	0.000658	-0.00894

By convention here and in the following tables the normalisation parameter $\bar{y}$ for all periods is the mean per capita consumption in quarter A. Other choices lead to qualitatively similar results.

Table 3: Variations of Atkinson inequality measures owing to price index distribution.

$u(y)$	*Period*	$\frac{\Delta AI_1}{AI(x/H)}$	$\frac{\Delta AI_2}{AI(x)}$	$\frac{\Delta AI_3}{AI(x/I)}$	$\frac{\Delta AI_4}{AI(x)}$
y^2	Y	0.0833	-0.00104	0.2756	-0.00105
y^2	A	0.0241	0.0000105	0.1415	0.00000424
y^2	B	0.0648	-0.00000201	0.1794	-0.0000154
y^2	C	0.2051	0.0000119	0.4041	-0.00000663
y^2	D	0.1096	-0.00000376	0.2096	-0.00000675
$\sqrt{y}$	Y	0.0602	0.0000647	-0.3214	-0.0000420
$\sqrt{y}$	A	0.0553	-0.0000188	-0.1535	-0.00000760
$\sqrt{y}$	B	0.0657	0.00000384	-0.1501	0.0000295
$\sqrt{y}$	C	0.1386	-0.0000216	-0.2060	0.0000119
$\sqrt{y}$	D	0.0694	0.00000670	-0.1045	0.0000120
$1-e^{-y/\bar{y}}$	Y	0.0649	-0.0291	-0.1210	-0.0395
$1-e^{-y/\bar{y}}$	A	0.0497	-0.0119	-0.0409	-0.0212
$1-e^{-y/\bar{y}}$	B	0.0634	-0.0312	-0.0462	-0.0408
$1-e^{-y/\bar{y}}$	C	0.1394	-0.0285	-0.0230	-0.0411
$1-e^{-y/\bar{y}}$	D	0.0791	-0.0422	-0.0219	-0.0494
$\ln y$	Y	0.0518	0.000296	-0.1383	0.000157
$\ln y$	A	0.0628	-0.00000908	-0.0370	-0.00000365
$\ln y$	B	0.0631	0.00000190	-0.0431	0.0000146
$\ln y$	C	0.1154	-0.0000103	-0.0483	0.00000574
$\ln y$	D	0.0518	0.00000299	-0.0255	0.00000536

Y = 'year'; A, B, C, D denote the four quarters.

Table 4: Atkinson inequality measures and lower and upper bounds

$u(y)$	*Period*	*AI*	Upper bound	Lower bound
y^2	Y	-0.1262	-0.1013	-0.1165
y^2	A	-0.1823	-0.1627	-0.1780
y^2	B	-0.1789	-0.1524	-0.1681
y^2	C	-0.2155	-0.1583	-0.1788
y^2	D	-0.2034	-0.1714	-0.1833
$\sqrt{y}$	Y	0.0523	0.06231	0.0493
$\sqrt{y}$	A	0.0820	0.08975	0.0777
$\sqrt{y}$	B	0.0746	0.08247	0.0700
$\sqrt{y}$	C	0.0882	0.09351	0.0774
$\sqrt{y}$	D	0.0856	0.08933	0.0801
$1-e^{-y/\bar{y}}$	Y	0.0977	0.1032	0.0918
$1-e^{-y/\bar{y}}$	A	0.1570	0.1592	0.1495
$1-e^{-y/\bar{y}}$	B	0.1273	0.1303	0.1197
$1-e^{-y/\bar{y}}$	C	0.1600	0.1535	0.1402
$1-e^{-y/\bar{y}}$	D	0.1317	0.1299	0.1220
$\ln y$	Y	0.0990	0.1065	0.0941
$\ln y$	A	0.1584	0.1601	0.1491
$\ln y$	B	0.1404	0.1437	0.1321
$\ln y$	C	0.1662	0.1638	0.1490
$\ln y$	D	0.1715	0.1714	0.1630

Table 5: Aversion and sensitivity parameters

$u(y)$	*period*	SDS	SDA	SNA1	SNS
y^2	A	3	3	12205	11890
y^2	B	3	3	9622	9366
y^2	C	3	3	11112	10729
y^2	D	3	3	9028	8848
$\sqrt{y}$	A	1.5	1.5	3739	3643
$\sqrt{y}$	B	1.5	1.5	3063	2982
$\sqrt{y}$	C	1.5	1.5	3400	3283
$\sqrt{y}$	D	1.5	1.5	2632	2579
$1-e^{-y/\bar{y}}$	A	0.974	0.964	2101	2074
$1-e^{-y/\bar{y}}$	B	1.220	1.112	2073	2039
$1-e^{-y/\bar{y}}$	C	1.023	1.010	2035	1998
$1-e^{-y/\bar{y}}$	D	1.161	1.155	1952	1928
$\ln y$	A	1	1	2112	2057
$\ln y$	B	1	1	1785	1737
$\ln y$	C	1	1	1920	1854
$\ln y$	D	1	1	1139	1116

Table 6: Explicit formulae of sensitivity and aversion parameters for the chosen functional forms

$u(y)$	SNS	SDS
y^2	$\frac{3}{\bar{I}^2}\frac{E(x^2)}{\bar{x}}$	$\frac{3}{\bar{I}}$
$\sqrt{y}$	$\frac{3}{2\bar{I}^2}\frac{E(x^{1/2})}{E(x^{-1/2})}$	$\frac{3}{2\bar{I}}$
$1-e^{-y/\bar{y}}$	$\frac{\int\left(-\frac{x^2}{\bar{I}^3\bar{y}}+\frac{2x}{\bar{I}^2}\right)e^{-x/(\bar{I}\bar{y})}\,dF_1(x)}{\int e^{-x/(\bar{I}\bar{y})}\,dF_1(x)}$	$\frac{\int\left(-\frac{x^2}{\bar{I}^2\bar{y}}+\frac{2x}{\bar{I}}\right)e^{-x/(\bar{I}\bar{y})}\,dF_1(x)}{\int x\,e^{-x/(\bar{I}\bar{y})}\,dF_1(x)}$
$\ln y$	$\frac{1}{\bar{I}^2E(1/x)}$	$\frac{1}{\bar{I}}$

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