UNIVERSITY OF NOTTINGHAM

Discussion Papers in Economics

Discussion Paper
No. 01/04

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March 2001 DP 01/04

ISSN 1360-2438

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Shall we meet halfway? Endogenous spillovers and locational choice*

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February 2001

Abstract

We propose two versions of an address model - namely "mill-pricing" and "discriminatory pricing" - with quality-enhancing R&D and spillovers that depend on firms' location. Our results show that the distance between location increases with the degree of product differentiation. Further, we find that minimal quality differentiation always occurs. Minimal product differentiation is associated with no investment in R&D whereas maximal product differentiation engenders the highest R&D effort. We relate these results to the literature on absorptive capacity.

Keywords: Endogenous spillovers, quality, R&D, location.
JEL Class.: L1, 03

*An earlier version of this paper was presented at the Workshop on Innovation and Product Differentiation, Bergamo, October 2000. We thank participants for their helpful comments. The usual disclaimer applies.

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1 Introduction

Recent empirical literature on spillovers suggests that their effect is primarily local in that it strongly decreases with increasing distance (e.g. Audretsch and Feldman (1996) and Jaffe et al. (1993)). Furthermore, Cohen and Levinthal (1989) have introduced the concept of absorptive capacity which refers to the efforts that a firm undertakes to enhance its ability to make use of the research results obtained by rivals through beneficial spillovers. In Kamien and Zang (2000) (henceforth KZ), absorptive capacity is not realised unless firms "meet halfway", that is, unless firms undertake a positive R&D effort, and this R&D investment is not firm-specific.

In this paper, we take KZ's recommendation to the letter and propose two versions of an address model with quadratic transport costs to study the circumstances under which firms may choose to "meet halfway" when the R&D spillover depends on the firms' choice of location. That is, we assume that the closer firms are to each other, the greater the benefit they receive from their rivals' efforts in quality enhancement (R&D). Using KZ's terminology, both firms locating at the market endpoints would correspond to the choice of a firm specific research approach, while sharing the same location would be tantamount to the choice of a non-specific research approach.

The assumption on the nature of the spillover enables us to tackle the issue of whether the incentive faced by the firms to locate closer to each other more than offsets that of pursuing a maximum differentiation strategy to relax price competition. In the first version, we consider a traditional Hotelling framework where transport costs are incurred by the consumers. We denote this as the "mill pricing" model. In the second version, the "discriminatory pricing" model, the firms pay the transport cost but can price discriminate by charging a price equal to the transport cost that the rival would incur if it served that location. The first type of pricing policy gives rise to what is referred to as a 'shopping' model while the second corresponds to a 'shipping' model. Although interesting, it is outside the scope of this paper to consider the strategic choice of pricing policy; we refer the interested reader to De Fraja and Norman (1993), Tabuchi (1999) and Thisse and Vives (1988).

In both contexts it turns out that a wide range of locational choice is possible, depending on the degree of product differentiation represented by the transport cost parameter. We generally find that the distance between location is an increasing function of the transport cost parameter. However, firms never choose maximum differentiation in a shipping model whereby firms price discriminate but bear the cost of delivering the product to the consumer's location, while in the shopping model firms choose to locate at the market endpoints for a high enough value of the transport cost parameter. Minimum differentiation is also an equilibrium result in both versions, for negligible transport cost: in this case, firms locate at the centre of the market, but choose zero effort in quality enhancement and make no profit as price is driven down to marginal cost.

The paper contributes to two areas of applied microeconomics, one dealing with strategic R&D and endogenous spillovers and the other with the issue of locational choice in spatial competition. Before presenting the model and its analysis we provide a very brief review of the relevant literature, in addition to Kamien and Zang (2000). Economides (1989) considers quality enhancement - a form of R&D - in the absence of spillovers. The relevant result states that in the location-quality-price game there is a unique equilibrium exhibiting maximal varietal differentiation (firms locate at the extremes), minimal quality differentiation and equal prices. Thus, the principle of maximal differentiation in location holds. In contrast, Mai and Peng (1999) show that the principle of maximal differentiation may not hold when firms incur a communication cost that depends on their relative proximity. They assume that spatial proximity facilitates communication and thus reduces R&D costs. However, they do not model R&D explicitly. Poyago-Theotoky (1999) which is closely related to Kamien and Zang (2000) shows that non-cooperating firms choose a non-specific R&D approach which is equivalent to an endogenous spillover equal to zero.

The literature on locational choice in the absence of R&D is extensive; for an excellent review see Gabsewicz and Thisse (1992). One of its main themes has been the examination of the validity or not of Hotelling's (1929) principle of minimum differentiation. Economides (1986) shows that the equilibrium locations depend on the convexity of the transport cost function and that interior (non maximal differentiation) equilibria can exist.[1] For the case of spatial discriminatory pricing, we note that Hamilton et al. (1989) have established, using a duopoly model with downward sloping demand, constant marginal production costs and linear transport costs, that under Bertrand competition firms locate at distinct locations inside the first and third quartiles. In contrast, when firms compete *à la* Cournot, agglomeration is the equilibrium location, a result also confirmed by Anderson and Neven (1991).

The remainder of the paper is organised as follows. In section 2 we present the model and explain in more detail how it relates to the notion of absorptive capacity as used in Kamien and Zang (2000). Then, in sections 3 and 4 we provide the analysis for the "mill pricing" and the "discriminatory pricing" versions of the model, respectively. Finally, in section 5 we provide some concluding remarks.

[1] In particular, internal locations obtain in equilibrium when the transportation cost (distance) function is $f(d) = d^{\alpha}, \frac{5}{4} < \alpha < \frac{5}{3}$.

2 The Model: Set-up and notation

Consider two firms, firm 1 and firm 2, each producing a homogenous good at constant marginal cost, $\overline{c} \geq 0$. There is a continuum of consumers uniformly distributed on a unit-length interval, with a mass normalized to unity without loss of generality. Denote the location of firm i as $y_i \in [0,1]$, $i = 1,2$, with $y_2 \geq y_1$. Then firm i's 'effective' R&D effort, X_i, can be represented as a function of both firms' R&D efforts, $x_i, i = 1,2$, and locational choice:

$$X_i = x_i + (1 - y_2 + y_1)x_j, \; i,j = 1,2; i \neq j. \tag{1}$$

This expression summarises the relationship between the spillover effect and the distance between the firms' locations. The spillover is at a maximum when firms share the same location; it is at a minimum when firms locate at the market endpoints.

The above specification for 'effective' R&D bears both differences and similarities with that in KZ, who define it as:

$$X_i^{KZ} = x_i + (1 - \delta_i)(1 - \delta_j)\beta x_i^{\delta_i} x_j^{1-\delta_i}, \tag{2}$$

where $0 \leq \beta \leq 1$ is the (exogenously given) spillover and $0 \leq \delta_i, \delta_j \leq 1$ is a parameter that captures a firm's research approach. Firm i cannot realise any beneficial spillover from firm j without engaging in (cost-reducing) R&D itself, i.e. $x_i > 0$, unless $\delta_i = 0$ as well.

The situation in which firms in KZ choose a firm-specific approach in R&D, i.e. $\delta_i = 1, i = 1,2$, is equivalent to our case when firms choose to locate at the market extremes: in both cases the realised spillover is equal to zero. The situation in which firms in KZ adopt a basic research approach, i.e. $\delta_i = 0, i = 1,2$, corresponds in our case to a situation where firms choose the same location and benefit from the maximum spillover. The main difference between the two specifications, (1) and (2), lies in the fact that the spillover for firm i in our specification does not depend upon having firm i investing in R&D. That is, in our approach firms may decide not to engage in any investment activity and yet benefit from the other firm's investment. However, the analogy between the two specifications is reinforced by the observation that in KZ the (subgame-perfect) equilibrium value of δ_i is either zero or one. Thus, *de facto*, the equilibrium value of the effective R&D in KZ is either $X_i = x_i + \beta x_j$ or $X_i = x_i$. That is, the realised spillover for firm i is independent of whether firm i invests in R&D or not, as in the case considered in this paper.

A consumer located at $s \in [0,1]$, who decides to buy (one unit) from firm i, derives surplus

$$v + X_i - p_i - t(s - y_i)^2, \tag{3}$$

where $v \geq 0$ is the basic reservation utility obtained by any consumer who purchases from any of the two firms and p_i is the (mill) price of firm i's product and $t \geq 0$ is an index of the transportation cost per unit (or disutility in the context of characteristics space).[2] The quadratic cost assumption is invoked because it guarantees existence of equilibrium in the price stage. We assume that v is large enough so that, in equilibrium, consumers always make a purchase and that a consumer buys (one unit) from the firm that offers her the highest surplus or, equivalently, the lowest full price. To avoid any arbitrage between consumers we assume that the transaction costs for the resale of goods are prohibitively high. Note that in (3), effective R&D, X_i, is equivalent to a quality enhancement for product i; thus $v + X_i$ is the highest price a consumer would pay for the product (Economides 1989). However, given our assumption of constant marginal cost, it is straightforward to verify that our analysis would yield identical results if we omitted X_i from (3) and assumed that R&D is cost-reducing: $MC_i = \overline{c} - X_i$. To save on notation and complexity, we set $\overline{c} = 0$ without loss of generality and to avoid having to consider the conditions for a non-negative marginal cost, we adopt the quality enhancement specification, as in (3), for the remainder of the paper.

3 Mill pricing

Following Hotelling's (1929) classic paper we present a spatial duopoly model where firms' geographical location (or choice of product characteristics) determines endogenously the extent of knowledge spillovers that a firm can benefit from. Note that in the context of mill pricing transport costs are borne by consumers. We formulate a three-stage non-cooperative game where in the first stage firms choose their location, then they choose R&D expenditure to improve the quality of their product in stage two, and finally, in the third stage they compete by setting price. As we are seeking the subgame-perfect equilibrium of this game we start by analysing the last stage.

3.1 The Price Game

The first step is to derive the demands for the two firms. The surplus from buying a unit from firm 1 to a consumer located at $s, s \in [0, 1]$, is $v + X_1 - p_1 - t(s - y_1)^2$, and the surplus for buying from firm 2 is $v + X_2 - p_2 - t(s - y_2)^2$. Thus, the 'indifferent' consumer is defined by the following equality

$$v + X_1 - p_1 - t(\widehat{s} - y_1)^2 = v + X_2 - p_2 - t(\widehat{s} - y_2)^2$$

which yields

$$\widehat{s} = \frac{(p_2 - p_1) - (X_2 - X_1)}{2t(y_2 - y_1)} + \frac{y_2 + y_1}{2}. \tag{4}$$

[2] A similar assumption is made in Piga (1998) in the context of a model of advertising.

For the two firms to be active in the market it must be the case that $y_1 \leq \widehat{s} \leq y_2$. This condition is satisfied if $|(p_2 - p_1) - (X_2 - X_1)| \leq t(y_2 - y_1)^2$. For $y_2 < \widehat{s}$ only firm 1 is active serving the whole market; this is so if $p_1 - X_1 < p_2 - X_2 - t(y_2 - y_1)^2$ while for $\widehat{s} < y_1$ or $p_1 - X_1 > p_2 - X_2 - t(y_2 - y_1)^2$ firm 2 takes the whole market. Hence, aggregate demand for firm 1 is

$$D_1 = \begin{cases} 1 \text{ if } p_1 - X_1 < p_2 - X_2 - t(y_2 - y_1)^2 \\ \widehat{s} \text{ if } |(p_2 - p_1) - (X_2 - X_1)| \leq t(y_2 - y_1)^2 \\ 0 \text{ if } p_1 - X_1 > p_2 - X_2 - t(y_2 - y_1)^2 \end{cases} . \tag{5}$$

An analogous expression holds for firm 2's demand, $D_2 \in [0, 1]$. The profit function for firm i, $i = 1, 2$, is given by

$$\pi_i = p_i D_i - c(x_i) \tag{6}$$

where $c(x_i)$ is the R&D cost and $c' > 0$, $c'' > 0$ to capture decreasing returns in R&D spending. In what follows we will use $c(x_i) = \frac{1}{2}x_i^2$. In the remainder of the paper we will concentrate on the quadratic part of the profit functions, i.e. we will consider the case when both firms are active, $D_i \in (0, 1)$. From the first order conditions we obtain the equilibrium prices:[3]

$$\begin{aligned} p_1 &= \frac{1}{3}\left[(X_1 - X_2) + t(y_2 - y_1)(2 + y_2 + y_1)\right] \\ &= \frac{1}{3}(y_2 - y_1)\left[(x_1 - x_2) + t(2 + y_2 + y_1)\right] \end{aligned} \tag{7}$$

$$\begin{aligned} p_2 &= \frac{1}{3}\left[(X_2 - X_1) + t(y_2 - y_1)(4 - y_2 - y_1)\right] \\ &= \frac{1}{3}(y_2 - y_1)\left[(x_2 - x_1) + t(4 - y_2 - y_1)\right]. \end{aligned} \tag{8}$$

Note that when firms locate at the same position, $y_1 = y_2$, prices are equal to the unit cost of production (normalized to zero), $p_1 = p_2 = 0$, i.e. the Bertrand outcome with homogeneous goods. Substituting the equilibrium prices, (7) and (8), into the profits expression, (6), we obtain:

$$\pi_1 = \frac{1}{2t(y_2 - y_1)}p_1^2 = \frac{\left[X_1 - X_2 + t(y_2 - y_1)(2 + y_2 + y_1)\right]^2}{18t(y_2 - y_1)} \tag{9}$$

and

$$\pi_2 = \frac{1}{2t(y_2 - y_1)}p_2^2 = \frac{\left[X_2 - X_1 + t(y_2 - y_1)(4 - y_2 - y_1)\right]^2}{18t(y_2 - y_1)}. \tag{10}$$

This concludes the analysis of the price game.

[3]The second order condition is satisfied, $\frac{\partial^2 \pi_i}{\partial p_i^2} = -\frac{1}{t(y_2 - y_1)} < 0$.

3.2 The R&D/Quality Game

In the second stage firms choose their R&D effort, x_i, $i, j = 1, 2, i \neq j$, noncooperatively, taking locations as given. Using the expressions for profits derived in the previous section, (9) or (10), and the expression for 'effective' R&D, (1), after taking first-order conditions we obtain the best-response functions:

$$x_1(x_2) = \frac{(y_2 - y_1)[t(2 + y_2 + y_1) - x_2]}{9t - (y_2 - y_1)} \tag{11}$$

$$x_2(x_1) = \frac{(y_2 - y_1)[t(4 - y_2 - y_1) - x_1]}{9t - (y_2 - y_1)} \tag{12}$$

and hence the equilibrium R&D efforts (qualities)

$$x_1 = \frac{(y_2 - y_1)[3t(2 + y_2 + y_1) - 2(y_2 - y_1)]}{3[9t - 2(y_2 - y_1)]} \tag{13}$$

$$x_2 = \frac{(y_2 - y_1)[3t(4 - y_2 - y_1) - 2(y_2 - y_1)]}{3[9t - 2(y_2 - y_1)]}. \tag{14}$$

Note that the second-order condition requires $9t - (y_2 - y_1) > 0$, for given locations $y_1(t)$ and $y_2(t)$. From the reaction functions, (11) and (12), it is easy to see that they are negatively sloped, $\frac{dx_1}{dx_2} = \frac{dx_2}{dx_1} - \frac{(y_2-y_1)}{9t-(y_2-y_1)} < 0$; i.e. R&D effort is perceived as a strategic substitute by firms. Further, stability requires that $\left|\frac{dx_i}{dx_j}\right| < 1, i, j = 1, 2, i \neq j$, which is satisfied if $9t - 2(y_2 - y_1) > 0$ and is stronger than the second-order condition. Note that from the equilibrium R&D efforts, (13) and (14), there is no quality improvement if the two firms choose identical locations, i.e. $x_1 = x_2 = 0$ if $y_1 = y_2$, as expected intuitively. To guarantee positive quality improvements the following conditions must hold:

$$\begin{aligned} x_1 &> 0 \Rightarrow 3t(2 + y_2 + y_1) - 2(y_2 - y_1) > 0 \\ x_2 &> 0 \Rightarrow 3t(4 - y_2 - y_1) - 2(y_2 - y_1) > 0. \end{aligned}$$

It is obvious that firms will choose the same quality improvement, $x_1 = x_2$, if they choose symmetric locations, $y_1 + y_2 = 1$. We can then state:

Lemma 1 *For any symmetric locations of the two firms, $y_1 + y_2 = 1$, the equilibrium R&D efforts are equal, $x_1 = x_2$, i.e. there is no quality differentiation.*

The content of Lemma 1 is similar to the result on minimal quality differentiation obtained by Economides (1989).

Next, using (13) and (14) we can compute equilibrium profits:

$$\widehat{\pi}_1^2 = \frac{(y_2 - y_1)[9t - (y_2 - y_1)][3t(2 + y_2 + y_1) - 2(y_2 - y_1)]^2}{18[9t - 2(y_2 - y_1)]^2} \tag{15a}$$

$$\widehat{\pi}_2^2 = \frac{(y_2 - y_1)[9t - (y_2 - y_1)][3t(4 - y_2 - y_1) - 2(y_2 - y_1)]^2}{18[9t - 2(y_2 - y_1)]^2}. \tag{15b}$$

We can then proceed to the analysis of the location choice.

3.3 The Location Game

In the first stage, firms choose their locations, $y_i, i = 1, 2$, anticipating how this choice will affect their subsequent choices of R&D and price. Using the expressions for profits obtained in the previous stage, (15a) and (15b), taking first-order conditions and then restricting the resulting solution to a symmetric one, i.e $y_1 + y_2 = 1$, we obtain two sets of candidate equilibrium locations:[4]

$$y_1^* = \frac{4 - 12t - 27t^2 + 9t^{3/2}\sqrt{6 + 9t}}{4(2 + 3t)} \quad (16)$$

$$y_2^* = \frac{4 + 24t + 27t^2 - 9t^{3/2}\sqrt{6 + 9t}}{4(2 + 3t)} \quad (17)$$

and

$$\widetilde{y}_1 = \frac{4 - 12t - 27t^2 - 9t^{3/2}\sqrt{6 + 9t}}{4(2 + 3t)}$$

$$\widetilde{y}_2 = \frac{4 + 24t + 27t^2 + 9t^{3/2}\sqrt{6 + 9t}}{4(2 + 3t)}.$$

In the previous section, we have established that the stability condition in the R&D stage requires that $9t - 2(y_2 - y_1) > 0$. It is easy (though tedious) to check that the stability condition is satisfied by the first set of location choices, y_1^* and y_2^*, but not by the second set, $\widetilde{y}_1$ and $\widetilde{y}_2$.Thus the equilibrium locations are given by y_1^* and y_2^*.

Using this solution we can compute the subgame-perfect equilibrium levels for all relevant variables. These values are reported in Table 1.

Table 1. Subgame-perfect Equilibrium Values-Mill pricing

Location	$y_1^* = \frac{4-12t-27t^2+9t^{3/2}\sqrt{6+9t}}{4(2+3t)}, y_2^* = 1 - y_1^*$
R&D	$x_1^* = x_2^* = \frac{3t(2+3t-\sqrt{t}\sqrt{6+9t})}{4+6t}$
Effective R&D	$X_1^* = X_2^* = \frac{3t[4-3t(1+9t)+(9t-2)\sqrt{t}\sqrt{6+9t}]}{4+6t}$
Profit	$\pi_1^* = \pi_2^* = \frac{9t^2}{8+12t}$
Price	$p_1^* = p_2^* = \frac{9t^2(2+3t-\sqrt{t}\sqrt{6+9t})}{4+6t}$

[4]Note that the second-order condition, evaluated at the symmetric equilibrium is satisfied, $\left.\frac{\partial^2 \pi_1}{\partial y_1^2}\right|_{y_1^*} = \left.\frac{\partial^2 \pi_1}{\partial y_1^2}\right|_{\widetilde{y}_1} = -(\frac{1}{2}t + \frac{1}{3}) < 0$. Similarly for firm 2.

Given symmetric locations, from Lemma 1 we already know that firms will choose the same quality/R&D level and it is easily checked from the expression for R&D that $x_1^*, x_2^* \geq 0$ as $t \geq 0$. It is also easy to establish that R&D, profits and price are increasing in the transportation cost, $\partial x_i/\partial t > 0, \partial \pi_i/\partial t > 0$ and $\partial p_i/\partial t > 0, i = 1, 2$. We then state our main results in the following propositions.

Proposition 1 *There exists a subgame-perfect symmetric location equilibrium given by* $y_1^*(t) = \frac{4-12t-27t^2+9t^{3/2}\sqrt{6+9t}}{4(2+3t)}$ *and* $y_2^*(t) = 1 - y_1^*(t)$. *Given continuity of* $y_1^*(t)$ *and* $y_2^*(t)$, *there exists a transportation cost* t *such that any symmetric location on the unit interval,* $y_i \in [0,1], i = 1, 2, y_1 + y_2 = 1$, *can be an equilibrium. In particular:*

(a) agglomeration or minimal product differentiation, $y_1^* = y_2^* \approx \frac{1}{2}$, *is an equilibrium as the transportation cost becomes negligible,* $t \to 0$;

(b) deglomeration or maximal product differentiation occurs, $y_1^* = 0$ *and* $y_2^* = 1$, *for relatively large transportation cost,* $t = \frac{1}{9}(5 + \sqrt{13}) \approx 0.9562$;

(c) an intermediate location is an equilibrium for $0 < t < 0.9562$.

Further, the subgame-perfect equilibrium exhibits minimal quality differentiation, $x_1^* = x_2^* = \frac{3t(2+3t-\sqrt{t}\sqrt{6+9t})}{4+6t}$, *equal prices,* $p_1^* = p_2^* = \frac{9t^2(2+3t-\sqrt{t}\sqrt{6+9t})}{4+6t}$ *and profits by* $\pi_1^* = \pi_2^* = \frac{9t^2}{8+12t}$.[5]

Turning to the equilibrium locations, from (16) and (17), it is apparent that location is a function of the transportation rate, t, and is intricately related to our notion of location-related spillover, $1 - y_2 + y_1$. The comparative static effect of an increase in t on the equilibrium locations can be evaluated as follows:

$$\begin{aligned} \frac{\partial y_1^*}{\partial t} &= \frac{9}{4}\left[-1 + \frac{3\sqrt{t}(1+t)\sqrt{6+9t}}{(2+3t)^2}\right] < 0 \\ \frac{\partial y_2^*}{\partial t} &= \frac{9}{4}\left[1 - \frac{3\sqrt{t}(1+t)\sqrt{6+9t}}{(2+3t)^2}\right] > 0. \end{aligned}$$

In addition, using (16) and (17), the endogenous spillover is given by $1 - y_2^* + y_1^* = \frac{4-3t(4+9t-3\sqrt{t}\sqrt{6+9t})}{4+6t}$ and the effect of an increase in t is negative, $\frac{\partial(1-y_2^*+y_1^*)}{\partial t} < 0$.[6] Hence we can state the following:

Proposition 2 *The higher the transportation cost, the further apart firms will choose to locate and the lower the endogenous (location-related) spillover will be.*

[5] Note that when $t \to 0$, in the SPNE firms locate in the centre, $y_1 = y_2$, price is set equal to marginal cost (zero by assumption) and profits are zero.

[6] This effect is $\frac{\partial(1-y_2^*+y_1^*)}{\partial t} = \frac{9}{2}\left(-1 + \frac{3\sqrt{t}(1+t)\sqrt{6+9t}}{(2+3t)^2}\right) < 0$.

An intuitive discussion of the two propositions follows. When the transportation cost is negligible, $t \to 0$, we know from Proposition 1 that firms will choose to locate at the centre of the unit interval, $y_1 \approx y_2$. In this case the location-related (endogenous) spillover is at a maximum, $1 - y_2 + y_1 = 1$, and the firms stand to benefit greatly from each other's R&D effort. However, a central location for both firms implies that they do not differentiate their product. The intensive price competition that ensues dissipates any incentive towards quality-enhancing R&D so that firms do no R&D at all, $x_1 = x_2 = 0$. As the transportation cost increases firms choose distinct locations, $y_1 \neq y_2$. Proposition 2 establishes that the higher the transportation cost the further apart firms locate, offering increasingly differentiated products, and the less they benefit from each other's R&D. Location at the endpoints, $y_1 = 0$ and $y_2 = 1$, corresponds to a zero spillover and maximal product differentiation. This latter result corresponds to Remark 2 of Kamien and Zang (2000) which states that firms choose firm-specific R&D approaches to offset exogenous spillovers. A firm-specific R&D approach in their model corresponds to maximal product differentiation in our model. In the same Remark, KZ find that a broad approach in R&D is chosen only when there is no danger for firms to confer a benefit to their rival. By the same token, our results show that minimal differentiation is accompanied by no investment in R&D.

Further, the higher the transportation cost the more firms differentiate their product and relax price competition. Relaxing price competition allows firms to invest in quality enhancement that increases consumer's surplus and willingness to pay and leads to higher profits.

Our results are similar to those obtained by Mai and Peng (1999) in that we also find that the equilibrium locations depend on the transportation cost and that firms will locate at an increasing distance from each other as the transportation cost increases. However, as mentioned in the Introduction, the defining difference of our approach is that we model R&D explicitly and endogenise the location-related spillover externality.

4 Discriminatory pricing

In this section of the paper we consider discriminatory pricing, i.e. firms bear the costs of transportation. Note that in the context of product competition in characteristics space (product differentiation in variety), discriminatory pricing corresponds to the selling custom-made products, e.g. mainframe computers, aircraft, PCs etc.

Recall that to avoid any arbitrage between consumers, we have assumed that the transaction costs for the resale of goods are prohibitively high. In the case of discriminatory pricing, the full price for a unit of the good is equivalent to the delivered price in each location. Following Tabuchi

(1999), we define the equilibrium price schedule of firm i as the second lowest delivered (transportation) cost net of 'effective' R&D:

$$\begin{aligned} p_i^d &= t(s-y_j)^2 - X_j, \text{ for } s \in [0, \overline{s}] \\ &= t(s-y_i)^2 - X_i, \text{ for } s \in [\overline{s}, 1] \end{aligned} \tag{18}$$

$i, j = 1, 2, i \neq j$, where $s \in [0,1]$ represents a consumer's location and $\overline{s}$ is the market boundary defined by $\overline{s} \equiv \{s \mid t(s-y_1)^2 - X_1 = t(s-y_2)^2 - X_2\}$. The market boundary is then $\overline{s} = \frac{t(y_2-y_1)+x_1-x_2}{2t}$. For each consumer location, s, a firm has to pay the transportation cost (net of 'effective' R&D) for the range of s where it has a positive market area. Hence, the profit for firm 1 is given by:

$$\pi_1 = \int_0^{\overline{s}} \left[t(s-y_2)^2 - X_2 - t(s-y_1)^2 + X_1\right] ds - \frac{1}{2}x_1^2 \tag{19}$$

and for firm 2:

$$\pi_2 = \int_{\overline{s}}^1 \left[t(s-y_1)^2 - X_1 - t(s-y_2)^2 + X_2\right] ds - \frac{1}{2}x_2^2. \tag{20}$$

Having described the price schedules and the resulting profit functions we can now turn to the choice of R&D efforts.

4.1 The R&D/Quality Game

In this stage firms choose their R&D efforts, $x_i, i, j = 1, 2, i \neq j$, taking locations as given. Using profits as given by (19), (20) and (1), after taking first-order conditions we can derive the best-response functions:[7]

$$x_1(x_2) = \frac{(y_2-y_1)\left[t(y_2+y_1) - x_2\right]}{2t - (y_2-y_1)} \tag{21}$$

$$x_2(x_1) = \frac{(y_2-y_1)\left[t(2-y_2-y_1) - x_2\right]}{2t - (y_2-y_1)} \tag{22}$$

and the equilibrium R&D efforts:

$$x_1 = \frac{(y_2-y_1)\left[t(y_2+y_1) - (y_2-y_1)\right]}{2(t-y_2+y_1)} \tag{23}$$

$$x_2 = \frac{(y_2-y_1)\left[t(2-y_2-y_1) - (y_2-y_1)\right]}{2(t-y_2+y_1)}. \tag{24}$$

From the best-response functions we can easily establish that in this case too R&D is a strategic substitute, $\frac{dx_i}{dx_j} = -\frac{(y_2-y_1)}{2t-(y_2-y_1)} < 0, i, j = 1, 2, i \neq j$, where

[7] Note that the second order condition requires that $2t-(y_2-y_1) > 0$ for given locations $y_1(t)$ and $y_2(t)$.

the denominator is positive from the second-order condition. Moreover, the stability condition, $\left|\frac{dx_i}{dx_j}\right| < 1, i, j = 1, 2, i \neq j$, requires that $2t - 2(y_2 - y_1) > 0$, which is a stronger condition than the second-order condition, $2t - (y_2 - y_1) > 0$. In the remainder of this and the following section we impose this stronger condition. Next, using the expressions for equilibrium R&D, (23) and (24), we obtain equilibrium profits:

$$\widehat{\pi}_1^2 = \frac{(y_2 - y_1)(2t - y_2 + y_1)\left[t(y_2 + y_1) - (y_2 - y_1)\right]^2}{8(t - y_2 + y_1)^2} \tag{25}$$

$$\widehat{\pi}_2^2 = \frac{(y_2 - y_1)(2t - y_2 + y_1)\left[t(2 - y_2 - y_1) - (y_2 - y_1)\right]^2}{8(t - y_2 + y_1)^2}. \tag{26}$$

Next we turn our attention to the choice of location.

4.2 The Location Game

In this stage firms choose a location, $y_i \in [0, 1], i = 1, 2$, non-cooperatively. Using the profit expressions from the previous stage, (25) and (26), taking the first-order conditions and then imposing symmetry, $y_1 + y_2 = 1$, we obtain two sets of solutions:[8]

$$\overline{y}_1 = \frac{1}{2}\left[1 - t + \frac{t^{\frac{3}{2}}}{\sqrt{1+t}}\right] \tag{27}$$

$$\overline{y}_2 = \frac{1}{2}\left[1 + t - \frac{t^{\frac{3}{2}}}{\sqrt{1+t}}\right] \tag{28}$$

and

$$\overline{y}_1^{\diamond} = \frac{1}{2}\left[1 - t - \frac{t^{\frac{3}{2}}}{\sqrt{1+t}}\right]$$

$$\overline{y}_2^{\diamond} = \frac{1}{2}\left[1 + t + \frac{t^{\frac{3}{2}}}{\sqrt{1+t}}\right].$$

Following the same methodology used in section 3.3 , we are left with only one stable solution, that is $\overline{y}_1$ and $\overline{y}_2$.[9] Using this solution, that is (27) and (28), we can calculate the subgame-perfect equilibrium levels for all relevant variables. Table 2 summarises these.

[8] The second order condition, evaluated at the symmetric equilibrium is satisfied, $\left.\frac{\partial^2 \pi_1}{\partial y_1^2}\right|_{\overline{y}_1} = \left.\frac{\partial^2 \pi_1}{\partial y_1^2}\right|_{\overline{y}_1^{\diamond}} = -\frac{3}{4}(1+t) < 0$. A similar condition holds for firm 2.

[9] The stability condition, $2(t - y_2 + y_1) > 0$ is satisfied for $\overline{y}_1^*$ and $\overline{y}_2^*$ but not for $\overline{y}_1^{\diamond}$ and $\overline{y}_2^{\diamond}$.

Table 2. Subgame-perfect Equilibrium Values-Discriminatory pricing

Location	$\overline{y}_1 = \frac{1}{2}[1 - t + \frac{t^{\frac{3}{2}}}{\sqrt{1+t}}], \overline{y}_2 = 1 - \overline{y}_1$
R&D	$\overline{x}_1 = \overline{x}_2 = \frac{t(1+t-\sqrt{t}\sqrt{1+t})}{2(1+t)}$
Effective R&D	$\overline{X}_1 = \overline{X}_2 = \frac{t[2+t-2t^2+2(t-1)\sqrt{t}\sqrt{1+t}]}{2(1+t)}$
Profit	$\overline{\pi}_1 = \overline{\pi}_2 = \frac{t^2}{8(1+t)}$
Price	see expression (18)

From Table 2 it is evident that, given symmetric locations, firms will choose the same level of quality enhancement, $\overline{x}_1 = \overline{x}_2$, leading to the same level of 'effective' R&D, $\overline{X}_1 = \overline{X}_2$ and equal profits. Further, it can be shown that $\frac{\partial x_i}{\partial t} > 0$ and $\frac{\partial \pi_i}{\partial t} > 0$. These results are in line with those obtained for the case of mill pricing.

The equilibrium locations, $\overline{y}_1$ and $\overline{y}_2$, depend on the transportation cost, t. It is easy to check that there does not exist $t \in R$ that would result in maximal product differentiation, i.e. $\overline{y}_1 = 0$ and $\overline{y}_2 = 1$. Further, note that $\lim_{t \to +\infty} \overline{y}_1 = \frac{1}{4}$ and $\lim_{t \to +\infty} \overline{y}_2 = \frac{3}{4}$. However, an infinite transportation cost is not consistent with a non-negative surplus (see [3]) so that, strictly speaking, the above values should be viewed as boundary locations: firms will never choose to locate at these points or beyond. Thus, for finite t firms will locate symmetrically within the interval $(\frac{1}{4}, \frac{3}{4})$. We can then state:

Proposition 3 *There exists a subgame-perfect symmetric location equilibrium given by* $\overline{y}_1(t) = \frac{1}{2}[1 - t + \frac{t^{\frac{3}{2}}}{\sqrt{1+t}}]$ *and* $\overline{y}_2(t) = 1 - \overline{y}_1(t)$. *Given continuity of* $\overline{y}_1(t)$ *and* $\overline{y}_2(t)$, *there exists a finite transportation cost* t *such that any symmetric location on the interval* $(\frac{1}{4}, \frac{3}{4})$, *can be an equilibrium. In particular:*

(a) agglomeration or minimal product differentiation, $\overline{y}_1 = \overline{y}_2 \approx \frac{1}{2}$, *is an equilibrium for negligible transportation cost,* $t \to 0$.

(b) deglomeration or maximal product differentiation is not an equilibrium.

Further, the subgame equilibrium exhibits minimal quality differentiation, $\overline{x}_1 = \overline{x}_2 = \frac{t(1+t-\sqrt{t}\sqrt{1+t})}{2(1+t)}$, *and equal profits,* $\overline{\pi}_1 = \overline{\pi}_2 = \frac{t^2}{8(1+t)}$.

In line with the case of mill-pricing (see Proposition 2), the effect of a higher transportation cost is to shift firms further apart so that they benefit less from each other's R&D. Indeed, $\frac{\partial \overline{y}_1}{\partial t} < 0$, $\frac{\partial \overline{y}_2}{\partial t} > 0$ and $\frac{\partial(1-\overline{y}_2+\overline{y}_1)}{\partial t} < 0$.[10] Hence, we can restate Proposition 2 as follows:

[10] These comparative static effects are: $\frac{\partial \overline{y}_1}{\partial t} = \frac{1}{2}[-1 + \frac{\sqrt{t}(3+2t)}{(1+t)^{\frac{3}{2}}}] < 0$, $\frac{\partial \overline{y}_2}{\partial t} = \frac{1}{2}[1 - \frac{\sqrt{t}(3+2t)}{(1+t)^{\frac{3}{2}}}] > 0$ and $\frac{\partial(1-\overline{y}_2+\overline{y}_1)}{\partial t} = -1 + \frac{\sqrt{t}(3+2t)}{2(1+t)^{\frac{3}{2}}} < 0$.

Proposition 2R *Whether firms use mill-pricing or discriminatory pricing, the higher the transportation cost the further apart firms will locate and the lower the endogenous (location-related) spillover will be.*

The intuition is mostly similar to that given for the mill-pricing version of the model. Therefore, we stress here the differences and offer some explanatory comments. First, with discriminatory pricing there can be no maximal product differentiation as firms never locate at the extremes. This is explained by noting that firms choose a lower level of quality enhancement for all values of transportation cost relative to the case of mill pricing, $\overline{x}_i < x_i^*$, thus incurring lower R&D expenditure given that they have to pay for the good to be delivered to consumers' locations.[11] Locating at the endpoints would lead to negative profits. Moreover, minimal quality differentiation is an equilibrium for zero transportation costs. Thus, our results complement and enhance those obtained by Hamilton et al. (1989) who, in the absence of R&D, found that agglomeration never occurs and that there is no maximal differentiation . Second, a simple comparison of equilibrium profits reveals that profits are lower in the case of discriminatory pricing, $\overline{\pi}_i < \pi_i^*$.[12] At first this result may seem surprising. However, it can be explained by observing that for given transportation cost, in the case of discriminatory pricing firms will locate nearer to each other than in the case of mill pricing (except for the case of zero transport cost when they choose the central location). Hence, they will be subjected to fiercer price competition and as they also have to bear the transportation costs they will earn lower profits.

5 Concluding Remarks

In this paper we have developed a three-stage game of an address model with quality-enhancing R&D and endogenous spillovers. The locational interpretation of our model enables us to identify the conditions under which firms "meet halfway", a metaphor used by Kamien and Zang (2000) to make operational the notion of absorptive capacity in models of R&D. Central to our findings is the role played by the transport cost parameter, usually interpreted as a measure of the importance of product differentiation. In our model firms will share the same location, i.e. "meet halfway", only when the products are not differentiated. However, when firms agglomerate their R&D effort is zero. Thus, our theoretical prediction indicates an inverse

[11] From Tables 1 and 2, $x_i^* - \overline{x}_i = t + \frac{1}{2}t^{\frac{3}{2}}\left(\frac{1}{\sqrt{1+t}} - \frac{9}{\sqrt{6+9t}}\right)$. It can be shown that this is positive for the relevant range of t.

[12] From Tables 1 and 2 respectively, we have $\pi_i^* = \frac{9t^2}{8+12t}$ and $\overline{\pi}_i = \frac{t^2}{8(1+t)}$. It is easily observed that $\overline{\pi}_i < \pi_i^*$. A similar result is obtained by Tabuchi (1999) for fixed locations though.

relation between agglomeration and R&D effort. While this may not provide an appropriate description of firms' R&D behaviour in a geographical context (e.g. industrial districts) it nevertheless generates a set of testable hypotheses on the relationship between product differentiation arising from a product's characteristics approach and R&D. Indeed, we find that the higher the transport cost parameter the further away firms choose to position their products. However, regardless of the transport cost we always obtain minimal quality differentiation, that is firms always exert equal R&D efforts.

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