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1 Introduction

It is well known that processes of growth based solely on factor accumulation cease at some point because of diminishing returns to scale of production. By contrast, introducing innovative products may create new opportunities on the production side. Innovation may be induced by the prospect of enjoying temporary monopoly profits. Such temporary monopoly power makes the law of diminishing returns to scale vanish and growth based on innovation sustainable [see e.g. the seminal papers by Romer (1986, 1990) and, for a general overview, Aghion and Howitt (1997)].

Matsuyama (1996, 1999), among others, has unified these two approaches in a single growth model, and has shown that "under empirically plausible conditions, the balanced growth path is unstable and the economy achieves sustainable growth through cycles, perpetually moving back and forth between two phases" (Matsuyama 1999, p. 335). These two phases capture the main features of the approaches mentioned above: the Solow (neoclassical) model and the neo-Schumpetarian approach.

In one phase, when the growth rates of output and investment are higher, there is no innovation. The economy is then competitive and the evolution of this economy is the one pointed out by Solow. In the other phase, when the growth rates of output and investment are lower, there is innovation and the economy is "more monopolistic" as in the Schumpeterian endogenous growth approach. In the long run, the growth rates of innovation and investment are equal, however they do not follow the same evolution: they move over the cycle in an a-synchronised way. That is, the economy alternates between periods of high innovation and periods of high investment.

The mechanism at work is linked to the timing of entry of innovators into the market for new goods. To start up production innovators need to ensure that the market for their product is sufficiently large to recoup the costs of innovation, and since they enjoy monopoly rents only for one period, innovators introduce new products into the market at the same time as their competitors. Delaying entry would mean losing temporary monopoly rents and make innovation not profitable enough. Therefore, innovative activities take place at the same time, and prevail until competition among innovators builds up and monopoly rents drop. As the economy becomes more competitive, more resources are available to manufacturing activities, and both output and investment growth increase. Higher output and investment will, in turn, build up a larger resource base in the economy, which stimulates another period of innovative activity. It is this asynchronicity of innovation and investment activities that creates sustainable growth through cycles.[1]

We rely on Matsuyama (1996, 1999) to study the effects on growth and welfare of non distortive tax/subsidy policies. In particular, we are interested in investigating the stabilising/destabilising effects of policies aimed at eliminating economic fluctuations. The focus on stabilisation originates from the observation that nations and governments aim to reach high permanent growth without incurring recessions. We concentrate our attention on the issue of how subsidising innovators (and taxing consumers) can generate stable sustained growth and higher welfare.

In this respect, our paper can be seen as a reformulation in a macroeconomic context of the branch of the R&D literature that deals with public aid to innovation (see e.g. Aghion and Howitt (1992), Davidson and Segerstrom (1998) and Segerstrom (2000)). In this literature it is often claimed that there exists a role for public intervention both in subsidising R&D activities and enforcing property rights to innovation as means to promote economic growth.

We wish to add a new macroeconomic perspective to this claim: we show that in fast growing economies, in which high factor accumulation plays a crucial role alongside innovative sectors that enjoy temporary rents,[2] stabilisation poli-

[1] Periods of high innovation are followed by periods of high investment, and in each phase of the economy either innovation or capital accumulation play the dominant role. Moreover, as shown by Matsuyama (1996, 1999) when the economy moves back and forth between the two phases growth is faster than along the (unstable) balanced growth path.

[2] What we have in mind here are economies like Japan and East Asian newly industrialised countries (NICs) like South Korea, Hong Kong, Singapore and Taiwan. Japan experienced fast growth since the aftermath of the second World War until the early 1990s. Until the 1960s the main engine of growth was undoubtedly rapid factor accumulation; from the 1960s until

cies should follow an unorthodox approach. Namely, these policies should tax consumers during recessions and reallocate resources to the innovative sectors.

In spite of the simplicity of the model, some interesting policy implications can be drawn. In recent times the world has witnessed several fast-growing economies falling into recession. The so-called East Asian tigers and Japan, in particular, are still struggling to find a way out of such a crisis, and the issue of the policy requirements needed to resume fast growth is widely debated among economists (see e.g. Crafts (1999), Dornbusch (1995) and Ito (1996)). Most of these studies focus on the need for financial and structural reforms as pre-requisites to resume sustained growth in the long run. Also, there is a general agreement among researchers that the high saving rates of East Asian and Japanese economies are becoming an impediment to their economic recovery. Indeed, with profitability depressed very little of the relatively large share of income that is saved is eventually invested; moreover, high saving depresses domestic consumption. As a result, standard expansionary fiscal and monetary policies seem unable to trigger enough stimulus in aggregate demand and investment. Our model suggests that taxing the consumers and using the receipts to subsidise innovators may help to overcome recessions and resume sustained growth.[3]

Our paper also demonstrates that a tax on innovators aimed at raising resources available to consumers during recessions is detrimental. On the one hand it can be destabilising; more specifically, if the steady state is globally sta-

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the 1990s the main engine of growth of Japanese economy can be identified as the development and expansion of high technology industry (see Odagiri and Goto (1993)). As regards East Asian NICs, a large part of their growth until 1990 was driven by rapid factor accumulation (see Young (1995)). However, there is extensive evidence of the emerging role of high tech industry such as semiconductor industry in East Asian Economies (see Mathews and Cho (2000)).

[3]As mentioned before, there are few papers in the R&D literature that emphasise the role of subsidies to R&D in promoting economic growth. For instance, Davidson and Segerstrom (1998) distinguish between the role of innovative R&D and imitative R&D, and conclude that subsidising innovative R&D promote growth while subsidising imitative R&D can be detrimental to growth. General R&D subsidies, on the other hand, always exhert positive effects on growth. Similar conclusions are reached by Segerstrom (2000) where the emphasis is between vertical versus horizontal innovation. Even though our paper is not directly comparable with this strand of the literature, in that we adopt the most simple model of innovation and look at different issues (i.e. macroeconomic stabilisation), it is interesting to note the similarities in the policy implications of the two approaches.

ble under laissez faire, the introduction of the tax makes cycles possible. On the other hand, such a tax has negative welfare effects as it depresses output growth along the transitional adjustment path. These results are at odds with the general presumption that policies aimed at raising private saving (and, therefore, at expanding the resource base of the economy) foster higher growth and welfare.

The remainder of the paper is as follows. In Section 2 we describe the basics of the economic model used. In Section 3 we derive the equilibrium dynamics, while in Section 4 we present our results on the stabilising and destabilising effects of the tax/subsidy. Finally we provide some concluding remarks in Section 5. Proofs of propositions are relegated in the Appendix.

2 The Model

As in Matsuyama (1996) we assume that agents live for two periods. They work, consume and save when young, and they only consume when old. There is a final good, taken as numeraire, which is competitively produced, and is either consumed or invested. The part invested is converted into a variety of differentiated intermediate products, and associated with labour (exogenously fixed) according to a Cobb-Douglas technology. The intermediate products are aggregated into a symmetric CES technology. The systematic study of the intermediate production process makes it possible to specify the final good production function according to the prevailing regime. In the case of the Solow (no innovation) regime, the production function follows the traditional assumption of diminishing returns to scale. In the case of the Romer (innovation) regime, the final good production function is of the "AK" type. We depart from Matsuyama by assuming that in case of recessions, the government intervenes by implementing tax and subsidy policy aimed at ensuring a sustained growth path without downturns in economic activity.

The details of the model are as follows. Time is discrete, $t = 0, 1, ...$ Agents when young work and receive an income $w_t L$, and pay (or receive) a lump sum tax (subsidy) B_t, whereas when old they use all their savings for consumption.

w_t represents the real wage, L is the exogenously fixed labour supply, and the population growth rate is zero. The utility function, U_t, of the representative consumer of each generation is given by $U_t = (1-s)\ln c_{1t} + s \ln c_{2t+1}$ where s is the saving rate, c_{1t} is the consumption when young and c_{2t+1} is the consumption when old. Maximising utility subject to the intertemporal budget constraint,[4] $c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = w_t L + B_t$ yields, a simple saving function, S_t

$$S_t = s\left(w_t L + B_t\right) . \tag{1}$$

In this economy there is one final good, taken as numeraire, which is competitively produced and is either consumed or invested. The part invested is converted into a variety of differentiated intermediate products, and the latter are aggregated into a symmetric CES technology. Technology in the final goods sector is Cobb-Douglas, hence the final goods production function is given by (see Matsuyama 1999),

$$Y_t = \widehat{A}(L)^{\frac{1}{\sigma}} \left\{ \int_0^{N_t} x_t(z) dz \right\}^{1-\frac{1}{\sigma}} \tag{2}$$

where $x_t(z)$ is the intermediate input of variety $z \in [0, N_t]$ and $\sigma \in (1, \infty)$ is the elasticity of substitution between each pair of intermediate goods. It also follows that $\frac{1}{\sigma}$ is the labour share of income, since $w_t L = \frac{Y_t}{\sigma}$. Turning to the specification of the intermediate goods, we define x_t^c as the intermediate input produced in the competitive sector (with no innovation), and x_t^m the intermediate input produced by the monopolistic innovative sector. It is assumed that from period 0 to period $t-1$ only 'old' intermediate goods are available in the market. The variety $z \in [0, N_{t-1}]$ is produced under perfect competition with constant returns to scale. Between $t-1$ and t a range of 'new' intermediate goods $z \in [N_{t-1}, N_t]$ may be introduced. In this case, the variety $z \in [N_{t-1}, N_t]$ is produced and sold exclusively by the respective innovators in period t. Innovators operate under monopolistic competition with no barriers to entry. New

[4] The real interest rate is denoted by r.

intermediate goods are produced by using a units of capital per unit of output and require F units of capital per variety. The profit function of each innovator Π_t^m reads as,

$$\Pi_t^m = p_t^m x_t^m - r_t \left[a x_t^m + F \right] .$$

From the FOC it follows that $p_t^m = \frac{a\sigma r_t}{\sigma - 1} = \frac{\sigma}{\sigma - 1}$. Since there are no barriers to entry, the profit (net of fixed cost) must be zero at all times, implying that 'new' intermediate products ($N_t \geq N_{t-1}$) are introduced if and only if $a x_t^m \leqslant (\sigma - 1)F$. If innovators receive a lump sum subsidy (or pay a lump sum tax) the effect is clearly to increase (or reduce) the incentive to enter by potential innovators. The break-even level of production is then $x_t^m = (\sigma - 1)(F + aT_{t-1})$ where $T_{t-1} > 0$ ($T_{t-1} < 0$) is a lump sum tax (subsidy) set by the government at the end of period $t-1$. Demands for intermediate inputs come from the maximization of the final good profit function, taking into account that all intermediate goods enter symmetrically into the production of the final good, i.e. $x_t(z) \equiv x_t^c$ for $z \in [0, N_{t-1}]$ and $x_t(z) \equiv x_t^m$ for $z \in [N_{t-1}, N_t]$. Under these assumptions,

$$\frac{x_t^c}{x_t^m} = \left(\frac{p_t^c}{p_t^m} \right)^{-\sigma} = \left(1 - \frac{1}{\sigma} \right)^{-\sigma} .$$

The above implies that the demand for each intermediate input is,

$$x_t^c = \frac{1}{a} \theta \sigma F \left(1 + \frac{aT_{t-1}}{F} \right), \qquad \theta \equiv \left(1 - \frac{1}{\sigma} \right)^{1-\sigma} \tag{3}$$

$$x_t^m = \frac{1}{a} (\sigma - 1) F \left(1 + \frac{aT_{t-1}}{F} \right) \tag{4}$$

where $\theta \in (1, e = 2.71..)$. This is a parameter related to the monopoly margin of the innovator (i.e. $\frac{1}{\sigma - 1}$). Thus, when σ is close to one $\theta \to 1$, and when $\sigma \to \infty$, $\theta \to e$. Using the above relationships, the economy resource constraint on capital in period t is,[5]

[5] K_t denotes the capital stock available at the end of period t. It corresponds to the amount of final goods left un-consumed at the end of period t and carried over to period $t+1$.

$$K_{t-1} = N_{t-1}(ax_t^c) + (N_t - N_{t-1})(ax_t^m + F)\,. \qquad (5)$$

It then follows that when $N_t = N_{t-1}$ (i.e. no innovation),

$$\frac{K_{t-1}}{N_{t-1}} = ax_t^c$$

whereas, when $N_t > N_{t-1}$ (i.e. innovation) substituting the expressions for demand, (3) and (4), into the economy resource constraint on capital gives,

$$N_t = N_{t-1} + \frac{K_{t-1}}{\sigma\,(F + aT_{t-1})} - \theta N_{t-1}\,.$$

Letting $k_t \equiv \frac{K_t}{\theta\sigma F N_t}$ we obtain an expression governing the introduction of new products,

$$N_t - N_{t-1} = Max\ N_{t-1}\left\{0, \frac{\theta k_{t-1}}{1 + \frac{aT_{t-1}}{F}} - \theta\right\}. \qquad (6)$$

The critical point at which innovation starts to be profitable is $k_{cr} \equiv 1 + \frac{aT_{t-1}}{F}$.

In a symmetric equilibrium, total output, as in (2), is equal to

$$Y_t = \widehat{A}(L)^{\frac{1}{\sigma}}\left[N_{t-1}(x_t^c)^{1-\frac{1}{\sigma}} + (N_t - N_{t-1})(x_t^m)^{1-\frac{1}{\sigma}}\right].$$

Using the demand for the intermediate inputs, (3) and (4), the reduced form aggregate production function for $N_t = N_{t-1}$ is

$$Y_t = \mathcal{A}_{t-1}\left(1 + \frac{aT_{t-1}}{F}\right)^{-\frac{1}{\sigma}}(k_{t-1})^{-\frac{1}{\sigma}}K_{t-1}\,, \qquad (7)$$

while for $N_t > N_{t-1}$ it becomes

$$Y_t = \mathcal{A}_{t-1}K_{t-1} \qquad (8)$$

where $\mathcal{A}_{t-1} \equiv \frac{\widehat{A}}{a}\left(\frac{aL}{(1+\frac{aT_{t-1}}{F})\theta\sigma F}\right)^{\frac{1}{\sigma}}$. Note that under laissez faire, $T_{t-1} = 0$, $\mathcal{A}_{t-1} = A \equiv \frac{\widehat{A}}{a}\left(\frac{aL}{\theta\sigma F}\right)^{\frac{1}{\sigma}}$ (as in Matsuyama (1999)). The reduced form aggregate production of the final good is of the 'AK' type if $N_t > N_{t-1}$ as in expression (8). Therefore, if the resource base of the economy is large enough, $k_{t-1} > k_{cr}$, new products are introduced and the economy evolves according to a 'Romer regime'. If the resource base is not large enough, no innovation takes place and the aggregate production function (i.e. expression (7)) exhibits decreasing returns to capital. Hence the economy evolves according to a 'Solow regime'.

As shown in the section below, the equilibrium dynamics of the model simplifies to a non-linear one-dimensional difference equation, well defined in the forward direction of time. The analysis of this one-dimensional equation (evaluation of the steady state and its stability properties) allows us to study the evolution of the economy between the two regimes within a single growth process.

3 Equilibrium Dynamics and Steady State

To derive the dynamic equilibrium we need to specify how capital accumulation evolves over time. At equilibrium, saving equals investment, $S_t = K_t$, and the government balances the budget, $B_t = (N_t - N_{t-1})\, T_{t-1}$. The latter expression implies that the lump sum tax on innovators at the ouset of period t ($T_{t-1} > 0$) is redistributed to the consumers in the form of a lump sum subsidy. Equivalently, lump sum taxes levied on the consumers ($T_{t-1} < 0$) finance the subsidy distributed to the innovators. Hence, the saving function, (1), can be written as

$$S_t = s\frac{Y_t}{\sigma} + s\,(N_t - N_{t-1})\, T_{t-1}\,.$$

Substituting for $N_t - N_{t-1}$ from (6) and Y_t from either (7) or (8) into the expression above we obtain capital K_t as

$$K_t = \frac{s}{\sigma}\left[\mathcal{A}_{t-1} Max\left(\left(1+\frac{aT_{t-1}}{F}\right)^{\frac{1}{\sigma}} k_{t-1}^{-\frac{1}{\sigma}},\ 1\right) K_{t-1}\right]$$
$$+sT_{t-1} Max\ N_{t-1}\left[0, \theta\left(\frac{k_{t-1}}{1+\frac{aT_{t-1}}{F}}-1\right)\right].$$

Dividing both sides of the above expression by $\theta\sigma F N_t$, the forward perfect foresight dynamics of the system can be expressed as a one-dimensional map in k, Λ: $\Re_+ \to \Re_+$,

$$k_t = \Lambda(k_{t-1}) \equiv G\,(k_{t-1})^{1-\frac{1}{\sigma}} \tag{9}$$
$$if \quad k_{t-1} \leqslant k_{cr}$$

$$k_t = \Lambda(k_{t-1}) \equiv G\left[\frac{\left((1+\frac{aT_{t-1}}{F})^{-\frac{1}{\sigma}}\right)k_{t-1}}{1+\theta\left(\frac{k_{t-1}}{1+\frac{aT_{t-1}}{F}}-1\right)}\right] + \frac{s}{\theta\sigma F}T_{t-1}\left(1-\frac{1}{1+\theta\left(\frac{k_{t-1}}{1+\frac{aT_{t-1}}{F}}-1\right)}\right) \tag{10}$$
$$if \quad k_{t-1} \geq k_{cr}$$

where $G \equiv \frac{sA}{\sigma}$ represents the growth potential of the economy at the laissez faire equilibrium.

We proceed by first studying the equilibrium properties of the steady state under laissez faire.[6] Table 1 below summarises the stability properties of the steady state under laissez faire.

	Solow	Romer	
SS value	$k^* = G^\sigma < 1$	$k^{**} = \frac{G-1}{\theta} + 1 > 1$	
Stability Properties of SS	*Monotonic convergence to SS*	$1 < G < \theta - 1$	*two − period cycles*
		$G > \theta - 1$	*Non monotonic convergence to SS*

Table 1 - Steady State (SS) properties under laissez faire

[6]Setting $T_{t-1} = 0$ the dynamical system (9)-(10) is equivalent to that of Matsuyama (1996).

The occurrence of two-period cycles depends on technology parameter values. Note, in particular, the role played by the parameter $\theta \equiv \left(1 - \frac{1}{\sigma}\right)^{1-\sigma}$ implying $\theta \in (1, e)$. This parameter measures the extent to which the innovator loses market power if he or she waits until the goods that he or she is competing with become competitively priced. If intermediate goods are poor substitutes, the rate of obsolescence is high. This triggers innovation. As intermediate goods become more substitutable the economy may fluctuate between periods of positive innovation and zero innovation. As substitutability of intermediate inputs increases the system switches to a regime with no innovation at all. Low values of θ and sufficiently high values of G imply an oscillatory convergence to a balanced growth path with innovation, whereas sufficiently small values of G imply a monotonic convergence to a stationary path with factor accumulation and no innovation. For $\theta > 2$ (implying $\sigma > 2$) there exists a range of values of G such that the equilibrium growth path of the economy fluctuates between a phase of capital accumulation and no innovation and a phase of no factor accumulation and innovation.[7]

We are now well equipped to address the main issue of our investigation, that is, to evaluate the growth and welfare effects of policies aimed at eliminating fluctuations.

4 Stabilisation

First, note that in the 'Solow regime' where the economy monotonically converges to the steady state there is clearly no scope for stabilisation. We focus therefore on equilibrium dynamics situated in the 'Romer regime'.

Fluctuations are generally not seen as beneficial for the economy. Ideally, nations would like to avoid a growth pattern characterised by frequent upswings and downturns in output growth. Moreover, they would aim at reaching high permanent growth. In other words, they would aim at reaching a balanced growth path where there is enough innovation and, at the same time avoid

[7]The empirical plausibility of the conditions for cycles is discussed in Matsuyama (1996), and we refer the reader to Section 5 of his paper.

cycles. In our model, this implies ensuring: (i) that the economy will be situated in the Romer regime where innovation is the engine of growth, rather than in the Solow regime with no innovation and factor accumulation; (ii) that in the event of fluctuations, policy should aim at bringing the system on a balanced growth path where the economy grows ($G > 1$), rather than on a balanced growth path where the economy stays stationary ($G < 1$).

To achieve a different dynamic allocation with respect to the case of laissez faire, the government has, in principle, a variety of policy options. We choose to focus on the non-distortive option of re-allocating resources between agents by implementing an appropriate system of subsidies to the innovators financed by lump-sum taxes on the consumer or vice versa. In this way, we can focus on purely stabilising/destabilising effects of policy and not on policy that affect the steady state as well. In particular, we assume that policymakers follow the following simple stabilisation principle,

$$T_{t-1} = \gamma(k^{**} - k_{t-1}), \qquad if \quad k_{t-1} < k^{**} \tag{11}$$

where the parameter γ represents the size of government intervention. This principle implies that governments intervene only in the case of recessions, and it is meant to represent a standard (countercyclical) macroeconomic policy. In particular if, at the outset of period $t-1$ (i.e. beginning of period t), the government observes a deviation of k from its long run trend, it may decide either to redistribute income to the consumer by means of a lump sum tax on the innovators ($\gamma > 0$) or, to subsidise the innovators by taxing the consumer ($\gamma < 0$).

Suppose that $k_{t-1} < k^{**}$; the government then decides that in order to increase output growth in the final good sector, innovators are to be subsidised by levying a lump sum tax on the young consumers. The rationale for such a move is to create an incentive for the innovators to produce new intermediate products which foster higher production of the final good. This implies in turn higher consumption of the representative consumer once old. Similarly, the case of a tax levied on innovators and redistributed to the young consumer reduces the potential growth of the final good. This implies in turn lower consumption by the representative consumer once old. To verify the validity of this conjecture in this particular economy we need to study the effects of implementing the rule above on the dynamic properties of the equilibrium.

When $k_{t-1} \leqslant k_{cr}$ (i.e. 'Solow regime') the dynamics remain the same as described by (9). When $k_{cr} \leqslant k_{t-1} \leqslant k^{**}$ (i.e. 'Romer regime') by using the proposed rule, i.e. (11), into (10) the dynamics becomes,

$$\begin{aligned} k_t \quad = \quad \Lambda(k_{t-1}) \equiv & G\left(\frac{\left((1+\frac{a\gamma}{F}(k^{**}-k_{t-1}))^{-\frac{1}{\sigma}}\right)k_{t-1}}{1+\theta\left(\frac{k_{t-1}}{1+\frac{a\gamma}{F}(k^{**}-k_{t-1})}-1\right)}\right) \\ & +\frac{s}{\theta\sigma F}\gamma(k^{**}-k_{t-1})\left(1-\frac{1}{1+\theta\left(\frac{k_{t-1}}{1+\frac{a\gamma}{F}(k^{**}-k_{t-1})}-1\right)}\right). \end{aligned} \tag{12}$$

When $k_{t-1} \geq k^{**}$ the dynamics remains the same as in (10).

By construction, the steady state solution of (12) gives the same steady state value of k as in the laissez faire equilibrium, i.e. k^{**}. The dynamic of adjustment, on the other hand, differs. Note that the expression (12) cannot be differentiated at k^{**} (indeed Λ has a kink at k^{**}), implying that $\Lambda'(k^{**})$ does not exist; however, we can always compute the value of $\Lambda'(k)$ when k tends to k^{**} from the left, i.e.,

$$\Lambda'_{-}(k^{**}) = \frac{a\gamma}{F}\left(\frac{k^{**}}{\sigma}-\left(1+\frac{\theta-1}{G}\right)k^{**}-\frac{G-1}{A\theta F}\right)+\frac{1-\theta}{G}. \tag{13}$$

The value of $\Lambda'(k)$ when k tends to k^{**} from the right is equal to $\Lambda'_{LF}(k^{**})$, i.e. $\Lambda'_{+}(k^{**}) = \Lambda'_{LF}(k^{**}) = \frac{1-\theta}{G}$. Therefore, for any infinitely small neighbourhood of the steady state the slope of the dynamics changes. Indeed, as discussed in Section 2, the introduction of a lump sum tax/subsidy changes the critical point at which the economy moves from growth driven by factor accumulation to growth driven by innovation. Taxing innovators and subsidising the consumer lowers the growth potential of the final output. Figure 1 illustrates by giving an example of the changes in the transitional adjustment path. It also shows that this policy, which is aimed at stabilising the economy, may on the contrary exert a destabilising effect. If (as in this example) the steady state is globally stable under laissez faire, the introduction of a tax makes cycles possible. Moreover, as the economy moves from being on the balanced growth path to a stable two-period cycle, growth is reduced, implying lower welfare along the transition. The following propositions summarise the results related to the implementation of the tax on innovators.

Proposition 1 *(Stability properties of the steady state under tax on innovators).*
If $G > \theta - 1$ *there is a* $\gamma^* > 0$, *such that for any* $\gamma > \gamma^*$ *and for any* $k_{cr} < k_{t-1} < k^{**}$, $\Lambda'_{-}(k^{**}) < -1$. *The steady state is unstable and there are equilibrium cycles of period two.*

Proof*:* See appendix.

Increasing γ as a means to stabilise downward fluctuations in output leads to equilibrium cycles. Hence it is highly destabilising. In particular, as γ crosses γ^*, a flip bifurcation occurs: the steady state looses stability and a stable cycle of period two appears.[8]

Note also that such policy has negative welfare effects as it depresses output growth along the transitional adjustment path.

Proposition 2 *(Welfare properties of the adjustment path under tax on innovators).*
The average growth rate of the economy over the two-period cycles under a tax on innovators is lower than the average growth rate of the economy under laissez faire in the 'Romer regime'.

[8]To check for the stability properties of the two-period cycle we have simulated the model. These simulations suggest that the cycle remains stable for a wide range of parameter values.

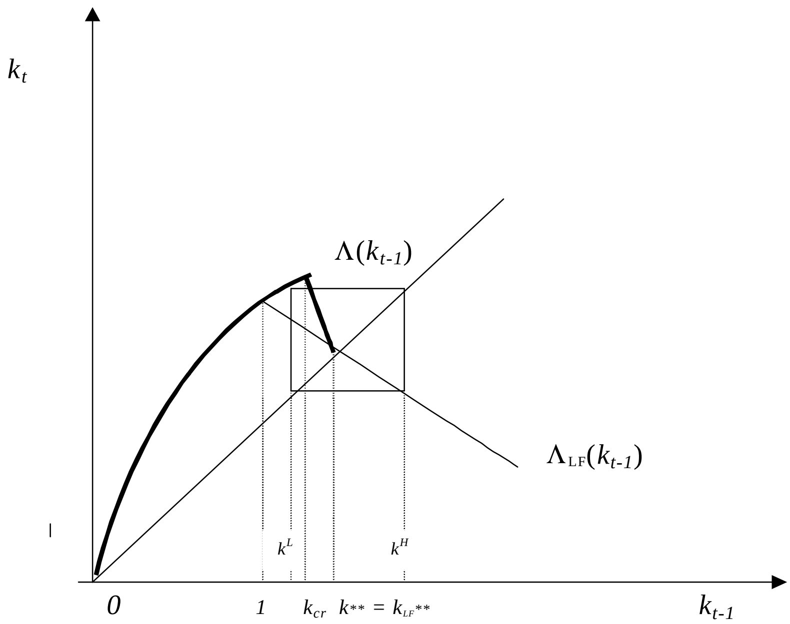

Figure 1: The graph of $\Lambda(k_{t-1})$ for $\gamma > 0$ and $G > \theta - 1$

Proof: See appendix.

Propositions 1 and 2 establish that implementing a tax on innovators whose receipts are redistributed to the consumer is detrimental for the economy. The basic intuition is that reallocating resources from the innovators to the consumer affects the balance between the two engines of growth, i.e. factor accumulation and innovation. As the subsidy to the consumer financed through the lump sum tax on the innovators increases, the economy moves from a situation in which innovation is highly profitable to one in which innovation is less profitable. This implies that, as the economy moves closer to a regime with no innovation and stationary growth it can be trapped in a phase where it cycles between high innovation (and low factor accumulation) and low innovation (and high factor accumulation).

However, if the government chooses the alternative option of taxing the young consumer and redistributing the receipts to the innovators as a lump sum subsidy the results are reversed. Figure 2 gives an example of the changes of the transitional adjustment path. It shows that this policy has a stabilising effect on the economy. If (as in this example) the steady state is unstable under laissez faire and a two-period cycle emerges, the introduction of a subsidy stabilises the economy and higher welfare along the adjustment path is also achieved.[9]

The next propositions summarise the results related to the implementation of the subsidy on innovators.

Proposition 3 *(Stability properties of the steady state under subsidy on innovators).*
If $G < \theta - 1$ there is a $\gamma^* < 0$, such that for any $\gamma^* < \gamma < -\frac{F}{a}$ and for any $k_{cr} < k_{t-1} < k^{**}$, $-1 < \Lambda'_{-}(k^{**}) < 0$. The economy monotonically converges towards the steady state in the 'Romer regime' .

Proof: See appendix.

As regards welfare, such policy has positive effects in that it promotes output growth along the transitional adjustment path.

Proposition 4 *(Welfare properties of the adjustment path under subsidy on innovators).*
The average growth rate of the economy under subsidy on innovators is higher than the average growth rate of the economy under laissez faire.

Proof: See appendix.

The basic intuition is as follows. When the economy is in the 'Romer regime' and exhibits equilibrium two-period cycles under laissez faire, a period of recession (i.e. $k_{t-1} < k^{**}$) corresponds to an equilibrium situated in the Solow regime and to an increase in saving (i.e. $\Lambda(k_L) > \Lambda(k^*)$). This is due to the a-synchronisation between innovation and investment activities that characterises this model (see Introduction). In fact, if saving increases while the equilibrium

[9] To ensure $0 < k_{cr} < 1$ we impose $\gamma < -\frac{F}{a}$.

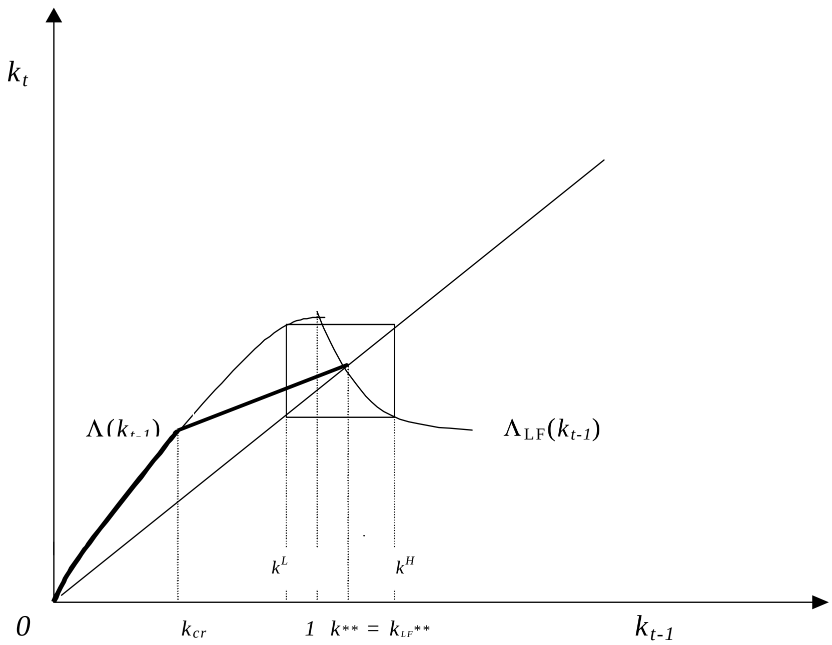

Figure 2: The graph of $\Lambda(k_{t-1})$ for $\gamma < 0$, $G < \theta - 1$ and $\Lambda(k_{cr}) < \Lambda(k^{**})$.

is in the low growth regime (i.e. the 'Solow' regime) more resources are directed to the production of 'old' intermediate goods to the detriment of the innovative sector which becomes less profitable. Hence, the mechanism behind the appearance of equilibrium cycles is amplified rather than reduced. As a consequence, to ensure stable sustained growth saving must be directed towards innovation during a recession. Taxing the young and distributing the proceeds to the innovators not only may bring stability but it also increases the amount of goods available for both young and old. Therefore total welfare in the economy is higher. In our model reallocating resources from the consumer to innovators affects the properties of the balanced growth path of the final output; more specifically, the economy switches from an equilibrium in which it cycles between high innovation (and low factor accumulation) and low innovation (and high factor accumulation) to the regime with innovation and sustained growth.

5 Conclusion

In this paper we have presented an OLG economy exhibiting sustained growth through the implementation of subsidies to innovative sectors (financed by a lump sum tax on young consumers). The OLG structure makes it possible to: (i) study the dynamic properties of the economy by use of a one-dimensional map, which makes the analysis simple and straightforward, (ii) explain how the reallocation of resources between sectors and consumers affects the intergenerational exchanges.

Taxing the young and redistributing the proceeds to the innovative sectors brings stability and increases the amount of goods available for both generations; hence it also increases total welfare of the economy. This implies that fast-growing economies, in which high factor accumulation plays a crucial role alongside innovative sectors that enjoy temporary monopoly rents, should follow an unorthodox stabilisation policy when they are facing recessions. They should tax the consumers and redistribute the receipts to the innovative sectors in form of subsidies.

APPENDIX

Proof of Proposition 1

By direct inspection of (13): (i) $\Lambda'_{-}(k^{**}) < \Lambda'_{LF}(k^{**})$, and (ii) $\Lambda'_{-}(k^{**})$ is decreasing in γ.■

Proof of Proposition 2

Assume: $k_{t-2} = k_H$, $k_{t-1} = k_L$, $k_t = k_H$, $k_{t+1} = k_L$, $k_{t+2} = k_H$ and so on, where k_L is situated in the 'Romer regime' and k_H is situated in the 'Solow regime' (see, e.g., Figure 1 in Section 4). This assumption implies that under our policy rule, see (11), $T_{t-2} = 0$, $T_{t-1} = \gamma(k^{**} - k_L)$, $T_t = 0$, $T_{t+1} = \gamma(k^{**} - k_L)$ and so on. In view of this fact and given the dynamics as in (9)-(10), the rates of growth of all relevant variables when the economy fluctuates every other period between the two regimes are,

$$\begin{aligned}
g_{N_{Solow}} &\equiv \frac{N_t}{N_{t-1}} = 1 \\
g_{N_{Romer}} &\equiv \frac{N_{t+1}}{N_t} = 1 + \theta\,(k_H - 1) \;\; if \;\; k_H > k^* \\
&= 1 + \theta\left(\frac{k_H}{1 + \frac{a\gamma}{F}(k^* - k_L)} - 1\right) \;\; if \;\; k_H < k^*,
\end{aligned}$$

$$\begin{aligned}
g_{K_{Solow}} &\equiv \frac{K_t}{K_{t-1}} = \frac{k_t}{k_{t-1}}.\frac{N_t}{N_{t-1}} = \frac{k_H}{k_L} = \frac{G(k_L)^{1-\frac{1}{\sigma}}}{k_L} = G(k_L)^{-\frac{1}{\sigma}} \\
g_{K_{Romer}} &\equiv \frac{K_{t+1}}{K_t} = \frac{k_{t+1}}{k_t}.\frac{N_{t+1}}{N_t} = \frac{k_L}{k_H}\frac{N_{t+1}}{N_t} \\
&= \frac{k_L}{k_H}\left(1 + \theta\left(\frac{k_H}{1 + \frac{a\gamma}{F}(k^* - k_H)} - 1\right)\right) \;\; if \;\; k_H < k^* \\
&= \frac{k_L}{k_H} G \;\; if \;\; k_H > k^*,
\end{aligned}$$

$$
\begin{aligned}
g_{Y_{Solow}} &\equiv \frac{Y_t}{Y_{t-1}} = \frac{\mathcal{A}_{t-1}\left(1+\frac{aT_{t-1}}{F}\right)^{\frac{1}{\sigma}}(k_{t-1})^{-\frac{1}{\sigma}}K_{t-1}}{\mathcal{A}_{t-2}K_{t-2}} \\
&= (k_{t-1})^{-\frac{1}{\sigma}}\frac{k_{t-1}}{k_{t-2}}.\frac{N_{t-1}}{N_{t-2}} = G\left(k_L\right)^{-\frac{1}{\sigma}} \\
g_{Y_{Romer}} &\equiv \frac{Y_{t+1}}{Y_t} = \frac{\mathcal{A}_t K_t}{\mathcal{A}_{t-1}\left(1+\frac{aT_{t-1}}{F}\right)^{\frac{1}{\sigma}}(k_{t-1})^{-\frac{1}{\sigma}}K_{t-1}} \\
&= (k_{t-1})^{\frac{1}{\sigma}}\frac{k_t}{k_{t-1}}.\frac{N_t}{N_{t-1}} = G \quad if \;\; k_H \geq k^* \\
&= \left(1+\frac{a\gamma}{F}(k^*-k_H)\right)^{-\frac{1}{\sigma}}.G \;\; if \;\; k_H < k^*
\end{aligned}
$$

The average growth rate over the two-period cycle is given by,

$$
g_{Y_{Tax}} \equiv \left(g_{Y_{Solow}}.g_{Y_{Romer}}\right)^{\frac{1}{2}} = G(k^L)^{-\frac{1}{2\sigma}}
$$

Under laissez faire (and $k > k_{cr}$) the rate of growth of output is $g_{Y_{Romer}} = G$. To demonstrate that the latter is always higher than the average growth rate over the cycle it suffices to show that $(k^L)^{-\frac{1}{2\sigma}} < 1$. This condition is always verified since, under the tax on innovators the critical point moves from a value of 1 to a value strictly higher than one, implying $k^L > 1$ and therefore $(k^L)^{-\frac{1}{2\sigma}} < 1$. ■

Proof of Proposition 3

Let us note that $k_{cr} < 1 < k^{**}$.

If Λ is increasing in k in the Romer regime (see Figure 2), then $\Lambda(k_{cr}) < \Lambda(1) < \Lambda(k^{**})$. In addition, for any $k \in (k_{cr}, k^{**})$, $\Lambda(k) > k$ then $k_{t+1} = \Lambda(k_t) > k_t$ and the economy monotonically converges to the steady state. ■

Proof of Proposition 4

From the section devoted to the proof of Proposition 2, we know the expressions for g_Y under laissez faire and under subsidy/tax. Hence, $g_{Y_{LF}} - g_{Y_{Sub}} =$

$(g_{Y_{Solow}}.g_{Y_{Romer}})^{\frac{1}{2}} - g_{Y_{Sub}} = G\left[(k_L)^{-\frac{1}{2\sigma}} - \left(1 + \frac{a\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}}\right]$. If $g_{Y_{LF}} - g_{Y_{Sub}} < 0$ then $\Longleftrightarrow (k_L)^{-\frac{1}{2\sigma}} < \left(1 + \frac{a\gamma}{F}(k^{**} - k_L)\right)^{-\frac{1}{\sigma}} \Longleftrightarrow 1 + \frac{a\gamma}{F}(k^{**} - k_L) < (k_L)^{\frac{1}{2}} \Longleftrightarrow \frac{1+\frac{a\gamma}{F}k^{**}}{1+\frac{a\gamma}{F}} < \frac{k_L\left(k_L^{-\frac{1}{2}} + \frac{a\gamma}{F}\right)}{1+\frac{a\gamma}{F}}$. Recall $k_{cr} = \frac{1+\frac{a\gamma}{F}(k^{**}-k_L)}{1+\frac{a\gamma}{F}} < k_L$. Now, let us show that $k_L^{-\frac{1}{2}} + \frac{a\gamma}{F} > 1 + \frac{a\gamma}{F} \Longleftrightarrow k_L^{-\frac{1}{\sigma}} > 1$ which is always true since $k_L < 1$. Therefore $g_{Y_{LF}} < g_{Y_{Sub}}$. ■

References

Aghion, P. and P. Howitt (1992), "A Model of Growth Through Creative Destruction", *Econometrica* 60, 323-351.

Aghion, P. and P. Howitt (1997), *Endogenous Growth Theory*, MIT Press.

Crafts, N. (1999), "Implications of financial crisis for East Asian trend growth", *Oxford Review of Economic Policy* 15, 110-131.

Davidson, C. and P. Segerstrom (1998), "R&D subsidies and economic growth", *Rand Journal of Economics* 29, 548-577.

Dornbusch, R. (1995), "Why is Japan Shooting Itself in the Foot?", *Business Week*, 8 May.

Ito, T. (1995), "Japan and the Asian Economics: A 'Miracle' in Transition", *Brookings Papers on Economic Activity* 2, 205-272.

Mathews, J. A. and D-S Cho (2000), *Tiger Technology: The Creation of a Semiconductor Industry in East Asia*, Cambridge University Press.

Matsuyama, K. (1996), "Growing through cycles", *CMS-EMS Working Paper* N. 1203, Northwestern University.

Matsuyama, K. (1999), "Growing through cycles", *Econometrica* 67, 335-347.

Odagiri, H. and A. Goto (1993), "The Japanese System of Innovation: Past, Present and Future", in R. Nelson (ed.) *National Innovation Systems*, Oxford University Press.

Romer, P. M. (1986), "Increasing returns and long run growth" *Journal of Political Economy* 94, 1002-37.

Romer, P. M. (1990), "Endogenous technological change" *Journal of Political Economy* 98, S71-S102.

Segerstrom, P. (2000), "The Long-Run Effects of R&D Subsidies", *Journal of Economic Growth* 5, 277-305.

Young, A. (1995), "The Tiranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience", *Quarterly Journal of Economics* 110, 641-680.