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by Marta Aloi and Huw D. Dixon

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1 Introduction

This paper is a theoretical exploration of the dynamic relationship between entry, capacity utilization and productivity in a monopolistic economy in the presence of demand and technology changes. By *capacity utilization* we mean the extent to which the output of the firm compares to some standard reference level[1] (the free-entry equilibrium or the technically efficient scale of production)[2]. We employ an entry model found in Das and Das (1996) and developed by Dixon (2000) and Datta and Dixon (2001) in which the cost of entry depends on the flow of entry due to some congestion effect or externality. There is an intertemporal zero-profit condition which equates the cost of entry in each instant to the net present value of incumbency, resulting in the number of firms gradually adjusting towards its long-run value. Output per firm and capacity utilization vary in the short term with output, whilst in the long-run there is a free-entry steady state condition which ties down output per firm and capacity utilization.

We consider an economy of monopolistic firms and find that there is a direct relationship between the level of capacity utilization and productivity which is stronger when monopoly power is greater. There are two elements to the equilibrium: first there is a markup of price over marginal cost resulting from the market power of the firms, $p > MC$; second in the long-run with free entry there will be zero profits so price equals average cost $p = AC$. Putting these two elements together implies that $p = AC > MC$: in long-run equilibrium average cost will be decreasing since it exceeds marginal cost. This is of course the standard Robinson-Chamberlin excess capacity result for free-entry monopolistic equilibrium: if firms have a $U-$shaped average cost curve, then equilibrium output will be on the downward sloping portion of the AC and below the efficient operating level defined by the minimum AC output. Hence there is a direct relationship between productivity and capacity utilization: if capacity utilization varies from the initial free-entry equilibrium due to a demand or technology shock, there will be a first order effect of capacity utilization on average costs; since average cost is simply the dual of factor productivity, a fall in costs represents an increase in productivity. This effect is absent in a Walrasian setting with free entry: since firms are at optimum scale, the AC curve is locally flat ($P = MC = AC$), with no relationship between capacity utilization and productivity.

Whilst other papers have emphasized the relationship between imperfect competition and increasing returns to scale (Hall 1986,1990, Rotemberg and Woodford 1995, Basu 1996, Devereux et al 1996, Ambler and Cardia 1998,

[1] Nelson (1989) provides a useful discussion and history of the concept of capacity utilization in this context and its relation to survey data. Since there is no capital in our model, the standard definitions of Cassel (1937) and Klein (1960) coincide, with the reference level of output being minimum average cost.

[2] The term "capacity utilization" is also sometimes used in a different meaning, to distinguish between the flow of capital services and the stock of capital (see Greenwood et al 1988, King and Rebelo 1999). In fact Burnside and Eichenbaum (1996) use the more accurate *capital utilization* for this, as is standard in the wider literature (Winston 1974).

Coto and Dixon 1999, Cook 2001), the contribution of this paper is to set this in the context of an explicit dynamic entry model. Existing papers have tended to assume either that there is instantaneous free entry (the actual or expected profit is driven to zero in each period) or that the number of firms is constant (possibly imposing a long-run zero profit condition as in Hornstein 1993). The dynamic entry model we employ has both instantaneous free-entry and no-entry as limiting cases. Of course, many papers with perfect competition assume that there are global constant returns to scale, so that entry is irrelevant. This is also true in our model in the case of perfect competition, even when the underlying firm technology has varying returns to scale, since in equilibrium there will be locally constant returns when $MC = AC$.

In this paper, we explore the role of entry dynamics in a simple small open economy with a Ramsey consumer. In order to focus on the dynamics induced by entry and exit, we eliminate all other sources of dynamics. There is no capital, only labor; there is an internationally traded bond with world interest rates equalling the household discount rate so that the household is able to perfectly smooth utility. We consider changes in demand and technology, changes which can be either permanent or transitory, anticipated or unanticipated. The entry dynamics induces an endogenous productivity dynamic through variations in output per firm, capacity utilization. The technology assumed at the firm level allows for the case of $U-$shaped AC curve with increasing MC, and for the cases of globally decreasing AC with either constant or decreasing MC.

In the case of a pure demand shock (modelled as a balanced budget fiscal policy), there is no actual technology change, so that all variations in measured labor productivity (such as the Solow residual) are caused by changes in capacity utilization and are transitory. However, even short-run changes can have long-run effects, since the stock of foreign bonds may be permanently affected. We find that the relationship between the stock of bonds and entry is influenced both by the nature of technology (whether the marginal product of labor is increasing or decreasing) and the degree of imperfect competition. Whilst in the Walrasian case with diminishing returns the accumulation of firms is always associated with a decline in bonds, this can be reversed if there is a high markup, and is always reversed if there is a non-diminishing marginal product.

In the case of a real technology shift, the transitory dynamic induced by capacity utilization is superimposed on the underlying technology change. This means that the measured productivity changes may not be a good guide to the real technology shock except in the long-run: the time profile of measured productivity will differ from that of the underlying technology. For example, in the case of an *anticipated permanent* technological improvement, there will be entry prior to the actual occurrence of the technological change: if there is imperfect competition this entry will increase the degree of capacity underutilization and reduce measured productivity. With a *permanent unanticipated* technological change, the impact effect can either be smaller or larger than the long-run effect. The crucial factor here is the response of the labor supply to the shock, which can take either sign due to the conflict of income and substitution effects. Thus we can either have the case where the capacity utilization effect reinforces the technological shock in the short run, leading to an overshooting of measured productivity over true productivity, or the opposite. Hence, even a simple step change in productivity causes a more complicated time profile in its aftermath (when unanticipated) and possibly before and after (when anticipated).

Indeed, the basic insight of the paper is that the output of the economy or industry depends not only on the total level of input (labor in this paper, but also capital[3] and intermediates in general), but also the way that this is divided between firms. Here consider the simplest of a symmetric industry where all firms are identical[4]: the number of firms should be viewed as an additional quasi-input, representing the organisation of the industry/economy. In the short-run, the number of firms can adjust only slowly, so that output changes only through changes in labor. At the firm level, this implies that output per firm varies. In the long-run, the number of firms can adjust alongside labor, so that output can change even if output per firm is constant. The key point is that we would expect a completely different relationship between output and employment in these two cases. This reinforces the applied work of Caves D *et al* (1981) and Bendt and Fuss (1986), which emphasized the need to focus on firm level data to understand productivity, not just industry or economy wide data.

Although the model we present is primarily theoretical and not intended to model data, it does have the following features, each of which is supported by empirical evidence:

- **Entry is procyclical.** Frank Portier (Portier 1995) used aggregate data from the French economy[5] and found that the number of firms was correlated with GDP: the correlation of current entry with current GDP using annual data was 53%, with lagged GDP 31% and lead GDP 42%. Ambler and Cardia (1998) used quarterly US data on net business formation (Citibase data series) from 1954-1991. They found that the correlation between output and net firm creation was 58%[6]. Campbell (1998) used US plant data[7] for both entry and exit over the period 1972-1988. He found that the flow of entry is procyclical and exit is countercyclical (*op. cit.* 376).

- **Productivity is procyclical**. Rotemberg and Summers (1990), Basu (1996). Ryan (2000) found not only is productivity procyclical, *but also that the relationship is stronger when the markup is larger.* His study used the NBER Manufacturing productivity database with annual data for 450 four-digit SIC level from 1951-1991.

[3] Brito and Dixon (2000) consider a closed economy model of entry with capital and labor.

[4] In a more general and realistic model, the distribution of firm sizes would also be crucial (see for example Hopenhayn 1992, Ericson and Pakes 1995 and Basu and Fernald 1997)

[5] Portier usedthe *Sirene* database for the period 1977 to 1989, using the number of firms less the self-employed. The correlations are log-deviations from a linear trend.

[6] Ambler and Cardia (1998) used HP filtered data.

[7] The U.S. Bureau of the Census compiles an Annual Survey of Manufacturers to construct a panel.

- **Productivity is correlated with capacity utilization.** Hulten (1986), Zegeye and Rosenblaum (2000), Ryan (2000), Bendt and Fuss (1986), Morrison (1986, 1992, 1993, 1997) find that there is a substantial positive relationship between capacity utilization and productivity.

- **Labor's share is countercyclical.** Ambler and Cardia (1998): the correlation is -71%.

The advantage of our model is that not only does it posses all these features, but also it is able to explain how they are all linked. For example, in the case of a demand shock, increasing returns at the firm level will cause capacity utilisation and productivity to increase in the short-run; this will increase profits and reduce labour share; the profits will encourage entry. Of course, whilst our model leaves out many things that are of importance (we have no nominal rigidities in wages or prices and a constant markup, no capital and no uncertainty), we have provided a framework and justification for the analysis of entry which can be developed into richer models.

The outline of the paper is as follows. First, in sections 2 and 3 we outline the optimization problem of the household and the firm, introducing the entry model in section 3.2. In section 4 we put these together into a dynamic general equilibrium model, exploring both the steady state and the linearized dynamics. In section 5 we use the model to analyze fiscal demand shocks, in section 6 we analyze technology shocks.

2 The household

There is a small open economy, with a world capital market interest rate r equal to the discount rate of the Ramsey household. Utility satisfies standard assumptions and depends on aggregate consumption C and leisure $\ell = 1 - L$, where L is the labour supply.

Assumption 1 $U(C, 1-L)$, is twice continuously differentiable and strictly concave with $U_C > 0 > U_{CC}$, $U_\ell > 0 > U_{\ell\ell}$ and $U_{C\ell} = 0$.

Leisure and consumption are normal goods, with U additively separable in C and l, ($U_{C\ell} = 0$). The household earns income from three sources: supplying labour at wage w, receiving interest income from net foreign bonds rb and receiving profit income Π. As is standard, the household treats profit income as a lump sum payment.

$$\max \quad \int_0^\infty U(C, 1-L) \exp[-rt] dt$$

subject to

$$\dot{b} = rb + wl + \Pi - C - G \tag{1}$$

Where we assume that the government finances its expenditure G by a lump-sum tax equal to expenditure in each instant[8].

We assume that the utility function takes the following form

$$U = \log C - \frac{1}{\beta} L^{\beta}, \quad \beta > 1 \tag{2}$$

The crucial feature of the utility function is that leisure and consumption are additively separable ($U_{C\ell} = 0$): in fact most of the results in the paper in the case of diminishing marginal productivity go through with this more general case.

The solution[9] to this is defined by the equations

$$\begin{aligned} U_C - \lambda &= 0 & \text{(3a)} \\ -U_\ell + \lambda w &= 0 & \text{(3b)} \\ \dot{\lambda} &= 0 & \text{(3c)} \end{aligned}$$

along with the Transversality condition (TVC)

$$\underset{t\to\infty}{Lim} \lambda b \exp[-rt] = 0 \tag{4}$$

The solution to the households problem is simple. Since $U_{C\ell} = 0$, we can write optimal consumption and labor supply in terms of Frisch demands:

$$\begin{aligned} C &= C(\lambda) = \frac{1}{\lambda} \\ L &= L^F(w, \lambda) = (\lambda w)^{\frac{1}{\beta - 1}} \end{aligned}$$

The presence of international capital markets means that the household can completely smooth its consumption ($\dot{\lambda} = 0$). λ is an index of the level of utility derived from consumption: a high λ means a low level of consumption and *vice versa*. For a given wage, L^F is increasing in λ.

The aggregate consumption good C is either imported or produced domestically by a perfectly competitive industry with a $CRTS$ production function using intermediate inputs which are monopolistically supplied. There is a continuum of possible intermediate products, $i \in [0, \infty)$. At instant t, there is a range of active products defined by $n(t) < \infty$, so that $i \in [0, n(t))$ are *active* and available, whilst $i > n(t)$ are inactive and not produced. Total domestic output Φ is related to inputs y_i by the following technology

$$\Phi = n^{\frac{1}{1-\theta}} \left[\int_0^n y_i^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{5}$$

[8] This is for convenience and avoids the need for introducing government bonds. Since Ricardian equivalence holds, the timing of taxation does not matter.

[9] These are the first order conditions for the current value Hamiltonian $H = U(C, 1-L) + \lambda[wL + \Pi - C]$.

where $\theta > 1$ is the elasticity of substitution between products. Notice that there is no *Ethier effect* here: an increase in the range of intermediates does not affect the unit cost function. Treating the unit price of the consumption good as the numeraire, the demand for each available product i takes the constant elasticity form

$$y_i = p_i^{-\theta}\Phi$$

3 Firms: Technology, entry and exit

There is a continuum of potential firms: each firm can produce only one product. the index of firms and products is therefore the same. At time t, firm $i \in [0, n(t))$ uses labor L_i to produce output y_i using the technology[10]

$$y_i = A.L_i^{\gamma} - F \tag{6}$$

where $\gamma > 0$ and $F \geq 0$ is a fixed flow overhead and $A > 0$ is a technology parameter which may be set at $A = 1$ unless otherwise stated. In this paper overhead represent foregone output: an alternative is to have the overhead in terms of labor as in Dixon and Lawler (1996). When $\gamma < 1$, $F > 0$ there is an *U-shaped* average cost (AC) with increasing marginal cost (MC); this is compatible with both perfect (Walrasian) and imperfect competition. When $\gamma = 1, F = 0$, we have constant returns to scale: $AC = MC$. When $\gamma = 1, F > 0$, we have constant MC and decreasing AC; when $\gamma > 1$ there is decreasing AC and MC (the extent to which γ can exceed 1 is limited - the upper bounds are stated below). In these last two cases with globally increasing returns to scale, equilibrium can only exist with imperfect competition.

The number of active firms at instant t is denoted $n(t)$: we will drop the time index when it does not lead to ambiguity. Throughout we will be assuming that labor markets function perfectly so that labor is allocated equally across firms, so that $L_i = L/n$. The *aggregate production function*, obtained from (6,5) under symmetry is Homogenous of degree 1 in n and L :

$$\Phi(L, n, A) = AL^{\gamma}n^{(1-\gamma)} - nF \tag{7}$$

Note that the aggregate marginal product of labor equals the firm-level marginal product (this is because labor is allocated equally across firms)

$$\frac{\partial y_i}{\partial L_i} = \frac{\partial \Phi}{\partial L} = \Phi_L = \gamma A \left(\frac{L}{n}\right)^{\gamma-1}$$

Clearly, Φ_{LL} is negative when $\gamma < 1$, zero when $\gamma = 1$, and positive when $\gamma > 1$.

The number of firms operating influences the level of output in the economy: it increases the level of overhead costs nF and decreases the level of employment

[10] This specification is similar to Hornstein (1993) and Ambler and Cardia (1998), the main difference being that there is no capital here.

per firm.

$$
\begin{aligned}
\Phi_n &= \left[(1-\gamma)A\left(\frac{L}{n}\right)^{\gamma} - F\right] \\
\Phi_{nn} &= -\frac{(1-\gamma)\gamma}{n}A\left(\frac{L}{n}\right)^{\gamma} \\
\Phi_{nL} &= \frac{\gamma(1-\gamma)}{n}A\left(\frac{L}{n}\right)^{\gamma-1}
\end{aligned}
$$

When $\gamma < 1$, an increase in the number of firms initially increases output, but eventually reduces it ($\Phi_{nn} < 0$). When $\gamma \geq 1$, more firms always reduce output. When $\gamma < 1$ additional firms *increase* the marginal product of labor (since it reduces employment per firm); with $\gamma = 1$ there is $CMPL$ so that $\Phi_{nL} = 0$; when $\gamma > 1$ more firms *reduce* the marginal product of labor.

In this economy there is a clear optimal firm size when $\gamma < 1$: the technically efficient level of employment and output *per firm* where $AC = MC$ are

$$
\begin{aligned}
\left(\frac{L}{n}\right)^{e} &= \left[\frac{F}{A\,(1-\gamma)}\right]^{1/\gamma} \\
y^{e} &= \frac{\gamma F}{1-\gamma}
\end{aligned}
$$

The efficient number of firms conditional upon employment is $n = L\left[A(1-\gamma)/F\right]^{1/\gamma}$. The case of the *U-shaped* AC is significantly different from the limiting case of $CRTS$ ($\gamma = 1, F = 0$) when $\Phi_{nn} = \Phi_{nL} = 0$ and the number of firms has no effect on aggregate output, there is no optimal firm size. In the cases of $\{\gamma = 1, F > 0\}$ and $\{\gamma > 1, F \geq 0\}$ there are globally increasing returns to scale $IRTS$, so that $\Phi_n < 0$ and the efficient firm size is unbounded.

In this paper we define $\mathcal{P}$ as *output per unit labor* ($\mathcal{P} = y/L$): since there is only one factor of production, $\mathcal{P}$ can be unambiguously interpreted as *productivity.* When firms are at their optimum size, the average and marginal product of labor are equated, so that for $\gamma < 1$

$$
\mathcal{P}^{e} = w^{e} = A^{\frac{1}{\gamma}}\gamma\left(\frac{F}{1-\gamma}\right)^{\frac{\gamma-1}{\gamma}}, \quad \gamma < 1
$$

When $\gamma = 1$, efficient production implies $\mathcal{P}^{e} = A$ (this occurs at any output if $F = 0, \infty$ if $F > 0$). If $\gamma > 1$ the efficient production is undefined (productivity is unbounded as L tends to infinity).

3.1 Profits

In this section, we determine the operating profits of an active firm, i.e. a firm that does not incur any entry costs. Due to imperfect competition, the firm maximizes profits given real wage w (using output price as the numeraire) by choosing employment to satisfy

$$w = (1-\mu)\Phi_L \tag{8}$$

Where μ is the Lerner index of monopoly. Now, the aggregate flow of operating profits given w equals $n\pi$, where π is firm level profit. Since Φ is homogenous of degree 1 in $\{n, L\}$ we have

$$\pi = \Phi_n + \mu\Phi_L \frac{L}{n} \tag{9}$$

$$n\pi = \Phi - (1-\mu)\Phi_L L \tag{10}$$

The zero operating-profit[11] condition $\pi = 0$ implies

$$\Phi_n = -\mu\Phi_L \frac{L}{n} \tag{11}$$

Under the functional form assumed, zero-profits implies that employment and output per firm are given by

$$\left(\frac{L}{n}\right)^* = \left(\frac{F}{A(1-\gamma(1-\mu))}\right)^{\frac{1}{\gamma}} \tag{12}$$

$$y^* = \frac{\gamma(1-\mu)}{1-\gamma(1-\mu)}.F \tag{13}$$

For a zero profit equilibrium to exist, we require that $\gamma < \frac{1}{1-\mu}$. This restriction stems from the requirement that for a profit maximizing output MR must cut MC from above: a higher μ means a steeper MR which allows a steeper fall in MC. Hence a higher degree of monopoly allows for larger γ: $\gamma < 1$ is compatible with any $\mu \geq 0$. If $\gamma > 1$, then it implies that the market cannot be too competitive: $\mu > \frac{\gamma-1}{\gamma}$. In fact there is a second restriction $\gamma < \beta$: this means that the slope of the labor supply curve in (w, L) space is greater than the slope of the marginal product schedule. Thus we have a clear upper bound on γ given μ, β:

$$\gamma < \min\left[\frac{1}{1-\mu}, \beta\right] \tag{14}$$

Note that whilst an increase in productivity A reduces employment per firm under zero profits, output per firm is unaffected. In the zero profit equilibrium productivity is

$$\mathcal{P}^* = A^{\frac{1}{\gamma}}\gamma(1-\mu)\left(\frac{F}{1-\gamma(1-\mu)}\right)^{\frac{\gamma-1}{\gamma}} \tag{15}$$

[11] Operating profit is the profit earned by firms already in the industry. Firms which enter pay an entry cost. This is dealt with in the next section.

The number of firms conditional upon employment is,

$$n = LA^{\frac{1}{\gamma}} \left(\frac{1-\gamma(1-\mu)}{F} \right)^{\frac{1}{\gamma}} \tag{16}$$

Comparing the efficient and the free-entry outcomes with $\gamma < 1$, we can see that in the Walrasian case ($\mu = 0$), the free-entry and zero-profit outcomes are the same. In both cases, firms are operating where $AC = MC$, at the bottom of the $U-$shaped AC curve. When $\mu > 0$ however, $\left(\frac{L}{n}\right)^* < \left(\frac{L}{n}\right)^e$,$y^* < y^e$, $\mathcal{P}^* < \mathcal{P}^e$: this is the standard *excess capacity* result of Chamberlin and Robinson. With monopolistic competition, free entry leads to excess entry and firms operate on the decreasing part of the AC curve (there are locally increasing returns to scale).

Under symmetry, aggregate free entry output of the consumption good is given by $\Phi = ny^*$, which can be written as a function of L only using (16)

$$\Phi = \left[A^{\frac{1}{\gamma}}.\gamma(1-\mu) \left(\frac{1-\gamma(1-\mu)}{F} \right)^{\frac{1}{\gamma}-1} \right] L \tag{17}$$

Since Φ is homogenous of degree 1 in (n, L), the fixing of the ratio (L/n) in (12) means that Φ becomes proportional to L. Free entry imposes long-run $CRTS$ on the relation between L and Φ irrespective of the value of γ at the firm level.

3.2 The entry decision

What determines the number of firms operating at each instant t? In this paper we employ the model developed from Das and Das (1996) by Dixon (2000) and Datta and Dixon (2001). At time t, there is a flow cost of entry $q(t)$ for each entrant (entry and exit are symmetric for simplicity, with $-q$ being the cost of exit at time t). The cost of entry is assumed to be increasing in the flow of entry $E = \dot{n}$

$$q = \nu E \tag{18}$$

The total entry costs incurred by entrants are therefore νE^2. The relationship between the flow of entry and the cost of entry is based on the notion that there is a congestion effect: when more firms are being set up, the cost of setting up is higher. We do not model this: however, this might be because of a direct externality in the production of new firms, or due to the fixed supply of some factor involved in the creation of new firms. In Dixon (2000) it is also shown that this model can be derived form a locational setup where firms are situated along a real line representing location in some technological/product or geographical space. If the cost of entry depends on the distance of the new entrant from the nearest incumbent firm, then the same relation between entry flow and cost exists: a higher flow of entry means that firms more distant from

$n(t)$ are setting up. Whilst exit and entry are treated symmetrically in this paper, this is not essential. It is possible to model exit differently (e.g. there is a fixed cost of exit, perhaps zero, as in Das and Das 1996 or Hopenhayn 1992), the alternatives being analysed in Dixon (2000).

The flow of entry in each instant is determined by an *arbitrage condition.* Suppose a firm is inactive: it can either set up in instant t or delay. The firm can either invest in setting up or not. The opportunity cost of funds is given by the return on the bond, r. This must equal the return on investing a dollar in setting up a new firm, given by the LHS of (19)

$$\frac{\pi}{q} + \frac{\dot{q}}{q} = r \tag{19}$$

where π is given by (9). The first LHS term is the number of firms per dollar $(1/q)$ times the flow operating profits the firm will make if it sets up: the second term reflects the change in the cost of entry. If $\dot{q}/q > 0$, then it means that the cost of entry is increasing, so that there is a capital gain associated with entry at time t; if $\dot{q}/q < 0$ it means entry is becoming cheaper, thus discouraging immediate entry. The arbitrage[12] condition equates the return on bonds with setting up a new firm, and is a differential equation in q, which determines the entry flow by (18).

With entry, the total profits are the operating profits of firms less the entry costs paid by the entrants

$$\Pi = n\pi - \nu E^2 = n\Phi_n + \mu\Phi_L L - \nu E^2 \tag{20}$$

In equilibrium, $q(t)$ represents the net present value of incumbency[13]: it is the present value of profits earned if you are an incumbent at time t. This arises since the entrants are indifferent between entering and staying out. When $q < 0$, the present value of profits is negative: in equilibrium this is equal to the cost of exit. In steady state, we have $E = q = 0$, so that the entry model implies the zero-profit condition. Entry costs are thus a disequilibrium phenomenon.

Note that our entry model has the standard models as limiting cases: when $\nu = 0$, we have instantaneous free entry so that (19) becomes $\pi = 0$ and there are zero profits each instant; if we have $\nu \to +\infty$, then changes in n become very costly and n moves little if at all which approximates the case of a fixed number of firms.

[12] The arbitrage equation can be written in a way directly analagous to the user cost of capital:

$$\pi = q(r - \frac{\dot{q}}{q})$$

[13] See Datta and Dixon (2000) for a proof.

4 The dynamic system

We are now ready to draw together the different elements of the economy in order to represent the economy as an integrated dynamic system. Since consumption is constant, we have three dynamic equations

$$\left.\begin{array}{c} \dot{n} = E = \frac{q}{\nu} \\ \dot{q} = rq - \left[\Phi_n + \mu\Phi_L \frac{L}{n}\right] = Q(n, \lambda, q) \\ \dot{b} = rb + wL + \Pi - C(\lambda) - G = B(b, n, \lambda, q) \end{array}\right\} \tag{21}$$

where

$$\begin{aligned} w &= (1-\mu)\Phi_L \\ \Pi &= n\Phi_n + \mu\Phi_L L - \nu\frac{E^2}{2} \end{aligned}$$

In addition to (21), we also have the household TVC (4). From (21) we have a subsytem in (n, q) which determines the dynamics of the whole system, the bond equation being a residual (see Brock and Turnovsky 1994, Turnovsky 1997). The bond equation along with the TVC then determines the equilibrium value of λ.

In order to understand the dynamics system, we will take two steps. First, we will specify the steady state for a given level of bonds b^*. Secondly, we will linearize and from the dynamics derive an expression for steady state bonds. The steady state depends on the path taken to equilibrium, since the stock of bonds will vary in response to any trade deficit or surplus along the path to equilibrium. This is a standard feature of open economy models (see Turnovsky 1997) which differentiates them from closed economy Ramsey models where the steady-state is not path dependent. As we shall see, the entry process in moving towards equilibrium has a long-run effect through its impact on bonds.

4.1 Steady-state

In steady state we have $\dot{n} = E = \dot{q} = \dot{b} = 0$. Before proceeding further, it is useful to write the labor supply as a function of (n, λ): this is obtained by combining the Frisch supply $L^F(\lambda, w)$ with the marginal productivity relationship defining w

$$L(\lambda, n, A) = L^F(\lambda, (1-\mu)\Phi_L(L, n)) = (\lambda\gamma(1-\mu).A)^{\frac{1}{\beta-\gamma}}.(n)^{\frac{1-\gamma}{\beta-\gamma}} \tag{22}$$

Whilst $L_\lambda, L_A > 0$ are unambiguous, the effect of n on L depends on $\gamma : L_n > 0$ for $\gamma < 1$; $L_n = 0$ for $\gamma = 1$; $L_n < 0$ for $\gamma > 1$. Entry alters employment per firm: this affects the real wage. With a decreasing MPL, entry increases the real wage and hence labor supply; with increasing MPL the opposite holds.

Let us first consider the steady state conditional on b^*. In this case we have three equations in three unknowns $\{n, \lambda, L\}$

$$\Phi_n(L^*, n^*) = -\mu\Phi_L(L(\lambda^*, n^*), n^*)\left(\frac{L^*}{n}\right) \quad (23a)$$

$$(1-\mu)\Phi_L(L^*, n^*) = \frac{U_\ell(1-L^*)}{U_C(C(\lambda^*))} \quad (23b)$$

$$C(\lambda^*) = w^*L^* + rb^* - G \quad (23c)$$

Equation (23a) implies that there are zero-profits in steady-state. Since Φ is homogeneous of degree 1 in $\{L, n\}$ the free entry condition (23a) determines both the ratio of $(L/n)^*$ and the wage w^*. Equation (23b) means that the wage equates the MRS and the real wage. The final equation (23c) comes from the steady state condition for bonds: $B(b, n, \lambda, q) = 0$. For an arbitrary value of b^*, we can thus solve the equations for the implied values of $\{n, \lambda, L\}$. We can represent the steady state in consumption-leisure space

Fig. 1 here

The Income Expansion Path (IEP) represents the consumption leisure choice given the equilibrium real wage w^*. The assumption of additive separability $U_{C\ell} = 0$ does not place any simple restriction on the shape of the IEP. Since both leisure and consumption are assumed normal, it is upward sloping. The steady-state budget constraint (23c) is linear with slope w and intercept[14] $rb^* - G$, the dynamics of the path to steady state being reflected in the divergence between steady state bonds b^* and initial bonds b_0. The steady state equilibrium is then the intersection of the IEP and the budget constraint at point A.

Given the functional forms we have assumed, free-entry condition determines the steady-state real wage and productivity (15) $w^* = \mathcal{P}^*$. Substituting into (3b) yields the reduced form equation for L,

$$\mathcal{L}(\lambda, A) = (\lambda(1-\mu)\gamma)^{\frac{1}{\beta-1}} A^{\frac{1}{\gamma(\beta-1)}} \left(\frac{1-\gamma(1-\mu)}{F}\right)^{\frac{1-\gamma}{\gamma(\beta-1)}}$$

The corresponding SS number of firms and output from (12) and (17) are

$$n(\lambda, A) = (\lambda(1-\mu)\gamma)^{\frac{1}{\beta-1}} A^{\frac{\beta}{\gamma(\beta-1)}} \left(\frac{1-\gamma(1-\mu)}{F}\right)^{\frac{\beta-\gamma}{\gamma(\beta-1)}}$$

$$\mathcal{Y}(\lambda, A) = (\lambda)^{\frac{1}{\beta-1}}((1-\mu)\gamma)^{\frac{\beta}{\beta-1}} A^{\frac{\beta}{\gamma(\beta-1)}} \left(\frac{1-\gamma(1-\mu)}{F}\right)^{\frac{\beta(1-\gamma)}{\gamma(\beta-1)}}$$

By use of the reduced form equation $\mathcal{L}(\lambda, A)$ we can then determine the equilibrium level of λ^*, conditional on A and b^*, by the output market clearing condition

[14] As depicted, we have $rb^* - G > 0$. Of course, the intercept can be negative if b^* is small or negative.

$$G + C(\lambda^*, A) - w\mathcal{L}(\lambda^*, A) - rb^* = 0 \tag{24}$$

where (24) can be viewed as an excess demand function for the steady state in terms of the price of marginal utility λ. The first two terms of the expression above, representing the expenditure side, are decreasing in λ, while the income terms, $w\mathcal{L}(\lambda) + rb^*$, are increasing in λ; hence there exists a $\lambda^* > 0$ such that the economy is at the steady state equilibrium. We have now defined the steady-state for a given value of the steady-state bonds b^*: we now need to turn to the dynamics to derive the steady state stock of bonds.

4.2 Linearized system

The analysis of the steady-state was conditional on the level of steady state bonds b^*. However, to determine b^* we need to know the path taken to equilibrium. The dynamics of the system will be analyzed by linearizing around the steady state:

$$\begin{bmatrix} \dot{n} \\ \dot{q} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\nu\lambda^*} & 0 \\ \frac{\lambda^*}{n(\lambda^*)}\frac{\gamma(\beta-1)F}{\beta-\gamma} & r & 0 \\ \gamma\left(\frac{F}{1-\gamma(1-\mu)}\right)(\varepsilon^*_{nL} - \mu) & 0 & r \end{bmatrix} \begin{bmatrix} n - n^* \\ q - q^* \\ b - b^* \end{bmatrix} \tag{25}$$

where $q^* = 0$ and $\varepsilon^*_{Ln} \equiv L_n \left(\frac{n}{L}\right)^* = \frac{1-\gamma}{\beta-\gamma}$ represents the elasticity of the labour supply to the number of firms evaluated at the steady state. The determinant of the sub-system (25) in $\{n, q\}$ is

$$\Delta = -\frac{1}{n(\lambda^*)}\frac{\gamma(\beta-1)F}{\nu(\beta-\gamma)} < 0 \tag{26}$$

which is negative since $\beta > \gamma$. Hence the system is saddle-path stable, with the negative real root $\Gamma < 0$ and a positive root Γ^+

$$\Gamma = \frac{r}{2}\left(-1 - \sqrt{1 - \frac{4\Delta(\lambda^*)}{r^2}}\right) < 0 < \Gamma^+ = \frac{r}{2}\left(-1 + \sqrt{1 - \frac{4\Delta(\lambda^*)}{r^2}}\right) \tag{27}$$

The solution to the linearized system is

$$n(t) = n^* + (n_0 - n^*)\exp[\Gamma t] \tag{28a}$$

$$q(t) = (n_0 - n^*)\Gamma\nu\lambda^* \exp[\Gamma t] \tag{28b}$$

$$b(t) = b^* + \frac{\Omega}{\Gamma_1 - r}(n_0 - n^*)\exp[\Gamma t] \tag{28c}$$

where Ω gives the effect of entry on the stock of bonds:

$$\Omega = \gamma\frac{F}{1-\gamma(1-\mu)}(\varepsilon^*_{nL} - \mu) \tag{29}$$

where $sign\ \varepsilon^*_{nL} = sign\ (1-\gamma)$.

Note that the sign of Ω is ambiguous

$$sign\ \Omega = sign\ \left(\left[\frac{1-\gamma}{\beta-\gamma}\right] - \mu\right)$$

in the Walrasian case ($\mu = 0, \gamma < 1$), $\Omega > 0$ and the accumulation of firms leads to a *reduction* in bonds. The main mechanism here is that there is a positive effect of n on labor supply and output ($\Phi_{Ln} > 0$), so that having too few firms means that wages, labor income and home production are below their steady state level. To maintain consumption, this low level of income is compensated by higher than steady state imports, financed by running down bonds[15]. However, given $\gamma < 1$, if μ is large enough then bonds will increase as firms are accumulated. This is because the level of profits along the path to equilibrium is large: whilst the number of firms is below equilibrium, the extra profits generated are enough to exceed the adjustment costs and lower wage. In addition, there is a capacity effect, so that productivity is higher whilst the number of firms is below equilibrium (for $\mu > 0$, free-entry leads to excessive number of firms in steady-state). In the case of $\gamma \geq 1$, the flow of entry leads to an increase in the stock of bonds: this is because n has a negative effect on wages and profits, so that n below its steady state implies income above the steady state.

Fig. 2 here

The phase diagram of the system in $\{n, q\}$ space is depicted in Figure 2. The downward sloping line represents the combinations of $\{n, q\}$ for which $\dot{q} = 0$ and the arbitrage condition is satisfied $\pi = qr$. Above the $\dot{q} = 0$ line, the arbitrage condition implies that $\dot{q} > 0$; below it implies $\dot{q} < 0$. The $\dot{n} = 0$ phase line corresponds to the $n-$axis, since $\dot{n} = 0$ whenever $q = 0$. The saddle-path is downward sloping between the horizontal axis and the arbitrage line. Note that from (28b) the growth (shrinkage) in the marginal cost of entry q is given in absolute terms by the stable eigenvalue

$$\left|\frac{\dot{q}}{q}\right| = \Gamma$$

with the sign being determined by whether profits are positive (firms are being accumulated or negative (decumulated).

The linearized dynamics gives an explicit solution for steady state bonds as a function of λ, A and the initial condition n_0.

$$b^* = b(\lambda^*, A) = b_0 - \frac{\Omega}{\Gamma - r}(n_0 - n(\lambda^*, A)) \tag{30}$$

[15]It is important to understand that an *increase* in firms per se makes wages higher. However, the number of firms is increasing because it is below the steady-state. The stock of bonds decreases not because the accumulation of firms lowers income, but because entry implies that the initial level of n was low in the first place.

where $sign\ b_\lambda = sign\ \Omega$. We can now rewrite the SS condition for bonds (23c) as a function of λ only

$$w\mathcal{L}(\lambda^*, A) + rb(\lambda^*, A) - C(\lambda^*) - G = 0 \tag{31}$$

Hence we now have three equations (23a,23b,31) to determine the three variables $\{\lambda, n, L\}$ in steady state.

The following is a useful result for what follows.

Lemma 1 $w\mathcal{L}_\lambda + rb_\lambda - C_\lambda > 0$

Proof. See appendix A. ■

Similarly to the case analysed in the previous section, we can interpret the equation (31) as a steady state market clearing condition: the LHS is the excess of income over expenditure. The Lemma shows that this is strictly monotonic in λ. Hence, if a steady-state exists it is a *unique* steady state solution *for* λ^*. Existence follows from the Inada conditions on $U(C, 1-L)$ and the fact that b^* is bounded in (30) since n^* is bounded[16]. When λ is close to zero, L is very small and C is very large, with C unbounded as $\lambda \to 0$: hence there expenditure exceeds income; when λ is very large, C is very small and L is close to 1, so that there is an excess of income over expenditure. Hence for some intermediate value of λ (31) is satisfied.

5 Fiscal Demand shock

We will consider first a demand shock in terms of a tax financed change in government expenditure. In order to properly understand this, we need to introduce a national income accounting framework. We define total consumption Y to consist of private and public consumption, and classify the expenditure incurred insetting up new firms as investment, $I = \nu E^2$.

- *Gross domestic product (GDP)*: $GDP = \Phi(L, n, A)$
- *Gross National product*: $GNP = \Phi + rb - \dot{b}$.
- *Total Consumption*: $Y = C + G$.

These measures are clearly related: since we are considering a small open economy, all of these measures capture different aspects of the behavior of the economy. In steady state note that $Y = GNP = GDP + rb$.

[16] n^* is proportional to L^*, which lies in $[0, 1]$.

5.1 Permanent unanticipated increase in government expenditure

Let us first look at the steady-state changes induced by a permanent and *unanticipated* change in G. Since in the long-run there is a zero-profit equilibrium, the basic properties of the long-run multipliers are not affected by the technology parameter γ or the degree of imperfect competition.

Proposition 2 *Long-run multipliers for government expenditure.*

(a) *GNP.* $1 > \frac{dY^*}{dG} > 0.$
(b) *GDP.* $1 > \frac{d\Phi^*}{dG} > 0$, *and*
$sign \left[\frac{d\Phi^*}{dG} - \frac{dY^*}{dG}\right] = -sign\ \Omega$
(c) $\frac{dL^*}{dG} > 0$; $sign\ \frac{db^*}{dG} = -\ sign\ \Omega.$

Proof. See appendix A. ■

The increase in government has two distinct effects on steady state consumption. First, there is the standard *resource withdrawal effect*: in Figure 3 this is represented by the vertical shift of the $LRBC$ by dG. Secondly, there is the *bond effect*: the increase in output and the number of firms causes a reduction in steady-state bonds if $\Omega < 0$ as depicted, since bond decumulation occurs along the path to the new steady state, represented by the further downward shift in the $LRBC$ by $r(db^*/dG)$.

Fig. 3 here

In the case of $\Omega > 0$ the bond effect will result in an outward shift in the $LRBC$. The overall reduction in leisure is given by the move from A to B: this is decomposed into the resource withdrawal effect A to A', and the bond effect A' to B. In terms of output, the decrease in private consumption is offset by the increase in government expenditure. The change in GNP which includes the bond effect is from A to B. The change in GDP excludes the bond effect, and is equivalent to the move from $A - A'$.

5.2 Impact Effects, capacity utilization and productivity

The instantaneous effects of an increase in G differ from the long-run effects since the number of firms is at its initial value. To analyze this, we use (22) which defines the labor supply conditional on n.

Proposition 3 *Impact compared to long-run multipliers for $\mu > 0$.*

$$\text{(a)} \quad \frac{dY(\infty)}{dG} = \frac{dY(0)}{dG}$$

$$\text{(b)} \quad \frac{dL(\infty)}{dG} > 0, \ \frac{dL(0)}{dG} > 0$$

$$sign\,[\gamma - 1] = sign\left[\frac{dL(0)}{dG} - \frac{dL(\infty)}{dG}\right]$$

$$\text{(c)} \quad \frac{d\Phi(0)}{dG} > 0, \ \frac{d\Phi(\infty)}{dG} > 0$$

$$For\ \gamma > 1 \ \ \frac{d\Phi(0)}{dG} > \frac{d\Phi(\infty)}{dG}$$

$$For\ \gamma < 1 \ \ sign\left[\frac{d\Phi(0)}{dG} - \frac{d\Phi(\infty)}{dG}\right] = sign\left[1 - \mu - \frac{dL(0)/dG}{dL(\infty)/dG}\right]$$

$$\text{(d)} \quad \frac{dw(\infty)}{dG} = 0, \ \ sign\frac{dw(0)}{dG} = sign\,[\gamma - 1]$$

Proof. See Appendix A. ■

The increase in taxes G causes an increase in labor supply and this leads to a reduction in the wage when $\gamma < 1$ and an increase if $\gamma > 1$. In the long run this is reversed as the economy moves to the free entry equilibrium and corresponding real wage w^*. The initial number of firms is below the new steady state. Firms are then accumulated which has the effect of stimulating the labor supply if $\gamma < 1$, or reducing it if $\gamma > 1$. Total consumption is constant (a). GDP jumps initially in response to jump in the labor supply. In the long-run there are two effects at work: as firms are accumulated it changes the real wage and alters labor supply (increasing it when $\gamma < 1$, reducing it when $\gamma > 1$), whilst the additional firms reduce output due to the additional fixed costs (unless $\mu = 0$). When $\gamma \geq 1$, both these effects work in the same direction, reducing GDP as firms are accumulated. If $\gamma < 1$ and there is a low level of imperfect competition, the first effect will dominate and the long-run multiplier is bigger than the impact. If the markup is large enough the long-run multiplier will be smaller.

The Walrasian case is a useful benchmark. When $\mu = 0$, or close to 0, we have the following:

Proposition 4 *Let $\gamma < 1$. There exists $\bar{\mu} > 0$ such that for $\mu < \bar{\mu}$,* $\frac{d\Phi(0)}{dG} < \frac{d\Phi(\infty)}{dG}$.

Proof. See appendix A. ■

By continuity, there exists $\bar{\mu} > 0$ such that the inequality is sustained.

Lastly, we can examine the effect of an increase in demand on productivity, measured as the average product of labor $\mathcal{P} = \Phi/L$. Before the increase in G, the industry is in free entry equilibrium and operating at normal capacity $y = y^*$, with productivity equal to the real wage, $\mathcal{P}^* = w^*$. The impact effect of an increase in demand is to increase output per firm.

Proposition 5 *Productivity and capacity utilization.*

(a) If $\mu = 0$ *then* $\frac{d\mathcal{P}(0)}{dG} = \frac{d\mathcal{P}(\infty)}{dG} = 0$
(b) If $\mu > 0$ *then* $\frac{d\mathcal{P}(0)}{dG} > \frac{d\mathcal{P}(\infty)}{dG} = 0$

Proof. See appendix A. ∎

When $\mu > 0$, the effect of an increase in government expenditure is to produce a transitory increase in the average product of labor, i.e. productivity. The long run average product is equal to the steady state real wage which is of course unaffected by changes in government expenditure.

The behavior of productivity is best understood in terms of the change in *capacity utilization* caused by the change in employment. We can represent the technology in terms of the cost function as in Fig.4, with output on the horizontal axis and cost on the vertical axis: we depict the marginal and average cost functions on the assumption of diminishing marginal productivity of labor to yield the traditional U-shaped average cost function.

Fig 4 here

In Fig.4(a) we depict the equilibrium in the Walrasian case: the zero-profit long-run equilibrium output per firm is the technically efficient level y^e. Average cost equals marginal cost, so that average cost is flat, there being no first-order effects of capacity utilization on productivity (there is constant returns to scale in the neighborhood of the equilibrium). However, in Figure 4(b) we depict the long-run equilibrium when $\mu > 0$. In this case, we have the standard Robinson-Chamberlin excess capacity in long-run equilibrium. The AC curve is downward sloping, there being increasing returns in the neighborhood of the equilibrium. As output increases, capacity utilization increases towards the efficient level and there is a resultant increase in productivity.

Fig 5 here

The time path of the productivity following an unanticipated demand shock at time T is depicted in figure 5. There is an initial jump as capacity utilization increases, followed by a gradual decay back to the free-entry value as entry occurs. Clearly, productivity is endogenous in this model, determined by capacity utilization, which in turn is affected by the fiscal shock.

The productivity dynamic induced by demand is endogenous. However, suppose we mistakenly assume the productivity shock to be *exogenous*: in this case although there has been no underlying change in the technology parameter, we would infer that there had been a technology shock that decayed over time. We can relate this to the Solow growth residual, often advocated as a measure of technological change. From the definition of GDP :

$$\frac{d(GDP)}{GDP} = \left(\frac{\Phi_L L}{y}\right)\frac{dL}{L} + \left(\frac{\Phi_n n}{y}\right)\frac{dn}{n} + \frac{dA}{A}$$

To derive the true measure of technical progress dA/A, we substitute the actual equilibrium values from the model $\Phi_L = w/(1-\mu)$ and $\Phi_n n = \Pi - \frac{\mu}{1-\mu} wL - \nu E^2$. Hence defining S^L as labor share of GDP and $S^\Pi = \left[\Pi - \nu E^2\right]/y$ as the share of dividends we have

$$\frac{dA}{A} = \frac{d(GDP)}{GDP} - \left(\frac{S^L}{1-\mu}\right)\frac{dL}{L} - \left(S^\Pi - \frac{\mu}{1-\mu}S^L\right)\frac{dn}{n}$$

The measured Solow residual (SR) is in this model

$$SR = \frac{d(GDP)}{GDP} - S^L \frac{dL}{L}$$

Hence, the difference between the true value of technological change and the measured SR is

$$SR - \frac{dA}{A} = \frac{\mu}{1-\mu} S^L \left[\frac{dL}{L} - \frac{dn}{n}\right] + S^\Pi \frac{dn}{n} \tag{32}$$

Assuming that we start from a free-entry equilibrium, so that $S^L = 1$, $S^\Pi = 0$, in the case of an unanticipated *demand* shock ($dA/A = 0$) on impact (since $dn/n = 0$), the Solow residual overestimates technological progress by

$$SR - \frac{dA}{A} = SR = \frac{\mu}{1-\mu}\frac{dL}{L}$$

That is, the conventional Solow residual will misinterpret the increase in productivity due to the capacity utilization effect as technological change. The size of this error is zero in the Walrasian case of $\mu = 0$, but is increasing in μ and proportional to the growth in employment when $\mu > 0$. *This is exactly as found by Ryan (2000), that the relationship between capacity utilization and productivity is stronger when there is more imperfect competition.* Over time, employment will fall and entry occurs, reducing the error, until in the long-run the growth in employment and the number of firms are equal (the term in square brackets in (32) becomes zero, as does S^Π).

5.3 Anticipated changes in government expenditure

In this section we will briefly examine the effect of an *anticipated* permanent change in government expenditure. The technical methodology follows Datta and Dixon (2001), so we will only illustrate the methodology here (see appendix B for an outline of the proof). First consider a permanent step increase in government expenditure that is to occur at time T but is announced at time

$t = 0$. Let us assume that the economy at time 0 is in steady state prior to the announcement. The dynamics breaks up into two periods: from $0 < t < T$, and $t \geq T$. For each of these two phases there is the corresponding phase diagram, appropriate for the prevailing level of G. In the second phase, the economy will converge to the new steady state along the new saddle-path. As shown in Figure 6, in the initial phase after the announcement but before the change occurs, the economy will follow an unstable path.

Fig. 6 here

When the announcement is made, the present value of the incumbent firm q jumps. It jumps to a level below the new saddle-path, since the net present value would be higher if the increase occurred immediately. From this the economy follows and unstable path with q and n rising together as we get nearer the time of the increase. At time T the unstable path from the initial dynamics system joins on to the new saddle-path at point A: from here both q and n fall together towards the steady state. Note that in the initial phase, entry occurs despite the fact that profits are negative and becoming more so. At time T the level of profits jumps in response to the increase in expenditure (output and employment jump). The arbitrage equation is still satisfied at point T: the jump in π is exactly offset by the fall in $\dot{q}$ reflected in the kink at point q. The behavior of productivity and capacity utilization along this path is that until T, both productivity and capacity utilization decline below their steady state values. At T, there is a jump in both to above their steady state values, after which they both decline to the initial steady-state.

5.4 Temporary changes in government expenditure

Consider now a temporary *unanticipated* increase in G. The increase occurs at time $t = 0$ and continues until T, after which time the expenditure falls back to the new steady state[17]. The dynamics breaks up into two periods: from $0 < t < T$, and $t \geq T$. As shown in Figure 7, the new steady state will usually differ from the initial steady state $n_B \neq n_0$: although the shock is temporary, it has a permanent effect through its impact on the stock of bonds[18].

Fig. 7 here

As in the case of an anticipated permanent change, q jumps at time 0. The jump is to a level below the temporary saddlepath, since the increased profitability is only temporary. This leads to the economy following an unstable and non-monotonic path until at time T. Initially, there is a fall in q and increase in n: at point A,the increase in n peaks when $q = 0$. Then there is a period where both n and q are falling. The reason behind this is that although firms

[17] The formal analysis of this case closely follows Turnovsky 1997 (pages 94-98) and we refer the readers to his book.

[18] In Figure 7 we have $n_B > n_0$: whether the shock will result in an increase or decrease in steady state n will depend on the overall effect on L and hence the sign of Ω.

are profitable (the temporary shock is still present), the firms are anticipating the future decline in profitability. At time T the path joins up with saddlepath to the new steady state at point B: there are now too many firms. Again, at the point where there is an anticipated decline in expenditure, there are equal and opposite jumps in π and $\dot{q}$. The time path of productivity and capacity utilization is that there is a jump at $t = 0$, followed by a decline until point A is reached, after which this decline is partially reversed, although still above its steady-state value. At time T, when the increase in G is reversed, there is underutilisation of capacity and productivity jumps to below $\mathcal{P}^*$ and gradually increases back to it as firms exit.

6 Technological change

In the previous section we analyzed the effect of a change in demand on the economy in the form of a change in government consumption funded by a balanced budget. The underlying technology, represented by parameter A was assumed to be unchanged: any variation in productivity was therefore solely due to the capacity utilization effect. In this section we will explore what happens when there is a change in technology, resulting in a superposition of the underlying technological change and transitory changes on productivity induced by changes in capacity utilization. In this section we will for simplicity set $G = 0$, so that $Y = C$.

6.1 The long run effects of a productivity change

To study the long run effect of a permanent unanticipated change in the technology parameter A, note that from the SS bond equation (31) we have

$$\frac{d\lambda^*}{dA} = \frac{-\left(w_A\mathcal{L} + rb_A\right)}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} = -\frac{\lambda^*}{A}\frac{\frac{\beta}{\gamma}\left(w\mathcal{L}_\lambda + rb_\lambda\right)}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} < 0 \tag{33}$$

where $w_A > 0$ is the derivative of the steady state wage (15) *w.r.t.* A. As we would expect, an increase in productivity reduces the steady state marginal utility: technological progress raises real income and the real wage.

At the firm level, the specification of the technology means that the free-entry firm output (13) is unaffected, but the firm's free-entry employment is *reduced* (12). First we will consider the long-run effect of a permanent technological improvement.

Proposition 6 *A permanent improvement in technology: the long-run effects.*

(a) $\frac{dC^*}{dA} > 0$; $\frac{dn^*}{dA} > 0$; $\frac{dw^*}{dA}$
(b) sign $\frac{dL^*}{dA} = sign\left[b_0 - \frac{\Omega}{\Gamma - r}n_0\right]$
(c) $\frac{d\Phi^*}{dA} > 0$ *if* $\frac{dL^*}{dA} > 0$
(d) $\frac{d\mathcal{P}^*}{dA} > 0$

Proof. See appendix A. ■

The effect on the labor supply is ambiguous because there is a conflict of income and substitution effects: the higher wage causes a substitution effect for less leisure and more consumption; the income effect is for more leisure. Which effect dominates depends on the level of initial wealth: $b_0 - \frac{\Omega}{\Gamma - r} n_0$ is the initial value of wealth in terms of bonds ($-\frac{\Omega}{\Gamma - r} n_0$ is the present value of the bonds that would have been decumulated/accumulated if $n_0 = 0$). If $\Omega > 0$, (i.e. $\gamma < 1$ and μ small enough) then a sufficient condition for $\frac{dL^*}{dA} > 0$ is that $b_0 \geq 0$: if $\Omega < 0$, (for which $\gamma \geq 1$ is sufficient) then a sufficient condition for $\frac{dL^*}{dA} < 0$ is $b_0 \leq 0$.

This ambiguity in the labor supply response may in principle carry over to GDP: whilst productivity tends to boost the output given employment, if employment falls enough it might lead to an overall reduction. Only if the labor supply increases can we be sure that GDP also increases. Note the contrast of part (a) and part (c): GNP always increases, irrespective of labor supply, with income and substitution effects working together to increase consumption.

6.2 Impact effect of technological change

In order to find the impact effect of technological change, we hold n fixed. The contrast between the impact and long-run effects depends on the effect of entry which has three elements. First there is the direct effect of increasing fixed cost nF. Second there is the effect of entry on lifetime income (the Ω effect on the bond stock). Third, there is the capacity utilization effect: on impact GDP varies with employment, so that the changes in capacity utilization will induce changes in productivity as in the case of demand shocks.

If we first consider the impact effect on employment, this like the long-run effect, is ambiguous for the same reason (income and substitution effects may clash). However, if we look at the difference between the impact and long-run effect, this turns to depend on whether there is an increasing or diminishing marginal product of labor. When $\gamma < 1$, on impact there is a negative relationship between the real wage and employment; when $\gamma > 1$ a positive relation; when $\gamma = 1$ no relation. We can thus get undershooting of employment ($\gamma > 1$) or overshooting ($\gamma < 1$) on impact depending on whether entry increases or decreases the marginal product (see Proposition 6 a,c). The change in GDP is also ambiguous on impact for the same reason, and there is no obvious ranking of impact and long-run effects. We summarize these results in Proposition 6.

Proposition 7 *Impact versus long-term effects of technological change on employment, output and wages.*

(a) $sign \frac{dL(0)}{dA} = sign\ [(\beta - \gamma)(w\mathcal{L}_\lambda + rb_\lambda) + \gamma C_\lambda]$
$sign\ \left[\frac{dL(\infty)}{dA} - \frac{dL(0)}{dA}\right] = sign\ L_n = sign\ [\gamma - 1]\,.$

(b) $\frac{d\Phi(0)}{dA} = \Phi_A + \Phi_L \frac{dL(0)}{dA}$
$\frac{d\Phi(\infty)}{dA} - \frac{d\Phi(0)}{dA} = \Phi_L \frac{dL(\infty)}{dA}\left[1 - \mu - \frac{dL(0)/dA}{dL(\infty)/dA}\right]$

(c) $\frac{dw(0)}{dA} = (1-\mu)\Phi_{LL}\frac{dL(0)}{dA} + \frac{w^*}{A\gamma}$
$sign\left[\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA}\right] = sign\left[\gamma - 1\right].$

Proof. See appendix A. ■

Lastly, we turn to the impact effect of the technological improvement on measured productivity $\mathcal{P}$.

$$\begin{aligned}\frac{d\mathcal{P}(0)}{dA} &= (1-\mu)\left(\frac{n}{L}\right)^{(1-\gamma)} + (\gamma-1)\frac{A}{L}\left[\frac{n}{L}\right]^{1-\gamma}\frac{dL(0)}{dA} + \left[\frac{n}{L}\right]\frac{F}{L}\frac{dL(0)}{dA} \\ &= \frac{\mathcal{P}^*}{\gamma A} + \frac{\mu}{1-\mu}\mathcal{P}^*\frac{dL(0)}{dA}\end{aligned}$$

The first term is the pure technology effect: with fixed employment and labor supply, an increase in technology boosts output. The second and third terms represent the *capacity utilization effect*, the impact of changes in employment: the second term is the effect on variable output per unit labor, the third the effect of the change in labor supply on spreading fixed costs.

This is in general ambiguous in sign ($dL(0)/dA$ can take either sign): when positive we have *overshooting*, and when negative *undershooting* of underlying measured productivity. A direct comparison of the impact and long-run effect indicates that:

Proposition 8 *Impact versus long-run productivity effects.*

$$\frac{d\mathcal{P}(0)}{dA} - \frac{d\mathcal{P}(\infty)}{dA} = \frac{\mu}{1-\mu}\mathcal{P}^*\frac{dL(0)}{dA}$$

Proof. See appendix A. ■

If the impact effect on employment is positive, then productivity overshoots its long-run value, if employment falls on impact, it undershoots. The interesting thing to note is that *this result holds irrespective of whether the marginal product of labor is increasing or decreasing*. This is easy to understand on further reflection. If we start from a free-entry equilibrium, then AC is flat: there is no capacity effect on employment. After the technology improvement has occurred, the new free-entry employment per firm is lower: at the initial level of employment, firms are over capacity (on the upward slope of the AC curve). If employment *falls* on impact, then you move back down the AC curve thus tending to increase productivity; if employment rises on impact, then you move further up the AC curve and tending to reduce productivity. Hence, it is the fact that there are locally increasing returns to employment that drives the simple relationship between employment and productivity independently of the technology parameter γ.

The fact that capacity utilization causes endogenous productivity dynamics is important, since it implies that the time profile of measured productivity will differ from, and may tend to mask, the true changes in underlying technology.

For example, if there is productivity overshooting, then a permanent change in productivity leads to an exaggerated instantaneous impact, that dies away to the permanent change. On the other hand, if there is a capacity effect which tends to reduce measured productivity at first, the measured technology increase will adjust more slowly up to the full effect. The deviations of measured productivity form technological change become small when there is near-perfect competition, and in the limiting Walrasian economy they disappear. This indicates that it may be misleading to use measured productivity as a guide to technological change except in the long-run unless the economy has almost perfect competition in the output market.

We show the two cases of measured productivity undershooting and overshooting in figure 8. In both cases, we have a permanent step change in technology with A rising to A_1 at time T.

Fig. 8 here

With overshooting the capacity effect reinforces the technology effect, leading to an increase in measured productivity that initially exaggerates the underlying increase. With undershooting, the capacity effect counteracts the technology improvement, leading to a gradual increase in measured productivity towards the long-term increase. This reinforces the notion that we have to take into account capacity utilization when we are using productivity measures to understand technological change.

Burnside and Eichenbaum (1996, p.1169) examined the response to a technology shock in the presence of variable factor utilization (factor hoarding). They found that there would be overshooting: the initial shock in technology is amplified by the fact that capital utilization and labor effort both increase in response to a positive technology shock. The mechanism differs: in our setup, the intensity of factor use is fixed, so that the mechanism is one of variation in *capacity* utilization, movements down the firm's average cost function.

6.3 Anticipated technological change

Lastly, we will briefly consider the impact of an *anticipated* permanent technological change from A to A_1 at T. The mathematical analysis is similar to the case of an anticipated demand shock, so that the discussion will be brief. When the technological advance becomes known, there is a jump in the flow of entry in anticipation of the additional profits to be made. In terms of the phase diagram, as in Figure 6, this follows an unstable path in terms of the initial steady state with the flow of entry increasing through time. When the new technology comes on line, the flow of entry is at its peak: thereafter it follows the new saddlepath to the new steady state. The time profile of productivity is depicted in Figure 9. In the initial announcement phase prior to the change, productivity is falling: it then jumps up when the change occurs and gradually converges to the new state. We have depicted the case where there is undershooting, when the initial jump takes it to a point below the new steady state: it is of course possible for overshooting to occur.

Fig. 9 here

6.4 Temporary technological change

The case of a temporary unanticipated change can also of course be analyzed. In this case, as in the case of a similar demand shock depicted in Figure 7, the improvement in technology causes a jump in the flow of entry along an unstable path relative to the better technology. Eventually there is exit and when the technology reverts to normal the saddlepath is followed. In terms of measured productivity, the long-run effect is zero. The impact effect will depend on how employment responds: entry will increase employment further if $\gamma < 1$ or decrease it if $\gamma > 1$. When the technology reverts, employment will jump again. The exact time pattern of measured productivity can thus vary overtime as employment responds to the technological change itself and the entry of firms affects the labor supply.

7 Concluding remarks

In this paper we have developed a dynamic model of entry in the context of a simple yet standard open economy macromodel. We have shown how the dynamics of entry is crucial to understanding the behaviour of measured productivity in the response to both demand and technology changes. The crucial insight is that variations in output per firm, capacity utilization, has an important role in the short-run. We have developed this insight in an integrated and consistent manner to explore both the long-run and short run effects of changes in demand and technology both when unanticipated and when anticipated, when permanent and when temporary.

Clearly, the model can be developed in many dimensions. First, it would be useful to include uncertainty in the evolution of both technology and demand. The role of uncertainty has been developed in several papers at the microeconomic level (Hopenhayn 1992, Dixit and Pindyk 1994, Ericson and Pakes 1995, Das and Das 1996 inter alia). The current model without uncertainty would serve as a useful reference point for developing more complex models. Second, it would be useful to develop the model to allow for oligopolistic markets, so that entry would be modelled with the number of firms as an integer which might be small as in Cook (2001). This would possibly raise many strategic issues such as entry deterrence and the optimal timing of entry by entrants, the latter which has yet to be modelled extensively at the microeconomic level[19]. Laslty, of course, we can develop the model to allow for capital accumulation (Brito and Dixon 2000 allow for capital and entry in a closed economy model)

[19]See Sadanand and Sadanand (1996) for one of the few papers on the topic of the optimal timing of entry.

and other features that would allow for the model to be applied to data. This task remains for future work.

A Proofs

A.1 Proof of Lemma 1

Proof. Since $C_\lambda < 0$, it suffices to show that $w\mathcal{L}_\lambda + rb_\lambda > 0$.

$$w\mathcal{L}_\lambda + rb_\lambda = \Phi_L \mathcal{L}_\lambda \left[\frac{\Gamma(1-\mu) - r(1-\varepsilon_{Ln})}{\Gamma - r}\right] = \Phi_L \mathcal{L}_\lambda \left[\frac{\Gamma(1-\mu) - r(\frac{\beta-1}{\beta-\gamma})}{\Gamma - r}\right] > 0$$

■

A.2 Proof of Proposition 1

Proof. From the steady state market clearing condition

$$G + C(\lambda^*, A) - w\mathcal{L}(\lambda^*, A) - rb(\lambda^*, A) = 0$$

hence from the Lemma

$$\frac{d\lambda^*}{dG} = [w\mathcal{L}_\lambda + rb_\lambda - C_\lambda]^{-1} > 0$$

(a) The output multiplier for GNP is

$$\begin{aligned} Y^* &= G + C(\lambda^*) \\ \frac{dY^*}{dG} &= 1 + C_\lambda . \frac{d\lambda^*}{dG} \\ &= \frac{w\mathcal{L}_\lambda + rb_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \in (0,1) \end{aligned}$$

(b) For GDP the multiplier is

$$\begin{aligned} \Phi^* &= \mathcal{Y}(\lambda^*, A) \\ &= \Phi\left(\mathcal{L}(\lambda^*, A), \left(\frac{n}{L}\right)^* \mathcal{L}(\lambda^*, A)\right) \\ \frac{d\Phi}{dG} &= \left[\Phi_L + \Phi_n \left(\frac{n}{L}\right)^*\right] \mathcal{L}_\lambda \frac{d\lambda^*}{dG} \\ &= \frac{w\mathcal{L}_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \end{aligned}$$

Hence the difference is

$$\frac{d\Phi^*}{dG} - \frac{dY^*}{dG} = \frac{-rb_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} = -\frac{\Gamma\Omega}{\Gamma - r}\left(\frac{n}{L}\right)^* \frac{\mathcal{L}_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda}$$

(c)

$$\begin{aligned}\frac{dL^*}{dG} &= \mathcal{L}_\lambda . \frac{d\lambda^*}{dG} > 0 \\ \frac{db^*}{dG} &= \frac{\Omega}{\Gamma - r}\left(\frac{n}{L}\right)^* \mathcal{L}_\lambda \frac{d\lambda^*}{dG}\end{aligned}$$

■

A.3 Proof of Proposition 2

Proof. (a) Both C and λ jump to their new values $\{C^*, \lambda^*\}$, and hence Y too.

(b) The impact effect of fiscal policy is differentiated form the long-run effect by the fact that the number of firms is unchanged. Hence:

$$\begin{aligned}\frac{dL(0)}{dG} &= L_\lambda(n,\lambda,A)\frac{d\lambda^*}{dG} > 0 \\ \frac{dL(\infty)}{dG} &= L_\lambda(n,\lambda,A)\frac{d\lambda^*}{dG} + L_n \frac{dn}{d\lambda}\frac{d\lambda^*}{dG} = \mathcal{L}_\lambda \frac{d\lambda^*}{dG} > 0 \\ sign\left[\frac{dL(\infty)}{dG} - \frac{dL(0)}{dG}\right] &= sign\ \left[L_n \frac{dn}{d\lambda}\frac{d\lambda^*}{dG}\right] \\ &= sign\ [1-\gamma]\end{aligned}$$

(c)

$$\begin{aligned}\frac{d\Phi(0)}{dG} &= \Phi_L \frac{dL(0)}{dG} = \frac{w^*}{1-\mu}\frac{dL(0)}{dG} > 0 \\ \frac{d\Phi(\infty)}{dG} &= \Phi_L \frac{dL(\infty)}{dG} + \Phi_n \left(\frac{n}{L}\right)^* \frac{dL(\infty)}{dG} \\ &= w^* \frac{dL(\infty)}{dG}\end{aligned}$$

$$\begin{aligned}\frac{d\Phi(\infty)}{dG} - \frac{d\Phi(0)}{dG} &= w^* \frac{dL(\infty)}{dG}\left[1 - \frac{1}{1-\mu}\frac{dL(0)/dG}{dL(\infty)/dG}\right] \\ &= \frac{w^*}{1-\mu}\frac{dL(\infty)}{dG}\left[1-\mu - \frac{dL(0)/dG}{dL(\infty)/dG}\right]\end{aligned}$$

When $\gamma \geq 1$, $\frac{dL(0)/dG}{dL(\infty)/dG} \geq 1$, so that for $\mu > 0$ the inequality is as stated. When $\gamma < 1$, the term in square brackets determines the sign of the RHS.

(d)

$$\begin{aligned}\Phi_{LL} L_\lambda(n,\lambda)\frac{d\lambda^*}{dG} &= \frac{dw(0)}{dG} \\ sign\,[\Phi_{LL}] &= sign\,[\gamma - 1]\end{aligned}$$

■

A.4 Proof of Proposition 3

Proof. When $\mu = 0$,

$$\frac{d\Phi(\infty)}{dG} - \frac{d\Phi(0)}{dG} = w^* \frac{dL(\infty)}{dG}\left[\frac{dL(\infty)}{dG} - \frac{dL(0)}{dG}\right] > 0.$$

■

A.5 Proof of Proposition 4

Proof. From the definition of productivity, we have

$$\frac{d\mathcal{P}}{dL} = \frac{d(\Phi/L)}{dL} = \frac{L\Phi_L - \Phi}{L^2} = \frac{\mu\Phi_L}{L} = \frac{\mu}{1-\mu}\frac{w^*}{L}$$

Hence

$$\frac{d\mathcal{P}(0)}{dG} = \frac{d\mathcal{P}}{dL}\frac{dL}{d\lambda}\frac{d\lambda^*}{dG} = \frac{\mu}{(1-\mu)}\frac{w^*}{L}L_\lambda(n,\lambda)\frac{d\lambda^*}{dG}$$

This is strictly positive if $\mu > 0$, zero if $\mu = 0$. Since $\Phi(L, n)$ is homogeneous of degree 1 in (L, n) and the ratio L/n fixed by free entry, we have $\frac{d\mathcal{P}(\infty)}{dG} = 0$. ■

A.6 Proof of Proposition 5

Proof. (a)

$$\begin{aligned}
\frac{dC^*}{dA} &= C_\lambda \frac{d\lambda^*}{dA} = \frac{\beta}{\lambda^* A\gamma}\frac{(w\mathcal{L}_\lambda + rb_\lambda)}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} > 0 \\
\frac{dn^*}{dA} &= \left[n_A + n_\lambda \frac{d\lambda^*}{dA}\right] = n_A\left(\frac{-C_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda}\right) > 0 \\
\frac{dw^*}{dA} &= \frac{w^*}{\gamma A} > 0.
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{dL^*}{dA} &= \mathcal{L}_A\left(1 - \frac{\beta\,(w\mathcal{L}_\lambda + rb_\lambda)}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda}\right) \\
&= \frac{\mathcal{L}_A}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda}\left((1-\beta)\,(w\mathcal{L}_\lambda + rb_\lambda) - C_\lambda\right) \\
&= \frac{\mathcal{L}_A}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda}\frac{1}{\lambda^*}\left(C - wL - r\frac{\Omega}{\Gamma - r}n\right)
\end{aligned}$$

However, since in steady state we have

$$C - wL = rb$$

$$\frac{dL^*}{dA} = \frac{\mathcal{L}_A}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \frac{1}{\lambda^*} \left(rb - r\frac{\Omega}{\Gamma - r} n \right)$$

From (28c) $b - \frac{\Omega}{\Gamma - r} n = b_0 - \frac{\Omega}{\Gamma - r} n_0$

$$\frac{dL^*}{dA} = \frac{\mathcal{L}_A}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \frac{r}{\lambda^*} \left(b_0 - \frac{\Omega}{\Gamma - r} n_0 \right)$$

Clearly, since $n_0 > 0$, if $\Omega > 0$, then the term in brackets is positive if initial bonds holdings are non-negative, so that employment must rise. Likewise if $\Omega < 0$, $b_0 \leq 0$ ensures that employment falls.

(c) From the zero-profit equilibrium expression for Φ (17), productivity has a direct effect on GDP, Φ_A, and an indirect effect on labor supply via λ and A:

$$\frac{d\Phi^*}{dA} = \frac{\Phi^*}{\gamma A} + w^* \frac{dL^*}{dA}$$

The first term is clearly positive, so an increase in the labor supply is sufficient for GDP to increase. However, if there is a reduction in the labor supply *and* this is sufficient to outweigh the positive effect then the overall effect might be negative.

(d) From (15)

$$\frac{d\mathcal{P}^*}{dA} = \frac{1}{\gamma} \frac{\mathcal{P}^*}{A} > 0$$

■

A.7 Proof of Proposition 6

Proof. (a) Totally differentiating $L = L(\lambda, n, A)$ keeping n fixed yields

$$\begin{aligned} \frac{dL(0)}{dA} &= L_\lambda \frac{d\lambda}{dA} + L_A \\ &= -L_A \left[\frac{(\beta - \gamma)(w\mathcal{L}_\lambda + rb_\lambda) - \gamma C_\lambda)}{\gamma \left(w\mathcal{L}_\lambda + rb_\lambda - C_\lambda \right)} \right] \end{aligned}$$

As in the long-run case, the income and substitution effects of a technological improvement work in opposite directions.

The difference between the long-run and impact multiplier is accounted for by the effect of entry, so that

$$\frac{dL(\infty)}{dA} - \frac{dL(0)}{dA} = L_n n_\lambda \frac{d\lambda}{dA} + L_n n_A = L_n n_A \left[\frac{-C_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \right]$$

$$sign\ \left[\frac{dL(\infty)}{dA} - \frac{dL(0)}{dA}\right] = sign\ L_n = sign\ [1-\gamma]$$

In the case of diminishing MPL ($\gamma < 1$), the short-run response of employment is less than the long-run response; the opposite is true with increasing MPL.

(b) The impact effect follows directly from (7) holding n constant. The long-run effect from the fact that n is given by (16).

(c)

$$\frac{dw(0)}{dA} = (1-\mu)\Phi_{LL}\frac{dL(0)}{dA} + \frac{w^*}{A\gamma}$$

Hence

$$\begin{aligned} \frac{dw(0)}{dA} - \frac{dw(\infty)}{dA} &= (1-\mu)\Phi_{LL}\frac{dL(0)}{dA} \\ sign\ \left[\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA}\right] &= sign\ [\gamma - 1] \end{aligned}$$

The difference between the long-run and short run wage effect depends on whether an increase in employment increases the MPL ($\gamma > 1, \Phi_{LL} > 0$), or decreases it ($\gamma < 1$, $\Phi_{LL} < 0$). ■

B Analysis of anticipated changes

The analysis here follows Datta and Dixon (2001) and uses standard techniques (e.g. Turnovsky 1997, pages 94-98). We will therefore just sketch the solution method taking the case of an anticipated step change, announced at time 0 to occur at time T. Once the change has occurred, the economy will follow the saddle-path to the new steady state. The initial (pre-announcement) stock of firms is in a steady state: we denote the initial steady state stock of firms as n_1. The eventual post-announcement steady state number of firms is n_2. Note that in this model, the eigenvalues depend on the steady state number of firms (26,27): the negative eigenvalues corresponding steady states are denoted Γ_i, $i = 1, 2$: the positive eigenvalue for the initial steady state is denoted Γ_1^+. Of course, the steady-state $q_i = 0$.

First, we describe the path over the initial phase over $t \in [0, T]$. When the announcement is made, before the actual increase in government expenditure occurs, the economy follows an unstable path relative to the initial equilibrium. Since the pre-announcement the economy is in equilibrium, $n(0) = n_1$, $b(0) = b_1$. Hence

$$n(t) = n_1 + A_1 e^{\Gamma_1 t} + A_2 e^{\Gamma_1^+ t} \tag{34a}$$

$$q(t) = A_1 \nu \Gamma_1 e^{\Gamma_1 t} + A_2 \nu \Gamma_1^+ e^{\Gamma_1^+ t} \tag{34b}$$

$$\begin{aligned} b(t) &= b_1 + \frac{\Omega_1}{\Gamma_1 - r} A_1 e^{\Gamma_1 t} + \frac{\Omega_1}{\Gamma_1^+ - r} A_2 e^{\Gamma_1^+ t} \\ &\quad - \left[\frac{\Omega_1}{\Gamma_1 - r} A_1 + \frac{\Omega_1}{\Gamma_1^+ - r} A_2 \right] e^{rt} \end{aligned} \tag{34c}$$

where Ω_1 is as in (29) evaluated at n_1. Note that setting $t = 0$, we have $A_1 = A_2$.

After the change in government expenditure occurs, $t \in [T, \infty)$, the economy follows a stable path to the new steady state

$$n(t) = n_2 + A_1' e^{\Gamma_2 t} \tag{35a}$$

$$q(t) = A_1' \nu \Gamma_2 e^{\Gamma_2 t} \tag{35b}$$

$$b(t) = b_2 + \frac{\Omega_2}{\Gamma_2 - r} A_1' e^{\Gamma_2 t} \tag{35c}$$

Since both $\{n, q\}$ are continuous at T, we have the two equalities in two unknowns $\{A_1, A_1'\}$.

$$\begin{gathered} A_1 \Gamma_1 e^{\Gamma_1 T} - A_1 \Gamma_1^+ e^{\Gamma_1^+ T} - A_1' \Gamma_2 e^{\Gamma_2 T} = 0 \\ A_1 e^{\Gamma_1 T} - A_1 e^{\Gamma_1^+ T} - A_1' e^{\Gamma_2 T} = n_2 - n_1 \end{gathered}$$

Now, simple substitution determines $\{A_1, A_1'\}$ conditional on n_2.

$$\begin{aligned} A_1' &= A_1 \left[\frac{\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T}}{\Gamma_2 e^{\Gamma_2 T}} \right] \\ A_1 &= (n_2 - n_1) \left[e^{\Gamma_1 T} - e^{\Gamma_1^+ T} - \left(\frac{\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T}}{\Gamma_2} \right) \right]^{-1} \\ &= (n_2 - n_1) \left[\frac{\Gamma_2}{\Gamma_2 \left(e^{\Gamma_1 T} - e^{\Gamma_1^+ T} \right) - \left(\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T} \right)} \right] \\ &= (n_2 - n_1) \left[\frac{\Gamma_2}{e^{\Gamma_1 T} (\Gamma_2 - \Gamma_1) + e^{\Gamma_1^+ T} (\Gamma_1^+ - \Gamma_2)} \right] > 0 \end{aligned}$$

Since $n_2 - n_1 > 0$ implies $\Gamma_2 - \Gamma_1 > 0$. Furthermore, both $\{A_1, A_1'\}$ can be seen as continuous functions of n_2 for $n_2 \geq n_1$.

The new steady state number of firms is determined by λ and comes from the dynamics for $b(t)$. Recall that we know b_1: hence from (34c), $b(T)$ is

$$b(T) = b_1 + A_1 \left[\frac{\Omega_1}{\Gamma_1 - r} e^{\Gamma_1 T} - \frac{\Omega_1}{\Gamma_1^+ - r} e^{\Gamma_1^+ T} - \left[\frac{\Omega_1}{\Gamma_1 - r} - \frac{\Omega_1}{\Gamma_1^+ - r} \right] e^{rT} \right] \tag{36}$$

where on the RHS of (36) $\{\Omega_1, \Gamma_1, \Gamma_1^+\}$ are known, only A_1 needs to be determined. Turning to b_2, from (35c), we have

$$b_2 = b(T) + \frac{\Omega_2}{\Gamma_2 - r} A_1' \tag{37}$$

where $\{\Omega_2, \Gamma_2\}$ are functions of n_2 and hence b_2.

The two equations (36,37) give us a relationship between $\{A_1, A_1', n_2\}$ and b_2. We thus have an additional equation to determine n_2, since (23c) gives us the level of λ_2 given b_2, and hence n_2. In effect, we can conceive of the following algorithm: assume an arbitrary level of n_2: this then ties down $\{\Omega_2, \Gamma_2, A_1, A_1'\}$: we can then from (37) determine b_2 and hence λ. If the implied level of n_2 equals the initial value, then we have found the equilibrium value and the full solution to the model.

Does such a solution exist? First, note that in effect we have a mapping from n_2 onto itself. The constants $\{\Omega_2, \Gamma_2, A_1, A_1'\}$ are continuous functions of n_2; b_2 is a continuous function of these variables and the known values $\{b_1, \Omega_1, \Gamma_1, \Gamma_1^+\}$ from (37), and n_2 is a continuous function of b_2 from (23c). Second, note that n_2 belongs to a compact convex set: we have the lower bound n_1 and the upper bound $(L/n)^*$ (the number of firms in the economy when $L = 1$): $n_2 \in [n_1, (L/n)^*]$. Hence we have a continuous mapping of a compact convex set onto itself, which must posses a fixed point.

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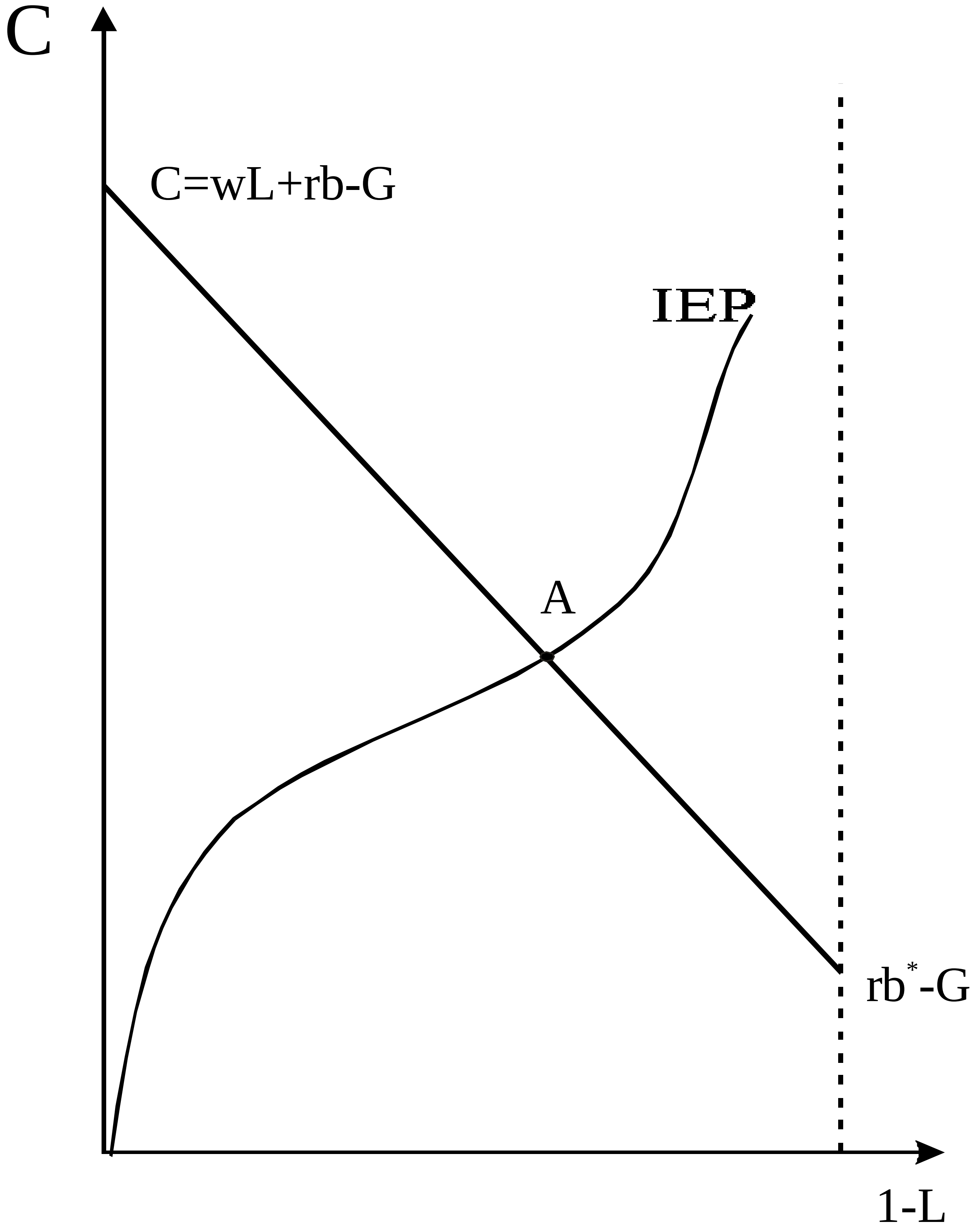

Figure 1: long run Equilibrium

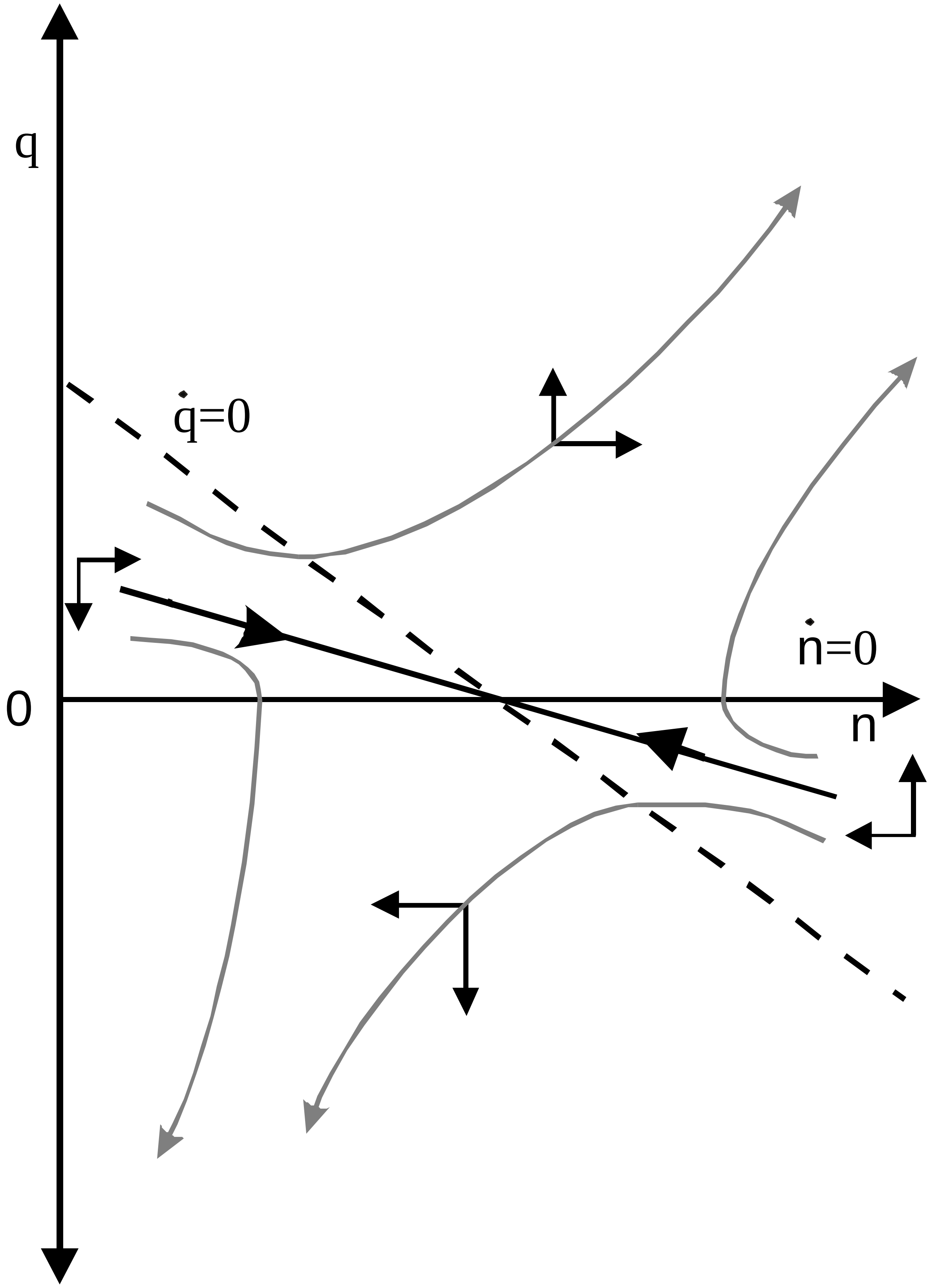

Fig.2: Phase Diagram in {n,q} space.

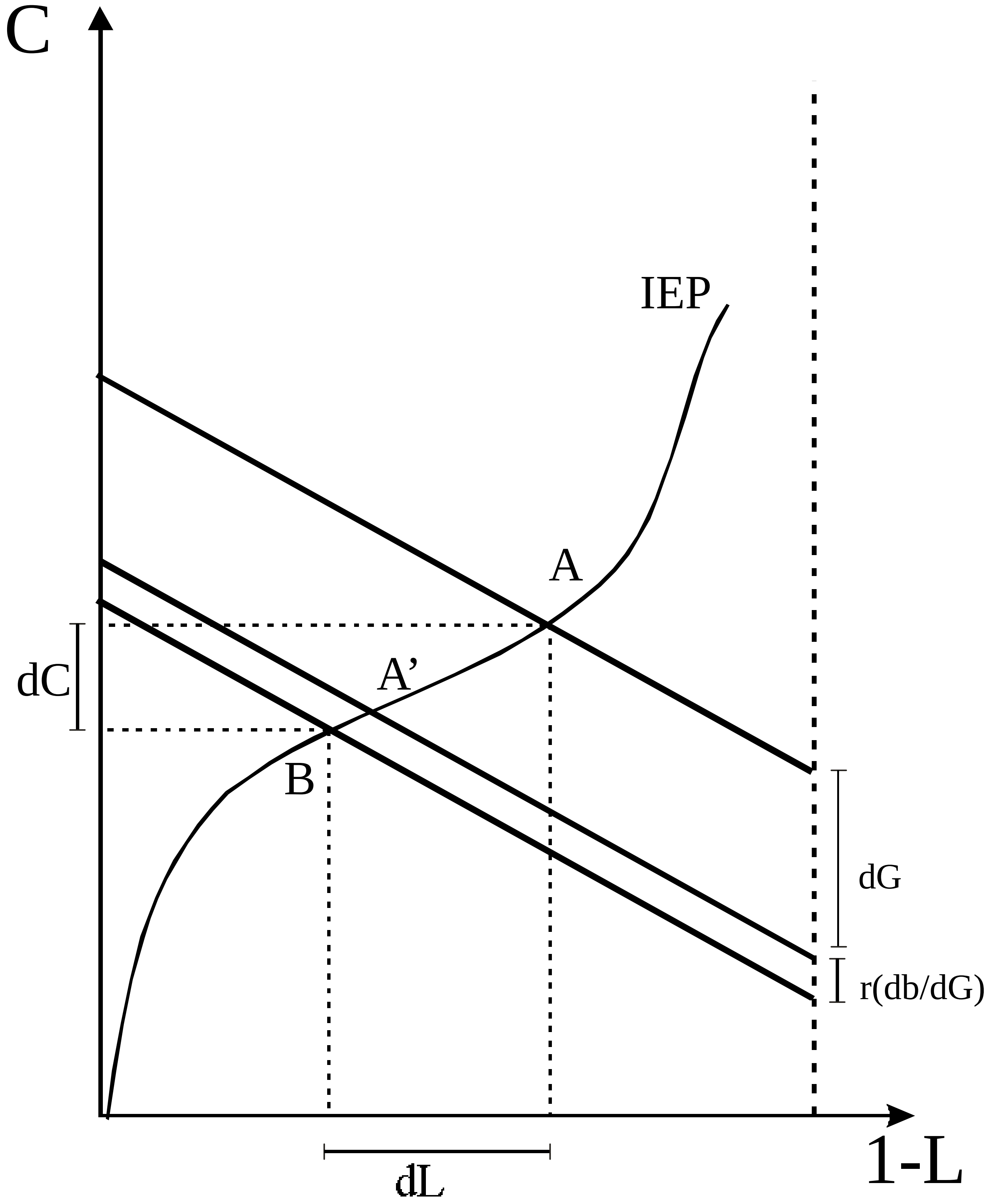

Figure 3: The Multiplier, Resource Withdrawal and Bond Effect.

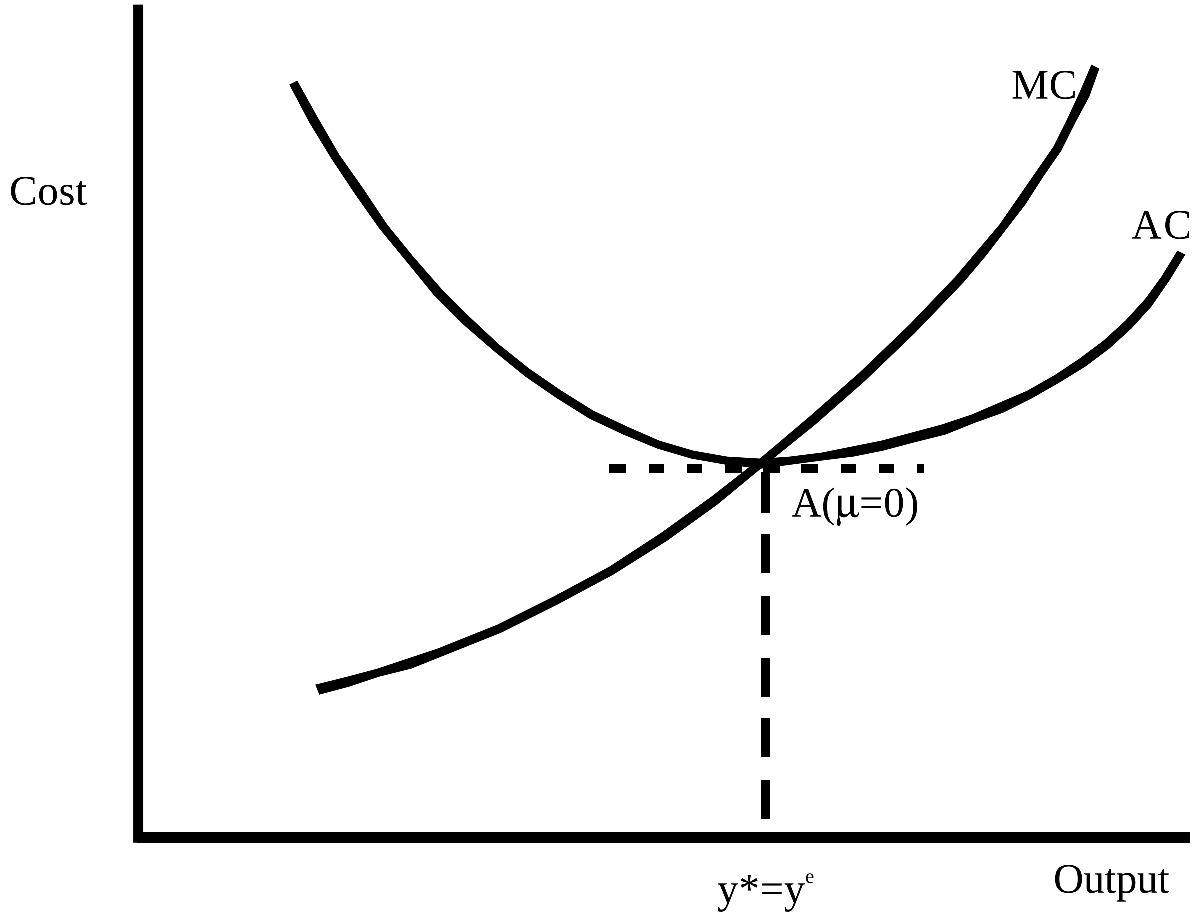

Fig 4(a) Capacity utilization and productivity when μ=0

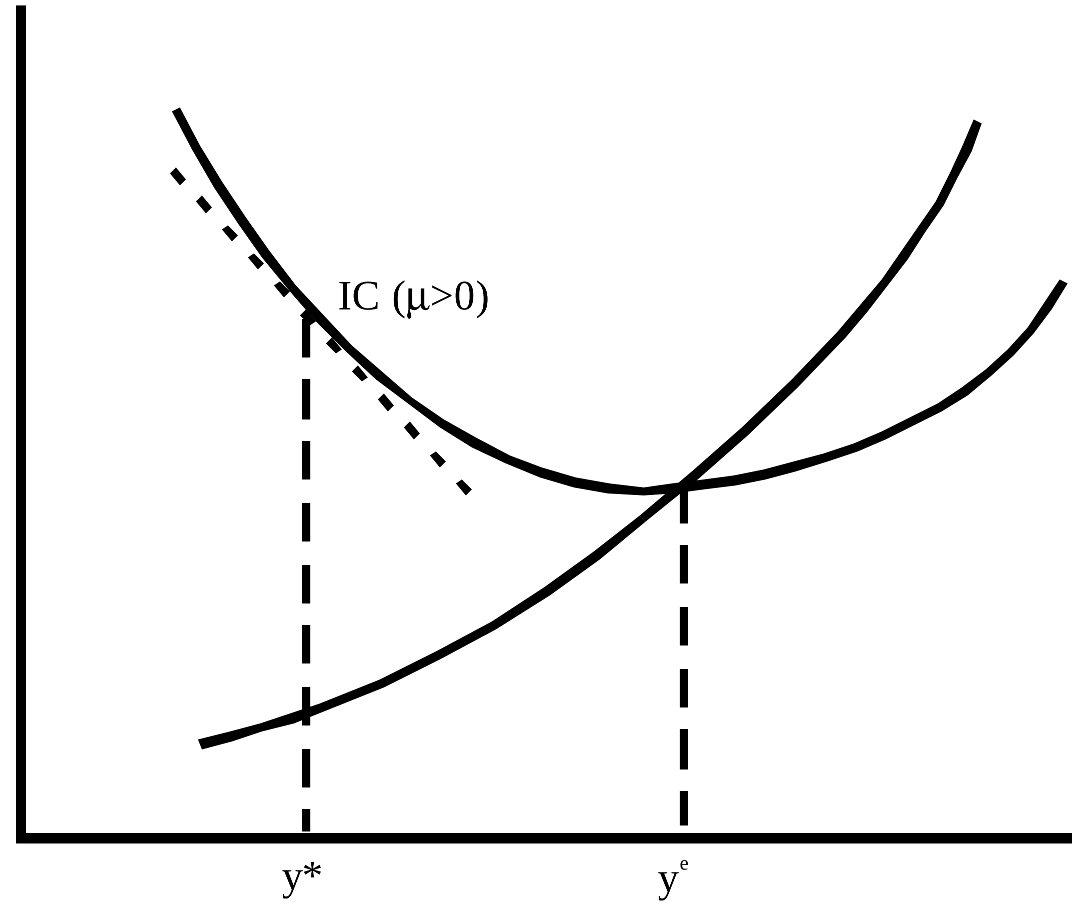

Fig 4(b) Capacity utilisation and productivity when μ>0

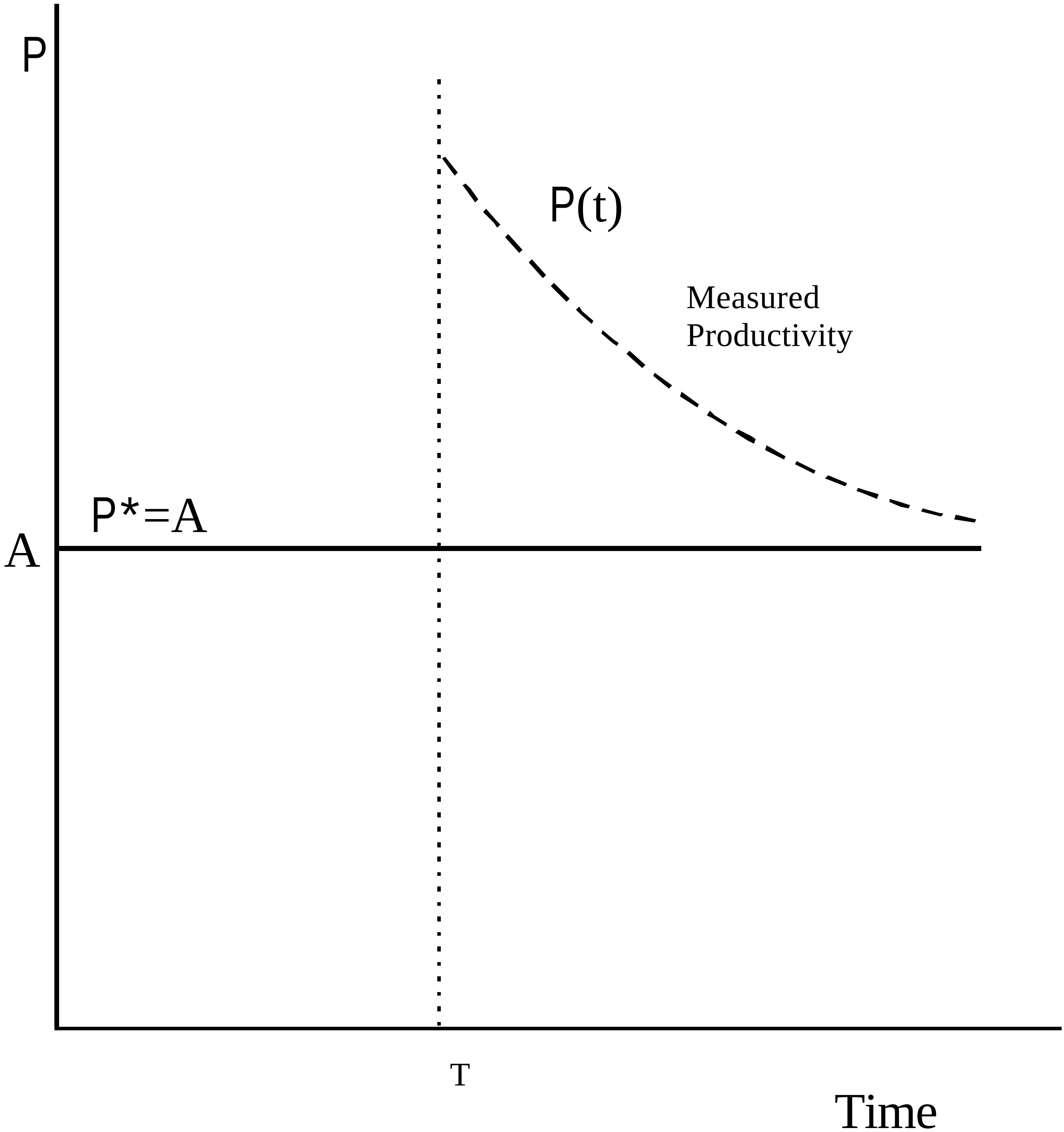

Figure 5: Measured productivity
After a demand shock at T.

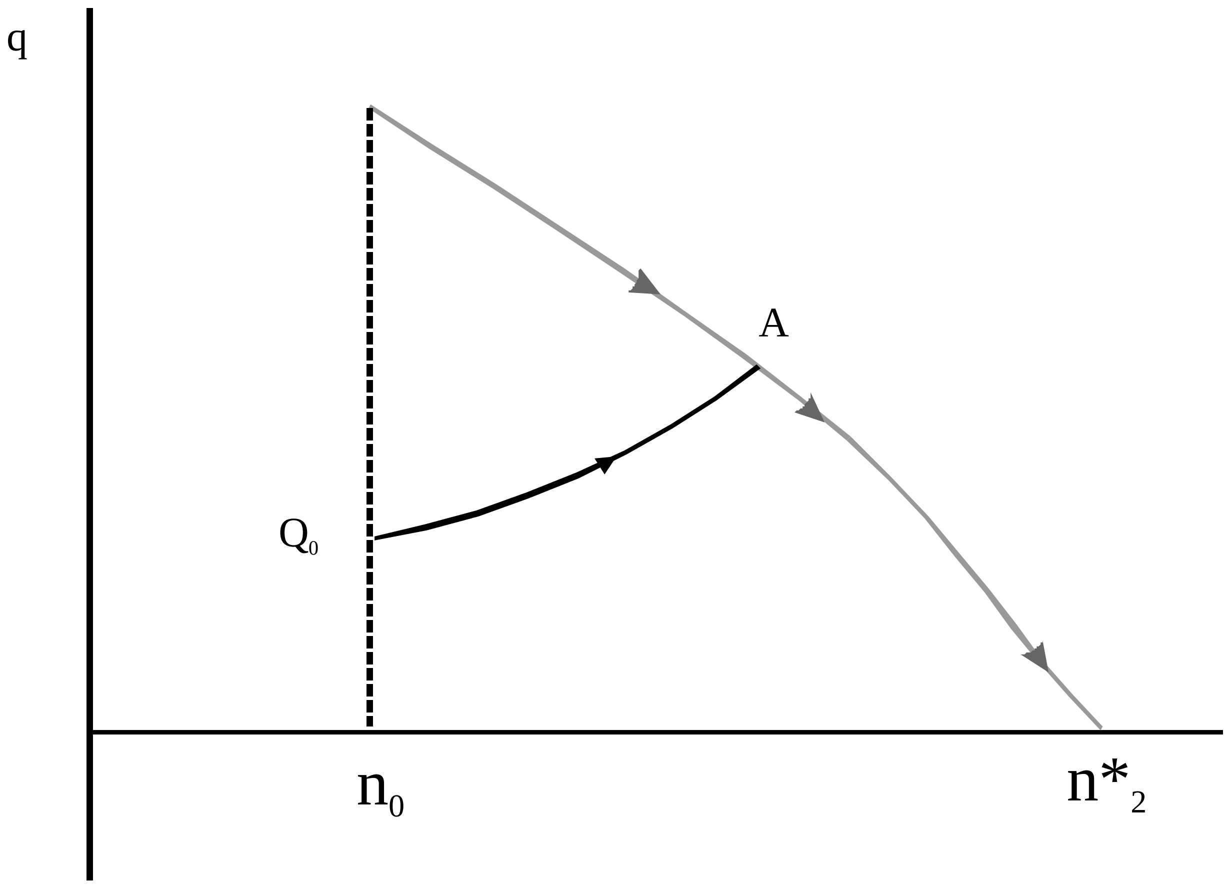

Figure 6: An anticipated Permanent Increase.

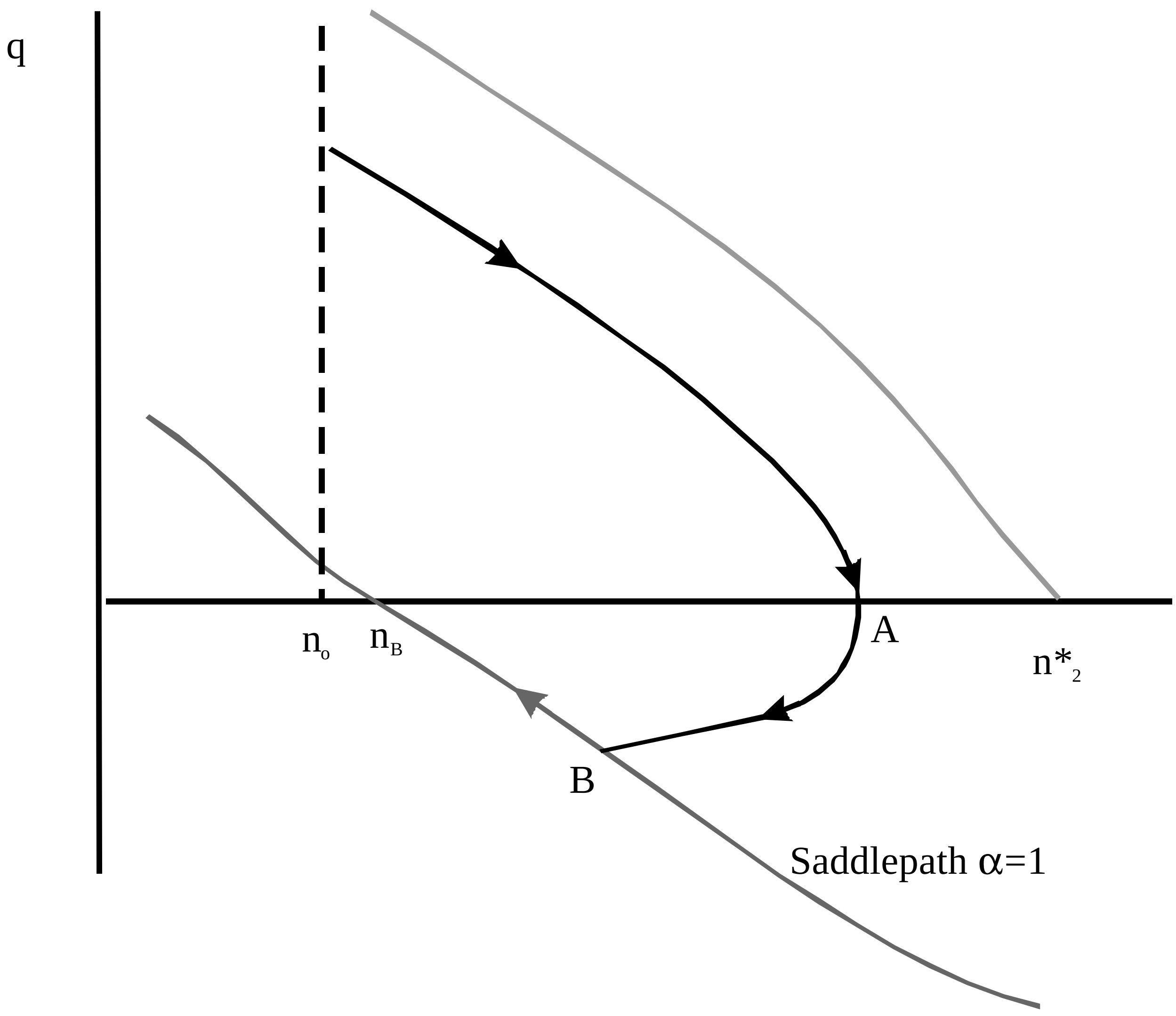

Figure 7: An unanticipated temporary change

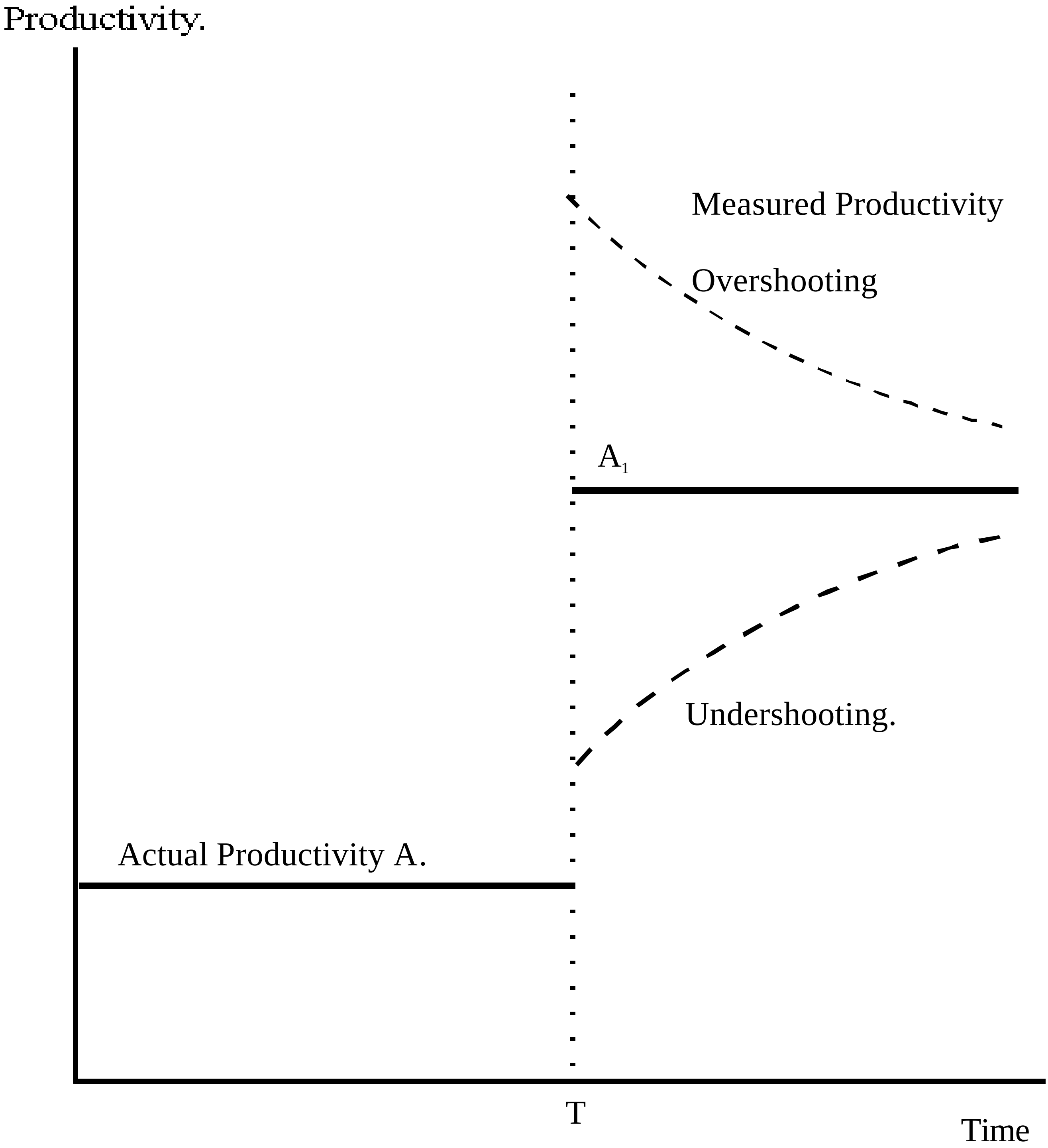

Figure 8: Undershooting and Overshooting of measured productivity: a permanent unanticipated technological improvement.

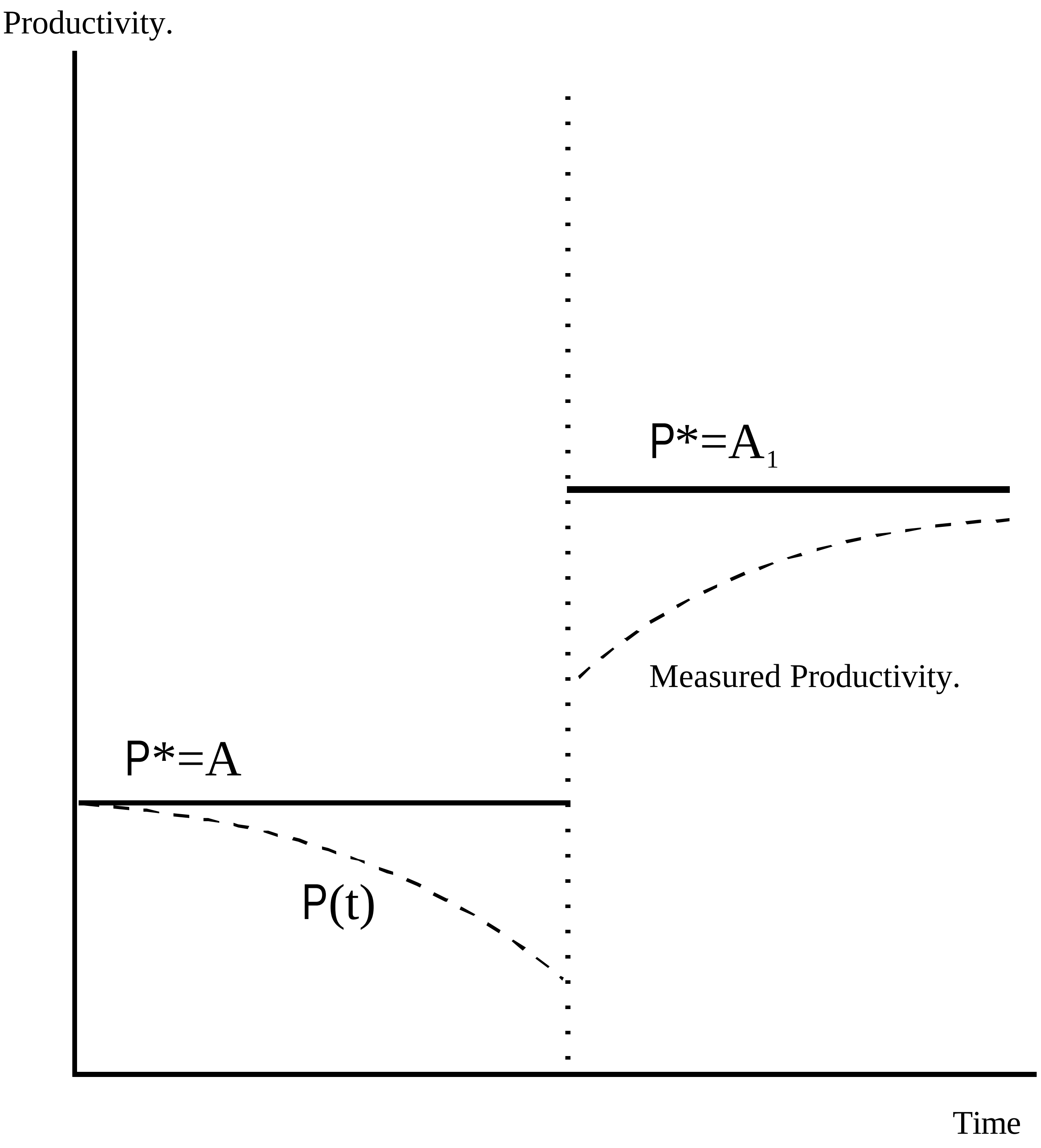

Figure 9: Measured productivity and technological change: an anticipated permanent advance in technology.