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by Mark A. Roberts

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Can the capital gains arising from an unfunded pensions reform make it Pareto-improving?[1]

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Abstract.

If unfunded pensions crowd-out private savings, pensions reform should raise the time path of capital. Even if reform has long-run benefits, there will still be a "double-burden" problem for a transitional generation. Assuming that there is an asset which discounts the present value of an income flow, which is positively related to the path of capital, then a surprise reform will generate unexpected capital gains. We consider whether the taxation of these extraordinary capital gains could raise enough revenue to fully compensate the transitional generation. An OLG model is presented with land, both as a store of value and a factor of production. In principal, the capital gains effect can be sufficient to generate windfall revenue for a Pareto-improving reform, but in practice, for plausible parameterisations, is that it will increase its likelihood by reinforcing other mechanisms.

1. Introduction.

[1] This paper is part of the research project, "Social security initiatives and economic growth" under the ***Evolving Macroeconomy Programme***, initated ESRC to which I am grateful for finance. I would also like the thank the Economic Policy Research Unit, Copenhagen, for hospitality and for further finance. I would like to thank Friedrich Breyer for comments on the present version of this particular paper. For comments on an earlier version, I would like to thank Spiros Bougheas, Richard Disney, Eric Fisher, Klaus Thustrup Kreiner, Karsten Staehr and the participants of seminars presented n at EPRU at Copenhagen University, *The Evolving Macroeconomy* Conference at the University of Warwick, the EEA conference at Geneva and the University of Konstanz for helpful comments without implicating them.

The recent demographics of ageing populations have shaken the fiscal foundations of state pensions schemes which operate on a "pay-as-you-go" (PAYG) basis, transferring resources from those currently active in the labour market to those retired from it. Apart from the issue of sustainability, life-cycle theory predicts that PAYG schemes depress savings and capital accumulation through a consumption smoothing response. This raises the question of pensions reform, which was first broached a quarter of a century ago by Feldstein (1975).

There are clear long-term advantages to moving from public PAYG to private funded pensions. Samuelson (1958) stated that the rate of return of PAYG is the same as the growth rate of the income tax base, which is equivalent to the underlying real growth rate. This is empirically much lower than the long-run rate of return on equity.[3]

A problem of transition remains, because while there are clearly long-run gains from abandoning PAYG, there are unavoidable short-run costs, which must be borne by a generation currently alive. Assuming that existing pension liabilities are honoured, this transitional generation must bear a "double burden" by financing pension payments both for current retirees through their labour taxes and for themselves through their own private savings, since they cannot expect to be requited in return with public transfers. The implication is that an existing PAYG pensions policy may be dynamically efficient in the sense that a reform would make at least one (transitional) generation worse-off.

In principle the problem may be resolved if pensions reform brings additional side benefits, which can be used to compensate the transitional generation. A number of papers have considered the fact that PAYG pensions are financed by taxes which invariably have a distortionary effect on labour supply. Against the result in Breyer (1989) that no reform can be Pareto-improving where labour supply is fixed, subsequent papers by Homburg (1990), Breyer and Straub (1993) and Raffelhueschen (1993) have showed that an improvement is possible with an endogenous labour supply. Demmel and Keuschnigg (2000) also show that a reform can also be Pareto-

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improving where employment is alternatively determined as the outcome of wage bargaining between firms and unions.

Another side benefit is considered by Belan, Michel and Pestieau (1998). Growth is growth is endogenous and technological knowledge is embodied in the capital stock, so that a reform of unfunded pensions will raise the growth rate.

A remaining problem is that while there may be significant side benefits, which are distributed over time, their current level may be insufficient to compensate the transitional generation. This may call for an intergenerational redistribution policy to complement the reform by running a government debt. Later generations would share some of the burden of honouring liabilities by paying higher taxes commensurate with the gains, which would arise over a longer horizon. Government debt, of course, leads to the crowding out of capital investment where finite-lived agents do not have Becker-altruistic preferences within an operative bequest equilibrium as in Barro (1974).

To summarise, a Pareto-improving reform may require two factors, additional side benefits to the reform and a complementary policy of intergenerational redistribution. In these two respects, the literature has focussed on the labour supply benefits from a reduction in distortionary taxation and on government debt as an intergenerational redistribution policy. This present paper presents an alternative model where the side benefit comes through the factor of land instead of labour. As land is an asset, the intergenerational distribution effect comes through the capitalization of future land rents in the current asset price.

Land is both a factor of production, used along with capital and labour, and a store of value which is traded between the generations. The store of value aspect holds because land rents are not taxed away at 100%. This gives rise to "non-productive savings" as in Tirole (1991). If there was a 100% rent tax, the asset price of land

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[3] Feldstein (1998) gives respective figures of 2.6% and 9.3% for the US.

would be zero, land would not be held as a store of value and economy would be dynamically efficient with no scope for a Pareto-improving pensions reform.

It might be argued that, in terms of this model, there would be no possibility of a Pareto-improving pensions reform if rents were taxed at 100%. This is correct, but given a starting position where rents are not fully taxed, it is never Pareto-improving to raise this tax, since individuals choosing to hold land would suffer a loss of both income and capital value. Consistency requires that if Pareto-efficiency is the criterion for pensions reform, it must also be used as a criterion for other policies. The criterion of Pareto-efficiency rules out the possibility of raising rental taxes and thus allows the other possibility of a pensions reform.

The reform of PAYG pensions will lead to increased savings and capital accumulation. The value of land rents, which is the marginal product of land, will then rise, if there are cross effects in the production function. Land is also an asset which is priced as the present discounted value of all future rents. A pensions reform will then cause capital gains in the price of land which may be taxed to reduce current level of labour taxation.

The side benefit to the reform is the capital gain on land from the rise in present value of the future land rents. An asset market for land with a capital gains tax substitutes for a government debt policy, since a forward-looking asset price brings forward the future gains of higher land rents into the present. Thus, an extraordinary capital gains tax on land at the moment of reform can kill both of these birds with one stone.

In a different model Belan, Michel and Pestieau (1998) argue that reform is a more a pretext than a requirement for internalizing an externality. It is certainly true in this present model, that the full taxation of land rents would remove any possibility of a Pareto-improving pensions reform. However, a separate policy to increase the taxation of land rents is not itself a Pareto-improving movement. The strict Paretian criterion points to a hands-off approach to land rents taxation, while allowing for the possibility of a pensions reform.

The paper is set up as follows. In Section 2, a version of the Diamond (1965) model is presented. The key feature is that young households save not only by holding deposits which are loaned to firms to raise physical capital but by buying land from old households. In Section 3 the model is solved in its steady-state. Section 4 considers the a PAYG reform enacted with a policy of taxing the extraordinary capital gains which arise from an unanticipated reform. It is shown that the double burden problem can be eliminated with the parameter values chosen. Section 5 extends the discussion somewhat and Section 6 provides a brief summary.

2. The model.

Households.

There is a representative household which buys a consumption good in each period of a two-period lifetime. The rate of time preference is $0 \le \theta < \infty$. It also supplies a fixed unit of labour without incurring a cost of effort. The utility function is:

$$V_t = \ln c_t^Y + \frac{1}{1+\theta} \ln c_{t+1}^O \tag{1}$$

where c_t^Y and c_{t+1}^O are the consumption levels of a household which is young at time *t* and old at time *t+1*. The budget constraint is:

$$c_t^Y + \frac{1}{1+r_{t+1}} c_{t+1}^O = x_t^Y + \frac{1}{1+r_{t+1}} x_{t+1}^O \tag{2}$$

where x_t^Y and x_{t+1}^O are the respective income levels at *t* and *t+1*.

The income taxes of the young workers are paid to finance pay-as-you-go pensions payments, b_t, to those who are currently old. There is no population growth and everyone lives for a full two periods, so the ratio of young to old is unity. Where x_t is the disposable income paid out at time t, the PAYG pensions system is defined by the rule:

$$x_t^Y = (1-t_t) w_t, \qquad x_{t+1}^O = t_{t+1} w_{t+1} \tag{3}$$

There is no inheritance or bequest income. Savings by individual i, s_t^i, may be held in two assets, fixed price deposits, a_t^i, and land, q_t^i, which has the variable price, v_t. [4] In aggregate, savings are

$$s_t = a_t + q_t v_t$$

where $s_t = \int_0^1 s_t^i di$, $\quad a_t = \int_0^1 a_t^i di$, $\quad q_t = \int_0^1 q_t^i di$ (4)

Land yields the rental income, z_t, and also a capital gain/loss where , $v_{t+1} \neq v_t$. It is purchased by the young from the old, who have already received the current rent, and who then hold it for one period before they sell it to the next generation of young. We allow for a possible capital gains tax on land at the rate, τ_{t+1}, where $v_{t+1} - v_t > 0$, and a rental tax at the rate, $\tilde{\tau}_{t+1}$, which gives second-period asset income as:

$$(1 + r_{t+1})a_t^i + \left((1 - \tilde{\tau}_{t+1})z_{t+1}/q_t + v_{t+1} - \tau_{t+1}(v_{t+1} - v_t)\right)q_t^i$$

$(1 - \tilde{\tau}_{t+1})z_{t+1}/q_t$ is total net rent payments per land holdings paid out at time *t+1*.

All second period income is consumed, as there no bequests. It comprises asset income plus potential income from a PAYG pension.[5] Using equation (4) and where $v_{t+1} - v_t > 0$, second-period consumption is given by:

$$c_{t+1}^O = (1 + r_{t+1})s_t^i + q_t^i\left(v_{t+1} - \tau_{t+1}(v_{t+1} - v_t) + (1 - \tilde{\tau}_{t+1})z_{t+1}/q_t - (1 + r_{t+1})v_t\right) + \tau_{t+1}w_{t+1}$$

(5)

Maximization of (1) subject to (2)-(5) with respect to total savings, the "consumption-wealth" decision, and land, the "portfolio decision", gives two first-order conditions:

$$c_{t+1}^O = \left(\frac{1 + r_{t+1}}{1 + \theta}\right)c_t^Y \tag{6}$$

$$v_t = \frac{\left((1 - \tilde{\tau}_{t+1})z_{t+1}/q_t + (1 - \tau_{t+1})v_{t+1}\right)}{1 + r_{t+1} - \tau_{t+1}} \tag{7}$$

[4] Generally, there may also be fixed price government debt which would accumulate at the same interest rate as deposits, r_{t+1}.

[5] Savings would be affected by any unexpected element of the capital gain which is not taxed away.

Equation (6) is the Euler equation which gives the time profile of individual consumption and equation (7) is the (risk-neutral) no arbitrage condition between the assets of land and deposits.

The land price *per unit* is solved from equation (7) as:

$$v_t = \frac{1}{(R_{t+1} - \tau_{t+1})}(1-\tilde{\tau}_{t+1})z_{t+1} + \frac{1-\tau_{t+1}}{(R_{t+1} - \tau_{t+1})(R_{t+2} - \tau_{t+2})}(1-\tilde{\tau}_{t+2})z_{t+2} + \frac{(1-\tau_{t+1})(1-\tau_{t+2})}{(R_{t+1} - \tau_{t+1})(R_{t+2} - \tau_{t+2})(R_{t+3} - \tau_{t+3})}(1-\tilde{\tau}_{t+3})z_{t+3} + .. , \text{ where } R_T \equiv 1 + r_T \quad (8)$$

The household savings function is solved as:

$$s_t^i \equiv a_t + v_t q_t = \frac{1}{2+\theta}\left((1-t_t)w_t - \frac{1+n}{1+r_{t+1}} t_{t+1} w_{t+1}\right) \quad (9)$$

From the point of the household saver, land and deposits are perfect substitutes, but only deposits get channelled through by financial intermediaries to firms which invest in physical capital. Thus "investment" in land leads to the crowding-out of productive investment. Assuming 100% depreciation, capital accumulation is equal to deposit savings:

$$k_{t+1} = a_t = s_t - q_t v_t = \frac{1}{2+\theta}\left((1-t_t)m_t w_t - \left(\frac{1}{1+r_{t+1}}\right) t_{t+1} m_{t+1} w_{t+1}\right) - q_t v_t \quad (10)$$

Production.

Each firm has a Cobb-Douglas production function and operates under constant internal returns to scale in three factors, land, q_t, capital, k_t, and labour, l_t[6]:

$$y_t = A q_t^{1-\alpha-\beta} k_t^{\alpha} l_t^{\beta} \quad (11)$$

The profits equation:

$$\pi_t = A q_t^{1-\alpha-\beta} k_t^{\alpha} l_t^{\beta} - z_t q_t - r_t k_t - w_t l_t$$

is maximized with respect to two factors, land and capital:

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[6] Rhee (1991) shows that dynamic inefficiency is not a possibility with a Cobb-Douglas and, hence, constant factor share production function.

$$\frac{\partial \pi_t}{\partial q_t} = (1-\alpha-\beta)Aq^{-\alpha-\beta}k_t^{\alpha}l_t^{\beta} - z_t = 0 \tag{12}$$

$$\frac{\partial \pi_t}{\partial k_t} = \alpha A q^{1-\alpha-\beta}k_t^{\alpha-1}l_t^{\beta} - r_t = 0 \tag{13}$$

There is a free-entry, zero profits condition:

$$\pi_t = 0 \tag{14}$$

Equations (12)-(14) determine the three factor demands. The aggregate supply of land is assumed to be fixed. Moreover, we also assume that there is no population growth, so that the aggregate supply labour is also exogenous. This implies that there the will be no side effects on labour supply emanating from a change in either the level or structure of taxation which would accompany pensions reform.

Using the following normalizations,

$$l_t = m_t = m = 1, \qquad q_t = q = 1 \tag{15}$$

gives the following respective solutions for the rental value of land, the interest rate and the wage:

$$z_t = (1-\alpha-\beta)Ak_t^{\alpha} \tag{16}$$

$$r_t = \alpha A k_t^{\alpha-1} \tag{17}$$

$$w_t = \beta A k_t^{\alpha} \tag{18}$$

The convenient assumption of no population growth is made to obtain a steady-state solution for the level of capital alongside the two other assumptions of constant returns to scale in all factors and of a fixed aggregate supply of land. The positive solution value for the interest rate and the steady-state solution for the level (not the growth rate of) the capital stock and, thus, the labour income base imply that the market rate of return exceeds the (zero) implicit rate of return on social security, so that reform is of undisputed long term benefit.

Substituting the solutions for the wage and interest rate into (10) gives:

$$k_{t+1} \equiv \left(\frac{1}{2+\theta} \left((1-t_t)A\beta k_t^{\alpha} - \left(\frac{1+n}{1+\alpha A k_{t+1}^{\alpha-1}} \right) t_{t+1} A\beta k_{t+1}^{\alpha} \right) \right) - v_t \tag{19}$$

It is also apparent that increases in the price of land will lead to the crowding-out of the capital stock. Recessions may arise from the possibility of short-lived property speculation. [7]

3. The steady-state.

Solution.

Equations (7), (16) and (17) show that in the steady-state the price of land is given by:

$$v_t = \phi k_{t+1}, \qquad \text{where } \phi \equiv \left(\frac{1-\alpha-\beta}{\alpha}\right) \tag{20}$$

It is shown later in the paper that the value of the parameter, ϕ, the land-capital ratio, is important for the possibility of a Pareto-improving reform of PAYG pensions.

Given that there will be no capital gains in the steady-state and that wages are constant, because of the convergence property due to decreasing returns in the capital stock, budgetary balance requires constant labour income taxes, $t = t_{t+i} \ \forall i$. We also assume that there are no rental taxes, $\tilde{\tau}_{t+i} = 0 \ \forall i$.

The steady-state for capital is then solved as a quadratic:

$$k^{1-\alpha} = \frac{\alpha A}{2}\left(\frac{\beta(1-t)}{(1-\beta)(2+\theta)} - 1 + \left(\left(\frac{\beta(1-t)}{(1-\beta)(2+\theta)}+1\right)^2 - \frac{4\beta t}{(1-\beta)(2+\theta)}\right)^{\frac{1}{2}}\right) \tag{21}$$

Only the positive rooted solution is considered to ensure both the non-negativity of the capital stock and plausible comparative static properties like an inverse relationship between the capital stock and the level of PAYG pensions.

Simulated values.

We assume the following parameter values: $\beta = 0.66$, $t = 20\%$ and $\theta = 2$. The labour share parameter, β, is set at its generally assigned value; the tax/pension

[7] Furthermore, the land price equation (7) admits bubbles which, in expectation, with a constant interest rate will grow exponentially at the rate $(1+r-t)/(1-t)$. The non-negativity of the capital stock requires that any positive land bubbles are short-lived. We could use the same argument as in Diba and Grossman (1988) to rule out short-lived probabilistic bubbles.

parameter, *t*, is initially set at a high level and the rate of time preference has been chosen to obtain an empirically plausible value for the interest rate, r_0, of 3.082. This translates into an annualized rate of 4.10%, where a half-life is assumed to last 50% of three-score years and ten.[8]

We consider the effect on the steady-state of abandoning PAYG social security by reducing the tax/pensions rate from 20% to zero. The initial steady-state capital stock (where $t = 20\%$) is normalized at unity: $k_0 = 1$. The new steady-state capital stock, $\tilde{k}$, with zero taxes and expenditure on PAYG pensions, $t = 0$, depends on the choice of the value for the capital share, α, which, given the normalization, $k_0 = 1$, implies a value for total factor productivity, A. We try $\alpha = 0.25$ and find that $\tilde{k} = 1.78$, roughly giving a three-quarters increase in the capital stock from the one steady-state to the other. This magnitude of change is fairly robust to variations in the value of α, since if $\alpha = 0.20$, $\tilde{k} = 1.72$, while if $\alpha = 0.1\dot{6}$, $\tilde{k} = 1.68$. The new steady-state interest rate, $\tilde{r}$, only depends on two parameters, β and θ. As $\tilde{r} = ((2+\theta)(1-\beta)/\beta)$, this is always equal to 2, which is at the annualized rate of 3.19%.[9] Thus, the reform of this PAYG pensions system with sizeable PAYG payments leads to a reduction in the annualized interest rate by almost a full percentage point.

4. A Pareto-improving reform?

There are undisputed long-run benefits to reform since the implicit rate of return on PAYG in the steady-state without population growth is zero while the rate of return on private savings is at the positive rate of interest. In addition reform will raise steady-state wages by increasing the capital stock. The question is, can the combination of PAYG reform and a complementary capital gains tax policy be Pareto-improving?

[8] Over a thirty-five year half-life, $(1+3.082)^{1/35} - 1 = 4.10\%$

[9] Over the same half-life, $(1+2)^{1/35} - 1 = 3.19\%$

We consider a reform from the starting position of the old steady-state where $t = 20\%$ and $k_0 = 1$. The conditions for reform to be Pareto-improving are:

$$t_0 w_0 \le b_t \tag{22}$$

$$(1-t_0)w_0 + \frac{1}{R_0} t_0 w_0 \le (1-t_T)w_T + \frac{1}{R_{T+1}} t_{T+1} w_{T+1} \qquad \forall T \ge t \tag{23}$$

where b_t is pension payments to retirees at time *t*.

These conditions state that for each generation the level of wealth with the reform must be at least as great as it would have been without the reform - in the old steady-state.

Pensions benefit and tax specification of the reform:

We make the four assumptions, which follow below as (A1)-(A4), to define a particular reform. These assumptions cover (i) the payment of existing pension liabilities, (ii) a labour income taxation policy, (iii) a capital gains taxation policy and (iv) a (no-) government debt policy.

First, it is assumed that existing pension liabilities to the old who are currently alive are just honoured, so that condition (22) is satisfied as an equality, and that there are no further pensions payments to future generations of retirees:

$$b_t = t_0 w_0, \qquad b_T = 0, \quad \forall T \ge t+1 \tag{A1}$$

Concerning labour taxation, the generation currently working at time *t* has to pay it the rate, t_C, without receiving a retirement pension. All future generations neither pay labour income taxes not receive retirement pensions. Labour income taxation policy is defined as:

$$t_t = t_C; \qquad t_T = 0, \quad \forall T \ge t+1 \tag{A2}$$

We also assume that capital gains on land at the time of the reform are taxed at the rate 100% and that, thereafter, there are no capital gains taxes:

$$\tau_t = 1; \qquad \tau_T = 0, \quad \forall T \ge t+1 \tag{A3}$$

Note that the 100% capital gains tax at time *t* does not only ensure maximum revenue from this source; it also ensures that there is no revision of household saving emanating from the unanticipated capital gain, as all of this is taxed away.

Finally, the labour income tax at time *t*, t_C, is set such that there is no government deficit at time *t*:

$$t_0 w_0 - t_C w_0 - (v_t - v_0) = 0 \tag{A4}$$

There are no later government deficits by the assumptions (A1)-(A4). As there is no initial government debt, the zero deficit assumption implies that there is no subsequent government debt.

Solution.

Assumptions (A2)-(A4) imply that capital accumulation over *t, t+1*, is given by:

$$k_{t+1} = \left(\frac{(1-t_C)}{2+\theta}\right) w_0 - v_0 \tag{24}$$

and beyond that:

$$k_{T+1} = \frac{1}{2+\theta}\left(\beta A k_T{}^{\alpha}\right) - v_T, \qquad \forall T \geq t+1 \tag{25}$$

and the price of land by

$$v_T = (1-\alpha-\beta)\left(\frac{1}{R_{t+1}} k_{T+1} + \frac{1}{R_{t+1}R_{t+2}} k_{T+2} + \frac{1}{R_{t+1}R_{t+2}R_{t+3}} k_{T+3} + ..\right) \tag{26}$$

It is necessary to solve the price of land during the transition to the new steady-state. The model is too nonlinear for an explicit solution, so we use the following approximation:

$$R_T \equiv 1 + r_T \approx (1+\bar{r}^{-1}) r_T, \qquad \forall T \tag{27}$$

This allows a solution for the transitional dynamics of the form,

$$k_{T+1} = G k_T{}^{\alpha} \tag{28}$$

where G can be solved as an undetermined coefficient within the approximation used in equation (27).

The undetermined coefficient solution method is now used to solve G. Equations (26)-(28) imply:

$$v_T = \frac{(1-\alpha-\beta)}{\alpha(1+\bar{r}^{-1})}\left(1 + \frac{G}{\alpha(1+\bar{r}^{-1})A} + \frac{G^2}{\alpha^2(1+\bar{r}^{-1})^2 A^2} + ..\right) k_{T+1} \qquad \text{or}$$

$$v_T = \left(\frac{1-\alpha-\beta}{\alpha(1+\bar{r}^{-1})-G/A}\right)k_{T+1} \tag{29}$$

which is substituted back into equation (25) to give:

$$k_{T+1} = \frac{1}{2+\theta}\left(\beta A k_T{}^{\alpha}\right) - \left(\frac{1-\alpha-\beta}{\alpha(1+\bar{r}^{-1})-G/A}\right)k_{T+1}$$

Rearranging and imposing consistency with equation (27) gives the solution:

$$G = A\left(\frac{X}{2} - \left(\left(\frac{X}{2}\right)^2 - Y\right)^{\frac{1}{2}}\right)$$

$$\text{where} \quad X \equiv 1+\alpha\bar{r}^{-1} - \left(\frac{1+\theta}{2+\theta}\right)\beta, \qquad Y \equiv \frac{\alpha\beta(1+\bar{r}^{-1})}{2+\theta} \tag{30}$$

Equation (29) where $T = t$ and equation (24) gives

$$v_t = \left(\frac{1-\alpha-\beta}{\alpha(1+\bar{r}^{-1})-G/A}\right)\left(\left(\frac{(1-t_C)w_0}{2+\theta}\right) - v_0\right) \tag{31}$$

which is substituted into the balanced budget assumption (A4) to give:

$$(t_0 - t_C)w_0 = \left(\frac{1-\alpha-\beta}{\alpha(1+\bar{r}^{-1})-G/A}\right)\left(\left(\frac{(1-t_C)w_0}{2+\theta}\right) - v_0\right) - v_0$$

This can be rearranged in terms of the current tax rate:

$$t_C = \frac{\left(t_0 + v_0/w_0\right)\left(\alpha(1+\bar{r}^{-1}) - G/A\right) - (1-\alpha-\beta)\left(\left(1/(2+\theta)\right) - v_0/w_0\right)}{\left(\alpha(1+\bar{r}^{-1}) - G/A\right) - \left((1-\alpha-\beta)/(2+\theta)\right)} \tag{32}$$

Equation (32) pins down the current labour income tax rate that satisfies the government budget equation, given the above assumptions, (A1)-(A4), on pension payments, labour taxes, capital gains taxes and debt/deficit policy.

Assumption (A1) and the fact that the wage at time *t* is predetermined with the capital stock, $w_t = w_0$, together imply that the condition for the Pareto-improvement in (23) requires that at time *t*:

$$(1-t_0)w_0 + \frac{1}{R_0}t_0 w_0 \le (1-t_C)w_0 \qquad \text{or} \qquad \left(\frac{r_0}{1+r_0}\right)t_0 \ge t_C \tag{33}$$

Assumption (A2) and condition (23) imply that, thereafter, the following condition is required:

$$(1-t_0)w_0 + \frac{1}{R_0}t_0 w_0 \leq w_T, \quad t+1 \leq T \quad \text{or}$$

$$1-\left(\frac{w_T}{w_0}\right) = 1-\left(\frac{k_T}{k_0}\right)^{\alpha} \leq \left(\frac{1}{1+r_0}\right)t_0, \quad t+1 \leq T \qquad (34)$$

Condition (33) states that the transitional generation is not made worse-off. This is also necessary for condition (34) that no subsequent generation is made worse-off, which requires only that the capital stock never falls below its initial value, $k_T \geq k_0$, $t+1 \leq T$. A higher utility level for the transitional generation requires they have a higher level of disposable income, which leads to higher savings and capital accumulation for all subsequent generations.

In particular, we show that condition (34) where $t+1 = T$ depends on condition (33). The requirement here is that $k_{t+1} > k_0$. The capital accumulation equation (10) where the capital stock is in the old steady-state is given by:

$$k_0 = \left(\frac{(1-t_0) - (1/(1+r_0))t_0}{2+\theta}\right)w_0 - v_0$$

Subtraction from equation (24) gives:

$$k_{t+1} - k_0 = \left(\frac{(1+1/(1+r_0))t_0 - t_C}{2+\theta}\right)w_0$$

Condition (33) implies that the right-hand-side is positive and, thus, $k_{t+1} > k_0$.

There should be no subsequently drop in the capital stock, because no generation from *t+1* onwards will pay labour income taxes. This implies that subsequent levels of households savings will be even higher. Furthermore, there will be no rise in the land price which is so high as to reverse the beneficial effect of savings on capital accumulation, since the price of land will already have discounted most of the future capital accumulation effects at time *t*. This allows us to focus exclusively on condition (33) as the condition for a Pareto-improving reform.

Simulated values.

We return to the values chosen in the previous section. An initial steady-state with an interest rate of 3.082, annualized to 4.10%, and a tax rate of 20% imply that the critical value for the new tax rate, t_C, is 15.1% from equation (34). If this is at least as great as the tax rate which will balance the budget in equation (32), then the reform is Pareto-improving.

We consider three different values of α and, by implication, three values for the steady-state land-capital ratio, ϕ, for a given labour share, β, at $0.\dot{6}$.

(i) The first case is where $\alpha = 0.25$, so that $\phi = 0.\dot{3}$. After substituting these values into equation (32), it can be shown that the labour income tax rate which balances the budget is 18%, which would make the current generation worse-off (as 18%>15.1%). (ii) Trying the case $\alpha = 0.20$, so $\phi = 0.\dot{6}$, reduces the budget balancing tax rate to 16%, but the policy is still not Pareto-improving (as 16%>15.1%). (iii) Finally, the value $\alpha = 0.1\dot{6}$, so $\phi = 1$, is more than sufficient because the budget balancing tax rate falls to 13.2% (as 13.2%<15.1%).

Small values of α are required because they imply large values of ϕ, the steady-state land price/capital ratio. Reform will be Pareto-improving if the capital gain is sufficiently large, which requires the steady-state ratio of the value of land to capital is sufficiently high.

5. Further considerations.

It has been shown, in principle at least, that broadening the basic overlapping generations model to include land may, with a capital gains tax on land prices, solve the transition problem of reforming PAYG pensions system. Arguably, the capital gains effect may in practise be too small to bring about a Pareto-improving reform. Although there may be little doubt about this, we suggest that the effect of capital gains on land may be of some worth in complementing the labour supply effects, which are more often considered.

In fact, as well as an additional effect from capital gains there may also be a useful synergy working between the capital gain and the labour supply effects. In a broader model embracing an endogenous labour supply, the increase in employment through a reduction in distortionary taxation may cause even larger capital gains through a cross effect between land and labour in the production function. Secondly, the replacement of distortionary taxes on labour by non-distortionary taxes on capital gains will lead to an even greater labour supply response.

We have introduced a forward-looking asset price as a means of bringing forward the long-run future gains to the transitional generation currently alive. One possibility would be to consider alternative forms of asset. There is a problem, however, with the most likely candidate of equity capital, because the equity price of the marginal firm will always equal an undetermined entry cost.[10] In the basic and standard case of zero entry costs, land alone is the factor earning a "rent" and the only asset generating taxable capital gains. Prospective capital gains to capital would immediately be consumed by new entry, although there may be other external benefits in an increase in the number of firms.

We have considered the case where reform comes as a complete surprize, so that there is a single unanticipated transition from a pre-reform tax rate of $\tau_T = \tau > 0$, $\forall T \geq t+1$, to a post-reform tax rate of $\tau_T = 0$, $\forall T \geq t+1$. If there are prior beliefs of reform but with uncertainty, the price of land will depend on the probability distribution of each τ_T, $\forall T \geq t+1$. Prior to the reform, the first moments of the distributions will generally satisfy $0 \leq E(\tau_T) \leq \tau$, $\forall T \geq t+1$, instead of $E(\tau_T) = \tau$, $\forall T \geq t+1$. The effect of uncertainty on these first-moments will be to reduce the value of the post-reform capital gain.

There is some ambiguity in the overall effects of uncertainty, because with uncertainty the higher moments of the τ_T-distributions may have an increasing effect on the capital gain. The effect of uncertainty in raising precautionary savings may raise the price of land through increased capital accumulation. Furthermore, land becomes a risky-asset and one with a price which is *negatively* correlated with PAYG pensions payments. This suggests that the demand for land may rise through the a portfolio effect intended to reduce the overall variance on second-period income, which comprises the return on deposits, and the total return on land and PAYG pensions.

Uncertainty muddies the waters also because there may in fact be *two* transitions. The first is from an initial pre-reform state where reform is not even contemplated to an intermediate state where reform is probabilistic. The second is from this intermediate probabilistic state to a final state where reform is actually enacted. The analysis of this paper has condensed these two transitions into one, eliminating the intermediate state.

6. Concluding comments.

A model has been presented where land is both a fixed factor of production in aggregate and an asset which is traded between the generations for its store of value property. As usual, a reform of state-run PAYG pensions will raise savings and capital accumulation. This in turn increases land rents, because of cross effects in the production function. The asset price of land then rises as it is determined as the present discounted value of all future land rents. An unanticipated pensions reform will lead to immediate capital gains which may then be taxed to reduce the level of current labour taxation in order to compensate the transitional generation.

It is important to highlight the fact that labour supply is fixed in this model. As such, it is unaffected by changes in either the level or the structure of taxation. It is rather the feature that land, which yields an economic rent, is held as an asset which is the driving force behind the model. Pensions reform causes (financial) capital gains on land from the standard (physical) capital accumulation effect and cross-effects in the production function. These gains may be taxed (in any way) to reduce taxes (of any kind) on labour income. We have assumed proportional taxes on labour income for plausibility and a once-off 100% tax on capital gains for analytical tractability.

[10] This is with perfect financial market where the prospective firm can borrow against future profits at a common market rate of interest.

In principle, for certain parameterizations, the tax revenue collected from the extraordinary capital gains may be sufficiently high to facilitate a Pareto-improving reform. In practise, it is probably more realistic to believe that the capital gains tax effect would be less generous. At worse, the effect would augment and reinforce the studied labour supply responses, thus tipping the balance further in favour of the possibility of Pareto-improving reform.

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Appendix.

It can be shown that the absence of a rental tax is dynamically efficient. We consider the effect if introducing a steady-state rental tax, $\tilde{\tau}_{t+i} = \tilde{\tau} \quad \forall i$. The new steady-state land price is given by:

$$v = (1-\tilde{\tau})\phi k \text{, where } \phi \equiv \left(\frac{1-\alpha-\beta}{\alpha}\right) \qquad (20A)$$

There are two effects:

$$\frac{\partial v}{\partial \tilde{\tau}} = -\left(1 - \frac{(1-\tilde{\tau})}{k}) \frac{\partial k}{\partial \tilde{\tau}}\right)\phi k = -\left(1-(1-\tilde{\tau})\frac{\partial \ln k}{\partial \tilde{\tau}}\right)\phi k$$

We consider the case with no payroll taxes and social security, that is where the capital stock and, hence, its derivative, $\frac{\partial k}{\partial \tilde{\tau}}$, will be highest. This gives the steady-state capital stock as

$$k = \left(\frac{\alpha\beta A}{\left(1-\beta-\tilde{\tau}(1-\alpha-\beta)\right)(2+\theta)}\right)^{\frac{1}{1-\alpha}} \qquad (21A)$$

$$sign\left(\frac{\partial v}{\partial \tau}\right) = sign\left(\frac{1-\alpha-\beta}{(1-\alpha)\left(1-\beta-\tilde{\tau}(1-\alpha-\beta)\right)} - 1\right)$$

The most favourable case for a positive sign is where $\tilde{\tau} = 0$; there

$$sign\left(\frac{\partial v}{\partial \tau}\right) < 0$$