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Can pay-as-you-go pensions raise the capital stock?[1]

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Abstract.
We reconsider pay-as-you-go pensions (PAYG) policy in a version of the Diamond (1965) overlapping generations model with an imperfectly competitive financial sector and with a low rate of tax on its profits. PAYG then has two effects on capital: the well-known negative crowding-out effect in displacing savings and a positive effect in reducing a second form of crowding-out caused by the displacement of "productive savings" in capital by "non-productive savings" in financial sector equity. These two countervailing effects may generate a hump-shaped steady-state relationship between the PAYG contribution rate and the capital stock. If the overriding goal of policy is to raise the level of economic activity, a radical overhaul of PAYG pensions may be ineffective or even counter-productive unless financial sector profits or dividends are taxed at a high rate. If this secondary policy is infeasible, the scale of any proposed reform of PAYG should be measured against the degree of competition in the financial sector.

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1. Introduction.

The reform of unfunded, pay-as-you-go (PAYG), pensions systems is of topical concern, largely because of the problem of fiscal sustainability caused by the demographic change of an ageing population, which places an increasing burden on younger tax payers. In addition, there are the known long-run financial benefits of reform where the market rate of return on savings exceeds the implicit rate of return on PAYG, which Samuelson (1958) first pointed out to be the growth rate of the tax base. Breyer (1989), however, shows that Pareto-losses are intrinsic to any reform of unfunded pensions, because there has to be a generation, which has contributed into a fund *pre-reform* and which will be unable to draw out from another fund *post-reform*.[3] These losses are also likely to raise political barriers to reform, especially in democratic systems where median voters bear the burden of transition.

Aside from these sustainability and relative rate of return issues, a prediction of standard life-cycle models is that PAYG pensions crowd-out private household savings and capital through consumption smoothing effects.[4] The crowding-out effect of PAYG, as an elemental feature of the analysis, has some bearing upon the issues of sustainability and relative rate of return. It has also been stated in its own right as an argument in favour of reform, namely by Feldstein (1995), which is valid whenever the level of economic activity is another policy objective.

This present paper is not concerned with the questions of whether reform is either necessary or desirable, although its conclusions will inevitably have implications for both of these concerns. Instead, it develops a theoretical model to test the general validity of the contention that unfunded pensions necessarily reduce the capital stock.[5]

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[3] Reform may still be Pareto-improving if there are various accompanying side-benefits, as in Homburg (1990) who considers the accompanying labour supply effect of lowering distortionary taxation. In a very strict theoretical sense, it is then not pensions reform *per se* which is Pareto-improving, since the side benefits could be introduced independently of the reform, eg. by switching from distortionary to non-distortionary taxation. In a less strict sense, pension reform may still open the door for a Pareto-improvement, if it is practically infeasible to obtain the side-benefits independently of the reform, eg. where taxes are inevitably distortionary.

[4] If the market rate of interest equals the rate of time preference, so that an individual's consumption is constant over time, the temporary equilibrium effect of a unit increase in PAYG will be a reduction of household savings by the $(2+n)/(2+r)$ per unit where r is the market rate of interest and n is the growth rate of the tax base.

[5] There are other reasons why PAYG may have either a small or a negative effect on the capital stock. Generally, much of savings may be unrelated to life-cycle factors and may be made from precautionary

We can also provide a theoretical rationale for the empirical research results, which have found that the extent of crowding-out is not nearly as great as the standard life-cycle models predict and that there may even be a *crowding-in*.

The time-series evidence in Feldstein (1974) suggests that social security reduces household saving only by 30-50%, and with more data, in Feldstein (1996), by 60%. Leimer and Lesnoy (1982) find either that social security can lead to small decrease or even an *increase* in private savings.[6] The theoretical results of this paper show both that the scale of crowding-out may be not be so large and that *crowding-in* may not be an aberration but a real possibility for certain economies.

We explicitly analyze the effects of the optimizing behaviour of *financial* firms which act as intermediaries between saving households and borrowing *productive* firms. A version of the Diamond (1965) overlapping generations model is considered with Nash-Cournot competition between N financial firms where the integer, N, can take any value. The model has some generality, since it embraces the extreme monopoly-monopsony or *monempory*[7] case of a single firm or of a cartel where $N = 1$ and the standard, implicitly competitive case where $N \to \infty$.

Recent literature has also investigated the economic effects of monopolistic finance on economic activity. Partial equilibrium models have focussed on considering how monopoly banking may help to deal with the moral hazard and adverse selection problems which arise under imperfect information. In particular, Petersen and Rajan (1995) argue that a higher concentration of firms supplying credit makes it easier for them to internalize externalities, leading to an easing of credit constraints.

More recent papers by Cetorelli (1997), Smith (1998) and Guzman (2000A) have looked at the effects of monopoly finance on macroeconomic performance in the

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and bequest motives. Concerning life-cycle factors, households facing borrowing constraints would be unable to offset a pensions increase by dissaving [Hubbard (1986)]. Pension increases may even increase savings by inducing early retirement [Feldstein (1974)] and by making workers more aware of their retirement needs [Cagan (1965)].

6 In their survey of the literature, Engen and Gale (2000) conclude that it is difficult to assess the effects from time-series analysis.

general equilibrium of overlapping generations models. A general conclusion in the survey by Guzman (2000B) is that the effects of banking monopoly on performance are more harmful in dynamic general equilibrium models, because of the additional and adverse effect on capital accumulation.

Although models of imperfectly competitive financial markets have established a welcomed beachhead within dynamic general equilibrium models, the macroeconomic analysis of pensions has generally been relegated to life-cycle models, which implicit assume perfect competition in the finance sector. This is by an *a priori* imposition of equality between the savings rate of interest and the marginal product of capital, which is the investment rate of interest where the production sector also is perfectly competitive. This present paper investigates the effects of unfunded pensions on capital accumulation in a setting of perfect information but with imperfect competition where both of these interest rates are endogenously determined to differ.

We find that imperfect competition and PAYG pension policy each potentially have two effects on capital accumulation. The first effect of imperfect competition is the *wedge effect*, where the savings rate of interest to be determined below the borrowing rate of interest. The wedge effect causes savings and investment to be lower than their levels under competitive finance. The lower savings rate of interest reduces the likelihood that the economy is dynamically efficient, which is usually[8] defined where the savings rate of interest exceeds the economic growth rate.

A second effect of imperfect competition is an *equity price effect*, which exists because of the assumption that financial profits are endogenously returned to households as the dividend returns on their holdings of financial sector equity.[9]

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[7] Nichol (1943) first used the terms "momempory" and "oligempory".
[8] This definition no longer applies under imperfect competition.
[9] The alternative is where profits are returned *exogenously* by a fiscal distribution rule where the authority seizes the totality of financial sector profits and then reallocates the proceeds to the household sector. The effect of the rule depends on to whom and how profits are returned and these choices inevitably have powerful effects in life-cycle models where aggregate savings are determined by the intergenerational distribution of income. Consumption smoothing implies higher savings where profits distributed to the young rather than the old, unless the old receive them in the form of a pro-rata savings subsidy [Roberts (1998)].

Households may either hold deposits or financial sector equity. These two assets are assumed to be perfect substitutes assets from the point of view of individual households, but are widely different in their effects on the economy. Only deposit savings generate the funds, which are transferred from the household sector to the production sector for the purpose of investment. The transfer is engineered by the intermediation process of the financial sector.

We assume that financial equity serves only as a store of value, because there are no new issues, since there are no investment needs in the financial sector.[10] Financial firms have already acquired their existing stock of non-depreciable operating capital by issuing shares at some starting point in time for perpetuity. Hence, financial equity plays a role equivalent to money, non-productive government debt, a fixed factor like land or a bubble. Consequently, savings in financial equity merely constitutes asset trading within the household sector, as the old sell this particular store of value on to the young. So while deposit saving raises economic activity, savings in financial equity leads to the crowding-out of capital.[11] Consequently, we may use the terminology of Tirole (1985) to refer to the holding of these respective assets as "productive" and "non-productive savings".

The extent of crowding-out positively depends on the market price of financial equity, because, in equilibrium, the higher the price the young pay for this store of value, the less total savings there are left over to go into deposit savings. The equity price is naturally the present value of financial sector profits. It then follows that the abnormal profits arising under imperfect competition generate a second form of crowding-out by inflating financial equity prices to reduce deposit savings.[12]

[10] This is a reasonable approximation, if the aggregate economic gains from investing in a new banking technology are insignificant compared with those from investing in productive capital. It is also possible to consider financial profits net of some operating or depreciation cost without losing the main point of the analysis.

[11] In a broader setting, financial intermediation may raise the activity level or growth rate by changing the composition of savings. In a key paper, Bencivenga and Smith (1991) argue that financial intermediaries provide liquidity, which enables a switch from savings held in liquid assets to productive assets. As per usual, this effect is already implicitly incorporated, since liquid assets are not considered.

[12] This second effect has, to our knowledge, not been considered in models with imperfectly competitive finance.

To recap, imperfect competition reduces capital through two routes. The *wedge effect* of imperfect competition in the financial sector reduces total savings, while the *equity price effect* reduces that part of total savings which goes into deposit holding and capital accumulation. In what follows, we show that PAYG pension policy also has two related effects on capital accumulation. The first is the well known *consumption smoothing effect* in reducing total savings and the second is another *wedge effect* which, this time, increases the both the amount of savings and the proportion of "productive savings".

First, an increase in the level of PAYG, by definition, raises later-period retirement income allocations and lowers initial-period working incomes, because of the requirement of higher payroll taxes to finance the pension payments. A consumption smoothing response then reduces total savings. This first effect does not depend on imperfect competition and is, in fact, even stronger under perfect competition, since the scale of crowding-out is magnified by an already larger capital stock.

The second effect is more interesting, because it can seriously undermine or even reverse the standard comparative static predictions of PAYG. It works through the positive effect of PAYG pensions on the interest elasticity of savings.[13] An increase in PAYG will cause a greater wealth effect of the rate of interest on savings by raising the later-period allocation. A rise in the interest elasticity of the savings function then reduces the scope for the monopsonistic exploitation of households by profit-maximizing financial firms. The effect will cause financial sector profits and equity prices to fall, causing a crowding-in of "productive savings" as households can put more of their saving into deposits. So, while the *consumption smoothing effect* of PAYG reduces total savings, the importance of the *wedge effect* is in increasing the proportion of reduced total savings, which is channeled into investment.[14]

These two countervailing effects of PAYG on the capital stock create the possibility of a non-monotonic relationship. We show that with standard function form

[13] The analysis is not symmetric regarding the interest elasticity of investment, since PAYG policy directly effects only the savings schedule. There would be a second-order strengthening of the result with a rise in the interest elasticity, but this is fixed by the assumption of a Cobb-Douglas technology.
[14] The wedge effect may also provide some offsetting rise in total savings.

specifications - logarithmic for the utility function and Cobb-Douglas for the production function- the capital stock is a hump-shaped function of the PAYG contribution rate. Thus, there is an interior level of the PAYG contribution rate, which maximizes the steady-state value of the capital stock. The most basic policy implications, where the level of economic activity is an objective, are that there is some intermediate level of PAYG provision that would benefit a developing economy without any provision and that only partial reform may desirable for a mature welfare state.

The rest of the paper is set up as follows. In Section 2 the basic model is laid out, presenting the behavior of the household, productive and financial sectors. Section 3 derives the main results, which are contained in two propositions. Section 4 discusses a range of other related issues and Section 5 provides a general summary.

2. The model.

The household sector.

There is a representative household, j, which has a the two-period utility function:

$$U_t^j = U\left(c_t^{Y,j}, c_{t+1}^{O,j}\right) \tag{1}$$

where $c_t^{Y,j}$ and $c_{t+1}^{O,j}$ are the consumption levels of a household, j, which is young at time t and old at time $t+1$. The budget constraint is

$$c_t^{Y,j} + \frac{1}{1+r_{t+1}} c_{t+1}^{O,j} = w_t - b_t + \frac{1}{1+r_{t+1}} b_{t+1} \tag{2}$$

where w_t is the common first-period wage income, b_t is the amount of payroll taxation raised to finance current PAYG payments and b_{t+1} is second-period retirement income, consisting of PAYG receipts.

Savings by an individual, j, at time t may be held in two assets, fixed price deposits, a_t^j, and in a portfolio of financial equity, e_t^j, which has the variable price, v_t. Thus j's savings are given by

$$s_t^j = a_t^j + e_t^j v_t \tag{3}$$

Individuals are distributed on the interval [0,1] and aggregate savings are given by

$s_t = a_t + e_t v_t$

where $s_t = \int_0^1 s_j^j dj$, $a_t = \int_0^1 a_j^j dj$, $e_t = \int_0^1 e_j^j dj$ (4)

Savings in fixed-price deposits at time t earn an interest rate of return, at time r_{t+1}^S at time $t+1$. The other asset, financial equity, is purchased by the young from the old. To simplify, the analysis it is assumed that there are no new issues of financial equity. At some initial point in time, the financial sector made a once–and-for-all equity issue to fund its own perpetual and non-depreciation stock of operating capital. Thus, young households acquire equity only from old households. At time t, after they have received the financial profit or dividend income, d_t^j, the old sell the financial equity on to the next generation of young at the price, v_t. Apart from the dividend payment, there is also the possibility of a capital gain or loss outside the steady-state, for the old who sell the equity at the price, v_t, would have purchased it when they were young at the price, v_{t-1}, and generally $v_t \neq v_{t-1}$.

First and second-period consumption levels are, respectively,

$$c_t^{Y,j} = w_t - b_t - a_t^j - v_t e_t^j = w_t - b_t - s_t^j \tag{5}$$

$$\begin{aligned} c_{t+1}^{O,j} &= b_{t+1} + (1 + r_{t+1}^S) a_t^j + (d_{t+1} + v_{t+1}) e_t^j \\ &= b_{t+1} + (1 + r_{t+1}^S) s_t^j + \left(d_{t+1} + v_{t+1} - (1 + r_{t+1}^S) v_t\right) e_t^j \end{aligned} \tag{6}$$

We assume certainly equivalence, as if the interest rate, the dividend income and the equity price at time $t+1$ are all known at time t. The maximization of utility in equation (1) subject to the budget constraint in the form of equations (5) and (6) with respect the choices of deposit holdings and equity holdings by the young give two-first-order conditions:

$$\frac{\partial U_t}{\partial c_t^Y} = (1 + r_{t+1}^S) \frac{\partial U_t}{\partial c_{t+1}^O} \tag{7}$$

$$v_t = \frac{d_{t+1} + v_{t+1}}{1 + r_{t+1}^S} \tag{8}$$

Equation (7) is the Euler equation which gives the representative individual's time-profile of consumption or asset accumulation and equation (8) is the risk-neutral no-arbitrage condition between deposits and equity. Certainty equivalence first implies that the composition of the final portfolio has no bearing on asset accumulation, which is given by equation (7). It may be inverted to give the savings equation:

$$s_t = s\left(w_t - b_t, b_{t+1}, r_{t+1}^S\right) \tag{9}$$

Secondly, as there is no incentive to hold a balanced portfolio, individuals will only hold one of the two assets in the presence even of infinitesimal transactions costs. Thus the household sector will be split between the deposit holders and equity holders, respectively, the customers and the owners of the financial sector. This feature has no implications for the solution, but does endorse the underlying assumption of value maximization, which is discussed further in Section 4.

The pre-determined stock of equity is normalized to unity, $e_t = 1$, $\forall t \geq 0$, so that equations (4) and (9) determine aggregate deposit savings as

$$a_t = s\left(w_t - b_t, b_{t+1}, r_{t+1}^S\right) - v_t \tag{10}$$

The production sector.

There is a representative firm operating under constant return to scale, which enables the production function to be presented in per capita terms,

$$y_t = f(k_t), \quad f' > 0,\ f'' < 0, \tag{11}$$

where y_t and k_t are output and the capital stock per head at time t. The production function has the usual properties.

Firms hire capital at the separate borrowing rate of interest, r_t^L; the variable w_t is both the wage they pay their workers and the per capita wage bill. There is perfect competition in the production sector. Per capita profits are:

$$\Psi_t = f(k_t) - r_t^L k_t - w_t$$

The first-order condition for a maximum with full depreciation over the period of production determines the investment function as:

$$f'(k_t) = r_t^L \tag{12}$$

and, with the constant returns and free entry assumptions, the wage is determined as:

$$w_t = f(k_t) - f'(k_t)k_t, \quad \text{where} \quad \frac{\partial w_t}{\partial k_t} = -f''(k_t)k_t > 0 \tag{13}$$

The financial sector.

There are N finance firms, indexed z, $z = 1,..N$, which engage in Nash-Cournot quantity competition. Each finance firm, z, creates deposits at time t, $a_{Z,t}$, by borrowing from households and then lends these funds to production firms, which then acquire capital at time $t+1$, $k_{Z,t+1}$.[15] At time $t+1$, the bank pays householders the market rate of interest on savings, r_{t+1}^S, and charges firms the market rate of interest on loans, r_{t+1}^L, where $r_{t+1}^L \geq r_{t+1}^S$. Each production firm uses its loan for investment; and with 100% depreciation, the aggregate capital stock is determined by:

$$k_{t+1} = a_t, \qquad \text{where} \quad k_{t+1} = \sum_{Z=1}^{N} k_{Z,t+1}, \qquad a_t = \sum_{Z=1}^{N} a_{Z,t} \tag{14}$$

Profits accruing at time $t+1$ for each financial firm, z, are

$$\Pi_{Z,t+1} = (r_{t+1}^L - r_{t+1}^S)k_{Z,t+1} \tag{15}$$

To confirm, there are no issues of equity, because the outstanding stock of non-depreciable working capital for each financial firm was introduced at some initial point in time for perpetuity. The assumption of symmetry across financial firms allows us to work with the assumption that households are also indifferent to the composition of the equity portfolio.[16]

Equations (10) and (14) imply that the aggregate capital stock is given by

$$k_{t+1} = s\left(w_t - b_t, b_{t+1}, r_{t+1}^S\right) - v_t \tag{16}$$

The acquisition of financial market equity by the young is captured by the valuation term, v_t, which crowds-out the capital stock.

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[15] To the extent that deposits are created for life-cycle savings, these financial firms will also consist of pension companies as well as investment banks.

[16] The normalization, $e = 1$, implies that $e_Z = 1/N \;\; \forall z$.

As all financial profits are paid out as dividends, $\Pi_{Z,t+1} = d_{Z,t+1}$, equations (8) and (15) together give

$$v_{Z,t} = \frac{\left(r_{t+1}^{L} - r_{t+1}^{S}\right)k_{Z,t+1} + v_{Z,t+1}}{1 + r_{t+1}^{S}} \tag{17}$$

Some properties of the steady-state.

At this point, it is possible to consider some of the steady-state properties of the model before proceeding with the financial sector optimization. Substituting the aggregate form of equation (17),

$$v_t = \frac{\left(r_{t+1}^{L} - r_{t+1}^{S}\right)k_{t+1} + v_{t+1}}{1 + r_{t+1}^{S}},$$

into (16) and then considering the steady-state levels, defined $x = x_t$, $\forall t$, where $x_t = k_t, v_t, b_t, w_t, r_t^S, r_t^L$, the capital stock and the financial equity valuation are:

$$k = \frac{r^S}{r^L} s\left(w - b, b, r^S\right) \tag{18}$$

$$v = \left(1 - \frac{r^S}{r^L}\right) s\left(w - b, b, r^S\right) \tag{19}$$

First, equation (18) shows that the proportion of crowding-out in the steady-state is $1 - r^S / r^L$. In the competitive finance case where, $r^S = r^L$, this is, of course, zero. Secondly, equation (19) shows that the equity price in the steady-state is a multiple of this same crowding-out ratio. Thirdly, returning to equation to (18), we see that PAYG pensions policy has potentially two effects on the capital stock: one in affecting total savings, $s(..)$, and the other in affecting the proportion of total savings, r^S / r^L, which goes into deposits and capital. To complete the model, it only remains to determine the savings rate of interest from optimization in the financial sector.

Financial sector optimisation.

Each firm, z, acts to maximize the value of its financial equity, $v_{Z,t}$ at time t, as given by equation (17). We consider the dynamic programming solution where the future value $v_{Z,t+1}$ is taken as given at time t, so that the current maximizer takes as given the decisions of the future maximizer and, by recursions, all future maximizers. Although, by assumption, no game is played over time, there is Nash-Cournot behaviour played within any period of time, because the interest rates are common prices determined by aggregate quantities to which each firm makes a non-negligible contribution where $N < \infty$.

It is convenient to invert equation (16) to express the household savings function in terms of the savings rate of interest,

$$r_{t+1}^S = r^S\left(k_{t+1} + v_t, w_t - b_t, b_{t+1}\right)$$

and then to use equation (12) for the borrowing rate of interest. Equation (17) becomes:

$$v_{Z,t} = \frac{\left(f'(k_{t+1}) - r^S\left(k_{t+1} + v_t, w_t - b_t, b_{t+1}\right)\right)k_{Z,t+1} + v_{Z,t+1}}{1 + r^S\left(k_{t+1} + v_t, w_t - b_t, b_{t+1}\right)} \tag{20}$$

Note that the *aggregate* financial value or the price of the aggregate equity portfolio price, v_t, enters the right-hand-side of equation (20) (twice), because the savings rate of interest is determined by aggregate savings and because some part of aggregate savings consists of aggregate equity. Consequently, we use *the implicit function theorem* to obtain the derivative.

Each financial firm, z, maximizes the value of its equity, $v_{Z,t}$ in equation (20) by choosing a level of deposits, $a_{Z,t}$, which is equivalent to choosing a level of lending, $k_{Z,t+1}$ [from equation (14)]. The firm takes into account the effect both of its individual choice, $k_{Z,t+1}$, on the aggregate outcome, k_{t+1}, and of its own equity valuation, $v_{Z,t}$, on the aggregate equity portfolio, v_t. The first derivative is:

$$\frac{\partial v_{Z,t}}{\partial k_{Z,t+1}} = \frac{\partial v_{Z,t}}{\partial a_{Z,t}} = \frac{f'(k_{t+1}) - r_{t+1}^S(k_{t+1},..) + f''(k_{t+1})k_{Z,t+1} - \left(v_{Z,t} + k_{Z,t+1}\right)r_1^S(k_{t+1},..)}{1 + r_{t+1}^S(k_{t+1},..) + \left(v_{Z,t} + k_{Z,t+1}\right)r_1^S} \tag{21}$$

We consider (i) an interior solution where $\partial v_{Z,t} / \partial k_{Z,t+1} = 0$, $\forall z$, and (ii) a symmetric equilibrium where $k_{Z,t+1} = \frac{1}{N} k_{t+1}$, $v_{Z,t} = \frac{1}{N} v_t$, $\forall z$. Using $s_t = v_t + k_{t+1}$, gives the first-order condition: $f'(k_{t+1}) - r_{t+1}^S (k_{t+1},..) + f''(k_{t+1}) \frac{k_{t+1}}{N} - r_1^S \frac{s_t}{N} = 0$

We also define the interest elasticities of savings and investment as

$$\varepsilon \equiv \frac{r^S}{r_1^S .s} \qquad \eta \equiv \left| \frac{f'(k)}{f''(k)k} \right| > 1 \tag{22}$$

so that the equilibrium may be presented as a relationship between the ratio of two interest rates, the two elasticities and the number of firms:

$$\frac{r^S}{r^L} = (\varepsilon/(1 + N\varepsilon))((1 + N\eta)/\eta) \tag{23}$$

With imperfect competition, $N < \infty$, and with less than infinite interest elasticities, $\varepsilon < \infty$ and $\eta < \infty$, the savings rate of interest will less than he borrowing rate of interest, so that the rate of profit is positive. The crowding-out ratio in the steady-state is calculated as $(1 + (\varepsilon/\eta))/(1 + N\varepsilon)$.

3. Analysis.

Equation (23) shows the ratio of the two interest rates is constant, if the two interest elasticities are constant (for a given number of firms). Equation (18) then implies that, in the steady-state, the rate at which non-productive savings crowds-out productive savings is also constant. In this constant elasticity case, PAYG policy will always reduce the capital stock through crowding-out private savings. The extent to which PAYG crowds-out capital is less under imperfect financial market competition, because the scale of capital is already lower by the factor r^S / r^L.

It is not only more plausible, but the more interesting case is also where the interest elasticities vary with the level of PAYG. There are two broad possibilities. If the ratio, r^S / r^L, fell with PAYG, it would cause even greater damage to capital accumulation. However, the existence of wealth or discounting effects of the rate of interest suggests the opposite. PAYG is a future income allocation, which is discounted by the savings rate of interest when current savings decisions are made. As

PAYG rises, any change in the savings rate of interest will have a larger negative effect on wealth and current consumption, so that the response of savings will become more positive.[17]

The rise in the interest elasticity of savings, ε, implies that there will be less scope for the monopsonistic exploitation of households by value-maximizing financial firms. This effect reduces the rate of profit, the gap between the borrowing and savings rates of interest. It is then clear, from equation (23), that for r^S/r^L to rise, it is necessary that there should be no countervailing decrease in the interest elasticity of investment, η - if there should be - of a magnitude which swamps the effect of the higher ε. This is guaranteed within the approximation of a Cobb-Douglas production technology where η is constant.[18] We consider also the following specification for the household utility and savings functions.

A logarithmic household utility function.

We specify a logarithmic household utility function:

$$V_t = \theta \ln(w_t - b_t - s_t) + (1-\theta)\ln\left(b_{t+1} + (1 + r_{t+1}^S)s_t\right) \qquad 0 < \theta < 1 \tag{24}$$

This is the most tractable case and one commonly used, since the income and substitution effects of the rate of interest exactly cancel out to leave the wealth effect of the rate of interest, the only one of concern. It also allows us to obtain an analytical solution to the model.

The weighting, θ, relates to the rate of time-preference, δ, where $\delta \equiv (2\theta - 1)/(1-\theta)$, where conventionally, $\theta \geq 1/2$ so that $\delta \equiv (2\theta - 1)/(1-\theta) \geq 0$. Household optimization generates a savings function of the form

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[17] There could be a sign reversal, so that the PAYG would generates a backward-bending savings function instead of a negatively-sloped function where the intertemporal elasticity of substitution is inelastic.

[18] If there is a general CES form for the production function, $y = A\left(\alpha^{-1}(1-\alpha) + k^{(\sigma-1)/\sigma}\right)^{\sigma/(1-\sigma)}$, then $\eta = \sigma\left(1 + \alpha(1-\alpha)^{-1} k^{(\sigma-1)/\sigma}\right)$. The effect of a rising ε will be dampened or reversed with an elastic degree of factor substitution, where $\sigma > 1$, and enhanced with an inelastic degree where $(0 <)\sigma < 1$. This follows from the reasoning that the temporary equilibrium effect of PAYG will reduce the capital stock and from the sign of the derivative, $\partial\eta/\partial k = (\sigma - 1)\alpha(1-\alpha)^{-1} k^{-1/\sigma}$.

$$s_t = (1-\theta)(w_t - b_t) - \frac{\theta}{1+r_{t+1}} b_{t+1} \qquad 0 \le \theta \le 1/2 \tag{25}$$

which has the steady-state interest elasticity:

$$\varepsilon = \frac{\theta b r^S}{(1-\theta)(1+r^S)^2(w-b) - (1+r^S)\theta b} \tag{26}$$

A constant elasticity, Cobb-Douglas production function is used,

$$y_t = A k_t^{\alpha}$$

which gives the following solutions for the borrowing rate of interest, the wage and the (constant) interest elasticity of investment:

$$r^L = \alpha A k^{\alpha-1}, \; w = (1-\alpha) A k^{\alpha}, \; \eta = -1/(1-\alpha) \tag{27}$$

We also consider the case of an exogenous contribution rate, β, where $\beta \equiv b/w$, so that the ratio of pensions to current wages rather than an level of the pension is exogenous. The pension level then is then also endogenous to the capital stock from equation (27):

$$b = \beta w = \beta(1-\alpha) A k^{\alpha} \tag{28}$$

This reflects the real-world feature that the level of welfare provision tends to increase as economies becomes richer.

Equations (18), (27) and (28) together give a solution for the savings rate of interest:

$$\frac{\alpha}{1-\alpha} = r^S \left((1-\theta)(1-\beta) - \frac{\theta\beta}{1+r^S} \right)$$

which has the quadratic form,

$$(1+r^S)^2 - g(1+r^S) + h = 0$$

with the solution,

$$1 + r^S = g + \left(g^2 - h\right)^{\frac{1}{2}}$$

where $g \equiv \dfrac{\alpha/(1-\alpha) + 1 - \theta + (2\theta-1)\beta}{2(1-\theta)(1-\beta)}$, $\quad h \equiv \dfrac{\theta\beta}{(1-\theta)(1-\beta)} \qquad \partial r^S / \partial\beta > 0 \qquad (29)$

We use the positive valued solution to ensure the general non-negativity of the interest rate.[19] Note that the savings rate of interest is increasing in β. As $\beta \to 0$, $r^S \to \alpha/(1-\alpha)(1-\theta)$ and as $\beta \to 1$, $r^S \to \infty$.

Returning to (26), we see the required property of the savings elasticity, $\partial\varepsilon/\partial b > 0$, obtains, at least locally, since as $\beta \to 0$, $\varepsilon \to 0$, while $\beta > 0$ implies $\varepsilon > 0$.

Equations (23) and (26)-(28) give

$$k^{1-\alpha} = \frac{(N-(1-\alpha))\alpha A\theta\beta}{(1-\theta)(1-\beta)(1+r^S)^2 - \theta\beta(1+r^S) + \theta\beta N r^S}$$

Eliminating the interest rate term by using both forms in equation (29), removing superfluous terms and applying the definitions, g and h, also used in equation (29), gives

$$k^{1-\alpha} = \frac{(N-(1-\alpha))\alpha A}{2\left(g^2/h\right)\left(1+\left(1-\left(h/g^2\right)\right)^{\frac{1}{2}}\right)\left(1+((N-1)/2)(h/g)\right)-N},$$

$$g \equiv \frac{\alpha/(1-\alpha)+1-\theta+(2\theta-1)\beta}{2(1-\theta)(1-\beta)} \text{ and } h \equiv \frac{\theta\beta}{(1-\theta)(1-\beta)}. \tag{30}$$

Equation (30) is the ultimate statement of the model which shows that the level of the capital stock in the steady-state depends on the parameters of the household savings function, θ and β, of the firms' production functions, A and α, the number of financial firms, N, and the government policy parameter, β. We now consider the effects of policy on the capital stock and savings in the steady-state.

Proposition One: There is a non-monotonic relationship between the PAYG contribution rate, β, and the capital stock (where $N < \infty$).

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[19] For the negative root, as $\beta \to 0$, $g - \left(g^2 - h\right)^{\frac{1}{2}} \to 0$, so $r^S \to -1$.

Proof: (i) Generally, where $0<\beta<1$, $(0<)g<\infty$, $0<h<\infty$, so that $k>0$. (ii) In the limit $\beta \to 0$, we find that $h \to 0$ and that $g \to \left(\alpha/(1-\alpha)+(1-\theta)\right)/2(1-\theta)$, so that $0<g<\infty$. This implies $k \to 0$ where $N<\infty$. (iii) In the other limit where $\beta \to 1$, we find that $g^2/h \to \infty$ and that $h/g \to 2\theta/\left((\alpha/1-\alpha)+\theta\right)$, so that $0<h/g<\infty$. This also implies that $k \to 0$ where $N<\infty$.

Therefore, there is a low range of values for β where $\partial k/\partial\beta > 0$ and a high range of values where $\partial k/\partial\beta < 0$.

The monopoly or cartel case, where $N=1$, is the most accessible to analysis, since the ratio term, h/g, then drops out of the denominator of equation (30) to give,

$$k^{1-\alpha}=\frac{\alpha^2 A}{2\left(g^2/h\right)\left(1+\left(1-\left(h/g^2\right)\right)^{\frac{1}{2}}\right)-1}$$

The remaining ratio term, g^2/h, which enters the denominator twice, has an unambiguously negative effect on k. Thus, the value of β which minimizes g^2/h is the same value which maximizes k and is determined at $\beta^*=\dfrac{1-\theta(1-\alpha)}{1+\alpha}$. If $\beta<\beta^*$, $\partial k/\partial\beta>0$ and if $\beta \geq \beta^*$, $\partial k/\partial\beta \leq 0$.

We consider the effects of varying the interest elasticity of investment. In the limiting case of an infinitely elastic investment function where the production function exhibits constant returns to scale, $\alpha \to 1$, $\beta^*=1/2$. We find the case of a unitary elasticity of investment, where the production function is logarithmic production, where $\alpha \to 0$, so that $\beta^* \to 1-\theta$. Here $\beta^*<1/2$ with a positive rate of time preference, where $\theta>1/2$. So generally, with a single bank or a cartel and positive rates of time preference, PAYG pensions will generally raise the steady-state capital stock up until a benefit rate at some point less 50%. The exact point depends positively on the interest elasticity of investment.

For more than a single cartel firm and non-cartel behaviour, where $N > 1$, the ratio, h/g, remains in the denominator of equation (30). Any positive effect of PAYG on the capital stock is then dampened, because $\partial(h/g)/\partial\beta > 0$. As N increases, the dampening increases, so that the level of the pension ratio which maximizes the steady-state capital stock is decreasing in N, the measure of financial market competition. As $N \to \infty$, the capital stock maximizing pensions ratio tends to zero.

Proposition Two: There is a non-monotonic relationship between the PAYG contribution rate, β, and savings (where $N < \infty$).

Proof: Equations (18) and (27) imply that steady-state savings are given by $s = \alpha A k^{\alpha} / r^{S}$. The value of β which maximizes savings will be lower than the corresponding value which maximizes steady-state capital, k because while the numerator of $s = \alpha A k^{\alpha} / r^{S}$ is a hump-shaped function of β from Proposition One, the denominator, r^{S}, is an increasing function of β from equation (29).

4. Further discussion.

Welfare considerations.

The model has been considered only in its steady-state, because of the analytical difficulties of the dynamics which arise when forward-looking asset prices are combined with backward-looking capital stock dynamics in a nonlinear model. Consequently, it is not possible to say a great deal about the welfare implications of pensions changes, although there are some basic statements which can be made.

In all models, PAYG generally has three effects on household utility: (i) a relative rate of return effect, which is favourable only where the growth rate (especially of the tax base) exceeds the market rate of interest on savings, (ii) a favourable effect from an increase in the savings rate of interest and (iii) an unfavourable effect from the decrease in capital and, hence, wages.

In the standard model, where both the goods and finance markets are competitive, if the implicit rate of return on pensions exceeds the marginal product of capital, the first effect becomes favourable and the second favorable effect dominates the third unfavourable effect. Thus, dynamic efficiency requires the implicit rate on pensions is no greater than the savings rate of interest, which is equal to the marginal productivity of capital. In this present model, this equality no longer holds, so that the dynamically efficiency condition becomes more complex. Furthermore, there is a sign change where PAYG pension increases raise the capital stock.

In all models, the generation alive at the time of a policy change will be unaffected by the third, wage effect, because the capital stock is predetermined. In this particular model if PAYG eventually raises the capital stock, the utility gains (losses) of later generations are likely to exceed (be less than) those of earlier generations. This suggests that a utility gain for the current generation, following an increase in PAYG, may be a sufficient condition for the economy to be dynamically inefficient.

It is known in the standard competitive model with overlapping generations that economy is dynamically inefficient if the capital stock is too high. We can demonstrate that in this present model there will be dynamic inefficiency if the capital stock is also too low. The proof of Proposition One demonstrates that where the pension rate tends to zero, so does the capital stock, so that the economy vanishes in the absence of any level of PAYG pensions (or of any other allocation for retirement). An obvious corollary is that there must be some low level of pension provision, which cannot make any generation worse off, since no one can do worse than zero. Thus, the economy must be dynamically inefficient for some very low range of values for the pension rate.

Taxing financial sector profits or dividends.

PAYG can increase the capital stock, only because high net-of-tax dividends on financial sector equity already draw savings away from deposits, which would get channeled into investment. If the overriding objective of policy is to raise economic activity and, hence, the capital stock, the first recourse should be to tax either financial profits or dividend payments at 100%.[20] With regard to financial equity price determination and capital accumulation, policies which tax financial sector profits,

[20] In an alternative environment of uncertainty, there may also be benefits in allowing financial firms a minimum level of expected net-of-tax profits, making dividend taxation the better option.

reducing gross and net dividends, and which tax gross dividends directly are equivalent.

The capital stock may generally be maximized at an intermediate level of the PAYG contribution rate in the absence of an adequate dividend tax. If dividends are completely taxed away, it will be highest under a zero contribution rate, because there will be no second equity price effect as equity prices would be constant at zero. If the objective is to raise the level of economic activity, a PAYG pensions reform may even be counter-productive in the absence of an accompanying policy to raise sufficiently the rate of taxation on either financial sector profits or on dividends. It will be beneficial if accompanied by this separate policy.

A policy to raise either of these taxes may not be pursued for the same reasons that there may be resistance to reforming an excessively burdensome pension system. Raising these taxes will lead to losses for the constituency locked itself into holding financial sector equity, which has intra-generational as well as intergenerational welfare implications. Furthermore, it has also been argued that raising the level of pensions from a very low level can be Pareto-improving. In this state, the same welfare and political economy considerations might point to a limited measure of pension provision along with a hands-off attitude towards financial profits or dividend taxation.

A resolution of the customer-owner dilemma.

There is a well known problem of general equilibrium with imperfect competition, because households are also the ultimate owners of profit-making firms as well as their customers. A conflict may arise which may be described as the customer-owner dilemma. Demand functions may become less well-behaved, but, more importantly, the underlying objective of profit or value maximization itself may be in question [Hart (1985)]. It even becomes valid to question the assumption of value-maximization and to ask whether instead the customer-owners would prefer a better deal on prices (interest rates) than profits?[21]

[21] This may be resolved by the existence of *mutual societies* in the UK sense of the term, which, in principle, have intended to offer better deals on interest rates.

This problem can be resolved by two intrinsic features of the model and by an additional, but very weak assumption. The first feature is that savings can be undertaken in two ways, by young households acquiring either financial sector equity or deposits. The second feature is that with certainty, or certainty-equivalence, the two assets, equity and deposits, are perfect substitutes. The additional assumption is that acquisition of both financial equity and financial deposits involve separate transactions, each requiring a negligible transaction cost. The negligibility of costs implies that households will always make asset transactions, but the mere presence of costs along with the perfect substitutability of the assets implies that each household will transact in only one of the two assets.

Thus, the economy will generally be divided into two groups of households, those who hold deposits and those who hold financial equity. This division avoids the customer-owner dilemma, since the customers, the holders of financial deposits, will not be the same people as the owners, the holders of financial equity. The owners will be quite happy with the objective of value-maximization, as they are not the customers, while the customers, as non-share-holders, will have no voting rights.

Fully funded pensions.

The model has demonstrated that the sign of the effect of changes in the level of *unfunded* PAYG pensions on the capital stock and economic activity cannot be taken for granted. An immediate question is whether the same analysis applies where state pensions are funded, where the fund is invested like any other form of savings? We note that the "financial sector" has been loosely defined in this model and will also consist of pensions companies insofar as household savings are made for the life-cycle. A large part of household deposit savings in this model will be in fact be the acquisition of private funded pensions.

An established result is that the provision of fully funded state pensions will not crowd-out total savings, because voluntary private savings by individuals will then be replaced one-for-one by forced savings through taxation, which go into the acquisition of a state savings fund. There may even be the crowding-in of total savings with other forms of financial market imperfection, such as lending constraints, especially the discriminatory lending constraints considered in the model with unemployment in Fisher and Roberts (2002).

There will be a further source of crowding-in comparable to the interest elasticity effect, which has been considered with respect to PAYG pensions. The managers of state pension funds will appear as big players to the financial institutions, so there will be little scope for the same monopsonistic exploitation, which individual savers, as small players, would be subjected to under imperfect competition. This effectively makes the aggregate supply of savings more interest elastic. The total provision of funded pensions and total savings will increase because *state* funded pensions will then crowd-out *private* funded pensions by a factor less than one - even in the absence of lending constraints. The fall in pension company profits, as before, will cause private savings to switch from "non-productive savings" to "productive savings". Furthermore, if the government is concerned with activity levels, the managers of the public pensions fund, as agents of the government, would ensure that the fund was invested in capital increasing activities rather than in investing in the profitability of other pension funds.

Functional form specifications.

The case of the logarithmic utility function has been considered. This has the property that income and substitution effects of the interest rate directly cancel out, which leaves only the discounting effect, which is itself zero in the absence of retirement income in the two-period model. This ensures that savings are infinitely inelastic in the absence of a pension and that the elasticity is increasing in the level of the pension. Generalizing the specification can either weaken or strengthen the results, depending on whether substitution or income effects dominate. If substitution come to dominate income effects as PAYG provision increases, the results are strengthened, but weakened *vice versa*.

The form of imperfect competition.

Finally, another generalization would be to consider other forms financial sector competition. In terms of quantity competition, one extension might be a Stackelberg game between a single large institution and the rest of the market. Another consideration would be see how the central bank determination of base interest rates and market intervention would affect the behaviour of the financial system. Price competition is also likely to be relevant because of the heterogeneity of financial services.

5. Summary and conclusion.

We have looked at the effects of PAYG pensions on the steady-state of the capital stock where the financial sector is imperfectly competitive. A key effect is the absorption of households savings into financial sector equity, drawn by the abnormal profits which are made under imperfect competition. This effect is greatest where the level of pension provision is lowest. Consequently, a non-monotonic, specifically hump-shaped, steady-state relationship may arise - with standard specifications of functional forms - between the PAYG contribution rate and the capital stock on one hand and total savings on the other.

The ambiguity of the sign of the response, which depends on the level of pensions, may help to explain some of variability in the empirical results to the extent that the steady-state properties of the economy will be reflected in the time-series data. There may be structural changes because of the nonlinear effects of changes in the level of PAYG over long time spans. There may also be other changes in factors, not least in the state of competition in the financial sector, as well as changes in public policy and the institutional framework regarding the regulation of this particular sector.

One of the main reasons for advocating pension reform is the belief that PAYG pensions inevitably reduce the capital stock. We have a presented a model, which shows that this can no longer be a foregone conclusion, but depends on the environment of financial market competition. If the overriding goal of policy is to raise the level of economic activity, then a radical overhaul of PAYG pensions may be ineffective or even counter-productive in the absence of a fuller taxation of financial sector profits. If this is infeasible, then the scale of any reform of PAYG should then be measured against the weight of competition in the financial sector and the level of PAYG provision. If a mature welfare state is so overburdened with a PAYG pension fund that the supply-side is substantially constricted, a partial reform may be the best outcome among the options of full reform and no reform, while a modest provision of PAYG in a developing economy without a welfare state may be a good prescription for raising the capital stock.

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