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Bertrand vs. Cournot competition in asymmetric duopoly:
the role of licensing

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Abstract: This paper shows the possibility of higher welfare under Cournot competition in an asymmetric cost duopoly when the firms have the option for technology licensing. We find that if there is licensing with up-front fixed-fee, welfare is higher under Cournot competition compared to Bertrand competition when the initial cost difference of these firms is moderate; but, for very small or very large cost initial cost differences, welfare is higher under Bertrand competition. If licensing occurs with per-unit output royalty, welfare is always higher under Bertrand competition.

Key words: Bertrand, Cournot, Licensing, Welfare
JEL Classifications: D34, L13, O33

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Bertrand vs. Cournot competition in asymmetric duopoly: the role of licensing

1   Introduction

The debate on the effect of competitiveness on social welfare is going back to Schumpeter (1943) and Arrow (1962). In a seminal work Schumpeter (1943) argued that market concentration is a stimulus to the innovation. So, a society might prefer to sacrifice 'static' efficiency for the 'dynamic' efficiency. The work by Arrow (1962) challenged this view. In his work Arrow (1962) argued that the incentive for innovation would be stronger for competitive industry than a monopolist.

Contrary to the earlier literature focusing on the competitive market and monopoly, the recent contributions focus on the oligopolistic markets and examine the effects of different types of competition (i.e., Bertrand competition and Cournot competition) on social welfare. The conventional wisdom suggests that for same cost of production under Bertrand and Cournot competitions, price of the product is lower under Bertrand competition compared to Cournot competition and hence, social welfare is higher under Bertrand competition (see, e.g., Deneckere, 1982). However, the possibility of innovation before production can generate higher welfare under Cournot competition (see, e.g., Delbono and Denicolò, 1990, Bester and Petrakis, 1993, Qiu, 1997, Bonanno and Haworth, 1998, Mukherjee, 2003 and Symeonidis, 2003). Different amounts of investment in R&D under Bertrand and Cournot competitions are responsible for this finding. So, even if the 'static' welfare is higher under Bertrand competition, 'dynamic' welfare can be higher under Cournot competition.[1]

In this paper we show that if the firms produce with different technology (hence, different cost of production) welfare can be higher under Cournot competition compared to Bertrand competition even if there is no possibility of innovation before production. More specifically, we show that if the firms have the option for technology[2] licensing and licensing contract consists of up-front fixed-fee only, welfare under Cournot competition is higher when the initial cost difference of these firms is moderate. Hence, our result shows the possibility of higher 'static' welfare under Cournot competition compared to Bertrand competition.

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[1] One may refer to Delbono and Denicolò (1990) for the meaning of 'static' and 'dynamic' welfare.

In what follows, in section 2 we describe our basic model with two firms having different technology and producing homogeneous products. In section 3 we compare welfare under Bertrand competition and Cournot competition, when there is no possibility of technology licensing. Here, we show that welfare is always higher under Bertrand competition. If there is no possibility of technology licensing then Bertrand competition always generates lower price of the product compared to Cournot competition and generates lower deadweight loss. As a result, welfare is always higher under Bertrand competition.

Section 4 extends our analysis by allowing these firms to engage in technology licensing. We consider two important types of licensing, viz., licensing with up-front fixed-fee and licensing with per-unit output royalty (see, e.g., Wang, 1998). In case of licensing with up-front fixed-fee, both firms produce with the efficient technology. Under Cournot competition, firms engage in licensing with up-front fixed-fee provided the initial cost of production of these firms is sufficiently close. If the firms engage in fixed-fee licensing then it increases production efficiency in the industry by allowing both firms to produce with the efficient technology. In contrast to this, licensing does not occur under Bertrand competition. Hence, under Bertrand competition both firms compete with their own technology. So, even if competition is fierce under Bertrand competition, licensing helps to increases production efficiency under Cournot competition. We show that the former effect dominates the later effect when the difference in the initial cost of production (i.e., cost of production without licensing) of these firms is sufficiently small and therefore, welfare is higher under Bertrand competition compared to Cournot competition. But, if the difference in the initial cost of production of these firms is not sufficiently small but small enough to encourage licensing under Cournot competition, the later effect dominates the former effect and welfare is higher under Cournot competition compared to Bertrand competition. Though, for simplicity, we show our result for homogeneous products, it is clear from our following discussion that our qualitative result showing higher 'static' welfare under Cournot competition in presence of licensing can hold even for some degree of horizontal product differentiation.

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[2] We define the technology by the marginal cost of production. Lower marginal cost implies better technology.

In case of licensing with per-unit output royalty, we find that licensing never occurs under Bertrand competition but licensing always occurs under Cournot competition. In case of licensing under Cournot competition the optimal output royalty will be such that the effect cost of production of the licensee will be equal to its cost of production without licensing. Hence, optimal outputs of the firms are same under licensing with output royalty and no-licensing but licensing with output royalty helps the technologically efficient firm to increase its profit. We find that this benefit from royalty income under Cournot competition is not enough to outweigh the benefit from fierce competition under Bertrand competition. Hence, welfare is always higher under Bertrand competition compared to Cournot competition even if the firms can engage in licensing with per-unit output royalty.

Rest of the paper is organized as follows. The next section gives a general framework for our analysis. In section 3, we do our analysis for Bertrand and Cournot competitions without the possibility of licensing. Section 4 examines the welfare implications under Bertrand and Cournot competitions in presence of licensing. Section 5 concludes the paper.

2   The Model

Consider an economy with two firms, called firm 1 and firm 2, producing homogeneous product. Assume that the constant marginal cost of production of firm 1 and firm 2 are c_1 and $c_2 > c_1$ respectively. For simplicity, we assume that $c_1 = 0$. It is needless to say that no qualitative result will change for c_1 to be positive but our simplifying assumption of $c_1 = 0$ will make the algebra less cumbersome. As another simplification, we assume that there neither firm needs to bear any other cost of production. Assume that inverse market demand function is

$$P = a - q,$$ (1)

where the notations have usual meaning.

In the following analysis we will consider two types of product market competition, viz., Cournot competition and Bertrand competition. Further, we will divide our analysis in two parts: (i) where licensing of technology is not feasible, and (ii) where licensing of technology is feasible.

We assume that $c_2 < \frac{a}{2}$. This assumption implies that the (unrestricted) monopoly price of firm 1 is greater than the constant marginal cost of production of firm 2 and hence, ensures duopoly market structure.

3   Comparing Bertrand and Cournot without licensing

In this subsection we will examine the implications of Bertrand and Cournot competitions on social welfare when the firms cannot engage in technology licensing. We define social welfare as the summation of consumer surplus and total industry profit.

3.1   Cournot competition

Given the demand and cost specifications, if the firms compete like Cournot duopolists in the product market, total output is $\frac{(2a-c_2)}{3}$. The profits of firm 1 and firm 2 are respectively $\pi_1^{c,nl} = \frac{(a+c_2)^2}{9}$ and $\pi_2^{c,nl} = \frac{(a-2c_2)^2}{9}$. Therefore, welfare under Cournot competition is

$$W^{c,nl} = \frac{(a+c_2)^2 + (a-2c_2)^2}{9} + \frac{(2a-c_2)^2}{18}. \tag{2}$$

3.2   Welfare under Bertrand competition

Since the firms are producing homogeneous products, the price under Bertrand competition is equal to c_2. Therefore, industry output is $(a-c_2)$. So, welfare under Bertrand competition is

$$W^{b,nl} = c_2(a-c_2) + \frac{(a-c_2)^2}{2}. \tag{3}$$

3.3   Welfare comparison under Bertrand and Cournot competitions

Comparing (2) and (3), we find that (3) is greater than (2) provided

$$-a^2 - 8ac_2 + 20c_2^2 < 0. \tag{4}$$

Left hand side (LHS) of (4) is continuous, quadratic and convex with respect to c_2 over $[0, \frac{a}{2}]$. Further, LHS of (4) is negative for $c_2 = 0$ and zero for $c_2 = \frac{a}{2}$. This implies that (4) holds for all $c_2 \in [0, \frac{a}{2}]$.[3]

The following proposition summarizes the above discussion.

Proposition 1: *If technology licensing is not feasible, welfare is always higher under Bertrand competition compared to Cournot competition.*

4   Comparing Bertrand and Cournot with licensing

Now we extend the analysis of the previous section by incorporating the possibility of licensing between the firms. In case of licensing, the technologically superior firm, firm 1, licenses its technology to the technologically inferior firm 2 and charges price for its technology. In the following analysis, we will consider two popular types of licensing contract (see, e.g., Wang, 1998), viz., licensing with up-front fixed-fee and licensing with per-unit output royalty.[4]

4.1   Fixed-fee licensing

In this subsection, we consider that in case of licensing the licensee licenses its technology with an up-front fixed-fee before production. Hence, in case of licensing, both firms produce with the technology of firm 1.

4.1.1   Licensing under Cournot competition

Let us first consider the situation under licensing. If licensing occurs, each firm will produce $\frac{a}{3}$. So, the profit of firm 1 and firm 2 are respectively

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[3] It is easy to check that industry output under Bertrand competition is greater than the industry output under Cournot competition for all $c_2 < \frac{a}{2}$.

[4] As shown in Rockett (1990), fixed-fee licensing is the optimal licensing contract when the licensee can imitate the licensed technology costlessly and licensing with output royalty is optimal when the licensee cannot imitate the licensed the technology due to sufficiently higher cost of imitation. If it is difficult to verify the output of the licensee, fixed-fee licensing is the only way of licensing the technology (see, Katz and Shapiro, 1985).

$$\pi_1^{c,l} = \frac{a^2}{9} + F,\qquad(5)$$

and

$$\pi_2^{c,l} = \frac{a^2}{9} - F.\qquad(6)$$

where F is the fixed-fee charged under licensing. In our analysis we assume that firm 1 gives a take-it-or-leave-it offer to firm 2 under licensing and firm 2 accepts the licensing contract if it is not worse off under licensing compared to no-licensing. Therefore, the maximum amount of fixed-fee that can be charged by firm 1 is the difference of firm 2's profit under licensing and no-licensing. So, the fixed-fee charged by firm 1 is

$$F^* = \frac{a^2}{9} - \frac{(a - 2c_2)^2}{9}.\qquad(7)$$

However, firm 1 will license its technology provided it is better off under licensing compared to no-licensing. This implies that firm 1 will license provided

$$\pi_1^{c,l} = \frac{a^2}{9} + F^* > \frac{(a + c_2)^2}{9} = \pi_1^{c,nl}.\qquad(8)$$

From (7) and (8) and after simplifying, we find that firm 1 will license its technology to firm 2 provided

$$c_2 < \frac{2a}{5},\qquad(9)$$

where $\frac{2a}{5} < \frac{a}{2}$.[5]

Hence, the following lemma is immediate

Lemma 1: *Under Cournot competition, fixed-fee licensing occurs for $c_2(0, \frac{2a}{5})$.*

Therefore, for $c_2 \in (0, \frac{2a}{5})$, licensing occurs and welfare is given by (after rearranging)

$$W^{c,fl} = \frac{4a^2}{9}.\qquad(10)$$

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[5] One can find similar result in Marjit (1990).

4.1.2 Licensing under Bertrand competition

Since these firms produce homogeneous products, licensing will encourage the firms to charge a price equal to zero. Hence, in case of licensing, both firms receive zero profit under Bertrand competition. So, it is trivial that firm 1 has no incentive to offer a licensing contract under Bertrand competition.[6] Hence, the welfare under Bertrand competition is given by the expression (3).

4.1.3 Welfare comparison under Bertrand and Cournot competitions

It is trivial that if $c_2 \in [\frac{2a}{5}, \frac{a}{2})$, fixed-fee licensing will not occur either under Bertrand competition or under Cournot competition. So, our conclusion about the relative welfare under Bertrand and Cournot competitions becomes similar to Proposition 1.

If $c_2 \in (0, \frac{2a}{5})$, licensing occurs under Cournot competition only and welfare under Bertrand and Cournot competitions is given by the expressions (3) and (10) respectively. After rearranging, we find that (3) is greater than (10) provided

$$c_2 < \frac{a}{3}. \tag{11}$$

Since, $\frac{a}{3} < \frac{2a}{5}$, it implies that, in presence of fixed-fee licensing, welfare under Bertrand competition is greater than that of under Cournot competition provided $c_2 \in (0, \frac{a}{3})$. But, welfare under Cournot competition, in presence of fixed-fee licensing, is greater than that of under Bertrand competition for $c_2 \in (\frac{a}{3}, \frac{2a}{5})$.[7]

The following proposition summarizes the above discussion under licensing with up-front fixed-fee.

Proposition 2: *Assume that the firms have the option for licensing with up-front fixed-fee only.*

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[6] Licensing can occur under Bertrand competition with homogeneous product if the firms play infinitely repeated game and try to collude tacitly (see, Lin, 1996).

[7] It is easy to check that industry output is higher under Cournot competition for $c_2 \in (\frac{a}{3}, \frac{2a}{5})$. For other values of c_2, industry output is higher under Bertrand competition.

(i) Welfare is higher under Bertrand competition compared to Cournot competition for either $c_2 \in (0, \frac{a}{3})$ or $c_2 \in [\frac{2a}{5}, \frac{a}{2})$.

(ii) Welfare is higher under Cournot competition compared to Bertrand competition for $c_2 \in (\frac{a}{3}, \frac{2a}{5})$.

If we consider that these firms produce imperfect substitutes then licensing is possible even under Bertrand competition. However, as evident from the previous work (see, Wang and Yang, 1999 and Wang, 2002), the possibility of licensing is lower under Bertrand competition compared to Cournot competition when licensing contract involves up-front fixed-fee only.[8] Hence, there are situations, where licensing is possible under Cournot competition but not under Bertrand competition even if the products are imperfect substitutes. So, it is easy to understand that our qualitative result like the above proposition holds even under some degree of horizontal product differentiation.

4.2 Licensing with per-unit output royalty

Now we examine another type of licensing contract, viz., licensing with per-unit output royalty.

4.2.1 Licensing under Cournot competition

If firm 1 licenses its technology with per-unit output royalty, the effective marginal cost of firm 2 is $(0 + r)$, where r shows the optimal per-unit output royalty. If firm 1 offers the licensing contract and firm 2 accepts the offer then, given the per-unit output royalty, the optimal outputs of firm 1 and firm 2 are $\frac{(a+r)}{3}$ and $\frac{(a-2r)}{3}$ respectively. Therefore, given r, the profits of firm 1 and firm 2 are $\frac{(a+r)^2}{9} + \frac{r(a-2r)}{3}$ and $\frac{(a-2r)^2}{9}$ respectively. So, while choosing the optimal amount of output royalty, firm 1 maximizes the following expression:

$$\underset{r}{Max} \frac{(a+r)^2}{9} + \frac{r(a-2r)}{3} \tag{12}$$

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[8] Mukherjee and Balasubramanian (2001) have examined the optimal patent licensing contract under Cournot duopoly and also examined the implications of imitation on the quality of the licensed technology.

subject to the constraint $r \leq c_2$.[9] Maximizing (12) and ignoring the constraint $r \leq c_2$, we find that the optimal output royalty is $\frac{a}{2}$. However, $\frac{a}{2}$ is greater than c_2 for all $c_2 < \frac{a}{2}$. This implies that the incumbent will charge the optimal output royalty equal to c_2.

With the optimal output royalty, the effective constant marginal cost of firm 2 becomes c_2. Therefore, optimal outputs of firm 1 and firm 2 are same under licensing and no-licensing. So, the profit of firm 2 and consumer surplus are same under licensing and no-licensing but the profit of firm 1 rises by the amount of royalty income, which is equal to $\frac{c_2(a-2c_2)}{3}$. This immediately gives us the following lemma.

Lemma 2: *Licensing with per-unit output royalty always occurs under Cournot competition.*

Therefore, if the firms have the option for licensing with per-unit output royalty, welfare is given by

$$W^{c,rl} = \frac{(a+c_2)^2 + (a-2c_2)^2}{9} + \frac{c_2)(a-2c_2)}{3} + \frac{(2a-c_2)^2}{18} \tag{13}$$

4.2.2 *Licensing under Bertrand competition*

Like licensing with up-front fixed-fee, it is easy to understand that even if we consider licensing with per-unit output royalty, licensing does not occur under Bertrand competition. For any positive per-unit output royalty that is less than c_2,[10] the output of firm 2 is zero and this licensing contract reduces the profit of firm 1 compared to the situation with no-licensing. This is because the optimal output of firm 1 under this type of licensing is greater than the optimal output of firm 1 under no-licensing, which is greater than the monopoly output of firm 1.

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[9] The entrant will be better off under no-licensing compared to licensing if $r > c_2$ and hence, will not accept the licensing contract with $r > c_2$.

[10] There is no meaning in considering the output royalty greater than c_2 since firm 2 can always produce with its own technology.

4.2.3   Welfare comparison under Bertrand and Cournot competitions

Comparing (3) with (13) and after rearranging, we find that (3) is greater than (13) provided

$$a^2 + 2ac_2 - 8c_2^2 > 0. \tag{14}$$

LHS of (14) is continuous, quadratic and concave with respect to c_2 over $[0, \frac{a}{2}]$. Further, LHS of (14) is positive at $c_2 = 0$ but zero at $c_2 = \frac{a}{2}$. This implies that (14) holds for all $c_2 \in (0, \frac{a}{2})$.

Hence, the following proposition is immediate.

Proposition 3: *Consider the possibility of licensing with per-unit output royalty. In this situation, welfare is always higher under Bertrand competition compared to Cournot competition.*

5   Conclusion

There is an existing debate on the effect of market competition on social welfare. Previous works on oligopolistic markets suggest that even if 'static' welfare is higher under Bertrand competition, dynamic welfare may be higher under Cournot competition when the firms have the possibility of innovation before production.

In this paper we show the possibility of higher 'static' welfare under Cournot competition compared to Bertrand competition in an asymmetric duopoly with homogeneous product. We show that 'static' welfare can be higher under Cournot competition if the firms can engage in technology licensing and licensing contract consists of up-front fixed-fee only. If the firms engage in licensing with up-front fixed-fee only, welfare is higher under Cournot competition for moderate initial cost difference of these firms. But, for very small or very large initial cost differences, welfare is higher under Bertrand competition compared to Cournot competition even if the firms have the option for licensing with up-front fixed fee only. If there is no possibility of licensing or licensing involves per-unit output royalty only, welfare under Bertrand competition is always higher than that of under Cournot competition.

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