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by Mark A Roberts

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Bismarckian and Beveridgean pay-as-you-go pension schemes where the financial sector is imperfectly competitive

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Abstract

A switch from a Bismarckian (BIS) earnings-related to a Beveridgean (BEV) flat rate pay-as-you-go (PAYG) pension scheme will raise the variance of personal replacement ratios and, hence, the variance of individual interest-saving elasticities. A monopolistic financial sector can then make greater profits through an increased scope for price-discrimination between household savers and borrowers. A proportion of households may opt out of participating with the financial sector, while savers receive a lower rate of return. Provided the growth rate is similar under both regimes, BIS-PAYG may emerge as the democratic outcome, if the median voter saves a significant amount, and BEV-PAYG, if the median voter saves little or nothing and receives an ordinary allocation of returned profits. The actual outcome depends both on the model specification, how profits are returned and on the underlying income distribution.

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Introduction

Public pensions are generally run on a pay-as-you-go (PAYG), unfunded basis as transfers from tax-paying workers to those who have retired from the labour market. There are, however, important differences concerning the calculation of benefits. "Bismarckian" (BIS) pensions are related to the earnings history of the recipient, while Beveridgean (BEV) pensions are calculated on a "fixed rate" basis. BIS-PAYG is regarded as a form of social insurance, while there is a re-distributive element to BEV-PAYG to the extent that pension contributions are raised from proportional income taxes.[2]

In a seminal paper, Browning (1975) showed that a democratic voting equilibrium may lead to a higher than optimal provision of PAYG. The reasoning is that the median age individual anticipates a relatively high level of net social security wealth, because while past taxes paid are regarded as a sunk cost, future benefits can be looked forward to in their entirety. Many papers have also considered the intra-generational re-distributive effects of PAYG, which may lead to coalitions of interest between the old and the young. Casammata, Cremer and Pestieau (2000) find in a two-period model that if the inter-temporal elasticity of substitution is less than unity, there may be a coalition between the old and the middle-income young. Tabellini (2000) finds that inter-generational linkages through two-way altruism will also support a high level of PAYG. A survey of the political economy of social security is provided by Galasso and Profeta (2002).

The general approach of these papers is to model the level of inter-generational re-distribution where the intra-generational element of re-distribution is taken as constitutionally set. That is to determine the level of PAYG for given BEV and BIS components. This present paper reverses this approach by considering the how voters might choose between BIS and BEV schemes where instead the level of PAYG is pre-determined. Following Casammata, Cremer and Pestieau (2000), we define a and $1-a$,

[2] Originally, Beveridgean pensions in the UK were not re-distributive, as they were financed through individuals paying fixed amount contributions.

as the BIS and BEV components, where $0 \leq a \leq 1$. We, likewise, consider two-period models.

Each generation of young determines the size of the a component pertaining to the pension that they will receive in the next period when they are old. We assume that each particular choice is honoured over the passage of time, so that the old receive a pension which is defined by level of a, which was predetermined by an earlier vote. In the absence of descending altruism, the old have no stake in determining the succeeding level of a, which defines the future pension benefits of the current generation of young. Consequently, in each period only the young would have the incentive to vote for the determination of a a, so that median voter preferences may be considered with reference to this single generation.

As social security is an alternative to private saving, the saving characteristics of the median voter are crucial to the democratic choice of social security system. A basic defining characteristic is whether this individual saves or not, and if so, whether it is a significant amount. Despite the fact that representative agent models necessarily assume that all households save, empirically only a proportion actually do. The main issue however is not the whether this is merely a proportion, but - assuming a correlation between saving behaviour and social security preferences - whether the median voter would be found within this group or whether this proportion is at least 50%.

Guaraglia (1998) reports that 55.88% of all individuals responding to a *BHPS* survey within the UK are savers, while research commissioned by the *Abbey National* find that for *regular* savers the figures are 41% for women and 37% for men. Evidence from the US *Survey of Consumer Finances* in Kennickell, Starr-McCluer and Sunden (1997) report a figure of 57% for 1991 and 55% for 1994, while a report for the *Financial Planning Association* of Australia Limited find that less than half of working age Australians are savers. There is also a tendency for a higher proportion of the young to save, so that it is 59.29% of the 25-35 age group in the *BHPS* study and more than half of the 25-34 age group for the same Australian data set. Our reading of the evidence is that it is highly marginal, country-specific and time-variable whether the median voter is a saver or not.

We present a model of imperfect competition within the financial sector. This characteristic is not only interesting as an alternative to the standard paradigm of a perfect competition, but it also captures the real world features of differential interest rates for savers and borrowers and of a tendency for a proportion of households to opt out of financial dealings altogether. Financial market imperfection is usually considered with reference to borrowing constraints. For example, Boadway and Wildasin (1989) use these constraints to determine an interior equilibrium for the level of social security in a voting model. In a different context, Fisher and Roberts (forthcoming 2003) show that that fully-funded pensions in conjunction with borrowing constraints may increase labour market participation and the endogenous growth rate.

Although there has been a considerable literature focussing on credit constraint effects, relatively little has been written on the case where agents face different interest rates. One notable exception, which is very relevant here, is the paper by Conde and Profeta (2002) where the rich can save on better terms than middle income individuals who, in turn, get a better deal than the poor.

We consider the case where there is only one saving rate of interest, but a higher one for borrowing, where both are endogenously determined by financial sector optimisation. Not all individuals will then smooth consumption, for although the (i) the rich may save at rate that is lower (ii) and the poor may borrow a rate that is higher than the competitive rate of interest, there may be (iii) a third group with intermediate incomes, may choose not to participate with the financial sector by neither saving nor borrowing.

This consideration of imperfect competition is important for at least the following four reasons. (i) There is a need to correct an imbalance of emphasis, since while there has been a burgeoning macroeconomic literature on imperfect competition in goods and labour markets[3], little has been written about imperfectly competitive finance. The latter is crucial to the financial intermediation, which is the basis of the process of capital accumulation driving dynamic models. (ii) The assumption of imperfectly competitive finance is actually more plausible than imperfect competition in product markets in the following sense. The entry of new firms may generally require the access to finance to cover set-up costs. While profit maximising banks may happily fund the entry costs of new production firms, they may see it as shooting themselves in the foot to fund the entry of prospective competitors in their own sector. (iii) The capital accumulation effects have always been an intrinsic part of pension analysis. A more recent literature has also explored the importance of banking competition for capital accumulation.[4] There is undoubted merit in integrating these two analyses in order to see how banking competition and pensions interact to determine the process of capital accumulation. (iv) And, last but not least, and especially for our purposes, imperfect competition in the financial sector gives a coherent account of why a sizeable proportion of individuals may opt out of dealing with the financial sector altogether.

We consider a single institution and call it a "monopoly[5] bank[6]". It charges borrowers a high rate on interest than it pays to savers for the source of its funds. A generalisation to the N-firm case of Nash-Cournot competition, where $1 \leq N < \infty$, as in Roberts (forthcoming 2003) will not effect the qualitative nature of the results, but only their magnitude, which is decreasing in N. In this same paper, PAYG is implicitly Bismarckian because of the homogeneous household assumption. A particular feature is that profits are returned endogenously as dividend payments to households, which hold financial sector equity. A general result that the economy is less likely to be dynamically efficient, since the profit-maximising wedge drives the savings rate of interest paid to households below the competitive level, which in turn, is below the lending rate of interest charged to firms. The special feature concerning profits return generates the

[3] A survey of this literature is given in Dixon and Rankin (1994).
[4] A survey of this separate literature is given in Guzman (2000).

[5] Technically the institution is a both a monopoly in the loan market and a monopsony in the savings market or, to coin the term started by Nicholl (1943), a monempory.

result that PAYG, over a low range of values, may actually raise the capital stock by reducing "unproductive saving" in financial sector equity to increase "productive saving" in the capital stock.

This present paper moves away in two directions. First, we consider intra-generational heterogeneity through a distribution of wage incomes. This provides a vehicle to analysing re-distributive or Beveridgean social security and to make a comparison with the Bismarckian alternative. The second direction is to consider the exogenous return of financial profits to households through a fiscal policy rule, where profits or dividends are taxed at a high rate and then returned through a lower labour income tax rate and a higher pension levels. This feature sits well with the underlying emphasis on median voter preferences, since, empirically, only a small proportion of savers, let alone households, have significant holdings in financial sector equity.

The analytical complexity of solving a financial sector optimisation problem in a world of heterogeneous agents has also forced us into another two directions. One is to assume a uniform distribution of characteristics, which necessarily implies that median voter preferences will under-predict the pressures for redistribution that would exist within an empirically skewed distribution. The other is to approach the analysis with two models. The first is exclusively of intra-generational asset trading and can be solved analytically. The second model is one of overlapping generations with capital, so it has the extra dimension of inter-generational financial trading. It has to be solved numerically. The solutions show that the degree of intra-generational trading in equilibrium is quantitatively insignificant, because of the uniform distribution property. So, while the first model is of intra-generational trading by construction, the second model is, as effectively one solely of inter-generational trading by default. This difference implies that under BEV-PAYG the median voter will choose not to participate financially in the first model, but will be a saver in the second model. The two models together throw up

[6] It is regarded an almalgam of all financial institutions. These include those administering private pension funds, given that is saving is made for smoothing consumption over the whole life-cycle.

enough intuition to point towards fairly strong conclusions about democratic choices between BIS- and BEV-PAYG.

The main intuition of the analysis is that BEV-PAYG, in raising the variance of personal replacement ratios, increases the variance of individual interest-saving elasticities. This gives the financial sector an increased scope for monopolistic price-discrimination, which allows it to offer savers a lower rate of return on saving than it would pay under BIS-PAYG. The higher profits that accrue are then returned to all households. In the first model the median voter does not participate financially, so is not hurt by the increased monopolistic price distortion, but does gain from increased monopoly profits under BEV-PAYG. In the second model, the parameterisation is such that the median voter is a saver and the better deal on interest rates under BIS-PAYG turns out to be an over-riding consideration. We conclude, with a caveat about similar growth rates, that BIS-PAYG is preferred where median voter saves a significant amount and BEV-PAYG where this individual saves little or nothing. The democratic outcome will then depend on the underlying income distribution and any factors which promote saving in general.

The rest of the paper is structured as follows. Section 2 considers how the nature of the PAYG scheme affects the forms and elasticities of the aggregate saving and borrowing functions, where households have different income or productivity endowments and where the financial sector is monopolistic. Section 3 presents a pure consumption-loan model (model 1) which by construction only allows intra-generational financial trading. Section 4 considers an overlapping generations model (model 2), which turns out to be one of almost exclusively inter-generational trading. Section 5 synthesises the analysis of the previous two sections and generalises the discussion. Section 6 gives the conclusions.

2. Household savings and borrowing

The household sector

There is a representative household, i, which lives for two periods and has a logarithmic utility function:

$$V_t(i) = \ln(c_t^Y(i)) + \frac{1}{1+\theta}\ln(c_{t+1}^O(i)) \tag{1}$$

where $c_t^Y(i)$ and $c_{t+1}^O(i)$ are the consumption levels of a household, i, which is young at time t and old at time $t+1$. Each household supplies an inelastic unit of labour only when young and receives an idiosyncratic net of tax wage, $(1-\tau)w_t(i)$, where τ is a tax rate, which is common to all households.

The household receives a pension benefit, $b_{t+1}(i)$ when old. The pension is unfunded on a pay-as-you-go basis (PAYG) and amounts to an inter-generational redistribution from the tax-paying young to the old. Two broad possibilities for PAYG are considered, Beveridgean (BEV) and Bismarckian pensions (BIS). A BEV pension is "flat rate" with respect to the recipient's earnings history, but at a level which is normally indexed to the general wage level at the time the pension is paid, w_{t+1},[7]

$$b_{t+1}^{BEV}(i) = \beta^{BEV} w_{t+1} \qquad \forall i,$$

while a BIS pension is related to the earnings history of the recipient, $w_t(i)$. BIS pensions are also indexed to movements in the current general wage level, so that,

$$b_{t+1}^{BIS}(i) = \beta^{BIS}\left(w_{t+1}/w_t\right)w_t(i)$$

Defining $\phi(i)$ as $i's$ relative income and G_{t+1} as the growth factor in the general wage level where

$$w_t(i) \equiv \phi(i)w_t \quad \text{and where} \quad 1 = \int_{\phi_{MIN}}^{\phi_{MAX}} \phi(i)f(i)di \tag{2}$$

and

$$G_{t+1} \equiv w_{t+1}/w_t \,, \tag{3}$$

for a common average replacement ratio, $\beta^{BIS} = \beta^{BEV} = \beta$, an we can present a general form of the benefit equation as in Casanatta, Cremer and Pestieau (2000),

$$b_{t+1}(i) = \beta\left(a\phi_t(i) + (1-a)\right)G_{t+1}w_t \tag{4}$$

where the BIS and BEV components have the respective weightings, a and $1-a$. An extension is the consideration of the effect on benefit levels of the growth factor, which may differ across pension schemes under imperfect competition. There are activity effects as well as distribution effects, so that not all individuals with incomes above average, $\phi(i) > 1$, will necessarily prefer restrictions on redistribution under BIS-PAYG, because the state pension fund may grow faster under BEV-PAYG.

We consider a constant tax rate, τ, on the labour income of the young:

$$t_t(i) = \tau w_t(i) \tag{5}$$

We simplify to the case of a constant population and of equal numbers of young and old, so that the social security budget constraint is defined where the income tax rate equals the average replacement rate,

$$\tau = \beta \tag{6}$$

The coefficients β and $1-a$ measure, respectively, policy choices with respect to *inter*-generational and *intra*-generational redistribution.

The financial sector generally makes profits. There are three broad possibilities for the return of profits: (i) profits are returned exogenously to households by a fiscal distribution rule after being taxed at the rate of 100%; (ii) profits are returned endogenously as tax-free dividends to those households that hold financial sector equity; or (iii) we abstract from this issue by assuming that profits are either expatriated or taxed at 100% and then spent on some wasteful government expenditure. In practice, we might expect to see some combination of all three possibilities. Profits or dividends might be taxed at a rate of less than 100% and the dividend tax receipts might be used to partially reduce labour income taxation and/or increase to pension benefits, while the remainder is spent on some wasteful item of government expenditure.

The way in which profits are returned to households is important to the analysis. The case considered is where profits are taxed at 100%, so if there is financial equity, the holders only receive ε, $\varepsilon \approx 0$, in dividend payments, and then returned both to young and old households through a reduction in the rate of labour income tax and an increase

[7] This link was broken in Britain under the Thatcher reforms.

in pension the respective proportions, λ_t, $1-\lambda_t$. The return of profits to the two generations is consistent with the overall policy choice on inter-generational redistribution, so that

$$\lambda_t = \frac{(1-\beta)}{\beta}\left(\frac{\eta_{t+1}}{\eta_t}\right)(1-\lambda_{t+1}) \quad \text{where} \quad \eta_t \equiv \Pi_t / w_t \tag{7}$$

Using the steady-state growth approximation, $\eta_t = \eta$, $\forall t$, gives

$$\lambda = 1-\beta = 1-\tau, \qquad 1-\lambda = \beta = \tau \tag{8}$$

This ensures that the level of profits has no effect either on aggregate or individual replacement rates, as the individual's first and second-period incomes are

$$(1+\eta)(1-\beta)\phi(i)w_t, \quad (1+\eta)\beta\big(a\phi(i)+(1-a)\big)Gw_t$$

which gives the budget constraint as

$$c_t^Y + \frac{1}{R_{t+1}}c_{t+1}^O \leq (1+\eta)\left((1-\beta)\phi_t(i) + \frac{1}{R_{t+1}}\beta\big(a\phi(i)+(1-a)\big)G\right)w_t \tag{9}$$

Maximisation of the household's utility subject to the budget constraint gives the individual's savings, $s_t(i)$, where $s_t(i) = (1-\beta)(1+\eta)\phi(i)w_t - c_t$, as follows:

$$s_t(i) = \frac{1+\eta}{2+\theta}\left((1-\beta)\phi(i) - \beta\left(\frac{1+\theta}{R_{t+1}^S(i)}\right)\big(a\phi(i)+(1-a)\big)G\right)w_t \tag{10}$$

Poorer individuals will tend to borrow and richer ones to save. The consistency of the rule governing the return of profits with the inter-generational re-distribution rule ensures that financial profits only have a scale effect and do not alter the tendency for any individual to be a either a saver or borrower.

The individual savings function has the interest elasticity

$$\varepsilon_t^S(i) \equiv \frac{\partial s_t(i)}{\partial R_{t+1}^S(i)}\frac{R_{t+1}^S(i)}{s_t(i)} = \frac{\beta(1+\theta)\big(a\phi(i)+1-a\big)G}{(1-\beta)\phi(i)R_{t+1}^S - \beta(1+\theta)\big(a\phi(i)+1-a\big)G} \tag{11}$$

The choice of social security policy has implications for the interest elasticity of the individual's savings function and, thus, for the distribution of elasticities across individuals. If $a=1$, the case of BIS-PAYG,

$$\varepsilon_t^{BIS}(i) = \frac{\beta(1+\theta)G}{(1-\beta)R_{t+1}(i) - \beta(1+\theta)G} \qquad \forall i,$$

all individuals have the same elasticity and in the absence of other sources of heterogeneity. This is not the case for BEV-PAYG, where $a = 0$, for the individuals elasticity is

$$\varepsilon_t^{BEV}(i) = \frac{\beta(1+\theta)G}{(1-\beta)\phi(i)R_{t+1}(i) - \beta(1+\theta)G}$$

Observation 1*: Under BEV-PAYG (or generally $a < 1$), the absolute value of the individual saving elasticity is decreasing in household relative income, $\phi(i)$, but under BIS-PAYG the saving elasticity is the same for all individuals.*

The savings of rich individuals will not only be higher, but less responsive to the rate of interest, while the marginal saver will have an infinitely elastic savings function.

Observation 2: *The variance of individuals' savings elasticities is greater under BEV-PAYG and depends on the income distribution.*

This is a corollary of Observation 1 with an income distribution.
Finally, in all cases the absolute value of the savings elasticity is increasing in the growth factor, as higher growth requires that the interest rate discounts a higher value for the pension, the second period endowment.

Aggregate savings and borrowing under different interest rates

If all individuals were net savers, a perfectly discriminating bank would be able to charge a higher interest rate on richer households with lower elasticities. However, we tend to see the opposite, probably because interest returns are generally paid net of fixed transactions cost, decreasing pro rata in savings size. We would also expect big borrowers to pay more interest than small borrowers. This is not generally observed, possibly because of risk differences between small and large loans.

Instead, we assume that the most obvious form of price discrimination, where savers receive a lower interest rate than the one charged to borrowers. This is, after all, fundamental to the profitability of the financial sector, since borrowing and lending constitute its input and output activities.

Assumption 1: *There is price discrimination between savers and borrowers:*

$R(i) = R^S$ if $s(i) > 0$, and $R(i) = R^L$ if $s(i) < 0$., where $R^L \geq R^S$.

An individual will save only if his relative income exceeds a critical high, ϕ^{CH}

$$\phi(i) > \phi^{CH} = \frac{\beta(1+\theta)(1-a)G}{(1-\beta)R_{t+1}^S - \beta(1+\theta)aG} \tag{12.S}$$

and, likewise, will borrow only if his relative income falls below a critical low, ϕ^{CL}

$$\phi(i) < \phi^{CL} = \frac{\beta(1+\theta)(1-a)G}{(1-\beta)R_{t+1}^L - \beta(1+\theta)aG} \tag{12.L}$$

There is an intermediate group for whom it will be too costly to borrow and not viable to save. These "non-participants" are somewhere in the interior of the income distribution:

$$\phi^{CL} \leq \phi(i) \leq \phi^{CH} \tag{12.N}$$

If relative income is continuously distributed on the support, (ϕ^{MIN}, ϕ^{MAX}), where, by definition, $E(\phi(i)) = 1$, aggregate household savings and borrowing are

$$s \equiv \int_{\phi^{CH}}^{\phi^{MAX}} s(\phi(i))(i) f(\phi(i)) di, \qquad l \equiv -\int_{\phi^{MIN}}^{\phi^{CH}} s(\phi(i)) f(\phi(i)) di \tag{13}$$

Equations (12) and (13) show that interest rate changes will generally affect the number of savers or borrowers as well as the amount any individual may save or borrow.

Assumption 2: *There is a uniform distribution of income:*

$\phi(i) \sim U[\phi^{MIN}, \phi^{MAX}]$, where $0 \leq \phi^{MIN} < 1 < \phi^{MAX} \leq 2$

This implies that the savers, borrowers and non-participants are in the following proportions,

$$0 < \lambda^S = \frac{\phi^{MAX} - \phi^{CH}}{\phi^{MAX} - \phi^{MIN}} < 1, \quad 0 < \lambda^L = \frac{\phi^{CL} - \phi^{MIN}}{\phi^{MAX} - \phi^{MIN}} < 1, \quad 0 < \lambda^N = \frac{\phi^{CH} - \phi^{CL}}{\phi^{MAX} - \phi^{MIN}} < 1, R^L > R^S \tag{14}$$

Aggregate household saving and borrowing per population are solved as

$$s_t = \frac{1}{2(\phi^{MAX} - \phi^{MIN})\Omega_{t+1}^S}\left(\phi^{MAX}\Omega_{t+1}^S - \Psi_{t+1}^S\right)^2, \qquad \text{where}$$

$$\Omega_{t+1}^S \equiv \frac{1+\eta}{2+\theta}\left(1-\beta-\beta\left(\frac{1+\theta}{R_{t+1}^S}\right)aG_{t+1}\right)w_t, \qquad \Psi_{t+1}^S \equiv \frac{1+\eta}{2+\theta}\left(\beta\left(\frac{1+\theta}{R_{t+1}^S}\right)(1-a)G_{t+1}\right)w_t \tag{15.S}$$

$$l_t = \frac{1}{2(\phi^{MAX} - \phi^{MIN})\Omega_{t+1}^L}\left(\Psi_{t+1}^L - \phi^{MIN}\Omega_{t+1}^L\right)^2, \qquad \text{where}$$

$$\Omega_{t+1}^L \equiv \frac{1+\eta}{2+\theta}\left(1-\beta-\beta\left(\frac{1+\theta}{R_{t+1}^L}\right)aG_{t+1}\right)w_t, \qquad \Psi_{t+1}^L \equiv \frac{1+\eta}{2+\theta}\left(\beta\left(\frac{1+\theta}{R_{t+1}^L}\right)(1-a)G_{t+1}\right)w_t \tag{15.L}$$

Aggregate (per capita) saving and borrowing under Bismarckian social security

Where $a=1$, in the Bismarckian case, equation (10) shows that the individual decision to be a saver or borrower does not depend on relative income. In the absence of other sources of heterogeneity, all households either save or borrow, giving per capita aggregate savings as

$$s_t^{BIS} = \frac{(1-\beta)(1+\eta)}{2+\theta}\left(1-\frac{\beta}{1-\beta}\left(\frac{1+\theta}{R_{t+1}^S}\right)G_{t+1}\right)w_t \qquad \text{where}$$

$$\varepsilon_t^{S,BIS} \equiv \frac{\partial s_t}{\partial R_{t+1}^S}\frac{R_{t+1}^S}{s_t} = \frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R_{t+1}^S - \beta(1+\theta)G_{t+1}} > 0 \tag{16.S}$$

Alternatively, per capita aggregate borrowing is

$$l_t^{BIS} = \frac{(1-\beta)(1+\eta)}{2+\theta}\left(\frac{\beta}{1-\beta}\left(\frac{1+\theta}{R_{t+1}^L}\right)G_{t+1} - 1\right)w_t \qquad \text{where}$$

$$\varepsilon_t^{L,BIS} \equiv \frac{\partial l_t}{\partial R_{t+1}^L}\frac{R_{t+1}^L}{l_t} \to \frac{-\beta(1+\theta)G_{t+1}}{(1-\beta)R_{t+1}^L - \beta(1+\theta)G_{t+1}} < 0 \tag{16.L}$$

Aggregate (per capita) saving and borrowing under Beveridgean social security

In the case of Beveridgean social security, where $a=0$, equations (12) give the following critical income levels, which determine whether an individual is a saver, borrower or non-participant:

$$\phi^{CH,BEV} = \frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R^S_{t+1}}, \qquad \phi^{CL,BEV} = \frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R^L_{t+1}} \tag{17}$$

The subset for whom $\phi^{CH,BEV} < \phi(i) < \phi^{MAX}$ save in aggregate

$$s_t^{BEV} = \frac{(1-\beta)(1+\eta)}{2(\phi^{MAX}-\phi^{MIN})(2+\theta)}\left(\phi^{MAX} - \frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R^S_{t+1}}\right)^2 w_t \qquad \text{where}$$

$$\varepsilon_t^{S,BEV} \equiv \frac{\partial s_t}{\partial R^S_{t+1}}\frac{R^S_{t+1}}{s_t} = \frac{2\beta(1+\theta)G_{t+1}}{(1-\beta)\phi^{MAX}R^S_{t+1} - \beta(1+\theta)G_{t+1}} \tag{18.S}$$

The other subset for whom $\phi^{MIN} < \phi(i) < \phi^{CL,BEV}$ borrow in aggregate

$$l_t^{BEV} = \frac{(1-\beta)(1+\eta)}{2(\phi^{MAX}-\phi^{MIN})(2+\theta)}\left(\frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R^L_{t+1}} - \phi^{MIN}\right)^2 w_t \qquad \text{where}$$

$$\varepsilon_t^{L,BEV} \equiv \frac{\partial l_t}{\partial R^L_{t+1}}\frac{R^L_{t+1}}{l_t} = \frac{2\beta(1+\theta)G_{t+1}}{(1-\beta)\phi^{MIN}R^L_{t+1} - \beta(1+\theta)G_{t+1}} \tag{18.L}$$

while the remaining subset, or whom $\phi^{CL.BEV} \le \phi(i) \le \phi^{CH,BEV}$, neither save nor borrow but exactly consume their current incomes.

Observation 3: *It is not possible to ascertain whether the elasticity of aggregate savings is higher or lower under BIS-PAYG.*

Even if the growth and interest factors, G_{t+1} and R^S_{t+1}, are common to both regimes, the elasticity is higher under BIS- (BEV-) PAYG if- $\frac{\beta(1+\theta)}{1-\beta}\frac{G_{t+1}}{R^S_{t+1}} > (<) 2-\phi^{MAX}$.

It is not possible to determine *a priori* which pension scheme will generate the greater aggregate interest elasticity. Under BEV-PAYG, aggregate savings may be more responsive to the interest rate, because it of the extra effect of changes in the number of savers. There is, however, generally a smaller subset of savers that have higher relative incomes and less elastic individual savings functions [Observation 1], so that the aggregating function itself may be less elastic. Generally, the relative size of the elasticities will depend on the functional form, the moments of the income distribution function and the position of the marginal saver within the distribution.

Observation 4: *Under BEV-PAYG, in the limiting case of convergence of the saving and borrowing interest factors,* $R_{t+1}^{S,BEV} \to R_{t+1}^{L,BEV} \to R_{t+1}^{BEV}$, the aggregate saving function converges to the form under BIS-PAYG,

$$s_t^{BEV} - l_t^{BEV} \to \left(\frac{1-\beta}{2+\theta}\right)\left(1 - \frac{\beta}{1-\beta}\left(\frac{1+\theta}{R_{t+1}^{BEV}}\right)G_{t+1}\right)w_t \,.$$

This demonstrates that in the form of PAYG does not matter for aggregate saving with a competitive finance sector.

3. A model of intra-generational financial trading (model 1)

Initially, we abstract from considerations of capital and production in order to consider intra-generational financial trading effects in isolation. Wages are here seen as exogenous endowments, which are generated from a stationary distribution, giving a unitary growth factor $G_{t+1} = 1$. Time-subscripting is suppressed, because everything else is regarded as time-invariant. The model is first solved from the benchmark of perfect competition in the financial sector. We then look at the model where there is a single monopoly bank and consider the effects of each of the PAYG schemes.

Competitive financial sector equilibrium benchmark in model 1

Competitive equilibrium requires that all individuals receive and pay the same rate of interest, $R(i) = R^*,\ \forall i$, and the rate of profit/wage ratio is zero, $\eta = 0$. Equation (10) becomes

$$s(i) = \frac{1}{2+\theta}\left((1-\beta)\phi(i) - \beta\left(\frac{1+\theta}{R^*}\right)(a\phi(i) + (1-a))\right)w$$

As there is no production sector and demand for investment funds, aggregate net household saving is zero,

$$s \equiv \int_{\phi^{MIN}}^{\phi^{MAX}} \phi(i) f(\phi(i)) di = \frac{1}{2+\theta}\left((1-\beta) - \beta\left(\frac{1+\theta}{R^*}\right)\right)w = 0 \qquad (19)$$

Observation 5: *If the financial sector is perfectly competitive, (i) aggregate saving does not depend on the PAYG scheme (BIS or BEV) and (ii) it coincides with the form under BEV-PAYG.*

The weighting a, does not appear in equation (19), which is a scaled-down version of equation (17.1).

The competitive interest rate is solved from (19) as

$$R^* = \left(\frac{\beta}{1-\beta}\right)(1+\theta) \tag{20}$$

Substitution back into the individual savings function (10) gives individual saving/borrowing as

$$s_t(i) = \frac{(1-\beta)(1-a)}{2+\theta}(\phi_t(i)-1)w_t \tag{21}$$

and individual indirect utility is solved from equation (1) as

$$V(i) = \frac{1}{1+\theta}\left(\ln\left(\frac{\beta}{1-\beta}\right) + (2+\theta)\ln\left(\left(\frac{(1-\beta)(1+\theta)}{2+\theta}\right)\left(\phi(i) + \left(\frac{1}{1+\theta}\right)(a\phi(i)+1-a)\right)w_t\right)\right) \tag{22}$$

As known, individuals with incomes that are above average incomes both save and have a prefer BIS-PAYG security. If the distribution of income is symmetric, then 50% will be savers and 50% will be borrowers and the preferred policy cannot be determined by majority voting, since the median voter will be indifferent between the two schemes under the assumption of a uniform distribution.

Monopoly finance in model 1

A single institution, termed "a monopoly bank". raises $s(R^S;..)$ funds and lends the amount $l(R^L;..)$ to borrowers in order to maximise the profit,

$$\Pi = (R^L-1)l(R^L;..) - (R^S-1)s(R^S;..), \qquad l \le s \tag{23}$$

As $R^L \ge 1$, it is optimal for lending must to be at its maximal, feasible level where $l = s$. There are two parts to the optimisation problem: the choice of a pair of interest factors, (R^L, R^S), which satisfy maximum lending,

$$l(R^L;..) = s(R^S;..) \tag{24}$$

and a choice of one of this pair to maximise profits $(R^L - R^S)l(R^L;..) = (R^L - R^S)S(R^S;..)$. As the savings and investment functions are each monotonic functions of their own interest factors, we can invert the lending-saving equality in (24) to get

$$R^L = f(R^S), \qquad \text{where } f(..) \equiv l^{-1}(s(..)), \quad f'(R^S) = \frac{\partial s(R^S;..)}{\partial R^S} \Big/ \frac{\partial l(R^L;..)}{\partial R^L} \tag{25}$$

Endogenising the lending-saving constraint gives the profit function as

$$\Omega = \left(f(R^S) - R^S\right)s\left(R^S\right)$$

The other interest factor, R^S, is then chosen to maximise this function. The first-order condition for an interior solution is,

$$\frac{\partial \Omega}{\partial R^S} = \left(f'(R^S) - 1\right)s\left(R^S\right) + \left(R^L - R^S\right)\frac{\partial s\left(R^S\right)}{\partial R^S} = 0$$

The interior solution may be expressed in terms of a ratio between the two interest factors

$$R^S = \left(\frac{1 - \varepsilon_L{}^{-1}}{1 + \varepsilon_S{}^{-1}}\right) R^L, \qquad \text{where}$$

$$\varepsilon_S \equiv \frac{\partial s(R^S;..)}{\partial R^S} \Big/ \frac{s(R^S;..)}{R^S} > 0, \qquad \varepsilon_L \equiv -\frac{\partial l(R^L;..)}{\partial R^L} \Big/ \frac{l(R^L;..)}{R^L} > 0 \tag{26}$$

The simultaneous solution of equations (26) and (27) determines the two interest factor *unknowns*.

Monopoly finance and BIS-PAYG in model 1

Inspection of equation (10) shows that in the limiting case of BIS-PAYG, $\alpha \to 1$ all individuals will either save, borrow or just consume from their current incomes. The condition of zero net aggregate saving implies that each individual will save zero, having an infinite elasticity (with zero savings), as $\varepsilon^S, \left|\varepsilon^L\right| \to \infty$, and so that $R^S \to R^L \to R^*$ and so profits are zero, $\eta = 0$. This is also the competitive solution.

Utility is maximised at

$$V(i)\big|_{a=1} = \ln(1-\beta) + \frac{1}{1+\theta}\ln\beta + \left(\frac{2+\theta}{1+\theta}\right)\left(\ln\phi(i) + \ln w\right) \tag{27}$$

A monopoly bank and BEV-PAYG in the model 1

We now consider BEV-PAYG, where $a=0$. Using (20) in (16) for $a=0$ and $G_{t+1}=1$ gives the critical relative income levels as

$$\phi^{CH}=R^*/R^S, \qquad \phi^{CL}=R^*/R^L \tag{28}$$

Substitution into the aggregate elasticities in equations (18) gives

$$\varepsilon_t^{S,BEV}=\frac{2}{\phi^{MAX}R^S/R^*-1}, \qquad \varepsilon_t^L\big|_{a=0}=\frac{2}{\phi^{MIN}R^L/R^*-1} \tag{29}$$

Substitution also into the aggregate saving and borrowing equations[8] in (18) gives

$$s^{BEV}=\frac{(1+\eta)(1-\beta)}{2(2+\theta)(\phi^{MAX}-\phi^{MIN})}\left(\phi^{MAX}-\phi^{CH}\right)^2 w_t$$

$$l^{BEV}=\frac{(1+\eta)(1-\beta)}{2(2+\theta)(\phi^{MAX}-\phi^{MIN})}\left(\phi^{CL}-\phi^{MIN}\right)^2 w_t$$

Condition (24) requires that $\phi^{MAX}-\phi^{CH}=\phi^{CL}-\phi^{MIN}$ or - since $\phi^{MIN}+\phi^{MAX}=2$ because of a symmetric equilibrium centred on unity - that

$$\phi^{CH}+\phi^{CL}=2 \tag{30}$$

Equations (28) and (29) give

$$\varepsilon^S=\left(\frac{2\phi^{CH}}{\phi^{MAX}-\phi^{CH}}\right), \qquad \varepsilon^L=-\left(\frac{2\phi^{CL}}{\phi^{CL}-\phi^{MIN}}\right)$$

which with equations (26) and (30) give a cubic solution for the critical relative income levels,

$$\phi^{CL^3}-\left(3+\frac{\phi^{MAX}-\phi^{MIN}}{2}\right)\phi^{CL^2}+2\left(1-\phi^{MIN}\right)\phi^{CL}+2\phi^{MIN}=0, \qquad \phi^{CH}=2-\phi^{CL} \tag{31}$$

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[8] Price discrimination implies that individual saving/borrowing is

$$s(i)=\frac{(1+\eta)(1-\beta)}{2+\theta}\left(\phi(i)-\phi^{CH}\right)w_t \qquad \text{if } \phi(i)>\phi^{CH}$$

$$s(i)=0 \qquad \text{if } \phi^{CH}\le\phi(i)\le\phi^{CH}$$

$$s(i)=\frac{(1+\eta)(1-\beta)}{2+\theta}\left(\phi(i)-\phi^{CL}\right)w_t \qquad \text{if } \phi(i)<\phi^{CL}$$

We consider the convenient case of maximal income dispersion, where $\phi^{MIN} = 0$, $\phi^{MAX} = 2$, which generates a quadratic solution,

$$\phi^{CL} = 0,\ 2 \mp \sqrt{2} = \ 0,\ 0.586,\ 3.414, \qquad \phi^{CH} = 2,\ \pm\sqrt{2} = \ 2,\ 1.414,\ -1.414 \tag{32}$$

Only the intermediate solutions, $\phi^{CL} = 0.586$ and $\phi^{CH} = 1.414$, are both feasible and consistent with financial activity. They are feasible because they satisfy $\phi^{CL} < 2$ and $\phi^{CH} > 0$ and consistent with a financial activity, which requires $0 < \phi^{CL}$ and $\phi^{CH} < 2$.

The introduction of Beveridgean social security causes the proportions of savers and borrowers to fall each from 50% to 29.3% and introduces a new proportion, 41.4%, of households, which consume just from their current incomes in each period. The savings rate of interest falls to 0.707R*, while the borrowing rate of interest rises to $R^L = 1.707R^*$. Assigning an annualised value of the competitive interest rate at 5% and thirty-five year length for a half life, implies that under monopoly finance the effect of BEV-PAYG is to determine annualised saving and borrowing interest rates at 3.97% and 6.62%.

The rate of profit is solved as

$$\eta = \left(R^L - R^S\right)\frac{(1+\eta)(1-\beta)}{(2+\theta)}\left(\phi^{MAX} - \phi^{CH}\right)^2 \quad \text{or} \quad 1+\eta = \left(1 - \left(R^L - R^S\right)\left(\frac{1-\beta}{2+\theta}\right)\left(\phi^{MAX} - \phi^{CH}\right)^2\right)^{-1}$$

Given the interest rate solutions from equations (20), (28) and (31), the profit-wage factor is solved as

$$1+\eta = \left(1 - \beta\left(\frac{1+\theta}{2+\theta}\right)\left(\frac{1}{\phi^{CL}} - \frac{1}{\phi^{CH}}\right)\left(\phi^{MAX} - \phi^{CH}\right)^2\right)^{-1}$$

$$\frac{1}{(1+\eta)}\frac{\partial \eta}{\partial \beta} = \left(\left(\frac{2+\theta}{1+\theta}\right)h - \beta\right)^{-1} \quad \text{where } h \equiv \left(\frac{1}{\phi^{CL}} - \frac{1}{\phi^{CH}}\right)^{-1}\left(\phi^{MAX} - \phi^{CH}\right)^{-2} \tag{33}$$

Although this is monotonically increasing in β, there will be no appreciable effect on the preferences of young household regarding its value.

Welfare and preferences in model 1

There are there are three effects of moving towards BIS-PAYG under monopoly banking:

(i) A direct redistribution effect from households for whom $\phi(i) > 1$ to those for whom $\phi(i) < 1$

(ii) Negative and positive effects on the saving and borrowing interest rates, $\frac{\partial R^S}{\partial(1-a)} < 0$ and $\frac{\partial R^L}{\partial(1-a)} > 0$.[9]

(iii) A positive profit effect, $\frac{\partial \eta}{\partial(1-a)} > 0$.

Result 1: *In Model 1 BEV-PAYG is democratically preferred to BIS-PAYG, if the financial sector is monopolistic and if profits are returned through the fiscal rule.*

We consider first the household with the lowest income, $\phi(i) = 0$. BEV-PAYG is strictly preferred as the difference between having some wealth [effects (i) and (iii)] - for any finite level of the interest factor [effect (ii)] and no wealth at all. Second, we consider the median income household, $\phi(i) = 1$. This not gain from redistribution effect (i), as $\phi(i) = 1$; effect (ii) is irrelevant to it, because it never saves nor borrows; but it gains, like all households from the return of profits effect (iii) under BEV-PAYG. Thirdly, a reasonable property is that household preferences between PAYG schemes remain ordered according to relative incomes. This then implies that all households for which $0 \le \phi(i) \le 1$ will prefer BEV-PAYG.[10] Households with relative incomes above but close to unity $\phi(i) > 1$[11], save very small amounts, so that both effects (i) and (ii) are very small. Effect (iii) then ensures a clear majority would vote for BEV-PAYG.

This majority could be quite high. Consider an individual for whom $\phi(i) = \phi^{CH}$, who is just indifferent between being saving and non-participating under BEV-PAYG. Respective indirect utility levels [See the Appendix] under BIS- and BEV-PAYG for this household are

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[9] This is deduced from monotonicity, where $R^{S,BEV} < R^{S,BIS} = R^{S,COMP}$ and $R^{L,BEV} > R^{L,BIS} = R^{L,COMP}$.

[10] To assume otherwise implies that there might be some low income households, which, implausibly, would lose more from paying a higher lending rate than they would gain both from the direct redistribution and the return of profits.

[11] They may not save at all with transaction costs

$$\ln(1-\beta)+\frac{1}{1+\theta}\ln\beta+\left(\frac{2+\theta}{1+\theta}\right)\left(\ln\phi^{CH}+\ln w\right),\quad \ln(1-\beta)+\frac{1}{1+\theta}\ln\beta+\ln\phi^{CH}+\left(\frac{2+\theta}{1+\theta}\right)\left(\ln(1+\eta)+\ln w\right)$$

Generally this household would prefer BEV-PAYG if $(1+\eta)^{2+\theta} > \phi^{CH}$. This is ensured if θ is sufficiently high. BEV-PAYG would then get a majority vote of at least 70.7% for the assigned parameter values.

BEV-PAYG increases saving and borrowing at the two ends of the income distribution, but on worst terms, so the poor like it less and the rich dislike it more. Profits are created by the bank, which, we assume critically, are returned to all households through fiscal policy. There are, therefore, two re-distributions, the standard one from the right-hand-side to the left-hand-side of the distribution and a further one from both sides to the centre, where the median voter is located. The two re-distributions imply that BEV-PAYG may be supported by a coalition of low- and middle-income households even if there is a uniform distribution of income.

This model has relied on the assumption that a sizeable proportion of low-income households are able to borrow against their future social security incomes. The assumption is fully consistent with the perfect information world being modelled, but, of course, is not supported empirically. It is a necessary feature to generate financial activity in a two-period model, which does not have a store of value. The assumption enables us to make a basic point about the effect of monopolistic financial profits on median voter preferences, which we believe extends to other models. In the next section, the model is generalised to include capital and inter-generational financial trading.

4. An overlapping generations model with endogenous growth

We now generalise the model by incorporating capital, productivity and growth. Wages and interest rates depend also on productivity. Three groups now enter the financial sector, firms that borrow for investment, households that borrow for consumption and saving households. The potential complexity of the financial sector optimisation problem points us to the Romer (1986) model of endogenous growth. One merit of this model is that a single financial institution may be able to internalise the investment externality, so that the demand for capital is infinitely elastic under the constant social returns to scale assumption. This pegs the borrowing rate of interest at the competitive level, so that it is easier to solve the other two household interest rates for functions, which have finite elasticities. Another merit is the absence of transitional dynamics in the growth process. Even, so we find that we still have to resort to numerical solutions for the case of BEV-PAYG and, hence, to make comparisons.

The model

There is a unit measure of firms, indexed z, which produce under the conditions of constant internal returns to scale in own-capital, $k_t(z)$ and labour, $n_t(z)$ capital and constant external returns to scale in own capital and aggregate capital, k_t, following Romer (1986):

$$y_t(z) = Ak_t^{1-\mu} k_t(z)^{\mu} n_t(z)^{1-\mu}, \qquad n_t = 1 = \int_{\phi^{MIN}}^{\phi^{MAX}} \phi(i) f(\phi(i)) di \tag{34}$$

The firm employs workers over a whole spectrum of abilities and we assume that there is competitive market for each level of ability, parameterised by its relative productivity level, $\phi(i)$.[12]

The firm is a price-taker and maximises its profit given by

$$\Omega_t(z) = Ak_t^{1-\mu} k_t(z)^{\mu} n_t(z)^{1-\mu} - R_t^F k_t - w_t \int_{\phi^{MIN}}^{\phi^{MAX}} \phi(i) f(\phi(i)) di \tag{35}$$

Maximisation requires that its interest cost of borrowing, R_t^F, is equated with the marginal revenue product of capital, which with the normalisation, $n_t = 1$, gives

$$R_t^F = \mu A k_t^{1-\mu} k_t(z)^{\mu-1} \tag{36}$$

The production sector is perfectly competitive and the *average* wage is solved under the conditions of constant returns to scale and of zero profit, $\Omega_t(z) = 0$, from no further entry or exit, to give

$$w_t(z) = (1-\mu) A k_t^{1-\mu} k_t(z)^{\mu} \tag{37}$$

Imposing the condition of symmetric equilibrium, $k_t(z) = k_t$, $\forall z$, across firms ensures that the interest factor at which at the firm borrows and the average wage paid out to its s workers are

$$R_t^F = \mu A \tag{38}$$

$$w_t(z) = (1-\mu)Ak_t, \quad \forall z \tag{39}$$

The investment-savings equilibrium condition under the assumption of 100% depreciation fixes the capital stock at the level of gross investment, gives

$$k_{t+1} = s_t \tag{40}$$

A competitive financial sector

If the finance market is perfectly competitive, households borrow and save at the same rate of interest and there is no discrimination between borrowing households and borrowing firms. In symmetric equilibrium, the common interest rate factor is pinned down by the conditions of profit maximisation, constant external returns to scale in capital and symmetric equilibrium in the production sector,

$$R_t^F = R_t^S = R_t^L = R_t = \mu A, \; \Pi_t = 0 \tag{41}$$

The form of the savings equation is

$$s_t = \left(\frac{1-\beta}{2+\theta}\right)\left(1 - \frac{\beta}{1-\beta}\left(\frac{1+\theta}{R_{t+1}}\right)G_{t+1}\right)w_t$$

The savings equation with equations (38)-(40) with the growth definition in (3) combine to give a linear solution for the growth factor under a competitive finance market,

$$G^{COMP} = \frac{(1-\beta)A}{\left((2+\theta)/(1-\mu)\right) + \left(\beta(1+\theta)/\mu\right)}, \tag{42}$$

There are two points to make. There is an absence of transitional dynamics in all cases. Secondly, the form of PAYG has no effect on aggregate saving, as before, and growth, where the financial sector is competitive.

Monopolistic finance in model 2

[12] Alternatively, firms could specialise in employing workers of a particular ability, parameterised by a firm-specific total factor productivity parameter, $A(z)$.

There are two complications which arise under the assumption of a single financial firm which generates profits that are redistributed to households. The distribution of profits over time will affect the savings decision of young households with inter-generational trading. The financial sector may perceive that behaviour may be manipulated through inter-generational effects from the return of profits. A single financial firm would potentially be able to internalise this effect, which will then alter the conditions for profit maximisation. We discount this complicating factor by imposing at the outset the condition that expectations of profit-wage ratio are constant over time.

The second issue concerns the fact that a monopoly bank single-handedly determines the capital stock, the levels of activity, wages and pension benefits. To avoid this complication, we discount this effect in the bank's optimisation. This departure from full rationality is made in the interests of analytical clarity. It is probably stretching credibility too far, if a single financial firm is able to consciously decide what the levels of these aggregate variables are going to be, although it is consistent with full rationality. The single-firm case is only after all a convenient alternative to the more usual benchmark of perfectly competitive banking.[13] A small departure from strict rationality makes it easier to analyse the model much without, we believe, overturning its main results.

Monopolistic finance and BIS-PAYG in model 2

BIS-PAYG ensures that all households have the same replacement ratio. As households are also assumed to be homogeneous in all other respects, households will either all save, borrow or just spend from their current incomes in each period. As firms are now present, which only borrow, households in aggregate must savers to ensure an aggregate equilibrium with zero net saving and each households saves given their homogeneous under BIS-PAYG. Households all face the same interest rate,

$$R_t^H = R_t^H(i), \ \forall i$$

[13] There is also another issue that the rate of interest at which firms borrow is the same under competition. This is because of the model-specific feature that the demand for capital in general equilibrium is in effect infinitely elastic.

The objective function of the monopoly bank is to maximise

$$\Pi_{t+1} = \left(\mu A - R^{H}_{t+1}\right)s_t$$

The savings function is given by equation (18) and the elasticities condition (32) for a maximum implies that the household rate of interest is

$$R^{H}_{t+1} = \left(\frac{\mu A \beta(1+\theta)G_{t+1}}{1-\beta}\right)^{\frac{1}{2}} \tag{43}$$

Substitution back into the savings function gives

$$s_t^{BIS} = \left(\frac{1-\beta}{2+\theta}\right)\left(1-\left(\frac{\beta(1+\theta)G_{t+1}}{(1-\beta)\mu A}\right)^{\frac{1}{2}}\right)w_t$$

which with equations (38)-(40) with the growth definition in (3), again, give a quadratic solution for growth under monopoly banking and Bismarckian social security,

$$G^{BIS} = (1-\mu)A\left(\frac{1-\beta}{2+\theta}\right)\left(1-\left(\frac{\beta(1+\theta)}{(1-\beta)\mu A}\right)^{\frac{1}{2}}G^{BIS\,\frac{1}{2}}\right) \quad \text{or}$$

$$G^{BIS} = h + \frac{j^2}{2} + j\left(h + \left(\frac{j}{2}\right)^2\right)^{\frac{1}{2}} \quad \text{where}$$

$$h \equiv (1-\mu)A\left(\frac{1-\beta}{2+\theta}\right), \qquad j \equiv (1-\mu)A\left(\frac{1-\beta}{2+\theta}\right)\left(\frac{\beta(1+\theta)}{(1-\beta)\mu A}\right)^{\frac{1}{2}} \tag{44}$$

The solution differs from the competitive one, because there is price discrimination between firms and households.

Monopolistic finance and BEV-PAYG in model 2

Households generally have different replacement ratios, so there is scope to charge borrowing households a rate of interest, which is higher than the return on savings. The infinite interest elasticity of investment also implies that households will also borrow on worst terms than firms. Some households will save, others will borrow and a proportion will choose not to participate at all. The bank's profit function is

$$\Pi_{t+1} = R^{F}_{t+1}k_{t+1} + R^{L}_{t+1}l_t - R^{S}_{t+1}s_t$$

where $k_{t+1} = s_t - l_t$

$$s_t^{BEV} - l_t^{BEV} = \frac{(1-\beta)(1+\eta)}{2(\phi^{MAX} - \phi^{MIN})(2+\theta)}\left(\left(\phi^{MAX} - \frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R_{t+1}^S}\right)^2 - \left(\frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R_{t+1}^L} - \phi^{MIN}\right)^2\right)w_t$$

$$G_{t+1}^{BEV} = \frac{(1-\beta)(1-\mu)A}{2(\phi^{MAX} - \phi^{MIN})(2+\theta)}\left(\left(\phi^{MAX} - \frac{\beta(1+\theta)G_{t+1}^{BEV}}{(1-\beta)R_{t+1}^S}\right)^2 - \left(\frac{\beta(1+\theta)G_{t+1}^{BEV}}{(1-\beta)R_{t+1}^L} - \phi^{MIN}\right)^2\right) \quad (45)$$

This may be split into two parts, showing the net gain in channelling savings to firms and the net gain in diverting funds from firms to households,

$$\Pi_{t+1} = (R_{t+1}^F - R_{t+1}^S)s_t + (R_{t+1}^L - R_{t+1}^F)l_t$$

The profit maximisation can also be solved in these two parts, because in symmetric general equilibrium, in effect, there is an infinitely elastic loan demand from firms, which pins down their rate of interest according to (46).

Equations (18) and (26) determine the household saving and borrowing interest factors as

$$R^{S,BEV} = -\frac{\beta(1+\theta)G^{BEV}}{2(1-\beta)\phi^{MAX}} + \left(\left(\frac{\beta(1+\theta)G^{BEV}}{2(1-\beta)\phi^{MAX}}\right)^2 + \frac{2\mu A\beta(1+\theta)G^{BEV}}{(1-\beta)\phi^{MAX}}\right)^{\frac{1}{2}} \quad (46.L)$$

$$R^{L,BEV} = -\frac{\beta(1+\theta)G^{BEV}}{2(1-\beta)\phi^{MIN}} + \left(\left(\frac{\beta(1+\theta)G^{BEV}}{2(1-\beta)\phi^{MIN}}\right)^2 + \frac{2\mu A\beta(1+\theta)G^{BEV}}{(1-\beta)\phi^{MIN}}\right)^{\frac{1}{2}} \quad (46.S)$$

Results 2: *In the OLG-growth model, for a value of the capital share parameter,* μ*, that is not extremely and implausibly low , the median voter is always a saver.*

Proof: This is certainly true in the competitive case and under BIS-PAYG where households are homogeneous with respect to whether they save or borrow and where firms always borrow, regardless of the value of μ. It applies in the Beveridgean case if

$R^{S,BEV} > \frac{\beta(1+\theta)G^{BEV}}{(1-\beta)}$ or from equation (46), after manipulation, if

$$\frac{G^{BEV}}{(1-\beta)} < \frac{2\mu A}{\beta(1+\theta)\left(1+\phi^{MAX}\right)}$$

This is certainly true where β is low, but uncertain where $\beta \to 1$, because there $G^{BEV} \to 0$. However, using the intuition that where profits are not returned to households, $G^{COMP} > G^{BEV} \ \forall\beta$, a sufficient condition for the inequality condition is that

$\frac{G^{COMP}}{(1-\beta)} < \frac{2\mu A}{\beta(1+\theta)\left(1+\phi^{MAX}\right)}$. Using equation (42), this becomes $\phi^{MAX} - 1 < 2\left(\frac{2+\theta}{1+\theta}\right)\left(\frac{\mu}{1-\mu}\right)$. The least favourable case is where $\phi^{MAX} = 2$ and $\theta \to \infty$, which reduces to an inequality condition for the capital share, $\mu > 1/3$. This is a sufficient, not a necessary condition, since generally, $\beta < 1$, $G^{COMP} > G^{BEV}$, $\phi^{MAX} \leq 2$, so that the capital share can be well less than its stylised value of one-third for the result still to hold. The median voter will remain a saver, if profits are returned to households according to the fiscal rule.

This result derives from the specification of a uniform distribution of income and the property of linear individual saving functions. A side result, which emerges from the numerical simulations below, is that the number of borrowers will be very small, so the main effect of the BEV-PAYG is to cause non-participation. This may suggest that BEV-PAYG necessarily reduces aggregate saving, but this is true, because it also induces high-income individuals to save high amounts, while low-income do not offset this by borrowing high amounts, if at all.

Rather, we find that saving and growth may actually be higher under BEV-PAYG. This point is difficult to demonstrate analytically in a direct way, so we approach it indirectly. We look instead at the effect on aggregate saving of changing the degree of income dispersion in the presence of BEV-PAYG. A change in the level of dispersion for a given BEV-PAYG is analogous to change in the BEV-PAY component for a given level of dispersion, because the effect of each of these is to alter the distribution of individual replacement ratios. We consider a change in dispersion, abstracting from the return of profits

Result 3: *In the OLG-growth model, increased dispersion has a positive effect on net aggregate saving under the Beveridgean scheme, if* β *is low, and a negative effect if* β *is high.*

We define $\phi^{MIN} \equiv 1-\sigma$, $\phi^{MAX} \equiv 1+\sigma$, where σ, $0 \leq \sigma \leq 1$, is a measure of dispersion in household incomes, and consider its effect on Beveridgean net aggregate saving. There are two effects, a direct effect *(E1)* and another indirect effect *(E2)* that works through interest rates. To isolate the first effect, we fix the interest factors, $R^S_{t+1} = \overline{R}^S_{t+1}$, $R^L_{t+1} = \overline{R}^L_{t+1}$. Equation (45) is presented as

$$s_t^{BEV} - l_t^{BEV} = \frac{(1-\beta)}{4(2+\theta)\sigma}\left(\left(1+\sigma-\frac{\beta(1+\theta)G_{t+1}}{(1-\beta)\overline{R}^S_{t+1}}\right)^2 - \left(\frac{\beta(1+\theta)G_{t+1}}{(1-\beta)\overline{R}^L_{t+1}} - (1-\sigma)\right)^2\right)w_t$$

We define $\Gamma^S \equiv 1+\sigma-\frac{\beta(1+\theta)G^{BEV}}{(1-\beta)\overline{R}^S_{t+1}}$, $\Gamma^L \equiv \frac{\beta(1+\theta)G_{t+1}}{(1-\beta)\overline{R}^L_{t+1}} - (1-\sigma)$

Differentiating with respect to dispersion, for fixed interest factors, gives

$$\frac{\partial\left(s_t^{BEV} - l_t^{BEV}\right)}{\partial\sigma}\Big|(E1) = \frac{-(1-\beta)}{4(2+\theta)\sigma^2}\left(\Gamma^{S2} - \Gamma^{L2}\right)w_t + \frac{(1-\beta)}{4(2+\theta)\sigma}\left(2\left(\Gamma^S - \Gamma^L\right)\right)w_t$$

Manipulation gives

$$\frac{\partial\left(s_t^{BEV} - l_t^{BEV}\right)}{\partial\sigma}\Big|(E1) = \frac{\beta(1+\theta)}{4(2+\theta)\sigma^2}\left(\Gamma^S - \Gamma^L\right)\left(\frac{1}{\overline{R}^S_{t+1}} - \frac{1}{\overline{R}^L_{t+1}}\right)w_t > 0$$

The first effect is positive in sign, because $\Gamma^S > \Gamma^L$ from the condition that firms borrow, so that $s_t^{BEV} - l_t^{BEV} > 0$, and because $\overline{R}^L_{t+1} > \overline{R}^S_{t+1}$ under imperfect competition.

The indirect effect works through the interest elasticities. Turning to the interest rate solutions in equations (46), we find that

$$\frac{\partial R^{S,BEV}_{t+1}}{\partial\sigma} = \frac{\beta(1+\theta)G^{BEV}}{(1-\beta)}\left(\frac{1}{2\phi^{MAX}} - \frac{1}{2}\left(\frac{1}{\sqrt{\left(..|_S\right)}}\right)\left(\frac{1}{\phi^{MAX^3}} + \frac{2\mu A(1-\beta)}{\beta(1+\theta)G^{BEV}_{t+1}\phi^{MAX^2}}\right)\right) < 0$$

$$\frac{\partial R^{L,BEV}_{t+1}}{\partial\sigma} = \frac{\beta(1+\theta)G^{BEV}}{(1-\beta)}\left(\frac{1}{2\phi^{MIN}} - \frac{1}{2}\left(\frac{1}{\sqrt{\left(..|_L\right)}}\right)\left(\frac{1}{\phi^{MIN^3}} + \frac{2\mu A(1-\beta)}{\beta(1+\theta)G^{BEV}_{t+1}\phi^{MIN^2}}\right)\right) > 0$$

An increase in dispersion will lower the saving rate of interest and, hence, saving, and raise the borrowing rate of interest, reducing borrowing. The effects on net saving are given by

$$s_t^{BEV} - l_t^{BEV} = \frac{(1-\beta)}{4(2+\theta)\sigma}\left(\left(1+\sigma-\frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R^{S,BEV}_{t+1}}\right)^2 - \left(\frac{\beta(1+\theta)G_{t+1}}{(1-\beta)R^{L,BEV}_{t+1}} - (1-\sigma)\right)^2\right)w_t$$

Differentiating with respect to dispersion,

$$\frac{\partial\left(s_t^{BEV}-l_t^{BEV}\right)}{\partial\sigma}\Big|(E2)=\frac{2\beta(1+\theta)G_{t+1}^{BEV}}{4(2+\theta)\sigma^2}\left(\frac{\Gamma^S}{\left(R_{t+1}^{S,BEV}\right)^2}\frac{\partial R_{t+1}^{S,BEV}}{\partial\sigma}+\frac{\Gamma^L}{\left(R_{t+1}^{L,BEV}\right)}\frac{\partial R_{t+1}^{L,BEV}}{\partial\sigma}\right)w_t$$

Even if there is no effect on the relative scale of saving and borrowing, net saving as a whole would fall. However, the reduction in savings will swamp any increase in borrowing, again because of the conditions of the model that $\Gamma^S > \Gamma^L$ and $\overline{R}_{t+1}^L > \overline{R}_{t+1}^S$.

It is apparent that the magnitude effect is increasing in the value of because β, which determines the interest elasticities of saving and borrowing. Therefore, we conclude that increased dispersion will raise net aggregate saving where β is low and raise it where β is high. However, where β is very low, the scale of any PAYG effects is also small.

A corollary is that the same inference can be made for the growth factor. Growth and saving always move together, because growth is a positive multiple of net savings - from the property of constant returns to scale in the capital stock - and because net savings is a monotonically decreasing function of the growth factor through consumption smoothing. The analysis has the same implications for the savings rate of interest of which savings and growth are monotonically increasing functions. To summarise, if β is low, increased dispersion in incomes will raise saving, growth and the saving rate of interest under Beveridgean PAYG, while these effects will be reversed for high values of β. Consequently, we cannot prove conclusively whether growth will be higher or lower under BEV-PAYG.

Simulations

The purpose of the simulations is merely to look at the relative comparative statics of BIS- and BEV-PAYG, illustrating the characteristics of the particular model specification and parameter values. We believe the results are highly sensitive to the specification and values, so we use them to point to possibilities rather than to make definitive statements of what we would expect to see.

Some simulated values where profits are not returned

In this subsection we abstract from the return of profits to make some conclusions about how preferences for a particular scheme may relate to the value of β. In the following subsection, we then show how incorporating the return of profits in to the analysis strengthens these results.

Throughout we assign the following parameter values. We fix the competitive interest rate at an annualised rate of 4.5%, which gives a compounded factor over a thirty-five year half-life of 5.516. We also consider the case of maximal income dispersion where $\phi^{MIN} = 0$, $\phi^{MAX} = 2$. This implies a constant borrowing elasticity of -2, so that the household borrowing interest factor is 11.032, which is annualised at the rate of 7.10%. The time preference parameter is set at $\theta = 4$. Initially we consider a low value for the pension of at $\beta = 0.10$ and then raise it to higher one at $\beta = 0.40$. Also, we abstract from the return of profits to households. In the tables, we report in bold-type the values which change across simulations. In the first simulation where $\beta = 0.10$, we calibrate the annualised growth rate at 2.50% for Beveridgean PAYG and determine total factor productivity, A, and the capital share, μ, endogenously. We then use these same determined values in all the following simulations.

The lack of transitional dynamics implies that the choice of PAYG scheme leads to a particular steady-state growth factor, which also determines the subsequent level of pension payments. It also means that that the starting wage at any time, which is normalised to unity, can be regarded not just as a common factor in any utility function, but as a factor which is common to utility levels under different PAYG schemes.

Table One: A low value of the pension, $\beta = 0.10$, where profits not returned

	Competitive finance ***With either scheme***	***Monopoly finance with Beveridgean scheme***	***Monopoly finance with Bismarckian scheme***
Growth factor	**3.126 (3.31%)**	**2.373 (2.50%)**	**2.343 (2.46%)**
Firms' borrowing interest factor	5.516 (5%)	5.516 (5.00%)	5.516 (5.00%)
Household savings interest factor	5.516 (5%)	**2.387 (2.52%)**	**2.680 (2.86%)**
Household borrowing interest factor	5.516 (5%)	11.032 (7.10%)	N/A
Financial profit-wage ratio (approx)	0	**10.29%**	**9.33%**
Median income household utility	**-0.252**	**-0.368**	**-0.359**

First, we see that the imperfect competition reduces the annual growth rate significantly by about 0.80%, but that the effect of particular PAYG scheme is relatively trivial. Equation (42) shows that if the finance sector is competitive, the annualised growth rate would be 4.74% in the absence of PAYG. Consequently, about 1/3 of the fall to the reported numbers of about 2.50% is attributed to imperfect competition, while the remainder to PAYG, whether it is BIS or BEV.

These values show that the critical relative income levels are $\phi^{CL} = 0.126$, $\phi^{CH} = 0.582$, so that 70.9% of households will save, 6.3% will borrow and 22.8% will choose not to participate financially. A uniform distribution with social security at $\beta = 0.10$ causes household aggregate borrowing to be a trivial 0.67% of household aggregate borrowing under BEV-PAYG. Consequently, little would change if borrowing constraints were imposed on households and it may be a rational response of the bank, given the thinness of this market with in the presence of operating costs.

We see that the median voter has a slight preference for Bismarckian social security, as the savings rate of interest is higher although the growth rate is very slightly lower.

In the next simulations we raise the value of PAYG to $\beta = 0.40$, so that the average net of tax replacement rises six-fold from 11.11% to 66.67%.

Table Two: A high value of the pension, $\beta = 0.40$, ***where profits not returned***

	Competitive finance with either scheme	***Monopoly finance with Beveridgean scheme***	***Monopoly finance with Bismarckian scheme***
Growth factor	**1.073 (0.20%)**	**0.894 (-0.31%)**	**0.855 (-0.45%)**
Firms' borrowing interest factor	5.516 (5%)	5.516 (5.00%)	5.516 (5.00%)
Household savings interest factor	5.516 (5%)	**3.377 (3.54%)**	**3.965 (4.01%)**
Household borrowing interest factor	5.516 (5%)	11.032 (7.10%)	N/A
Financial profit-wage ratio (approx)	0	**7.04%**	**5.10%**
Median income household utility	**-0.666**	**-0.715**	**-0.718**

Increased social security reduces the growth rate, as expected. These values show that the critical relative income levels are $\phi^{CL} = 0.270$, $\phi^{CH} = 0.882$, so that a smaller proportion, 55.9%, of households save, 13.5% borrow and 30.6% choose not to participate financially. This time, the proportion of household borrowing to saving is still small, but significantly larger at 5.84%, so there will be a discernible crowding-out effect on investment. In this case borrowing constraints would have some effect on the solution under BIS-PAYG.

A calculation of the values and a comparison of Tables One and Two shows that the rise in the level of β from 0.10 to 0.40 with BEV-PAYG causes a drop in the proportion of savers from 70.9% to 55.9%, a rise in the proportion of borrowers from borrowing from 6.3% to 13.5% and a rise in the non-participation proportion from 22.8% to 32.6%.

The rise in β causes a substantial drop in the growth factor for both schemes, but causes the savings interest factors to rise through an increase in the elasticities. The overall effect is to reduce utility under both schemes, but we see that the utility of the median income household under monopoly finance is not so low relative to that under competition with a higher level of PAYG.

If there were also social preferences for a moderate element of income redistribution, it is possible that BEV-PAYG would be preferred where $\beta = 0.10$, but BIS-PAYG where $\beta = 0.40$. This would then be another way of explaining the association between higher levels of PAYG and the operation of a BIS scheme.
Finally, the minority of non-participants will have unambiguously increased preferences for BEV-PAYG, since both the growth factor and the profit rate will be higher.

Where financial profits are returned

We have abstracted from the return of financial sector profits. The analysis could stand with at least one of the rationalisations that profits are expatriated or appropriated to fund wasteful government expenditure. If they are not used to fund a beneficial public good, there are two other broad possibilities. One is that profits are endogenously returned as dividend payments to households, which have acquired financial sector equity. Roberts (2003) shows that this causes a crowding-out of productive saving and may reduce the economy's capital stock. The other is that they are returned exogenously through fiscal policy to reduce labour income taxation[14] and/or to increase spending on pension benefits.

In general the level of taxation on dividend income would determine the weightings of the endogenous and exogenous return of profits. Profits are returned endogenously with a zero rate of tax on dividends and exogenously with a 100% rate. We assume the latter, which is consistent with the focus of the present analysis: even if the median voter is a saver, a very small proportion of savers actually hold equities.

The further assumption is that the return of profits through a combination of raising the net of tax incomes of the young and the benefit incomes of the old is consistent with the inter-generational redistribution rule, parameterised by β. The implication of this is that whereas the average young household gross income, before the profit return, was the wage given by $(1-\mu)Ak$, it is now the wage in addition to profits, which are approximately, $(\mu A - R^S)k$, giving an approximate average gross income of $(A - R^S)k$. The implication is that the growth factor under Bismarck is now given by

$$G_{t+1}^{BEV} = \frac{(1-\beta)(A - R_{t+1}^S)}{2(\phi^{MAX} - \phi^{MIN})(2+\theta)}\left(\left(\phi^{MAX} - \frac{\beta(1+\theta)G_{t+1}^{BEV}}{(1-\beta)R_{t+1}^S}\right)^2 - \left(\frac{\beta(1+\theta)G_{t+1}^{BEV}}{(1-\beta)R_{t+1}^L} - \phi^{MIN}\right)^2\right)$$

while

$$G_{t+1}^{BIS} = A\left(c - \left(c^2 - d\right)^{\frac{1}{2}}\right) \text{ where}$$

$$c \equiv (1-\beta)\left(\frac{1}{\beta(1+\theta)-(2+\theta)}\right)^2\left(2+\theta-\beta(1+\theta)+\frac{1}{2}\left(\beta(1+\theta)\right)\left(\mu^{\frac{1}{2}} + \mu^{\frac{-1}{2}}\right)^2\right)$$

$$d \equiv \left(\frac{1-\beta}{\beta(1+\theta)-(2+\theta)}\right)^2$$

[14] Or, capital income taxation, which we do not consider.

The consequence of this profit return is some expansion of the interest and growth factors under both pension schemes without altering the relative median voter preferences between them.

Table Three: A low value of the pension, $\beta = 0.10$ ***where profits are returned***

	Competitive finance with either scheme	***Monopoly finance with Beveridgean scheme***	***Monopoly finance with Bismarckian scheme***
Growth factor	**3.126 (3.31%)**	**2.530 (2.69%)**	**2.491 (2.64%)**
Firms' borrowing interest factor	5.516 (5%)	5.516 (5.00% pa)	5.516 (5.00% pa)
Household savings interest factor	5.516 (5%)	**2.455 (2.60%)**	**2.763 (2.95%)**
Household borrowing interest factor	5.516 (5%)	11.032 (7.10% pa)	N/A
Financial profit-wage ratio (approx)	0	**10.07%**	**9.06%**
Median income household utility	**-0.252**	**-0.243**	**-0.240 (-0.231)**

Returning profits where $\beta = 0.10$, has a negligible effect on the various proportions as $\phi^{CL} = 0.127$, $\phi^{CH} = 0.573$. Comparison with Table 1 shows that the main effect is to reduce the overall utility cost of monopoly. The figures reported are for the steady-steady-states. The figure in parenthesis makes allowance for the fact that the profit-wage ratio is already predetermined when the young decide on the PAYG scheme, so that the movement to BIS-PAYG gives even greater utility, since they would receive a lower profit-wage ratio only in the second period.

Hypothetically, there could be a preference reversal if the general scale of profits were sufficiently high, because profits are always higher under BEV-PAYG. A final point is that although the median voter may opt for BIS-PAYG, this will reduce the growth rate and the utility of future generations. The assumption of descending altruism would modify the results. Furthermore, although PAYG may be dynamically efficient, because of the endogenous growth feature [Saint-Paul (1992)], a change in its BEV and BIS components may raise the average utility levels of future generations for the same reason.

5. Further discussion

Two models of financial sector interaction have generated clear and distinct results regarding preferences between BIS- and BEV-PAYG. In a model where financial trades are exclusively intergenerational, middle income households are not hurt by interest rate distortions, as they have little incentive to either save or borrow, but they gain like everyone else, from the profit return. Households with middle incomes would form a coalition with low income households to vote for BEV-PAYG, which gives greater scope for the profit-making activities of the financial sector. This depends on the exogenous return of profits through the fiscal rule. If profits were also returned through dividends, this result would be weakened.

In a model with the potential of intra- and inter-generational trading and a store of value, the young save in aggregate. A convenient specification of a uniform income distribution and the absence of other sources of heterogeneity causes there to be a majority of savers who are less concerned with profits and more with getting a better deal on interest rates. The simulation results pointed to the importance of higher saving rates under BIS-PAYG.

One reason for this result is that the profit differences between these two schemes were relatively small within the specification used where the financial sector can also lend to the corporate sector. Potentially, BEV-PAYG could lead to significantly grater profits with other specifications and assigned numerical values. Another, related reason is that differences in the growth factors were even smaller very, so this variable had no significance on the results.

Nevertheless, the two sets of results do point to a general conclusion that that majority voting would uphold BIS PAYG if the median voter saves a significant amount, but BEV PAYG if this individual either saves a small amount or not at all. It is truism that the behaviour of the saving behaviour of the median voter depends on general saving behaviour and where the median voter lies within the distribution. These features have been simplified in the interest of tractability.

Empirical income distributions are skewed, so that the median income household is less likely to be a saver, the greater the degree of income dispersion.[15] Other things being equal, societies with less equal (skewed) income distributions will have a greater tendency to vote for BEV PAYG than more egalitarian societies. Conde-Ruiz and Profeta (2002) point out that this prediction is consistent with the finding of the *World Bank's* World Development Indicators (2000).

Interestingly, if the tax-benefit system is geared towards reducing income inequality for working age individuals, the median voter will have a greater predilection for BIS-PAYG. This suggests that the median voter is more likely for BEV PAYG, if there is no opportunity to vote for another, more fundamental policy of income re-distribution.

The income of the median voter also depends on the voting franchise, which historically has depended on property ownership. A wider franchise would work in favour of BEV PAYG by increasing the income and saving propensity of the median voter. The historical process of franchise extension would suggest that earlier pension institutions would be BIS and later ones BEV[16] and that there is a tendency to move towards greater measures of re-distribution.

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[15] The median income is $\exp\left(-0.5\sigma^2\right)$ of the average income, if the distribution is log-normal.

[16] However, in 1867, also the year of the second Reform Bill in Great Britain, Bismarck introduced universal manhood suffrage in the North German Confederation. Historians have questioned the political significance of this measure, because of the lack of privacy in voting and aristocratic control.

The only form of saving behaviour considered is that which is undertaken to smooth consumption over the life-cycle. In practice, a lot of saving may also made for precautionary and bequest motives. Uncertainty of future income, necessary expenditures and longevity long with descending altruism all increase the tendency of the median voter to favour BIS-PAYG.

Against this, other forms of individual heterogeneity will promote BEV-PAYG. If households have different rates of time preference, there is no guarantee that the median voter will be a saver even in the most favourable case, where income is equally distributed. If, individuals with higher incomes also save a greater proportion of their income, there van be an aggregate equilibrium, which is characterised by middle-income savers. Dynan, Skinner and Zeldes (7906) make this finding and they also attribute it to precautionary saving and the bequest motive.

Finally, the results are weakened, but not overturned, if profits are only partially returned through reduced labour taxes and increased pensions. The results would be over-turned, if profits are returned through dividends and if investment in the financial sector is inherently non-productive as in Roberts (forthcoming 2003). This possibility would raise the growth rate under BIS-PAYG and switch preferences accordingly, since lower profits would lead to less crowding-out of productive investment. Dividend taxation policy will, therefore, have some effect on preferences concerning social security. The political economy focus is valid, if the median voter is not only a saver, but also one with substantial holdings in financial sector equity.

7. Concluding comments

We have almost certainly only scratched the surface of an area, which integrates the analyses of social security and imperfect financial sector competition. Social security, as a forced form of saving, is a substitute - good or bad - for voluntary private saving. The presence of financial market imperfections, which discourage or worsen the terms of saving, may sometimes tip the balance further in favour of social security.

The feature of price discrimination under financial sector imperfect competition gives a coherent account of why, empirically, only about 40-60% of individuals save. The model specification then determines whether the median voter will be included or excluded within this group of individuals. A basic intuition is that the savings position of the median voter is crucial in determining democratic preferences for the social security system. This paper has presented two theoretical models, one with a democratic preference for BEV-, another for BIS-PAYG. A majority of individuals may vote for BEV-PAYG in order to share the gains of imperfect competition, while elsewhere another majority may vote for BIS-PAYG in order to mitigate its costs.

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Appendix

Indirect utilities in model 1

The indirect utilities of individuals who are non-participants in the financial market, savers and borrowers are

$$V^{BIS}(i) = \ln(1-\beta) + \frac{1}{1+\theta}\ln\beta + \left(\frac{2+\theta}{1+\theta}\right)(\ln\phi(i) + \ln w) \tag{A1}$$

$$V^{N,BEV}(i) = \ln(1-\beta) + \frac{1}{1+\theta}\ln\beta + \ln\phi(i) + \left(\frac{2+\theta}{1+\theta}\right)(\ln(1+\eta) + \ln w) \tag{A2N}$$

$$V^{S.BEV}(i) = \ln(1-\beta) + \frac{1}{1+\theta}\left(\ln\beta - \ln\phi^{CH}\right) + \left(\frac{2+\theta}{1+\theta}\right)\left(\ln\left(\frac{1+\theta}{2+\theta}\right) + \ln\left(\phi(i) + \frac{\phi^{CH}}{1+\theta}\right)\ln(1+\eta) + \ln w\right) \tag{A2S}$$

$$V^{L,BEV}(i) = \ln(1-\beta) + \frac{1}{1+\theta}\left(\ln\beta - \ln\phi^{CL}\right) + \left(\frac{2+\theta}{1+\theta}\right)\left(\ln\left(\frac{1+\theta}{2+\theta}\right) + \ln\left(\phi(i) + \frac{\phi^{CL}}{1+\theta}\right)\ln(1+\eta) + \ln w\right) \tag{A2L}$$