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Sharing Transboundary Rivers Fairly and Efficiently

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Abstract

This paper provides an analytical framework for exploring the relationship between equity and efficiency for two riparians sharing a transboundary river. Decreasing marginal productivities of water make a noncooperative approach inefficient when water is scarce. If the upstream riparian uses its optimal quantity the downstream riparian suffers disproportionately. The undesirable properties can be avoided by means of a cooperative agreement in which the two riparians share the water equally. Equal quota is at least as efficient as noncooperation when riparians are identical or when the downstream riparian is relatively cost-effective and generates a cooperative surplus when water is sufficiently scarce. The equity-efficiency trade-off is found to be insignificant, in magnitude as well as prevalence, limited to the case where the downstream riparian has a high relative cost disadvantage and water is very scarce.

Keywords: efficiency, equity, equity-efficiency trade-off, regional public goods, transboundary rivers, unidirectional externality.

JEL classification: D62, D63, Q25

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1. Introduction

River basins shared by two or more riparians constitute an important class of regional public goods or open access resources that are nonexcludable but rival in consumption. There are close to 261 international river basins in the world, covering almost half of the total land surface of the globe (Wolf, 1999). The quality and quantity of freshwater is increasingly under pressure mainly because of economic development and population pressures.

Transboundary rivers can elicit either cooperation or conflict depending on the perceptions of their relative benefits (Sadoff and Grey, 2002). A recent empirical study suggests that cooperation over shared water is relatively more common than conflict (Wolf, Yoffe and Giordano, 2003). Nevertheless, water has indeed been a cause of political tensions between the Arabs and Israelis; Indians and Bangladeshis; Americans and Mexicans; and all ten riparian states of the Nile River. At the heart of regional water disputes is the lack of internationally accepted criteria for sharing water resources between riparians (Wolf, 1999). The identification of suitable equity criteria is essentially a political question, but economists can add value to the debate by analysing the efficiency implications of the various criteria. Indeed, if economic analysis is to make an important contribution to policy formulation in transboundary water cooperation it must give due attention to distributive issues in addition to its traditional focus on efficiency (Just, Frisvold, Harrison, Oppenheimer and Zilbeman, 1998). The purpose of this paper is therefore to present a theoretical framework within which the equity-efficiency relationship on transboundary rivers can be explored further.

Several authors emphasise the intrinsic and instrumental importance of 'reasonable and equitable use' of transboundary rivers (Barrett 1994; Sadoff et. al. 2002; Wolf 1996, 1999; Wolf and Dinar 1994). These papers take a discursive approach by presenting and contrasting the vague, numerous and sometimes contradictory principles of international law. The most comprehensive treatment is given by Wolf (1999) who compares international practice with legal principles in an analysis of the 145 existing treaties on international freshwater resources. Wolf observes a general shift in negotiations over time away from rights-based (i.e. absolute territorial sovereignty or unlimited territorial integrity[2]) towards needs-based criteria (i.e. for agricultural production).

There are relatively few contributions in the economics literature on efficient water sharing agreements among countries within an international river basin. Efficiency is typically approached either via *market solutions* (e.g. Kilgour and Dinar 1995, 2001), or via cooperation in the form of *joint development projects* (e.g. Rogers 1997). Kilgour and Dinar identify Pareto-optimal allocations for every possible flow volume in a river, but they do not directly address questions

[2] The doctrine of absolute territorial sovereignty gives a country the right over the management of waters within its territory. In comparison, the doctrine of unlimited territorial integrity gives a country the right of uninterupted water flow upstream of its territory.

of equity. Dinar and Wolf (1994) find that a welfare-enhancing market scheme for the Middle East is theoretically feasible under certain conditions, but acknowledge that political objections are likely to occur. This is partly due to unbalanced allocations of the regional gains, most of which would benefit Israel.

The joint development approach lends itself directly to equity analysis because an agreement between the riparians must be reached on which projects to pursue and how to distribute their benefits and costs. Several authors take the view that the objectives of equity and efficiency are inseparable and potentially at odds with each other. One example is Rogers (1997) who applies Baumol's (1986) concept of *Superfairness* (an extended theory of non-envy). He demonstrates how the simultaneous pursuance of the two objectives can lead to situations where one riparian might prefer a Pareto-inferior project solution to one which is Pareto optimal, even though this would give it (and other riparians) a lower net benefit. The potential problem with some Pareto-optimal allocations, according to Rogers, is that they might induce envy between the parties, for instance regarding the geographical location of infrastructure investments. The policy implication is that of a second-best solution where the development of the river basin is planned under the restriction of non-envy. A related point is found in Sadoff et. al. (2002) who criticise the conventional economic argument that, first, aggregate benefits to society should be maximised, and thereafter issues of distribution can be addressed. Redistribution of economic gains, especially over international borders, is extremely complex in reality and there are few successful precedents anywhere in the world, argue the authors. The absence of side payments implies a recommendation of second-best policies which do not necessarily maximise social welfare, but lead to equitable agreements, acceptable to all parties.

Finally, a few authors apply cooperative game theory to identify fair and efficient water allocations in river sharing problems involving more than two countries. Barrett (1994a) analyses how two different rights-based doctrines (territorial sovereignty vs. territorial integrity) would affect the set of core allocations in problems with three riparians. The Shapley value is also invoked, although primarily as a means to select a unique, stable and efficient allocation rather than as a means to achieve equity. Ambec and Sprumont (2002) analyse a model of n identical riparians who have quasi-linear preferences over water and money (thus allowing for side payments). They identify exactly one welfare distribution in the core, and this fair and efficient outcome represents a compromise between the two conflicting doctrines analysed by Barrett. The authors apply Moulin's (1990) fairness axiom for negative group externalities, which essentially implies that since 'no particular agent bears the distinguished responsibility for these externalities, it is a natural to ask that everyone takes up a share of them: no one ends above his aspiration welfare.' (Ambec and Sprumont, 2002).

In summary, the existing literature on transboundary rivers presents several different views on how the objectives of equity and efficiency interrelate. These differences are partly due to the interpretation of water as an economic good, and partly due to alternative ways of defining equity. The market approach interprets water as a private good with clearly defined property rights. Guided by second welfare theorem and the Coase theorem, this approach emphasises *separability* and the corresponding policy implication of 'efficiency first - equity afterwards'. In the joint development approach, water is perceived primarily as a public good with undefined property rights and this leads to the conclusion of *inseparability* of objectives. Trade-offs may occur if equity is defined as 'non-envy' or 'agreement acceptability', but the possibility of objective compatibility also exists if an appropriate measure of equity can be identified within the core.

The present paper represents an alternative, and hitherto ignored, possibility of separable compatibility where the introduction of equity considerations potentially leads to efficiency gains. We perceive water as a public good with *a priori* undefined property rights and allow for the possibility of side payments. Our interpretation of equity as 'equal sharing of the waters' is consistent with several prominent theories of distributive justice. The remainder of the paper is organised as follows: Section 2 presents the model. Section 3 presents the results. Section 4 draws policy implications. Finally, section 5 concludes.

2. The Model

An upstream and a downstream riparian share a transboundary river with an average annual flow of $\overline{Q} > 0$ units of water originating entirely within the former[3]. Sharing takes place when the upstream riparian does not use the entire volume, instead passing some of it on to the downstream neighbor. Any unused water is eventually discharged into the sea[4]. Each riparian uses water $q_i \geq 0$ $(i = u, d)$ for irrigation to produce an agricultural output $y_i \geq 0$ which may be thought of as rice or cotton[5]. The agricultural output is sold at competitive world markets that both riparians are too small to influence, thus $p_i = p$. Riparians are described in terms of an agricultural production function exhibiting diminishing returns to scale $y_i = y_i(q_i)$ where $y_i(0) = 0$, $\frac{\partial y_i}{\partial q_i} > 0$ and $\frac{\partial^2 y_i}{\partial q_i^2} < 0$. Each riparian incurs a constant unit cost of using water: $c(q_i) = c_i q_i$[6]. Thus, riparian profit is defined as $\pi_i = p y_i(q_i) - c_i q_i$.

[3] In addition to its analytical simplicity, a two-country model is an empirically important case study relevant to two-thirds of the existing 261 river basins in the world (www.transboundarywaters.orst.edu).

[4] A common approach to modelling this situation is to represent riparians with water demand functions and let them exchange water on a market (see Kilgour and Dinar, 1995). This paper takes an alternative approach by giving the water market a less explicit treatment.

[5] A majority of global freshwater is used by the agricultural sector (Gleick et. al., 2002). Around 40 percent of global food crop production originates from irrigated agriculture (Merrett, 2003).

[6] A more general specification with increasing unit costs introduces uneccessary complications without yielding any further insights.

2.1 Non-cooperative equilibrium

Riparian i chooses an optimal level of water input q_i^* to maximise profit π_i subject to a water availability constraint $\overline{q}_i$, where $\overline{q}_u = \overline{Q}$ and $\overline{q}_d = \overline{Q} - q_u^*$. The upstream riparian chooses q_u^* first thereby affecting the constraint of the downstream riparian $\overline{q}_d$[7]. The optimisation problem of riparian i is:

$$\max_{q_i} \{\pi_i = py_i(q_i) - c_i q_i \mid q_i \leq \overline{q}_i\},\ i = (u, d) \tag{1}$$

which yields the necessary and sufficient Kuhn-Tucker first-order conditions:

$$py_i'(q_i^*) - c_i \geq 0,\ q_i^* \leq \overline{q}_i, \tag{2}$$
$$(py_i'(q_i^*) - c_i)(\overline{q}_i - q_i^*) = 0 \tag{3}$$

At an interior solution for optimal water we get $q_i^* = y'^{-1}\left(\frac{c_i}{p}\right)$. To simplify let $p = 1$ and get $q_i^* = y'^{-1}(c_i) \equiv g(c_i)$ where $g(\cdot) > 0$, $g'(\cdot) < 0$, $g''(\cdot) > 0$. Since we are only interested in relative unit costs, we normalise the cost differential by assuming $c_u = 1$ and $c_d = c$. Thus, for $c > 1$ the upstream riparian has the relative cost advantage, while the opposite is the case for $c < 1$. Table 1 gives an overview of the noncooperative solution for the three possible scenarios N_H, N_M and N_L[8]. In scenario N_H water is abundant and the riparians coexist peacefully without any need for cooperation. To some extent, this reflects the historical situation in many river basins decades ago. In scenarios N_M and N_L water is scarce relative to demand and this introduces a conflict of interest which most riparians are familiar with today. In this model, the upstream riparian imposes a negative unidirectional externality upon the downstream riparian by preventing the latter from reaching an unconstrained, profit maximising water input level (see also Mäler, 1990)[9]. In scenario N_L water is so scarce that the upstream riparian leaves a volume insufficient for the downstream riparian to engage in any agricultural irrigation[10]. Table 2 presents a more precise definition of the three noncooperative scenarios as sets in $(c, \overline{Q})$-space.

[7] Note that the geographical reality of who is upstream and downstream is sometimes secondary to the strategic reality of who moves first (or has done so historically). Consider the River Nile, for instance, where the downstream riparian and hegemon Egypt was first to develop its water infrastructure, to the regret of many latecomers upstream.

[8] We adopt the following labelling convention: N stands for *N*oncooperation and the subscript refers to the water level (*H*igh, *M*edium or *L*ow).

[9] Note that the externality enters in the optimisation constraint rather than the objective function.

[10] Zero input refers to a situation where irrigation is commercially nonviable - not necessarily to one where the downstream river is dried out.

Table 1. Noncooperative equilibria

	N_H	N_M	N_L
q_u^*	$g(1)$	$g(1)$	$\overline{Q}$
q_d^*	$g(c)$	$\overline{Q}-g(1)$	0
π_u^*	$y\left(g(1)\right)-g(1)$	$y\left(g(1)\right)-g(1)$	$y\left(\overline{Q}\right)-\overline{Q}$
π_d^*	$y\left(g(c)\right)-cg(c)$	$y\left(\overline{Q}-g(1)\right)-c\left(\overline{Q}-g(1)\right)$	0

Table 2. Definition of noncooperative scenarios

$N_H = \left\{c > 0 \mid g(1) + g(c) \leq \overline{Q}\right\}$
$N_M = \left\{c > 0 \mid g(1) < \overline{Q} < g(1) + g(c)\right\}$
$N_L = \left\{c > 0 \mid 0 < \overline{Q} \leq g(1)\right\}$

Figure 1 illustrates the noncooperative equilibrium for two identical riparians at different levels of water availability $\overline{Q}$. The width of the diagram is determined by the size of $\overline{Q}$. The water use by the upstream riparian q_u is measured from left to right and that of the downstream riparian q_d in the opposite direction. The two lines are the respective agricultural production functions. When water is abundant both riparians can choose their unconstrained optimal water use levels where marginal product equals marginal cost (points A and B in panel N_H). As water becomes increasingly scarce the diagram contracts. In N_M, the upstream riparian chooses an unconstrained level of water at A', but there is insufficient water in the basin for the downstream riparian to choose its optimum B', so it chooses the constrained optimum C. Finally, in N_L when water is very scarce, the upstream riparian is also constrained at D, while the downstream riparian is forced to a situation of zero water input (E).

2.2 Pareto efficiency

There is broad agreement in the water resources management literature that the most appropriate unit of analysis is the river basin itself rather than the territory of any individual riparians[11]. Integrated river management effectively internalises all externalities within the planning area (Sadoff et. al., 2002). A *Pareto efficient* allocation is thus defined as *a feasible water allocation that maximises basinwide profit*[12]. We can write the social planner's problem as:

$$\max_{q_u,q_d}\left\{\Pi = \pi_u + \pi_d = y(q_u) - q_u + y(q_d) - cq_d \mid q_u + q_d \leq \overline{Q}\right\} \quad (4)$$

At an interior solution the first-order conditions of the associated Lagrangian yields $\frac{\partial y_u}{\partial q_u} = 1$ and $\frac{\partial y_d}{\partial q_d} = c$, i.e. each riparian must equalise marginal product and marginal cost. Moreover, marginal products must also equal marginal cost across riparians. Thus, we get the following condition for Pareto efficiency:

$$\frac{\partial y_u}{\partial q_u} = \frac{\partial y_d}{\partial q_d} + (1-c). \quad (5)$$

[11] An important exception is Just and Netanyahu (1998) who favour a less ambitous management concept.

[12] Note that we refer to productive efficiency rather than social efficiency. The latter would neccessitate the maximisation of a social welfare function.

What are the efficiency properties of noncooperation? We consider here the simple case of identical riparians. The Pareto condition then states that marginal products must be equalised. If water is abundant (N_H) then noncooperation is also efficient. Basinwide profit is maximised by letting each riparian maximise their individual profits. However, the noncooperative solution is not efficient when water is scarce (N_M and N_L) because the marginal product of the upstream riparian is lower (because its water use is higher) than for the downstream neighbor. Basinwide profit can thus be increased by transferring water units from the upstream to the downstream riparian until the marginal product is equalised across riparians. We therefore have the following result:

Proposition 1 *Suppose two identical riparians (one upstream and one downstream) share a transboundary river and there are diminishing returns to scale, constant marginal costs and an exogenous output price. The equal quota allocation is Pareto efficient.*

Proof. (by contradiction). Denote riparian profit as $\pi_i = py_i(q_i) - cq_i,\ i = (j,k)$, basinwide profit as $\Pi = \pi_j + \pi_k$ and $q_j = q_k = q$ as the equal quota allocation where $q_j + q_k \leq \overline{Q}$. We must show that $\Pi(q,q) > \Pi(q_j, q_k)$ $(j \neq k)$. Consider any allocation different from equal quota such as $(\widetilde{q}_j, \widetilde{q}_k)$ where $\widetilde{q}_j > \widetilde{q}_k$ and assume that this allocation maximises Π. This proposition leads to a logical contradiction. Because of diminishing returns to scale we have $\frac{\partial y_i}{\partial q_i} > 0$ and $\frac{\partial^2 y_i}{\partial q_i^2} < 0$. This implies that $p\frac{\partial y_j(\widetilde{q}_j)}{\partial q_j} < p\frac{\partial y_k(\widetilde{q}_k)}{\partial q_k}$ and therefore that $\frac{\partial \pi_j(\widetilde{q}_j)}{\partial q_j} < \frac{\partial \pi_k(\widetilde{q}_k)}{\partial q_k}$. Suppose that we redistribute a small quantity of water $\varepsilon > 0$ from j to k so that $\widetilde{q}_j - \varepsilon > \widetilde{q}_k + \varepsilon$. This implies that $\pi_j(\widetilde{q}_j) - \pi_j(\widetilde{q}_j - \varepsilon) < \pi_k(\widetilde{q}_k + \varepsilon) - \pi_k(\widetilde{q}_k)$. Rearranging yields $\Pi(\widetilde{q}_j, \widetilde{q}_k) < \Pi(\widetilde{q}_j - \varepsilon, \widetilde{q}_k + \varepsilon)$ which is a contradiction to the statement that $(\widetilde{q}_j, \widetilde{q}_k)$ maximises Π. ■

2.3 Equity and equal quota

The noncooperative equilibrium emerges as an undesirable solution to the water conflict if water is scarce (N_M and N_L) for two reasons. First, it is productively inefficient, i.e. it does not maximise basinwide profit. This result was demonstrated for identical riparians in proposition 1 and will be generalised later in the paper. Secondly, recall the tendency observed by Wolf (1999) that international water negotiations are now focusing more on needs-based allocations. Apart from differences in location and cost structure, our model assumes that riparians are identical, i.e. they have the same population size, geographical area etc. They also therefore have the same need for water for agricultural production. This begs the question why one riparian should be granted a larger share of the water than the other. In light of the undesirable properties of the noncooperative solution one must identify alternative criteria which can enhance basinwide economic efficiency and satisfy reasonable notions of equity. In the absence of a supra-national authority, any such criteria would need to be embodied in a cooperative agreement which the two parties sign up to voluntarily.

The criterion of equal quota emerges, on efficiency grounds, as a superior alternative to noncooperation, at least if riparians are identical. Is the equal quota allocation equitable? We shall not attempt a definitive answer to this difficult philosophical question here. Rather, we are interested in making explicit the value judgments that are embodied in an affirmative answer. Equal quota can be supported by adopting an appropriate theory of distributive justice[13], such as the egalitarian standard of *equal treatment of equals*[14]. But 'equality of what?' to paraphrase Sen (1982). In practice, negotiators have had to decide between sharing the water or sharing the benefits of the water (Wolf, 1999). We choose water as the equity metric in this paper for the following reasons. First, water is more easily measured in practice than benefits. Secondly, and possibly related to the first point, equal water sharing is relatively more common in existing international agreements[15]. Thirdly, it is trivial to analyse the efficiency implications of benefit sharing since the principle implies separability between equity and efficiency. In an effective benefit sharing agreement, riparians would aim to maximise efficiency first and distribute benefits afterwards.

Alternative theories of distributive justice, such as *equality of opportunity* can also be adopted in support equal quota. Foley's (1967) principle of *no-envy* would support equal quota if water (not benefits) is accepted as the equity metric. Even if benefits was the metric, a move from noncooperation to equal quota would often imply more fairness, i.e. less envy. A similar argument can be made in the case of Rawls' (1971) *Maximin Rule* and the utilitarian principle of Pareto optimality, i.e. the maximisation of basinwide profit[16]. On a final note, Nozick's (1974) *Entitlement Theory* would be inapplicable in this case since it deals with the rights to *private* rather than *public* goods. In sum, by interpreting equal quota as an equitable allocation, we implicitly subscribe to principles of egalitarianism and equality of opportunity.

2.4 The equal quota allocation

In the remainder of this paper we focus our attention on the efficiency properties of equal quota vis-a-vis noncooperation, particularly when unit costs are allowed to differ. It should be emphasised, however, that the theoretical framework presented here is sufficiently flexible to deal with a wide range of exogenously defined sharing rules. Under equal quota each riparian choose an optimal level of water input q_i^* to maximise profit π_i subject to the constraint

[13] See Roemer (1996) for an introduction.

[14] There is some empirical support for choosing an egalitarian standard as an equity principle in water disputes. A household survey undertaken in the urban areas of Western Australia where groundwater allocation was an ongoing salient issue found relatively strong support for this principle (Syme et. al., 1999). However, the universality of this finding to other types of water disputes and locations remains to be tested.

[15] Only 33 out of 149 international water agreements specify an explicit sharing rule. Nine hereof use the principle 'half of flow to each of the two' while two agreements specify 'equal allocation of benefits'. All, but one, of the nine 'equal water sharing agreements' relate to boundary waters with Niagara (US and Canada) being the only exception (Wolf, 1999).

[16] Rawls' least advantaged riparian often benefits from equal quota. As shown later, equal quota does sometimes (but not always) coincide with Pareto optimality.

$\overline{q}_i = \frac{\overline{Q}}{2}$. The optimisation problem of riparian i is otherwise similar to (1). We note how the equal quota entitlement has neutralised the strategic first-mover advantage of the upstream riparian.

An important tool of the analysis is the definition of the various scenarios. We define nine possible scenarios for the equal quota allocation in $(c, \overline{Q})$-space[17]. Scenarios E_{HU}, E_{HD} and E_{HI} all refer to Equal quota allocations where there is sufficient water for both riparians to maximise profits unconstrained. Scenarios E_{MU} and E_{MD}, where one of the two riparians is constrained, differ from each other because costs differ. The riparian with the cost advantage will choose higher levels of input/output and will thus be the first to become constrained under an equal quota allocation. Finally, both riparians are constrained in E_{LU}, E_{LD}, E_{LI} and E_{MI} (identical riparians become constrained simultaneously). Table 3 gives an overview of the equal quota allocation and table 4 summarises the nine definitions.

Table 3. The equal quota allocation

	E_{HU}, E_{HD}, E_{HI}	E_{MU}	E_{MD}	$E_{LU}, E_{LD}, E_{LI}, E_{MI}$
q_u^*	$g(1)$	$\frac{\overline{Q}}{2}$	$g(1)$	$\frac{\overline{Q}}{2}$
q_d^*	$g(c)$	$g(c)$	$\frac{\overline{Q}}{2}$	$\frac{\overline{Q}}{2}$
π_u^*	$y\left(g(1)\right) - g(1)$	$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2}$	$y\left(g(1)\right) - g(1)$	$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2}$
π_d^*	$y\left(g(c)\right) - cg(c)$	$y\left(g(c)\right) - cg(c)$	$y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2}$	$y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2}$

Table 4. Equal quota definitions

$E_{HU} = \left\{c > 1 \mid 2g(1) \leq \overline{Q}\right\}$
$E_{MU} = \left\{c > 1 \mid 2g(c) < \overline{Q} < 2g(1)\right\}$
$E_{LU} = \left\{c > 1 \mid 0 < \overline{Q} \leq 2g(c)\right\}$
$E_{HD} = \left\{c < 1 \mid 2g(c) \leq \overline{Q}\right\}$
$E_{MD} = \left\{c < 1 \mid 2g(1) < \overline{Q} < 2g(c)\right\}$
$E_{LD} = \left\{c < 1 \mid 0 < \overline{Q} \leq 2g(1)\right\}$
$E_{HI} = \left\{c = 1 \mid 2g(1) < \overline{Q}\right\}$
$E_{MI} = \left\{c = 1 \mid \overline{Q} = 2g(1)\right\}$
$E_{LI} = \left\{c = 1 \mid 0 < \overline{Q} < 2g(1)\right\}$

2.5 Water loans

We shall make an additional assumption about the nature of possible interaction between the two riparians. If one riparian has an unutilised water quota which is in demand by the other riparian then the former can lend the unutilised water units to the latter against a lump-sum interest payment. This arrangement is in accordance with existing practices as discussed by Wolf (1996) in the case of the 1959 Nile River Waters Treaty: This Treaty allocated 55.5

[17] We label the Equal quota scenarios as follows: Subscript U refers to a situation where the Upstream riparian has the relative cost advantage ($c > 1$), D when Downstream has the lowest cost and I when the riparians have Identical unit cost.

billion cubic meter (BCM) per year for Egypt and 18.5 BCM per year for Sudan. Since Sudan could not absorb that much water at the time, the treaty provided for a Sudanese water loan to Egypt of up to 1.5 BCM per year through 1977. Water loans have also been made from the US to Mexico in 1966 on the Colorado River (Wolf, 1998). According to the original treaty texts[18], Egypt was not required to make any interest payments to Sudan, while Mexico agreed to reimburse the United States. This difference can be explained by considering the marginal value of water to the lending riparian (in both cases located upstream). Sudan could not make use of its entitlement - a fact which weakened its bargaining position fundamentally and resulted in zero payment from Egypt. In the Colorado River case, the US was already making productive use of its water entitlement. Thus, when Mexico made a plea for a water loan due to a serious drought, the US could effectively charge Mexico at market value for any decrease in US power generation. In what follows we define a water loan in the context of the model and highlight some of its properties.

Definition 1 *Let* $(\overline{q}_j, \overline{q}_k)$ *be the water entitlement of two riparians* $(j \neq k)$*. If* $\widehat{q}_j$ *denotes the unconstrained profit maximum of riparian* j *then* $\delta = \max\{\widehat{q}_j - \overline{q}_j, 0\}$ *expresses riparian* j*'s demand for a water loan. The supply of water loans from* k *is given by* $\sigma = \max\{\overline{q}_k - q_k^*, 0\}$*. A water loan transaction is a transfer of* $\Delta = \min\{\delta, \sigma\}$ *units of water from riparian* k *to riparian* j *in exchange for a lump sum interest payment* $i\Delta \in [0; \pi_j(\overline{q}_j + \Delta) - \pi_j(\overline{q}_j)]$ *determined as a result of mutual bargaining.*

Lemma 1 *A water loan transaction will only take place if the water constraint of riparian* j *is binding while that of riparian* k *is non-binding and if the riparians are non-identical. In the case of equal quota this implies that the riparians are in scenario* E_{MU} *or* E_{MD}*.*

Proof. Neither riparian has a demand ($\delta = 0$) in scenarios E_{HU}, E_{HD} and E_{HI} since they are both unconstrained. In scenarios E_{LU}, E_{LD} and E_{LI} both riparians are constrained so $\sigma = 0$. In scenarios E_{MU} and E_{MD} one of the two have unutilised water units which is in demand by the other and a transaction can occur. If riparians are identical (E_{MI}) they will both face a simultaneous constraint so $\sigma = 0$. ■

Lemma 2 *A water loan transaction increases basinwide profit.*

Proof. Ignore the transfer payment $i\Delta$ since it does not affect the size of basinwide profit - only its distribution. We must show that $\Pi(\overline{q}_j + \Delta, \overline{q}_k - \Delta) > \Pi(\overline{q}_j, \overline{q}_k)$ when $\Delta > 0$. If riparian k lends water to j its profit (excluding the transfer payment) is unaffected by the transaction, hence $\pi_k(\overline{q}_k - \Delta) = \pi_k(q_k^*)$. It is therefore sufficient to show that $\pi_j(\overline{q}_j + \Delta) > \pi_j(\overline{q}_j)$. If there is excess supply then $\Delta = \delta = \widehat{q}_j - \overline{q}_j$ so $\pi_j(\overline{q}_j + \widehat{q}_j - \overline{q}_j) > \pi_j(\overline{q}_j)$ which is true since $\widehat{q}_j > \overline{q}_j$ and $\frac{\partial \pi_j(q_j)}{\partial q_j} > 0$ on the relevant interval. If there is excess demand then

[18] Available at http://www.transboundarywaters.orst.edu/

$\Delta = \sigma = \overline{q}_k - q_k^*$ then $\pi_j(\overline{q}_j + \overline{q}_k - q_k^*) > \pi_j(\overline{q}_j)$ which is true since $\overline{q}_k > q_k^*$ and $\frac{\partial \pi_j(q_j)}{\partial q_j} > 0$ on the relevant interval. ■

The water loans institution is essentially a tradeable quota scheme with upper bounds on transferable quantities. Riparians are only allowed to trade their unutilised water entitlement but not the entire quota they already use. We are therefore assuming that they possess enough information to realise a mutual interest in engaging in a water loan, but that they do not have the information necessary to realise how the total amount of water $\overline{Q}$ is most efficiently allocated between them. The lump sum interest payment is determined on the basis of a process of mutual bargaining not specified explicitly in the model. The marginal value of water to the lending country and its ability to make credible threats in terms of alternative use of the water are important determinants of the interest payment. Finally, we note that water loans only apply to a situation where a riparian has an unutilised quota to which it is entitled by bilateral agreement. Notably, it does not allow the upstream riparian to hold back all the water in a noncooperative situation and subsequently lend it to the downstream riparian in exchange for payment.

2.6 An illustration

We conclude this section by illustrating how the equal quota allocation works in the presence of cost differences and water loans. Consider figure 2 where the downstream riparian has a relative cost advantage ($c < 1$). Both riparians are unconstrained by the equal quota $\frac{\overline{Q}}{2}$ when water is relatively abundant as shown in panel E_{HD} - upstream chooses A and downstream chooses B. As water becomes more scarce (scenario E_{MD}) the downstream riparian can no longer choose its unconstrained optimum B' but must initially suffice with C. Upstream is at a sufficiently low input/output level not to be affected by the water scarcity and chooses its optimum A'. In this situation the downstream riparian would prefer to increase the scale of production while upstream only uses a fraction of its entitlement. Allowing for water loans, upstream can supply $\sigma = \frac{\overline{Q}}{2} - q_u^*$ units of water to meet downstream's demand of $\delta = q_d^* - \frac{\overline{Q}}{2}$. In this case there is excess supply on the water loans market ($\sigma > \delta$) and the two riparians efficiently transact $\Delta = \delta$ units. This enables downstream to attain its preferred input/output level at B' while upstream stays put. Note that this transaction is efficiency enhancing (cf. Lemma 2) and that surplus water remains in the basin which neither of the riparians can make productive use of. In the third panel we consider the case where the water level $\overline{Q}$ is further reduced (still in scenario E_{MD}). The riparians are in a situation where there is excess demand in the water loans market. In brief, downstream would prefer B'', but must initially choose D and upstream chooses unconstrained at A''. Downstream has an unmet demand of δ which is higher than the unused quota σ of upstream. The water loan brings downstream to point E which is lower than the preferred point B''. The final possibility is that where water is so scarce that both riparians are constrained by their equal quota allocations

(E_{LD}). Both riparians must choose the point E which is inferior to their preferred optima A''' and B'''.

3. Results

3.1 Identical riparians

Proposition 1 established that the equal quota allocation is Pareto efficient for identical riparians. Since the noncooperative equilibrium is efficient only in N_H, but inefficient when water is scarce (N_M and N_L) it follows that equal quota is at least as efficient as noncooperation. (As shown later, identical riparians implementing equal quota also generate a cooperative surplus when water becomes scarce). To what extent does this result hold when riparians are allowed to differ in cost? As demonstrated formally below, we find that equal quota is at least as efficient as the noncooperative equilibrium when the downstream riparian has the relative cost advantage and that equal quota generates a cooperative surplus when water is sufficiently scarce. When the upstream riparian has the lowest unit cost, the results become less straightforward, and will ultimately depend on the functional form of the production function.

To arrive at these results we need to consider all possible combinations of noncooperative and equal quota scenarios. Recall the definitions of the noncooperative and equal quota sets in tables 2 and 4. The combination of the two types of sets yield a number of joint sets, such as $E_{LD} \cap N_M = \{0 < c < 1, g(1) < \overline{Q} \leq 2g(1)\}$. This reads as follows: The values of c and $\overline{Q}$ are such that the two riparians would be in scenario N_M in a noncooperative situation (i.e. downstream is constrained) but in scenario E_{LD} under equal quota (both riparians constrained). As usual, subscript D refers to a situation where the downstream riparian has the cost advantage. One can identify eleven joint sets, as illustrated in figure 3 (for simplicity we have omitted the three sets for identical riparians E_{HI}, E_{MI}, E_{LI} on the vertical line where $c = 1$). The purpose of the diagram is to clearly identify the relevant combinations of noncooperation and equal quota to aid the proof of the subsequent propositions.

3.2 Downstream riparian has the cost advantage

Proposition 2 *Suppose the downstream riparian has the relative cost advantage ($c < 1$) and a market for water loans exists. The equal quota allocation is at least as efficient as the noncooperative allocation.*

Proof. We must show that $\Pi^E \geq \Pi^N \ \forall(c, \overline{Q})$ on the relevant domain. Using the definitions of the noncooperative and equal quota sets we identify five relevant joint sets (see also figure 3): $E_{HD} \cap N_H$, $E_{MD} \cap N_H$, $E_{MD} \cap N_M$, $E_{LD} \cap N_M$, $E_{LD} \cap N_L$. Since the noncooperative equilibria and equal quota allocations differ on each of these sets we must check five different (weak) inequalities. A full proof is given in Appendix A. ■

The intuition behind proposition 2 is straightforward. Equal quota essentially implies a redistribution of water from upstream to downstream, and this generates a relative productivity effect and an absolute cost effect. The first effect emerges as relative differences in marginal products across the riparians are reduced by letting the downstream riparian reach a higher scale of production while reducing the scales of production upstream. This effect is most substantial when $\overline{Q}$ is relatively small since this is when differences in marginal profit are large (the production function is steep). The second effect occurs because more water is being used by the low cost riparian, thus the agricultural output is produced more cost effectively.

A more careful analysis of each of the five possible cases reads as follows: If water is sufficiently abundant then the riparians would be unconstrained by equal quota, i.e. the noncooperative and equal quota basinwide profits are identical ($E_{HD} \cap N_H$). The second case is that where there is sufficient water in the noncooperative situation, but where the downstream riparian is constrained by the equal quota ($E_{MD} \cap N_H$). Here the downstream riparian borrows water from its upstream neighbor to attain exactly the same profit level as under noncooperation and basinwide profits again become identical. The third case where the downstream riparian is constrained in the noncooperative as well as the equal quota allocation yields the same qualitative result ($E_{MD} \cap N_M$). Fourthly, if water is so scarce that the downstream riparian is constrained under noncooperation but both riparians are constrained in equal quota ($E_{LD} \cap N_M$) then the two parties generate a cooperative surplus by signing up to equal quota. In the fifth and final case, the two riparians are constrained under noncooperation as well as equal quota ($E_{LD} \cap N_L$). The marginal profit of the downstream riparian is very high under noncooperation - possibly infinite depending on the functional specification of $y_i(q_i)$ - because it is producing zero output. By sharing the water in this situation the riparians can attain substantial cooperative benefits.

Corollary 1 *If the downstream riparian has the relative cost advantage and water is sufficiently scarce ($\overline{Q} \leq 2g(1)$) then the equal quota allocation is more efficient than noncooperation thus generating a cooperative surplus.*

Proof. Follows from the proof of proposition 2. ■

3.3 Upstream riparian has the cost advantage

Proposition 3 *Suppose the upstream riparian has the relative cost advantage and a market for water loans exists. The efficiency effect of introducing equal quota is ambiguous.*

Proof. The proof follows that of proposition 2. We must show $\Pi^E \geq \Pi^N$ $\forall(c, \overline{Q})$ on the relevant domain which consists of the following six joint sets:

$E_{HU} \cap N_H$, $E_{MU} \cap N_H$, $E_{MU} \cap N_M$, $E_{MU} \cap N_L$, $E_{LU} \cap N_M$, $E_{LU} \cap N_L$. A full proof is given in Appendix A. ■

It is the conflict between the relative productivity and absolute cost effects which is causing this ambiguity. The relative productivity effect is positive, however, the absolute cost effect is now negative, because the water recipient is a high cost producer. To avoid this ambiguity more structure needs to be introduced to the problem. We do this by making additional assumptions about the functional form of the production function $y_i(q_i)$. In what follows we analyse two of the most relevant production functions. A brief survey of the literature on water productions for irrigated agriculture suggests the theoretical and empirical relevance of at least two functional forms: The Cobb Douglas and the quadratic form (Hexem and Heady 1974, Caswell and Zilberman 1986).

3.3.1 Cobb Douglas

Suppose agricultural production can be described with a Cobb Douglas function of the form $y_i = q_i^\alpha$. The riparian profit expression is $\pi_i = pq^\alpha - c_i q_i$ the interior solution of which yields $q_i^* = \left(\frac{\alpha p}{c_i}\right)^{\frac{1}{1-\alpha}}$. Applying the normalisations introduced earlier ($c_u = p = 1$, $c_d = c$) we get the following noncooperative equilibria and sets where $D = \{(c, \alpha) \mid c > 0,\ 0 < \alpha < 1\}$:

Table 5. Noncooperative equilibria (Cobb Douglas)

	N_H	N_M	N_L
q_u^*	$\alpha^{\frac{1}{1-\alpha}}$	$\alpha^{\frac{1}{1-\alpha}}$	$\overline{Q}$
q_d^*	$\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$	$\overline{Q} - \alpha^{\frac{1}{1-\alpha}}$	0
π_u^*	$\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$	$\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$	$\overline{Q}^\alpha - \overline{Q}$
π_d^*	$\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}} - c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$	$\left(\overline{Q} - \alpha^{\frac{1}{1-\alpha}}\right)^\alpha - c\left(\overline{Q} - \alpha^{\frac{1}{1-\alpha}}\right)$	0

Table 6. Noncooperative domain (Cobb Douglas)

$$N_H = \left\{(c, \alpha) \in D \mid \alpha^{\frac{1}{1-\alpha}} + \left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} \leq \overline{Q}\right\}$$
$$N_M = \left\{(c, \alpha) \in D \mid \alpha^{\frac{1}{1-\alpha}} < \overline{Q} < \alpha^{\frac{1}{1-\alpha}} + \left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right\}$$
$$N_L = \left\{(c, \alpha) \in D \mid 0 < \overline{Q} \leq \alpha^{\frac{1}{1-\alpha}}\right\}$$

In a similar vein, we introduce equal quota allocations and definitions using the same methodology as described previously.

Table 7. Equal quota allocation (Cobb Douglas)

	E_{HU}, E_{HD}, E_{HI}	E_{MU}	E_{MD}	$E_{LU}, E_{LD}, E_{LI}, E_{MI}$
q_u^*	$\alpha^{\frac{1}{1-\alpha}}$	$\frac{\overline{Q}}{2}$	$\alpha^{\frac{1}{1-\alpha}}$	$\frac{\overline{Q}}{2}$
q_d^*	$\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$	$\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$	$\frac{\overline{Q}}{2}$	$\frac{\overline{Q}}{2}$
π_u^*	$\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$	$\left(\frac{\overline{Q}}{2}\right)^\alpha - \frac{\overline{Q}}{2}$	$\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$	$\left(\frac{\overline{Q}}{2}\right)^\alpha - \frac{\overline{Q}}{2}$
π_d^*	$\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}} - c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$	$\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}} - c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$	$\left(\frac{\overline{Q}}{2}\right)^\alpha - c\frac{\overline{Q}}{2}$	$\left(\frac{\overline{Q}}{2}\right)^\alpha - c\frac{\overline{Q}}{2}$

Table 8. Equal quota domain (Cobb Douglas)

$$E_{HU} = \left\{0 < \alpha < 1,\ c > 1 \mid\ 2\alpha^{\frac{1}{1-\alpha}} \leq \overline{Q}\right\}$$
$$E_{MU} = \left\{0 < \alpha < 1,\ c > 1 \mid\ 2\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} < \overline{Q} < 2\alpha^{\frac{1}{1-\alpha}}\right\}$$
$$E_{LU} = \left\{0 < \alpha < 1,\ c > 1 \mid\ 0 < \overline{Q} \leq 2\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right\}$$
$$E_{HD} = \left\{0 < \alpha < 1,\ c < 1 \mid\ 2\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} \leq \overline{Q}\right\}$$
$$E_{MD} = \left\{0 < \alpha < 1,\ c < 1 \mid\ 2\alpha^{\frac{1}{1-\alpha}} < \overline{Q} < 2\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right\}$$
$$E_{LD} = \left\{0 < \alpha < 1,\ c < 1 \mid\ 0 < \overline{Q} \leq 2\alpha^{\frac{1}{1-\alpha}}\right\}$$
$$E_{HI} = \left\{0 < \alpha < 1,\ c = 1 \mid\ 2\alpha^{\frac{1}{1-\alpha}} < \overline{Q}\right\}$$
$$E_{MI} = \left\{0 < \alpha < 1,\ c = 1 \mid\ \overline{Q} = 2\alpha^{\frac{1}{1-\alpha}}\right\}$$
$$E_{LI} = \left\{0 < \alpha < 1,\ c = 1 \mid\ 0 < \overline{Q} < 2\alpha^{\frac{1}{1-\alpha}}\right\}$$

Proposition 4 *For a Cobb Douglas production function the equal quota is at least as efficient as noncooperation except for the special case where the upstream riparian has the relative cost advantage* ($c > 1$)*, water is very scarce* ($\overline{Q} \leq \alpha^{\frac{1}{1-\alpha}}$) *and* $\alpha > \frac{1}{2}$.

Proof. It follows from propositions 1 and 2 that this is true for $c \leq 1$ for $\overline{Q} > 0$ and $0 < \alpha < 1$. However, as indicated by proposition 3 the result is not obvious when the upstream riparian has the relative cost advantage ($c > 1$). We must show $\Pi^E \geq \Pi^N\ \forall(c, \overline{Q})$ on the relevant domain which consists of the same six joint sets as identified in proposition 3. A full proof is presented in appendix A. ■

The result in proposition 4 can be explained by considering the two efficiency effects of introducing equal quota: relative differences in marginal product and absolute differences in cost. In the case of Cobb Douglas we note that the difference in marginal product is a function of the parameter α. The marginal productivity difference for the allocation (q_i, q_j) , $q_i \neq q_j$ is relatively small when α is high (the production function is a straight line if $\alpha = 1$). Consider the situation where the upstream riparian has the relative cost advantage. This implies that the relative productivity effect is positive while the absolute cost effect is negative (cf. proposition 3). As α increases, the productivity effect diminishes and (since the negative cost effect is unchanged) the total effect eventually becomes negative when water is sufficiently scarce. It emerges from the proof of proposition 4 that this happens when the riparians are in N_L.

Figures 4.1 - 4.5 give a graphical interpretation of proposition 4 where we have assumed $c = 2$ and $\alpha = \frac{1}{2}$, i.e. a situation where equal quota is at least as efficient as noncooperation. Consider figure 4.1 which illustrates case $E_{HU} \cap N_H$. Panel (a) illustrates agricultural output with A and B representing unconstrained optima for each riparian. In panel (b) below the relevant riparian and basinwide profit curves have been depicted. Riparian profit is increasing in q_i until the optimum q_i^* is reached after which it remains constant. Hence, we assume free disposal whereby a riparian can release unused surplus water further downstream thereby ignoring the possibility of flooding. The regional profit is the vertical sum of riparian profits. We see that equal quota and noncooperative profits are identical. Both allocations are, in fact, Pareto optimal as are all other allocations on the horizontal part of the $\Pi-$curve. Finally, note that all Pareto optimal states in the diagram have identical distributional implications. Regional profit is distributed according to relative cost differences alone because water is abundant.

Case $E_{MU} \cap N_H$ is illustrated in figure 4.2. The upstream riparian is constrained by equal quota and must take a water loan to move from C to A. Panel (b) illustrates how this loan leads to a small efficiency gain. We see that $\Pi^E = \Pi^N$ and since there is still unused water in the basin there are many other Pareto optimal states with identical distribution of Π.

In figure 4.3 upstream chooses A under noncooperation and downstream is left with E ($E_{MU} \cap N_M$). Under equal quota upstream would choose C (and D after a loan) while downstream can reach its optimum at B. In this case the equal quota allocation (after the water loan) is a unique Pareto optimum. This implies that a cooperative surplus is generated. It also follows that any perpetuation of the equal quota would have distributional as well as efficiency implications.

In scenario $E_{MU} \cap N_L$, illustrated in figure 4.4, Upstream is constrained in the noncooperative, as well as the equal quota allocation. As a consequence, it can never attain its unconstrained optimum A. It chooses F noncooperatively, while downstream is left at zero input E. In equal quota (after loan) upstream is at D and downstream at B. Equal quota is now substantially more efficient than noncooperation. Despite this, the equal quota (after loan) is no longer Pareto optimal. A marginal redistribution of water from downstream to upstream would yield further, albeit small, efficiency improvements.

Situation $E_{LU} \cap N_M$ does not exist because we have assumed $c = 2$ (see figure 3 to verify this). The final case to analyse is therefore $E_{LU} \cap N_L$ in figure 4.5 which is qualitatively very similar to figure 4.4 apart from the fact that water loans are no longer possible.

Figure 5 illustrates the cooperative surplus of introducing equal quota for a Cobb Douglas production function with $\alpha = \frac{1}{2}$ (note that the horisontal axis is inversed). We make a number observations. First, the cooperative surplus is present when water is sufficiently scarce. Secondly, as $\overline{Q}$ decreases the surplus reaches its maximum when the total amount of water available in the basin equals the total water needs of the upstream riparian ($\overline{Q} = \alpha^{\frac{1}{1-\alpha}}$), i.e. at the $N_M - N_L$ border. Thirdly, it falls gradually thereafter as water becomes increasingly scarce. Finally, the cooperative surplus is increasing in the relative cost advantage of the downstream riparian (decreasing in c) and decreasing in the parameter α (not illustrated).

3.3.2 Quadratic form

The quadratic production function $y_i = aq_i - bq_i^2$ satisfies $y_i(0) = 0$ and exhibits diminishing returns $\frac{\partial y_i}{\partial q_i} > 0$ and $\frac{\partial^2 y_i}{\partial q_i^2} < 0$ on the interval $0 \leq q_i \leq q_i^*$. A riparian which uses more water than its unconstrained optimum experiences declining production $\frac{\partial y_i}{\partial q_i} < 0$ for $q_i > q_i^*$. This would be the case if the agricultural lands were flooded. We shall, however, ignore that possibility in the following analysis by continuing to assume free disposal[19]. Setting $p = 1$ we can write riparian profit as $\pi_i = (a-c)q_i - bq_i^2$ where we assume $a > c$. At the interior solution $q_i^* = \frac{a-c}{2b}, b \neq 0$. Suppose for simplicity that $a - c = 1$ for the upstream riparian and $b = \frac{1}{2}$ so that riparians have identical production technology (but differ in cost) then we get $q_u^* = 1$ and $q_d^* = a - c$. We then have three situations:

A: $a - c < 1$: upstream riparian has relative cost advantage.
B: $a - c > 1$: downstream riparian has relative cost advantage.
C: $a - c = 1$: identical riparians.

The noncooperative scenarios and equal quota allocations and domain are:

Table 9. Noncooperative equilibria (Quadratic)

	N_H	N_M	N_L
q_u^*	1	1	$\overline{Q}$
q_d^*	$a-c$	$\overline{Q}-1$	0
π_u^*	$\frac{1}{2}$	$\frac{1}{2}$	$\overline{Q} - \frac{1}{2}\overline{Q}^2$
π_d^*	$\frac{1}{2}(a-c)^2$	$(a-c)(\overline{Q}-1) - \frac{1}{2}(\overline{Q}-1)^2$	0

Table 10. Noncooperative domain (Quadratic)

$N_H = \left\{a - c > 0 \mid 1 + a - c \leq \overline{Q}\right\}$
$N_M = \left\{a - c > 0 \mid 1 < \overline{Q} < 1 + a - c\right\}$
$N_L = \left\{a - c > 0 \mid 0 < \overline{Q} \leq 1\right\}$

Table 11. Equal quota allocation (Quadratic)

	E_{HU}, E_{HD}, E_{HI}	E_{MU}	E_{MD}	$E_{LU}, E_{LD}, E_{LI}, E_{MI}$
q_u^*	1	$\frac{\overline{Q}}{2}$	1	$\frac{\overline{Q}}{2}$
q_d^*	$a-c$	$a-c$	$\frac{\overline{Q}}{2}$	$\frac{\overline{Q}}{2}$
π_u^*	$\frac{1}{2}$	$\frac{\overline{Q}}{2} - \frac{\overline{Q}^2}{8}$	$\frac{1}{2}$	$\frac{\overline{Q}}{2} - \frac{\overline{Q}^2}{8}$
π_d^*	$\frac{1}{2}(a-c)^2$	$\frac{1}{2}(a-c)^2$	$(a-c)\frac{\overline{Q}}{2} - \frac{\overline{Q}^2}{8}$	$(a-c)\frac{\overline{Q}}{2} - \frac{\overline{Q}^2}{8}$

[19] This assumption is reasonable since both riparians use water for irrigation. It would have been less plausible if, say, the upstream riparian had a hydropower plant since this could cause serious flow disruptions downstream, and possibly flooding.

Table 12. Equal quota domain (Quadratic)

$E_{HU} = \{0 < a - c < 1,\ 2 \leq \overline{Q}\}$
$E_{MU} = \{0 < a - c < 1,\ 2(a - c) < \overline{Q} < 2\}$
$E_{LU} = \{0 < a - c < 1,\ 0 < \overline{Q} \leq 2(a - c)\}$
$E_{HD} = \{a - c > 1,\ 2(a - c) \leq \overline{Q}\}$
$E_{MD} = \{a - c > 1,\ 2 < \overline{Q} < 2(a - c)\}$
$E_{LD} = \{a - c > 1,\ 0 < \overline{Q} \leq 2\}$
$E_{HI} = \{a - c = 1,\ 2 < \overline{Q}\}$
$E_{MI} = \{a - c = 1,\ \overline{Q} = 2\}$
$E_{LI} = \{a - c = 1,\ 0 < \overline{Q} < 2\}$

The cooperative surplus for a quadratic production function is shown in figure 6. We note a number of similarities with the Cobb Douglas form. First, the surplus arises when water is sufficiently scarce. Secondly, it reaches it maximum at the $N_M - N_L$ border. Thirdly, it is increasing in the relative cost advantage of the downstream riparian (decreasing in $(a - c)$). Finally, a cooperative loss occurs from introducing equal quota when the downstream riparian has a cost disadvantage $(a - c < 1)$ and the water level in the basin is very low (N_L). More precisely we have the following result for the quadratic production function:

Proposition 5 *If the agricultural production function is quadratic then equal quota is at least as efficient as noncooperation except for the special case where the upstream riparian has the relative cost advantage* $(a - c) < 1$ *and water is very scarce* $\overline{Q} \leq 1$.

Proof. It follows from propositions 1 and 2 that this is true for $(a - c) \geq 1$, i.e. when downstream is more cost effective or when riparians are identical. We must show $\Pi^E \geq \Pi^N\ \forall(a - c, \overline{Q})$ on the relevant domain which consists of five joint sets derived from tables 10 and 12. A full proof is presented in appendix A. ■

The major difference between the two functional forms is that equal quota is more likely to have a positive efficiency impact if agricultural production can be described with Cobb Douglas technology rather than a quadratic form. This is because the marginal product at $q_i = 0$ is infinite for a Cobb-Douglas function and finite (equal to a) for the quadratic form. The size of the marginal product at $q_i = 0$ is important because the cooperative surplus is maximised at the $N_M - N_L$ border, i.e. when the downstream riparian is at zero output under noncooperation.

3.4 Agreement stability

The principal analytical concern thus far has been whether equal quota represents a superior alternative to noncooperation in terms of its equity and efficiency properties. As previously highlighted equal quota can only be attained by means of a cooperative agreement between the two riparians. In the absence of a supranational body to enforce the agreement, neither of the signatories must find it in their own interest to deviate and act unilaterally. In other words, the agreement must be self-enforcing (Barrett, 1994b). Since equal quota implies a redistribution of water, and hence profit, from upstream to downstream the key question is whether the upstream riparian would find it individually rational to enter the agreement. Clearly, were the downstream riparian to keep all its additional profits under an equal quota agreement then the upstream riparian would never sign. The analysis therefore presupposes the possibility of lump sum side payments payable from the winner of cooperation to the loser[20]. This is why the size of the cooperative surplus matters. If the surplus is non-positive then it would be undesirable from an efficiency point of view for any of the riparians to enter the agreement although this could be justified on equity grounds provided the trade-off is politically acceptable.

Suppose in the following example that there are benefits from cooperation. The subsequent analysis gives an illustration of how a stable agreement can be reached. We assume identical Cobb Douglas production functions where $\alpha = \frac{1}{2}$, $c = 2$ (upstream riparian has the cost advantage) and $Q = \frac{1}{5}$ which implies that the riparians are in $E_{MU} \cap N_L$. For simplicity we set the interest payment for the water loan equal to zero ($i\Delta = 0$). Table 13 gives an overview:

Table 13. Example of a stable equal quota agreement

	q_u^*	π_u^*	q_d^*	π_d^*	Π
1. Noncooperation	0.20	0.247	0.00	0.00	0.247
2. Equal quota (before loan)	0.10	0.216	0.06	0.125	0.341
3. Equal quota (after loan)	0.14	0.233	0.06	0.125	0.358
4. Stable agreement	0.14	0.247	0.06	0.111	0.358
5. Splitting the surplus	0.14	0.302	0.06	0.056	0.358

Given the noncooperative payoffs, the individual rationality constraint of the upstream riparian is $\pi_u^E \geq \pi_u^N = 0.247$, and, $\pi_d^E \geq \pi_d^N = 0$ for the downstream riparian. The downstream riparian gains substantially under equal quota (before loan) while downstream incurs a minor loss. By allowing for water loans the upstream riparian can make productive use of the 0.04 units of water not utilised by downstream. While equal quota (after loan) is more efficient than noncooperation ($\Pi^E = 0.358 > \Pi^N = 0.247$) it is not individually rational for the upstream riparian to cooperate. To make the upstream riparian sign the agreement, the downstream riparian must pay a side payment of at least $\pi_u^N - \pi_u^E = 0.247 - 0.233 = 0.014$. This side payment is sufficient to guarantee a stable agreement. Ultimately, the size of the side payment will be a matter of negotiation between the two riparians. The Nash bargaining solution provides a theoretical solution to this problem. Supposing both riparians are risk neutral we must solve the following problem $\max\limits_{u_u + u_d \leq 0.358} \{(u_u - 0.247)(u_d - 0)\}$. The result embodies the popular notion of splitting the cooperative surplus.

[20]It is exactly this type of transfer which Sadoff et. al. (2002) is sceptical about.

To what extent can the final outcome of the Nash bargaining solution be said to be equitable? After all, the upstream riparian gets 70 percent of the water and 84.3 percent of basinwide profits. It is important here to reemphasise that our notion of equity is the *egalitarian standard of equal shares of water to equals,* or alternatively, *equality of opportunity.* The cooperative agreement does indeed reflect these principles: Both countries were given an equal share of the water. The reason why the downstream riparian only uses 30 percent is not due to the agreement but rather to a lack of innate capability of making productive use of its entitlement. Because our equity metric is water - not final outcomes such as profits - we refrain from making any evaluative judgement of the fairness of the basinwide profit distribution.

Before concluding the analysis of stability it is worth pointing out that riparians do sometimes sign (and respect) international agreements even though they are unstable in a narrow economic sense. As pointed out in the international relations literature such behavior can be entirely rational if one also considers the broader political benefits from signing such an agreement (LeMarquand, 1977). First, the signatories to an agreement may want to project a positive international image of themselves as in the case of the decision by the US Government to build a desalting plant on the Lower Colorado River in the 1970s. Secondly, river sharing agreements are only one of many ways in which countries interact, thus one country might accept a 'bad deal' if a linkage has been made to another bilateral agreement on which it stands to gain more substantially (see Bennett et. al., 1998). Finally, a reluctant upstream riparian may be downstream to the same or other countries on other rivers and this produces a more flexible stance (Sadoff et. al., 2002). On the other hand, there are also examples of economically rational, but politically infeasible agreements, as exemplified by Dinar and Wolf's (1994) analysis of water markets for the Middle East.

4. Policy implications

Suppose the model presented in this paper gives a sufficiently reasonable description of reality. How could it be used for practical purposes to guide the negotiation of a water sharing agreement between two riparians? What are the informational requirements? What are the policy implications? The answers to these questions are best addressed by proposing a simple algorithm that negotiators can adopt. Obviously, this solution represents a substantial simplification of what in most cases would be a complex negotiation scenario. Nevertheless, it constitutes a basic prescription of the steps necessary to determine whether an equal quota agreement (or any other exogenous share) is worth pursuing vis-à-vis noncooperation when riparians aspire for an efficient and equitable solution.

In step 1 the riparians must collect all relevant information. First, this includes an estimate of the water production functions of each riparian. The paramount interest is to estimate the values of parameters such as α, a or b. Secondly, riparian cost functions must be estimated. What matters here is the relative difference in unit costs between riparians. Thirdly, a reliable estimate must be made over the mean annual water flow. The annual flow volume is usually stochastic (affected by weather) and fluctuations of 25 percent above or below the mean annual flow are quite common (Kilgour and Dinar, 1995). Flow data is often (but not always) available to negotiators and can be estimated more easily than production or cost functions. Step 2 involves a re-specification of the theoretical model in light of the available data. For instance, it is highly likely that riparians will have production or cost functions which are different from those presented in this paper. On the basis of the data collected in step 1 negotiators must, in step 3, make an overall assessment of which of the many possible scenarios the riparians are most likely to find themselves in. This may not be a unique scenario, such as $E_{LD} \cap N_M$, but rather a range of possible scenarios. The uncertainty derives partly from the statistical uncertainty of the parameter values of the production and cost functions, but most importantly the substantial variation in annual water flow. Step 4 involves an estimation of the expected cooperative surplus of introducing equal quota. This obviously depends on the conclusions of step 3 and riparians could also account for possible transaction costs. In step 5, the riparians must decide whether it is worthwhile to share the water equally. If the cooperative surplus (net of transaction costs) is positive then equal quota is a first-best policy. If the cooperative surplus is negative then the equal quota is the second-best policy and riparians trade-off efficiency to attain equity. Finally, in step 6 the riparians must embark on negotiations of how to share the cooperative surplus.

5. Conclusion

Economic efficiency and social equity are both valid policy objectives in the management of transboundary rivers. This paper has dealt with the question of how these two objectives interrelate and whether, for instance, they are at odds with each other. The theoretical results contain a relatively optimistic policy message: Although equity and efficiency are inseparable objectives this does not necessarily imply a trade-off. Under certain circumstance, cooperating riparians can be rewarded with a cooperative surplus. Trade-offs do exist, but they are predictable, less frequent and relatively small in magnitude.

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Appendix A. Proof of Propositions

Proof. (of proposition 2). Suppose the downstream riparian has the relative cost advantage ($c < 1$).

1) $\Pi^E \geq \Pi^N \ \forall \ (c, \overline{Q}) \in E_{HD} \cap N_H \Leftrightarrow$
$y\left(g(1)\right) - g(1) + y\left(g(c)\right) - cg(c) \geq y\left(g(1)\right) - g(1) + y\left(g(c)\right) - cg(c)$
which holds with equality.

2) $\Pi^E \geq \Pi^N \ \forall \ (c, \overline{Q}) \in E_{MD} \cap N_H \Leftrightarrow$
$y\left(g(1)\right) - g(1) + y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2} \geq y\left(g(1)\right) - g(1) + y\left(g(c)\right) - cg(c) \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2} \geq y\left(g(c)\right) - cg(c)$
Allowing for a water loan (downstream borrows) there is excess supply ($\Delta = \delta = g(c) - \frac{\overline{Q}}{2}$) which gives:
$y\left(\frac{\overline{Q}}{2} + \Delta\right) - c\left(\frac{\overline{Q}}{2} + \Delta\right) \geq y\left(g(c)\right) - cg(c) \Leftrightarrow$
$y\left(g(c)\right) - cg(c) \geq y\left(g(c)\right) - cg(c)$
which holds with equality.

3) $\Pi^E \geq \Pi^N \ \forall \ (c, \overline{Q}) \in E_{MD} \cap N_M \Leftrightarrow$
$y\left(g(1)\right) - g(1) + y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2} \geq y\left(g(1)\right) - g(1) + y\left(\overline{Q} - g(1)\right) - c\overline{Q} + cg(1) \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2} \geq y\left(\overline{Q} - g(1)\right) - c\overline{Q} + cg(1)$
Allowing for a water loan (downstream borrows) there is excess demand ($\Delta = \sigma = \frac{\overline{Q}}{2} - g(1)$) which gives:
$y\left(\frac{\overline{Q}}{2} + \Delta\right) - c\left(\frac{\overline{Q}}{2} + \Delta\right) \geq y\left(\overline{Q} - g(1)\right) - c\overline{Q} + cg(1) \Leftrightarrow$
$y\left(\overline{Q} - g(1)\right) - c\left(\overline{Q} - g(1)\right) \geq y\left(\overline{Q} - g(1)\right) - c\overline{Q} + cg(1)$
which holds with equality.

4) $\Pi^E \geq \Pi^N \ \forall \ (c, \overline{Q}) \in E_{LD} \cap N_M \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2} + y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2} \geq y\left(g(1)\right) - g(1) + y\left(\overline{Q} - g(1)\right) - c\overline{Q} + cg(1) \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2} + y\left(\frac{\overline{Q}}{2}\right) + c\frac{\overline{Q}}{2} \geq y\left(g(1)\right) - g(1) + y\left(\overline{Q} - g(1)\right) + cg(1) \Leftrightarrow$
$2y\left(\frac{\overline{Q}}{2}\right) + (1-c)\frac{\overline{Q}}{2} \geq y\left(g(1)\right) + y\left(\overline{Q} - g(1)\right) + (1-c)g(1)$
which is true with strict inequality since
$2y\left(\frac{\overline{Q}}{2}\right) > y\left(g(1)\right) + y\left(\overline{Q} - g(1)\right)$ cf. proposition 1 and
$(1-c)\frac{\overline{Q}}{2} > (1-c)g(1)$ since the upstream riparian is unconstrained in N_M.

5) $\Pi^E \geq \Pi^N \ \forall \ (c, \overline{Q}) \in E_{LD} \cap N_L \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2} + y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2} \geq y\left(\overline{Q}\right) - \overline{Q} \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) + \frac{\overline{Q}}{2} + y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2} \geq y\left(\overline{Q}\right)$

which holds with strict inequality since
$2y\left(\frac{\overline{Q}}{2}\right) \geq y\left(\overline{Q}\right)$ and $(1-c)\left(\frac{\overline{Q}}{2}\right) > 0$. ■

Proof. (of proposition 3). Suppose the upstream riparian has the relative cost advantage ($c > 1$).

1) $\Pi^E \geq \Pi^N \; \forall \; (c, \overline{Q}) \in E_{HU} \cap N_H \Leftrightarrow$
$y\left(g(1)\right) - g(1) + y\left(g(c)\right) - cg(c) \geq y\left(g(1)\right) - g(1) + y\left(g(c)\right) - cg(c)$
which holds with equality.

2) $\Pi^E \geq \Pi^N \; \forall \; (c, \overline{Q}, \alpha) \in E_{MU} \cap N_H \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2} + y\left(g(c)\right) - cg(c) \geq y\left(g(1)\right) - g(1) + y\left(g(c)\right) - cg(c) \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2} \geq y\left(g(1)\right) - g(1)$
Allowing for a water loan (upstream borrows) we have excess supply ($\Delta = \delta = g(1) - \frac{\overline{Q}}{2}$) and can write:
$y\left(\frac{\overline{Q}}{2} + \Delta\right) - \left(\frac{\overline{Q}}{2} + \Delta\right) \geq y\left(g(1)\right) - g(1) \Leftrightarrow$
$y\left(g(1)\right) - g(1) \geq y\left(g(1)\right) - g(1)$
which holds with equality.

3) $\Pi^E \geq \Pi^N \; \forall \; (c, \overline{Q}, \alpha) \in E_{MU} \cap N_M \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2} + y\left(g(c)\right) - cg(c) \geq y\left(g(1)\right) - g(1) + y\left(\overline{Q} - g(1)\right) - c\left(\overline{Q} - g(1)\right)$
Allowing for a water loan (upstream borrows) we have excess demand ($\Delta = \sigma = \frac{\overline{Q}}{2} - g(c)$) and can write:
$y\left(\frac{\overline{Q}}{2} + \Delta\right) - \left(\frac{\overline{Q}}{2} + \Delta\right) + y\left(g(c)\right) - cg(c) \geq$
$y\left(g(1)\right) - g(1) + y\left(\overline{Q} - g(1)\right) - c\left(\overline{Q} - g(1)\right) \Leftrightarrow$
$y\left(\overline{Q} - g(c)\right) - \left(\overline{Q} - g(c)\right) + y\left(g(c)\right) - cg(c) \geq$
$y\left(g(1)\right) - g(1) + y\left(\overline{Q} - g(1)\right) - c\left(\overline{Q} - g(1)\right)$
which is ambiguous without further assumptions.

4) $\Pi^E \geq \Pi^N \; \forall \; (c, \overline{Q}, \alpha) \in E_{MU} \cap N_L \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2} + y\left(g(c)\right) - cg(c) \geq y\left(\overline{Q}\right) - \overline{Q}$
Allowing for a water loan (upstream borrows) we have excess demand ($\Delta = \sigma = \frac{\overline{Q}}{2} - g(c)$) and can write:
$y\left(\frac{\overline{Q}}{2} + \Delta\right) - \left(\frac{\overline{Q}}{2} + \Delta\right) + y\left(g(c)\right) - cg(c) \geq y\left(\overline{Q}\right) - \overline{Q} \Leftrightarrow$
$y\left(\overline{Q} - g(c)\right) - \left(\overline{Q} - g(c)\right) + y\left(g(c)\right) - cg(c) \geq y\left(\overline{Q}\right) - \overline{Q} \Leftrightarrow$
$y\left(\overline{Q} - g(c)\right) + g(c) + y\left(g(c)\right) - cg(c) \geq y\left(\overline{Q}\right)$
which is ambiguous without further assumptions.

5) $\Pi^E \geq \Pi^N \; \forall \; (c, \overline{Q}, \alpha) \in E_{LU} \cap N_M \Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right) - \frac{\overline{Q}}{2} + y\left(\frac{\overline{Q}}{2}\right) - c\frac{\overline{Q}}{2} \geq y\left(g(1)\right) - g(1) + y\left(\overline{Q} - g(1)\right) - c\overline{Q} + cg(1) \Leftrightarrow$

$y\left(\frac{\overline{Q}}{2}\right)+y\left(\frac{\overline{Q}}{2}\right)+(c-1)\frac{\overline{Q}}{2}\geq y\left(g(1)\right)+(c-1)g(1)+y\left(\overline{Q}-g(1)\right)$
which is ambiguous since
$2y\left(\frac{\overline{Q}}{2}\right)>y\left(g(1)\right)+y\left(\overline{Q}-g(1)\right)$ cf. proposition 1, but also
$(c-1)\frac{\overline{Q}}{2}<(c-1)g(1)$ because we know that the upstream riparian is constrained by equal quota.

6) $\Pi^E\geq\Pi^N\ \forall\ (c,\overline{Q})\in E_{LU}\cap N_L\Leftrightarrow$
$y\left(\frac{\overline{Q}}{2}\right)-\frac{\overline{Q}}{2}+y\left(\frac{\overline{Q}}{2}\right)-c\frac{\overline{Q}}{2}\geq y\left(\overline{Q}\right)-\overline{Q}$
which is ambiguous since we have that $2y\left(\frac{\overline{Q}}{2}\right)>y\left(\overline{Q}\right)$, but also
$(1-c)\frac{\overline{Q}}{2}<\overline{Q}$. ■

Proof. (of proposition 4 - Cobb Douglas). Suppose the upstream riparian has the relative cost advantage ($c>1$).
1) $\Pi^E\geq\Pi^N\ \forall\ (c,\overline{Q},\alpha)\in E_{HU}\cap N_H\Leftrightarrow$
$\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}+\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\geq\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}+\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$
which holds with equality.

2) $\Pi^E\geq\Pi^N\ \forall\ (c,\overline{Q},\alpha)\in E_{MU}\cap N_H\Leftrightarrow$
$\left(\frac{\overline{Q}}{2}\right)^{\alpha}-\frac{\overline{Q}}{2}+\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\geq$
$\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}+\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\Leftrightarrow$
$\left(\frac{\overline{Q}}{2}\right)^{\alpha}-\frac{\overline{Q}}{2}\geq\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}$
Allowing for a water loan (upstream borrows) we have excess supply ($\Delta=\delta=\alpha^{\frac{1}{1-\alpha}}-\frac{\overline{Q}}{2}$) and can write:
$\left(\frac{\overline{Q}}{2}+\Delta\right)^{\alpha}-\left(\frac{\overline{Q}}{2}+\Delta\right)\geq\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}\Leftrightarrow$
$\left(\frac{\overline{Q}}{2}+\alpha^{\frac{1}{1-\alpha}}-\frac{\overline{Q}}{2}\right)^{\alpha}-\left(\frac{\overline{Q}}{2}+\alpha^{\frac{1}{1-\alpha}}-\frac{\overline{Q}}{2}\right)\geq\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}$
which holds with equality.

3) $\Pi^E\geq\Pi^N\ \forall\ (c,\overline{Q},\alpha)\in E_{MU}\cap N_M\Leftrightarrow$
$\left(\frac{\overline{Q}}{2}\right)^{\alpha}-\frac{\overline{Q}}{2}+\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\geq$
$\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}+\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}-c\overline{Q}+c\alpha^{\frac{1}{1-\alpha}}$
Allowing for a water loan (upstream borrows) we have excess demand ($\Delta=\sigma=\frac{\overline{Q}}{2}-\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$) and can write:
$\left(\frac{\overline{Q}}{2}+\Delta\right)^{\alpha}-\left(\frac{\overline{Q}}{2}+\Delta\right)+\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\geq$
$\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}+\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}-c\overline{Q}+c\alpha^{\frac{1}{1-\alpha}}\Leftrightarrow$
$\left(\overline{Q}-\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}-\left(\overline{Q}-\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right)+\left(\frac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\geq$
$\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}+\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}-c\overline{Q}+c\alpha^{\frac{1}{1-\alpha}}\Leftrightarrow$

$$\left(\overline{Q}-\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}+\left(\tfrac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{\alpha}{1-\alpha}}-\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}\geq$$
$$\overline{Q}-\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}+c\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}-c\overline{Q}+c\alpha^{\frac{1}{1-\alpha}}\Leftrightarrow$$
$$\left(\overline{Q}-\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}+\left(\tfrac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{\alpha}{1-\alpha}}-\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}\geq$$
$$(c-1)\left(\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}+\alpha^{\frac{1}{1-\alpha}}-\overline{Q}\right)$$
which is true as can be verified numerically.

4) $\Pi^E \geq \Pi^N \ \forall\ (c,\overline{Q},\alpha)\in E_{MU}\cap N_L \Leftrightarrow$
$$\left(\tfrac{\overline{Q}}{2}\right)^{\alpha}-\tfrac{\overline{Q}}{2}+\left(\tfrac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\geq\overline{Q}^{\alpha}-\overline{Q}$$
Allowing for a water loan (upstream borrows) we have excess demand ($\Delta=\sigma=\frac{\overline{Q}}{2}-\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}$) and can write:
$$\left(\tfrac{\overline{Q}}{2}+\Delta\right)^{\alpha}-\left(\tfrac{\overline{Q}}{2}+\Delta\right)+\left(\tfrac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\geq\overline{Q}^{\alpha}-\overline{Q}\Leftrightarrow$$
$$\left(\overline{Q}-\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}-\left(\overline{Q}-\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right)+\left(\tfrac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-c\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\geq\overline{Q}^{\alpha}-\overline{Q}\Leftrightarrow$$
$$\left(\overline{Q}-\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}+(1-c)\left(\tfrac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}+\left(\tfrac{\alpha}{c}\right)^{\frac{\alpha}{1-\alpha}}-\overline{Q}^{\alpha}\geq 0$$
which is true for $\alpha\leq\frac{1}{2}$ and $c>1$ as can be verified numerically.

5) $\Pi^E \geq \Pi^N \ \forall\ (c,\overline{Q},\alpha)\in E_{LU}\cap N_M \Leftrightarrow$
$$2\left(\tfrac{\overline{Q}}{2}\right)^{\alpha}-\tfrac{\overline{Q}}{2}-c\tfrac{\overline{Q}}{2}\geq\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}+\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}-c\overline{Q}+c\alpha^{\frac{1}{1-\alpha}}\Leftrightarrow$$
$$2\left(\tfrac{\overline{Q}}{2}\right)^{\alpha}-\tfrac{\overline{Q}}{2}+c\tfrac{\overline{Q}}{2}\geq\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}+\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}+c\alpha^{\frac{1}{1-\alpha}}\Leftrightarrow$$
$$2\left(\tfrac{\overline{Q}}{2}\right)^{\alpha}+(c-1)\tfrac{\overline{Q}}{2}\geq\alpha^{\frac{\alpha}{1-\alpha}}+\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}+(c-1)\alpha^{\frac{1}{1-\alpha}}$$
which is true with strict inequality since
$2\left(\frac{\overline{Q}}{2}\right)^{\alpha}>\alpha^{\frac{\alpha}{1-\alpha}}+\left(\overline{Q}-\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}$ and
$(c-1)\frac{\overline{Q}}{2}<(c-1)\alpha^{\frac{1}{1-\alpha}}$.

6) $\Pi^E \geq \Pi^N \ \forall\ (c,\overline{Q},\alpha)\in E_{LU}\cap N_L \Leftrightarrow$
$$2\left(\tfrac{\overline{Q}}{2}\right)^{\alpha}-\tfrac{\overline{Q}}{2}-c\tfrac{\overline{Q}}{2}\geq\overline{Q}^{\alpha}-\overline{Q}\Leftrightarrow$$
$$2\left(\tfrac{\overline{Q}}{2}\right)^{\alpha}+\tfrac{\overline{Q}}{2}-c\tfrac{\overline{Q}}{2}\geq\overline{Q}^{\alpha}\Leftrightarrow$$
$$2\left(\tfrac{\overline{Q}}{2}\right)^{\alpha}+(1-c)\tfrac{\overline{Q}}{2}\geq\overline{Q}^{\alpha}\Leftrightarrow$$
$$2\tfrac{\overline{Q}^{\alpha}}{2^{\alpha}}+(1-c)\tfrac{\overline{Q}}{2}\geq\overline{Q}^{\alpha}\Leftrightarrow$$
$$2^{1-\alpha}+(1-c)\tfrac{\overline{Q}^{1-\alpha}}{2}\geq 1\Leftrightarrow$$
$$2^{(2-\alpha)}-2+(1-c)\overline{Q}^{1-\alpha}\geq 0\Leftrightarrow$$
$\overline{Q}^{1-\alpha}\leq\frac{2-2^{(2-\alpha)}}{1-c}\Leftrightarrow$ (note the sign change since $1-c<0$)
$$\overline{Q}\leq\left(\tfrac{2-2^{(2-\alpha)}}{1-c}\right)^{\frac{1}{1-\alpha}}$$
Since $E_{LU}\cap N_L=\{\overline{Q}\leq\alpha^{\frac{1}{1-\alpha}}$ and $\overline{Q}\leq 2\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}}\}$ we get two conditions (only one of which is binding at any one time):

$\alpha^{\frac{1}{1-\alpha}} \leq \left(\frac{2-2^{(2-\alpha)}}{1-c}\right)^{\frac{1}{1-\alpha}}$ and $2\left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} \leq \left(\frac{2-2^{(2-\alpha)}}{1-c}\right)^{\frac{1}{1-\alpha}}$
which is true for $\alpha \leq \frac{1}{2}$ and $c > 1$ as can be verified numerically.
It is, however, false for some values of c when $\alpha > \frac{1}{2}$. ■

Proof. (of proposition 5 - quadratic). Suppose that the upstream riparian has the relative cost advantage ($a - c < 1$):

1) $\Pi^E \geq \Pi^N \; \forall \; (a, c, \overline{Q}) \in E_{HU} \cap N_H \Leftrightarrow$
$\frac{1}{2} + \frac{1}{2}(a-c)^2 \geq \frac{1}{2} + \frac{1}{2}(a-c)^2$ which holds with equality.

2) $\Pi^E \geq \Pi^N \; \forall \; (a, c, \overline{Q}) \in E_{MU} \cap N_H \Leftrightarrow$
$\frac{\overline{Q}}{2} - \frac{1}{2}\left(\frac{\overline{Q}}{2}\right)^2 + \frac{1}{2}(a-c)^2 \geq \frac{1}{2} + \frac{1}{2}(a-c)^2 \Leftrightarrow$
$\frac{\overline{Q}}{2} - \frac{1}{2}\left(\frac{\overline{Q}}{2}\right)^2 \geq \frac{1}{2}$

Allowing for water loans (upstream borrows) we have excess supply ($\Delta = \delta = 1 - \frac{\overline{Q}}{2}$):
$\left(\frac{\overline{Q}}{2} + \Delta\right) - \frac{1}{2}\left(\frac{\overline{Q}}{2} + \Delta\right)^2 \geq \frac{1}{2} \Leftrightarrow$
$\left(\frac{\overline{Q}}{2} + 1 - \frac{\overline{Q}}{2}\right) - \frac{1}{2}\left(\frac{\overline{Q}}{2} + 1 - \frac{\overline{Q}}{2}\right)^2 \geq \frac{1}{2} \Leftrightarrow$
$1 - \frac{1}{2} \geq \frac{1}{2}$ which holds with equality.

3) $\Pi^E \geq \Pi^N \; \forall \; (a, c, \overline{Q}) \in E_{MU} \cap N_M \Leftrightarrow$
$\frac{\overline{Q}}{2} - \frac{1}{2}\left(\frac{\overline{Q}}{2}\right)^2 + \frac{1}{2}(a-c)^2 \geq \frac{1}{2} + (a-c)(\overline{Q}-1) - \frac{1}{2}(\overline{Q}-1)^2 \Leftrightarrow$
Allowing for water loans (upstream borrows) we have excess demand ($\Delta = \sigma = \frac{\overline{Q}}{2} - (a-c)$) and letting $(a-c) = k$:
$\overline{Q} - k - \frac{1}{2}\left(\overline{Q} - k\right)^2 + \frac{1}{2}k^2 \geq \frac{1}{2} + k(\overline{Q}-1) - \frac{1}{2}(\overline{Q}-1)^2 \Leftrightarrow$
$\overline{Q} - k - \frac{1}{2}\left(\overline{Q}^2 + k^2 - 2Qk\right) + \frac{1}{2}k^2 \geq \frac{1}{2} + k\overline{Q} - k - \frac{1}{2}(\overline{Q}^2 + 1 - 2Q) \Leftrightarrow$
$\overline{Q} - k - \frac{1}{2}\overline{Q}^2 - \frac{1}{2}k^2 + Qk + \frac{1}{2}k^2 \geq \frac{1}{2} + k\overline{Q} - k - \frac{1}{2}\overline{Q}^2 - \frac{1}{2} + Q \Leftrightarrow$
$-\frac{1}{2}k^2 + Qk + \frac{1}{2}k^2 \geq k\overline{Q}$ which holds with equality.

4) $\Pi^E \geq \Pi^N \; \forall \; (a, c, \overline{Q}) \in E_{MU} \cap N_L \Leftrightarrow$
$\frac{\overline{Q}}{2} - \frac{1}{2}\left(\frac{\overline{Q}}{2}\right)^2 + \frac{1}{2}(a-c)^2 \geq \overline{Q} - \frac{1}{2}\overline{Q}^2$
Allowing for water loans (upstream borrows) we have excess demand ($\Delta = \sigma = \frac{\overline{Q}}{2} - (a-c)$) and letting $(a-c) = k$:
$\overline{Q} - k - \frac{1}{2}\left(\overline{Q} - k\right)^2 + \frac{1}{2}k^2 \geq \overline{Q} - \frac{1}{2}\overline{Q}^2 \Leftrightarrow$
$\overline{Q} - k - \frac{1}{2}\left(\overline{Q}^2 + k^2 - 2Qk\right) + \frac{1}{2}k^2 \geq \overline{Q} - \frac{1}{2}\overline{Q}^2 \Leftrightarrow$
$\overline{Q} - k - \frac{1}{2}\overline{Q}^2 - \frac{1}{2}k^2 + Qk + \frac{1}{2}k^2 \geq \overline{Q} - \frac{1}{2}\overline{Q}^2 \Leftrightarrow$
$-k + \overline{Q}k \geq 0 \Leftrightarrow$

$k(\overline{Q}-1) \geq 0$ which is false $\forall\ (a,c,\overline{Q}) \in E_{MU} \cap N_L$ since $\overline{Q} < 1$ on N_L. Note, however, that $E_{MU} \cap N_L$ does not exist for $\frac{1}{2} < (a-c) < 1$.

5) $\Pi^E \geq \Pi^N\ \forall\ (a,c,\overline{Q}) \in E_{LU} \cap N_L \Leftrightarrow$
$\frac{\overline{Q}}{2} - \frac{\overline{Q}^2}{8} + (a-c)\frac{\overline{Q}}{2} - \frac{\overline{Q}^2}{8} \geq \overline{Q} - \frac{1}{2}\overline{Q}^2 \Leftrightarrow$
$\overline{Q}^2 + 2\overline{Q}(a-c-1) \geq 0 \Leftrightarrow$
$\overline{Q} \geq \underline{Q}(a-c) \equiv -2(a-c-1)$ ■

Figure 1. Noncooperative equilibrium for identical riparians

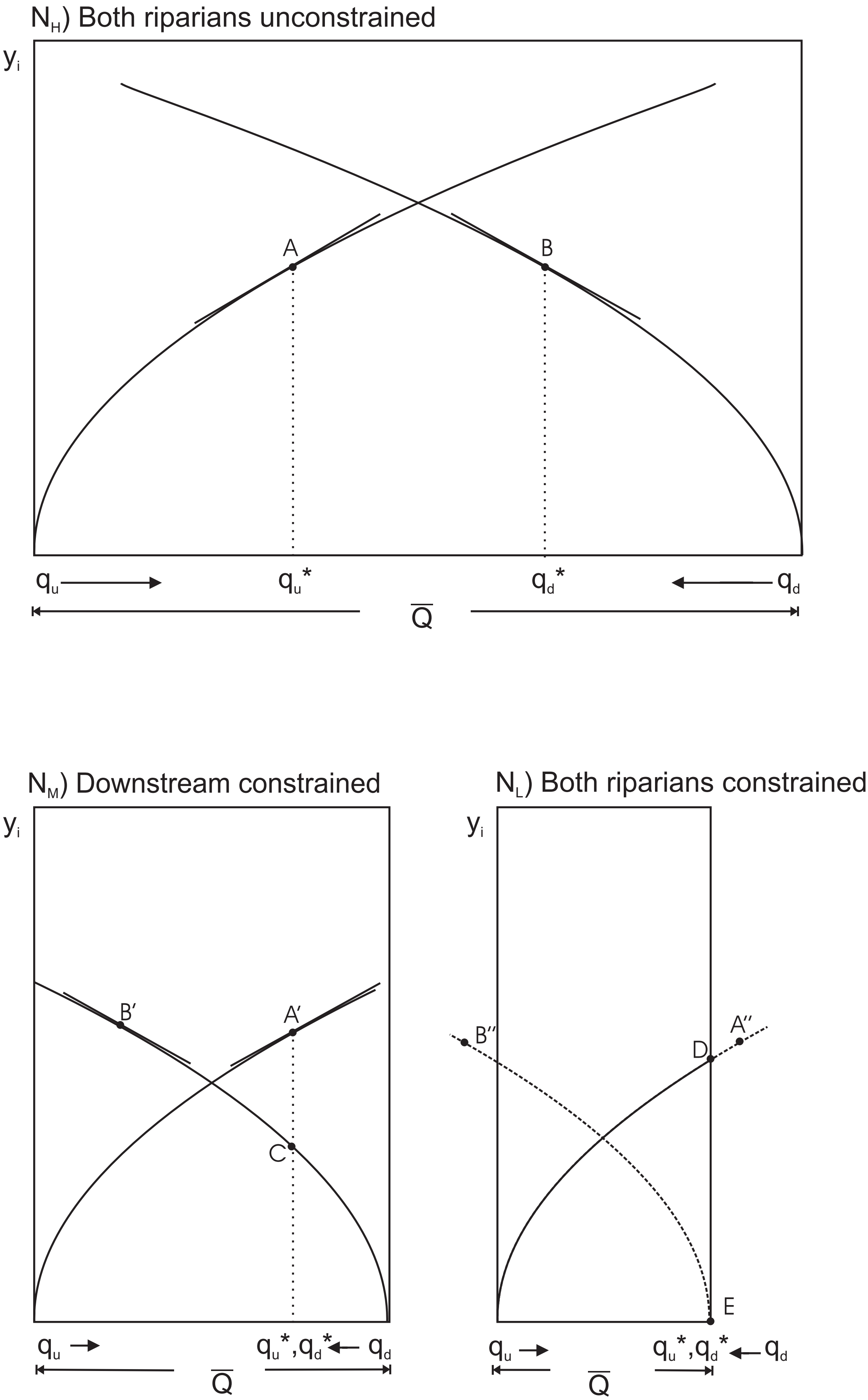
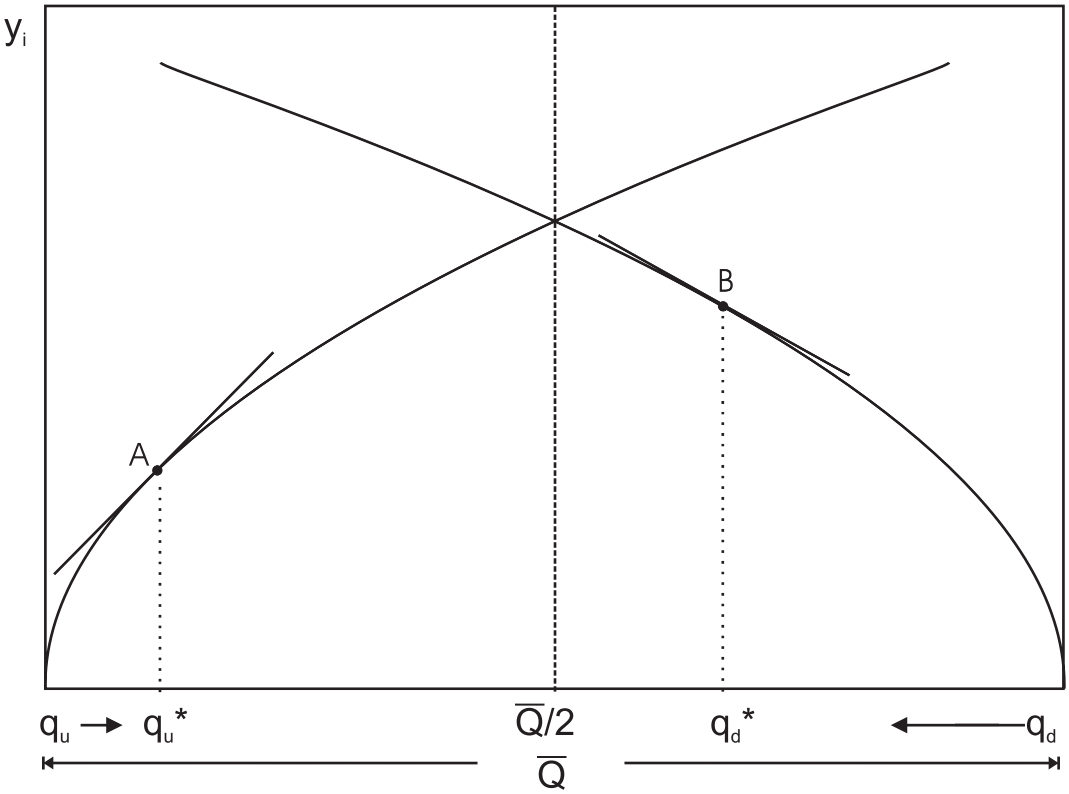


Figure 2. Equal quota (downstream riparian has cost advantage)

E_{HD}) Both riparians unconstrained



E_{MD}) Downstream constrained (excess supply for water loan).

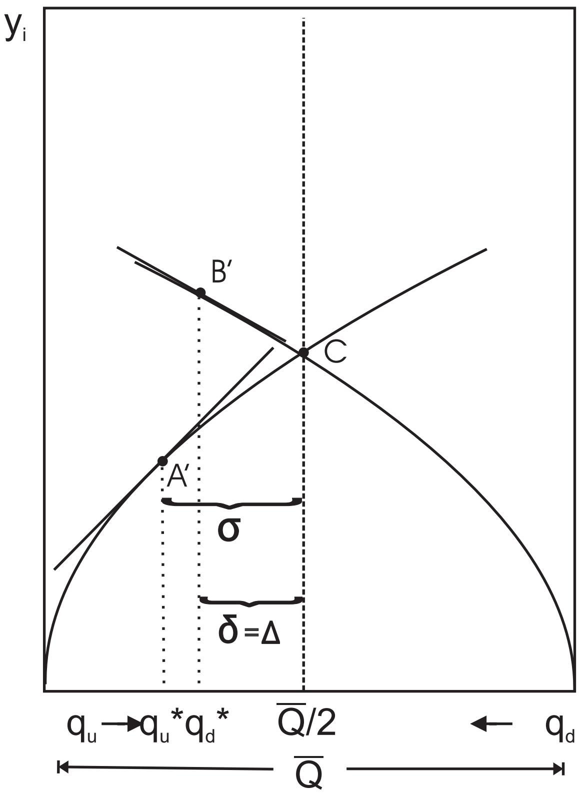
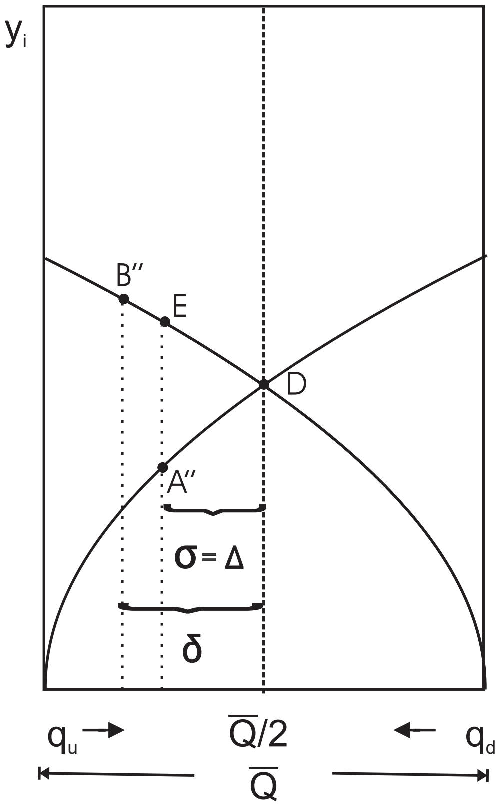


Figure 2. Equal quota (downstream riparian has cost advantage)

E_{MD}) Downstream constrained (excess demand for water loan).



E_{LD}) Both riparians constrained

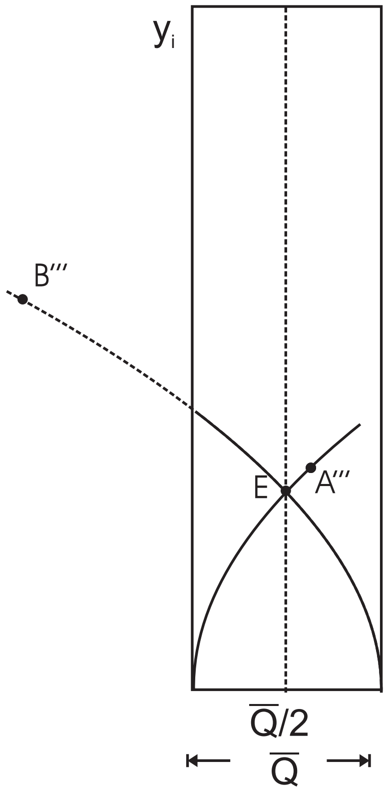


Figure 3. The Domain of the equal quota and noncooperative sets in $(c,\overline{Q})$-space.

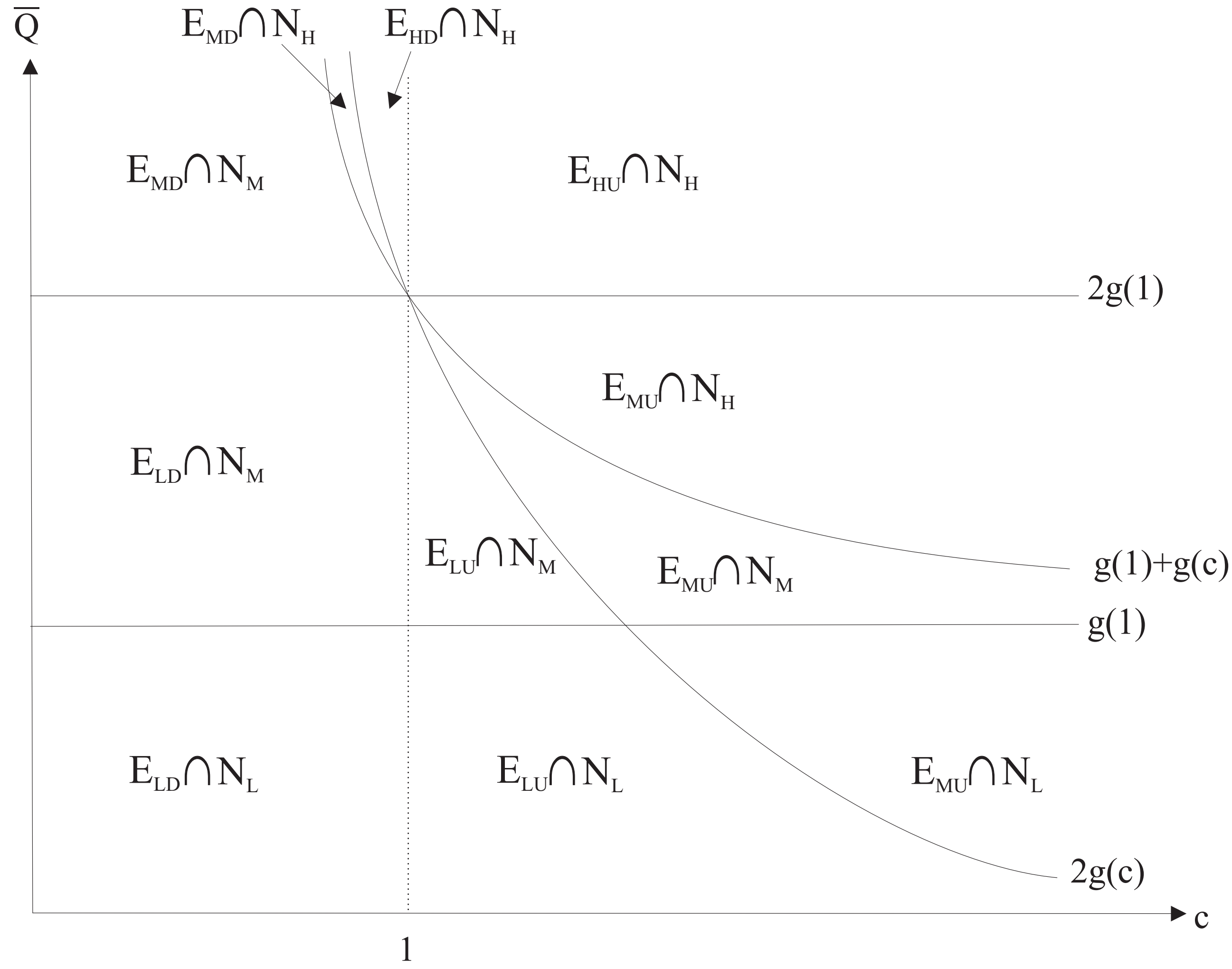
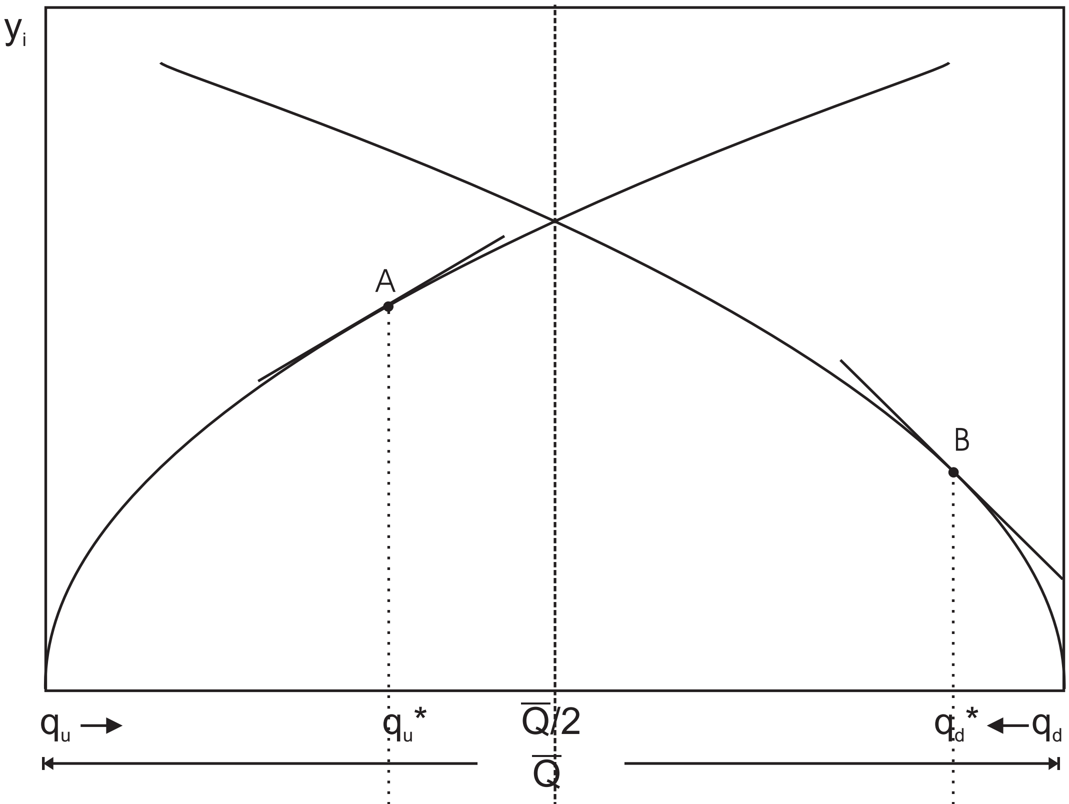


Figure 4.1 Upstream has cost advantage: $E_{HU} \cap N_H$

a) Agricultural Output



b) Riparian and basinwide profit

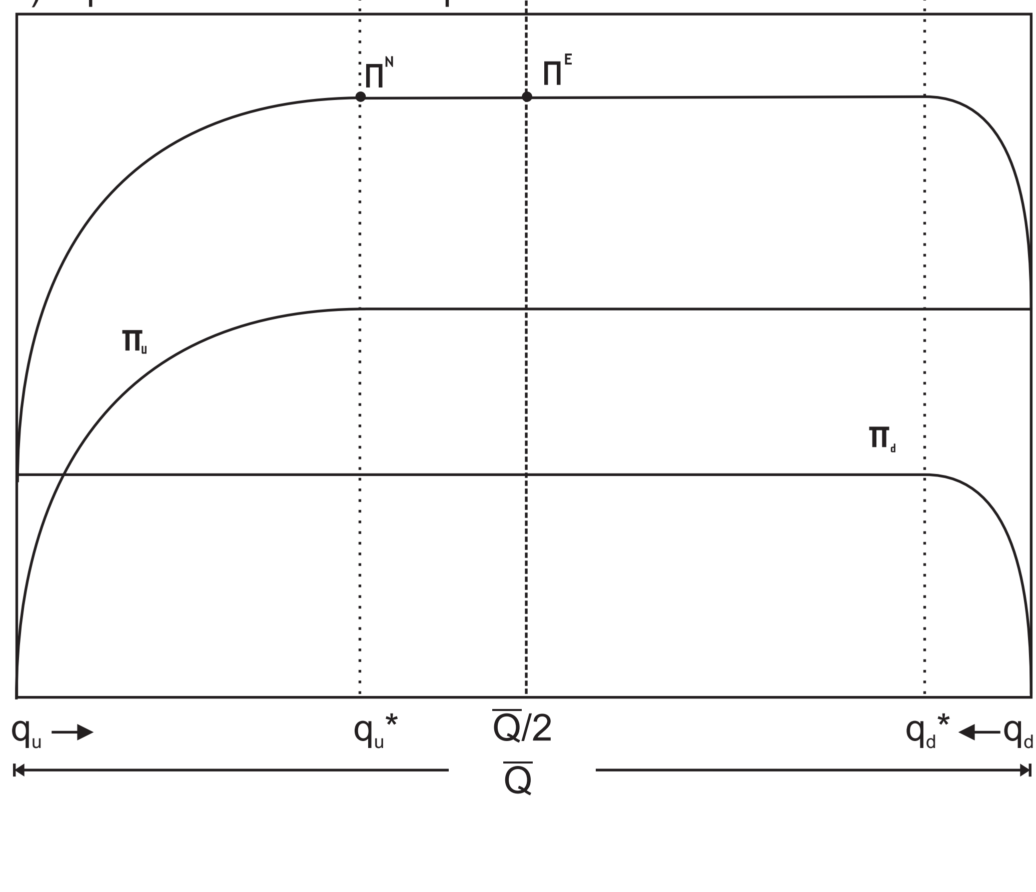


Figure 4.2 Upstream has cost advantage: $E_{MU} \cap N_H$

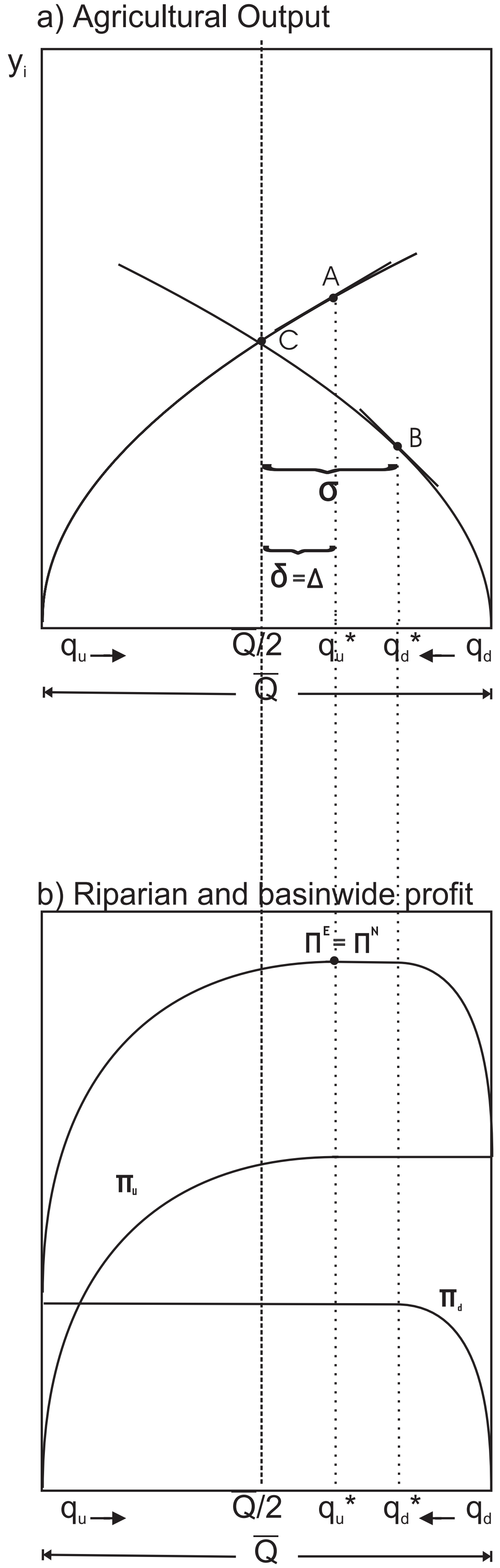


Figure 4.3 Upstream has cost advantage: $E_{MU} \cap N_M$

a) Agricultural Output

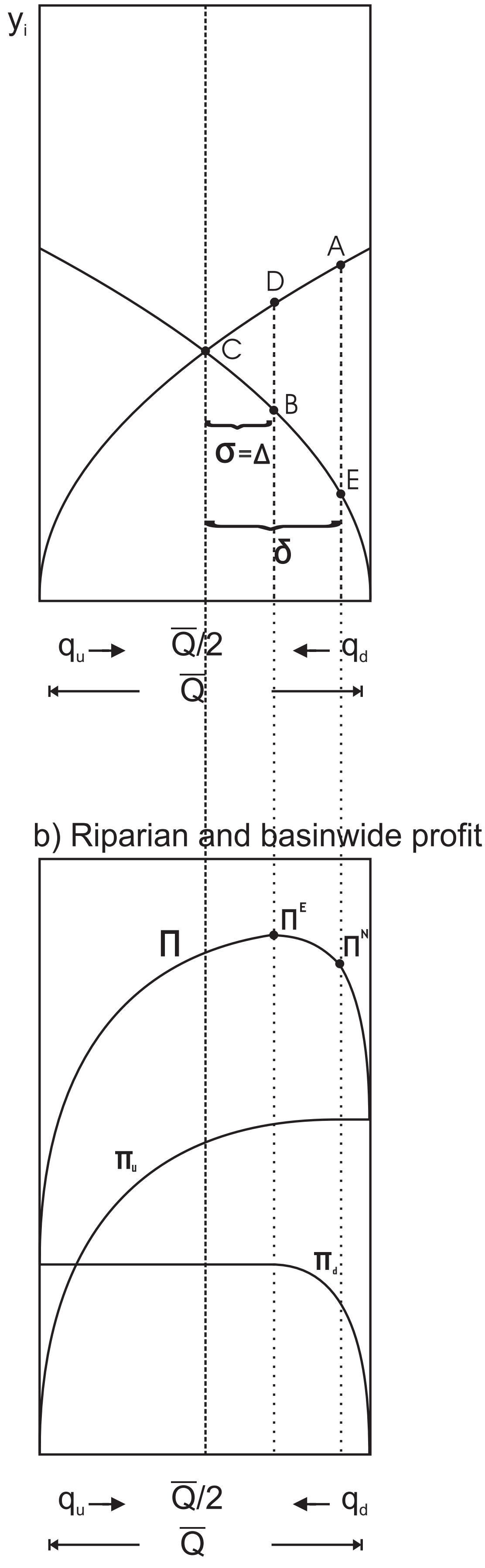
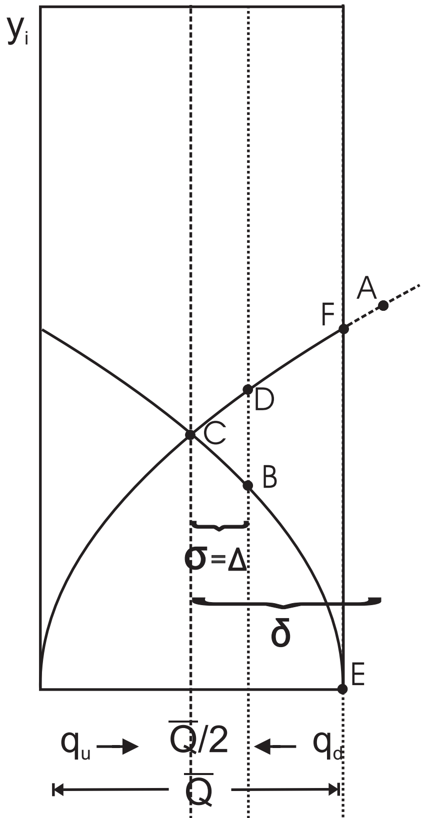


Figure 4.4 Upstream has cost advantage: $E_{MU} \cap N_L$

a) Agricultural Output



b) Riparian and basinwide profit

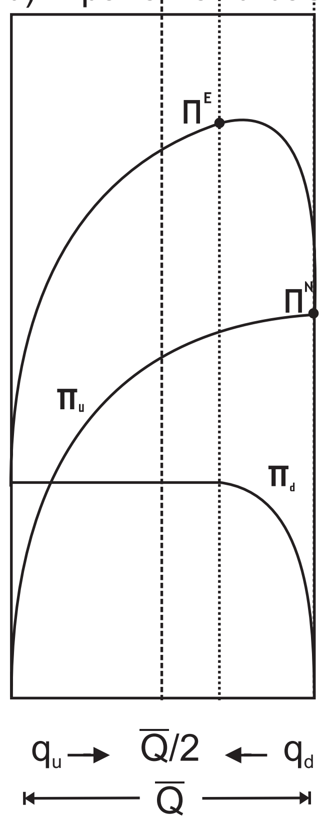
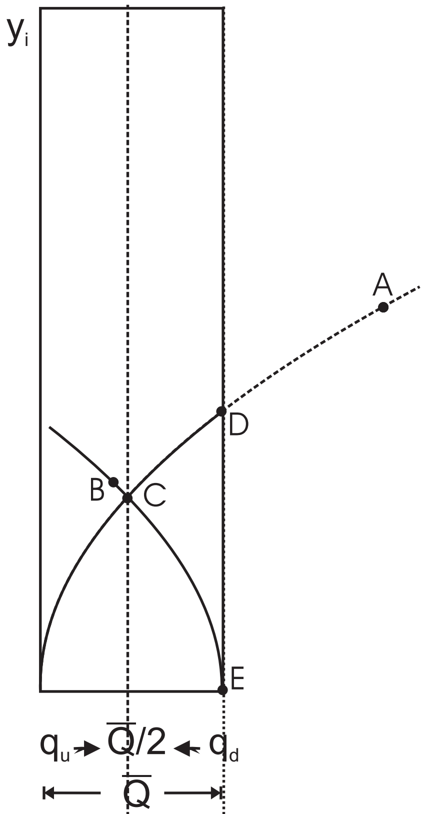


Figure 4.5 Upstream has cost advantage: $E_{LU} \cap N_L$

a) Agricultural Output



b) Riparian and basinwide profit

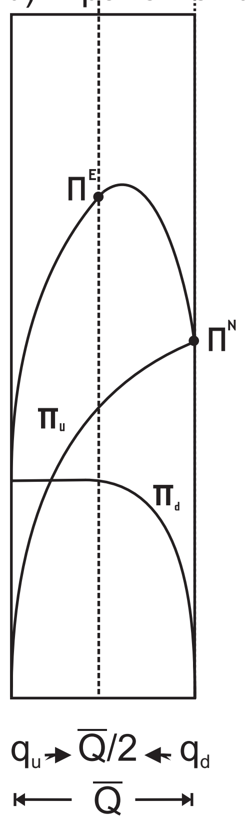


Figure 5. Cooperative Surplus (Cobb Douglas alfa=0.5)

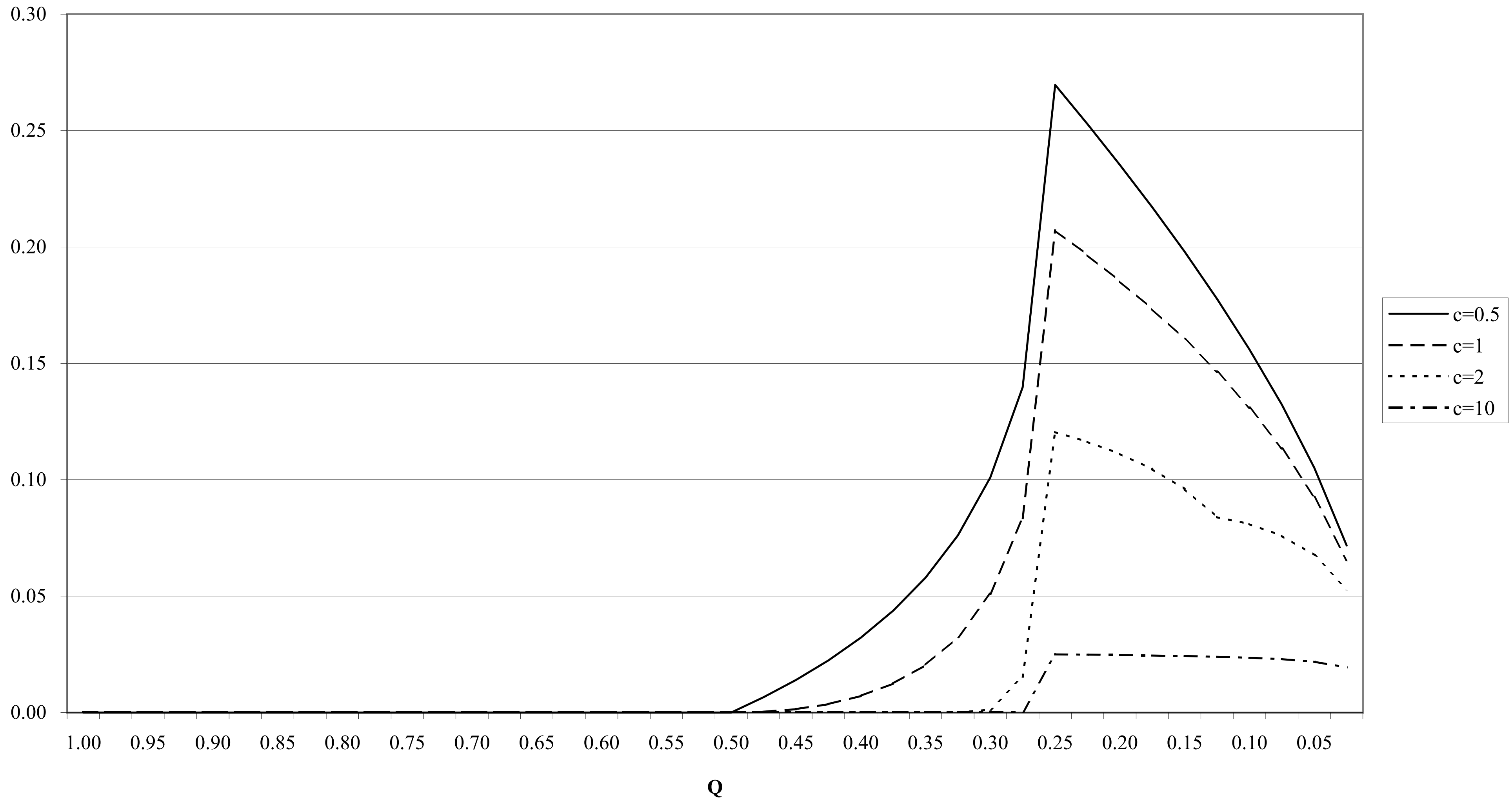


Figure 6. Cooperative Surplus (Quadratic Production Function)