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## COLLUSION WITH ASYMMETRIC R&D

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# Collusion with asymmetric R&D

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**JEL Classification:** L10, L13

**Key Words:** Consumer surplus, Collusion, Profit, Uncertain R&D

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## **Collusion with asymmetric R&D**

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### **1. Introduction**

Whether firms will be allowed to cooperate in the product market is a concern of the anti-trust bodies and is important for competition policies. The textbook view says that cooperation in the product market increases profits of the cooperating firms and reduces welfare of the consumers. However, this view considers the same market demand and cost structure of the firms and ignores the effects of other non-production activities such as R&D.

In this paper we consider the effects of product market cooperation on technology, profits, consumer surplus and welfare when a firm does R&D. We show that if the degree of knowledge spillover is sufficiently small, R&D investment is higher (lower) under product market cooperation for sufficiently larger (smaller) slope of the marginal cost of R&D. Therefore, whether relatively better technology is

used under cooperation depends on the degree of knowledge spillover and the cost of R&D.

While industry profit is higher under product market cooperation, we show that the effects of product market cooperation are ambiguous on consumer surplus and social welfare. Our results suggest that consumer surplus and social welfare are higher under cooperation if the degree of knowledge spillover is sufficiently large and the slope of the marginal cost of R&D is sufficiently small. Therefore, given the negative relationship between the degree of knowledge spillover and the degree of patent protection, a welfare maximizing government has the incentive to encourage the firms to cooperate in the product market if there is weak patent protection (which generates large knowledge spillover) in the economy.

Though there is an existing literature on R&D looking at the effects of cooperation on R&D investment, profits and welfare, this literature is focusing on the effects of *cooperation in R&D rather than the effects of cooperation in the product market* (see, e.g., d'Aspremont, C. and A. Jacquemin, 1988, Kamien et al. 1992, and Suzumura, K., 1992). More recently, Fershtman and Gandal (1994) and Brod and Shivakumar (1999) consider the effects of product market cooperation on R&D investments, profits and consumer surplus.

Though we address a similar question of Fershtman and Gandal (1994) and Brod and Shivakumar (1999), we differ from their analyses in an important way. While they consider R&D by both firms in a duopolistic industry, we consider R&D by a single firm.<sup>1</sup> Hence, the present paper is more appropriate for industries with a technology leader as in Gallini (1990), Bester and Petrakis (1993), Mukherjee (2003)

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<sup>1</sup> The implications of asymmetric R&D firms considered in this paper are directly comparable to Fershtman and Gandal (1994) for the case of no knowledge spillover of our analysis and to Brod and Shivakumar (1999) for the case of homogeneous products of their analysis.

and Mukherjee and Pennings (2004), whereas their analyses are applicable to those industries where the firms have symmetric R&D capabilities. Unlike Fershtman and Gandal (1994) and Brod and Shivakumar (1999), we find that R&D investments may be lower under cooperation and the industry profit is always higher under cooperation. Further, while in contrast to Fershtman and Gandal (1994), we show that consumer surplus may be higher under cooperation, unlike Brod and Shivakumar (1999), we show that consumers are worse off under cooperation for sufficiently low knowledge spillover. Hence, our results suggest that antitrust authority needs to be careful about the R&D capabilities of the firms while considering the proposals for product market cooperation. Further, unlike those papers, we also consider the effects on social welfare.

The remainder of the paper is organized as follows. Section 2 describes the model and shows the results. Section 3 concludes.

## **2. The model and the results**

Consider an economy with two firms, 1 and 2, producing homogeneous products. Assume that the firms have similar technologies at the beginning and each of them faces constant average cost of production  $c$ . Like Gallini (1990), Bester and Petrakis (1993), Mukherjee (2003), Mukherjee and Pennings (2004) and many others, we assume that one of these firms, say firm 1, is a technology leader in the economy<sup>2</sup> and it does R&D to reduce its cost of production.<sup>3</sup>

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<sup>2</sup> Often strategic interaction between the firms may be a reason for creating a technology leader in the economy. See, Mills and Smith (1996) on this issue.

<sup>3</sup> Alternatively, following La Manna (1994), we may think that only firm 1 is successful in inventing a new technology and it needs further investment in developing the technology to make it commercially viable. The benefit of this technology increases as firm 1 invests more in developing the technology for commercial use. So, the R&D process of our analysis is the development stage of research and development.

Assume that the inverse market demand function is

$$P = a - q, \quad (1)$$

where  $q$  is the total output of firms 1 and 2.

We consider a two-stage game. At stage 1, firm 1 invests in R&D and reduces its constant average cost of production. We assume that  $x$  amount of investment in R&D reduces the cost of production to  $(c - x)$ . However, investment in R&D is costly and the cost function for R&D is  $C(x) = \frac{\tau x^2}{2}$ . As  $\tau$  increases it increases the cost of R&D. We further assume that there may be knowledge spillover from R&D and  $x$  amount of R&D investment of firm 1 reduces firm 2's cost by  $\beta x$  amount, where  $\beta \in [0,1]$ . While  $\beta = 0$  implies no knowledge spillover,  $\beta = 1$  implies complete knowledge spillover. The degree of knowledge spillover may show the effectiveness of patent protection in the economy or it may capture the ex-ante perceived probability by the firms that the effective patent protection will not be granted to firm 1 to protect its innovation<sup>4</sup> (se, e.g., Amir and Wooders, 1999). We assume that there are no other costs of doing R&D. At stage 2, the firms produce their outputs and the profits are realized. We solve the game through backward induction.

We will consider two different scenarios for the product market competition. First, we will consider the situation where the firms choose their outputs non-cooperatively like Cournot duopolists to maximize their own profit. Second, we will consider the situation where the firms cooperate in the product market and choose output to maximize their joint profit in the product market.

For simplicity, we will do our analysis under the following assumption:

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<sup>4</sup> Griliches (1990) provides the evidences for the success rates of getting the patent protections. For example, it is 65% in the US, 90% in France, 80% in the UK and 35% in Germany.

$$\mathbf{A1.} \tau > \text{Max}\left\{\frac{2(1-\beta)(2-\beta)}{3}, \frac{a}{2c}, \frac{2(2-\beta)(a+c(1-\beta))}{9c}\right\}.$$

As we will see, the assumption A1 will ensure that the optimal output of firm 2 is positive under product market competition and also  $(c-x) \geq 0$  under both product market competition and product market cooperation.

### 2.1 Non-cooperation in the product market

Let us first consider the situation where the firms choose their outputs non-cooperatively like Cournot duopolists to maximize their own profits. We call this situation as non-cooperation.

Given the R&D investment at stage 1, firms 1 and 2 maximize following objective functions respectively to maximize their own profits:

$$\text{Max}_{q_1}(a - q - c + x)q_1 \quad (2)$$

$$\text{Max}_{q_2}(a - q - c + \beta x)q_2, \quad (3)$$

where  $q_1$  and  $q_2$  are the outputs of firms 1 and 2 respectively.

Maximizing (2) and (3), we find that the optimal outputs of firms 1 and 2 are respectively

$$q_1 = \frac{(a - c + 2x - \beta x)}{3} \quad \text{and} \quad q_2 = \frac{(a - c + 2\beta x - x)}{3}. \quad (4)$$

Optimal profits of firms 1 and 2 are respectively

$$\pi_1^n = \frac{(a - c + 2x - \beta x)^2}{9} - \frac{\tau x^2}{2} \quad \text{and} \quad \pi_2^n = \frac{(a - c + 2\beta x - x)^2}{9}. \quad (5)$$

Therefore, at stage 1, firm 1 maximize following expression to determine the optimal R&D investment:

$$\text{Max}_x \frac{(a - c + 2x - \beta x)^2}{9} - \frac{\tau x^2}{2}. \quad (6)$$



We find that the optimal R&D investment is

$$x^n = \frac{2(a-c)(2-\beta)}{(9\tau - 2(2-\beta)^2)}. \quad (7)$$

We get that  $x^n \leq c$  when  $\frac{2(2-\beta)(a+c(1-\beta))}{9c} \leq \tau$  and we assume that this holds.

The restriction  $\frac{2(2-\beta)(a+c(1-\beta))}{9c} \leq \tau$  also satisfies the second order condition of

the above maximization problem, which is  $\frac{2(2-\beta)^2}{9} < \tau$ .

We find from (4) and (7) that optimal outputs of firms 1 and 2 are respectively

$$q_1^n = \frac{3\tau(a-c)}{(9\tau - 2(2-\beta)^2)} \quad \text{and} \quad q_2^n = \frac{(a-c)(3\tau - 2(1-\beta)(2-\beta))}{(9\tau - 2(2-\beta)^2)}. \quad (8)$$

Note that optimal output of firm 2 is positive if and only if  $\tau > \frac{2(1-\beta)(2-\beta)}{3}$  and

we assume that it holds. Therefore, total output and consumer surplus are respectively

$$q^n = \frac{2(a-c)(3\tau - (1-\beta)(2-\beta))}{(9\tau - 2(2-\beta)^2)} \quad \text{and} \quad CS^n = \frac{2(a-c)^2(3\tau - (1-\beta)(2-\beta))^2}{(9\tau - 2(2-\beta)^2)^2}. \quad (9)$$

We find from (5) and (7) that optimal profits of firms 1 and 2 are respectively

$$\pi_1^n = \frac{\tau(a-c)^2(9\tau - 2(2-\beta)^2)}{(9\tau - 2(2-\beta)^2)^2} \quad \text{and} \quad \pi_2^n = \frac{(a-c)^2(3\tau - 2(1-\beta)(2-\beta))^2}{(9\tau - 2(2-\beta)^2)^2}. \quad (10)$$

Total industry profit is

$$\pi^n = \frac{(a-c)^2(\tau(9\tau - 2(2-\beta)^2) + (3\tau - 2(1-\beta)(2-\beta))^2)}{(9\tau - 2(2-\beta)^2)^2}. \quad (11)$$

Therefore, social welfare, which is the summation of industry profit and consumer surplus, under non-cooperation in the product market is

$$W^n = \frac{(a-c)^2(\tau(9\tau - 2(2-\beta)^2) + (3\tau - 2(1-\beta)(2-\beta))^2 + 2(3\tau - (1-\beta)(2-\beta))^2)}{(9\tau - 2(2-\beta)^2)^2}. \quad (12)$$

## 2.2 Cooperation in the product market

Now, consider that the firms cooperate in the product market and choose output to maximize their joint profit. We call this situation as collusion.

In case of collusion, given the positive R&D investment of firm 1, it is optimal for the firms to produce in firm 1 only, since the cost of production is lower in firm 1. Therefore, under product market cooperation, firm 1 will produce as a monopolist in the product market.

We assume that if the firms cooperate in the product market then, at stage 1, firm 1 gives a take-it-or-leave-it offer of a lump-sum payment  $F$  to firm 2 and firm 2 accepts the offer if it is not worse off compared to non-cooperation. Therefore, it is clear that optimal offer of firm 1 is equal to firm 2's profit under non-cooperation, i.e.,

$$F^* = \frac{(a-c)^2(3\tau - 2(1-\beta)(2-\beta))^2}{(9\tau - 2(2-\beta)^2)^2}. \text{ Then, at stage 2, firm 1 decides the R\&D}$$

investment. At stage 3, firm 1 choose its output to maximize the joint profit and the profit is realized.

Therefore, given the R&D investment of firm 1, the optimal output of firm 1 maximizes the following joint profit of the firms:

$$\text{Max}_q((a - q - c + x)q - F^*) + F^*. \quad (13)$$

Maximizing (13), we find that the optimal output and profit of firm 1 are respectively

$$q^c = \frac{(a - c + x)}{2} \quad (14)$$

$$\pi_1^c = \frac{(a - c + x)^2}{4} - F^* - \frac{\alpha x^2}{2}. \quad (15)$$

Therefore, at stage 1, firm 1 maximizes following expression to determine optimal R&D investment:

$$\text{Max}_x \frac{(a-c+x)^2}{4} - F^* - \frac{\tau x^2}{2}. \quad (16)$$

Optimal R&D investment is given by

$$x^c = \frac{(a-c)}{(2\tau-1)}. \quad (17)$$

We get that  $x^c \leq c$  when  $\frac{a}{2c} \leq \tau$  and we assume that it holds. The restriction  $\frac{a}{2c} \leq \tau$

immediately satisfies the second order condition of the above maximization problem,

which is  $\frac{1}{2} < \tau$ . It is also worth noting that  $F^*$  does not affect R&D investment of

firm 1 since it is a lump-sum payment offered by firm 1.

We find from (14) and (17) that total output and consumer surplus under collusion are respectively

$$q^c = \frac{\tau(a-c)}{(2\tau-1)} \quad \text{and} \quad CS^c = \frac{\tau^2(a-c)^2}{2(2\tau-1)^2}. \quad (18)$$

We find from (15) and (17) that profits of firms 1 and 2 are respectively

$$\pi_1^c = \frac{\tau(a-c)^2}{2(2\tau-1)} - F^* \quad \text{and} \quad \pi_2^c = F^*. \quad (19)$$

So, total profit and social welfare under collusion are respectively

$$\pi^c = \frac{\tau(a-c)^2}{2(2\tau-1)}. \quad (20)$$

$$W^c = \frac{\tau(3\tau-1)(a-c)^2}{2(2\tau-1)^2}. \quad (21)$$

### 2.3 Comparison between non-cooperation and collusion

Now, we are in a position to compare the situations under product market competition and product market cooperation. However, since the expressions under product

market competition are quite cumbersome, it is very difficult to compare the results in general. In the following analysis, we will consider two special cases: (i) no knowledge spillover (i.e.,  $\beta = 0$ ) and (ii) complete knowledge spillover ( $\beta = 1$ ). These comparisons are enough for our purpose and, since the R&D investments, profits, consumer surplus and welfare are continuous in  $\beta$ , the results will be affected in a predictable way for the intermediate values of knowledge spillover.

### 2.3.1 No knowledge spillover ( $\beta = 0$ )

Let us first consider the situation for no knowledge spillover. In this situation, A1

reduces to  $\tau > \text{Max}\{\frac{4}{3}, \frac{a}{2c}, \frac{4(a+c)}{9c}\}$

**Proposition 1:** *Suppose,  $\beta = 0$ .*

(i) *If  $a > 8c$ ,  $x^c > x^n$ .*

(ii) *If  $a \in (2c, 8c)$ ,  $x^c > x^n$  for  $\tau > 4$  and  $x^c < x^n$  for  $\tau \in (\frac{4(a+c)}{9c}, 4)$ .*

(iii) *If  $a < 2c$ ,  $x^c > x^n$  for  $\tau > 4$  and  $x^c < x^n$  for  $\tau \in (\frac{4}{3}, 4)$ .*

**Proof:** Comparing (7) and (17), we find  $x^c \begin{matrix} \geq \\ < \end{matrix} x^n$  if and only if

$$\tau \begin{matrix} \geq \\ < \end{matrix} 4. \tag{22}$$

(i) If  $a > 8c$ , we find  $\frac{a}{2c} = \text{Max}\{\frac{4}{3}, \frac{a}{2c}, \frac{4(a+c)}{9c}\}$ . So, assumption A1 implies that

$\tau > \frac{a}{2c}$ . Since,  $\frac{a}{2c} > 4$  when  $a > 8c$ , we get  $x^c > x^n$  in this situation.

(ii) If  $a \in (2c, 8c)$ , we find  $\frac{4(a+c)}{9c} = \text{Max}\{\frac{4}{3}, \frac{a}{2c}, \frac{4(a+c)}{9c}\}$ . So, assumption A1

implies  $\tau > \frac{4(a+c)}{9c}$ . Since,  $4 > \frac{4(a+c)}{9c}$  for  $a \in (2c, 8c)$ , we get  $x^c > x^n$  for  $\tau > 4$

and  $x^c < x^n$  for  $\tau \in (\frac{4(a+c)}{9c}, 4)$ .

(iii) If  $a < 2c$ , we find  $\frac{4}{3} = \text{Max}\{\frac{4}{3}, \frac{a}{2c}, \frac{4(a+c)}{9c}\}$ . So, assumption A1 implies  $\tau > \frac{4}{3}$ .

Since,  $4 > \frac{4}{3}$ , we get  $x^c > x^n$  for  $\tau > 4$  and  $x^c < x^n$  for  $\tau \in (\frac{4}{3}, 4)$ . Q.E.D.

The above result shows that if the market size<sup>5</sup> is sufficiently small, the R&D investment is lower under collusion than non-cooperation if the slope of the marginal cost of R&D is sufficiently low (i.e.,  $\tau$  is small). This result is in sharp contrast to Fershtman and Gandal (1994) and Brod and Shivakumar (1999).

For a given R&D investment, profit is higher under collusion compared to non-cooperation, which encourages firm 1 to invest more in R&D. On the other hand, there is a strategic effect under non-cooperation. If firm 1 invests more in R&D under non-cooperation, it increases its market share and profit under non-cooperation. If  $\tau$  is sufficiently small, the strategic effect under non-cooperation dominates the effect of higher profit under collusion and creates higher R&D investment under non-cooperation. As  $\tau$  increases, it makes R&D more costly. If  $\tau$  is sufficiently high, the effect of higher profit under collusion dominates the strategic effect under non-cooperation and the R&D investment is higher under collusion. Since, the R&D investments are continuous in  $\beta$ , the above result suggests that the R&D investment

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<sup>5</sup> The intercept term of the demand function is the proxy for the market size.

may be higher or lower under collusion for sufficiently lower degree of knowledge spillover.

R&D investment increases with the market size under both non-cooperation and collusion. However, it is easy to check that the increment is higher under collusion when  $\tau > 4$ . Further, as the market size increases, the requirement for the interior solution of the R&D investment eliminates relatively lower values of  $\tau$ , which, in turn, shows that the R&D investment is higher under collusion if the market is sufficiently large.

**Proposition 2:** *Suppose,  $\beta = 0$ . Industry profit is always higher under product market cooperation than product market competition.*

**Proof:** Comparison of (11) and (20) shows that  $\pi^c > \pi^n$  if and only if

$$9\tau^3 + 20\tau^2 - 64\tau + 32 > 0, \quad (23)$$

and it holds for  $\tau > \frac{4}{3}$ . This proves the result since the second order condition for

maximizing the R&D investment under cooperation requires  $\tau > \frac{4}{3}$  when  $\beta = 0$ .

Q.E.D.

Industry profit is higher under collusion than non-cooperation for a given R&D investment. However, R&D investment differs under non-cooperation and collusion. While higher R&D investment reduces the cost of production by a larger extent, it also increases the cost of R&D. The above result shows that industry profit is always higher under collusion than non-cooperation irrespective of the differences in R&D investments under these situations. This result is also in contrast to Fershtman and Gandal (1994) and Brod and Shivakumar (1999).

So far, we have done our analysis under the assumption that the firms have the incentive to cooperate in the product market. Though we have seen that  $F^*$  makes firm 2 indifferent under non-cooperation and collusion, we have not checked the incentive for collusion by firm 1. However, it is immediate from the above proposition that firm 1 is always better off under collusion than non-cooperation, since firm 2 is indifferent between non-cooperation and collusion while industry profit is higher under collusion compared to non-cooperation.

**Proposition 3:** *Suppose,  $\beta = 0$ . Consumer surplus and social welfare are always higher under product market competition than product market cooperation.*

**Proof:** We find  $CS^n > CS^c$  if and only if

$$3\tau^2 - 6\tau + 4 > 0, \quad (24)$$

which holds for  $\tau > \frac{4}{3}$ .

Comparison of (12) and (21) shows that  $W^n > W^c$  if and only if

$$45\tau^4 + 65\tau^3 + 88\tau^2 - 240\tau + 48 > 0, \quad (25)$$

which holds for  $\tau > \frac{4}{3}$ . Q.E.D.

Though R&D investment may be higher under collusion, the effect of competition under non-cooperation dominates the effect of R&D investment for consumer surplus. Hence, consumers are always better off under product market competition. Propositions 2 and 3 suggest that there is a conflict between the interests of the producers and the consumers if the degree of knowledge spillover is sufficiently small. While the firms prefer collusion, consumers prefer non-

cooperation. The above result shows that the effect of consumer surplus dominates the effect of profit on social welfare and creates higher social welfare under non-cooperation.

### 2.3.2 Complete knowledge spillover ( $\beta = 1$ )

Now, we consider the other extreme situation, where knowledge spillover is complete. Here, the assumption A1 reduces to  $\tau > \frac{a}{2c}$ .

**Proposition 4:** *Suppose,  $\beta = 1$ . Then,  $x^c > x^n$ .*

**Proof:** Comparison of (7) and (17) proves the result.

Q.E.D.

If there is complete knowledge spillover, it reduces firm 1's incentive for R&D significantly under product market competition, since firm 2 benefits significantly from the R&D of firm 1. So, the strategic effect under non-cooperation on R&D investment is sufficiently small. However, the degree of knowledge spillover does not affect the R&D investment under product market cooperation. If  $\beta = 1$ , the effect of higher profit on R&D investment under collusion always dominates the strategic effect of non-cooperation and generates higher R&D investment under collusion compared to non-cooperation. Since, the R&D investments are continuous in  $\beta$ , the above result suggests that the R&D investment is always higher under collusion for sufficiently higher degree of knowledge spillover.

**Proposition 5:** *Suppose,  $\beta = 1$ . Industry profit is always higher under collusion than non-cooperation.*



**Proof:** Comparison of (11) and (20) shows that  $\pi^c > \pi^n$ .

Q.E.D.

We have shown that the R&D investment is higher under collusion, i.e., the cost of production is lower under collusion. Further, collusion eliminates competition in the product market. The above result shows that both these positive effects of collusion dominate the negative effect of higher R&D investment under collusion, and collusion increases the industry profit. So, the result of Proposition 2 holds even for large degree of knowledge spillover.

**Proposition 6:** *Suppose,  $\beta = 1$ .*

(i) *Consumer surplus is higher under collusion (non-cooperation) for  $\tau \in (\frac{a}{2c}, \frac{4}{3})$*

*( $\tau > \frac{4}{3}$ ).*

(ii) *Social welfare is higher under collusion (non-cooperation) for*

*$\tau \in (\frac{a}{2c}, \frac{(23 + \sqrt{241})}{18})$  ( $\tau > \frac{(23 + \sqrt{241})}{18}$ ).*

**Proof:** (i) We find from (9) and (18) that  $CS^c \underset{<}{\geq} CS^n$  for  $\tau \underset{>}{\leq} \frac{4}{3}$ . However, the

assumption A1 implies that  $\tau$  must be greater than  $\frac{a}{2c}$ . Therefore, consumer surplus

is higher under collusion (non-cooperation) for  $\tau \in (\frac{a}{2c}, \frac{4}{3})$  ( $\tau > \frac{4}{3}$ ).

(ii) We find from (12) and (21) that  $W^c \underset{<}{\geq} W^n$  for

$$9\tau^3 - 23\tau^2 + 8\tau \underset{>}{\leq} 0. \quad (26)$$

We find that  $\tau = 0$ ,  $\tau = \frac{(23 - \sqrt{241})}{18}$  and  $\tau = \frac{(23 + \sqrt{241})}{18}$  are the roots of the equation  $9\tau^3 - 23\tau^2 + 8\tau = 0$ . However, since  $\tau$  must be greater than  $\frac{a}{2c}$  (which is greater than  $\frac{1}{2}$ ), the only root that is relevant for our analysis is  $\tau = \frac{(23 + \sqrt{241})}{18}$ . We further find that left hand side of (26) is continuous and convex in  $\tau$  for  $\tau > \frac{1}{2}$  and it is negative at  $\tau = \frac{1}{2}$ . Hence, social welfare is higher under collusion (non-cooperation) for  $\tau \in (\frac{a}{2c}, \frac{(23 + \sqrt{241})}{18})$  ( $\tau > \frac{(23 + \sqrt{241})}{18}$ ). Q.E.D.

We show that consumers may be better off even if the products are homogeneous but the firms differ with respect to R&D capability and this result is in sharp contrast to Fershtman and Gandal (1994) and Brod and Shivakumar (1999). When knowledge spillover is complete, it reduces R&D investment significantly under non-cooperation, which reduces consumer surplus and social welfare. So, even if collusion increases concentration in the product market, the positive effect of higher R&D investment under collusion dominates the negative effect of product market concentration for sufficiently low  $\tau$ , and creates higher consumer surplus and welfare under collusion compared to non-cooperation.

Propositions 5 and 6 suggest that both the consumers and the producers can be better off under collusion when the degree of knowledge spillover is sufficiently large. In this situation, the anti-trust authority has the incentive to encourage the firms to cooperate in the product market.

### **3. Conclusion**

The textbook view says that while firms benefit from product market cooperation, consumer surplus and social welfare is higher under product market competition. This view has been challenged by Fershtman and Gandal (1994) and Brod and Shivakumar (1999). We show that their results may change significantly if the firms are not symmetric with respect to R&D capabilities.

We consider a duopolistic market with a single innovating firm and show that the R&D investments may be higher or lower under product market cooperation. Though, the industry profit is always higher under cooperation, the effects of cooperation on consumer surplus and social welfare are ambiguous and depend on the degree of knowledge spillover and the slope of the marginal cost of R&D. We find that if the degree of knowledge spillover is sufficiently large and the slope of the marginal cost of R&D is sufficiently small, cooperation in the product market makes the producers, the consumers and the economy better off compared to non-cooperation in the product market. Hence, in this situation, government has the incentive to encourage the firms to cooperate in the product market.

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