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Trade, firm selection, and innovation: the competition channel

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Abstract

The availability of rich firm-level data has led researchers to uncover new evidence on the effects of trade liberalization. First, trade openness forces the least productive firms to exit the market; secondly, it induces surviving firms to increase their innovation efforts; thirdly, it increases the degree of product market competition. In this paper, we propose a model aimed at providing a coherent interpretation of these findings, and use it to asses the role of firm selection in shaping the aggregate welfare gains from trade. We introduce firm heterogeneity into an innovation-driven growth model where incumbent firms operating in oligopolistic industries perform cost-reducing innovation. In this environment, trade liberalization leads to lower markups level and dispersion, tougher firm selection, and more innovation. Calibrated to match US aggregate and firm-level statistics, the model predicts that moving from a 13% variable trade costs to free trade increases the stationary annual rate of productivity growth from 1.19 to 1.29% and increases welfare by about 3% of steady state consumption. Selection accounts for about $1/4^{th}$ of the overall growth increase and $2/5^{th}$ of the welfare gains from trade.

JEL Classification: F12, F13, O31, O41

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1 Introduction

An interesting set of empirical regularities about international trade has recently emerged from a large numbers of studies using firm-level data. Firstly, strong evidence suggests that trade liberalization induces the least productive firms to exit the market, reallocating both demand and resources to surviving, more productive firms; this is the so-called *selection effect* of trade resulting in an increase in aggregate productivity (e.g. Pavcnik, 2002, Topalova, 2011, and Tybout, 2003 for a survey). A second line of research has highlighted the joint selection and *innovation* effect of trade, showing that trade liberalization cleans the market of inefficient firms and forces the surviving firms to innovate more (e.g. Bustos, 2010, Bloom, Draca and Van Reenen, 2011, Aw, Roberts and Xu, 2010).[1] A third piece of evidence shows that trade liberalization has *pro-competitive* effects (reduces prices and markups) potentially leading to more selection and more innovation (e.g. Chen, Imbs and Scott, 2009, Corcos, Del Gatto, Ottaviano and Mion, 2010, Bugamelli, Fabiani and Sette, 2010, Griffith, Harrison and Macartney, 2006).[2]

Our paper has two main goals: first, to present a tractable model providing a coherent interpretation of these empirical regularities. Secondly, to perform a quantitative evaluation of the productivity and welfare gains from trade, as well as to measure the share of these gains attributable to selection under heterogeneous firms' decisions to exit, export, and innovate.

We set up a model in which trade liberalization has pro-competitive effects through reduced markups leading to firm selection and increased innovation. Industry dynamics with heterogeneous firms is added to a growth framework with innovation by incumbents and oligopolistic competition. There are two goods, a homogeneous good produced under constant returns, and a differentiated good produced with a continuum of varieties, each of them facing both variable and fixed production costs. As in Hopenhayn (1992) and Melitz (2003), productivity differs across varieties. Firms in the differentiated good sector allocate labor to the production of a

[1] Focusing on innovation instead of directly looking at productivity has the advantage of identifying one specific channel through which improvements in productivity take place. Other studies have instead estimated productivity as a residual in the production function, facing the problem that together with technological differences, residuals captures also other differences such as market power, factor market distortion, and change in the product mix. (see i.e. Foster, Haltiwanger and Syverson, 2008, Hsieh and Klenow, 2009, and Bernard, Redding and Schott, 2007).

[2] Further evidence from firm-level studies on the pro-competitive effect of trade can be found in Levinson (1993) for Turkey, Harrison (1994) for the Ivory Coast, Roberts (1997) for Colombia, Krishna and Mitra (1998) for India. Boulhol (2010) surveys the most recent literature.

specific variety and to innovation activities aimed at reducing their production costs. Each variety is produced by a small number of identical firms operating in an oligopolistic market; thus quantities produced and innovation activities result from the strategic interaction among them. The oligopolistic market structure and the cost-reducing innovation features are borrowed from static trade models of trade under oligopoly (e.g. Brander and Krugman, 1983, Neary, 2009) and from growth models with homogeneous oligopolistic firms (e.g. Peretto, 2003, Aghion, Bloom, Blundell, Griffith and Howitt, 2005, and Licandro and Navas-Ruiz, 2011). The open economy features two symmetric countries engaging in costly trade.

In order to clarify the analysis, we fist present a simple, analytically tractable, version of the model. It assumes that each variety of the differentiated good is manufactured by a fixed number of oligopolistic producers, and no fixed export costs, implying that all operating firms export. Trade liberalization, by allowing foreign firms to operate in the domestic market, increases product market competition reducing markups in the differentiated sector, thus increasing market efficiency and ultimately leading to an expansion of the quantities produced by oligopolistic firms. Since innovation is cost-reducing, the trade-induced increase in firms' size raises their incentive to innovate. This is the *direct pro-competitive effect* of trade already pointed out in models of trade and innovation with a representative firm (e.g. Licandro and Navas-Ruiz, 2011).

The decline in markups, moreover, forces the least productive firms out of the market, reallocating resources toward surviving firms, increasing their average size and their incentive to invest in cost-reducing innovation. Hence, trade-induced firm selection increases not only average productivity as in Melitz (2003), but also firms' innovation, ultimately affecting the growth rate of productivity. This is the *indirect pro-competitive effect* of trade which strictly depends on the presence of heterogeneous firms. Since, trade increases product market competition, triggers firm selection, and promotes innovation, the model is consistent with the patterns of firm exit, innovation, and pricing decisions shown in recent empirical research referred above.

The welfare effects of trade can also be decomposed into a direct and an indirect channel. Moreover, since innovation leads to endogenous productivity dynamics, each channel yields static and dynamic gains. The direct channel of welfare gains from trade has a static component, coming from the improved market efficiency generated by lower markups, the standard pro-competitive effect, and a dynamic component, related to the increase in firm size which stimulates innovation and productivity growth. The indirect channel, triggered by trade-induced selection, affects welfare positively through an increase in the average *level* of productivity produced by reallocations of market shares across heterogeneous firms (static component), and through an increase in the *growth rate* of productivity triggered by faster innovation (dynamic component). Moreover, firms' exit due to selection has also a negative effect on welfare by reducing the variety of goods produced and consumed in the economy. This negative effect of selection, as in Atkeson and Burstein (2010), could potentially offset the positive effects discussed above. We show analytically that, under plausible parameter restrictions, this negative effect does not dominate, thus obtaining positive welfare gains from the indirect channel.

A quantitative evaluation of our theory is performed in a more general version of the model, in which we endogenize the number of oligopolistic firms competing in each product line by introducing a sunk entry cost, and add a fixed cost of exporting leading to a trade equilibrium in which not all firms export. We calibrate this version of the model to match salient firm-level and aggregate statistics of the US economy, and solve it numerically. We find that trade liberalization slightly increases the number of firms per product, which provides an additional source of markups reduction and an additional selection margin not featured in the simple model: the reduction in variable trade costs and the increase in the number of firms per product line lower markups for exporters and non-exporters, thus forcing both the marginal domestic producer and the marginal exporter to be more productive. These changes lead to higher long-run growth and welfare: going from the benchmark value for variable cost of 13% to free trade increases the annual growth rate from 1.19 to 1.29%. About $1/4^{th}$ of this change is attributable to the indirect effect triggered by selection into the domestic (exit) and foreign market (export), and the associated reallocation of market shares across heterogeneous firms. Similarly, this reduction in the trade cost leads to a long-run welfare gain corresponding to 2.8% of lifetime consumption, about $2/5^{th}$ of which accounted for by selection and its interaction with innovation. Hence, the presence of heterogeneous firms accounts for a non-negligible share of the growth and welfare effects of trade liberalization.

This paper is related to the emerging literature on the joint effect of trade liberalization on selection and innovation. A first line of research introduces a one-step technological upgrading choice into a heterogeneous firm framework. Examples are Yeaple (2005), Costantini and Melitz (2007) and Bustos (2010). Atkeson and Burstein (2010) set up a dynamic model of process and product innovation, and show that selection has positive welfare effects from trade through process innovation that are offset by negative effects through product innovation.[3] While Atkeson and Burstein (2010) does not feature long-run growth, Baldwin and Robert-Nicoud (2008) and Gustaffson and Segerstrom (2008) introduce firm heterogeneity in an endogenous growth model of expanding product varieties (Romer, 1990). They show that the selection effect of trade on innovation and growth depends on the form of international knowledge spillovers.[4] In order to focus on the pro-competitive effect of trade and its implications for growth and welfare, our model does not feature international knowledge spillovers.[5] Moreover, while in our oligopolistic economy markups respond endgenously to changes in trade costs, all these papers adopt a monopolistically competitive market structure with constant markups, thus they cannot account for the pro-competitive effects of trade observed in the data.

Brander (1981) and Brander and Krugman (1983) devised a pioneering model of trade under oligopoly, followed by Venables (1985) and Horstman and Markusen (1986).[6] Van Long, Raff and Stähler (2011) analyze a static oligopoly model of innovation and ex-post firm heterogeneity. Since firms are identical prior to the realization of the cost shock and they innovate only once before entry, equilibrium innovation investment is the same across firms. In our model instead, innovation is an ongoing decision undertaken by incumbent firms and, consistently with empirical evidence, in equilibrium more productive firms innovate more.[7] Moreover, a dynamic model of innovation and growth allows us to capture both static and intertemporal (dynamic)

[3] Benedetti (2009) finds positive effects of trade liberalization on both types of innovation.

[4] Klette and Kortum (2004) and Lentz and Mortensen (2008) introduce a dynamic industry model with heterogeneous firms into a quality ladder growth model (Grossman and Helpman, 1991). They limit the analysis to the interaction between firm heterogeneity and creative destruction in closed economy, without exploring the effects of trade. Haruyama and Zhao (2008) explore the interaction between trade liberalization, selection and creative destruction in a quality ladder model of growth.

[5] Baldwin and Robert-Nicoud (2008) show that a key feature of the innovation technology that can directly influence the effect of trade on growth is *international knowledge spillovers*. In order to focus on the selection effect of trade on growth, we did not introduce any international knowledge spillovers. In our model instead, spillovers have only the role of generating positive long-run growth and stationarizing the productivity distribution, but they do not directly affect the impact of trade on growth via international knowledge diffusion.

[6] See Neary (2003) and Eckel and Neary (2010) for recent applications, and Neary (2010) for a review of the literature on oligopoly and trade.

[7] Van Long et al.'s prediction that firms with different productivity have the same investment in innovation is strongly counterfactual. From the early contribution by Klette and Kortum (2004) to the more recent models of innovation with heterogeneous firms (e.g. Akcigit and Kerr, 2011), the stylized fact that more productive firms invest more in innovation is one of the key fact targeted by the theoretical models. More evidence can be found in Lentz and Mortensen (2008) and Aw, Roberts and Xu, 2010). Moreover, the empirical evidence on the effects of trade on firm-level innovation is mostly based on the incumbent firms' decision to innovate when trade is liberalized (see e.g. Bloom, Draca, and Van Reenen, 2011, Aw, Roberts and Xu, 2010). A model where firms only innovate once before entry is not suitable to study the effects of trade on ongoing innovation decisions by existing firms.

gains from trade, whereas a static model cannot seize the latter. Melitz and Ottaviano (2008) jointly study the pro-competitive and selection effects of trade on welfare, using a linear demand system to obtain endogenous markups under monopolistic competition. We move from monopolistic competition to oligopoly and add a third channel of welfare gains: the dynamic gains coming from innovation and growth. Each one of these channels has been previously studied in the literature. In line with Melitz and Ottaviano, our contribution is not to uncover a new channel of welfare gains, but to provide a unified framework to analyze them jointly.

A new line of research has analyzed whether the presence of firm heterogeneity and the related selection channel in recent trade models such as Melitz (2003) leads to welfare gains from trade larger than those obtainable in standard models with a representative firm such as Krugman (1980). Arkolakis, Costinot, and Rodriguez-Clare (2012) show that welfare gains in a wide class of old and new trade models depend only on the change in trade share and on the Armington elasticity of trade to changes in trade costs. Atkeson and Burstein (2010) perform a similar exercise but adding innovation-driven productivity dynamics, and show that the overall welfare contribution of "new" margins –exit, export and process innovation decisions– can be offset by changes in product innovation (entry). A key difference between the two experiments is that while Atkeson and Burstein study the welfare effects of a given change in trade costs across models, Arkolakis et al. analyze the welfare gains of a change in trade costs, provided that this change produces the same change in trade shares across models. We follow Atkeson and Burstein in focusing on the effects of changes in trade costs, and complement their analysis by introducing pro-competitive effects of trade and endogenous growth. Our quantitative results show that the new firm-level margins produced by selection can have non-negligible aggregate effects on steady-state welfare and growth. The presence of *markup dispersion* –and its response to trade liberalization– turns out to be the necessary condition for obtaining substantial welfare gains due to trade-induced selection. Along the same line, Edmond, Midrigan and Xu (2012) in a quantitative version of the model of firm heterogeneity and endogenous markups by Atkeson and Burstein (2008) find substantial welfare gains due to reallocation when markup dispersion is high.[8] The second original ingredient of our model, knowledge spillovers leading to sustained lung-run growth, does not prove to be necessary for obtaining positive gains from

[8]Similarly, Alessandria and Choi (2011) find sizable welfare gains attributable to the response of heterogeneous firms to trade liberalization in a business cycle model with constant markups and exogenous productivity dynamics.

selection. Nevertheless, once the presence of markup dispersion makes these gains positive, the contribution of growth to the overall welfare gains from trade, and to the part due to selection, is quantitatively large.

Summarizing, our paper contributes to the existing literature by building a model consistent with three key features of the data: trade liberalization has a pro-competitive effect on prices and markups, forces inefficient firms out of the market, and induces surviving firms to innovate more. This model allows us to study the competition, selection and innovation effects of trade in a unified framework. Our contribution here is very much along the line of papers such as Melitz and Ottaviano (2008), which provides a unified framework to study the pro-competitive and selection effect of trade, and Atkeson and Burstein (2010), which provides an alternative framework to study the innovation and selection effects of trade. In particular, the contribution of the quantitative exercise can be understood as an extension of Atkeson and Burstein (2010). In the conclusion of their paper, they single out four important generalizations potentially affecting their main results. We follow their suggestion and incorporate two of them, endogenous markups and endogenous growth, and show how these additional features reverse their neutrality result on the indirect welfare gains from trade.

Section 2 describes the simple model with an exogenous number of competitors and studies its autarkic equilibrium. Section 3 analyses the equilibrium with two symmetric countries incurring in an iceberg trade costs but without fixed export costs. The simple model is extended in Section 4, allowing for an endogenous number of oligopolistic firms in each product line and for selection into the export market driven by fixed export costs. Section 5 presents the calibration of the generalized model and a numerical simulation of the effects of trade liberalization on innovation and welfare. Section 6 contains a discussion of how the two innovative features of the model, endogenous growth and endogenous markups, shape the contribution of selection to the aggregate welfare gains from trade. Section 7 concludes.

2 Simple Model: Autarky

This section presents a simple version of the model economy designed to illustrate the basic properties of the suggested theory. The general version of the model used to tackle the data is developed in Section 4. In both cases, this paper only explores the properties of the steady state equilibrium.

2.1 Economic Environment

The economy is populated by a continuum of identical consumers of measure one. Time is continuous and denoted by t, with initial time $t = 0$. Preferences of the representative consumer are described by

$$U = \int_0^\infty (\ln X_t + \beta \ln Y_t)\ \mathrm{e}^{-\rho t}\ \mathrm{d}t, \tag{1}$$

with $\beta > 0$ and discount factor $\rho > 0$. There are two types of goods, a homogeneous good Y and a differentiated good X. Consumers are endowed with a unit flow of labor, which among other uses can be transformed one-to-one into the homogeneous good. In this sense, Y can also be interpreted as leisure. The labor endowment, or equivalently the homogeneous good, is taken as the numeraire.

The differentiated good is composed of a continuum of varieties of endogenous mass M_t, $M_t \in [0, 1]$, according to

$$X_t = \left(\int_0^{M_t} x_{jt}^\alpha\ \mathrm{d}j \right)^{\frac{1}{\alpha}}, \tag{2}$$

where x_{jt} represents consumption of variety j, and $1/(1-\alpha)$ is the elasticity of substitution across varieties, $\alpha \in (0, 1)$. Each variety $j \in [0, M_t]$ is produced by n identical firms, manufacturing perfectly substitutable goods.[9] Firms use labor to cover both variable production costs and a fixed production cost λ, $\lambda > 0$. Productivity is assumed to differ across varieties, but firms producing the same variety are equally productive. Let us omit index j and identify varieties with their productivity, which we denote by $\tilde{z}_t$. A firm with productivity $\tilde{z}_t$ has the following production technology

$$l_t = \tilde{z}_t^{-\eta} q_t + \lambda, \tag{3}$$

where l represents labor and q production. Variable costs are assumed to be decreasing in the firm's state of technology, with $\eta > 0$. Irrespective of their productivity, varieties exogenously exit at rate $\delta > 0$; in which case, the variety becomes obsolete and all firms exit simultaneously.

We may think of this technological structure as a streamlined representation of a real economy in the following way: first, the set of firms is divided in small groups producing the closest possible goods in terms of their substitutability; substitutability has to be almost perfect. We

[9] In a more general framework, the degree of substitution across these n goods may be finite but larger than the degree of substitution across varieties. However, introducing another degree of imperfect substitutability across goods would complicate notation without adding any key insight.

call the goods they produce a 'variety.' Second, we assume that the degree of substitutability across varieties is constant, as in the Dixit-Stiglitz model of monopolistic competition. Finally, to keep the model tractable, we assume homogeneity in productivity within varieties, but heterogeneity across varieties. This simplified technological structure does not capture only heterogeneity across the 'few' observable sectors in the data, but also the productivity difference across the 'many' firms producing imperfectly substitutable goods.

As in Atkeson and Burstein (2010), changes in productivity come from both process and product innovation. The former is undertaken by incumbent firms and the latter by potential entrants. The process innovation technology for a firm with productivity $\tilde{z}_t$ is given by

$$\dot{\tilde{z}}_t = A k_t h_t, \tag{4}$$

where h_t represents labor allocated to innovation and $A > 0$ is an efficiency parameter. The externality k_t is defined as

$$k_t = D_t \; \tilde{z}_t^c. \tag{5}$$

It encompasses an increasing difficulty component and a knowledge spillover component. The spillover component comes from the average productivity of direct competitors –those producing the same variety– which is denoted by $\tilde{z}^c$: the more productive direct competitors are the more effective a firm's current innovation is in enhancing productivity. This specification of innovation technology is commonly used in the endogenous growth literature to generate a constant growth rate in steady state.[10] Since we are assuming that all firms producing the same variety have the same productivity, in a symmetric equilibrium $\tilde{z}^c$ is equal to $\tilde{z}$.

The term D_t represents the degree of difficulty in innovation, under the assumption that innovating is harder for firms producing highly productive varieties. The degree of difficulty in innovation is measured as the distance between the average productivity of the overall economy

$$\tilde{Z}_t = \frac{1}{M_t} \int_0^{M_t} \tilde{z}_{jt}^{\hat{\eta}} \; \mathrm{d}j,$$

$\hat{\eta} \equiv \eta\alpha/\left(1-\alpha\right)$, and the productivity of direct competitors, $D_t = \tilde{Z}_t/(\tilde{z}_t^c)^{\hat{\eta}}$. This particular definition of the average productivity, as it will be shown below, naturally emerges from our

[10] Aghion and Howitt (1992) and Grossmand and Helpman (1991), for example, adopt a similar knowledge spillovers structure to obtain sustained growth in Schumpeterian growth models. While Peretto (1996) uses this innovation technology to obtain positive long-run growth in an oligopolistic model of innovation and growth.

technological assumptions. Since, in the symmetric Cournot equilibrium that we will characterize, $\tilde{z}^c = \tilde{z}$, the definition of D implies that more productive firms find it harder to innovate. This assumption on the innovation technology makes productivity growth rates equal across varieties, yielding an equilibrium stationary distribution of productivity. A similar assumption is commonly used in R&D-driven growth models with homogeneous firms to eliminate counterfactual scale effects, and stationarize models with growing population (e.g. Jones, 1995, Kortum, 1997, and Segerstrom, 1998).[11] In recent models of endogenous growth with heterogeneous firms, such as Klette and Kortum (2004), increasing innovation difficulty is introduced to stationarize the distribution of productivity and to match a robust stylized fact in the data: more productive/larger firms invest more in R&D, but the growth rate of productivity does not scale with size.[12] As we show later, in equilibrium more productive firms innovate more but the steady state growth rate is the same for all firms.

Finally, let us define the product innovation technology. The cost of producing a new variety is assumed to be zero and there is a mass of unit measure of potential varieties of which $M_t \in [0, 1]$ are operative. At any time t, the n firms associated to any potential new variety wait outside the economy, and jointly draw a productivity z from a time-invariant initial productivity distribution $\Gamma(z)$, which is assumed to be continuous in $(z_{\min}, \infty)$, with $0 \leq z_{\min} < \infty$. This productivity distribution is defined on $z_t = \tilde{z}_{jt}^{\hat{\eta}} e^{-\hat{\eta} g t}$, where g is the endogenous growth rate of average productivity –to be computed at equilibrium. Note that the entry distribution Γ is assumed to depend on detrended productivity z. This assumption is crucial for the economy to be growing at a stationary equilibrium. Incumbent firms are involved in innovation activities making their productivity grow at the endogenous rate g. This makes the distribution of incumbent firms move permanently to the right at rate g. By defining the entry distribution as a function of detrended productivity z, we allow the productivity of entrants to grow on average at the same rate as that of incumbent firms. This is a form of technological spillover from incumbents to new entrants. A similar assumption has been previously used to support a stationary equilibrium in models of random (exogenous) growth with heterogeneous productivity such as Luttmer (2007), Poschke (2009) and Gabler and Licandro (2009). In the subsections below, we derive the steady state equilibrium of this economy.

[11] Empirical evidence supports decreasing returns to innovation. The evidence surveyed in Kortum (1993) suggests point estimates for the patent/R&D elasticity that range between 0.1 and 0.6. More recently, Blundell et al. (2002) find a long-run elasticity of 0.5.

[12] See Klette and Kortum (2004), fact 1, and Griliches (2000) for the supporting empirical evidence.

2.2 Households

The representative household maximizes utility subject to its instantaneous budget constraint. The corresponding first order conditions are

$$Y = \beta E \tag{6}$$

$$\frac{\dot{E}}{E} = r - \rho \tag{7}$$

$$p_{jt} = \frac{E}{X_t^{\alpha}}\, x_{jt}^{\alpha-1}, \tag{8}$$

where r is the interest rate and p_{jt} is the price of good j. For variables that are constant in steady state, such as r, Y, E and M, index t is omitted to simplify notation. Total household expenditure on the composite good X is

$$E = \int_0^M p_{jt} x_{jt} \, \mathrm{d}j.$$

Because of log preferences, total spending in the homogeneous good is β times total spending in the differentiated good. Equation (7) is the standard Euler equation implying $r = \rho$ at the stationary equilibrium, and (8) is the inverse demand function for variety j.

2.3 Cournot Equilibrium

Let us assume that the n identical firms competing in the production of each variety j play a dynamic Cournot game. They behave non-cooperatively and maximize the expected present value of their net cash flow, denoted by V_{ijs} for firm i producing variety j at time s. This differential game is solved focusing on Nash Equilibrium in open loop strategies. Let $a_{ijt} = (q_{ijt}, h_{ijt})$, $t \geq s$, be a strategy for firm i producing j at time t. Let us denote by a_{ij} firm i strategy path for quantities and innovation. At time s a vector of strategy path $(a_{1j}, ..., a_{ij}, ..., a_{nj})$ is an equilibrium in market j if

$$V_{ijs}(a_{1j}, ..., a_{ij}, ..., a_{nj}) \geq V_{ijs}(a_{1j}, ..., a'_{ij}, ..., a_{nj}) \geq 0,$$

for all firms $\{1, 2, ..., n\}$, where in $(a_{1j}, ..., a'_{ij}, ..., a_{nj})$ only firm i deviates from the equilibrium path. The first inequality states that firm i maximizes its value taking the strategy paths of the others as givens, and the second requires firm i's value to be positive.[13]

[13] We choose the open loop equilibrium because it allows for closed form solution. The drawback of focusing on the open loop equilibriun is that it does not generally have the property of subgame perfection, as firms

The characterization of the open loop Nash equilibrium proceeds as follows: at time s a firm producing a particular variety solves (let us suppress indexes i and j to simplify notation)

$$V_s = \max_{[q_t, h_t]_{t=s}^{\infty}} \int_s^{\infty} \Big((p_t - \tilde{z}_t^{-\eta}) q_t - h_t - \lambda \Big) \mathrm{e}^{-(\rho+\delta)(t-s)} \, \mathrm{d}t, \qquad s.t. \tag{9}$$

$$\begin{aligned} p_t &= \frac{E}{X_t^{\alpha}} \, x_t^{\alpha-1} \\ x_t &= \hat{x}_t + q_t \\ \dot{\tilde{z}}_t &= A k_t h_t \\ \tilde{z}_s &> 0, \end{aligned}$$

where the firm cash flow is discounted with the steady-state interest rate ρ and the exogenous firm death rate δ. The first constraint is the indirect demand function for each variety; the second is the quantity constraint that splits the total size of the market for a variety x_t between this firm and its direct $(n-1)$ competitors in the industry, $\hat{x}_t$. The third constraint is the innovation technology. In a Cournot game a firm takes as given the path of its competitors' production $\hat{x}_t$, the path of the externality k_t, as well as the path of the aggregates E and X_t, and the exit shock δ. The current value Hamiltonian for this problem is:

$$\begin{aligned} \mathcal{H}_t &= \Big((p_t - \tilde{z}_t^{-\eta}) q_t - h_t - \lambda \Big) + v_t A k_t h_t \\ &= \left(\left(\frac{E_t}{X_t^{\alpha}} \, (\hat{x}_t + q_t)^{\alpha-1} - \tilde{z}_t^{-\eta} \right) q_t - h_t - \lambda \right) + v_t A k_t h_t, \end{aligned}$$

where v_t is the costate variable. The first order conditions for the problem above, under

choose their optimal time-paths strategies at the initial time and stick to them forever. In closed loop equilibria, instead, firms do not pre-commit to any path and their strategies at any time depend on the whole past history. The Nash equilibrium in this case is strongly time-consistent and therefore sub-game perfect. Unfortunately, closed loop or feedback equilibria generally do not allow for a closed form solution and, often, they do not allow for a solution at all. The literature on differential games has uncovered classes of games in which the open loop equilibrium degenerates into a closed loop and therefore is subgame perfect (e.g. Reingaum, 1982, Fershtman, 1987, and Cellini and Lambertini, 2005). A sufficient condition for the open loop Nash equilibrium to be subgame perfect is that the state variables of other firms do not apper in the first order conditions for each firm.

symmetry, are

$$\frac{\partial \mathcal{H}_t}{\partial q} = 0: \quad \tilde{z}_t^{-\eta} = \theta \underbrace{\frac{E}{X_t^{\alpha}} x_t^{\alpha-1}}_{p_t} \tag{10}$$

$$\frac{\partial \mathcal{H}_t}{\partial h} = 0: \quad v_t A k_t = 1 \tag{11}$$

$$\frac{\partial \mathcal{H}_t}{\partial \, \tilde{z}} = \eta \tilde{z}^{-\eta-1} q_t = -\dot{v}_t + (\rho + \delta)\, v_t. \tag{12}$$

From (10), firms charge a markup over marginal costs, with $\theta \equiv (n-1+\alpha)/n$ being the inverse of the markup. This is the well known result in Cournot-type equilibria that the markup depends on the perceived demand elasticity, which is a function of both the demand elasticity and the number of competitors.[14]

Firms producing the same variety are assumed to face the same initial conditions, resulting in a symmetric equilibrium with $x_t = nq_t$. As shown in Appendix A, substituting (10) into (2), we obtain the demand for variable inputs

$$\tilde{z}_t^{-\eta} q_t = \theta e \, \frac{z}{\bar{z}}, \tag{13}$$

where $e \equiv E/nM$ is expenditure per firm, z is the measure of firm detrended productivity defined in the previous section, and

$$\bar{z} \equiv \frac{1}{M} \int_0^M z_j \; \mathrm{d}j$$

is the average of detrended productivity z. Notice that the amount of resources used by a firm in (13) is the product of average expenditures per firm, the inverse of the markup and the relative productivity of the variety the firm produces. When the environment becomes more competitive, θ increases, prices decline, produced quantities increase and firms demand more inputs. Moreover, (13) shows that more productive firms produce more.

The optimal growth rate of productivity is

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A \theta e - \rho - \delta, \tag{14}$$

[14] It can be easily shown that condition (10) is the solution of the corresponding static Cournot game with given productivities. Consistently with Cellini and Lambertini (2005), the solution to the open loop equilibrium coincides with the closed loop when $\hat{\eta} = 1$, since in this case the externality k in the FOC (11) does not depend on the productivity of direct competitors.

the same for all $\tilde{z}$. As shown in Appendix A, this is obtained using (5), (11), (12), (13) and the definition of D_t. Equilibrium innovation for firm z can be derived using (4), (5) and (14),

$$h = (\eta\theta e - \hat{\rho})\,\frac{z}{\bar{z}}, \tag{15}$$

where $\hat{\rho} = (\rho + \delta)/A$. Labor resources allocated to innovation h are directly proportional to the firm's relative productivity $z/\bar{z}$. Equation (15) shows that more productive firms innovate more: since they are larger, as shown in (13), and innovation is cost-reducing, they have higher incentives to innovate. This is consistent with the empirical evidence showing that more productive firms spend more on innovation without featuring higher growth rates of productivity (e.g. Lentz and Mortensen, 2008, and Aw, Roberts and Xu, 2010, Akcigit and Kerr, 2011). The specific form of the externality k in (5) allows for the growth rate to be equal across varieties, offsetting the positive effect that the relative productivity has on innovation.

Since there is no innovation in the homogeneous good sector, (1) and (3) imply that the growth rate of output is

$$g_{gdp} = \frac{\eta}{1+\beta}\,g, \tag{16}$$

where $1/(1+\beta)$ represents the share of the composite good in total consumption expenditure.

In a stationary equilibrium, the productivity of all firms grow at the same rate. As a consequence, their demand for variable inputs, as described by (13), is constant along the balance growth path. More importantly, even when firms do R&D and their productivity endogenously grow, in a stationary equilibrium they all stay in their initial position in the productivity distribution, and the model remains highly tractable.

2.4 Exit and Entry

From the previous section, it can be easily shown that profits are a linear function of the relative productivity $z/\bar{z}$

$$\pi(z/\bar{z}) = (1-\theta)\,e\;z/\bar{z} - \underbrace{(\eta\theta e - \hat{\rho})\,z/\bar{z}}_{h} - \lambda. \tag{17}$$

Produced quantities and innovation effort depend both on the distance from average productivity $z/\bar{z}$. In the following, we assume η to be small enough such that $1-(1+\eta)\theta > 0$, a sufficient condition for the cash flow to depend positively on z. Let us denote by z^* the stationary cutoff productivity below which varieties exit. At a stationary state, the cutoff productivity makes firm's profits and firm's value equal to zero, implying

$$e = \frac{\frac{\lambda}{z^*/\bar{z}} - \hat{\rho}}{1 - (1+\eta)\theta}. \tag{EC}$$

We refer to it as the exit condition.[15] We can now write $\bar{z}$ as a function of z^*

$$\bar{z}(z^*) = \frac{1}{1 - \Gamma(z^*)} \int_{z^*}^{\infty} z f(z) \, \mathrm{d}z. \tag{18}$$

Let us denote by $\mu(z)$ the stationary equilibrium density distribution defined on the z domain. The endogenous exit process related to the cutoff productivity z^* implies $\mu(z) = 0$ for all $z < z^*$. Since the equilibrium productivity growth rates are the same irrespective of z, in a stationary environment, surviving firms remain always at their initial position in the distribution Γ. Consequently, the stationary equilibrium distribution is $\mu(z) = f(z)/(1-\Gamma(z^*))$, for $z \geq z^*$, where f is the density associated to the entry distribution Γ.

Since varieties exit at the rate δ, stationarity requires

$$(1 - M)(1 - \Gamma(z^*)) = \delta M. \tag{19}$$

This condition states that the exit flow, δM, equals the entry flow defined by the number of entrants, $1 - M$, times the probability of surviving, $1 - \Gamma(z^*)$. Consequently, the mass of operative varieties is a function of the productivity cutoff z^*,

$$M(z^*) = \frac{1 - \Gamma(z^*)}{1 + \delta - \Gamma(z^*)}. \tag{20}$$

It is easy to see that M is decreasing in z^*, going from $1/(1+\delta)$ to zero.

2.5 Stationary Equilibrium

The labor market clearing condition can be written as

$$n \int_0^M (l_j + h_j) \, \mathrm{d}j + Y = n \int_0^M \left(\bar{z}_j^{-\eta} q_j + h_j + \lambda \right) \, \mathrm{d}j + \beta E = 1.$$

The labor endowment is allocated to production and R&D activities in the composite sector, as well as to production in the homogeneous sector. The first equality is obtained substituting

[15] Notice that problem (9) does not explicitly include positive cash flow as a restriction. By doing so and then imposing the exit condition (EC), we implicitly forbid firms with $z < z^*$ to innovate and potentially grow at some growth rate smaller than g. If they were allowed to do so, they will optimally invest in innovation up to the point in which the cash flow would be zero. In such a case, firms with initial productivity smaller than the cutoff value will be growing at a rate smaller than g, moving to the left of the distribution and eventually exiting. Such an extension would make the problem unnecessarily cumbersome without affecting the main results.

l from (3) and Y from (6). Let us change the integration domain from varieties $j \in [0, M]$ to productivities $z \in [z^*, \infty)$ and use (4), (13) and (15) to rewrite the labor market clearing condition as

$$\int_{z^*}^{\infty} \Big((1+\eta)\,\theta e\, z/\bar{z} - \hat{\rho} z/\bar{z} + \lambda \Big)\, \mu(z)\ \mathrm{d}z + \beta e = \frac{1}{nM}.$$

Since $\int_{z^*}^{\infty} \mu(z)\,\mathrm{d}z = \int_{z^*}^{\infty} z/\bar{z}\,\mu(z)\,dz = 1$, after integrating over all varieties we obtain

$$e = \frac{\frac{1}{nM(z^*)} + \hat{\rho} - \lambda}{\beta + (1+\eta)\theta}, \tag{MC}$$

a positive relation between e and z^*.

The market clearing condition is characterized by labor allocation between the homogeneous and the composite good sectors, and between production and R&D. Note that θe is the amount of labor allocated by the average firm to cover variable production costs. While E is total consumer expenditure on the differentiated good, price distortions imply that θE units of labor are required as variable costs to produce it, where $E = nMe$. Recall that we have taken labor as the numeraire. Because of log preferences, when E is spent in the consumption good, βE units of labor are allocated to the homogeneous good. Since the marginal return on innovation depends on firm's production, $\eta(\theta E)$ is allocated to R&D (minus the user cost of innovation, as measured by $\hat{\rho}$). Finally, firms also assign labor to cover the fixed cost λ.

Assumption 1. The entry distribution is such that

$$z^*/\bar{z}(z^*) \text{ is increasing in } z^*, \tag{a}$$

and the following parameter restrictions hold:

$$\bar{z}_e/z_{\min} > \hat{\rho} \tag{b}$$

$$(1+\eta)\theta > \Psi \tag{c}$$

where

$$\Psi = \frac{\frac{(1+\delta)}{n} + \hat{\rho}(1+\beta) - \lambda\left(1 + \beta\frac{\bar{z}_e}{z_{\min}}\right)}{\frac{(1+\delta)}{n} + \lambda\left(\frac{\bar{z}_e}{z_{\min}} - 1\right)}$$

and $\bar{z}_e$ is the average productivity at entry.

Assumption (a) makes the (EC) curve decreasing in z^*. As discussed in Melitz (2003), many common distributions satisfy condition (a).[16] Indeed, if the productivity distribution is

[16] More precisely, condition (a) in assumption 1 is satisfied by the Lognormal, Exponential, Gamma, Weibul, or truncation on $(0, +\infty)$ of the Normal, Logistic, Extreme value, or Laplace distributions. See Melitz (2003).

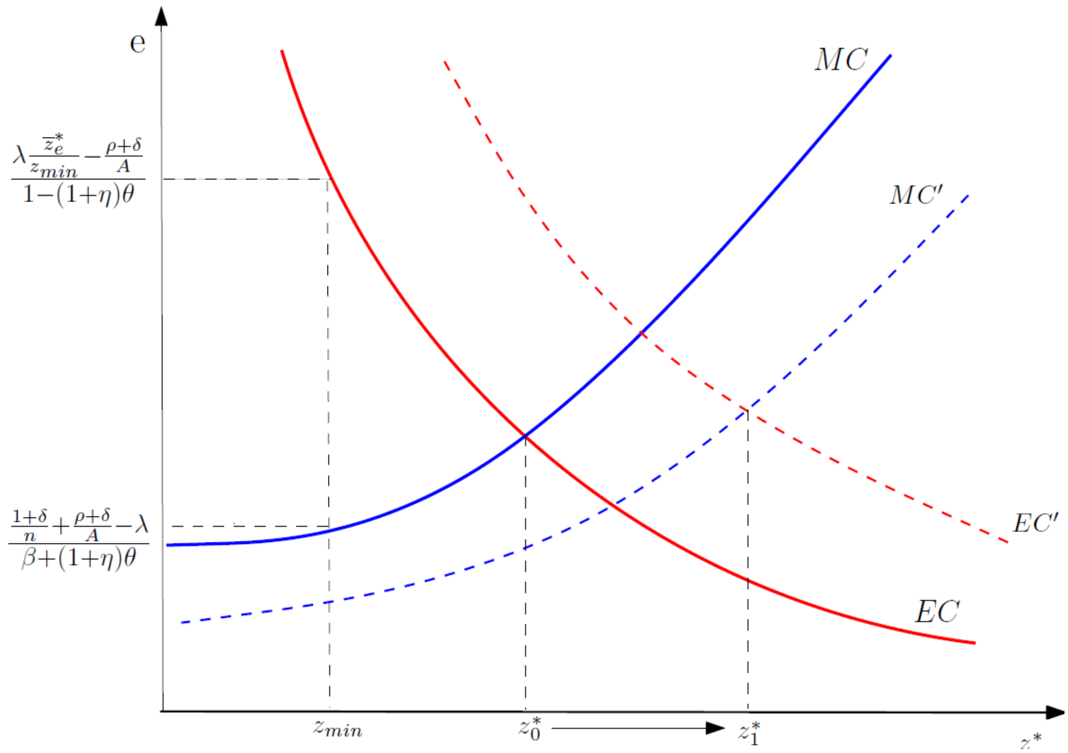

Figure 1: Equilibrium in closed economy

Pareto, (EC) is horizontal.[17] As stated in Proposition 1 below, under assumptions (b) and (c) the (EC) curve cuts the (MC) curve from above, which is sufficient for existence and unicity of equilibrium. Figure 1 provides a graphical representation.

Proposition 1 *Under Assumption 1, there exits a unique interior solution* (e, z^*) *of (MC) and (EC), with* M *determined by (20).*

Proof. See Appendix B. ■

Proposition 2 below states the main effects of an exogenous increase in product market competition as measured by θ.[18]

Proposition 2 *An increase in* θ *raises the productivity cutoff* z^**, reduces the number of operative varieties* $M(z^*)$*, has an ambiguous effect on the labor resources allocated to the homogeneous sector* e *and increases the growth rate* g*.*

[17] Consistently with evidence on US firm size distribution (e.g. Axtell, 2001, and Luttmer, 2007), in the quantitative analysis we will assume that firms' size/productivity is distributed Pareto.

[18] Notice that a reduction in the markup rate $1/\theta$, as $\theta \equiv (n - 1 + \alpha)/n$, can potentially be produced by either an increase in the substitutability parameter α, or by an increase in the number of firms n.

Proof. Figure 1 shows the effect of an increase in the degree of competition (reduction in the markup $1/\theta$) on the equilibrium values of z^* and e. An increase in θ shifts both the (EC) and the (MC) curves to the right, thereby increasing the equilibrium productivity cutoff z^*. Depending on the relative strengths of the shift of the two curves e can increase or decrease, but the average growth rate g always increases. From (14), the effect on g of a change in θ is determined by its effect on θe. Multiplying the market clearing condition (MC) by θ, we obtain θe as a function of θ and $M(z^*)$, and since in equilibrium $M(z^*)$ is decreasing in θ, we can conclude that θe is increasing in θ. ■

Two mechanisms contribute to the rise in growth, a direct growth effect and an indirect growth effect. In a Cournot equilibrium, an increase in competition reduces markups and allows for an increase in produced quantities; this can be easily seen from (13) which shows that the quantity produced is positively related to θ. The increase in quantities is feasible since the homogeneous good becomes relatively more expensive (i.e., the relative efficiency of the differentiated sector increases), and consumers' demand moves away from it towards the differentiated sector. Since the benefits of cost-reducing innovation are increasing in the quantity produced, but the innovation cost does not scale with quantity sold, the higher static efficiency associated with lower markups affects positively innovation and growth. This mechanism does not depend on firm heterogeneity: it is easy to check that assuming away the dependence of M on z^*, setting $M = 1$, the equilibrium growth rate becomes independent of the cutoff z^*, but is still increasing in θ.[19] This *direct growth effect* of competition can in fact be found in representative firm models of growth with endogenous market structure (see e.g. Peretto, 1996 and 2003, and Licandro and Navas-Ruiz, 2011).

Notice that, in this simple economy the direct growth effect of competition depends entirely on the presence of the homogeneous good. For a given z^*, the right-hand side of (MC) is constant. When $\beta = 0$, any change of θ has to be compensated by a proportional change in e, therefore having no effect on the growth rate g in (14). In the absence of the homogeneous good, any improvement in competition through an increase in θ will reduce expenditure per firm e proportionally. Since innovation incentives are proportional to firm's output, improvements in competition do not have any direct effect on innovation. In this sense, the homogeneous good is necessary in the simple model for competition to have a direct effect on growth. We show

[19] Shutting down firm heterogeneity the only equilibrium variable is expenditure e and is pinned down by the market clearing condition (MC). It is easy to see from (MC) that $|de/d\theta| < 1$, hence g is increasing in θ.

later that in the general model the direct growth effect is positive even in the absence of the homogeneous good.

The *indirect growth effect* is instead specifically related to the heterogeneous firms structure of the model. A reduction in the markup raises the productivity threshold above which firms can profitably produce, the cutoff z^*, thus forcing the least productive firms to exit the market. As a consequence, market shares are reallocated from exiting to surviving firms, thereby increasing their market size and their incentive to innovate. Therefore this selection mechanism leads to higher aggregate productivity level and higher innovation and productivity growth. The indirect effect is also at work in the extreme case of $\beta = 0$, since it would simply reallocate labor from low to high productivity firms.

3 Simple Model: Open Economy

Consider a world economy populated by two symmetric countries with the same technologies, preferences, and endowments as described in the previous section. We assume that both countries produce exactly the same varieties. Hence gains from trade are only triggered by changes in competition, with no gains coming from increasing varieties. We assume that trade costs are of the iceberg type: τ units of goods must be shipped abroad for each unit sold at destination, $\tau \geq 1$. They represent transportation costs and trade barriers created by policy. For simplicity, we do not assume entry costs in the export market, thus all operating firms sell both to the domestic and foreign markets.

3.1 Stationary Equilibrium

Since the two countries are perfectly symmetric, we can focus on one of them. Let q_t and $\tau\breve{q}_t$ be the quantities produced by a firm for the domestic and the foreign markets, respectively, and let $q_{x,t}$ be total firm's output. Firms solve a problem similar to the one in the closed economy (see Appendix C). The first order conditions are:

$$\tilde{z}_t^{-\eta} = \left((\alpha-1)\frac{q_t}{x_t}+1\right)p_t$$
$$\tau\tilde{z}_t^{-\eta} = \left((\alpha-1)\frac{\breve{q}_t}{x_t}+1\right)p_t$$
$$1 = v_t A k_t(\tilde{z})$$
$$\frac{\eta\tilde{z}_t^{-\eta-1}}{v_t}\underbrace{(q_t+\tau\breve{q}_t)}_{q_{x,t}} = \frac{-\dot{v}_t}{v_t}+\rho+\delta.$$

Because of the trade costs, firms face different marginal costs and have different sales the domestic and foreign markets. Under Cournot competition countries export and import goods that are perfectly substitutable to domestic production, even in the presence of positive variable trade costs.[20] Total consumption in the domestic market is $x_t = nq_t + n\breve{q}_t$, with $x_t \leq nq_{x,t}$ the difference being equal to the trade cost.

In Appendix C, we show that the first two conditions above yield

$$\tilde{z}_t^{-\eta}q_{x,t} = \theta_x e\, z/\bar{z}, \tag{21}$$

where $\tilde{z}$ and $\bar{z}$ are defined as in autarky, and

$$\theta_x = \frac{2n-1+\alpha}{n(1+\tau)^2(1-\alpha)}\left(\tau^2(1-n-\alpha)+n(2\tau-1)+(1-\alpha)\right) \tag{22}$$

is the inverse of the average markup faced by a firm in both the domestic and foreign market. Notice that θ_x is decreasing in variable trade costs τ, with θ_x reaching its maximum value $\theta_{\max} \equiv (2n-1+\alpha)/2n$ when $\tau = 1$, the polar case of no iceberg trade costs; the autarky value $\theta = (n-1+\alpha)/n$ is reached when $\tau = \bar{\tau} \equiv n/(n+\alpha-1)$, the alternative polar case of prohibitive trade costs implying that both economies do not have any incentive to trade.

Using the last two first order conditions above and proceeding as in the closed economy, we find that the growth rate of productivity

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A\theta_x e - \rho - \delta \tag{23}$$

[20] This is a standard result in the literature of trade under oligopoly since the pioneering contribution of Brander and Krugman (1983). Intuitively, in imperfectly competitive markets firms equal marginal revenues (not prices) to marginal costs. In the presence of variable export costs, marginal costs of exporting are higher than those of selling domestically. Hence, setting marginal revenues equal to marginal costs leads exporters to sell a lower quantity in the foreing market, compared to domestic sales. This leads to "cross-hauling", i.e., intra-industry trade of highly similar goods. See Neary (2010) for a recent overview of models of oligopoly and trade.

takes the same functional form as in the closed economy. Consequently, opening to trade only affects the equilibrium growth rate through changes in the markup θ_x. As in the closed economy case, we focus on steady state equilibria. The productivity cutoff is determined by the exit condition (see Appendix D)

$$\pi(z^*/\bar{z}) = (1-\theta_x)\, e\; z^*/\bar{z} - \underbrace{(\eta\theta_x e - \hat{\rho})\, z^*/\bar{z}}_{h} - \lambda = 0,$$

which yields

$$e = \frac{\frac{\lambda}{z^*/\bar{z}(z^*)} - \hat{\rho}}{1-(1+\eta)\,\theta_x}. \tag{EC^T}$$

Since firms fully compensate losses in local market shares by increased shares in the foreign market, profits are only affected by the change in the markup. Consequently, the exit condition has the same functional form as in (EC) except for θ_x.

The market clearing condition, proceeding as in the closed economy, becomes

$$e = \frac{\frac{1}{nM(z^*)} + \hat{\rho} - \lambda}{\beta + (1+\eta)\,\theta_x}, \tag{MC^T}$$

which is equal in all aspects to (MC) except for the markup, with θ_x instead of θ. Equations (EC^T) and (MC^T) yield the equilibrium (e, z^*) in the open economy, with $M(z^*)$ determined by (20). The equilibrium growth rate is characterized by (23).

Proposition 3 *Under Assumption 1 and for $\tau \in [1, \bar{\tau}]$, there exists a unique interior solution $(e, \tilde{z}^*)$ of (MC^T)-(EC^T).*

Proof. At the prohibitive trade cost $\bar{\tau}$, markups under trade and autarky are equal, $\theta_x = \theta$. Thus, for $\tau \geq \bar{\tau}$ firms do not have incentives to export, and trade does not take place. For $\tau < \bar{\tau}$ the proof of existence and unicity is similar to that in the closed economy, and we omit it for brevity. ■

3.2 Trade Liberalization

When countries are symmetric, trade openness does not affect firms' market shares because the reduction in local market sales due to foreign competition is perfectly offset by increased participation in the foreign market. This is related to the fact that for each variety the number of firms in the global market is $2n$ irrespective of the degree of trade openness. As an implication, a reduction in trade barriers makes local markets more competitive without changing the global number of firms operating in each market. For this reason, (MC^T) and (EC^T) are formally equivalent to (MC) and (EC) except for θ. We can then apply Proposition 2 to study the effects of trade liberalization. The economy with costly trade is characterized by a level of product market competition higher than in autarky, with $\theta_x > \theta$, due to the participation of foreign firms in the domestic market. A larger number of firms in the domestic market raises product market competition, thus lowering the markup rate.

From the definition of θ and θ_x we obtain

$$\theta_x - \theta = \frac{\tau\left(1-\alpha\right)^2 - n\left(\tau-1\right)^2\left(n+\alpha-1\right)}{n\left(1+\tau\right)^2\left(1-\alpha\right)}, \tag{24}$$

which is positive for any non-prohibitive level of trade costs ($\tau \leq \bar{\tau}$). Differentiating the expression above, it is easy to see that the distance between θ_x and θ is decreasing in τ, since θ_x is decreasing in τ (see Appendix E). Hence we have two results, first, when a country goes from autarky to costly trade, it experiences an increase in product market competition. Secondly, incremental trade liberalization increases product market competition as well. When trade is completely free, $\tau = 1$, product market competition reaches its maximum level, $\theta_{\max} \equiv (2n-1+\alpha)/2n$. Notice that $\theta_{\max}$ has the same functional form as the inverse of the markup in autarky, θ, but with twice the number of firms. Once established that trade reduces markups, it is easy to see that trade liberalization has the same effects on selection and innovation as those produced by an exogenous change in the markup in the closed economy –see Proposition 2 and Figure 1. These results can be summarized in the following proposition.

Proposition 4 *Trade liberalization makes markets more competitive by lowering markups, and then increases the productivity cutoff z^* and the productivity growth rate g.*

Proposition 4 states one of the main predictions of the model by matching and providing a coherent interpretation of the three pieces of evidence reported and discussed in the introduction. First, trade liberalization increases product market competition thereby reducing prices and markups. This is the pro-competitive effect of trade found in an extensive body of empirical analysis using firm-level data. Second, the trade-induced reduction in markups forces the less productive firms out of the market, reallocating market shares toward more productive surviving firms, thereby increasing the aggregate level of productivity; this is the selection effect found in the data. Finally, by reallocating market shares toward surviving firms, trade increases their incentive to innovate, as recently found in empirical studies.

The effects of trade on innovation and growth can be decomposed into two: a direct growth effect, which can be obtained also in a representative firm economy, and an indirect growth effect triggered by firm selection. From the steady state equilibrium growth rate (23), we obtain

$$\frac{\partial g}{\partial\left(1/\tau\right)}=\underbrace{g_1\frac{\partial\theta_x}{\partial\left(1/\tau\right)}}_{\texttt{Direct Effect}}+\underbrace{g_2\frac{\partial z^*}{\partial\theta_x}\frac{\partial\theta_x}{\partial\left(1/\tau\right)}}_{\texttt{Indirect Effect}},\tag{25}$$

where $g_1=\eta A\left(e+\theta_x\partial e/\partial\theta_x\right)$ and $g_2=\eta A\theta_x\partial e/\partial z^*$ are the derivatives of the growth rate with respect to θ_x keeping z^* constant and with respect to z^* respectively. The partial derivatives $\partial e/\partial z^*$ and $\partial e/\partial\theta_x$ result from differentiating (MC^T), and $\partial z^*/\partial\theta_x$ from differentiating the implicit function that results from combining (EC^T) and (MC^T).

The *direct growth effect* of trade liberalization is mainly produced by the reduction of oligopolistic distortions, which reallocates labor from the homogeneous to the composite sector. As innovation is cost reducing, lower markups trigger higher innovation. This effect can be found in models of trade and growth with endogenous market structure and homogeneous firms, such as Peretto (2003) and Licandro and Navas-Ruiz (2011). Notice that, as pointed out in Section 2.5, in the absence of the homogeneous good ($\beta=0$), the direct growth effect of increasing competition is zero. This happens because a trade-induced increase in θ_x is perfectly compensated by a decline in expenditure e, leaving g in (23) unchanged. In terms of the decomposition in (25), $\beta=0$ implies that g_1 is zero.

The *indirect growth effect* works through the reallocation of market shares produced by selection: the market shares of exiting firms are reallocated towards surviving firms, resulting in an increase in both average firms' productivity (static effect) and the incentives to innovate (dynamic effect). Thus, the selection effect of trade liberalization not only raises the level of productivity as in Melitz (2003), but also its growth rate.

Let us now move to the analysis of the welfare effects of trade. As pointed out by Atkeson and Burstein (2010), as long as love-for-variety matters, the positive welfare effect of improved selection may be offset by the reduction in the mass of varieties produced by selection. Aggregate steady state welfare can be written as (see Appendix E)

$$\rho U=\underbrace{\frac{1-\alpha}{\alpha}\ln(\bar{z}M)}_{\texttt{Productivity}}+\underbrace{\ln(\hat{\theta}E)}_{\texttt{Labor}}+\underbrace{\frac{\eta g}{\rho}}_{\texttt{Growth}}+\underbrace{\beta\ln(\beta E)}_{\texttt{Homogeneous good}}\tag{26}$$

where $\hat{\theta}$, $\hat{\theta}=\left(2n+\alpha-1\right)/n(1+\tau)$, measures aggregate output distortions generated by both the use of resources to cover trade costs and the inefficiency due to oligopolistic competition. It can be easily shown that $\hat{\theta} < \theta_x$. It does reflect the fact that aggregate consumption $\hat{\theta}E$ is smaller than total firms' production $\theta_x E$, because some resources are allocated to cover trade costs. Steady-state welfare in (26) has four terms. The first three correspond to composite good consumption, which is decomposed into its average labor productivity, total labor allocated to its production and its growth component. The last term represents homogeneous good consumption.

Before analyzing the indirect effect of trade on welfare, we should notice that the direct effect does not disappear in the absence of the homogenous good; as it is the case for the direct growth effect discussed above. In (26) a reduction in trade costs has a direct effect on welfare, since $\hat{\theta}e$ does change even if $\theta_x e$ does not. Trade liberalization reduces the direct losses associated to trade costs, thereby increasing $\hat{\theta}e$ and consumer welfare.

Selection has three indirect effects on steady-state welfare, as can be seen by differentiating (26) with respect to z^*, after substitution of E by nMe,

$$\mathtt{IE} = \underbrace{\frac{1-\alpha}{\alpha}\left(\frac{1}{\bar{z}}\frac{\partial \bar{z}}{\partial z^*} + \frac{1}{M}\frac{\partial M}{\partial z^*}\right)}_{\texttt{Productivity Level}} + \underbrace{(1+\beta)\left(\frac{1}{e}\frac{\partial e}{\partial M} + \frac{1}{M}\right)\frac{\partial M}{\partial z^*}}_{\texttt{Fixed Cost Savings}} + \underbrace{\frac{\eta^2 A\theta_x}{\rho}\frac{\partial e}{\partial M}\frac{\partial M}{\partial z^*}}_{\texttt{Productivity Growth}}, \tag{27}$$

where $\partial e/\partial M$ is computed by differentiating (MC^T), $\partial M/\partial z^*$ by differentiating (19) and $\partial \bar{z}/\partial z^*$ by differentiating the definition of $\bar{z}$.

First, selection has a static effect by increasing average labor productivity in the production of the composite good. This term embeds the two key offsetting forces in Atkeson and Burstein (2010): on the one hand selection increases the average productivity of firms $\bar{z}$, thus raising welfare, on the other it kills varieties M, thus reducing welfare. We show in Appendix E that the *productivity level* channel is strictly positive if the exit rate δ is small enough or equivalently if the production fixed cost λ is small enough. Second, since selection forces some firms out of the market, it reallocates resources towards the composite and the homogenous goods by reducing the amount of resources needed to cover fixed production costs. As shown in Appendix E, this welfare effect is positive. The reduction in the mass of varieties following selection, reduces the number of operative firms and consequently the labor allocated to cover total fixed operating costs, thereby freeing resources for production. Finally, the last component features the dynamic gains from trade triggered by a larger *growth rate* of productivity, which is specific to our economy with firm heterogeneity and endogenous technical change. Selection increases the growth rate which is unambiguously welfare improving (dynamic effect). The sum of these three channels pins down the *indirect welfare effect* of trade liberalization. Since only the productivity level channel can potentially affect welfare negatively, the restrictions on δ and λ discussed above provide sufficient conditions for the overall indirect effect to be positive.

In the next section, we provide a quantitative evaluation of the growth and welfare gains from trade and of their transmission channels. Before performing our quantitative analysis we generalize the model along two relevant dimensions: first, we allow vertical entry, that is we assume that in order to enter the market firms pay a fixed cost $\phi > 0$. This implies that the number of firms per product n will be endogenously determined. Second, we introduce a sunk cost of exporting leading to a market structure in which not all firms export, thus introducing the equilibrium decision of firms to serve the export market.

4 General Model

Following Melitz (2003), we assume that exporting firms face not only a variable trade cost but also a fixed export cost λ_x.[21] Under symmetric countries, this assumption implies that in equilibrium some operative varieties are exported and others not, splitting the product space into traded and non-traded goods. In equilibrium, markets for non-exporters behave as in the simple autarky model of Section 2, but markets for exporters behave as under the costly trade economy discussed in Section 3. The only difference between these markets is in the markup, $1/\theta$ for non-exporters and $1/\theta_x$ for exporters, with θ and θ_x as defined above. With this difference in mind, we proceed as in Section 3.

Non-exporters and exporters demand for variable inputs, are structurally similar to (13) and (21),

$$\tilde{z}_t^{-\eta} q_t = \theta e \left(\frac{\bar{p}}{p(z)} \right)^{\frac{\alpha}{1-\alpha}} \tag{28}$$

$$\tilde{z}_t^{-\eta} q_{x,t} = \theta_x e \left(\frac{\bar{p}}{p_x(z)} \right)^{\frac{\alpha}{1-\alpha}}, \tag{29}$$

where $q_{x,t}$ is now the production of exporting firms including domestic and foreign sales. Detrended prizes derive from (10) and the definition of stationary productivity $z \equiv \tilde{z}_t^{\hat{\eta}} \ e^{-\hat{\eta} g t}$,

[21] As in Melitz, this is equivalent to a sunk cost for entering the export market: since productivity is known when firms decide whether to export or not, firms are indifferent on whether to pay a sunk export cost f_x or its annualized value $\lambda_x \equiv f_x/(\rho + \delta)$. Sunk export costs can be costs of setting distribution channels abroad, learning about foreign regulatory system, advertising, etc.

yielding $p(z)^{\frac{\alpha}{\alpha-1}} = \theta^{\frac{\alpha}{1-\alpha}} z$ and $p_x(z)^{\frac{\alpha}{\alpha-1}} = \theta_x^{\frac{\alpha}{1-\alpha}} z$. The average detrended price is

$$\bar{p}^{\frac{\alpha}{\alpha-1}} = \left(\theta^{\frac{\alpha}{1-\alpha}} \int_{z^*}^{z_x^*} z\mu(z)\mathrm{d}z + \theta_x{}^{\frac{\alpha}{1-\alpha}} \int_{z_x^*}^{\infty} z\mu(z)\mathrm{d}z \right).$$

In the particular case where $\theta = \theta_x$, the ratios $(\bar{p}/p_x(z))^{\frac{\alpha}{1-\alpha}}$ and $(\bar{p}/p(z))^{\frac{\alpha}{1-\alpha}}$ become both equal to $z/\bar{z}$ as in (13) and (21) in the basic formulation of the previous sections.

In order to keep the model stationary, we assume that the degree of innovation difficulty in the externality (5) follows

$$D_t = \frac{\theta_x e}{(\tilde{z}_t^c)^{-\eta}\, y_t^c}, \tag{30}$$

where production y is q for non exporters and q_x for exporters. Both productivity $\tilde{z}$ and production y refer to direct competitors. This assumption is equivalent to the corresponding assumption in Section 2. By definition of average and detrended productivity, in the simple model the degree of difficulty D, defined as $\tilde{Z}/(\tilde{z}^c)^{\hat{\eta}}$, becomes equal to $\bar{z}/z^c$, where z^c is the detrended productivity of direct competitors. Using $\tilde{z}_t^{-\eta} q_t = \theta e\, z/\bar{z}$ from (13), we can obtain $D_t = \theta e/(\tilde{z}_t^c)^{-\eta} q_t^c$, which has the same structure as in (30) except for a different scale factor, θ instead of θ_x, and does not account for quantity differences between exporters and non-exporters.

As it was the case for the simple model of Section 2, the difficulty index in (30) makes innovation harder for more productive/larger firms and, as we show below, allows us to obtain symmetric growth across varieties and a stationary distribution of productivity in the steady state. Since in our model more productive firms are also larger we can interpret D both in terms of productivities (as we did in Section 2) or in terms of the firm size, as it is more appropriate for (30). Hence, we can think of D as measuring the distance between the labor size of the average firm, θe, and the labor resources employed by firms producing the variety $\tilde{z}$, $\tilde{z}_t^{c-\eta}\, y_t^c$.[22] Larger (more productive) firms face higher innovation difficulty than the average firm.

The first order conditions of the firms' problem are similar to (11)-(12). Using the new definition of k, it can be easily shown that the growth rate of productivity is the same for exporters and non-exporters and reads

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A \theta_x e - \rho - \delta. \tag{31}$$

[22]Recall that in symmetric Cournot equilibrium $\tilde{z}^c = \tilde{z}$, hence all firms producing the same variety have the same productivity.

The steady state innovation employment is

$$h_x\left(z\right)=\left(\frac{\bar{p}}{p_x(z)}\right)^{\frac{\alpha}{1-\alpha}}\left(\eta\theta_x e-\hat{\rho}\right) \tag{32}$$

for exporters and

$$h\left(z\right)=\left(\frac{\bar{p}}{p(z)}\right)^{\frac{\alpha}{1-\alpha}}\left(\eta\theta_x e-\hat{\rho}\right)\frac{\theta}{\theta_x} \tag{33}$$

for non-exporters. Since $p_x(z)<p(z)$ and $\theta<\theta_x$, exporting firms invest in innovation more than non-exporters. The productivity cutoff for exporters, z_x^*, is implicitly defined by the condition

$$\pi_x(z_x^*)=\Big(\big(1-(1+\eta)\theta_x\big)e+\hat{\rho}\Big)\left(\frac{\bar{p}}{p_x(z_x^*)}\right)^{\frac{\alpha}{1-\alpha}}-\lambda-\lambda_x=0,$$

and the one for non-exporters, z^*, by the exit condition

$$\pi(z^*)=\left(\big(1-(1+\eta)\theta\big)e+\hat{\rho}\frac{\theta}{\theta_x}\right)\left(\frac{\bar{p}}{p(z^*)}\right)^{\frac{\alpha}{1-\alpha}}-\lambda=0.$$

They lead to the following equilibrium cutoffs conditions

$$e=\frac{\lambda\left(\frac{p(z^*)}{\bar{p}}\right)^{\frac{\alpha}{1-\alpha}}-\hat{\rho}\frac{\theta}{\theta_x}}{1-\left(1+\eta\right)\theta}. \tag{EC'}$$

and

$$e=\frac{\left(\lambda+\lambda_x\right)\left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{1-\alpha}}-\hat{\rho}}{1-\left(1+\eta\right)\theta_x}. \tag{XC}$$

Firms entering the economy pay a fixed entry cost $\phi>0$ before they observe the productivity of the variety they will produce. We can think about this entry process as one in which n firms draw a technology z for producing a variety and they enter altogether. Free entry implies that the expected value of the firm must be equal to the entry cost

$$\left(1-\Gamma(z^*)\right)\frac{\bar{\pi}}{(\rho+\delta)}=\phi,$$

where expected profits are given by

$$\bar{\pi}=\int_{z^*}^{z_x^*}\Big((p(\tilde{z})-\tilde{z}^{-\eta})q-h\left(z\right)-\lambda\Big)\mu(z)dz+\int_{z_x^*}^{\infty}\Big((p(\tilde{z})-\tilde{z}^{-\eta})q_x-h_x\left(z\right)-\lambda-\lambda_x\Big)\mu(z)dz\,.$$

Using (28) and (29) the free entry condition can be written as

$$\left(1-\left(1+\eta\right)\bar{\theta}\right)e+\hat{\rho}\frac{\bar{\theta}}{\theta_x}-\lambda-\frac{1-\Gamma(z_x^*)}{1-\Gamma(z^*)}\lambda_x=\frac{\rho+\delta}{1-\Gamma(z^*)}\phi, \tag{FE}$$

where

$$\bar{\theta} = \theta \int_{z^*}^{z_x^*} \left(\frac{\bar{p}}{p(z)}\right)^{\frac{\alpha}{1-\alpha}} \mu(z)\mathrm{d}z + \theta_x \int_{z_x^*}^{\infty} \left(\frac{\bar{p}}{p_x(z)}\right)^{\frac{\alpha}{1-\alpha}} \mu(z)\mathrm{d}z.$$

As in the simple model of Section 2, firms/products exit the market at the exogenous destruction flow δ, and the stationarity condition for the mass of firms M is the same as in (20).

Finally, the labor market clearing condition

$$\int_{z^*}^{z_x^*} \left(\tilde{z}^{-\eta} q + h(z) + \lambda\right) \mu(z)\mathrm{d}z + \int_{z_x^*}^{\infty} \left(\tilde{z}^{-\eta} q_x + h_x(z) + \lambda + \lambda_x\right) \mu(z)\mathrm{d}z + \beta e + \frac{1 - M(z^*)}{M(z^*)}\phi = \frac{1}{nM(z^*)}$$

equals the labor resources used by domestic and exporting firms plus those devoted to entry, $(1-M(z^*))\phi$, to the labor endowment of the economy. From (19) and the definition of $\bar{\theta}$ above, the market clearing condition can be written as

$$\left(\beta + (1+\eta)\,\bar{\theta}\right) e + \left(\lambda + \frac{1-\Gamma(z_x^*)}{1-\Gamma(z^*)}\lambda_x + \frac{\delta}{1-\Gamma(z^*)}\phi\right) - \hat{\rho}\frac{\bar{\theta}}{\theta_x} = \frac{1+\delta/(1-\Gamma(z^*))}{n}. \qquad \text{(MC')}$$

A stationary equilibrium for this economy is a vector $\{z^*,\ z_x^*,\ e,\ n\}$ solving the system (EC')-(XC)-(FE)-(MC'), with $M(z^*)$ determined by (20).

5 Quantitative Analysis

The purpose of this section is twofold. First, we explore numerically the equilibrium properties of the generalized model, showing that the core results in Proposition 4 hold and that a richer sets of results can be obtained with this extended version. Secondly, we measure the quantitative effects on productivity growth and welfare of a reduction in variable trade costs, and asses the relevance of our mechanisms by breaking down the contribution of the direct and the indirect effect to trade-induced productivity growth and welfare gains. For this purpose, we refer to the calibrated version of the general model as the *benchmark* economy.

5.1 Calibration

Similarly to most calibrated models of firm dynamics, we target the US economy, for which micro data are widely available (see i.e. Bernard, Jensen, Eaton and Kortum, 2003, Luttmer, 2007, Alessandria and Choi, 2011). Consistent with the available evidence on firm size distribution, we assume that the entry distribution is Pareto with shape parameter κ, and scale parameter $z_{\min}$ (see e.g. Axtell, 2001, and Luttmer, 2007). We have to calibrate 12 parameters $(\alpha, \tau, \delta, \rho, \beta, \eta, A, \lambda, \lambda_x, \phi, \kappa, z_{\min})$. The discount factor ρ is equal to the interest rate at steady state, thus we set it to 0.05 following the business cycle literature. Anderson and van Wincoop (2002) summarize the tariff and non-tariff barriers to trade using TRAINS (UNCTAD) data: for industrialized countries tariffs are on average 5% and non tariff barriers are on average 8%. We take the sum of these two costs and set $\tau = 1.13$. Using Census 2004 data, we set $\delta = 0.09$ to match the average enterprise annual death rate in manufacturing observed in period 1998-2004.[23] Rauch (1999) classifies goods into homogeneous and differentiated, and finds that differentiated goods represents between 64.6 and 67.1 percent of total US manufactures, depending on the chosen aggregation scheme. We set $\beta = 0.5$ to get the share of differentiated goods $1/(1+\beta)$ equal to 2/3. We normalize the minimum value of the productivity distribution $z_{\min}$ to 1, without loss of generality.

Parameters (α, η, A, λ, λ_x, κ, ϕ) are jointly calibrated in order to match seven steady state moments predicted by the model to the corresponding firm-level and aggregate statistics. The annual growth rate of productivity is set equal to 1.19 percent, following evidence in Corrado, Hulten and Sichel (2009).[24] The R&D to GDP ratio is set equal to 2.5 percent, the US average in the post-War period (National Science Foundation, 2011). These targets are relevant in calibrating the technology parameters A and η. Bernard, Jensen, Eaton and Kortum (2003) using 1992 Census data for US manufacturing firms report the following statistics: first, exporters are about 33 percent more productive than non-exporters on average; second, the standard deviation of firm productivity is 0.75. Using World Bank World Development Indicators (2011) we compute an export share of output of 7.9 percent in 2009. We target these statistics since they are relevant in determining the fixed costs λ and λ_x, and the shape parameter of the productivity distribution κ. The average markup is set to 22 percent, an intermediate value in the range of estimates reported in Basu (1996), which is useful in calibrating the elasticity parameter α. Finally, Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002) find that the total regulatory entry cost for the U.S. in 1999 were 1.6 percent of GDP per capita, which we target in order to calibrate the entry cost ϕ. This leads to the following calibrated parameters: $\alpha = 0.6$, $\eta = 0.0655$, $A = 20.1$, $\lambda_x = 0.023$, $\lambda = 0.038$, $\kappa = 3.33$, $\phi = 0.06$. Table 1 shows that the calibrated model provides a sufficiently good fit of the targeted statistics.

[23] For each year the death rates are computed as follows: taking year 2000 as an example, the death rate is the ratio of firms dead between March 2000 and March 2001 to the total number of firms in March 2000. Data can be downloaded at http://www.sba.gov/advo/research/data.html#ne, file data_uspdf.xls.

[24] Since the model does not include tangible capital, investment in tangible capital has to be subtracted from total income in the data to compute labor productivity. After this adjustment, Corrado, Hulten and Sichel (2009) report an average growth of labor productivity of 1.19% a year in the period 1973-2003.

Table 1
MODEL FIT

Target	Data	Source	Model
Growth rate (%)	1.19	Corrado et al. (2009)	1.19
Innovation/GDP (%)	2.5	NSF (2011)	2.4
Productivity std.dev.	0.75	Bernard et al. (2003)	0.77
Export share	0.079	WDI (2011)	0.079
Exporters premium	0.33	Bernard et al. (2003)	0.35
Entry costs/GDP (%)	1.6	Djankov et al.(2002)	1.6
Markup rate (%)	22	Basu (1996)	22

The benchmark numerical solution also matches the following constraints. Firstly, the equilibrium number of firms $n = 2.12$ is small enough for the equilibrium prohibitive tariff, $n/(n+\alpha-1) = 1.23$, to be higher than the calibrated variable trade cost $\tau = 1.13$. Secondly, we check that conditions (b) and (c) in Assumption 1 hold, which as in the simple model make the profit function increasing in z. Under a Pareto distribution, condition (b) becomes $\kappa/(\kappa-1) > (\rho+\delta)/A$. As expected, the calibrated model yields $z^* > z_{\min} = 1$, implying that not all entrants get the chance to produce profitably, and $z_x^* > z^*$ stating that only the most productive firms export.

5.2 Trade Liberalization

Here, we use the calibrated economy to simulate the steady state equilibrium response to a reduction in trade costs τ. More precisely, we analyze the response of product market competition, selection and innovation, when the iceberg trade cost goes from its benchmark value to free trade, $\tau \to 1$. Since we are doing steady state welfare comparison, we interpret this exercise as comparing the welfare of two global economies similar in all features except for the variable trade cost. Figure 2 shows the results.

There are four main features in these results. First, trade liberalization has a first order pro-competitive effect for traded varieties: the markup $1/\theta_x$ declines, inducing larger selection into the export market, thereby increasing the export cutoff productivity z_x^*. Second, the pro-competitive effect of trade induces a first order increase in both the growth rate of productivity, g, and welfare. Third, trade liberalization also has a positive effect on the number of competitors n, inducing a reduction in the markup of non-traded varieties θ and an increase in the production cutoff productivity z^*, which ultimately leads to a reduction in the mass of operative varieties

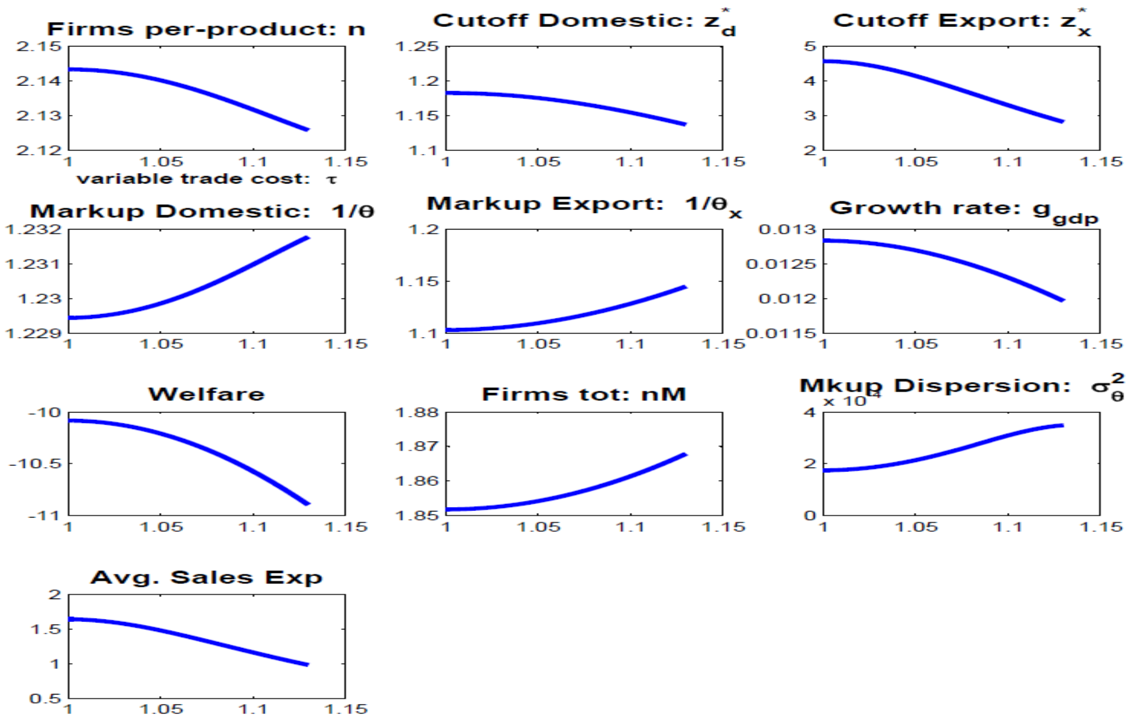

Figure 2: Trade Liberalization

M. However, all these changes are of a small magnitude. Finally notice that lower trade costs lead to lower markup dispersion.

We now explore the economic mechanism behind these results. A reduction in variable trade costs makes the markets for traded varieties more competitive, thereby reducing export markups and inducing the marginal exporters to exit the foreign market. As a consequence the productivity cutoff z_x^* increases. Notice that a reduction in the iceberg trade cost affects the export cutoff z_x^* in the opposite direction compared to Melitz (2003). In that paper, countries produce and trade different varieties, implying that there is no direct competition between domestic and foreign goods. Therefore, reducing trade costs implies that exporters benefit from an expansion of their market which leads to larger profits and a lower productivity threshold for exporting. In our model, the increase in revenues, produced by cheaper foreign market access, is more than compensated by the pro-competitive effect that reduces markups and profits making the export market more selective.

The entry decision depends on expected profits $(1-\Gamma(z^*))\,\bar{\pi}/(\rho+\delta)$, where $\bar{\pi}$ is an average of domestic and export profits. From the simple model we know that $\theta < \theta_x$, therefore in the neighborhood of z_x^* the profits of exporters are lower than those of non-exporters. A reduction in τ lowers the share of exporters in the economy, thus increasing expected profits for entering firms, stimulating entry and ultimately leading to a higher equilibrium number of competitors per variety. A larger n then reduces the domestic markup $1/\theta$ and raises the domestic cutoff z^*, thereby forcing the least productive domestic firms to exit. Tougher selection reduces the mass of operative firms M. A higher n also strengthens the reduction in the export markup produced by trade liberalization, thus further increasing the export cutoff z_x^*. As we can see from Figure 2, the changes in n, $1/\theta$, and z^* are quantitatively marginal, while the bulk of the effect of trade liberalization falls on the export markup $1/\theta_x$ and cutoff z_x^*.

Another interesting result is that although lowering trade costs makes the export market more competitive, there are more exporters per variety, each exporting more. In Figure 2, we can see that both the total number of domestic firms producing each variety, n, and the average sales of exporters increase. These two predictions are in line with the empirical evidence on US firms.[25] Interestingly, although trade liberalization increases the level of competition and reduces markups, there is 'market-size' effect that increases average firm size, sales and profits. Similarly to Melitz and Ottaviano (2008), the endogenous market structure of our model implies that trade liberalization has a positive effect on firms' production that outweighs the competition effect on prices and markups and allows surviving firms to be bigger, sell more, and earn higher profits on average.

The effect of trade on markup dispersion will be discussed in detail in section 6 below.

5.3 Growth and Welfare Decomposition

In this section, we assess the contribution of the direct and indirect channels to the overall productivity growth and welfare gains from trade. As in the simple model, trade-induced increases in competition affect growth and welfare directly by increasing market efficiency and indirectly through firm selection. Compared to the decomposition in Section 3.2, selection in the general model operates through two different margins, the domestic and export cutoffs, which the simple model merged in a unique margin. In assessing the contribution of selection to productivity growth and welfare, we perform two exercises. First, we follow Atkeson and Burstein (2010) procedure to evaluate the contribution of selection, the so called indirect channel: we compute

[25] Bernard, Jensen and Schott (2006) find that a reduction in trade costs increases the volume of export. Bernard, Redding and Schott (2007) find that the number of firms per product increases with a reduction in trade costs. Although they find that the number of both exporting firms and products increases, with the former increasing more than the latter.

the welfare and growth gains from a given reduction in τ in the full model simulated above; we repeat the same experiment keeping the cutoffs z^* and z_x^* constant, in order to pin down the direct effect; the indirect effect is then obtained as a residual. We perform this exercise for a reduction in trade costs τ from its benchmark value of 1.13 to 1, a world without variable trade costs. Secondly, we compare the gains in the benchmark model with those obtainable in a specification of the model with exogenous exit decision, all firms exporting, and no heterogeneity in productivity. We calibrate this version of the model in order to obtain the same export share and the same growth rate of the benchmark model. This second exercise goes beyond the break down of the direct and indirect channel of gains from trade, performing a comparison between a model of trade and growth with a representative firm and one featuring heterogeneous firms.

Direct and indirect effect. The decrease in the iceberg trade cost τ raises the growth rate from its benchmark value 1.19% to about 1.29%, which represents a 7.2% increase. Selection accounts for 25% of the total increase in growth generated by trade liberalization. Therefore about $1/4^{th}$ of the effect on aggregate growth comes from the indirect channel driven by heterogeneous firms' response to liberalization. Table 2 shows the robustness of the results to variations of parameters around their benchmark values.[26]

Table 2
DECOMPOSING GROWTH AND WELFARE GAINS
(Large trade liberalization: $\tau = 1.13 \rightarrow \tau = 1$)

(%)	Bench	$\underline{\kappa}$	$\overline{\kappa}$	$\underline{\beta}$	$\overline{\beta}$	$\underline{\phi}$	$\overline{\phi}$	$\underline{\lambda}$	$\overline{\lambda}$	$\underline{\lambda_x}$	$\overline{\lambda_x}$
Δ Growth	7.2	6.1	7.2	8.4	7	9	6.6	12	5.8	7.5	7
Indirect Effect	25	24	15	30	11	38	19	12	33	31	19
Welfare Gains ($\omega - 1$)	2.8	2.9	2.1	6.7	0.1	3	2.7	3	2.5	3.2	2.4
Indirect Effect	40	25	30	44	0	36	40	41	35	44	34

Robustness: $(\underline{\kappa}, \overline{\kappa}) = (2.1, 4.1)$, $(\underline{\beta}, \overline{\beta}) = (0, 4)$, other doubled and halved.

With the same procedure used in Section 3.2, we break down the overall welfare effect of trade into its direct and indirect component. In order to make welfare comparisons between the stationary solutions of the benchmark economy and the counterfactual economy, as described in the previous section, we use a consumption equivalent measure. Variable $\omega - 1$ in Table 2

[26] The robustness is performed halving and doubling the benchmark parameter values, except for κ and β. We limit our lower bond of κ above 2 because it is a necessary condition for the variance of the Pareto distribution of productivity to exist. Moreover, since we are interested in a thourough exploration of the role of the homogeneous good, we perform sensitivity for a wider interval of values of β, including $\beta = 0$, an economy without the homogeneous good.

measures the percentage gains in terms of lifetime total consumption triggered by the reduction in the iceberg trade cost.[27] The results show that, in the benchmark economy, a reduction in τ from 1.13 to free trade is equivalent to a permanent increase in lifetime consumption of 2.8%, about $2/5^{th}$ of which attributable to the indirect channel. Hence, the negative impact on welfare of the reduction in the mass of varieties does not completely offset the positive welfare effects generated by improvements in the level and growth rate of productivity.[28]

Table 2 allows us to explore the quantitative role of the homogeneous good in this economy. In the extreme case where there is no homogenous good ($\beta = \underline{\beta} = 0$) both the growth and welfare effect of trade increase, together with the importance of the indirect component. Intuitively, a higher share of the differentiated good in the economy leads to larger growth and welfare gains from trade, as well as a larger contribution of selection. Recall that in the simple model of Section 3, in the absence of the homogeneous good the direct growth effect vanishes and the direct welfare effect is only due to firms saving on variable trade costs when trade is liberalized. In the general model, there is an new source of direct gains from trade along both the growth and welfare dimension: the asymmetric effect of trade on domestic and foreign markups. As we can see from Figure 2, the export markup decreases substantially while the reduction in the domestic markup is small. A similar outcome is obtained when $\beta = 0$.[29] Hence, the direct effect is produced by the reallocation of market shares from domestic to exporting firms, triggered by the asymmetric effect of trade on domestic and export markups. As Table 2 shows, the direct effect is smaller than in the benchmark economy but still important. On the other hand, when $\beta = 4$ and thus the share of the differentiated good in the economy drops from 2/3 to 20 percent, the welfare gains from trade decrease substantially, as one would expect. Trade liberalization increases lifetime consumption by a meagre 0.1 percent, all attributable to the

[27] The compensating variation is computed as follows: let us denote by Ω the steady state equilibrium allocation $\{X, Y\}$ of the benchmark calibration and by $U(\Omega)$ the corresponding present-value welfare function, which results from substituting the equilibrium path in the utility function (1). Let us do the same for the counterfactual economy and denote by $U_c = U(\Omega_c)$ the level of welfare attained at equilibrium, where Ω_c represents the solution path of the counterfactual economy. Finally, let us define our consumption equivalent measure as the real number ω that makes $U(\Omega_\omega) = U_c$, where Ω_ω is the equilibrium allocation $\{\omega X, \omega Y\}$, that results from increasing consumption in the stationary state of the benchmark economy at the rate $\omega - 1$. It measures the percentage gains in terms of lifetime total consumption of comparing the benchmark with the counterfactual economy. From utility (1), $(1 + \beta)/\rho \log(\omega) = U(\Omega_c) - U(\Omega)$.

[28] Alessandria and Choi (2011) set up a version of Melitz (2003) with exogenous productivity dynamics and calibrate it to the US economy. In line with our results, they find that reducing the variable trade cost from 8% to free trade leads to a welfare gain of 1.03% of lifetime consumption, about 30% of which attributable to firm heterogeneity.

[29] We do not include the figure for $\beta = 0$ for brevity. It is available upon request.

direct component with no role left for selection any longer. Similarly, most of the growth effect comes from the direct channel with selection playing a marginal role.

Comparing with a representative firm economy. In our second experiment, we explore a different decomposition strategy. We compare the gains in the benchmark economy with those in a *representative firm* economy with exogenous exit decision, all firms exporting, and no heterogeneity in productivity, calibrated to match the same initial export share and growth rate of the benchmark economy. In the representative firm model, as reported in Table 3, a move from the benchmark τ to free trade leads to negligible growth effects and small welfare gains. Precisely, trade liberalization increases the growth rate by less than a percentage point, yielding a welfare gain of about 0.1% of long-run consumption. Hence, the comparison of these outcomes with those in the benchmark model suggests that introducing firm heterogeneity in a model with oligopoly trade and growth increases the quantitative gains from trade substantially.

Table 3

GROWTH AND WELFARE GAINS ACROSS MODELS

(Large trade liberalization: $\tau = 1.13 \rightarrow \tau = 1$)

(%)	Benchmark	Representative Firm
Δ Growth	7.2	0.06
Welfare ($\omega - 1$)	2.8	0.01

We can draw two conclusions from these experiments. First, the results seem to suggest that introducing pro-competitive effects and endogenous growth into a trade model with heterogeneous firms can potentially tame the neutrality result found in Atkeson and Burstein (2010), showing that firm-level responses to trade (i.e. selection into domestic and export market) can have non-negligible effects on aggregate innovation and welfare.[30] Second, we show that introducing firm heterogeneity in a model of oligopoly trade and growth (e.g. Peretto, 2003, and Licandro and Navas-Ruiz, 2011) yields substantial additional gains from trade spurring from the static and dynamic effects of firm selection.

[30] Atkeson and Burstein (2010) show that slow transitional dynamics can reduce the welfare effects attributable to the indirect channel, even in those specification of their model in which trade has large effects on steady state productivity and output. Since we focus on steady state analysis, we cannot check whether the speed of transition affects our quantitative results. Nevertheless, comparing our steady state results to theirs, we find noticeable size differences and less indications of potential neutrality of firm-level responses to trade. Computing the transitional dynamics of the model is certainly an important task for future research.

6 Discussion

In this section, we dig deeper into the role of two key features of the model in driving the quantitative results shown above: endogenous markups and endogenous productivity growth. We first argue that the presence of endogenous markups, and more precisely of markup dispersion, ultimately breaks Atkeson and Burstein (2010)'s neutrality result that indirect welfare gains from trade are small enough to be neglected. Then, we analyze the role of productivity growth.

Endogenous markups. We develop our argument in two stages. We start showing that trade does not generate welfare gains through selection in the simple model of Section 3 when we allow the number of firm per product line to be endogenously determined by the free entry condition. This result echoes the neutrality result in Atkeson and Burstein (in particular, Section IV.A). We then explain how the pro-competitive effect of trade breaks neutrality in the general model where not only the number of firms is endogenous but there is also selection of firms into the export market leading to heterogeneous markups between exporters and domestic firms.

The role of free entry. The absence in Atkeson and Burstein of indirect welfare gains from trade is fundamentally due to the role played by the free entry condition, which by shrinking the mass of operative varieties generates welfare losses that compensate the gains due to selection. We show below that the free entry condition plays a similar, even more extreme, role in our simple model. This can be easily seen by adding the free entry condition to the simple model of Section 3. [31]Notice that, when all varieties are traded, the free entry condition becomes

$$(1-(1+\eta)\theta_x)e+\hat{\rho}-\lambda=\frac{\rho+\delta}{1-\Gamma(z^*)}\phi$$

where the left-hand-side represent the average profit at entry. Combining this with the exit condition (EC^T) and rearranging terms, we obtain

$$\frac{\bar{z}(z^*)}{z^*}\lambda=\lambda+\frac{\rho+\delta}{1-\Gamma(z^*)}\phi, \tag{34}$$

which determines z^* independently of θ_x and, consequently, independently of the iceberg trade cost τ. Hence, changes in trade costs do not affect selection and, consequently, cannot have any indirect welfare effect. This result can be easily understood in terms of arbitrage. Incumbent

[31] Free entry is introduced to endogenize the number of firms. An equivalent conclusion would be reached if the number of firms remains constant, but the the mass of varieties is determined by the free entry condition, as in Atkeson and Burstein.

firms face two alternatives: operating their current technology or exiting and entering again by paying the fixed entry cost and draw an new productivity level. This trade-off is represented in (34), which combines the entry and exit conditions. Since both exiting and entering firms set the same markup, arbitrage makes the marginal firm indifferent to markup changes and, consequently, the market does not show up in equation (34). Hence, in our simple model, free entry implies that the selection effect of trade vanishes. However, as we show below, when exporting entails a fixed cost, the presence of different markup for exporters and non exporters implies that the marginal firm is not indifferent to markup changes any longer. Exiting and exiting firms face different profit opportunities and the neutrality mechanism breaks down. Finally notice that, although trade does not trigger horizontal exit (selection) and therefore does not have any indirect effect, it induces vertical exit, leading to a lower equilibrium number of firms per product line n.[32] The reduction in the number of firms, can potentially offset the trade-induced decrease in the markup, thus reducing the pro-competitive effects of trade.

The role of markup dispersion. While in the simple model the markup is the same for all firms, the general model features exporting firms charging lower markups than non-exporters, as shown in (28) and (29). As can be seen by comparing the free entry condition (FE) and the exit condition (EC$'$), the marginal firm is not indifferent anymore to changes in profits induced by trade liberalization, and the entry/exit arbitrage does not imply that selection is independent of the markup any longer. Hence, free entry does no shut down the selection effect of trade and, as shown in Table 2, the indirect welfare effect of trade is quantitatively substantial. Why is the presence of markup dispersion crucial in delivering a positive indirect welfare effect?

Markup dispersion provides an additional channel of distortion. As shown in the misallocation literature (e.g. Restuccia and Rogerson, 2008, Hsieh and Klenow, 2009, Epifani and Gancia, 2011), markup dispersion reduces aggregate efficiency. Edmond, Midrigan and Xu (2012) set up a model of heterogenous firms operating in oligopolistic markets and show that pro-competitive welfare gains from trade are large when inefficiencies associated with markup

[32] Using the exit condition (EC^T) and the market clearing condition (MC^T) we obtain

$$\frac{\frac{1}{nM(z^*)} + \hat{\rho} - \lambda}{\frac{\lambda}{z^*/\bar{z}(z^*)} - \hat{\rho}} = \frac{\beta + (1+\eta)\,\theta_x}{1-(1+\eta)\,\theta_x}.$$

Since z^* is not affected by a change in the markup, and since a trade induced increase in θ_x raises the right-hand side of this equation, it follows that for the left-hand side to increase, the number of firms n must decrease.

dispersion are large and there is a weak pattern of cross-country comparative advantages. Our model fulfills the latter condition since countries are identical (both facing the same distribution of firms across productivities), implying that comparative advantages play no role. When all firms export, they all set the same markup and, as shown above in this section, the selection effect disappears and the direct effect is tamed by a reduction in the number of firms per variety. Our general model with markup dispersion instead predicts positive and substantial welfare gains from trade, both direct and indirect, as shown in Figure 2 and Table 2. In order to provide a better understanding of the role of markup dispersion in our general model, let us express the variance of markups as follows (see Appendix F for derivation)

$$\sigma_\theta^2 = \left(\frac{1}{\theta} - \frac{1}{\theta_x}\right)^2 \left[1 - \left(\frac{z^*}{z_x^*}\right)^\kappa\right] \left(\frac{z^*}{z_x^*}\right)^\kappa .$$

It can be easily seen that $\sigma_\theta^2 = 0$ in the two extreme cases of autarky ($z^*/z_x^* = 0$ since $z_x^* = \infty$) and when all firms export $(z^*/z_x^* = 1)$. For a given τ, the variance of markups is hump-shaped reaching a maximum when fifty percent of firms export, $(1 - \Gamma(z_x^*)) / (1 - \Gamma(z^*)) = (z^*/z_x^*)^\kappa = 1/2$. In our calibrated economy the share of exporting firms is less than 10 percent. Hence we are on the increasing side of the hump. As can be seen from Figure 2, trade liberalization reduces z^*/z_x^*, thereby decreasing markup dispersion and the associated oligopolistic distortions. However, trade liberalization increases the distance between the markups of non-exporters and exporters, thereby potentially offsetting the effect of a reduction in z^*/z_x^* on σ_θ^2. Our benchmark simulations in Figure 2, show that markup dispersion declines with trade liberalization, implying that the effect of the reduction in z^*/z_x^* dominates the increase in $(1/\theta - 1/\theta_x)$. This results suggests that in our model trade liberalization reduces the inefficiency due to markup dispersion, thereby leading to substantial welfare gains from selection.

Endogenous growth. Finally, we compute the contribution of long-run growth, the second key original ingredient of our model, to direct and indirect welfare gains from trade. As shown in footnote 27, the compensating variation ω produced by changes in the iceberg cost τ can be expressed as

$$\frac{1+\beta}{\rho} \log(\omega) = U(\Omega_c) - U(\Omega).$$

Hence, from (16) and (26), we can express the overall (direct and indirect) contribution to welfare of changes in the growth rate as

$$\log(\omega) = \frac{\Delta_{g_{gdp}}}{\rho},$$

where $\Delta_{g_{gdp}}$ is the change in output growth produced by a unit reduction in trade costs. Using the results in Table 2, we find that about 61% of the total increase in welfare is due to growth, from which about $1/4^{th}$ is attributable to selection. Hence, although endogenous growth does not play the key role of markup dispersion in shaping the indirect welfare gains from trade, once these gains are positive, growth contributes to make them larger.

In conclusion, we have shown that endogenous markup dispersion, one main innovative feature of our model, is fundamental for selection to produce welfare gains from trade. The second original feature of our model, endogenous productivity growth, although not necessary for obtaining a positive indirect effect, turns out to be quantitatively relevant in an economy where trade reduces markup dispersion. In other words, once the presence of markup dispersion makes the indirect effect positive, the contribution of growth to the overall welfare gains from trade, and to the part due to selection, is quantitatively non-negligible.

7 Conclusion

In this paper, we built a rich and tractable model of trade with heterogeneous firms and innovation-driven productivity dynamics, in order to account for a set of empirical regularities on the effects of trade liberalization. In our framework, the competition channel is at the root of the selection and innovation effects of trade. Consistent with empirical evidence, trade liberalization increases product market competition, drives inefficient firms out the market, and forces surviving firms to innovate more. Endogenous markups are derived directly from oligopolistic competition among firms. The post-entry innovation activity of firms in the presence of knowledge spillovers generates endogenous growth in the economy.

Trade liberalization, by reducing the inefficiency due to markup dispersion, yields both direct and indirect (selection) welfare gains from trade. Endogenous growth implies that trade affects both the level and the growth rate of productivity, leading to static and dynamic welfare gains, both direct and indirect. In the absence of markup dispersion, trade-induced firm selection does not affect either welfare or growth, and the indirect channel disappears.

Calibrating the model to match US firm-level and aggregate statistics, we show that trade liberalization has first order growth and welfare effects. About $1/4^{th}$ of the growth and $2/5^{th}$ of the welfare gains can be attributed to firm selection, and that around 60 percent of the welfare gains are dynamic gains (due to the increase in the growth rate). Thus suggesting that firm-level static and dynamic decisions play an important role in shaping the aggregate response of our economy to trade liberalization.

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A Derivation of equation (13), the stationary growth rate, and the welfare function

Equation (13). Rearranging (10), we obtain $x_t = \tilde{z}_t^{\eta \frac{1}{1-\alpha}} \left(\theta E / X_t^{\alpha}\right)^{\frac{1}{1-\alpha}}$. Substituting it into (2) yields

$$X_t^{\alpha} = \left(\int_0^M \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j \right)^{1-\alpha} (\theta E)^{\alpha} ,$$

where $\hat{\eta} \equiv \eta\alpha/(1-\alpha)$. Using this into the expression for x above, we find

$$x_t^{\alpha} = (\theta E)^{\alpha} \, \tilde{z}_t^{\hat{\eta}} \left(\int_0^M \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j \right)^{-\alpha} .$$

Substituting these expressions for x and X into (10), considering that in a symmetric equilibrium $x = nq$, and using the definition of stationary productivity $z \equiv \tilde{z}_t^{\hat{\eta}} \, e^{-\hat{\eta} g t}$ we obtain (13).

Steady state growth. The stationary growth rate (14) is obtained differentiating (11) with respect to time, which yields $\dot{v}/v = \dot{k}/k = \dot{\tilde{z}}/\tilde{z}$, where the second equality is obtained using $k_t(\tilde{z}) = (\bar{z}/z)\tilde{z}_t$ in which by definition $\bar{z}$ and z are stationary. Plugging $\dot{v}/v = \dot{k}/k = \dot{\tilde{z}}/\tilde{z}$, (13), and $1/v = Ak$ from (11) into (12) we obtain (14).

B Equilibrium existence

Proof of proposition 1. Since M is decreasing in z^*, the (MC) locus is increasing starting at

$$\frac{\frac{(1+\delta)}{n} + \hat{\rho} - \lambda}{\beta + (1+\eta)\theta},$$

when $z^* = z_{min}$, and going to infinity when z^* goes to infinity. Under Assumption 1(a), the (EC) locus is decreasing, starting at

$$\frac{\lambda \frac{\bar{z}_e}{z_{\min}} - \hat{\rho}}{1 - (1+\eta)\theta},$$

for $z^* = z_{min}$, and going to $\big(\lambda - (\rho+\delta)/A\big)/\big(1-(1+\eta)\theta\big)$ when z^* goes to ∞. Assumption 1.b implies $\Psi < 1$ and substituting this into 1.c leads to $1+\eta < 1/\theta$, which guarantees that profits (17) are increasing in productivity z. Since $\Psi < 1$ it is easy to show that 1.c is a sufficient condition for the intercept of the (EC) curve be larger than the (MC) curve at $z^* = z_{min}$, which implies single-crossing of the two equilibrium conditions.

C Firm problem in open economy

Each firm solves the following problem

$$V_s = \max_{\left(q^D_{D,t}, q^F_{D,t}, z_{D,t}\right)_s^\infty} \int_s^\infty \left(\left(p_{D,t} - \frac{1}{\tilde{z}^\eta_{D,t}} \right) q^D_{D,t} + \left(p_{F,t} - \frac{\tau}{\tilde{z}^\eta_{D,t}} \right) q^F_{D,t} - h_{D,t} - \lambda \right) e^{-\int_s^T (r_z+\delta)\ dz}\ dt$$

$$\begin{aligned} & s.t. \\ & p_{D,t} = \frac{E_{D,t}}{X^\alpha_{D,t}}\ x^{\alpha-1}_{D,t} \qquad \textit{and} \quad p_{F,t} = \frac{E_{F,t}}{X^\alpha_{F,t}}\ x^{\alpha-1}_{F,t} \\ & x_{D,t} = \hat{x}^D_{D,t} + q^D_{D,t} + x^D_{F,t} \quad \textit{and} \quad x_{F,t} = \hat{x}^F_{D,t} + q^F_{D,t} + x^F_{F,t} \\ & \dot{\tilde{z}}_{D,t} = A\ \hat{z}_{D,t} h_{D,t} \\ & z_{D,s} > 0, \end{aligned}$$

where $p_{j,t}$, $E_{j,t}$ and $X_{j,t}$ are the domestic price, expenditure and total composite good respectively for country $j = D, F$, and q^j_i is the quantity sold from source country i to destination country j. Writing down the current-value Hamiltonian and solving it yields the following first order conditions

$$\left((\alpha - 1) \frac{q^D_{D,t}}{x_{D,t}} + 1 \right) p_{D,t} = \frac{1}{\tilde{z}^\eta_{D,t}} \tag{35}$$

$$\left((\alpha - 1) \frac{q^F_{D,t}}{x_{D,t}} + 1 \right) p_{F,t} = \frac{\tau}{\tilde{z}^\eta_{D,t}} \tag{36}$$

$$1 = v_{D,t} A \hat{z}_{D,t}, \tag{37}$$

$$\frac{\eta \tilde{z}^{-\eta-1}_{D,t}}{v_{D,t}} \left(q^D_{D,t} + \tau q^F_{D,t} \right) = \frac{-\dot{v}_{D,t}}{v_{D,t}} + r_t + \delta, \tag{38}$$

Since the two countries are symmetric, $q^D_{D,t} = q^F_{F,t} \equiv q_t$, $q^F_{D,t} = q^D_{F,t} = \breve{q}_t$, $x_{D,t} = x_{F,t} \equiv x_t$, $E_{D,t} = E_{F,t}$, $X_{D,t} = X_{F,t}$, $p_{D,t} = p_{F,t}$. From (35) and (36) and using $q_t/x_t +\ \breve{q}_t/x_t = 1/n$ yields

$$\left((\alpha - 1) \frac{q_t}{x_t} + 1 \right) = \frac{2n - 1 + \alpha}{n(1+\tau)} \equiv \theta_D \tag{39}$$

$$\left((\alpha - 1) \frac{\breve{q}_t}{x_t} + 1 \right) = \tau \frac{2n - 1 + \alpha}{n(1+\tau)} \equiv \theta_F = \tau \theta_D \tag{40}$$

which allows us to rewrite (35) and (36) as follows

$$\theta_D \frac{E_t}{X^\alpha_t}\ x^{\alpha-1}_t = \frac{1}{\tilde{z}^\eta_t} \text{ and } \tau \theta_D \frac{E_t}{X^\alpha_t}\ x^{\alpha-1}_t = \frac{\tau}{\tilde{z}^\eta_t}.$$

Multiplying the above equations by q_t and $\breve{q}_t$ and summing up we obtain

$$\frac{q_t + \tau \breve{q}_t}{\tilde{z}^\eta_t} = n \left(\theta_D \frac{q_t}{x_t} + \tau \theta_D \frac{\breve{q}_t}{x_t} \right) \frac{E_t}{n} \left(\frac{x_t}{X_t} \right)^\alpha.$$

Using $x_t = \{(1/\tilde{z}_t^{\eta})(X_t^{\alpha}/\theta_D E_t)\}^{\frac{1}{\alpha-1}}$, it is easy to prove that $(x_t/X_t)^{\alpha} = z/\bar{z}$. From (39) and using $q_t/x_t + \breve{q}_t/x_t = 1/n$ we obtain

$$\frac{q_t + \tau \breve{q}_t}{\tilde{z}_t^{\eta}} = \theta_x e_t \frac{z}{\bar{z}} \tag{41}$$

where

$$\theta_x = \frac{2n - 1 + \alpha}{n(1+\tau)^2(1-\alpha)} \left(\tau^2(1 - n - \alpha) + n(2\tau - 1) + 1 - \alpha\right)$$

is the inverse of the markup in the open economy.

D Exit in open economy

The productivity cutoff is determined solving the following equation

$$\pi_t(\tilde{z}^*) = \left(p_t - \frac{1}{\tilde{z}_t^{*\eta}}\right) q_t + \left(p_t - \frac{\tau}{\tilde{z}_t^{*\eta}}\right) \breve{q}_t - h_t - \lambda = 0$$

Using $p_t = 1/\theta_D \tilde{z}_{D,t}^{\eta}$ and $h_t = \eta \theta_x e_t \tilde{z}_t - \hat{\rho}$ obtained from (37) and (38) yields

$$\frac{1}{\theta_D} \frac{q_t + \breve{q}_t}{\tilde{z}_t^{*\eta}} - \left(\frac{q_t + \tau \breve{q}_t}{\tilde{z}_t^{*\eta}}\right)(1 + \eta) + \hat{\rho} - \lambda = 0.$$

With the same procedure used to derive (41) we obtain

$$\frac{q_t + \breve{q}_t}{\tilde{z}_t^{*\eta}} = \theta_D e_t \, z_t/\bar{z}_t$$

which, together with (41), yields

$$(1 - (1 + \eta)\theta_x) e_t z_t^*/\bar{z}_t + \hat{\rho} - \lambda = 0.$$

This expression is similar to (EC) except for the markup $1/\theta_x$ in the place of $1/\theta$.

E Welfare and the pro-competitive effect

Pro-competitive effect. Differentiating θ_x with respect to τ

$$\frac{\partial \theta_x}{\partial \tau} = -\frac{2(\tau - 1)(2n - 1 + \alpha)^2}{n(1+\tau)^3(1-\alpha)} \leq 0,$$

thus trade liberalization reduces the markup. Moreover, taking the absolute value of this derivative and differentiating it with respect to n we find

$$\frac{\partial\left(\left|\partial \theta^T/\partial \tau\right|\right)}{\partial n} = \frac{2(\tau - 1)(2n - 1 + \alpha)}{n^2(1+\tau)^3} > 0,$$

which implies that the competition effect of incremental trade liberalization is decreasing in the number of firms n.

Welfare equation. In the open economy, optimal quantities are

$$q_t = \left(1 - \frac{1}{\bar{z}_t^{\eta} p_t}\right) \frac{x_t}{1-\alpha}$$
$$\breve{q}_t = \left(1 - \frac{\tau}{\bar{z}_t^{\eta} p_t}\right) \frac{x_t}{1-\alpha}.$$

Since total consumption $x_t = n(q_t + \breve{q}_t)$, multiplying both sides of both equations by n, adding up right and left hand side terms and substituting p_t by the inverse demand function, we get

$$x_t^{1-\alpha} = \breve{\theta} \frac{E}{X_t^{\alpha}} \bar{z}_t^{\eta},$$

where $\breve{\theta}$,

$$\breve{\theta} = \frac{2n + \alpha - 1}{n(1+\tau)},$$

measures the aggregate distortions of output reductions due to both the use of resources to cover the trade costs and the inefficiency of increased markups –the anti-competitive effect of larger trade barriers. Under the assumption that τ is smaller than the prohibitive tariff $\tau_{\max} = n/(n + \alpha - 1)$, it can be easily proved that $\breve{\theta} < \theta_x$. It does reflect the fact that consumption is smaller than firms' production because of the trade cost. Since under free trade the trade cost is nil by assumption, $\breve{\theta} = \frac{2n+\alpha-1}{2n} = \theta_x$. In the other extreme, at the prohibitive tariff, since there is no trade, $\breve{\theta} = \frac{n+\alpha-1}{n} = \theta$.

Substituting x_t in the definition of composite consumption X_t and using the definition of average productivity $\bar{z}$, we get

$$X_t = (M\bar{z})^{\frac{1-\alpha}{\alpha}} \breve{\theta} E \; e^{\eta g t}.$$

The solution for X_t can now be substituted into the discounted utility (1). Using (6) yields the steady state welfare function

$$\begin{aligned} \rho U &= \rho \int_0^{\infty} (\ln X_t + \beta \ln Y_t) \; \mathrm{e}^{-\rho t} \; \mathrm{d}t \\ &= \frac{1-\alpha}{\alpha} \ln(\bar{z}M) + \ln(\breve{\theta} n M e) + \beta \ln(\beta n M e) + \frac{\eta g}{\rho}. \end{aligned}$$

The indirect welfare gains of a reduction on trade costs result from deriving (26) with respect to z^*, after substitution of E by nMe,

$$\texttt{IndirectEffect} = \underbrace{\frac{1-\alpha}{\alpha}\left(\frac{1}{\bar{z}}\frac{\partial \bar{z}}{\partial z^*} + \frac{1}{M}\frac{\partial M}{\partial z^*}\right)}_{\text{productivity}(>0)} + \underbrace{(1+\beta)\left(\frac{1}{e}\frac{\partial e}{\partial M} + \frac{1}{M}\right)\frac{\partial M}{\partial z^*}}_{\text{labor reallocation}(>0)} + \underbrace{\frac{\eta^2 A \theta_x}{\rho}\frac{\partial e}{\partial M}\frac{\partial M}{\partial z^*}}_{\text{growth}(>0)}.$$

Differentiating the definition of $\bar{z}$, we get

$$\frac{\partial \bar{z}}{\partial z^*} = (\bar{z} - z^*)\frac{f(z^*)}{1-\Gamma(z^*)} > 0.$$

Differentiating (19), we get

$$\frac{\partial M}{\partial z^*} = -\delta \frac{f(z^*)}{(1+\delta-\Gamma(z^*))^2} < 0.$$

Differentiating (MC^T), we get

$$\frac{\partial e}{\partial M} = -\frac{1}{(\beta + (1+\eta)\theta_x)nM^2} < 0.$$

Consequently,

$$\frac{\partial e}{\partial M}\frac{\partial M}{\partial z^*} = \frac{\delta}{(\beta + (1+\eta)\theta_x)n}\frac{f(z^*)}{(1-\Gamma(z^*))^2} > 0,$$

implying that the sign of the *growth effect* is strictly positive. Given that $\partial M/\partial z^* < 0$, the sign of the *reallocation effect* is the opposite of the sign of

$$\frac{1}{e}\frac{\partial e}{\partial M} + \frac{1}{M} = \frac{1}{M} - \frac{1}{(\beta+(1+\eta)\theta_x)nM^2 e} = \frac{1}{M}\frac{(\hat{\rho}-\lambda)nM}{(\hat{\rho}-\lambda)nM+1},$$

which is strictly negative since $\hat{\rho} < \lambda$ by Assumption 1 (b).

Finally, the *productivity effect*

$$\frac{1}{\bar{z}}\frac{\partial \bar{z}}{\partial z^*} + \frac{1}{M}\frac{\partial M}{\partial z^*} = \frac{(M - z^*/\bar{z})f(z^*)}{(1-\Gamma(z^*))(1+\delta-\Gamma(z^*))}$$

which is positive iff $M > z^*/\bar{z}$. From (19), $M > z^*/\bar{z}$ iff $\delta < \bar{\delta}$, where $\bar{\delta} \equiv (\bar{z}/z^* - 1)(1-\Gamma(z^*))$.

The productivity effect is strictly positive also if λ is small not enough: Let determine the equilibrium M in the case where the distribution is Pareto with tail parameter $\gamma > 1$. Then e is determined by the (EC^T) condition

$$e = \frac{\frac{\gamma}{\gamma-1}\lambda - \hat{\rho}}{1-(1+\eta)\,\theta_x}. \tag{EC^T}$$

Then, M is determined by

$$e = \frac{\frac{1}{nM} + \hat{\rho} - \lambda}{\beta + (1+\eta)\,\theta_x}. \tag{MC^T}$$

Substituting one condition in the other, after some algebra,

$$M = \frac{(1-(1+\eta)\theta_x)n}{\left(1 + \frac{\gamma\beta}{\gamma-1} + \frac{(1+\eta)\theta_x}{\gamma-1}\right)\lambda - (1+\beta)\hat{\rho}} \tag{42}$$

The condition $M > z^*/\bar{z} = \frac{\gamma-1}{\gamma}$ holds iff

$$\lambda < \frac{\frac{\gamma}{\gamma-1}(1-(1+\eta)\theta_x)n + (1+\beta)\hat{\rho}}{\left(1 + \frac{\gamma\beta}{\gamma-1} + \frac{(1+\eta)\theta_x}{\gamma-1}\right)} \equiv \bar{\lambda}. \tag{43}$$

F Derivations for the generalized model

Variable costs. We want to derive the variable costs for non-exporters $\tilde{z}^{-\eta}q$ and exporters $\tilde{z}^{-\eta}q_x$. The first order condition for non-exporters will be again (10), simply stating that their price, given by the inverse market demand, is equal to a markup θ over marginal costs $\tilde{z}^{-\eta}$

$$\tilde{z}_t^{-\eta} = \theta \frac{E}{X_t^{\alpha}} x_t^{\alpha-1} \tag{44}$$

The exporter will solve the same problem as in the open economy version for the benchmark model (Section 3), and face a price equal to a markup θ_x over marginal costs $\tilde{z}^{-\eta}$, that is

$$\tilde{z}_t^{-\eta} = \theta_x \frac{E}{X_t^{\alpha}} x_{xt}^{\alpha-1} \tag{45}$$

Rearranging we obtain $x_t^{\alpha} = \tilde{z}_t^{\hat{\eta}} \left(\theta E / X_t^{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$ and $x_{xt}^{\alpha} = \tilde{z}_t^{\hat{\eta}} \left(\theta E / X_t^{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$, where $\hat{\eta} \equiv \eta\alpha/(1-\alpha)$. Substituting these into (2) we obtain

$$X_t^{\alpha} = \left(\theta^{\frac{\alpha}{1-\alpha}} \int_0^{M_d} \tilde{z}_{jt}^{\hat{\eta}} \,\mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}} \int_0^{M_x} \tilde{z}_{jt}^{\hat{\eta}} \,\mathrm{d}j \right)^{1-\alpha} E^{\alpha}$$

where with a slight abuse of notation we temporarily define the mass of exporting firm M_x and the mass of non-exporters M_d. Substituting back into the expressions for x_t^{α} and x_{xt}^{α} yields

$$x_t^{\alpha} = \frac{\theta^{\frac{\alpha}{1-\alpha}} E^{\alpha} \tilde{z}_t^{\hat{\eta}}}{\left(\theta^{\frac{\alpha}{1-\alpha}} \int_0^{M_d} \tilde{z}_{jt}^{\hat{\eta}} \,\mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}} \int_0^{M_x} \tilde{z}_{jt}^{\hat{\eta}} \,\mathrm{d}j \right)^{\alpha}} \text{ and } x_t^{\alpha} = \frac{\theta_x^{\frac{\alpha}{1-\alpha}} E^{\alpha} \tilde{z}_t^{\hat{\eta}}}{\left(\theta^{\frac{\alpha}{1-\alpha}} \int_0^{M_d} \tilde{z}_{jt}^{\hat{\eta}} \,\mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}} \int_0^{M_x} \tilde{z}_{jt}^{\hat{\eta}} \,\mathrm{d}j \right)^{\alpha}}.$$

Plugging these into (44) and (45) and using $z \equiv \tilde{z}_t^{\hat{\eta}} \, e^{-gt}$ and the symmetric equilibrium condition $x = nq$ we obtain (28) and (29).

Markup dispersion. Under the assumption of Pareto productivity distribution adopted in the quantitative analysis, the variance of markups is

$$\begin{aligned}
\sigma_\theta^2 &= \int_{z^*}^{z_x^*} \left(\frac{1}{\theta} - \bar{\mu}\right)^2 \mu(z)dz + \int_{z_x^*}^{\infty} \left(\frac{1}{\theta_x} - \bar{\mu}\right)^2 \mu(z)dz \\
&= \left(\frac{1}{\theta} - \bar{\mu}\right)^2 \left[1 - \left(\frac{z^*}{z_x^*}\right)^{\kappa}\right] + \left(\frac{1}{\theta_x} - \bar{\mu}\right)^2 \left(\frac{z^*}{z_x^*}\right)^{\kappa}
\end{aligned}$$

and the average markup is

$$\bar{\mu} = \int_{z^*}^{z_x^*} \frac{1}{\theta} \mu(z)dz + \int_{z_x^*}^{\infty} \frac{1}{\theta_x} \mu(z)dz = \frac{1}{\theta}\left[1 - \left(\frac{z^*}{z_x^*}\right)^{\kappa}\right] + \frac{1}{\theta_x}\left(\frac{z^*}{z_x^*}\right)^{\kappa}$$

which substituted into σ_θ^2 gives the expression in the text.

G Calibration

Here we derive the moments used in the internal calibration of parameters α, η, A, λ, λ_x, κ, and ϕ. Since the model assumes no productivity growth in the homogeneous good sector, the overall growth rate of labor productivity that we match to the data is

$$\bar{g} = \frac{1}{(1+\beta)}\eta g_\tau,$$

were g_τ is the growth rate of the extended model derived in (31) and η comes from (3). The average R&D/GDP is

$$\bar{h} = \int_{z^*}^{z_x^*} h(z)\mu(z)dz + \int_{z_x^*}^{\infty} h_x(z)\mu(z)dz = \frac{\bar{\theta}}{\theta_x}\left(\eta\theta_x e - \left(\frac{\rho+\delta}{A}\right)\right)nM,$$

where $h(z)$ and $h_x(z)$ are taken from (32) and (33), the equilibrium productivity density under Pareto distribution is $\mu(z) = \kappa z^{*\kappa} z\kappa^{-\kappa-1}$, and the national income is pinned down by the size of population normalized to one. The standard deviation of the productivity distribution is

$$std(z) = \frac{z^*}{\kappa-1}\left(\frac{\kappa}{\kappa-2}\right)^{1/2},$$

and the share of exporters is

$$expshare = \frac{1-\Gamma(z_x^*)}{1-\Gamma(z^*)} = \left(\frac{z^*}{z_x^*}\right)^{\kappa}.$$

The average markup is

$$\bar{\mu} = \int_{z^*}^{z_x^*} \frac{1}{\theta}\mu(z)dz + \int_{z_x^*}^{\infty} \frac{1}{\theta_x}\mu(z)dz = \frac{1}{\theta}\left(1-\left(\frac{z^*}{z_x^*}\right)^{\kappa}\right) + \frac{1}{\theta_x}\left(\frac{z^*}{z_x^*}\right)^{\kappa}.$$

Finally the productivity advantage of exporters is computed as the percentage difference between the average productivity of exporters $\bar{z}_x = \frac{1-F(z_x^*)}{1-F(z^*)}\int_{z_x^*}^{\infty} z\mu(z)dz$ and the average productivity of non-exporters $\bar{z} = \int_{z^*}^{z_x^*} z\mu(z)dz$, yielding

$$expremium = \left(\frac{z^*}{z_x^*}\right) - \left(\frac{z^*}{z_x^*}\right)^{\kappa}.$$

G.1 Export share

Full model. Notice that in equation (29) we have

$$\tilde{z}_t^{-\eta} q_{x,t} = \theta_x e\left(\frac{\bar{p}}{p_x(z)}\right)^{\frac{\alpha}{1-\alpha}}$$

where $q_x = q + \tau\breve{q}$ is the sum of the quantity sold domestically q and the quantity exported $\tau\breve{q}$. In order to write down exporters' sales we need to find the quantity they sell on the export market. We follow the procedure used in the appendix to derive the total variable cost of exporters.

From exporters FOC

$$\tau\theta_D \frac{E_t}{X_t^\alpha}\ x_t^{\alpha-1} = \frac{\tau}{\tilde{z}_t^\eta},$$

and multiplying both sides by $\breve{q}$ we get

$$\frac{\tau\breve{q}}{\tilde{z}_t^\eta} = E_t\tau\theta_D \frac{\breve{q}}{x_t}\frac{x_t^\alpha}{X_t^\alpha}$$

where total quantity sold on a market $x/n = q + \breve{q}$.

Using the price of export $p_x\tau\breve{q} = \frac{1}{\theta_x \tilde{z}_t^\eta}$ and (44) we can compute export sales per firm

$$p_x\tau\breve{q} = \frac{1}{\theta_x \tilde{z}_t^\eta}\tau\breve{q} = E_t\tau\frac{\theta_D}{\theta_x}\frac{\breve{q}}{x_t}\frac{x_t^\alpha}{X_t^\alpha}$$

where $\theta_D = \frac{2n-1+\alpha}{n(1+\tau)}$ is derived in the appendix. In the appendix we can also see that $\frac{\breve{q}}{x_t} = \frac{\theta_D - 1}{\alpha-1}$ and $\frac{x_t^\alpha}{X_t^\alpha} = z\bar{p}^{\frac{\alpha}{1-\alpha}}$ (notice that in the appendix we have $\bar{z}$ in the place of $\bar{p}$ because we focus on the baseline model). Hence the sales are

$$p_x\tau\breve{q} = \frac{1}{\theta_x \tilde{z}_t^\eta}\tau\breve{q} = ne\tau\frac{\theta_D}{\theta_x}\frac{\theta_D - 1}{\alpha - 1}z\bar{p}^{\frac{\alpha}{1-\alpha}}. \tag{46}$$

Assuming Pareto we can compute the value of average of export/import

$$avgex = \int_{z_x^*}^{\infty} p_x(z)\tau\breve{q}(z)\frac{f(z)}{1-F(z_x^*)}dz = ne\tau\frac{\theta_D}{\theta_x}\frac{\theta_D - 1}{\alpha - 1}\bar{p}^{\frac{\alpha}{1-\alpha}}\kappa z_x^{*\kappa}\int_{z_x^*}^{\infty} z^{-\kappa}dz =$$
$$= ne\tau\frac{\theta_D}{\theta_x}\frac{\theta_D - 1}{\alpha - 1}\bar{p}^{\frac{\alpha}{1-\alpha}}\frac{\kappa}{k-1}z_x^*$$

Total export is equal to export of the average firm times the probability of exporting, times the number of firms

$$EXP = nM\left(\frac{z_d^*}{z_x^*}\right)^\kappa avgex = n^2Me\tau\frac{\theta_D}{\theta_x}\frac{\theta_D - 1}{\alpha - 1}\bar{p}^{\frac{\alpha}{1-\alpha}}\frac{\kappa}{k-1}z_d^{*\kappa}z_x^{*1-\kappa},$$

this is also the export share since the size of the economy is total labor set to 1.

Representative firm model. The representative firm model can be obtained from our baseline model where all firms export by assuming away firm heterogeneity and using free entry to endogenize n. The representative firm model has two equilibrium conditions, the market clearing and the free entry, and two unknowns $(n,\, e)$:

$$e = \frac{\frac{1}{nM(z^*)} + \frac{\rho+\delta}{A} - \lambda}{\beta + (1+\eta)\,\theta_x}$$

where $M = 1/(1+\delta)$, and the free entry is

$$\pi = e\left[1 - (1+\eta)\,\theta_x\right] + \frac{\rho+\delta}{A} - (\lambda + \lambda_x) = \frac{\phi}{\rho+\delta},$$

we could even shut down the destruction rate δ because it doesn't mean much here. We could have M exogenous and calibrate to the same M of the full model. We are going to normalize the productivity of the representative firm to one ($z = 1$).

The import penetration ratio can be determined using (46)

$$p_x \tau \breve{q} = \frac{1}{\theta_x \tilde{z}_t^{\eta}} \tau \breve{q} = ne\tau \frac{\theta_D}{\theta_x} \frac{\theta_D - 1}{\alpha - 1} z \bar{p}^{\frac{\alpha}{1-\alpha}}.$$

where $z = 1$ and $\bar{p}^{\frac{\alpha}{1-\alpha}} = \theta_x^{\frac{\alpha}{\alpha-1}}$ leading to

$$p_x \tau \breve{q} = \frac{1}{\theta_x \tilde{z}_t^{\eta}} \tau \breve{q} = ne\tau \frac{\theta_D}{\theta_x^{\frac{1}{1-\alpha}}} \frac{\theta_D - 1}{\alpha - 1}$$

and the total exporters sales is

$$EXP = n^2 Me\tau \frac{\theta_D}{\theta_x^{\frac{1}{1-\alpha}}} \frac{\theta_D - 1}{\alpha - 1}.$$