No-Envy and Equality of Economic Opportunity

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Abstract

Equality of economic opportunity based on the notion of envy-freeness in the context of opportunity sets in the k-dimensional real space is examined in this paper. We first examine the issue of equality of economic opportunity for a setting in which each agent's opportunity sets are linear budget sets. We show that, given certain properties, equality of economic opportunity is equivalent to requiring that all agents' budget sets have the same volume. The same issue is then examined for the case of exchange economies and economies with production. We show that equality of economic opportunity in an exchange economy is equivalent to equalizing money income for all agents, whereas in an economy with production, equality of economic opportunity is equivalent to all agents having the same full income. In both economies, the equality of economic opportunity is also equivalent to all agents having identical budget sets. We also discuss the compatibility of equality of economic opportunity and Pareto efficiency.

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1 Introduction

The purpose of this paper is to formulate and study a notion of equality of economic opportunity and apply it to exchange economies and economies with production, both with private goods.

The concept of equal opportunities has been widely used in ordinary life[1] and has also figured prominently in academic debates (see, for example, Arneson (1989), Cohen (1993), Dworkin (1981), Klappholz (1972), and Sen (1980) for different notions of equality or equality of opportunities). One notion of equal opportunities is to start with the presumption that all agents face identical choice sets (see, for example, Archibald and Donaldson (1979), Kolm (1973) and Thomson and Varian (1985)). However, there is no *a prior* reason why we should start with 'equality of opportunity as identical choice sets'. In this paper, we use the notion of *envy-free* opportunities to develop a theory of equality of opportunity[2] and show that equality of opportunity as identical choice sets is merely a consequence in some settings. Therefore, we provide a theoretical justification for equality of opportunity as identical choice sets.

The notion of envy-free allocations has been extensively discussed in economics (see, for example, Feldman and Kirman (1974), Foley (1967), Goldman and Sussangkarn (1978), Pazner (1977), Pazner and Schmeidler (1974, 1978), Thomson and Varian (1985), and Varian (1974, 1976)). The same concept, equity as no-envy, has also been discussed in social choice theory (see, for example, Suzumura (1981) and Tadenuma (1998)). There has been some attempts in the literature to formalize the idea of no-envy opportunities. The most notable ones are Varian (1976) and Thomson (1994). However, both of them formulated the notion of no-envy opportunities in terms of allocations and the notion of equal opportunities in their framework is merely *an instrument* to achieve an envy-free allocation. In this paper, we use the recent development in ranking opportunity sets (see, for example, Arrow (1995), Bossert, Pattanaik and Xu (1994), Gravel (1994), Pattanaik and Xu (1990, 1997, 1998), Puppe (1996), Sen (1991, 1992), Sugden (1998), and Suppes (1987), for discussions of ranking finite opportunity sets, and Pattanaik and Xu (1999) and Xu (1999) for discussions of ranking opportunity sets in economic environments) to formulate the concept of

[1] We have just to recall that in most companies, government agencies, or educational units, there is the *Office of Equal Opportunities.*

[2] See Kranich (1996, 1998) for a different approach to the issue of equality of opportunities.

envy-free opportunities directly. We assume that each agent is endowed with a class of opportunity sets and there is a ranking over these opportunity sets for each agent. In such a framework, therefore, it makes sense for the agent to say that one opportunity set offers more opportunities than another. At any given time, each agent in the society has an opportunity set from which he/she can make a choice. A profile of opportunity sets specifies one opportunity set for each individual in the society and is said to be envy-free if no agent envies other agent's opportunity sets.

With our notion of envy-free opportunities, we confine our attention to equality of economic opportunities in the sense that we propose a framework in terms of claims upon commodities and services rather than in terms of any index of utility or welfare. As a consequence, we work in the k-dimensional real space which is to be interpreted as the space of commodities and services. We first show that if the class of opportunity sets for each agent is confined to that of linear budget sets and if there are no interactions among the agents, then, under certain conditions, equality of economic opportunity requires that all agents have budgets sets with equal volumes. We then extend our analysis to exchange economies and economies with production, both with private goods. We show that in such economies, equality of economic opportunity is equivalent to requiring that all agents have the same money income in an exchange economy and all agents have the same full (or potential) income in a productive economy. We also show that a type of Walrasian mechanism guarantees both equality of economic opportunity and Pareto efficiency. The spirit of such mechanisms is to divide the aggregated social endowments of goods and services equally among all individuals and then to let them trade.

The remainder of the paper is organized as follows. In Section 2, we present some preliminary notation and definitions. Section 3 develops a theory in a setting without considering interactions among the agents in the society. Section 4 considers equality of economic opportunity in exchange economies with private goods, while Section 5 considers the same issue in economies with production. We conclude in Section 6.

2 The Basic Notation and Definitions

There are k commodities. Let $\mathbb{R}^k_+$ be the non-negative orthant of the k-dimensional real space. The points in $\mathbb{R}^k_+$ will be denoted by $x, y, z, a, b, \cdots$ and will be called commodity bundles. There are n agents and they are indexed by the set $N = \{1, \cdots, n\}$.

At any given point of time, the set of all commodity bundles that may be available to an agent i is a subset of $\mathbb{R}^k_+$. Such a set will be called agent i's *opportunity set*. We will use A_i, B_i, C_i etc. to denote the opportunity sets for agent i.

In this paper, we confine our attention to opportunity sets that are *linear budget sets*. Let $K =: \{\{x \in \mathbb{R}^n_+ | \sum_{l=1}^k x_l/a_l \leq 1\} | a_l \in \mathbb{R}_+, 0 < a_l < \infty, l = 1, \cdots, k\}$. For all $A \in K$, let vol(A) denote the volume of A. Given that A is a linear budget set, if $A = \{x \in \mathbb{R}^k_+ | \sum_{l=1}^k x_l/a_l \leq 1\} \in K$, then, vol$(A) = a_1 a_2 \cdots a_k/k!$. For all $i \in N$, let $\succeq_i$ be a reflexive, transitive and complete binary relation on K. The symmetric and asymmetric parts of $\succeq_i$ are denoted by $\sim_i$ and $\succ_i$, respectively. The intended meaning of $\succeq_i$ is the following: for all $A, B \in K, [A \succeq_i B]$ is to be interpreted as "agent i regards the opportunity set A is at least as good as the opportunity set B."

Let K^n be the n-fold Cartesian product of K. A profile of opportunity sets is an n-tuple $\mathbf{A} = (A_1, \cdots, A_n) \in K^n$. For all $\mathbf{A} = (A_1, \cdots, A_n) \in K^n$, we say that i *envies* j's opportunities iff $A_j \succ_i A_i$. A profile of opportunity sets $\mathbf{A} = (A_1, \cdots, A_n)$ is said to be *envy-free* iff no agent envies other agent's opportunities, i.e., $A_i \succeq_i A_j$ for all $i, j \in N$. Since the profile that all agents have identical opportunity sets is envy-free, the concept is not vacuous. From time to time, when $\mathbf{A}$ is envy-free, we also say that it is equitable in terms of economic opportunities.

3 Properties of Envy-Free Opportunities and Their Implications

Let Π be the set of all *permutations* of $\{1, \cdots, k\}$. Elements of Π will be denoted by π, π', etc. For all $A \in K$ and for all $\pi \in \Pi$, let $\pi(A) =: \{x \in \mathbb{R}^k_+ | x_l = y_{\pi(l)}, \exists y \in A\}$. For all $A, B \in K$, if $A = \pi(B)$ for some $\pi \in \Pi$, then A and B are said to be *symmetric*. For example, the sets $\{x \in \mathbb{R}^k_+ | x_1/a_1 + x_2/a_2 + x_3 + \cdots + x_k \leq 1\}$ and $\{x \in \mathbb{R}^k_+ | x_1/a_2 + x_2/a_1 + x_3 + \cdots + x_k \leq 1\}$, where $a_1 > 0$ and $a_2 > 0$, are symmetric. For all $A = \{x \in \mathbb{R}^k_+ | \sum_{l=1}^n x_l/a_l \leq 1\} \in K$, all $l \in \{1, \cdots, k\}$ and all $\alpha_l > 0$, let $\alpha_l(A) =: \{x \in \mathbb{R}^k_+ | \sum_{j=1}^k x_j/b_j \leq 1$ where $b_j = a_j$ for $j \neq l$ and $b_l = a_l/\alpha_l\}$.

We impose the following properties on each and every $\succeq_i$ $(i = 1, \cdots, n)$ over K (see Xu (1999)).

Symmetry: For all A_i, B_i and $C_i \in K$, if A_i and B_i are symmetric, then $A_i \succeq_i C_i$ if and only if $B_i \succeq_i C_i$.

Monotonicity: For all $A_i, B_i \in K$, if B_i is a proper subset of A_i, then $A_i \succ_i B_i$.

Invariance of Scaling Effects: For all $A_i, B_i \in K$, all $l = 1, \cdots, k$, and all $\alpha_l > 0$, $A_i \succeq_i B_i$ if and only if $\alpha_l(A_i) \succeq_i \alpha_l(B_i)$.

The property of Symmetry is simple and easy to explain. It essentially requires that, when evaluating opportunity sets in terms of opportunities offered, two symmetric budget sets A and B be treated similarly. It thus suggests that there is no *discrimination* among different commodities in ranking budget sets in terms of opportunities.

Monotonicity is intuitive. It requires that if B is a proper subset of A, then A offers more opportunities than B. It reflects that each agent is not averse to more opportunities.

The property of Invariance of Scaling Effects requires that, by re-scaling the unit of any commodity, say commodity l, the relative opportunities offered by the two opportunity sets A and B should exactly correspond to those before re-scaling: if A is ranked at least as high as B before rescaling, then A should be ranked at least as high as B after re-scaling and *vice versa*. To explain the intuition involved in the property, let us consider a simple economy where there are two commodities: bread and milk. The unit of measuring bread is *loaf* and the current practice of measuring milk is *gallon* as a unit and consider two budget sets for the agent: $A = \{x \in \mathbb{R}^2_+ | x_1/2 + x_2 \leq 1\}$ and $B = \{x \in \mathbb{R}^2_+ | x_1 + x_2/2 \leq 1\}$ where x_1 is for bread measured in loaves and x_2 is for milk measured in gallons. Now, suppose that the unit of measurement for milk will be *quart*, while the unit of measurement for bread will remain the same. Suppose that the agent's situation will remain unchanged and other aspects of the economy will remain unchanged. As a consequence, the budget sets A and B will become $A' = \{y \in \mathbb{R}^2_+ | y_1/2 + y_2/4 \leq 1\}$ and $B' = \{y \in \mathbb{R}^2_+ | y_1 + y_2/8 \leq 1\}$, respectively, where y_1 is for bread measured in loaves and y_2 is for milk measured in quarts (recall that 1 gallon = 4 quarts). Given the agent's situation, therefore, there is every reason to believe that the opportunity sets A and B should be ranked exactly the same as the opportunity sets A' and B'.

With the help of these properties, we are ready to examine the nature of envy-free profiles of opportunity sets. Our first result in this section gives a characterization of envy-free profiles of opportunity sets.

Theorem 3.1. Suppose for all $i \in N$, $\succeq_i$ over K satisfies Symmetry, Monotonicity and Invariance of Scaling Effects. Then, for all $\mathbf{A} \in K^n$, $\mathbf{A}$ is envy-free if and only if vol(A_i) =vol(A_j) for all $i, j \in N$.

Proof. For all $i \in N$, let $\succeq_i$ over K satisfy Symmetry, Monotonicity and Invariance of Scaling Effects. Then, as shown in Xu (1999), we have the following:

(3.1) for all $i \in N$, for all $A_i, B_i \in K$, $A_i \succeq_i B_i$ if and only if vol$(A_i) \geq$ vol(B_i).

Now, Let $\mathbf{A} = (A_1, \cdots, A_n) \in K^n$. If $\mathbf{A}$ is envy-free, then for all $i, j \in N$, $A_i \succeq_i A_j$. By (3.1), it then follows that vol$(A_i) \geq$ vol(A_j) for all $i, j \in N$. Therefore, vol(A_i) =vol(A_j) for all $i, j \in N$. Conversely, if vol(A_i) =vol(A_j) for all $i, j \in N$, then, from (3.1), $A_i \sim_i A_j$ for all $i, j \in N$. Therefore, $\mathbf{A}$ is envy-free. ■

In the following result, the nature of profiles of opportunity sets that are not envy-free is clarified.

Theorem 3.2. Suppose for all $i \in N$, $\succeq_i$ over K satisfies Symmetry, Monotonicity and Invariance of Scaling Effects. Then, for all $\mathbf{A} \in K^n$, if $\mathbf{A}$ is not envy-free, then there exists $i_w \in N$ such that $[A_j \succeq_j A_{i_w}$ for all $j \in N$ and $A_i \succ_i A_{i_w}$ for at least one $i \in N]$, and there exists $i_b \in N$ such that $[A_{i_b} \succeq_j A_j$ for all $j \in N$ and $A_{i_b} \succ_i A_i$ for at least one $i \in N]$.

Proof. For all $i \in N$, let $\succeq_i$ over K satisfy Symmetry, Monotonicity and Invariance of Scaling Effects. Let $\mathbf{A} \in K^n$ be such that agent i envies agent j's opportunity, that is, $A_j \succ_i A_i$. From (3.1), vol$(A_j) >$ vol(A_i). Let A_{i_w} be such that vol$(A_{i_w}) \leq$ vol(A_k) for all $k \in N$ and A_{i_b} be such that vol$(A_{i_b}) \geq$ vol(A_k) for all $k \in N$. Clearly, given the definition of vol$(\cdot)$, both A_{i_w} and A_{i_b} are well-defined. Further, we have vol$(A_{i_w}) <$ vol(A_j) and vol$(A_{i_b}) >$ vol(A_i). From (3.1), for all $k \in N$, $[A_k \succeq_k A_{i_w}$ and $A_j \succ_j A_{i_w}]$, and $[A_{i_b} \succeq_k A_k$ and $A_{i_b} \succ_i A_i]$. Therefore, Theorem 3.2 is proved. ■

The messages of Theorems 3.1 and 3.2 are clear. Theorem 3.1 suggests that equality of economic opportunity in the current setting is equivalent to requiring all agents have equal-volume opportunity sets. Theorem 3.2, on the other hand, suggests that there is a natural way of placing agents from the worst-off to the best-off in terms of opportunities: the worst-off agent in terms of opportunities is the one that no agent envies his/her opportunities and the best-off agent is the one that envies no other agent's opportunities. Therefore, a procedure leading to an envy-free profile of opportunity sets is to make the worst-off agents to be envied by no agent.

4 Exchange Economies

The analysis so far has ignored the joint feasibility of the choices of all agents. In economic environments, however, one agent's opportunity set is often constrained by other agents' opportunity sets. In this section, therefore, we consider the issue of equality of economic opportunity based on the notion of no-envy in a pure exchange economy. We show that, under a mild condition, equality of economic opportunity is equivalent to equalizing money income for all agents in the economy.

For all $i \in N$, let $\omega^i \in \mathbb{R}^k_+$ be agent i's initial endowment. We assume that for all $i \in N$, $\omega^i \neq 0$. For all $i \in N$, we use $x^i \in \mathbb{R}^k_+$ to denote agent i's consumption bundle. Let $p = (p_1, \cdots, p_k) \in \mathbb{R}^k_+$ be the price vector. To simplify our analysis, we assume that $p_l > 0$ for all $l = 1, \cdots, k$. Since in a pure exchange economy, all agents face the same price vector, for a given price vector $p = (p_1, \cdots, p_k)$ and a given initial endowment $\omega^i \in \mathbb{R}^k_+$, each agent i's opportunity set can then be written as: $O^e_i(p, \omega^i) = \{x \in \mathbb{R}^k_+ | p \cdot (x^i)' \leq p \cdot (\omega^i)'\}$ where $(x^i)'$ and $(\omega^i)'$ are the transposes of vectors x^i and ω^i, respectively (from now on, for any vector $a \in \mathbb{R}^k_+$, we use a' to denote its transpose).

Given the agents' initial endowments $\omega = (\omega^1, \cdots, \omega^n)$, and given a price vector, let $\mathbf{O}^e(p, \omega) = (O^e_1(p, \omega^1), \cdots, O^e_n(p, \omega^n))$ be the profile of the opportunity sets for the economy, where $O^e_i(p, \omega^i) = \{x \in \mathbb{R}^k_+ | p \cdot (x^i)' \leq p \cdot (\omega^i)'\}$ $(i \in N)$.

Theorem 4.1. Let $\omega = (\omega^1, \cdots, \omega^n)$ be the initial endowment and $p \in \mathbb{R}^k_+$ be the price vector. Suppose for all $i \in N$, $\succeq_i$ satisfies Monotonicity. Then, for all $\mathbf{O}^e(p, \omega) = (O^e_1(p, \omega^1), \cdots, O^e_n(p, \omega^n))$, $\mathbf{O}^e(p, \omega)$ is envy-free if and only if $p \cdot (\omega^i)' = p \cdot (\omega^j)'$ for all $i, j \in N$.

Proof. Let $\omega = (\omega^1, \cdots, \omega^n)$ be the initial endowment and $p \in \mathbb{R}^k_+$ be the price vector. For all $i \in N$, let $\succeq_i$ satisfy Monotonicity. Note that for all $i \in N$, $O^e_i(p, \omega^i) = \{x \in \mathbb{R}^k_+ | p \cdot (x^i)' \leq p \cdot (\omega^i)'\}$. Clearly, we have the following:

(4.1) for all $i, j \in N$, $O^e_i(p, \omega^i) \subseteq O^e_j(p, \omega^j)$ or $O^e_j(p, \omega^j) \subseteq O^e_i(p, \omega^i)$

Let $\mathbf{O}^e(p, \omega) = (O^e_1(p, \omega^1), \cdots, O^e_n(p, \omega^n))$. If $\mathbf{O}^e(p, \omega)$ is envy-free, then $O^e_i(p, \omega^i) \succeq_i O^e_j(p, \omega^j)$ for all $i, j \in N$. If $p \cdot (\omega^i)' \neq p \cdot (\omega^j)'$, or $O^e_i(p, \omega^i) \neq O^e_j(p, \omega^j)$, then, from

(4.1), we must have either $O_i^e(p,\omega^i)$ is a proper subset of $O_j^e(p,\omega^j)$, or $O_j^e(p,\omega^j)$ is a proper subset of $O_i^e(p,\omega^i)$. In the former, agent i envies agent j, and in the latter, agent j envies agent i. Both are in contradiction with the fact that the profile $\mathbf{O}^e(p,\omega)$ is envy-free. Therefore, $p\cdot(\omega^i)' = p\cdot(\omega^j)'$ for all $i,j \in N$. Conversely, if $p\cdot(\omega^i)' = p\cdot(\omega^j)'$ for all $i,j \in N$, then, we must have $O_i^e(p,\omega^i) = O_j^e(p,\omega^j)$ for all $i,j \in N$. It then follows that $O_i^e(p,\omega^i) \sim_i O_j^e(p,\omega^j)$ for all $i,j \in N$. That is, the profile $\mathbf{O}^e(p,\omega)$ is envy-free. ■

Therefore, according to Theorem 4.1, equality of economic opportunity is to make all agents having the same money income, or to make all agents having identical opportunity set. This is exactly the starting point for Kolm (1973) and Archibald and Donaldson (1979) where they require that agents be equal "if and only if they face identical choice set." In other words, Theorem 4.1 provides a justification for the concept of equality put forward by Kolm, and Archibald and Donaldson.

We now discuss the issue of the compatibility of equality of economic opportunity and Pareto efficiency. For all $i \in N$, let R_i be agent i's preference ordering over $\mathbb{R}_+^k$. For any exchange economy, we define a *Walrasian Equilibrium Procedure* as a triple $(\mathbf{O}^e(p,\omega),p,x)$ that specifies a collection of opportunity sets $\mathbf{O}^e(p,\omega) = (O_1^e(p,\omega^1,\cdots,O_n^e(p,\omega^n))$, an equilibrium price vector $p \in \mathbb{R}_+^k$ and an equilibrium allocation $x = (x^1,\cdots,x^n)$, where $O_i^e(p,\omega^i)$ is agent i's budget set corresponding to the initial endowment ω^i and the equilibrium price vector p, and $x^i \in \mathbb{R}_+^k$.

<u>Theorem 4.2.</u> Suppose that for all $i \in N$, $\succeq_i$ satisfies Monotonicity and R_i is nonsatiated. If $(\mathbf{O}^e(p,\omega),p,x)$ is a Walrasian Equilibrium Procedure with $p\cdot(\omega^i)' = p\cdot(\omega^j)'$ for all $i,j \in N$, then $\mathbf{O}^e(p,\omega)$ is envy-free and x is Pareto efficient.

Proof. First, Pareto efficiency of x follows from the standard argument. To see that $\mathbf{O}^e(p,\omega)$ is envy-free, we note that $p\cdot(\omega^i)' = p\cdot(\omega^j)'$ holds for all $i,j \in N$. Then, from Theorem 4.1, it follows easily that $\mathbf{O}^e(p,\omega)$ is envy-free. ■

This theorem shows that, under the mild condition requiring each and every agent's ranking of opportunity sets satisfy Monotonicity, assumptions that guarantee the existence and the efficiency of Walrasian allocations are sufficient for the existence of a Walrasian equilibrium procedure with an envy-free profile of opportunity sets and a Pareto efficient allocation. The theorem also provides an explicit way of constructing such an equilibrium procedure: making sure that, to start with, all agents have the same money income which is evaluated at the equilibrium price vector and let them trade to a Walrasian allocation. One way of guaranteeing that all agents have the same money income evaluated at the equilibrium price vector is to simply divide the aggregated initial endowment of goods equally among the agents.

From Theorem 4.1, the following result, which is a converse of Theorem 4.2, is immediate.

Theorem 4.3. Suppose that for all $i \in N$, $\succeq_i$ satisfies Monotonicity and R_i is non-satiated. If $(\mathbf{O}^e(p,\omega), p, x)$ is a Walrasian Equilibrium Procedure such that $\mathbf{O}^e(p,\omega)$ is envy-free and x is Pareto efficient, then $p \cdot (\omega^i)' = p \cdot (\omega^j)'$ for all $i, j \in N$, where $\omega^i = x^i$ for all $i \in N$.

Thus, an equal-money-income Walrasian equilibrium procedure guarantees equality of economic opportunity and yields an efficient allocation. Furthermore, a Walrasian equilibrium procedure that respects equality of economic opportunity equalizes all agents' money income. Therefore, from the view point of equality of economic opportunity, Theorems 4.2 and 4.3 provide some justifications for equal-money-income Walrasian equilibrium procedures.

To conclude this section, we make a comparison of our results with the results in the literature on envy-free allocations. An allocation $x = (x^1, \cdots, x^n)$ is envy-free if no agent envies other agents' commodity bundles, that is, $x_i R_i x_j$ for all $i, j \in N$. The following result is straightforward.

Theorem 4.4. Suppose for all $i \in N$, $\succeq_i$ satisfies Monotonicity. Then, if $(\mathbf{O}^e(p,\omega), p, x)$ is a Walrasian Equilibrium Procedure such that $\mathbf{O}^e(p,\omega)$ is envy-free and x is Pareto efficient, then x is envy-free as well.

Therefore, for all Walrasian equilibrium procedures $(\mathbf{O}^e(p,\omega), p, x)$, the envy-freeness of $\mathbf{O}^e(p,\omega)$ guarantees the envy-freeness of the equilibrium allocation x. In a sense, the notion of envy-free allocation is weaker than that of envy-free opportunities. Therefore, our results of Theorems 4.2 and 4.3 are stronger than the results obtained for envy-free allocations.

5 Economies with Production

The results of Section 4 are simple and straightforward, and are tied well with results of envy-free allocations in exchange economies. In this Section, we consider how far they can be extended in economies involving production.

Firms in a productive economy are indexed by the set F. For each firm $f \in F$, it produces a vector of outpts $y_f \in \mathbb{R}^k_+$. It is assumed that agent i owns a fraction, d_{fi}, of firm f and shares to that extend in the profits, where $d_{fi} \geq 0$ and $\sum_{i \in N} d_{fi} = 1$.

Again, for all $i \in N$, we use $\omega^i \in \mathbb{R}^k_+$ to denote agent i's initial endowment (it should be noted that labor services are among the endowments of the agent). Let $p \in \mathbb{R}^k_+$ be a given price vector. Then, for all agent $i \in N$, a given price vector $p \in \mathbb{R}_+ k$, and a given initial endowment $\omega^i \in \mathbb{R}^k_+$, agent i's opportunity set $O_i(p, \omega^i)$ can be written as:

$$O_i(p, \omega^i) = \{x \in \mathbb{R}^k_+ | p \cdot (x^i)' \leq p \cdot (\omega^i)' + \sum_{f \in F} d_{fi}(p \cdot y'_f)\}.$$

Theorem 5.1. Let $\omega = (\omega^1, \cdots, \omega^n)$ be the initial endowment and $p \in \mathbb{R}^k_+$ be the price vector. Suppose for all $i \in N$, $\succeq_i$ satisfies Monotonicity. Then, for all $\mathbf{O}(p, \omega) = (O_1(p, \omega^1), \cdots, O_n(p, \omega^n))$, $\mathbf{O}(p, \omega)$ is envy-free if and only if $p \cdot (\omega^i)' + \sum_{f \in F} d_{fi}(p \cdot y'_f) = p \cdot (\omega^j)' + \sum_{f \in F} d_{fj}(p \cdot y'_f)$ for all $i, j \in N$.
Proof. The proof is similar to that of Theorem 5.1, we omit it. ■

Theorem 5.1 says that equality of economic opportunity in a productive economy is equivalent to equalization of all agents' full income, or potential income (see, for example, Pazner and Schmeidler (1978) and Varian (1974)). In other words, equality of economic opportunity leads to equalization of the value of each agent's commodity-cum-leisure bundle and *vice versa*. It is also interesting to note that, under equality of economic opportunity, all agents face identical opportunity set. This is, once again, a justification for the concept of equality put forward by Kolm (1973), and Archibald and Donaldson (1979) in a productive economy.

We now move on to the discussion of the compatibility of equality of economic opportunity and Pareto efficiency in a productive economy. For a productive economy, we define a *Competitive Equilibrium Procedure* as a triple $(\mathbf{O}^e(p, \omega), p, (x, y))$ that specifies a collection of opportunity sets $\mathbf{O}^e(p, \omega) = (O^e_1(p, \omega^1, \cdots, O^e_n(p, \omega^n))$, an equilibrium price vector $p \in \mathbb{R}^k_+$ and an equilibrim allocation (x, y), where $O^e_i(p, \omega^i)$

is agent i's budget set corresponding to the initial endowment ω^i and the equilibrium price vector p.

Theorem 5.2. Suppose that for all $i \in N$, $\succeq_i$ satisfies Monotonicity, and standard assumptions for the existence of a competitive equilibrium hold for the productive economy. If $(\mathbf{O}^e(p,\omega), p, (x,y))$ is a Competitive Equilibrium Procedure with $p\cdot(\omega^i)'+\sum_{f\in F} d_{fi}(p\cdot y'_f) = p\cdot(\omega^j)'+\sum_{f\in F} d_{fj}(p\cdot y'_f)$ for all $i,j \in N$, then $\mathbf{O}^e(p,\omega)$ is envy-free and (x,y) is Pareto efficient.

Proof. First, Pareto efficiency of (x,y) follows from the standard argument. To see that $\mathbf{O}^e(p,\omega)$ is envy-free, we note that $p\cdot(\omega^i)'+\sum_{f\in F} d_{fi}(p\cdot y'_f) = p\cdot(\omega^j)'+\sum_{f\in F} d_{fj}(p\cdot y'_f)$ for all $i,j \in N$. Then, from Theorem 5.1, it follows easily that $\mathbf{O}^e(p,\omega)$ is envy-free. ■

The message of this theorem is clear. If each agent's ranking of opportunity sets satisfies Monotonicity, under standard assumptions guaranteeing the existence and the efficiency of competitive allocations in a productive economy, there exists a competitive equilibrium procedure with an envy-free profile of opportunity sets and a Pareto efficient allocation. The way to generating such a competitive equilibrium procedure is to equalizing all agents' full (or potential) income by dividing the aggregated social endowment including labor services equally among all agents. Under such a scheme, each agent is assigned an equal share of (physical) endowment of each good and an equal property right in everybody's endowment of time. Thus, the distribution of productive skills is viewed as a common pool of resources to be shared equally among all agents in the society. If the aggregated social endowment is divided in such a way, the usual competitive mechanism guarantees equality of economic opportunity and produces an equilibrium allocation that is efficient. In view of this result, one can argue that, on the grounds of equality of economic opportunity and efficiency, the equal full income competitive equilibrium procedure is justified. In fact, this argument can be re-enforced by the following result which is a converse of Theorem 5.2 and which follows from Theorem 5.1 straightforwardly.

Theorem 5.3. Suppose that for all $i \in N$, $\succeq_i$ satisfies Monotonicity, and standard assumptions for the existence of a competitive equilibrium hold for the productive economy. If $(\mathbf{O}^e(p,\omega), p, (x,y))$ is a Competitive Equilibrium Procedure such that $\mathbf{O}^e(p,\omega)$ is envy-free and (x,y) is Pareto efficient, then $p\cdot(\omega^i)'+\sum_{f\in F} d_{fi}(p\cdot y'_f) =$

$p \cdot (\omega^j)' + \sum_{f \in F} d_{fj}(p \cdot y'_f)$ for all $i, j \in N$.

Finally, in relation to envy-free allocations, we have the following result.

Theorem 5.4. Suppose that for all $i \in N$, $\succeq_i$ satisfies Monotonicity, and standard assumptions for the existence of a competitive equilibrium hold for the productive economy. If $(\mathbf{O}^e(p,\omega), p, (x,y))$ is a Competitive Equilibrium Procedure such that $\mathbf{O}^e(p,\omega)$ is envy-free and (x,y) is Pareto efficient, then (x,y) is an envy-free allocation.

Once again, the result of Theorem 5.4 suggests that the notion of envy-free allocations is a weaker concept than that of envy-free opportunities.

6 Concluding Remarks

We have formulated and studied equality of economic opportunity based on the notion of envy-free opportunities. Our results suggest that equality of economic opportunity is closed linked with the 'sizes' of opportunity sets which in our current setting are linear budget sets. In both exchange economies and economies with production, the equality of economic opportunity is equivalent to equalizing full income for all agents in the economy. We have shown that some sort of equal income Walrasian mechanisms gives us both equality of economic opportunity and Pareto efficiency. In a pure exchange economy, in order to have both equality of economic opportunity and Pareto efficiency, all we need to do is to divide the aggregated social initial endowment equally among all agents in the economy and then let agents trade to equilibrium. This is rather appealing. However, in a productive economy, in order to have both equality of economic opportunity and Pareto efficiency, we have to divide the aggregated social initial endowment including labor services equally among all agents. This may creat some unacceptable problems. For example, one implication of such a scheme is that a more able agent would likely consume less of his own leisure, while a less able agent could afford to consume more of his leisure. For discussions of other problems, see Pazner (1977).

Given the difficulty of dividing initial endowments of labor services, the full income Walrasian mechanism may not be feasible in practice. Then, as our results suggest, if there are at least two agents who have different full incomes to start with, we cannot achieve equality of economic opportunity. Therefore, alternative formulations of equality of economic opportunity may be called for. Or, we need a theory of *in*equality of economic opportunity to analyse economies with production. These aspects deserve further investigation.

Finally, we note that our analysis in the paper has been confined to private goods economies with competitive behavior. One implication of this setting is that all agents face parallel budget lines. However, in some economies involving public goods, or with non-competitive behaviors, different agents may not face parallel budget lines. It would be interesting to see how far our analysis can then be extended.

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