The Built-in Flexibility of Consumption Taxes*

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1 Introduction

A progressive tax system is one in which the marginal tax rate exceeds the average tax rate facing individuals at all income levels. The associated property of an increasing average tax rate as income increases produces what is referred to as the built-in ‡exibility of a tax system. This ‡exibility is measured in unit-free terms using the concept of the income elasticity of tax revenue. Convenient analytical expressions have been derived for the revenue elasticity of income taxes which are tractable and allow estimates of tax elasticities to be obtained; see Creedy and Gemmell (1998) for a survey of results. Expressions for indirect taxes have not, however, previously been available so that simulation methods have been used to measure their revenue responsiveness.[1]

The purpose of this paper is to extend the analysis of built-in ‡exibility to various forms of consumption taxation. This is useful in view of the extensive use of indirect taxes. Section 2 begins with basic de…nitions and a discussion of income taxation, concentrating on the multi-step case. This is needed in view of the fact that consumption is from income net of income taxation. Section 3 extends the analysis to include ad valorem and unit consumption taxes. Section 4 examines the implications for the revenue elasticity of changes in indirect tax rates. Some illustrative calculations are reported in section 5. Section 6 brie‡y concludes.

2 Income Taxation

2.1 Basic De…nitions

Let $T(y_i)$ denote the income tax paid by individual i with an income of y_i: The revenue elasticity of the income tax with respect to a change in income,

[1] See, for example, Creedy and Gemmell (1984, 1985). Simulation methods were use to examine the elasticity of income taxation by Dorrington (1974), Spahn (1975) and Hutton and Lambert (1982).

η_{T_y, y_i}, is defined as:

$$\eta_{T_y, y_i} = \frac{dT(y_i)/dy_i}{T(y_i)/y_i} = \frac{mtr_i}{atr_i} \tag{1}$$

where mtr_i and atr_i are the marginal and average tax rates faced by i. The first subscript of the revenue elasticity, η, refers to the type of tax revenue considered and the second subscript refers to the income that is considered to change (so that in the present case of the individual elasticity, the first subscript T_y is used as a shorthand for $T(y_i)$). In a progressive tax structure, $mtr_i > atr_i$ for all i, so that $\eta_{T_y, y_i} > 1$. This elasticity is also the measure of liability progression defined by Musgrave and Thin (1948).

2.2 A Multi-step Tax Function

The most common form of income tax function in use is the multi-step function defined by:

$$\begin{aligned} T(y_i) &= 0 && 0 < y_i \leq a_1 \\ &= t_1(y_i - a_1) && a_1 < y_i \leq a_2 \\ &= t_1(a_2 - a_1) + t_2(y_i - a_2) && a_2 < y_i \leq a_3 \end{aligned} \tag{2}$$

and so on.[2] Hence if y_i falls into the kth tax bracket, so that $a_k < y_i \leq a_{k+1}$, and $a_0 = t_0 = 0$, $T(y_i)$ can be written for $k \geq 1$ as:

$$T(y_i) = t_k y_i - \sum_{j=1}^{k} a_j (t_j - t_{j-1}) \tag{3}$$

Hence:

$$T(y_i) = t_k (y_i - a'_k) \tag{4}$$

where:

$$a'_k = \sum_{j=1}^{k} a_j \left(\frac{t_j - t_{j-1}}{t_k} \right) \tag{5}$$

The implication of (4) and (5) is that for taxpayers the tax function facing any individual is equivalent to a tax function with a single marginal tax rate, t_k, applied to income measured in excess of a single threshold, a'_k. The term, a'_k, is the effective threshold for individuals in the kth class, and is a weighted

[2] On the mult-step function, see also Hutton and Lambert (1980), Fries et al. (1982), Caminada and Goudsward (1996) and Creedy and Gemmell (1998).

sum of the a_js, with weights, $\frac{t_j - t_{j-1}}{t_k}$, determined by the structure of marginal rate progression.

It is therefore convenient to re-define the tax function as depending on income and the effective threshold, so that $T = T(y_i; a'_k)$.[3] The elasticity of tax with respect to a change in income, $\eta_{T_y;y_i}$, is given, following (1) as:

$$\eta_{T_y;y_i} = \frac{dT(y_i; a'_k)/dy_i}{T(y_i; a'_k)/y_i} \tag{6}$$

It can be shown that $\eta_{T_y;y_i} = 1 - \eta_{T_y;a'_k}$, where $\eta_{T_y;a'_k}$ is the elasticity of income tax revenue with respect to a'_k. An increase in the effective threshold reduces taxable income, so that $\eta_{T_y;a'_k} < 0$.[4]

From (4), for $k \geq 1$, differentiation gives, for income changes which do not involve a movement between tax brackets:

$$\eta_{T_y;y_i} = \frac{1 - da'_k/dy_i}{1 - a'_k/y_i} \tag{7}$$

The term da'_k/dy_i allows for a change in the level of effective deductions as y_i increases. Such a change is less important in the context of income taxation than in the case of consumption taxes where, as shown below, a change in income is associated with a change in the proportion of total expenditure devoted to taxed goods. In the case where $da'_k/dy_i = 0$, the elasticity becomes:

$$\eta_{T_y;y_i} = 1 + \frac{a'_k}{y_i - a'_k} \tag{8}$$

Furthermore, $\eta_{atr;y_i} = -\eta_{T_y;a'_k} = a'_k/(y_i - a'_k)$. This result, that the elasticity of the average tax rate with respect to income is the negative of the elasticity of tax with respect to the effective threshold, allows the tax elasticity for a multi-step function to be calculated solely from information on income levels and the effective allowance.

[3] The function, $T(y_i; a'_k)$ is homogeneous of degree 1, since $\lambda T = T(\lambda y_i; \lambda a'_k)$

[4] In the context of aggregate revenue, Fries et al. (1981) demontrated a similar result for a multi-step function having a variety of allowances and income-related deductions; see also Lambert (1993, pp.216-218).

2.3 Aggregate Revenue Elasticity

For tax policy purposes the aggregate, rather than the individual, tax revenue elasticity is often more relevant. Suppose there are N individuals, with incomes $y_1, \ldots, y_N$, so that total income, Y, is $\sum_{i=1}^{N} y_i$, and total income tax revenue, T_Y, is $\sum_{i=1}^{N} T(y_i)$. Taking the multi-step income tax function in (2) and totally differentiating gives:

$$\begin{aligned} dT_Y &= \sum_{i=1}^{N} \frac{\partial T(y_i; a_k^0)}{\partial y_i} dy_i \\ &= \sum_{i=1}^{N} \eta_{T_{y_i}, y_i} T(y_i; a_k^0) \frac{dy_i}{y_i} \end{aligned} \tag{9}$$

Therefore the elasticity of aggregate revenue with respect to a change in aggregate income, $\eta_{T_Y, Y}$, cannot be defined without specifying the distribution of proportionate income changes, dy_i/y_i, associated with any increase in total income.

Consider $\eta_{T_Y, Y}$ defined in relation to a situation in which all individuals experience an equal proportionate increase, so that any relative measure of inequality is unchanged.[5] In this case $dy_i/y_i = dY/Y$ and substituting into (9) gives:

$$\eta_{T_Y, Y} = \frac{dT_Y}{dY} \frac{Y}{T_Y} = \sum_{i=1}^{N} \eta_{T_{y_i}, y_i} \frac{T(y_i; a_k^0)}{T_Y} \tag{10}$$

so that the aggregate elasticity is a weighted average of the individual elasticities, with weights equal to the proportion of total revenue paid by each individual. Given that, in the case where the as are constant, the individual revenue elasticities exceed 1, the aggregate elasticity must also exceed 1.

3 Income and Consumption Taxes

This section extends the previous results to include consumption tax revenue. This is complicated by the need to allow for consumers' responses to changes in disposable income. Analytical expressions are obtained for the tax

[5] For treatments of non-equiproportional growth, see Lambert (1993, pp.209-212) and Creedy and Gemmell (1998)

revenue elasticities of both ad valorem and unit consumption taxes. Elasticity expressions for combined income and consumption tax revenues are also obtained. In many cases these revenue elasticities can be calculated using limited information on expenditure patterns in addition to data on the tax parameters.

3.1 Individual Revenue Elasticity

Consider an individual with income of y_i and facing a multi-step income tax function. It has been seen that such a tax is equivalent to a single-step function having a marginal rate, t_k, imposed on the individual's income in excess of an effective threshold of a'_k. For any tax-payer, $T(y_i; a'_k) = t_k(y_i - a'_k)$, where t_k and a'_k are the effective marginal tax rate and threshold faced by the individual, as in equation (4). Ignoring savings, total consumption expenditure, m_i, is equal to net or disposable income, so that:[6]

$$
\begin{aligned}
m_i &= y_i - T(y_i; a'_k) \\
&= a'_k t_k + y_i(1 - t_k) \qquad (11)
\end{aligned}
$$

Suppose that the tax-exclusive indirect tax rate imposed on the ℓth good (for $\ell = 1, \ldots, n$) is v_ℓ, giving rise to the equivalent tax-inclusive rate of $v'_\ell = v_\ell/(1 + v_\ell)$. Let p_ℓ be the tax-inclusive price of the ℓth good and define w_ℓ as the budget share of the ℓth good. The consumption tax paid on all goods, $T_v(y_i)$, is given by:

$$
T_v(y_i) = m_i \sum_{\ell=1} v'_\ell w_\ell \qquad (12)
$$

The total consumption and income tax paid by the individual, $R(y_i)$, is therefore $T(y_i; a'_k) + \{y_i - T(y_i; a'_k)\} \sum_{\ell=1}^{n} v'_\ell w_\ell$, or:

$$
R(y_i) = t_k(y_i - a'_k) + \{a'_k t_k + y_i(1 - t_k)\} \sum_{\ell=1} v'_\ell w_\ell \qquad (13)
$$

If the income tax thresholds are fixed, then the change in total revenue resulting from an increase in the individual's income, assuming that the individual

[6] It would be possible to allow for a propensity to save, but this complication is not considered here.

does not move into a higher tax bracket, is given by:

$$\frac{dR\,(y_i)}{dy_i} = t_k + (1 - t_k) \sum_{\ell=1} v'_\ell w_\ell + m_i \sum_{\ell=1} v'_\ell \frac{dw_\ell}{dy_i} \tag{14}$$

The last term in (14) involves changes in budget shares as income increases. In order to obtain a convenient expression for dw_ℓ/dy_i, first use the fact that, if e_ℓ is the total expenditure elasticity of demand for the ℓth good:

$$e_\ell = 1 + \frac{dw_\ell/w_\ell}{dm_i/m_i} \tag{15}$$

so that:

$$\frac{dw_\ell}{dm_i} = \frac{w_\ell\,(e_\ell - 1)}{m_i} \tag{16}$$

Then write $\frac{dw_\ell}{dy_i} = \frac{dw_\ell}{dm_i}\frac{dm_i}{dy_i}$, so that by using $dm_i/dy_i = 1 - t_k$ from (11), it can be seen that:

$$\frac{dw_\ell}{dy_i} = \frac{w_\ell\,(e_\ell - 1)\,(1 - t_k)}{m_i} \tag{17}$$

Substituting the right hand side of (17) into (14) and rearranging gives:

$$\frac{dR\,(y_i)}{dy_i} = t_k + (1 - t_k) \sum_{\ell=1} v'_\ell w_\ell e_\ell \tag{18}$$

The revenue elasticity, η_{R,y_i}, is therefore found to be:

$$\eta_{R,y_i} = \frac{1 + \frac{1-t_k}{t_k} \sum_{\ell=1}^{n} v'_\ell w_\ell e_\ell}{1 - \frac{a'_k}{y_i}\left(1 - \sum_{\ell=1}^{n} v'_\ell w_\ell\right) + \frac{1-t_k}{t_k} \sum_{\ell=1}^{n} v'_\ell w_\ell} \tag{19}$$

It can also be shown that η_{R,y_i} is the weighted average of the elasticities for income and consumption taxes, η_{T_y,y_i} and η_{T_v,y_i} respectively, where the weights are the shares of each tax in total tax revenue. Hence:

$$\eta_{R,y_i} = \frac{T_v\,(y_i)}{R}\,\eta_{T_v,y_i} + \left(1 - \frac{T_v\,(y_i)}{R}\right)\eta_{T_y,y_i} \tag{20}$$

The η_{R,y_i} expressions in (19) and (23) can therefore be decomposed into their income and consumption tax components. The former were given in equations (7) and (8). In general, the consumption tax elasticities are:

$$\eta_{T_v,y_i} = \frac{\left(\frac{1-t_k}{t_k}\right) \sum_{\ell=1}^{n} v'_\ell w_\ell e_\ell}{\left(\frac{a'_k}{y_i} + \frac{1-t_k}{t_k}\right) \sum_{\ell=1}^{n} v'_\ell w_\ell} \tag{21}$$

which can be expressed as:

$$\eta_{T_v,y_i} = \frac{(1-t_k)\, y_i \sum_{\ell=1}^{n} v_\ell' w_\ell e_\ell}{m_i \sum_{\ell=1}^{n} v_\ell' w_\ell} \tag{22}$$

These results show that the evaluation of the η_{R,y_i} requires, in addition to the tax parameters, information about individuals' budget shares and the total expenditure (income) elasticity of demand for each good. This information can be obtained from cross-sectional household budget surveys. Any indirect taxes that are not imposed as ad valorem rates can be expressed as equivalent to ad valorem rates. For example, a unit tax of t_u imposed on the tax-exclusive price, p_E, so that the tax-inclusive price is $p = p_E + t_u$, is equivalent to an ad valorem rate of $(t_u/p)/(1 - t_u/p)$.

3.2 Uniform Taxes

The result in (19) can be applied to any structure of indirect tax rates, but consider the special case where $v_\ell' = v'$ for all ℓ. Using the additivity properties that $\sum_{\ell=1}^{n} w_\ell e_\ell = \sum_{\ell=1}^{n} w_\ell = 1$, the elasticity becomes:

$$\eta_{R,y_i} = 1 + \frac{a_k'}{\beta y_i - a_k'} \tag{23}$$

where:

$$\beta = \frac{1}{1-v'} + \left(\frac{v'}{1-v'}\right)\left(\frac{1-t_k}{t_k}\right) \tag{24}$$

It can be seen that $\beta > 1$, so that comparison of η_{R,y_i} in (23) with η_{T_y,y_i} in (8) reveals that the latter must exceed the former; that is, overall tax revenue is less elastic than income tax revenue.

In this special case, substitution of $v_\ell' = v'$ into (22) gives:

$$\eta_{T_v,y_i} = 1 - \frac{a_k'}{\left(\frac{1-t_k}{t_k}\right) y_i + a_k'} = 1 - \frac{t_k a_k'}{m_i} \tag{25}$$

where $t_k a_k'$ is the effective allowance. Hence it can be seen that, for the uniform consumption tax rate, $0 < \eta_{T_v,y_i} < 1$ in the presence of a progressive income tax (that is, where $a_k' > 0$). However, as (23) shows, inelastic consumption tax revenue cannot outweigh the elastic responsiveness of income taxes, given in (8), such that total revenue remains income elastic: hence $\eta_{R,y_i} > 1$. The special case of a two-rate structure is of more policy relevance, and is examined in the following subsection.

3.3 The Two-rate Case

Consider the common situation where a sub-set of goods is untaxed, or taxed at a zero rate, while others are taxed at a common rate, v'.[7] If goods are ordered such that the first s goods have a zero rate, then the consumption tax revenue, $T_v(y_i)$, is:

$$T_v(y_i) = v' m_i \left(1 - \sum_{\ell=1} w_\ell\right) \tag{26}$$

The term in brackets in (26) is the proportion of expenditure on taxed goods. This kind of zero-rating is usually imposed in order to introduce some progressivity into the indirect tax structure, by selecting the first s goods to be those where budget shares decline as total expenditure increases.

Letting $w^* = \sum_{\ell=1}^{s} w_\ell$ denote the proportion of total expenditure devoted to untaxed goods, differentiation of (26) gives:

$$\frac{dT_v(y_i)}{dm_i} = v'(1 - w^*) - v' m_i \frac{\partial w^*}{\partial m_i} \tag{27}$$

From (16), $\partial w^* / \partial m_i$ can be written as:

$$\frac{\partial w^*}{\partial m_i} = \frac{\sum_{\ell=1}^{s} (e_\ell w_\ell - w_\ell)}{m_i} = \frac{(e^* - w^*)}{m_i} \tag{28}$$

where $e^* = \sum_{\ell=1}^{s} e_\ell w_\ell$. Hence, e^* is a budget-share weighted sum of the total expenditure (income) elasticities of those goods for which the tax rate is zero. Using (28) to substitute for $\partial w^* / \partial m_i$ in (27) gives:

$$\frac{dT_v(y_i)}{dm_i} = v'(1 - e^*) \tag{29}$$

and the consumption tax revenue elasticity at the individual level with respect to total expenditure, η_{T_v, m_i}, is:

$$\eta_{T_v, m_i} = \frac{1 - e^*}{1 - w^*} \tag{30}$$

[7] In a value added type of tax system, a distinction must of course be made between tax exempt and zero-rated goods, but this can be ignored here.

This elasticity takes the familiar form that might be expected from (7).[8] The denominator is simply the proportion of total expenditure that is subject to taxation, while the numerator is the (budget-share weighted) sum of the expenditure (income) elasticities of the taxed goods.[9] The numerator can also can be expressed as $(1 - w^*) - m_i(\partial w^*/\partial m_i)$, such that (30) becomes:

$$\eta_{T_v,m_i} = 1 - \frac{m_i(\partial w^*/\partial m_i)}{1 - w^*} \tag{31}$$

where $m_i(\partial w^*/\partial m_i)$ reflects the endogenous change in zero-rated expenditure (equivalent to an effective allowance) as total expenditure rises.

In order to obtain the income elasticity of indirect tax, it is necessary to use the multiplicative property of elasticities, such that any elasticity can be written as the product of the effect on tax payments of a change in the tax base and the effect on the tax base of a change in income. Writing the consumption tax revenue elasticity, η_{T_v,y_i} as $\eta_{T_v,m_i}\eta_{m_i,y_i}$, and using the result that $\eta_{m_i,y_i} = (1 - t_k^0)y_i/m_i$, it can be found that:

$$\eta_{T_v,y_i} = \frac{(1 - t_k)y_i(1 - e^*)}{m_i(1 - w^*)} \tag{32}$$

where the denominator is taxed expenditure. Further insight can be gained by making use of the fact that $m_i = a_k^0 t_k + y_i(1 - t_k)$, to rewrite (32) as:

$$\eta_{T_v,y_i} = \left[1 - \frac{a_k^0 t_k}{m_i}\right]\left\{\frac{1 - e^*}{1 - w^*}\right\} \tag{33}$$

From (33) it can be seen that a higher total expenditure elasticity of demand for zero-rated goods, e^*, implies a lower revenue elasticity, while a higher proportion of total expenditure devoted to zero-rated goods, w^*, is associated with a higher revenue elasticity. The first property is intuitively clear, since a higher elasticity implies that consumption moves more rapidly away from untaxed goods as disposable income rises; in practice, zero-rated goods are

[8] In the trivial case where all goods are zero-rated, then $n = s$ and $e^* = 1$, so that, as expected, the elasticity is zero.

[9] Since the budget-share weighted sum of the elasticities of all goods must equal unity, that is $\sum_{\ell=1}^{n} e_\ell w_\ell = 1$, it can be seen that $1 - e^* = \sum_{\ell=1}^{n} e_\ell w_\ell - \sum_{\ell=1}^{s} e_\ell w_\ell = \sum_{\ell=s+1}^{n} e_\ell w_\ell$ and $1 - w^* = \sum_{\ell=s+1}^{n} w_\ell$.

likely to have relatively low elasticities. The second property suggests that a policy change which reduces the number of zero-rated goods, in order to increase tax revenue initially, has the effect of reducing the revenue elasticity, thereby reducing revenue growth.

Equation (33) also indicates the conditions under which consumption tax revenue will be elastic or inelastic. Consider the two multiplicative terms in square brackets in (33). The former, $\left[1-\frac{a_k^0 t_k}{m_i}\right]$, is determined by the progressivity of the income tax and must be less than unity for a progressive tax, and $a_k^0>0$. The second term, $\left[\frac{1-e^*}{1-w^*}\right]$, reflects the progressivity of the consumption tax, as captured by the expenditure elasticities of the zero-rated goods, e_ℓ. Where only necessities are zero-rated ($e_\ell<1$ for all $\ell=1,...,s$) it can be shown that $(1-e^*)>(1-w^*)$.[10] Thus in (33) greater progressivity of the income tax exerts an 'inelastic' influence on the consumption tax revenue elasticity while greater progressivity of the consumption tax exerts an elastic influence. The overall outcome is ambiguous.

In some cases it may be possible to specify $w^*=\sum_{\ell=1}^{s} w_\ell$ directly as a function, say $r(m_i)$, of total expenditure. In the simple, though not realistic, case where the proportion of zero-rated expenditure is inversely proportion to total expenditure, so that $r(m_i)=\alpha/m_i$; then $T_v(y_i)=v'(m_i-\alpha)$. Here the use of zero-rated goods acts as a fixed allowance against total expenditure.[11]

3.4 Aggregate Consumption Tax Revenue Elasticity

As in the case of income taxes, the aggregate rather than the individual consumption tax elasticity is often more important for tax policy. For the equi-proportional income growth case, aggregate consumption tax revenue elasticities can be obtained analogously to those given in section 2.3 for the

[10] From the definitions of e^* and w^* above, it can be found that $(1-e^*)/(1-w^*) = (1-\sum_{\ell=1}^{s} e_\ell w_\ell)/(1-\sum_{\ell=1}^{s} w_\ell)$. A sufficient condition for this to exceed unity is therefore that all $e_\ell<1$ $(\ell=1,...,s)$. The necessary condition is that the (budget-share weighted) elasticity of taxed goods, $(1-e^*)$, exceeds the sum of their budget shares. Thus, within the subset of zero-rated goods, provided those with inelastic demands dominate (in terms of budget shares) those with elastic demands, $(1-e^*)/(1-w^*)$, exceeds unity.

[11] The reciprocal case was implicit in Kay and Morris (1979). A double-log specification was used by Creedy and Gemmell (1984, 1985) and Gemmell (1985), and this was extended so that $r(m)$ takes a finite value when $m=0$, in Creedy (1992).

multi-step income tax.

Let $T_V = \sum_{i=1}^{N} T_v(y_i)$ where, as earlier, $T_v(y_i) = m_i \sum_{\ell=1}^{n} v_\ell' w_\ell$: Total differentiation gives:

$$
\begin{aligned}
dT_V &= \sum_{i=1}^{N} \frac{\partial T_v(y_i)}{\partial y_i} dy_i \\
&= \sum_{i=1}^{N} \eta_{T_v;y_i} T_v(y_i) \frac{dy_i}{y_i}
\end{aligned}
\tag{34}
$$

For the equi-proportional income growth case, substitute dY/Y for dy_i/y_i and divide by T_V to get:

$$
\eta_{T_V;Y} = \sum_{i=1}^{N} \eta_{T_v;y_i} \frac{T_v(y_i)}{T_V}
\tag{35}
$$

Thus, as in the case of the income tax, the aggregate consumption tax elasticity is a consumption-tax-share weighted average of the individual revenue elasticities.

The aggregate revenue elasticity, $\eta_{T_R;Y}$; including both income and consumption taxes, is given (again in the equi-proportional growth case) from (9) by:

$$
\eta_{T_R;Y} = \sum_{i=1}^{N} \eta_{T_R;y_i} \left\{ \frac{T_y(y_i) + T_v(y_i)}{T_Y + T_V} \right\}
\tag{36}
$$

and using (10) and (35) it can be found that (36) becomes:

$$
\eta_{T_R;Y} = \left(\frac{T_Y}{T_Y + T_V} \right) \eta_{T_Y;Y} + \left(\frac{T_V}{T_Y + T_V} \right) \eta_{T_V;Y}
\tag{37}
$$

Hence the aggregate revenue elasticity is simply a revenue–share weighted average of the aggregate income tax and consumption tax elasticities.

4 Changes in Consumption Tax Rates

This section examines the effect on the elasticities of changes in the indirect tax rates. The revenue elasticity for each income, y_i; in (19) is influenced by the budget shares, w_ℓ; and the income elasticities, e_ℓ; in addition to the income tax parameters, a_k'; t_k; (appropriate to that income level) and the consumption tax parameters, the set of tax-inclusive rates $v'_\ell = v_\ell/(1 + v_\ell)$. The budget shares and total expenditure elasticities are also likely to vary with total expenditure, m_i. Any discretionary change in the consumption tax rates, designed perhaps to increase total tax revenue, therefore has a direct effect on the revenue elasticity in (19) and an indirect effect through endogenous changes in the budget shares.

Consider a change in prices arising from a change in indirect taxes. Given that the relationship between tax-inclusive and tax-exclusive prices, p_ℓ and $p_{E,\ell}$ respectively for good ℓ, is given by $p_\ell = (1 + v)\, p_{E,\ell}$, the proportionate change in the tax-inclusive price of the ℓth good, $\dot p_\ell$ resulting from a change in v_ℓ of dv_ℓ is given by:

$$\dot p_\ell = \frac{dv_\ell}{1 + v_\ell} \qquad (38)$$

The proportionate change in the quantity consumed of good ℓ, $\dot q_\ell$, is given by:

$$\dot q_\ell = \sum_{r=1} e_{\ell r}\dot p_r \qquad (39)$$

where $e_{\ell r}$ is the elasticity of demand for good ℓ with respect to a change in the price of the rth good, and $\dot p_r = dp_r/p_r$. The new expenditure on the ℓth good is given by $p_\ell q_\ell + d(p_\ell q_\ell)$ where the latter total differential is $p_\ell dq_\ell + q_\ell dp_\ell$ or $p_\ell q_\ell(\dot q_\ell + \dot p_\ell)$. Hence the proportional change in budget share of good ℓ resulting from a set of price changes is given by:

$$\dot w_\ell = \dot p_\ell + \sum_{r=1} e_{\ell r}\dot p_r \qquad (40)$$

Since indirect tax revenue is given by $m_i \sum_{\ell=1}^{n} w_\ell v'_\ell$, the change in revenue resulting from a set of indirect tax changes is given by:

$$m_i \left\{ \sum_{\ell=1} \left(v'_\ell \frac{dw_\ell}{dv_\ell} + w_\ell \frac{dv'_\ell}{dv_\ell} \right) \right\}$$

and the proportional change in indirect tax revenue is therefore:

$$\frac{\sum_{\ell=1}^{n} w_\ell v'_\ell (\dot w_\ell + \dot v'_\ell)}{\sum_{\ell=1}^{n} w_\ell v'_\ell} \qquad (41)$$

where $\dot w_\ell$ is obtained from (40). These results can be used to examine the effect of discretionary changes in taxation on total revenue and the revenue elasticity. However, the evaluation of such changes requires substantially more information than that of the revenue elasticities given above. In particular, information about the own-price and cross-price elasticities of demand are required for all goods, in order to compute the budget share changes, $\underline{w}_\ell$; associated with any change in consumption tax rates (and hence prices of goods).

5 Illustrative Examples

Section 3 demonstrated that both income and consumption tax revenue elasticities can be calculated, for the individual taxpayer, from information on relatively few parameters. These include effective income tax rates, t_k, the effective allowance, a'_k, and indirect tax rates, v_ℓ, together with the budget shares and expenditure elasticities for goods bearing different tax rates. With the restriction of equi-proportionate income growth, equivalent aggregate expressions are also tractable. These also require information about the income distribution in order to calculate revenue weights; see (10) and (35)).

This section provides illustrations of alternative revenue elasticities. The calculations are based on the use of budget shares for Australian households, using the Household Expenditure Survey (HES) for 1993. The calculation of total expenditure elasticities is described in the Appendix. The income tax parameters used approximate the 1993/94 schedule (the thresholds have been rounded for convenience). The thresholds are 6000, 21000, 38000, and 50000 (A$), and marginal income tax rates applied to taxable income above these thresholds, t_k; are 0.20, 0.355, 0.44 and 0.47 respectively.

Consumption tax calculations are based on the fourteen expenditure categories used by the HES, and listed in Table 1. This table also shows three alternative consumption tax regimes. The column headed (i) gives the estimated effective consumption tax rates arising from the complex range of indirect taxes operating in Australia, on the assumption that the taxes are fully shifted forward and using the method devised by Scutella (1997), which makes use of the matrix of inter-industry transactions. In contrast, column (ii) simply lists a uniform 15 per cent ad valorum rate, while column (iii) is

Table 1: Indirect Tax Rates

no.	Commodity Group	Tax Structure (i)	(ii)	(iii)
1	Current housing costs	0.1437	0.15	0.00
2	Electricity, gas and other fuels	0.0956	0.15	0.15
3	Food and beverages	0.1289	0.15	0.00
4	Spirits, beer and wine	0.4224	0.15	0.15
5	Tobacco	2.1510	0.15	0.15
6	Clothing and footwear	0.0731	0.15	0.15
7	Furniture and appliances	0.1201	0.15	0.15
8	Postal and telephone charges	0.0993	0.15	0.15
9	Health services	0.0603	0.15	0.15
10	Motor vehicles and parts	0.3126	0.15	0.15
11	Recreational items	0.1677	0.15	0.15
12	Personal care products	0.1441	0.15	0.15
13	Miscellaneous	0.1644	0.15	0.15
14	House building payments	0.1296	0.15	0.00

similar to (ii) but with three zero-rated categories. These are current housing costs, food and beverages, and house building costs. These categories are selected in view of the fact that, in countries using value added consumption taxes, they are commonly zero-rate.

5.1 Individual Revenue Elasticities

This section illustrates the properties of individual elasticities in the uniform consumption tax case. Using the income tax expression (8) and the expression (25), elasticities may be calculated for alternative income levels. Illustrative results are shown in Table 2. More details of the variation in the elasticities with income are given in Figure 1. This ...gure shows the tendency for the income tax elasticity to decline within each marginal rate band, with step increases as thresholds are crossed. The reverse occurs for consumption taxes: the elasticity increases within income tax bands with step decreases across thresholds. Table 2 also highlights the tendency (at least for income levels at the mid-point within tax bands) for the income tax elasticity to

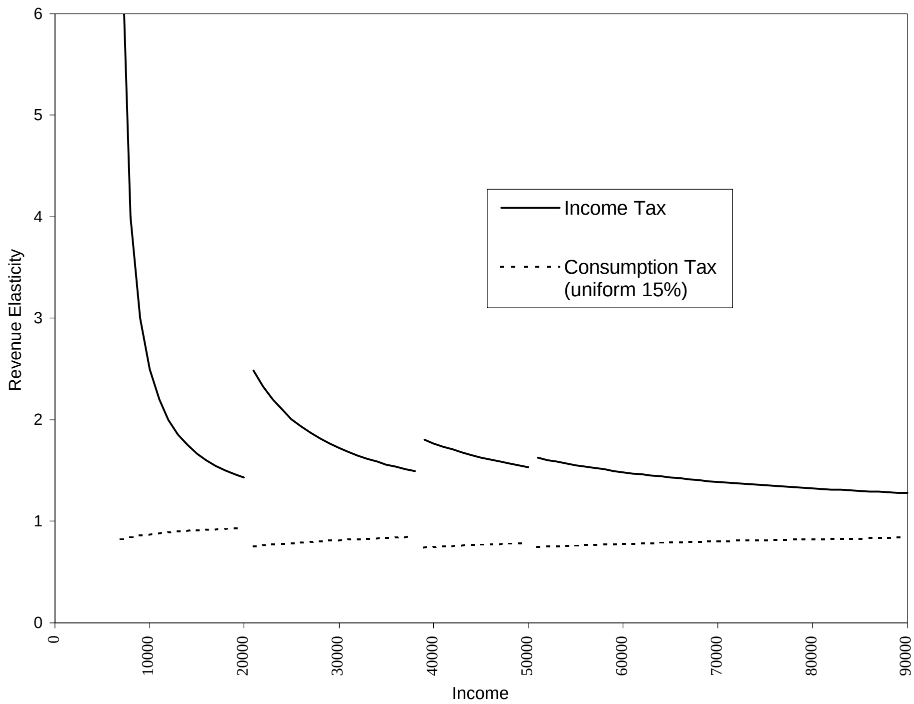

Figure 1: Individual Income and Consumption Tax Elasticities

decline but for the consumption tax elasticity to initially decline and then increase at high income levels. This consumption tax phenomenon (which is also characteristic of aggregate consumption tax elasticities - see below), arises here because the increase in $t_k a_k^0 = m_i$ in (25) as a result of crossing into higher marginal tax bands (and which reduces the consumption tax elasticity) is reversed above the highest threshold when $t_k a_k^0 = m_i$ falls as y_i, and pari passu m_i, rises further. As is evident from (25) however, the consumption tax elasticity cannot exceed unity (for this uniform consumption tax rate case).

Table 2: Individual Revenue Elasticities: Uniform Taxes

	Income Tax Band					
	0-6000	6001-21000	21001-38000	38001-50000	50001+	
t_k	0	0.2	0.355	0.44	0.47	0.47
a_k^0	0	6000	12560	17380	19560	19560
y_i	3000	13500	29500	44000	75000	150000
m_i	3000	12000	23480	32290	48900	88700
η_{T_y,y_i}	0	1.80	1.74	1.65	1.35	1.15
η_{T_v,y_i}	0	0.90	0.81	0.76	0.81	0.89

5.2 Aggregate Revenue Elasticities

The calculation of aggregate revenue elasticities is based on the use of a random sample of 10,000 individuals from a lognormal income distribution having a variance of logarithms of 0.5; this is obviously a simplification but, for present purposes, provides a reasonable approximation of the Australian income distribution. The simulated population is initially obtained using a mean of logarithms of income of 9.5; this gives an initial sample mean income of A$17,205, which is well below (about half of) the average income in Australia in 1993. Equi-proportionate growth of the 10,000 pre-tax incomes is then imposed.[12] Resulting profiles for the income tax, consumption tax and total tax revenue elasticities are shown in Figure 2 for the actual effective consumption tax rate case of column (i) of Table 1. Figure 3 compares consumption tax elasticity profiles for the three cases listed in Table 1.

The aggregate elasticities in Figure 2 display a similar pattern to the individual revenue elasticities shown in Figure 1: income tax elasticity revenue is elastic but declining, while consumption tax revenue is inelastic and relatively constant. As average income doubles from around A$17,000 to A$34,000 the income tax elasticity declines from around 1.75 to 1.5, while the consumption tax elasticity declines slightly from 0.88 to 0.82. The effect on the total tax

[12] The individual income tax revenue elasticities used to calculate the aggregate equivalents were obtained simply by calculating the average and marginal tax rates facing each individual, since the elasticity is the ratio of the former to the latter, as in equation (1). This avoids the need to calculate a_k^0 for each individual and tax schedule.

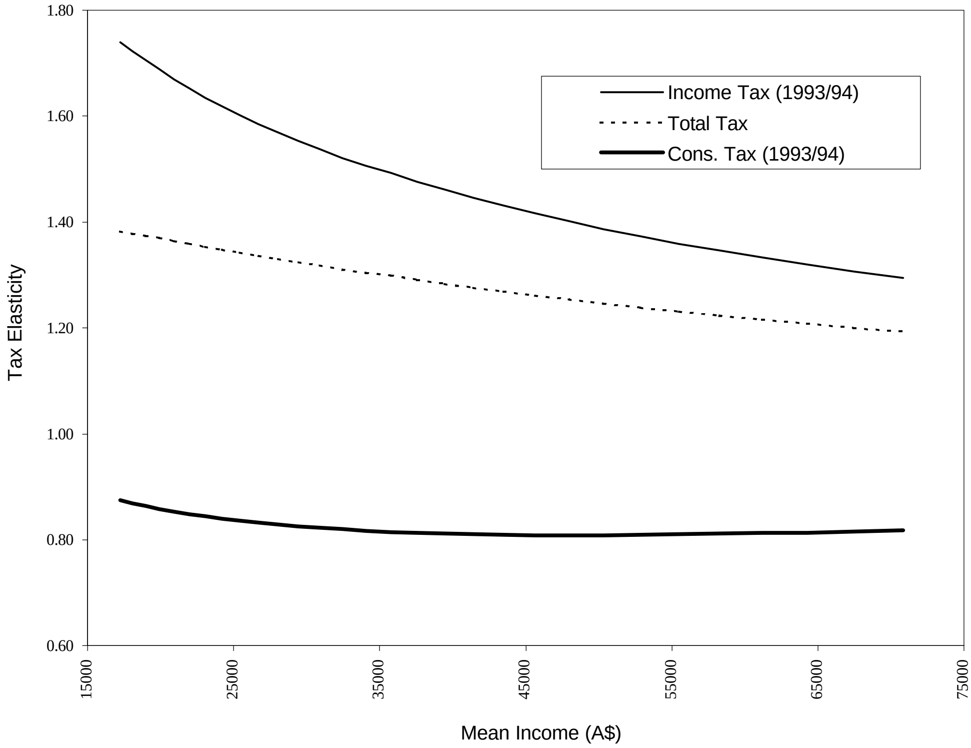

Figure 2: Income and Consumption Tax Elasticities

revenue elasticity is to produce a relatively ‡at schedule with the elasticity within the 1.2–1.4 range over a wide range of mean income levels.

Comparing the consumption tax elasticity pro…les shown in Figure 3, pro…le B (the uniform rate case) is the aggregate equivalent of the individual elasticity pro…le in Figure 1 and displays a similar pattern, rising for mean incomes above about A$35,000 for the reasons outlined earlier. Introducing exemptions (pro…le C) increases the elasticity at all income levels, as predicted for the two-rate case in section 3.2. However, increases are greatest at lower income levels and the pro…le is almost ‡at at high average incomes. This di¤erence from pro…le B re‡ects the fact that two of the zero-rated categories have expenditure elasticities less than unity (falling budget shares) and this element of progressivity serves to increase the consumption tax revenue

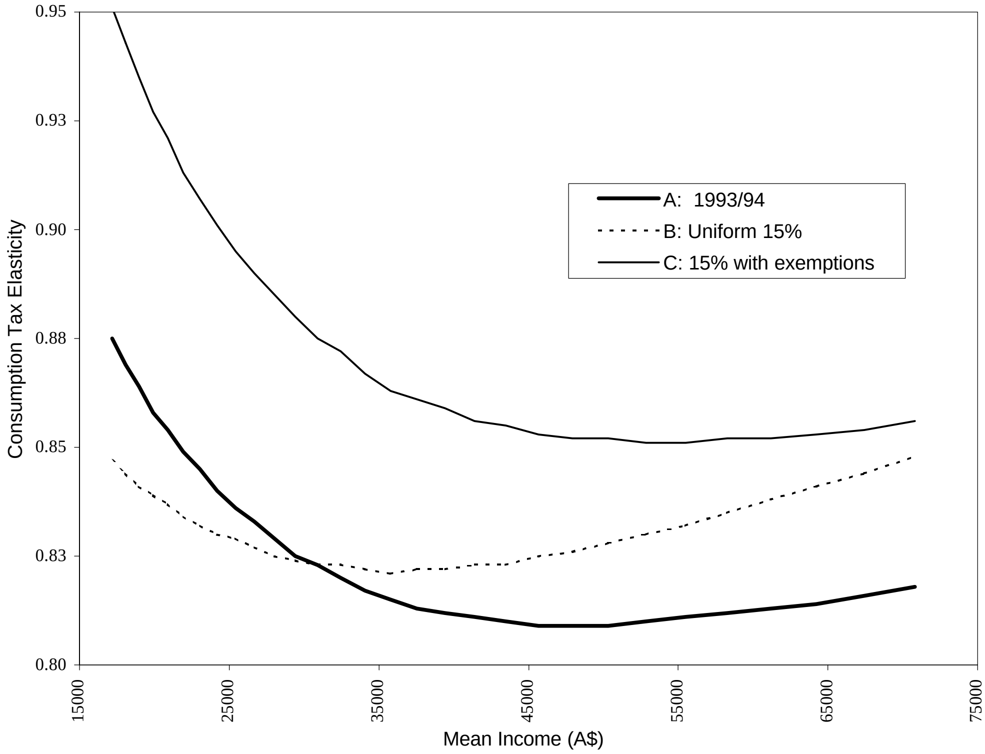

Figure 3: Consumption Tax Elasticities

elasticity, especially at low mean incomes.

Pro…le A, for the actual e¤ective tax rates in Australia, reveals intermediate elasticity levels at lower mean income levels compared with the other two cases, but has the lowest elasticity at high incomes. As mentioned earlier, unlike the uniform case B, both cases A and C could, in principle, produce consumption tax elasticities in excess of unity. However, in these particular cases any progressivity arising from the consumption tax exemptions is insuf…cient to outweigh the (inelastic) e¤ect arising from the progressive income tax. Simulating the case in which the four main categories with expenditure elasticities less than unity (categories 1, 2, 3, and 9 in Table 1) are zero-rated, with all other goods at 15 per cent, is found to yield consumption tax revenue elasticities slightly above unity.

6 Conclusions

How does tax revenue respond to income growth over the long run and what determines that responsiveness? These questions have been addressed extensively in the literature in relation to income taxes allowing their general built-in ‡exibility and redistributive properties to be understood. Surprisingly, a similar analytical approach to indirect taxes has not been pursued previously, with investigations limited to case-speci…c empirical exercises or simulations. This paper has shown that tractable analytical expressions can be produced for the built-in ‡exibility (elasticities) of various indirect taxes, where the indirect tax system is combined with a general multi-step income tax structure. These clarify the determinants of the revenue responsiveness properties of di¤erent taxes, and demonstrate that indirect tax elasticities can be estimated from information that is generally readily available for most tax systems: summary tax parameters and commodity demand (expenditure) elasticities. The e¤ects of changes in indirect tax rates on revenue elasticities were also examined analytically, though to estimate these in practice would require considerably more information, particularly relating to price elasticities of demand.

Using available tax data and expenditure elasticity estimates for Australia, income and consumption tax revenue elasticities were calculated for a simulated income distribution. These con…rmed the tractability of the analytical expressions and demonstrated the inelasticity of consumption taxes, both absolutely and relative to income tax elasticities. While it is possible for consumption tax revenue elasticities (modestly) to exceed unity, in practice this would appear to require low or zero rates on a substantial proportion of consumption expenditure, concentrated on those goods which form large budget shares at low income levels.

The various tax parameters in the elasticity expressions obtained here also highlight the potential impact of tax reform on built-in ‡exibility. When governments make discretionary changes to tax parameters, such as tax rates and allowances, the direct e¤ect on tax revenue is of immediate concern both to the treasury and taxpayers. However, the e¤ects of such changes on the built-in ‡exibility of individual taxes or the tax system as a whole are often less well appreciated. Year-to-year changes will generally have small impacts on revenue elasticities, but these can build into more substantial e¤ects over longer periods, especially when compounded by subsequent reforms. For example, from the early 1980s a number of industrialised countries undertook signi…cant tax reform (often involving a shift towards a less progressive income tax and greater use of indirect taxes), as well as experiencing rising mean incomes relative to tax thresholds, and increased inequality of pre-tax incomes. Each of these changes can be expected to reduce tax revenue elasticities, with important implications for future automatic revenue growth.

Appendix: Income Elasticities Using HES Data

Expenditure (income) elasticities were obtained using the 1993 Australian Household Expenditure Survey (HES). The 7590 households were divided into $K = 30$ total expenditure groups, and within each group the average budget shares for each of $n = 14$ commodity groups were obtained. Denote the arithmetic mean expenditure of the kth total expenditure group by y_k $(k = 1, \ldots, K)$ and the average expenditure weight, or budget share, for the ith commodity group and kth total expenditure group by w_{ki} $(i = 1, \ldots, n)$. Using the basic definition $w_{ki} = p_{ki}q_{ki}/y_k$, differentiation gives:

$$\dot{w}_{ki} = \dot{y}_k (e_{ki} - 1) \tag{42}$$

where $\dot{w}_{ki}$ and $\dot{y}_k$ denote the proportional changes in the budget share of ith good and total expenditure in the kth group. Hence:

$$e_{ki} = 1 + \dot{w}_{ki}/\dot{y}_k \tag{43}$$

Define the following discrete proportionate changes, for $k = 2, \ldots, K$:

$$\dot{y}'_k = (y_{k-1}/y_k) - 1 \tag{44}$$

$$\dot{w}'_{ki} = (w_{k-1,i}/w_{ki}) - 1 \tag{45}$$

so that although the 'dot' notation has been used, the changes are obtained by comparing values in adjacent total expenditure groups. These can be used to substitute into equation (43) to get the set of total expenditure elasticities for $k = 2, \ldots, K$ and $i = 1, \ldots, n$, giving:

$$e'_{i(k)} = 1 + \dot{w}'_{ki}/\dot{y}'_k \tag{46}$$

A similar set of elasticities can be obtained, for $k = 1, \ldots, K - 1$, using reductions in total expenditure, such that $\dot{y}^*_k = (y_{k+1}/y_k) - 1$ and so on. Arithmetic mean values were used for $k = 2, \ldots, K - 1$, while elasticities corresponding respectively to downward and upward changes in y were used for $k = K$ and $k = 1$.

However, the raw values of the average budget shares could not be used because the sampling variations would give rise to a large number of negative elasticities. The shares were 'smoothed' by first using the data to estimate, for each total expenditure group, k, a regression of the form:

$$w_{ki} = a_{ik} + b_{ik} \log (y_k) + \frac{c_{ik}}{y_k} \tag{47}$$

for each commodity group, i. This specification has the advantage that predicted weights, based on the estimated parameters, add to unity. However,

Table 3: Budget Shares: All Households Combined

Commodity Group	a_i	b_i	c_i	$\overline{R}^2$
Current housing costs	0.8439	-0.0628	-311.4117	0.9603
Electricity, gas and other fuels	0.1688	-0.0130	328.4054	0.9874
Food and beverages	0.9671	-0.0697	-495.8344	0.9773
Spirits, beer and wine	0.1546	-0.0078	-431.7199	0.5450
Tobacco	0.2110	-0.0169	-335.5932	0.9281
Clothing and footwear	-0.0570	0.0096	-91.8081	0.8194
Furniture and appliances	-0.1774	0.0204	124.5691	0.8643
Postal and telephone charges	0.2572	-0.0185	64.6643	0.9580
Health services	0.2507	-0.0180	-395.9763	0.8101
Motor vehicles and parts	-0.0151	0.0153	-751.3042	0.8265
Recreational items	0.0937	0.0039	-826.4007	0.8096
Personal care products	0.0653	-0.0041	-67.4465	0.5269
Miscellaneous	-0.0782	0.0133	-200.5533	0.8326
House building payments	-1.6847	0.1514	3390.4360	0.8099

it does not guarantee that the predicted weights always lie in the range $0 < w < 1$: At the lowest levels of total expenditure, there were very small negative weights for some categories. These were suitably adjusted, and the required additions were subtracted from the groups with the largest expenditure weights (food, housing and fuel) so that the effects of the adjustments on other groups was minimal. The smoothed budget shares were then used to calculate the income, or total expenditure, elasticities.

Differentiating (47), and dropping the k subscript, gives:

$$\frac{dw_i}{dy} = \frac{b_i y - c_i}{y^2} \tag{48}$$

so that w_i unequivocally falls as y rises if $b_i < 0$ and $c_i > 0$; or if $c_i < 0$; so long as $y > c_i/b_i$: Alternatively, the share rises as income rises (that is, the income elasticity exceeds 1) if $b_i > 0$ and $c_i < 0$; or if $c_i > 0$; so long as $y > c_i/b_i$: From the results shown in Table 3, the shares fall, over most of the relevant range of total expenditure[13], in the cases of: current housing costs; electricity, gas and other fuels; food and non-alcoholic beverages; postal and telephone charges, health services; and personal care products. Alternatively, the shares rise as income rises in the case of: clothing and footwear; furniture and appliances; motor vehicles and parts; recreational items; miscellaneous; and house building payments. In the cases of alcohol and tobacco the budget shares initially rise before falling in the higher-income groups.

[13] In carrying out the regressions, y was measured in cents per week.

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