Two-way migration between similar countries[†]

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Abstract

We develop a model to explain two-way migration of high-skilled individuals between countries that are similar in their economic characteristics. High-skilled migration is explained by a combination of two features: In both countries there is a continuum of workers with differing abilities, which are private knowledge, and the production technology gives incentives to firms for hiring workers of similar ability. In the presence on migration cost, high-skilled workers self-select into the group of migrants, thereby ensuring they are hired together with other high-skilled migrants. The laissez-faire equilibrium features too much migration, explained by a negative migration externality, and as a result all individuals are worse off than in autarky. We also show that for sufficiently low levels of migration cost the optimal level of migration is strictly positive. In extensions to our basic model, we consider the presence of an internationally immobile factor and find that in this case the possibility of aggregate gains from migration in the laissez-faire equilibrium emerges. We also show that our basic results are robust with respect to small differences in countries' technologies.

1 Introduction

In this paper we develop a model to explain two-way migration of high-skilled individuals between developed countries. While there is strong evidence that this phenomenon is quantitatively important, it has attracted surprisingly little attention in the theoretical literature.[1]

The key challenge in explaining two-way migration of similar (highly skilled) individuals within a group of similar (high-income) countries – rather than one-way migration from developing to developed countries – lies in the fact that country differences cannot be expected to play a central role. The model we develop in the main part of this paper therefore uses the assumption that countries are identical in all respects (this assumption is relaxed later on). In both countries there is a continuum of workers with differing abilities, which are private knowledge. Migration is costly, and the cost is the same for all individuals. The production technology, borrowed from Kremer (1993), exhibits complementarities between the skill levels of individual workers, and profit maximising firms therefore aim for hiring workers of identical skill. High-skilled individuals from both countries self-select into emigration in order to separate themselves from low-skilled co-workers at home. Firms can distinguish natives and immigrants, and the resulting segmentation at labour markets allows them to form more efficient matches.[2]

The welfare effects of migration in our model are stark: In the laissez-faire equilibrium all individuals are worse off than in autarky. This result is due to a negative migration externality, which leads to too much migration in equilibrium. In particular, for the marginal migrant the cost of migration is equal to the expected individual wage gain due to the prospect of being matched with more able co-workers. The marginal migrant rationally ignores the fact that by

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[1]Lowell (2007) reports a net increase from 1975 and 2005 of two million tertiary-educated adults who migrated between developed countries, equivalent to an increase of 40 %. Figure 3.8 in OECD (2008) shows that the largest destination countries for high-skilled migration, the U.S. and Canada, have substantial emigration of high-skilled individuals as well. Even more remarkably, for the United Kingdom and Germany inward and outward migration of high-skilled individuals are very similar.

[2]While the implication of our basic model that there is no cross-matching between natives and migrants within firms is extreme and should not be taken literally, the more general prediction that there is a clustering of migrants within certain firms is well supported by empirical evidence (Portes and Wilson, 1980; Hellerstein and Neumark, 2008; Gárcia-Péréz, 2009; Andersson et al., 2010).

migrating he moves from being the most able native to being the least able migrant, thereby lowering the average skill of both population groups and hence the expected wage of individuals in both groups. We also show that for sufficiently low migration cost the level of migration chosen by an omniscient social planner is strictly positive (but of course lower than in the laissez-faire equilibrium), since the existence of migrants as a distinct group of individuals enables firms to match workers of more similar expected skill.[3] While aggregate gains from migration exist in the social planner equilibrium, the distributional effects are strong: all migrants gain relative to autarky, while all natives are worse off. These distributional effects are mitigated if the social optimum is implemented via a migration tax, since in this case the possibility of redistributing part of the gains to non-migrants exists.

In an extension we add an internationally immobile factor ("capital") that is essential in production for all firms. We show that migration is potentially more benign in this case than in our basic model since it allows for the more efficient allocation of capital between domestic firms, with firms hiring migrants having a higher capital intensity due to a capital-skill complementary that is well known from many models of migration. There is a potentially offsetting effect, however, since capital reallocation towards firms that hire migrants makes migration more attractive, ceteris paribus. It is shown that aggregate welfare gains exist if capital is sufficiently important in production, and at the same time migration cost is sufficiently low.

Another extension allows for small differences in countries' technologies. By gaining access to a better technology, workers from the low-tech country then have an additional incentive to migrate, while the opposite holds true for workers from the high-tech economy. Incorporating this modified incentive structure, we still find two-way migration which is, however, biased towards the technologically superior country, i.e. the high-tech country experiences net immigration while the low-tech country faces net emigration. Given cross country technology differences and the associated gains from arbitrage, welfare prospects for workers from the low-tech county brighten up, while workers from the high-tech country are even more worse off than they were in the baseline model. Most notably, the main result of our baseline model, that too much migra-

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[3]The option to migrate in our model effectively serves as a costly screening device. See Spence (1973) and Stiglitz (1975) for two of the seminal papers on screening.

tion occurs in the laissez faire equilibrium, is robust with respect to this change and continues to hold.

The only other theoretical paper we are aware of that aim to explains two-way migration of high-skilled individuals are Schmitt and Soubeyran (2006). In their model, individuals differ in their abilities, and they choose to be either entrepreneurs or workers, as in Lucas (1978). The career choice of individuals depends not only on their own ability, but also on relative ability distribution within each country. Migration occurs only if countries' ability distributions are sufficiently different, and interestingly the equilibrium may feature two-way migration of both entrepreneurs and workers. In an equilibrium with two way migration these individuals may well have the same role (entrepreneur or worker, respectively), but they have different skill levels. Two-way migration of similar individuals in Schmitt and Soubeyran (2006) is therefore of a very different nature than in our model. Another paper dealing with two-way migration between identical countries is Fan and Stark (2011). In this paper workers have no skill dimension and migrate because they do not want people within their reference group to observe them working in a job to which a certain social stigma is attached. Our paper is also related to Hendricks (2001) and Giannetti (2003), who use the same basic selection mechanism of high-skilled individuals into emigration as we do. But neither do these papers analyse two-way migration of high-skilled individuals, nor are they interested in a welfare analysis, which is the focus of our paper.

The paper proceeds as follows. Section two derives the baseline model of two-way, high-skilled migration between identical countries. The welfare effects are derived in section three. Section four considers two extension of the basic model to allow for non-mobile factors of production as well as country asymmetries. Section five concludes.

2 The Model

Consider a world with two perfectly symmetric countries, each populated by a heterogeneous mass of workers, which we normalize to one without loss of generality.[4] Workers in each country differ with respect to their skills, s, which are uniformly distributed over the interval $[0, 1]$, and which are assumed to be private information. We assume workers to be risk neutral, such that

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[4]Since countries are assumed to be symmetric, we suppress all country indices.

utility, $u(x) = x$, can be expressed as a linear function of consumption, x. Each country is a single sector economy producing a homogeneous *numéraire* good, y, under perfect competition, which is costlessly traded.

Following Kremer (1993) we assume a production technology which reflects two basic assumptions: *(i)* the production process consists of $i = 1, ..., n$ tasks, which *(ii)* all must be completed successfully in order to get some output. If only one task in a series of production steps is not performed correctly, the value of the final output good drops down to zero. The production technology hence features an extreme form of complementarity. For simplicity we assume that there are only two tasks, $n = 2$, and each task needs to be performed by a single worker. Whether a worker performs the task entrusted to him correctly depends on his skill level, $s \in [0, 1]$, which serves as a proxy for the individual probability of performing a task successfully.

We denote $2A$ as potential output that results when all tasks are performed correctly. Alternatively, one can think of $2A$ as a term capturing total factor productivity. With the number of tasks equal to 2 the parameter A would then be output per task if all tasks are performed successfully. Output of a firm that performs the production process exactly once can then be written as

$$y = f(s_1, s_2) = 2As_1s_2. \tag{1}$$

Wages cannot be based on individual ability, since this is private information. Consequently each worker is paid half the firm's output independent of their actual contribution. In an equilibrium that features migration, firms can identify an individual worker as a member of either the group of natives or the group of immigrants. This is the only information they can base their hiring decision on, and this information is valuable since, as we show below, the average skill of the two groups is different. Firms maximize their expected profits by choosing the optimal skill mix of their employees:

$$\max_{\bar{s}_1, \bar{s}_2} \pi(\bar{s}_1, \bar{s}_2) = 2A\bar{s}_1\bar{s}_2 - w(\bar{s}_1) - w(\bar{s}_2), \tag{2}$$

with $\bar{s}_i$, $i = 1, 2$, denoting the average skill of the group from which the worker for task i is hired, and $w(\bar{s}_i)$ the expected wage paid to this worker. Lemma 1 gives the solution to this optimization problem.

Lemma 1 *Firms maximize expected profits by hiring workers of the same expected skill.*

Proof See the appendix.

Using our profit sharing rule, the expected wage rate of an individual worker with skill s equals

$$w\left(s\right) = A\bar{s}_\ell s. \tag{3}$$

where $\bar{s}_\ell$ with $\ell \in \{L, H\}$ is the average skill of the group to which the individual belongs. We assume that migration is costly, and the cost is equal to c. It is now straightforward to show that our model leads to self-selection of the most able individuals into emigration.

To see this, consider some arbitrary cutoff ability $\tilde{s}$ that separates high-skill and low-skill individuals. The average skills in the two groups L and H are $\bar{s}_L = \tilde{s}/2$ and $\bar{s}_H = (1 + \tilde{s})/2$ due to our assumption of a uniform distribution, and the resulting difference between the averages of both groups, $\bar{s}_H - \bar{s}_L$, is equal to $1/2$ for all values of $\tilde{s}$. The expected wage gain for an individual worker of being paired with a co-worker from group H is now given by $A(\bar{s}_H - \bar{s}_L)s = As/2$, and it follows immediately that this gain is increasing in an individual's skill. With identical migration cost for each individual, and assuming an interior solution, $0 < \tilde{s} < 1$, it follows that high-skilled individuals self-select into migrating abroad, while low-skilled individuals are deterred from migration by the cost attached to it. For the indifferent worker with skill $\tilde{s}$ the condition $A\tilde{s}/2 = c$ holds, which immediately gives the migration cutoff in the *laissez-faire* equilibrium as

$$\tilde{s}_{\text{lf}} = \frac{2c}{A}. \tag{4}$$

Self-selection into migration, $\tilde{s} \in (0, 1)$, then obviously requires $c \in (0, A/2)$. Proposition 1 summarizes :

Proposition 1 *With strictly positive but not prohibitively high migration cost, all workers with skill $s > \tilde{s}_{\text{lf}} = 2c/A$ migrate abroad, while all workers with skill $s \le \tilde{s}_{\text{lf}} = 2c/A$ stay in their home country. Migration flows increase for a higher level of technology, A, and lower migration cost, c.*

Taking stock, our model is able to explain two-way, high-skilled migration flows between two ex ante and ex post symmetric countries, which are driven by the desire of high-skilled workers to get separated from their low-skilled counterparts.

3 Welfare

In order to analyze the welfare effects of migration, the natural comparison is the scenario of prohibitive migration cost $c \geq A/2$, which leads to $\tilde{s} = 1$ (the "autarky case"). The value of aggregate production equals total wage income, which in turn is given by

$$Y(\tilde{s}) = A \left[\int_0^{\tilde{s}} \left(\frac{\tilde{s}}{2} \right)^2 \mathrm{d}s + \int_{\tilde{s}}^1 \left(\frac{1+\tilde{s}}{2} \right)^2 \mathrm{d}s \right] = \frac{A\left[1 + \tilde{s}(1 - \tilde{s})\right]}{4}. \tag{5}$$

Total output is therefore minimized at autarky ($\tilde{s} = 1$) and maximized if exactly half the individuals become migrants ($\tilde{s} = 1/2$). Aggregate welfare equals the difference between total output and total migration cost:

$$W(\tilde{s}, c) = \frac{A[1 + \tilde{s}(1 - \tilde{s})]}{4} - c(1 - \tilde{s}). \tag{6}$$

We can now use the link between $\tilde{s}$ and c provided by (4) to express aggregate welfare in the laissez-faire equilibrium as a function of either variable:

$$W_{\mathrm{lf}}(\tilde{s}) = \frac{A\left[1 - \tilde{s}(1 - \tilde{s})\right]}{4}, \tag{7}$$

$$W_{\mathrm{lf}}(c) = \frac{A}{4} - c \left(\frac{1}{2} - \frac{c}{A} \right). \tag{8}$$

Aggregate welfare is therefore maximized at autarky ($\tilde{s} = 1$) and minimized if exactly half the individuals become migrants ($\tilde{s} = 1/2, c = A/4$).

Lastly, we look at individual welfare, which is identical to an individual's expected wage rate, net of migration cost if applicable. Non-migrants' and migrants' welfare is given by

$$w_{\mathrm{lf}}^L(\tilde{s}, s) = \frac{A\tilde{s}s}{2} \qquad \text{and} \qquad w_{\mathrm{lf}}^H(\tilde{s}, s) = \frac{A(1 + \tilde{s})s}{2} - c = \frac{A[s - \tilde{s}(1 - s)]}{2},$$

respectively. We see that all individuals are worse off than in the autarky equilibrium, where the expected wage rate of an individual with skill s is equal to $As/2$. For non-migrants, this simply happens because the pool of co-workers available for matching now has a lower average skill. For migrants, this is explained by a negative external effect induced by migration that can best be seen by a thought experiment, in which individual migration occurs sequentially, in the order of decreasing ability of migrants: Every migrant, apart from the most skilled one, in
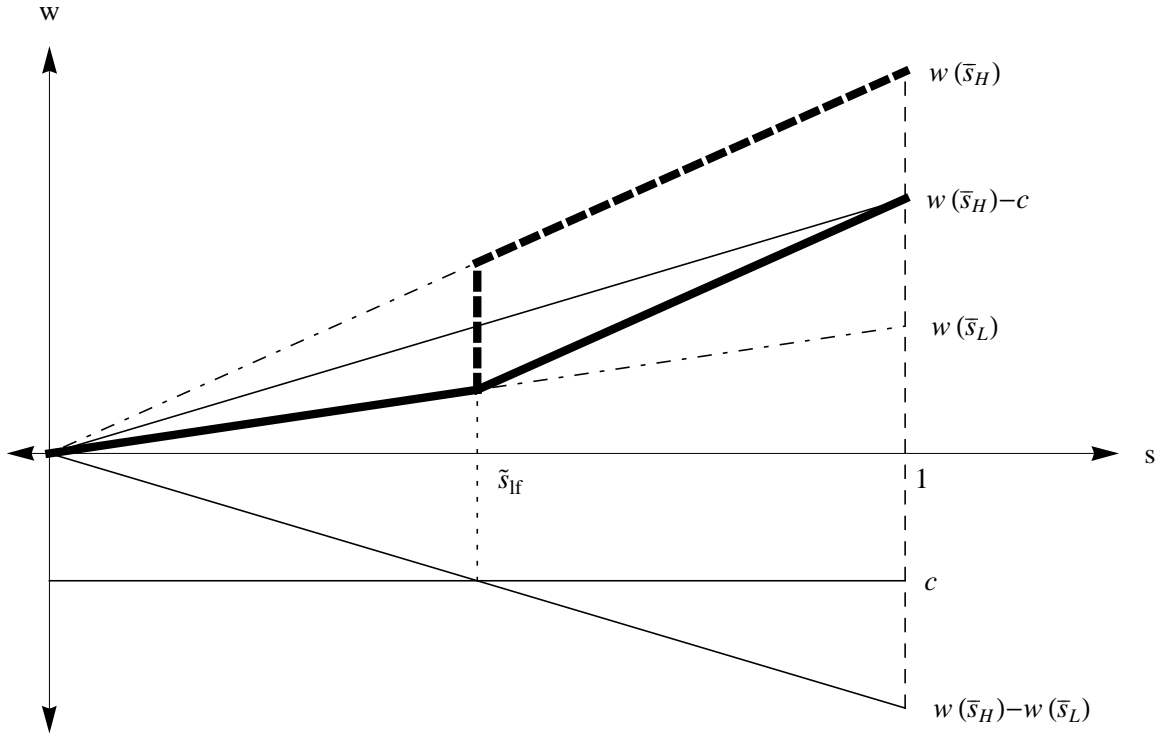
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Figure 1: *Laissez-faire equilibrium*

this case reduces the average skill of individuals in the migrant pool, thereby inflicting losses on infra-marginal migrants' wages. This effect is rationally ignored by individual migrants.

Figure 1 illustrates the results. The bottom quadrant shows how the migration cutoff is determined by the equality of migration cost and expected migration gain for the marginal migrant. The top quadrant shows in bold the resulting wage profile in the open economy as a function of individual ability s, where for migrants a distinction is made between the gross wage (bold dashed) and the net wage, which subtracts migration cost (bold solid). The wage profile in autarky is given by the thin solid line for comparison. Aggregate welfare is measured by the area under the autarky wage profile and open economy wage profile, respectively.

The main welfare implications of high-skilled migration are summarized as follows:

Proposition 2 *International migration leads to aggregate production gains, and to losses in aggregate welfare. Furthermore, all individuals are worse off in a migration equilibrium than in the autarky equilibrium.*

We now look at the social planner equilibrium. The social planner can freely choose the migration cutoff $\tilde{s}$, taking as given migration cost c, but disregarding individuals' migration incentives, which link $\tilde{s}$ to c in the *laissez-faire* equilibrium. Maximizing (6) with respect to $\tilde{s}$ gives the optimal migration cutoff $\tilde{s}_{\text{soc}}$, and hence the socially optimal extent of migration, as a function of c:

$$\tilde{s}_{\text{soc}} = \frac{1}{2} + \frac{2c}{A}. \tag{9}$$

Hence, while there is "too much" migration in the laissez-faire equilibrium due to the negative migration externality, the optimal level of migration is strictly positive if migration costs are sufficiently low. Note also that $\tilde{s}_{\text{soc}} > 1/2$ and therefore it is never socially optimal to have more than half the population emigrate. Finally, zero migration is enforced by the social planner ($\tilde{s}_{\text{soc}} = 1$) whenever $c \geq A/4$. Substituting (9) into (6) leads to

$$W_{\text{soc}}(c) = \frac{5A}{16} - c\left(\frac{1}{2} - \frac{c}{A}\right), \tag{10}$$

and it is easily checked that $W_{\text{soc}}(c)$ is strictly larger than autarky welfare $A/4$ for all non-prohibitive levels of c.

We now look at the effect that a socially optimal level of international migration has on individual wages. Clearly, non-migrants are worse off with any level of high-skill emigration, since the expected quality of their co-workers falls. Hence, we can restrict our attention to comparing the expected net wage of migrants in the social optimum with the respective wage in autarky. The net wage of migrants is given by

$$w_{\text{soc}}^H(\tilde{s}, c, s) = \frac{A(1 + \tilde{s}_{\text{soc}})s}{2} - c,$$

and, substituting for $\tilde{s}_{\text{soc}}$, it is immediate that there is a wage gain relative to autarky for migrants with skill $s > 4c/(4c + A)$. Simple algebra shows that this threshold value is strictly smaller than $\tilde{s}_{\text{soc}}$ as derived in (9), and therefore in the social optimum all migrants are better off than under autarky. Figure 2, which is directly analogous to Figure 1 (but for expositional purposes considers a smaller migration cost c), illustrates. In constructing Figure 2, we use the fact that from our results (4) and (9) we know that $\tilde{s}_{\text{soc}} = \tilde{s}_{\text{lf}} + 1/2$. Furthermore, the size of the jump in the wage profile in the upper quadrant at $\tilde{s}_{\text{soc}}$ is determined by the the wage gain

Figure 2: *Social planner equlibrium*

for the marginal migrant, which is determined in the lower quadrant. Proposition 3 summarizes the results:

Proposition 3 *The socially optimal level of migration is lower than in the laissez-faire equilibrium, if the latter features positive migration levels. For $c < A/4$ the socially optimal level of migration is strictly positive. In the social optimum, all migrants are better off than under autarky, while all non-migrants are worse off.*

The social optimum can alternatively be implemented by a tax on migration by both countries. In this case, individual incentives to migrate are again relevant, of course. We assume that a country's tax revenue is distributed equally to all nationals, independent of their residence, and hence does not affect the migration decision. Condition (4) now holds in a modified form, with effective migration cost $c + t$ replacing c:

$$\tilde{s} = \frac{2(c + t)}{A}$$

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Substituting for $\tilde{s}$ using $\tilde{s}_{\text{soc}}$ from (9), this implies $t = A/4$. Since countries are identical and the tax is imposed by both countries, it can alternatively be interpreted as an emigration tax or an immigration tax.[5]

We compare this scenario now with one where an immigration tax rate is set by a national government that wants to maximize domestic (rather than global) welfare. Domestic welfare equals the total wage bill of nationals, independent of their residential status, plus total transfer income. The latter is financed from immigration tax revenue, which is assumed to be distributed equally to all nationals, and hence does not affect their decision to emigrate. Since there is no cross-matching between immigrants and natives in our model, domestic welfare is independent of the number of immigrants. Hence the sole purpose of the immigration tax is to redistribute income from immigrants to natives, and the tax rate t_{noc} is set at its revenue maximizing level:

$$t_{\text{noc}} = \arg\max_{t} \left[(1 - \tilde{s})\, t\right] = \arg\max_{t} \left[\left(1 - \frac{2(c + t)}{A}\right) t\right] = \frac{A - 2c}{4}, \qquad (11)$$

with the resulting migration cutoff being equal to

$$\tilde{s}_{\text{noc}} = \frac{1}{2} + \frac{c}{A}. \qquad (12)$$

Comparison to (4) and (9) shows that this migration cutoff lies between the respective cutoffs in the laissez-faire equilibrium and the social optimum. Furthermore, the prohibitive migration cost is equal to $A/2$, as in the laissez-faire equilibrium. At this level of migration cost, the non-cooperative immigration tax, t_{noc}, is zero.

Aggregate welfare for each country in the non-cooperative equilibrium is given the difference between the value of total output and total migration cost, as in the previous two cases considered. Tax revenue has no net effect in the case where countries are symmetric. Substituting (11) into (6) leads to

$$W\left(\tilde{s}_{\text{noc}}\right) = \frac{5A}{16} - \frac{c^2}{4A} - c\left(\frac{1}{2} - \frac{c}{A}\right). \qquad (13)$$

It is easily checked that welfare under non-cooperative tax setting

(i) approaches the social optimum for $c \to 0$, and autarky welfare for $c \to A/2$

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[5]Note that t exactly matches the difference between the wage gain from migration and the corresponding migration costs at $\tilde{s}_{\text{soc}}$ in Figure 2.
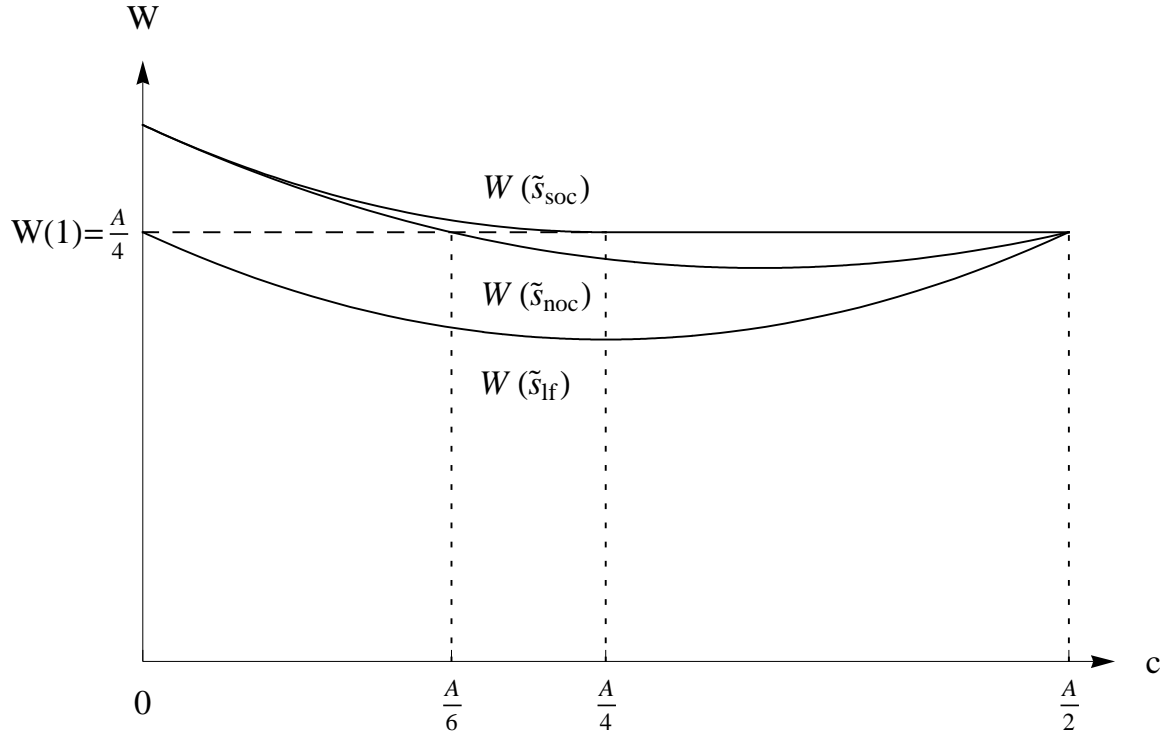
Figure 3: *Aggregate Welfare*

(ii) is lower than welfare under autarky if and only if $A/6 < c < A/2$

(iii) is minimized for $c = A/3$

The results are summarized as follows.

Proposition 4 *The bilateral migration flows in the non-cooperative equilibrium are smaller than in the laissez-faire equilibrium and larger than in the social optimum. Aggregate welfare is higher than in the laissez-faire equilibrium, and also higher than in autarky if migration cost is sufficiently low.*

Figure 3 illustrates the aggregate welfare results derived in this section.

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4 Extensions

Two important characteristics of the model presented in sections 2 and 3 are that *(i)* internationally mobile labour is the only factor of production, and that *(ii)* countries are ex ante identical in all respects. We now check the robustness of our results by changing those key assumptions. In section 4.1 we add an internationally immobile factor of production to the model. In section 4.2 we consider the effect of country asymmetries in our benchmark model without an immobile factor.

4.1 Internationally immobile factors of production

In this subsection, we add internationally immobile capital to our model. Capital is modeled as an essential input in all firms, and hence we introduce an interaction between migrants and domestic factors of production that is standard in most migration models, but has not been a feature of our basic model. The production technology is unchanged with respect to labour, i.e. there are two tasks, which have to be performed by exactly one worker each, and following Kremer (1993) we assume that capital is combined with labour in a Cobb-Douglas fashion. The resulting production function is given by

$$y = f\left(s_1, s_2, k\right) = 2As_1 s_2 k^\alpha, \tag{14}$$

with $\alpha \in [0,1]$ denoting the partial production elasticity of capital and k being the per capita capital stock used in production. With firms knowing only the average skill within the groups, L and H, lemma 1 implies positive assortative matching of group members. We can therefore separately write down the reduced form profit maximization problem of both types of firms as

$$\max_{k_\ell} \ \pi_\ell\left(k_\ell\right) = 2A\bar{s}_\ell^2 k_\ell^\alpha - 2w\left(\bar{s}_\ell\right) - rk_\ell \ \forall \ \ell \in \{L, H\}. \tag{15}$$

The profit maximising level of capital depends on whether the firm employs individuals from group H or L. It follows from

$$\frac{\partial \pi_\ell\left(k_\ell\right)}{\partial k_\ell} = 2A\alpha\bar{s}_\ell^2 k_\ell^{\alpha-1} - r \overset{!}{=} 0 \ \forall \ \ell \in \{L, H\}, \tag{16}$$

and we get the standard result that the rate of return to capital, r, equals its value marginal product. Using equation (16) in combination with the full employment condition

$$\bar{k} = \tilde{s} k_L + (1 - \tilde{s}) k_H, \tag{17}$$

as well as $\bar{s}_L = \tilde{s}/2$ and $\bar{s}_H = (1 + \tilde{s})/2$ allows us to solve for the amount of capital used by firms solely employing natives or migrants, respectively:

$$k_L = \left[\tilde{s} + (1 - \tilde{s}) \left(\frac{1 + \tilde{s}}{\tilde{s}} \right)^{\frac{2}{1-\alpha}} \right]^{-1} \bar{k}, \tag{18}$$

$$k_H = \left[(1 - \tilde{s}) + \tilde{s} \left(\frac{\tilde{s}}{1 + \tilde{s}} \right)^{\frac{2}{1-\alpha}} \right]^{-1} \bar{k}, \tag{19}$$

and it is easily checked that $k_H \geq \bar{k} \geq k_L$. Hence, firms employing workers of a higher expected ability, which in equilibrium will be firms employing migrants, have a higher capital intensity.

In analogy to section 2, wages are determined by splitting available revenue (now the difference between total firm revenue and payments to capital) equally between the two workers. The expected wage of a worker with ability s is then given by

$$w^\ell(s) = A \bar{s}_\ell s (1 - \alpha) k_\ell^\alpha \ \ \forall \ \ell \in \{L, H\}, \tag{20}$$

using (15), (16) and the fact that profits are zero due to perfect competition and free market entry. Capital returns are distributed equally among the nationals of a country, and hence capital ownership does not distort the decision to migrate. In analogy to the baseline model, the laissez-faire migration equilibrium is then determined by the condition that the wage gain for the marginal migrant is equal to the migration cost. We get

$$\tilde{s}_{\text{lf}} = \frac{2c}{A(1 - \alpha)\bar{k}^\alpha} (\Phi)^{-1} \tag{21}$$

with

$$\Phi \equiv (1 + \tilde{s}) \left(\frac{k_H}{\bar{k}} \right)^\alpha - \tilde{s} \left(\frac{k_L}{\bar{k}} \right)^\alpha \geq 1,$$

where the inequality is strict whenever $\alpha > 0$. Comparison with (4) shows that the relative size of the laissez-faire migration cutoffs in the two models depends on two effects. A larger value for $(1 - \alpha)\bar{k}^\alpha$ increases migration flows since the migration cost falls in relation to average

income. The second effect is given by Φ^{-1}, and it shows that an additional incentive to migrate exists in the extended model, which stems from the reallocation of domestic capital towards firms employing (more productive) migrants.

Turning to the welfare implications that migration has in the framework described above it can be shown that, as it was the case in the baseline model, migration increases total output in both countries. Deriving $Y_{\text{lf}}(\tilde{s}) \geq Y(1)$ analytically requires a lengthy proof which is delegated to the appendix. However, the intuition for this result should be clear. Migration not only leads to a more efficient matching of workers within their groups it also enables a more profitable use of capital which can be reallocated toward more productive firms and therefore unleashes additional efficiency gains.
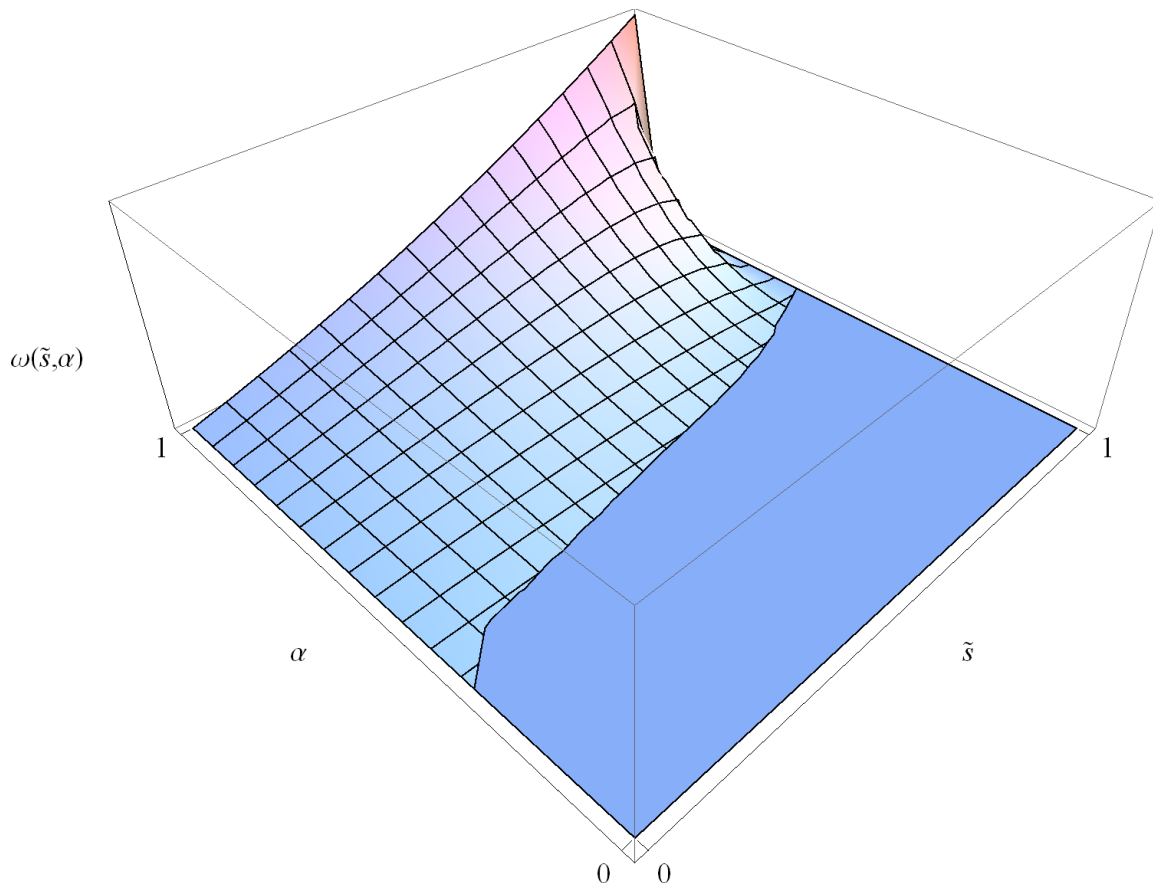

Figure 4: *Aggregate welfare in a model with capital*

For aggregate welfare the negative aspects of migration no longer dominate. Going through the same steps as in the baseline model, we find that aggregate welfare in the laissez-faire migration equilibrium is given by

$$W_{\text{lf}}(\tilde{s}, \alpha) = \frac{A \left[k_H^\alpha - \left(2\Phi(1-\alpha)\bar{k}^\alpha - k_H^\alpha \right) \tilde{s}(1-\tilde{s}) - (k_H^\alpha - k_L^\alpha) \tilde{s}^3 \right]}{4}, \tag{22}$$

and it is easily checked that autarky welfare is equal to $W(1, \alpha) = A\bar{k}^\alpha/4$. We can now compute the relative welfare levels in the migration equilibrium and in autarky, $\omega(\tilde{s}, \alpha) \equiv W_{\text{lf}}(\tilde{s}, \alpha) / W(1, \alpha)$, where aggregate migration gains exist whenever $\omega(\tilde{s}, \alpha) > 1$.

Figure 4 plots $\omega(\tilde{s}, \alpha)$ for all combinations of $\tilde{s}$ and α, where combinations that lead to $\omega(\tilde{s}, \alpha) > 1$ are highlighted in the figure by a grid surface. All other combinations lead to aggregate welfare losses from migration. We find that in contrast to our baseline model that abstracts from complementarities in production between internationally mobile and immobile factors, there exists now a non-trivial parameter space where welfare losses from the negative migration externality are overcompensated by the efficiency gains resulting from the reallocation of capital towards migrant-employing firms. The results are summarised as follows:

Proposition 5 *International migration leads to aggregate production gains. For high (low) values of α the model features aggregate welfare gains (losses).*

Turning to the social planner's solution, one can show that the socially optimal level of migration will be lower than the one in the *laissez faire* equilibrium given by equation (21).[6] It is easy to see why: Adding capital to the model opens up a new channel for gains from migration, but does not add a new distortion. Hence, the migration externality discussed in the previous section remains the only distortion in the model. As an immediate consequence migration levels in the *laissez faire* equilibrium will in general be too high.

4.2 Country-specific technology

We now introduce country asymmetries to our baseline model without capital by assuming $A^D \neq A^F$, where A^D denotes the technology level of the domestic economy while A^F refers to

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[6]The formal proof is available from the authors upon request.

the corresponding technology parameter in the foreign economy. Recalling equation (3), the indifference conditions for the marginal migrant can be written as

$$\frac{A^j}{2}\left(\tilde{s}^j\right)^2 = \frac{A^l}{2}\left(1 + \tilde{s}^j\right)\tilde{s}^j - c \quad \forall \quad j,l \in \{D,F\} \quad \text{with} \quad j \neq l, \tag{23}$$

in which we have used $\bar{s}_L^j = \tilde{s}^j/2$ and $\bar{s}_H^j = (1 + \tilde{s}^j)/2$. Solving for $\tilde{s}_{\text{lf}}^j$ yields

$$\tilde{s}_{\text{lf}}^j = \frac{A^l - \sqrt{\left(A^l\right)^2 + 8\left(A^l - A^j\right)c}}{2\left(A^j - A^l\right)} \quad \forall \quad j,l \in \{D,F\} \quad \text{with} \quad j \neq l. \tag{24}$$

It is now easy to check that the technologically superior country experiences net immigration, i.e. for $A^l > A^j$ we have $\tilde{s}_{\text{lf}}^l > \tilde{s}_{\text{lf}}^j$. Moreover, taking the total differential of equation (23) with respect to $\tilde{s}_{\text{lf}}^j$, c, A^j and A^l reveals that $\frac{\partial \tilde{s}_{\text{lf}}^j}{\partial c} > 0$ and $\frac{\partial \tilde{s}_{\text{lf}}^j}{\partial A^j} > 0 > \frac{\partial \tilde{s}_{\text{lf}}^j}{\partial A^l}$, if countries are not too dissimilar, i.e. $2/3 < A^D/A^F < 3/2$. This is the case we focus on henceforth. Thus, emigration increases if the technology in the destination country gets better while it falls if the same occurs in the source country. For higher cost of migration, c, the flows of workers between both economies decline. The prohibitive level migration cost is now also country-specific: Setting $\tilde{s}_{\text{lf}}^j = 1$ in (24), we find that emigration occurs from country j whenever $c < (2A^l - A^j)/2$.

Turning to the welfare implications of migration, aggregate output of nationals from country $j \in \{D,F\}$ can be expressed analogously to equation (5) as

$$Y^j\left(\tilde{s}^j\right) = \frac{\left(A^j - A^l\right)\left(\tilde{s}^j\right)^3}{4} + \frac{A^l\left[1 + \tilde{s}^j\left(1 - \tilde{s}^j\right)\right]}{4}, \tag{25}$$

with $j,l \in \{D,F\}$ and $j \neq l$. In an equilibrium with strictly positive migration levels, aggregate output produced by the natives of country j is higher than under autarky if (25) is larger than $Y^j(1) = A^j/4$ for all $\tilde{s}^j < 1$. To obtain aggregate welfare, migration cost c must be taken into account, such that

$$W^j\left(\tilde{s}^j, c\right) = \frac{\left(A^j - A^l\right)\left(\tilde{s}^j\right)^3}{4} + \frac{A^l\left[1 + \tilde{s}^j\left(1 - \tilde{s}^j\right)\right]}{4} - \left(1 - \tilde{s}^j\right)c, \tag{26}$$

for all $j,l \in \{D,F\}$ with $j \neq l$. In analogy to the baseline model we can use the link between migration cost and the laissez-faire migration cutoff in (24) to express aggregate welfare as a

function of either $\tilde{s}^j$ or c:

$$W_{\text{lf}}^j \left(\tilde{s}^j \right) = \frac{A^j \left(\tilde{s}^j \right)^2 \left(2 - \tilde{s}^j \right)}{4} + \frac{A^l \left[1 - \left(\tilde{s}^j \right)^2 \right] \left(1 - \tilde{s}^j \right)}{4}, \tag{27}$$

$$W_{\text{lf}}^j \left(c \right) = \frac{A^l \left(A^l - 10c - 2A^j \right) + 8A^j c + \left(A^l + 2c \right) \sqrt{\left(A^l \right)^2 + 8 \left(A^l - A^j \right) c}}{8 \left(A^l - A^j \right)}. \tag{28}$$

Migration leads to aggregate welfare gains for the natives of country j whenever (27) is maximised for $\tilde{s}^j < 1$. In this case, (28) can be used to derive the necessary condition for the migration cost. We find the following results:

Proposition 6 *Migration generates aggregate production gains for the natives of both countries. Aggregate welfare is lower in a migration equilibrium than under autarky for nationals of the country with the better technology. For nationals of the technologically inferior country, aggregate welfare gains from migration exist if migration costs are sufficiently low.*

Proof See the appendix.

From the baseline model we know that aggregate welfare in a migration equilibrium is driven by two facts. On the one hand, there are output gains from more efficient matching at labor markets, while, on the other hand, the negative migration externality induces too much migration, such that in the case of identical countries output gains are completely eaten up migration cost. Allowing for country asymmetries adds an additional source for welfare gains or losses. By migrating to another country migrants might gain/lose by accessing a superior/inferior production technology. Not surprising there exist no aggregate welfare gains for nationals from the technological superior country, since they have to give up their technologically advantage when going abroad. Conversely nationals from the technologically inferior country may gain in the aggregate from the better technology access, provided migration cost is not too high and, hence, the negative external effect of migration is not too strong.

As in the baseline model, we now turn to the social planner's solution. The migration cutoff that a social planner would chose results from the differentiation of the aggregate welfare functions, $W^j \left(\tilde{s}^j, c \right)$, given by equation (26) with respect to $\tilde{s}^j$. Doing so yields

$$\tilde{s}_{\text{soc}}^j = \frac{A^l - \sqrt{\left(A^l \right)^2 + 3 \left(A^l + 4c \right) \left(A^l - A^j \right)}}{3 \left(A^j - A^l \right)} \quad \forall \quad j, l \in \{D, F\} \quad \text{with} \quad j \neq l. \tag{29}$$

Note that the socially optimal migration cutoff, $\tilde{s}^j_{\text{soc}}$, has the same basic properties as the migration cutoff in the *laissez faire* equilibrium, $\tilde{s}^j_{\text{lf}}$. More precisely, we have $\tilde{s}^l_{\text{soc}} > \tilde{s}^j_{\text{soc}}$ for $A^l > A^j$ and $\frac{\partial \tilde{s}^j_{\text{soc}}}{\partial c} > 0$ as well as $\frac{\partial \tilde{s}^j_{\text{soc}}}{\partial A^j} > 0 > \frac{\partial \tilde{s}^j_{\text{soc}}}{\partial A^l}$, provided that countries are not too dissimilar,[7] i.e. $3/4 < A^D/A^F < 4/3$. Comparing the socially optimal migration cutoff, $\tilde{s}^j_{\text{soc}}$, with the one obtained in the *laissez faire* equilibrium, $\tilde{s}^j_{\text{lf}}$, results in the following proposition.

Proposition 7 *The socially optimal levels of migration are lower than in the laissez-faire equilibrium, if the latter features positive migration levels.*

Proof See the appendix.

According to proposition 7 the main result from the baseline model continues to hold also in the richer environment of asymmetric countries. As before the individual migrant rationally ignores the negative external effect of his migration decision on the average skill of natives and other migrants, such that in the *laissez faire* equilibrium too much migration occurs. Taking the biased incentive structure in a model with country asymmetries into account the social planner corrects for the migration externality by setting lower migration levels than in the *laissez faire* equilibrium.

5   Conclusion

In this paper we have developed a model that can explain two-way migration of high-skilled individuals between countries at the same level of economic development. The baseline model is extremely simple, but for this very reason it is transparent as well, and it furthermore lends itself to a comprehensive welfare analysis. We identify a negative externality from migration, resulting from the fact that the marginal migrant ignores the negative effect his migration decision has on

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[7]Note that the parameter constraint given here is more restrictive than the one imposed on equation (24). However, there is a simple explanation for this. Suppose we have the case of either $2/3 < A^D/A^F < 3/4$ or $4/3 < A^D/A^F < 3/2$. It is then easy to show that the social planner would not allow emigration from the technologically superior country irrespective of the underlying migration cost, c, and hence chooses $\tilde{s}^j_{\text{soc}} = 1$ for $A^j > A^l$. As a consequence the comparative static results derived above are no longer valid for $\tilde{s}^j_{\text{soc}} = 1$, but continue to hold for $\tilde{s}^l_{\text{soc}}$.

expected wages of both natives and migrants. As a consequence, there is too much migration in the laissez-faire equilibrium with positive migration cost, and aggregate welfare is lower than in autarky. We even find that all individuals in this case lose from migration. This does not mean, however, that *all* migration is socially harmful. We find that if migration cost is sufficiently low a social planner would choose strictly positive migration levels. The negative migration externality in this case has to be traded off against the better quality of matches within firms that can be achieved due to the existence of a well-defined high-skill group, comprising the migrants.

Aggregate gains from migration re-emerge as a possible feature of the laissez-faire equilibrium once our baseline framework is amended by standard features known from other migration models. Once we introduce a second factor of production that is internationally immobile and a complement to labour in the production function, aggregate gains from migration exist despite the persistent negative externality, provided the income share of this factor is sufficiently high and migration cost is sufficiently low. The welfare gains in this case result from a more efficient domestic allocation of internationally immobile factors of production, notably in the absence of any country asymmetries that would normally be responsible for positive welfare effects of migration.

Allowing for country asymmetries in form of small technology differences allows workers from the technologically inferior country to realize additional gains from migration by accessing a better technology. As a consequence we find two-way, high-skilled migration which is biased towards the technologically superior country. Gaining a better technology access, additionally pushes the wages of migrants from the low-tech country, such that for workers from this country aggregate welfare gains may arise even in the laissez faire equilibrium. This should, however, not hide the fact that due the negative migration externality aggregate welfare in the laissez faire equilibrium will be still smaller than in the social planner's solution which features restricted migration.

6 Appendix

6.1 Proof of lemma 1

In order to prove lemma 1 it suffices to show that given production function (2) firms optimally decide to match only workers of the same expected skill, such that $\bar{s}_i = \bar{s}_\ell$ with $i = 1, 2$ and $\ell \in \{L, H\}$. The simple proof presented here is taken from Basu (1997, pp. 35). For a more general proof of positive assortative matching see Becker (1991, p. 130) or Sattinger (1975).

Consider two different arbitrary average skill levels, $\bar{s}_L$ and $\bar{s}_H$, with $\bar{s}_H > \bar{s}_L$. A firm facing optimization problem (2) now has three different possibilities of pairing workers:

$$\pi\left(\bar{s}_H, \bar{s}_H\right) = 2A\bar{s}_H^2 - 2w\left(\bar{s}_H\right), \tag{A.1}$$

$$\pi\left(\bar{s}_L, \bar{s}_L\right) = 2A\bar{s}_L^2 - 2w\left(\bar{s}_L\right), \tag{A.2}$$

$$\pi\left(\bar{s}_H, \bar{s}_L\right) = 2A\bar{s}_H\bar{s}_L - w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right). \tag{A.3}$$

Let us first suppose $\pi\left(\bar{s}_H, \bar{s}_L\right) \geq \pi\left(\bar{s}_H, \bar{s}_H\right)$ which results in the following chain of inequalities

$$2A\bar{s}_H\bar{s}_L - w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right) \geq 2A\bar{s}_H^2 - 2w\left(\bar{s}_H\right), \tag{A.4}$$

$$2A\bar{s}_H\left(\bar{s}_H - \bar{s}_L\right) \leq w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right), \tag{A.5}$$

$$2A\bar{s}_L\left(\bar{s}_H - \bar{s}_L\right) < w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right), \tag{A.6}$$

$$2A\bar{s}_H\bar{s}_L - w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right) < 2A\bar{s}_L^2 - 2w\left(\bar{s}_L\right), \tag{A.7}$$

where $\bar{s}_H > \bar{s}_L$ has been utilized to derive inequality (A.6) from (A.5). Note that inequality (A.7) implies $\pi\left(\bar{s}_L, \bar{s}_L\right) \geq \pi\left(\bar{s}_H, \bar{s}_L\right)$. Now imagine $\pi\left(\bar{s}_L, \bar{s}_H\right) \geq \pi\left(\bar{s}_L, \bar{s}_L\right)$ giving rise to

$$2A\bar{s}_H\bar{s}_L - w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right) \geq 2A\left(\bar{s}_L\right)^2 - 2w\left(\bar{s}_L\right), \tag{A.8}$$

$$2A\bar{s}_L\left(\bar{s}_H - \bar{s}_L\right) \geq w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right), \tag{A.9}$$

$$2A\bar{s}_H\left(\bar{s}_H - \bar{s}_L\right) > w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right), \tag{A.10}$$

$$2A\left(\bar{s}_H\right)^2 - 2w\left(\bar{s}_H\right) > 2A\bar{s}_H\bar{s}_L - w\left(\bar{s}_H\right) - w\left(\bar{s}_L\right), \tag{A.11}$$

where again $\bar{s}_H > \bar{s}_L$ has been utilized to derive inequality (A.10) from (A.9). Inequality (A.11) implies $\pi\left(\bar{s}_H, \bar{s}_H\right) \geq \pi\left(\bar{s}_H, \bar{s}_L\right)$. Taking stock profits from positive assortative matching always surpass profits from cross matching, such that firms always decide to pair workers of identical skill, i.e. $\bar{s}_i = \bar{s}_\ell$. ∎

6.2 Proof of aggregate output gains from migration

In order to show that output in a migration equilibrium is at least as high as in an autarky equilibrium

$$Y_{\text{lf}}(\tilde{s}) = \tilde{s}Y_L(\tilde{s}) + (1-\tilde{s})Y_H(\tilde{s}) \geq Y(1), \tag{A.12}$$

must hold, where $Y_\ell(\tilde{s}) \; \forall \; \ell \in \{L, H\}$ is total output of firms employing only type ℓ workers. By inspection of (15) and (16) it follows that $\alpha Y_\ell(\tilde{s}) = r_{\text{lf}}(\tilde{s})k_\ell \; \forall \; \ell \in \{L, H\}$, as well as $\alpha Y(1) = r(1)\bar{k}$ for $\tilde{s} = 1$. In combination with $\bar{k} = \tilde{s}k_L + (1-\tilde{s})k_H$, which follows from the full employment condition (17), the above relationships can be used to rewrite equation (A12) as

$$r_{\text{lf}}(\tilde{s}) \geq r(1). \tag{A.13}$$

Using expression (18) in combination with $\bar{s}_H = \frac{1+\tilde{s}}{2}$ in equation (16) allows to solve for

$$r_{\text{lf}}(\tilde{s}) = \psi(\alpha, \tilde{s})^{1-\alpha} \frac{A\alpha}{2}\bar{k}^{\alpha-1} \geq r(1) = \frac{A\alpha}{2}\bar{k}^{\alpha-1}, \tag{A.14}$$

which holds true if

$$\psi(\alpha, \tilde{s}) \equiv \left(1 - \tilde{s}^2\right)(1+\tilde{s})^{\frac{1+\alpha}{1-\alpha}} + \tilde{s}^2\tilde{s}^{\frac{1+\alpha}{1-\alpha}} \geq 1. \tag{A.15}$$

Note that $\psi(\alpha, \tilde{s}) \geq 1 \; \forall \; \alpha, \tilde{s} \in [0, 1]$ if $\psi(0, \tilde{s}) \geq 1$ and $\frac{\partial \psi(\alpha, \tilde{s})}{\partial \alpha} \geq 0 \; \forall \; \alpha, \tilde{s} \in [0, 1]$. It is easy to verify that

$$\psi(0, \tilde{s}) = 1 + \tilde{s}(1-\tilde{s}) \geq 1 \; \forall \; \tilde{s} \in [0, 1]. \tag{A.16}$$

Furthermore note that

$$\frac{\partial \psi(\alpha, \tilde{s})}{\partial \alpha} = \frac{2\left[\left(1 - \tilde{s}^2\right)\ln(1+\tilde{s})(1+\tilde{s})^{\frac{1+\alpha}{1-\alpha}} + \tilde{s}^2\ln(\tilde{s})\tilde{s}^{\frac{1+\alpha}{1-\alpha}}\right]}{(1-\alpha)^2} \geq 0, \tag{A.17}$$

requires

$$\left(1 - \tilde{s}^2\right)\ln(1+\tilde{s})(1+\tilde{s})^{\frac{1+\alpha}{1-\alpha}} + \tilde{s}^2\ln(\tilde{s})\tilde{s}^{\frac{1+\alpha}{1-\alpha}} \geq$$
$$\left(1 - \tilde{s}^2\right)\ln(1+\tilde{s})(1+\tilde{s})^{\frac{1+\alpha}{1-\alpha}} + \tilde{s}^2\ln(\tilde{s})(1+\tilde{s})^{\frac{1+\alpha}{1-\alpha}} \geq 0, \tag{A.18}$$

where we have exploited the fact that the first term in inequality above is positive while $\ln(\tilde{s}) \leq 0 \; \forall \; \tilde{s} \in [0, 1]$. The inequality above can be transformed into

$$\zeta(\tilde{s}) \geq \xi(\tilde{s}), \tag{A.19}$$

with

$$\zeta\left(\tilde{s}\right) \equiv \ln\left(1+\tilde{s}\right) \geq 0 \quad \text{and} \quad \xi\left(\tilde{s}\right) \equiv \tilde{s}^2 \ln\left(\frac{1+\tilde{s}}{\tilde{s}}\right) \geq 0, \tag{A.20}$$

for all $\tilde{s} \in [0,1]$. Note that $\zeta\left(\tilde{s}\right) \geq \xi\left(\tilde{s}\right) \; \forall \; \tilde{s} \in [0,1]$, if $\zeta\left(0\right) = \xi\left(0\right)$, $\zeta\left(1\right) = \xi\left(1\right)$, $\zeta'\left(\tilde{s}\right) > 0$, $\xi'\left(\tilde{s}\right) \geq 0$ as well as $\zeta''\left(\tilde{s}\right) < 0 < \xi''\left(\tilde{s}\right) \; \forall \; \tilde{s} \in [0,1]$. Figure 4 illustrates the specification described above. In the following we will show that all the conditions imposed above hold true
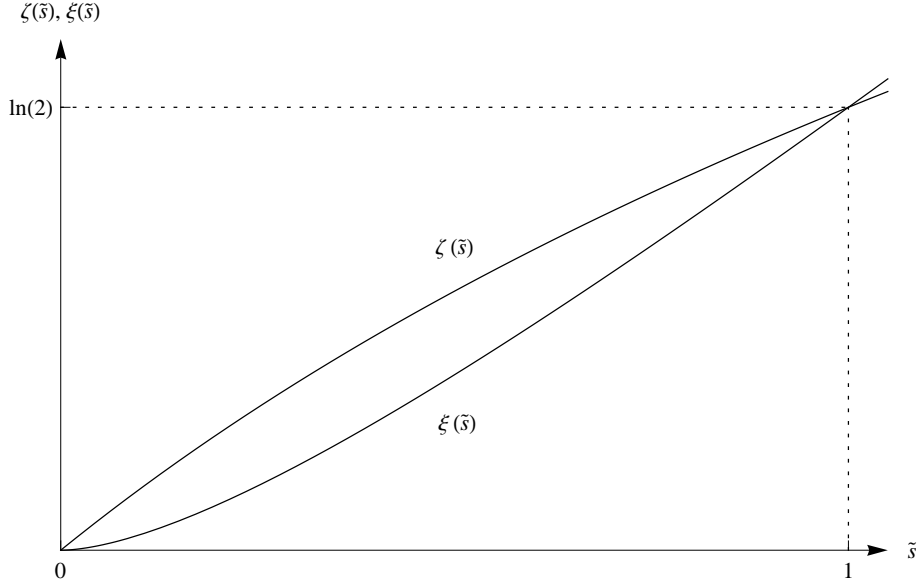

Figure 5: *Illustration of $\zeta\left(\tilde{s}\right) \geq \xi\left(\tilde{s}\right)$*

for $\tilde{s} \in [0,1]$. We start with $\zeta\left(0\right) = 0$ which follows immediately from (A.20). Note that for $\lim_{\tilde{s}\to 0} \xi\left(\tilde{s}\right)$ a undefined expression results. In order to use De l'hôpital's rule we introduce

$$\chi\left(\tilde{s}\right) \equiv \ln\left(\frac{1+\tilde{s}}{\tilde{s}}\right) \quad \text{and} \quad \eta\left(\tilde{s}\right) \equiv \frac{1}{\tilde{s}^2}, \tag{A.21}$$

with $\xi\left(\tilde{s}\right) = \chi\left(\tilde{s}\right)/\eta\left(\tilde{s}\right)$. Now applying De l'hôpital's rule yields

$$\lim_{\tilde{s}\to 0} \xi\left(\tilde{s}\right) = \lim_{\tilde{s}\to 0}\left[\frac{\chi'\left(\tilde{s}\right)}{\eta'\left(\tilde{s}\right)}\right] = \lim_{\tilde{s}\to 0}\left[\frac{\tilde{s}^2}{2\left(1+\tilde{s}\right)}\right] = 0, \tag{A.22}$$

such that $\zeta\left(0\right) = \xi\left(0\right)$ is fulfilled. By inspection of (A.20) it follows that $\zeta\left(1\right) = \xi\left(1\right) = \ln\left(2\right) > 0$. In the next step we compute $\zeta'\left(\tilde{s}\right) = 1/\left(1+\tilde{s}\right) > 0$ for all $\tilde{s} \in [0,1]$. Not that in order to show that $\xi'\left(\tilde{s}\right) \geq 0 \; \forall \; \tilde{s} \in [0,1]$ we can express $\xi'\left(\tilde{s}\right)$ as

$$\xi'\left(\tilde{s}\right) = \tilde{s}\left[\delta\left(\tilde{s}\right) - \gamma\left(\tilde{s}\right)\right], \tag{A.23}$$

with

$$\delta\left(\tilde{s}\right) \equiv 2\ln\left(\frac{1+\tilde{s}}{\tilde{s}}\right) > 0 \quad \text{and} \quad \gamma\left(\tilde{s}\right) \equiv \frac{1}{1+\tilde{s}} > 0. \tag{A.24}$$

Note that $\xi'\left(\tilde{s}\right) \geq 0 \ \forall \ \tilde{s} \in [0,1]$ requires $\delta\left(0\right) > \gamma\left(0\right)$, $\delta\left(1\right) > \gamma\left(1\right)$ and $\delta'\left(\tilde{s}\right), \gamma'\left(\tilde{s}\right) < 0 \ \forall \ \tilde{s} \in [0,1]$. We find $\lim_{\tilde{s}\to 0}\delta\left(\tilde{s}\right) \to \infty$ and $\gamma\left(0\right) = 1$ as well as $\delta\left(1\right) = 2\ln\left(2\right) > \gamma\left(1\right) = \frac{1}{2}$. Furthermore note that $\delta'\left(\tilde{s}\right) = -2/\left[\left(1+\tilde{s}\right)\tilde{s}\right] < 0$ and $\gamma'\left(\tilde{s}\right) = -1/\left(1+\tilde{s}\right)^2 < 0$. Hence $\xi'\left(\tilde{s}\right) \geq 0 \ \forall \ \tilde{s} \in [0,1]$. When computing the second derivatives of $\xi\left(\tilde{s}\right)$ and $\zeta\left(\tilde{s}\right)$, we find $\zeta''\left(\tilde{s}\right) = -1/\left(1+\tilde{s}\right)^2 < 0$, while $\xi''\left(\tilde{s}\right)$ can be written as

$$\xi''\left(\tilde{s}\right) = \rho\left(\tilde{s}\right) - \lambda\left(\tilde{s}\right), \tag{A.25}$$

with

$$\rho\left(\tilde{s}\right) \equiv 2\ln\left(\frac{1+\tilde{s}}{\tilde{s}}\right) + \frac{\tilde{s}}{\left(1+\tilde{s}\right)^2} \quad \text{and} \quad \lambda \equiv \left(\tilde{s}\right)\frac{3}{1+\tilde{s}}. \tag{A.26}$$

As before it suffice to show that $\rho\left(0\right) > \lambda\left(0\right)$, $\rho\left(1\right) > \lambda\left(1\right)$ and $\rho'\left(\tilde{s}\right), \lambda'\left(\tilde{s}\right) < 0$. By inspection of (A.26) it follows that $\lim_{\tilde{s}\to 0}\rho\left(\tilde{s}\right) \to \infty$ and $\lambda\left(0\right) = 3$ as well as $\rho\left(1\right) = 2\ln\left(2\right) + \frac{1}{4} > \lambda\left(1\right) = \frac{3}{2}$. Furthermore note that

$$\rho'\left(\tilde{s}\right) = \frac{\tilde{s} - 2\left(1+\tilde{s}\right)^2 - \tilde{s}^3}{\left(1+\tilde{s}\right)^3\tilde{s}} < 0 \ \forall \ \tilde{s} \in [0,1], \tag{A.27}$$

$$\lambda'\left(\tilde{s}\right) = -\frac{3}{\left(1+\tilde{s}\right)^2} < 0, \tag{A.28}$$

which implies $\xi''\left(\tilde{s}\right) > 0 \ \forall \ \tilde{s} \in [0,1]$. It follows that $r_{\text{lf}}\left(\tilde{s}\right) \geq r\left(1\right)$ and $Y_{\text{lf}}\left(\tilde{s}\right) \geq Y\left(1\right)$. ∎

6.3 Proof of Proposition 6

The proof of proposition 6 proceeds in two steps. Before proving the statements concerning aggregate welfare we first turn to the proof of the results regarding aggregate output. In order to show that $Y^j\left(\tilde{s}^j\right) \geq Y^j\left(1\right) = A^j/4 \ \forall \ 2/3 < A^D/A^F < 3/2$ note that this inequality can be rewritten as

$$\left(A^l - A^j\right)\left[1 - \left(\tilde{s}^j\right)^3\right] + A^l\tilde{s}^j\left(1 - \tilde{s}^j\right) \geq 0, \tag{A.29}$$

where $Y^j\left(\tilde{s}^j\right)$ has been substituted from equation (25), while $Y^j\left(1\right) = A^j/4$. Clearly for $A^l \geq A^j$ inequality (A.29) always holds. Now suppose $A^l < A^j$. We then have

$$\left(A^l - A^j\right)\left[1 - \left(\tilde{s}^j\right)^3\right] + A^l\tilde{s}^j\left(1 - \tilde{s}^j\right) \geq$$
$$\left(A^l - A^j\right)\left(1 - \tilde{s}^j\right) + A^l\left(1 - \tilde{s}^j\right) \geq 0, \tag{A.30}$$

which holds whenever $1/2 \leq A^D/A^F \leq 2$ and hence completes the first part of the proof.

Now we prove that $W^j\left(\tilde{s}^j\right) \leq W^j\left(1\right) = A^j/4 \ \forall \ A^j \geq A^l$. Note that for $\tilde{s}^j = 1$ we obtain $W^j\left(1\right) = A^j/4$ from equation (27). Using this together with $W^j\left(\tilde{s}^j\right)$ from equation (27) we obtain

$$
\begin{aligned}
W^j\left(\tilde{s}^j\right) - W^j\left(1\right) &= \left(A^l - A^j\right)\left[1 + \left(\tilde{s}^j\right)^3\right] - A^l\left(1 + \tilde{s}^j\right)\tilde{s}^j + A^j 2\left(\tilde{s}^j\right)^2 \\
&\leq \left(A^l - A^j\right)\left[1 + \left(\tilde{s}^j\right)^3 - 2\left(\tilde{s}^j\right)^2\right],
\end{aligned}
\tag{A.31}
$$

where the last line is non-positive whenever $1 + \left(\tilde{s}^j\right)^3 - 2\left(\tilde{s}^j\right)^2 \geq 0$ and $A^j \geq A^l$. Since $1 + \left(\tilde{s}^j\right)^3 - 2\left(\tilde{s}^j\right)^2$ has a local maximum at $\tilde{s}^j = 0$ and intersects the abscissa at $\tilde{s}^j = 1$ and $\tilde{s} = 1/2 \pm \sqrt{5/4}$, we have $1 + \left(\tilde{s}^j\right)^3 - 2\left(\tilde{s}^j\right)^2 \geq 0 \ \forall \ \tilde{s} \in [0,1]$ and, hence, $W^j\left(\tilde{s}^j\right) \leq W^j\left(1\right) \ \forall \ A^j \geq A^l$. In order to complete the proof of proposition 6 it remains to show that for $A^l > A^j$ we have $W^j\left(c\right) \geq A^j/4 \ \forall \ 0 \leq c \leq \frac{1}{4}\left(2A^l - 3A^j + \sqrt{4\left(A^l\right)^2 - 8A^j A^l + 5\left(A^j\right)^2}\right)$, while $W^j\left(c\right) < A^j/4 \ \forall \ \frac{1}{4}\left(2A^l - 3A^j + \sqrt{4\left(A^l\right)^2 - 8A^j A^l + 5\left(A^j\right)^2}\right) < c < \frac{1}{2}\left(2A^l - A^j\right)$. Using equation (28) it can be shown that $W\left(c\right) - A^j/4 = 0$ has three solutions, which are

$$
c_1 = \frac{1}{2}\left(2A^l - A^j\right),
\tag{A.32}
$$

$$
c_2 = \frac{1}{4}\left(2A^l - 3A^j + \sqrt{4\left(A^l\right)^2 - 8A^j A^l + 5\left(A^j\right)^2}\right),
\tag{A.33}
$$

$$
c_3 = \frac{1}{4}\left(2A^l - 3A^j - \sqrt{4\left(A^l\right)^2 - 8A^j A^l + 5\left(A^j\right)^2}\right).
\tag{A.34}
$$

Note that since $2/3 < A^D/A^F < 3/2$, soltution (A.34) is negative and therefore economically irrelevant. Solution (A.32) equals the prohibitive migration cost at which $\tilde{s}^j = 1$. Finally, it is easily checked that $0 < \frac{1}{4}\left(2A^l - 3A^j + \sqrt{4\left(A^l\right)^2 - 8A^j A^l + 5\left(A^j\right)^2}\right) < \frac{1}{2}\left(2A^l - A^j\right)$. Since equation (28) implies $W\left(0\right) = A^l/4 > A^j/4$ we can immediately infer that for low migration cost, i.e. $0 \leq c \leq \frac{1}{4}\left(2A^l - 3A^j + \sqrt{4\left(A^l\right)^2 - 8A^j A^l + 5\left(A^j\right)^2}\right)$ aggregate welfare gains exist, while for high migration cost, $\frac{1}{4}\left(2A^l - 3A^j + \sqrt{4\left(A^l\right)^2 - 8A^j A^l + 5\left(A^j\right)^2}\right) < c < \frac{1}{2}\left(2A^l - A^j\right)$, aggregate losses result. ∎

6.4 Proof of Proposition 7

In order to show that $\tilde{s}^j_{\text{soc}} > \tilde{s}^j_{\text{lf}}$ we have to consider two scenarios where either $A^j > A^l$ or $A^j < A^l$. We start with $A^j < A^l$ which allows us to rewrite $\tilde{s}^j_{\text{soc}} > \tilde{s}^j_{\text{lf}}$ as

$$-4c\left(A^l - c\right)\left(A^l - A^j\right) < A^j \left(A^l\right)^2, \tag{A.35}$$

which holds true for the economically relevant parameter space, $0 \leq c \leq \frac{1}{2}\left(2A^l - A^j\right)$ and, thus, completes the first part of the proof. Now suppose $A^j > A^l$, such that $\tilde{s}^j_{\text{soc}} > \tilde{s}^j_{\text{lf}}$ can be expressed as

$$-4c\left(A^l - c\right)\left(A^l - A^j\right) - A^j\left(A^l\right)^2 < -\frac{4}{3}c\left(A^l - c\right) - \left(A^l\right)^2 < \frac{4}{3}c^2 - \left(A^l\right)^2 < 0, \tag{A.36}$$

where we have made use of the fact that $\frac{2}{3}A^j < A^l < A^j$. The last inequality in (A.36) holds true, since the economically relevant parameter space $0 \leq c \leq \frac{1}{2}\left(2A^l - A^j\right)$ translates into $0 \leq c \leq \frac{1}{2}A^l$ for $\frac{2}{3}A^j < A^l < A^j$, such that we can be sure that for $0 \leq c \leq \frac{1}{2}\left(2A^l - A^j\right)$ we have $\tilde{s}^j_{\text{soc}} > \tilde{s}^j_{\text{lf}}$. $\blacksquare$

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