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Globalization, Employment, and Income: Analyzing the Adjustment Process

By C. Davidson and S. Matusz

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Globalization, Employment, and Income:

Analyzing the Adjustment Process

by

C. Davidson and S. Matusz

Abstract

In this paper we construct and analyze a general equilibrium trade model that explicitly accounts for the dynamic aspects of labor market adjustment that occur when trade is liberalized. We show how empirically observable parameters of the labor market determine the rate at which labor is released from the contracting sector and is absorbed into the expanding sector and therefore influence the magnitude and extent of the losses associated with trade reform. We also show that the economy may overshoot the new steady state during adjustment and that the length of the adjustment process is likely to be non-trivial.

Outline

Non-Technical Summary

Textbook models of international trade always posit well-functioning, frictionless factor markets. Policy-induced changes in relative output prices lead to the instantaneous reallocation of resources. While trade may influence the distribution of income, resulting in some winners and losers, factors are never unemployed and (ignoring terms-of-trade effects) the efficiency gains from trade liberalization always result in aggregate net benefits.

Rather than focusing on the well-understood benefits of liberalization, some policy makers and editorialists tend to focus on the potentially costly aspects of resource reallocation. Most workers who lose their jobs due to liberalization will find new employment opportunities, but there is typically a period of active search before such opportunities are found. Indeed, some workers may find that they have to re-tool before qualifying for employment in growing sectors. At the other end of the spectrum, some workers with little training and little innate ability may find themselves facing employment prospects so bleak that they choose to exit the labor force. Depending on the magnitude of the various effects, it is conceptually possible for the losses that occur during transition to outweigh the steady-state benefits of trade reform.

Our purpose in this paper is to construct and analyze a general equilibrium trade model that explicitly accounts for the dynamic aspects of labor market adjustment. Unlike earlier work in this area, we show how empirically observable parameters of the labor market determine the rate at which labor is released from the contracting sector and is absorbed into the expanding sector and therefore influence the magnitude and extent of the losses associated with trade reform. As a byproduct of the analysis, we are also able to show how the same parameters exert their own independent influence on the pattern and volume of trade.

After developing our model, we choose plausible parameters with the intent of using the model to simulate the employment effects resulting from the removal of a five percent import tariff. We find that the unemployment rate overshoots its new steady-state level and that the value of net output (measured at world prices) falls below what it would have been had the tariff not been removed. Moreover, we show that the value of output remains below that benchmark level for an extended length of time. However, the value of output ultimately rises above the level that would have been obtained had the tariff not been removed. In our theoretical model, adjustment costs can never be large enough to outweigh the gross benefits of liberalization. However, as we find in our numeric exercise, adjustment costs can be almost as large as the gross benefits of liberalization

In one final application of our model, we show how it can be used to shed some light on the Bhagwati-Dehejia thesis that increased globalization has led to increased job turnover, and therefore has affected the distribution of income.

1. Introduction

Textbook models of international trade always posit well-functioning, frictionless factor markets. Policy-induced changes in relative output prices lead to the instantaneous reallocation of resources. While trade may result in some winners and losers (ala Stolper-Samuelson), factors are never unemployed and (ignoring terms-of-trade effects) the efficiency gains from trade liberalization always result in aggregate net benefits.

Rather than focussing on the well-understood benefits of liberalization, some policy makers and editorialists tend to focus on the potentially costly aspects of resource reallocation. Most workers who lose their jobs due to liberalization will find new employment opportunities, but there is typically a period of active search before such opportunities are found. Indeed, some workers may find that they have to re-tool before qualifying for employment in growing sectors. At the other end of the spectrum, some workers with little training and little innate ability may find themselves facing employment prospects so bleak that they choose to exit the labor force. Depending on the magnitude of the various effects, it is conceptually possible for the losses that occur during transition to outweigh the steady-state benefits of trade reform.

Our purpose in this paper is to construct and analyze a general equilibrium trade model that explicitly accounts for the dynamic aspects of labor market adjustment. Unlike earlier work in this area, we show how empirically observable parameters of the labor market determine the rate at which labor is released from the contracting sector and is absorbed into the expanding sector and therefore influence the magnitude and extent of the losses associated with trade reform.[1] As a byproduct of the analysis, we are also able to show how the same parameters exert their own independent influence on the pattern and volume of trade.

After developing the model in the next section, we parameterize it in section 3 and use it to trace out the movement of the unemployment rate subsequent to the removal of a five percent import tariff. We find that the unemployment rate overshoots its new steady-state level as expected. We turn to the welfare analysis in section 4 where we show that subsequent to

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[1] For example, see Lapan (1976), Magee (1976), Baldwin, Mutti, and Richardson (1980), and Neary (1982). Winters and Takacs (1991) examine the likely impact on employment in the British footwear industry should trade restrictions be lifted. Their study, based upon the natural rate of voluntary separations observed in that industry, is similar in spirit to ours. However, they miss some important general equilibrium effects by focussing on a single industry.

liberalization, the value of net output (measured at world prices) falls below what it would have been had the tariff not been removed and remains below that benchmark level for an extended length of time. However, the value of output ultimately rises above the level that would have been obtained had the tariff not been removed. In our numeric exercise, the present discounted value of output under free trade is higher than it is with the tariff in place. In section 5, we show how our model can be used to shed some light on the Bhagwati-Dehejia thesis that increased globalization has led to increased job turnover, and therefore has affected the distribution of income.[2] Finally, we provide some suggestions for future research in section 6.

2. The Model

2.1 Labor Market Dynamics

To keep the model simple and to focus on labor market dynamics, we assume that labor is the only input in the production process. To allow us to examine the issues of interest, we introduce training costs and search frictions into the labor market. In particular, we assume that a worker must first undertake a period of training in order to obtain a job in either sector. Once training is complete, the worker must conduct a time-consuming search for employment.[3]

We formulate the model in a continuous-time framework and assume that transitions from one employment or training status to another follow a Poisson process. Eight parameters completely specify all transitions in this economy (four parameters for each of two sectors).[4] Unemployed workers searching for employment in sector i find jobs at rate $e_i > 0$. Employed workers in sector i lose their jobs at rate $b_i > 0$. Finally, workers training for employment in sector i exit the training process at rate $t_i > 0$. The Poisson process allows the nice

[2] See Bhawati (1998).

[3] We introduce worker heterogeneity in the next section where we assume that workers differ in their basic ability and therefore productivity differs across the population.

[4] Generalization to any number of sectors is a fairly trivial task. None of our qualitative results depend on the number of sectors.

interpretation that, for example, the expected duration of a spell of unemployment in sector i is $\frac{1}{e_i}$.[5]

We want the model to be able to capture the notion that some skills are general while others are job-specific. General skills are those that transfer across jobs while job-specific skills do not. We therefore assume that in each sector there is a probability $f_i \in [0,1]$ that a sector-*i* worker who loses his job can forego retraining and can immediately begin searching for a new job in the same sector.

The dynamics that occur within a given sector are illustrated in Figure 1 and are made explicit in (1) - (3). Define L_i^E, L_i^S, L_i^T as the measure of workers employed, searching for employment, or training for employment in the i^{th} sector. Let a dot above a variable indicate the derivative of that variable with respect to time. Then the measure of workers in each category evolve as follows:

(1) $$\dot{L}_i^E = e_i L_i^S - b_i L_i^E$$

(2) $$\dot{L}_i^S = f_i b_i L_i^E + t_i L_i^T - e_i L_i^S$$

(3) $$\dot{L}_i^T = (1 - f_i) b_i L_i^E - t_i L_i^T .$$

The change in employment over time equals the measure of workers who successfully complete the search process ($e_i L_i^S$) less the measure of separations ($b_i L_i^E$). The pool of searchers expands when workers lose their jobs but retain their skills ($f_i b_i L_i^E$) and when workers complete training ($t_i L_i^T$). On the other hand, the pool contracts when searchers successfully find employment ($e_i L_i^S$). Finally, the measure of workers in training expands

[5] In this paper, we treat the labor market turnover rates as exogenous. In reality, these rates can be affected by a variety of factors. Workers can affect their reemployment probability by varying search intensity, the rate at which jobs dissolve may be affected by trade policy, and the length of the training period may depend on worker effort and/or the amount of resources devoted to training by firms. Including these features in order to make the model more realistic would cause us to sacrifice tractability. In particular, adding these features would make it impossible to obtain a closed form solution for the adjustment path. Future research will be required in order to determine how sensitive our results are to these assumptions.

when workers lose their jobs and require retraining prior to search ($(1-f_i)b_i L_i^E$) and shrinks when workers complete training ($t_i L_i^T$).

Let L_i represent the total measure of workers attached to sector *i*. By definition, $L_i = L_i^T + L_i^S + L_i^E$. In writing (1) - (3), we have implicitly assumed that L_i is held constant. For example, all workers who are training in sector *i* were once employed in that sector. There are no inflows from the other sector. In fact, intersectoral flows play a critical role in the adjustment process and we explicitly consider such flows below when we discuss the impact of trade liberalization. Until then, we have two adding-up constraints, expressed as (4) and (5):

$$(4) \qquad \dot{L}_i^E + \dot{L}_i^S + \dot{L}_i^T = \dot{L}_i = 0$$

$$(5) \qquad L_1 + L_2 \leq L$$

where *L* is the total measure of labor available in the economy. The inequality in (5) allows for the possibility that some workers may choose to opt out of the labor force.[6]

Given L_i, it is straightforward to solve (1) - (3) for the steady-state values of L_i^E, L_i^S, and L_i^T. Doing so, we obtain (6) - (8).

$$(6) \qquad L_i^E = \left(\frac{e_i t_i}{(1-f_i)e_i b_i + (e_i + b_i)t_i} \right) L_i$$

$$(7) \qquad L_i^S = \left(\frac{b_i t_i}{(1-f_i)e_i b_i + (e_i + b_i)t_i} \right) L_i$$

$$(8) \qquad L_i^T = \left(\frac{(1-f_i)e_i b_i}{(1-f_i)e_i b_i + (e_i + b_i)t_i} \right) L_i .$$

[6] In calculating the economy's unemployment rate, we exclude those who are training from the definition of the labor force. When in the text we refer to someone who is not in the labor force, we mean to refer to someone who is neither training, nor searching, nor employed.

2.2 The Allocation of Labor Across Sectors

In the previous section, we took the allocation of labor (L_1 and L_2) across sectors as given. We show in this section how these values are determined endogenously by the behavior of income-maximizing workers.[7] In our model, workers cannot choose to become employed. Rather, they choose a sector in which to train. Each worker makes this decision based on the discounted lifetime income that he could expect to earn if he were to train in a particular sector. Once this decision is made, the worker undertakes training until its (exogenously determined) completion, at which time he becomes a searcher, and then ultimately an employee. The purpose of this section is to formalize this decision process.

As stated in the introduction, we would like our model to account for worker heterogeneity in terms of innate abilities. To this end, we define the ability level of a type-j worker as a_j and assume that this parameter is uniformly distributed over the interval [0,1]. We then assume that higher-ability workers are more productive than lower-ability workers are.[8] In particular, we assume that a worker with ability a_j can produce $q_i a_j$ units of output when employed in sector *i*. In what follows, we assume that labor is the only input, so that

(9) $$w_i(a_j) = p_i q_i a_j$$

where p_i is the price of the i^{th} good and $w_i(a_j)$ is the wage earned by a type-j worker employed in sector *i.*

We now have all of the assumptions necessary to determine the discounted expected lifetime income of a type-j worker contingent upon his current labor market status. Let $V_i^E(a_j)$ represent the discounted expected lifetime income of a type-j worker who is currently employed in sector i, let $V_i^S(a_j)$ represent the discounted expected lifetime income of a

[7] Assuming risk-neutrality, there is no difference between the decisions made by income-maximizing workers and those made by utility-maximizing workers in our model. To lighten the already cumbersome notation, we therefore formulate the decision-making process based on income maximization. However, we will have to deflate income by an appropriate price index when we consider welfare effects resulting from a change in prices.
[8] We could also allow training costs to vary by ability without changing any of the substantive results that follow.

worker who is searching for a job in sector *i*, and let $V_i^T(a_j)$ represent the discounted expected lifetime income of a type-j worker who is currently training for a job in sector *i*. Given the discount rate *r* and the wage rate w_i, the asset-value equation for a worker who is employed in the i^{th} sector can be written as

$$(10) \qquad rV_i^E(a_j) = w_i(a_j) + b_i\left[f_i V_i^S(a_j) + (1 - f_i)V_i^T(a_j) - V_i^E(a_j)\right]$$

To interpret (10), think of the discounted expected income generated by employment as an asset. Then $rV_i^E(a_j)$ is the flow income that is generated by the asset. This is equal to the instantaneous wage adjusted by the capital loss that would be realized if employment were terminated. The capital loss is represented by the expression in brackets. In the event of job loss, there is a probability (f_i) that the worker will not have to retrain before searching for a new job. In that event, the worker has an asset with a value of $V_i^S(a_j)$. Otherwise, the worker does have to retrain and therefore has an asset worth $V_i^T(a_j)$. The capital loss is multiplied by b_i, the rate at which losses are realized.[9]

For simplicity, we assume that workers who are currently searching for employment earn no income and incur no explicit costs. As such, the asset value equation for a searching worker can be expressed as in (11):

$$(11) \qquad rV_i^S(a_j) = e_i\left[V_i^E(a_j) - V_i^S(a_j)\right]$$

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[9] Shapiro and Stiglitz (1984) provide the generic technique for deriving the asset-value equations in their footnote 8. Consider a small interval of time [0,t]. During this period of time, the expected lifetime utility of a worker employed in sector *i* is

$$V_i^E(a_j) = w_i t + e^{-rt}\left[b_i t f_i V_i^S(a_j) + b_i t(1 - f_i)V_i^T(a_j) + (1 - b_i t)V_i^E(a_j)\right].$$

Substitute $1 - rt$ for e^{-rt}, solve for $V_i^E(a_j)$ as a function of $V_i^S(a_j)$ and $V_i^T(a_j)$ and take the limit of the resulting expression as $t \to 0$ to obtain (1). All remaining asset-

Since searchers earn no income, the flow value of the asset just equals the capital gain (the expression in square brackets) multiplied by the rate at which the gain is realized.

Finally, we assume that those engaged in training earn no income, but must pay an instantaneous cost equal to $p\, c_i$, where c_i is measured in units of sector-*i* output. Workers exit training and begin searching at a flow rate of t_i. Given these assumptions, the asset value equation for a worker who is currently training becomes:

$$(12) \qquad rV_i^T(a_j) = -p_i c_i + t_i\left[V_i^S(a_j) - V_i^T(a_j)\right]$$

Given a worker's level of ability, equations (10) - (12) can be solved for the six endogenous variables $\left(V_i^E(a_i), V_i^T(a_i), \text{and } V_i^S(a_i)\right)$ in terms of the exogenous parameters of the model. Defining $D_i \equiv (r+t_i)(r+e_i+b_i)+(1-f_i)e_i b_i$ to lighten the notation, we then have:

$$(13) \qquad rV_i^E(a_i) = \left\{\frac{(r+t_i)(r+e_i)}{D_i}\right\} w_i(a_j) \quad - \quad \left\{\frac{(1-f_i)(r+e_i)b_i}{D_i}\right\} p_i c_i$$

$$(14) \qquad rV_i^S(a_j) = \left\{\frac{(r+t_i)e_i}{D_i}\right\} w_i(a_j) \quad - \quad \left\{\frac{(1-f_i)e_i b_i}{D_i}\right\} p_i c_i$$

$$(15) \qquad rV_i^T(a_j) = \left\{\frac{e_i t_i}{D_i}\right\} w_i(a_j) \quad - \quad \left\{\frac{r(r+e_i+b_i)+(1-f_i)e_i b_i}{D_i}\right\} p_i c_i$$

Equations (13) - (15) have fairly clean interpretations. First, suppose that the discount rate is zero and that all skills are job-specific, so that $f_i = 0$. Then the lifetime income for a worker is independent of the worker's current status. This follows from the fact that the Poisson process implies that the expected durations of employment, search, and training are equal to $\frac{1}{b_i}, \frac{1}{e_i}$, and $\frac{1}{t_i}$, respectively. Therefore the ratio of time spent on the job relative to total time (employed, searching, or training) is $\frac{e_i t_i}{D_i}$. Likewise, the ratio of time spent training relative to total time is $\frac{e_i b_i}{D_i}$.

Putting the pieces together implies that the flow rate of income is a weighted average of the income earned when employed and the costs incurred while training, where the weights equal the share of time spent in each activity.[10] The existence of a strictly positive discount rate implies that more weight is placed on the current activity at the expense of the weight placed on the future activity. For example, an employed person places more weight on the current wage than on the future costs of training, while a person who is training places more weight on training costs than on the wage. Finally, greater transferability of skills (higher values for f_i) implies a smaller share of a worker's lifetime spent in training.[11]

From (12) - (15), it is clear that, regardless of current status, discounted expected lifetime income is *increasing* in the wage rate, the share of skills that are transferable, the rate at which training is completed, and the rate at which searching workers become employed. Regardless of current status, discounted expected lifetime income is *decreasing* in the job separation rate and in the cost of training. An increase in the discount rate places more weight on the current activity. Therefore, an increase in the discount rate increases the discounted expected lifetime income of a worker who is currently employed, but reduces the discounted expected lifetime income of a worker who is currently training or searching.

Now begin by considering an untrained worker of ability a_j who is trying to determine whether to train for a job in sector *i* or whether to opt out of the labor force. A worker will choose to train for a job in sector *i* if and only if the following two conditions are satisfied:

$$(16) \qquad V_i^T(a_j) \geq V_k^T(a_j) \qquad i \neq k$$

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[10] The share of time spent searching (during which no income is earned) is $\frac{t_i b_i}{D_i}$.

[11] For example, when $f_i = 1$, no worker ever returns to training once training is completed. All elements of (13) and (14) associated with training costs vanish.

$$(17) \qquad V_i^T(a_j) \geq 0.$$

At this point, it is useful to provide an interpretation of our two sectors so that we may sensibly place more structure on the model.

We can think of sector 1 as the "low-tech" sector where jobs are plentiful. In the limiting case, studied below, e_1 tends to infinity and jobs are instantaneously found. Furthermore, we can envision the skills necessary to perform any particular task in this sector as being very much specific to the job. For example, a store clerk may need to learn the layout of the store in which he is employed, the procedures involved in opening the store in the morning, the functioning of a particular type of cash register, and so on. These sorts of skills do not transfer across jobs. We capture this notion by setting $f_1 = 0$. By contrast, we can think of sector 2 as the "high-tech" sector where more training is necessary for employment and where jobs are not instantaneously available upon completion of the training. Formally, this means $c_2 > c_1, t_2 < t_1$ and e_2 is strictly finite. In addition, the sector-2 jobs that are available require relatively more general, and therefore transferable, knowledge. For example, the most important part of a lawyer's training is learning the law. Therefore we assume that $0 < f_2 < 1$.

Given our interpretation of the two sectors, it seems reasonable to set up the model so that higher-ability workers sort into the high-tech sector and lower-ability workers sort into the low-tech sector. This will be the case only if expected lifetime training costs are higher in sector 2 than in sector 1 and if expected lifetime income increases more rapidly with ability in sector 2 than it does in sector 1.[12] These restrictions are implicit in the way we have drawn Figure 2.

We have drawn in Figure 2 a representative worker's discounted expected lifetime income if he obtains training in sector *i*. It is easily seen from inspection of (14) that $V_i^T(a_j)$ is linear and increasing a_j.

[12] These conditions are necessary, but not sufficient. It might be possible, for example, that all workers would choose to train in sector 1 if training costs are extremely high in sector 2.

Using (9) to substitute for the wage, letting e_1 tend to infinity and setting $f_1 = 0$, we can solve (15) for the level of ability at which the discounted expected lifetime income of training in sector 1 equals zero. Denoting this level of ability by a_L, we have

$$(18) \qquad a_L = \frac{r+b_1}{t_1}\frac{1}{q_1}.$$

A worker with ability $a_j < a_L$ would earn negative lifetime income if he were to enter the labor force. That is, the income he could expect to earn while actually employed cannot compensate for the costs incurred while training. There exist no appealing job opportunities in this economy for these low-ability workers.

The ability level denoted by a_H in Figure 2 is the solution to $V_1^T(a_j) = V_2^T(a_j)$. This is the critical level of ability below which workers choose to train in sector 1 (or opt out of the labor force for sufficiently low ability) and train in sector 2 otherwise.

Recall that we have assumed a uniform distribution of ability. Therefore, the proportion of the labor force sorting into the low-tech sector is $(a_H - a_L)$ and the proportion sorting into the high-tech sector is $(1 - a_H)$. Defining L_0 to be the measure of workers who opt out of the labor force, we then have

$$(19.a) \qquad L_0 = a_L L$$

$$(19.b) \qquad L_1 = (a_H - a_L)L$$

$$(19.c) \qquad L_2 = (1 - a_H)L.$$

2.3 Equilibrium

Substituting (9) into (15), it is easy to see that $V_i^T(a_j)$ is proportional to p_i. This implies that the ability level at which $V_i^T(a_j) = 0$ is independent of price. In the context of Figure 2, a higher value of p_i merely rotates the $V_i^T(a_j)$ curve counterclockwise about its intercept with respect to the ability axis.

Consider the steady-state supply side of the economy. Take good 1 to be the numeraire and define $p \equiv p_2$. For a sufficiently low value of *p*, the discounted expected lifetime income of training in sector 2 just equals that for training in sector 1 for the person with the highest ability level (i.e., for the person for whom $a_j = 1$). In terms of Figure 2, this means that $a_H = 1$. Given the assumed parameters of the model, this means that in the steady state no one would choose to train in sector 2 and therefore there would ultimately be no employment or production in that sector. As *p* increases, a_H falls and the proportion of the labor force training in sector 2 expands while the proportion training in sector 1 falls. This leads to more output of good 2 and less of good 1 in the steady state. As *p* tends to infinity, the $V_2^T(a_j)$ curve becomes vertical. There are some workers who just do not have sufficiently high ability to profitably train in sector 2. To an extent, this is an artifact of our assumption that training costs in sector *i* are paid in units of that sector's output. Increases in p_i raise both the wage and the cost of training simultaneously.[13]

Combining the logic in the previous paragraph with an assumption that all workers have identical homothetic preferences, we can sketch the relative demand (RD) and steady-state relative supply curves (RS) for this economy (see Figure 3). The intersection of the two curves determines the steady-state value of autarkic prices.

Observe that demand shifts change both outputs *and* prices despite the fact that we have assumed a Ricardian production technology. This follows from the fact that workers have heterogeneous abilities, of which some are better suited to the low-tech sector and some of which are better suited to the high-tech sector.

Note also that there is a role for absolute advantage (in terms of the production technology) in determining the pattern of trade. To see this, imagine two countries identical in every respect except that one country is proportionately more productive in both sectors compared with the other country. At any given price, $\frac{w_2}{w_1}$ will be the same in both countries (holding worker

[13] As we will see below, the same logic implies that changes in trade policy that result only in changing prices cannot squeeze workers out of the labor market.

ability constant). However, in the high productivity country w_i is relatively large compared with training costs in both sectors. As such, some workers who would opt out of the labor force in the low-productivity country choose to train in sector 1 in the high productivity country. Furthermore, some workers with moderate ability who would train in sector 1 in the low-productivity country will choose to train in sector 2 in the high productivity country. Output in both sectors expands, but the expansion is not necessarily proportional. Which sector experiences the greater expansion depends on the complex interaction of all of the remaining parameters in the model.

The final point to make is that labor market characteristics exert their own independent influence on autarky prices and therefore the pattern of comparative advantage.[14] Suppose, for example, that there is an increase in the rate at which workers in sector 1 complete training. This change leads to an upward shift of the $V_1^T\left(a_j\right)$ curve, drawing low-ability workers into sector 1 who had originally opted out of the labor force and drawing moderate-ability workers into sector 1 who had originally chosen to train in sector 2. In the new steady state, the output of good 1 will have increased, while the output of good 2 will have fallen.

As an alternative, suppose that b_2 increases. That is, the job separation rate for workers in sector 2 increases. This reduces the discounted income from working in sector 2, making such employment less attractive. The marginal moderate-ability workers who had been training in this sector immediately switch to train in sector 1. Over time, as workers become separated from their sector 2 jobs, they too switch. In the end, the output of good 2 falls because fewer workers choose sector 2, and because (for any worker who stays in sector 2) a smaller fraction of time is actually spent employed. On the other hand, the output of good 1 increases. Both effects work to increase the price of good 2 relative to good 1.

An increase in e_2 would have exactly the opposite effect. By increasing discounted expected lifetime income, more workers are drawn to sector 2 at the expense of sector 1. Furthermore, each worker attached to sector 2 spends a larger fraction of his life employed. Similar results hold for an increase in f_2.

In summary, we have constructed a model that allows us to study the impact of trade on the distribution of labor between high-tech and low-tech jobs and, by implication, the steady state levels of output, unemployment, and training. The model also allows us to show how labor market parameters can exert an independent influence on the pattern of comparative advantage. Furthermore, the parameters of our model are generally observable. However, changes in the economic environment that lead to a new steady state take time to play themselves out. For proper policy analysis, it is important to know the behavior of the economy along the adjustment path. For example, removal of an import tariff might very well yield long-run benefits, but the short-term adjustment costs could potentially dominate. This is the issue that we address in the next section.

3. Gradual Adjustment Following Trade Reform.

3.1 The Time Path of Employment Following Tariff Removal

Suppose that the country under consideration is a small importer of the low-tech good (i.e., an importer of x_1). Further suppose that the country begins from a steady-state equilibrium and is considering the removal of an import tariff (T_1). With the tariff in place, $p_1 = 1 + T_1$. After removal of the tariff, $p_1 = 1$. As shown in Figure 4, the $V_1^T(a_i)$ curve rotates clockwise while the $V_2^T(a_i)$ curve remains unchanged.[15] The net result is that a_H falls, meaning that more workers will choose to train in sector 2 and fewer will choose to train in sector 1. But the shift between sectors occurs gradually.

Recall that for simplicity we have assumed that the rate at which workers become employed in sector 1 is infinite so that there is no period of search. At the instant of trade liberalization, all workers with ability $a_j \in [a'_H, a_H]$ who were training in sector 1 will switch to training in sector 2. However, assuming that the tariff was initially small enough, workers in that ability

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[14] See Davidson, Martin, and Matusz (1999) where we make the same point in the context of a very different model.

[15] If we defined $V_i^T(a_j)$ as the value of discounted *real* income for a worker who is currently training, then the $V_i^T(a_j)$ curve would shift up, because the wage for workers in sector 2 would increase in terms of x_1 while remaining constant in terms of x_2. It does not matter if we use real or nominal income when we discuss resource allocation. However, discussion of welfare effects should obviously be based on changes in real income if prices change.

range who are employed in sector 1 at the moment of liberalization will choose to remain so until they are exogenously separated. That is, as long as $V_1^E(a_j) > V_2^T(a_j)$, no one will voluntarily quit his or her job. However, once separated, these workers will begin training in sector 2. Therefore, the measure of trainers in sector 2 will jump up at the instant of liberalization, then gradually continue to expand until the new steady state equilibrium is attained.

The path taken by the measure of sector 2 searchers is less clear and depends on the relative magnitudes of the parameters in that sector. Suppose, for instance, that training is relatively quick, but searchers take a long time to find employment. Then the initial bulge in trainers will transmit itself to the pool of searchers, which will climb rapidly, overshoot its steady-state level, then return to the steady-state level. On the other hand, if training is lengthy compared with the time required by searchers to find a job, the bulge of trainers will be released only gradually into the search pool and the measure of searchers will increase monotonically toward the new steady-state level.[16] In any event, the steady-state level of unemployment is bound to expand, since the measure of unemployed workers is a fraction of the measure of people tied to sector 2, which is larger in the new steady state (there is no unemployment in sector 1).[17]

We want to stress that we are able to provide an analytic closed-form solution for the entire adjustment path. This is a unique feature of our model. A more typical approach might require linearization of the adjustment path near the new steady state. We do not need to make such an approximation. The closed-form solution to the complete system of differential equations is contained in Appendix A. Unfortunately, the solution is rather opaque and does not provide much insight on its own. We therefore close this section by providing results from a numerical exercise in order to gain some sense regarding the likely speed of adjustment.[18]

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[16] However, the labor force (which excludes those training) will take a longer time to return to its steady-state level. Therefore, it is still possible for the unemployment rate to first rise above its steady-state level.

[17] It is conceivable that some parameter configurations could result in a lower unemployment rate. This follows since the number of trainers (who would not be counted as part of the labor force) is a fraction of the number of workers tied to sector 1 plus a (different) fraction of the number of workers tied to sector 2. Clearly, the number of trainers could be lower in the new steady state, meaning that the labor force would be higher. We don't think that this case is likely, however, since we view sector 2 as entailing more training than sector 1.

[18] We used Mathcad 2000 Professional to calculate all of the numeric results. Our calculation routine is available on request.

3.2 A Numeric Example

We do not claim that this particular parameterization mimics an actual economy. In particular, it would be too much to ask of this simple two-sector model to accurately reflect *all* characteristics of a particular economy (e.g., the unemployment rate, the average duration of employment, the average duration of unemployment, the share of workers in the high-tech sector, and so on), but the numbers we have chosen strike us as lying in the range one might expect to see in many industrial countries.

In this exercise, we assume that we are dealing with a small country that begins with a 5% tariff on imports of x_1. At time zero, the tariff is fully removed. We assume that tariff revenues had been redistributed in a lump-sum fashion, so their loss does not affect anyone's decision regarding sector in which to train. Without loss of generality, we assume that units are normalized so that the world price of good 2 equals the world price of good 1. We summarize the values chosen for the remaining parameters that are used in this exercise in Table 1.

Table 1

q_1	q_2	b_1	b_2	e_2	t_1	t_2	c_1	c_2	f_2	r
1	7	1	.1	2	12	.5	1	4	.5	.03

In our numeric example, we interpret one period as one year. Therefore, setting $b_1 = 1$ implies that the expected tenure for a low-skill job is one year. By contrast, the expected tenure for a high skill job is 10 years. This latter figure is roughly in line with the average job destruction rate in U.S. manufacturing as reported by Davis, Haltiwanger, and Schuh (1996). A value of 2 for e_2 means that the average duration of a spell of unemployment is 6 months. This is probably too long for the U.S., with actual unemployment spells being closer to 3 months. However, doubling e_2 (and therefore halving the expected duration of unemployment) would lead to excessively low values of unemployment in this simple version of the model.

The values for t_1 and t_2 imply training periods of 1 week and 3 months for low-skill and high-skill jobs, respectively. Combining this with the values of $q_1, q_2, c_1,$ and c_2 implies training costs of approximately one week of wages for a worker of average productivity in sector 1, (for low-skill jobs) and approximately 10 months worth of wages for a worker of average productivity in sector 2. To calculate these figures, just note that the total amount of training cost for a worker training in sector i equals $\frac{c_i}{t_i}$. A worker of average ability in sector 1 produces $\left(\frac{a_L + a_H}{2}\right)q_1$ during the course of a year. A worker of average ability in sector 1 produces $\left(\frac{1+a_H}{2}\right)q_2$ in one year. Given the parameterization provided in Table 1, $a_L \approx 0$ and $a_H \approx .192$. Therefore, a worker of average ability in sector 2 produces approximately .8344 units of output in a year, whereas a worker of average ability in sector 1 produces approximately .096 units of output. That is, the average high-skilled worker produces between 8 and 9 times more output per year as the average low-skilled worker.

The outcome of this exercise is illustrated in Figures 5 - 7. Figure 5 shows that the measure of unemployed workers shoots up immediately following liberalization.[19] Given this parameterization, overshooting does occur, with the measure of the unemployment pool declining after just one period and nearing the new steady state within about 5 periods.

Figure 6 shows that the labor force dips immediately following liberalization, returning fairly quickly to the steady-state level. The dip is caused by workers exiting sector 1 employment to begin training in sector 2 and by workers who had been training in sector 1 (where the duration of training was short) starting to train in sector 2 (where training takes longer). Compared with the time required to return the measure of searchers to the steady state, a slightly shorter period is required for the labor force to return to the steady state. This follows because, in our parameterization, training is a less time-intensive process than searching.

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[19] In this exercise, we normalize the number of potential workers to equal 100. Therefore a number such as 4.0 can be interpreted to mean that 4.0 percent of all potential workers (including those not in the labor force) are looking for jobs.

Figure 7 combines the information contained in Figures 5 and 6 to illustrate the movement of the unemployment rate over time. Again, the unemployment rate overshoots the steady-state level, but begins coming down after 1 period and closely approximates the new steady-state level after roughly 4 periods.

In examining Figures 5 - 7, two features stand out. First, the length of the adjustment period is neither trivial, nor does it appear to be excessive. Second, the magnitude of the short-run effect is noticeable. The unemployment rate climbs nearly three-quarters of a percentage point during the first year after liberalization before returning to a level less than two tenths of a percentage point higher than in the initial steady state.

4. The Welfare Impact of Trade Reform

Even though labor is the only input in our model, workers are heterogeneous and therefore trade reform will benefit some workers while harming others.

First consider those workers with ability levels below a_L. In our model, such workers receive no income and therefore they are unaffected by reform. In a more elaborate version of the model, we might assume that such workers receive public assistance. We would then have to ask how that assistance is funded and how the funding changes with the removal of the tariff. These individuals benefit if the nominal amount of the assistance remains unchanged since the real value would increase in terms of x_1 while remaining the same in terms of x_2.

Clearly all workers who remain in sector 1 are harmed by the reform. When employed, their wage is unchanged in terms of x_1 but falls in terms of x_2. Similarly, those workers who are initially in sector 2 clearly benefit, with their wage increasing in terms of x_1 while remaining constant in terms of x_2.[20]

Now consider workers who switch sectors. On the one hand, sector 2 is more attractive after trade reform, so some workers are drawn into that sector. On the other hand, sector 1 is less attractive after reform, so some workers are pushed out of sector 1 into sector 2. We can

[20] For both sets of workers, the value of training costs moves in the same direction as the value of wages, but it is easy to demonstrate that the wage effect dominates the welfare consideration.

envision these effects in two stages. First, hold constant the function $V_1^T(a_j)$ while shifting $V_2^T(a_j)$ upward by the amount corresponding to trade reform. Any workers moving under these circumstances are better off. Now, shift $V_1^T(a_j)$ down, commensurate with trade reform. The remaining movers are worse off than they would have been had the tariff not been removed, but the reduction in welfare is softened by the ability to switch sectors.

Perhaps a more interesting question regards the impact on overall welfare. A complete analysis of this question would require us to postulate some utility function so that we might talk about both the consumption and production aspects of reform. However, we can obtain some interesting results by focussing exclusively on the production side of the economy.

Define $Y(t)$ as the value of output (net of training costs) produced at time t and measured using world prices.

Even though the new steady state is plagued by higher unemployment, we show in Appendix B that the free trade steady-state equilibrium is efficient. As such, steady state "welfare" with free trade is higher than steady state "welfare" with the tariff. In this context, "welfare" is measured by the value of output net of training costs measured at world prices. However, welfare along the adjustment path is certainly lower than it is at the new steady state, and possibly lower than it was in the initial steady state. Could it be that the losses during the adjustment path outweigh the long-run gains from liberalization? The answer to this is no, since we have shown in Appendix B that the free trade equilibrium is dynamically efficient (which means that a small movement away from the free trade equilibrium lowers welfare, taking into account the adjustment path). But, it is conceivable that the short-run losses during the adjustment process might eat away almost all of the long-run gains. This is an empirical question, but the model provides clear guidance regarding the proper data necessary to evaluate the experiment.

Formally, define $R_i^E(t)$ as the measure of workers who *remain* employed in sector *i* subsequent to liberalization. Define $S_{12}^{E_1}(t)$ as the measure of workers who *eventually switch* from sector 1 to sector 2, but who are employed in sector 1 at time *t*. A similar expression,

$S_{12}^{E_2}(t)$, represents the measure of workers who were employed in sector 1 prior to liberalization, but are employed in sector 2 at time *t*. We then have the following:

$$(20) \qquad Y(t) = \left\{ R_1^E \left(\frac{a_L + a'_H}{2} \right) q_1 + S_{12}^{E_1}(t) \left(\frac{a'_H + a_H}{2} \right) q_1 - L_1^T(t) c_1 \right\} \quad +$$

$$p^* \left\{ R_2^E \left(\frac{1 + a_H}{2} \right) q_2 + S_{12}^{E_2}(t) \left(\frac{a'_H + a_H}{2} \right) q_2 - L_2^T(t) c_2 \right\}.$$

To derive (20), note that the average ability for workers who remain in sector 1 is $\frac{a_L + a'_H}{2}$, implying that average output per worker employed in that sector is $\left(\frac{a_L + a'_H}{2} \right) q_1$. The other terms are obtained similarly.

We show $Y(t)$ as the solid line in Figure 8 for $t = 0, \ldots, 10$. The dashed horizontal line represents the value of steady-state output prior to liberalization. Conforming with the intuition discussed above, the value of output falls below its pre-liberalization level for the first several years after liberalization. In this example, based on the parameters of Table 1, output does not climb above the pre-liberalization level until sometime during the fourth year after the reform.

Reform is beneficial only if net losses during the early years of reform are compensated by the future gains. Formally, let W_{FT} represent the present discounted value of welfare under free trade, and let W_{SS} represent the present discounted value of welfare consistent with the tariff-distorted steady state. Then removal of the tariff is beneficial if and only if $\hat{W} = \frac{W_{FT} - W_{SS}}{W_{SS}} > 0$, where

$$(23.a) \qquad W_{SS} = \int_0^\infty e^{-rt} Y_{SS} \, dt$$

$$(23.b) \qquad W_{FT} = \int_0^\infty e^{-rt} Y_{FT}(t) \, dt$$

Based on the parameters in Table 1, $\hat{W} = .0002$. That is, there is a welfare gain, but the gain is less than .02 percent of pre-liberalization income. The measured welfare gain would be larger if we simply compare steady states. In this example, steady-state income with free trade is approximately .14 percent higher than steady-state income in the distorted economy.[21] While adjustment costs do not reverse the benefits of tariff reform, they are substantial.[22] Thus, although Appendix B tells us that this economy will benefit from liberalization, our numerical exercise tells us that the gains will be almost trivial.

5. The Bhagwati-Dehejia Thesis

Our main concern in this paper has been to develop a tractable model suited to predicting adjustment costs based on observable parameters. We can also use our model, however, to shed some light on the raging debate regarding the impact of globalization on the distribution of income.

As is well documented, income distributions within OECD countries have worsened, with the rich getting richer and the poor getting poorer. At the same time, imports from developing countries have exploded. The natural inclination among many economists is to apply the Stolper-Samuelson theorem to argue that increased globalization caused deterioration of the income distribution. The problem is that (depending upon one's interpretation) the data do not seem to support this hypothesis.[23]

Jagdish Bhagwati and Vivek Dehejia have suggested the possibility that globalization might impact relative income even without Stolper-Samuelson effects. They hypothesize that increased globalization means increasing competition, with razor-thin profit margins. Firms that are competitive today might be out of business tomorrow. They refer to this phenomenon

[21] This value is surprisingly similar to Krugman's view that the efficiency gain due to removal of the main trade barriers in the U.S. would be roughly .25% of income. See Krugman (1990, p. 104).
[22] Our results contrasts sharply with the findings of Magee (1972) and Baldwin, Mutti, and Richardson (1980) where discounted adjustment due to liberalization are estimated to be well under 5% of the discounted efficiency gains. Of course, both of these earlier studies treated the depth and length of the adjustment process in a rather ad hoc manner.

[23] See, for example, Lawrence and Slaughter (1993). There is significant debate regarding the proper way to examine the data, with some economists arguing that the only way for globalization to affect relative incomes is via price changes, and others arguing for the relevance of the quantity of labor embodied in the trade bundle. It is way beyond the scope of our paper to examine this debate.

as "kaleidoscopic comparative advantage" and argue that one implication is that job turnover rates might have increased due to this effect. In turn, the higher rates of job turnover might reduce incentives for workers to acquire human capital, flattening out the growth profile of earnings. They argue that this could result in an increase in the income differential between skilled and unskilled workers if skilled workers have greater transferability of workplace skills than do unskilled workers (as we model).[24] While our model is not tailored to address this thesis head on, we clearly have the machinery to explore some possibilities.

Suppose that globalization implies an increase in b_1 and b_2, all else equal.[25] Consider first what happens when b_1 increases. Sector 1 becomes less attractive. Some low-ability workers are pushed into economic inactivity. Some higher-ability workers are pushed into sector 2. Workers who remain in sector 1 spend a higher fraction of their lifetime in training and a smaller fraction actually employed. Whether the total amount of training goes up or down depends on the interaction of a smaller sector size with more time spent in training by those who continue to work in the sector.

Similar effects are seen in sector 2. An increase in b_2 will make sector 2 less attractive. Some lower-ability workers will be pushed into sector 1. Those workers who remain in sector 2 will spend a smaller fraction of their lifetime working, and a larger fraction searching and in training. Whether sector 2 shrinks or expands depends upon the magnitude by which b_2 increases compared with the increase in b_1. Even here, there is no simple comparison because all of the other parameters have a role to play.

We can use our model to derive three results, all of which are consistent with the Bhagwati-Dehejia thesis. First, if both turnover rates increase proportionately, it is likely that workers will shift out of sector 2 and into sector 1. If such is the case, there will be a reduction in the aggregate amount of training. Second, higher turnover rates imply lower lifetime incomes for all agents, but the impact is proportionately less for agents with higher ability. Therefore we can infer that such an increase in turnover rates will improve the welfare of the highest ability workers with respect to the welfare of the lowest ability workers. Finally, the degree to which

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[24] See Bhagwati and Dehejia (1994) or Bhagwati (1998) for a more detailed exposition.

income falls as a result of higher turnover is inversely related to the ease with which skills transfer between jobs.

To illustrate these results, we differentiate (15) with respect to b_i, holding all other variables constant. Doing so, we find

$$(24.a) \qquad \hat{V}_2^T(a_j) = -\left\{\left(1+\frac{p_2c_2}{rV_2^T(a_j)}\right)\left(\frac{rb_2+(1-f_2)e_2b_2}{D_2}\right)-\frac{t_2b_2}{D_2}\right\}\hat{b}_2$$

$$(24.b) \qquad \hat{V}_1^T(a_j) = -\left\{\left(1+\frac{p_1c_1}{rV_1^T(a_j)}\right)\left(\frac{b_1}{D_1}\right)\right\}\hat{b}_1.$$

At the initial steady state, $rV_1^T(a_H) = rV_2^2(a_H)$. By assumption, training costs are low in sector 1, so $p_1c_1 < p_2c_2$. The fractions $\frac{b_1}{D_1}$ and $\frac{rb_2+(1-f_2)e_2b_2}{D_2}$ are not directly comparable. Given the parameters in Table 1, $\frac{b_1}{D_1}$ has the slightly larger value. However, in our example, $\frac{\hat{V}_2^T(a_H)}{\hat{b}_2} < \frac{\hat{V}_1^T(a_H)}{\hat{b}_1} < 0$. A proportionate increase in b_1 and b_2 reduces the expected lifetime income from training in sector 2 by more than the reduction in sector 1 for the person with ability a_H. Workers therefore shift gradually from sector 2 to sector 1, illustrating our first result.

Inspection of (24.a) and (24.b) reveals that the proportionate impact on expected lifetime income of an increase in b_i diminishes as ability increases, therefore confirming our second result. Higher turnover rates in both sectors hurt everyone, but mostly impact those with the lowest abilities (i.e., workers in the bottom tail of the income distribution).

Finally, differentiating the right hand side of (24.a) with respect to f_2 shows that the coefficient on $\hat{b}_2$ becomes smaller in absolute value as skills become more transferable across

[25] More generally, increased turnover might also imply a higher job acquisition rate in sector 2.

jobs (i.e., as f_2 increases), therefore reducing the elasticity of $V_2^T(a_j)$ with respect to b_2. This confirms our third result.[26]

6. Conclusion

The vast majority of public debate about trade policy centers on its impact on the jobless and the poor and the short run adjustment costs generated by changes in the pattern of trade. With only a few notable exceptions, the vast majority of the academic literature on trade policy ignores such issues. In this paper, we have offered a simple model of trade that incorporates some of the more important features that the public seems concerned about. In our opinion, the two most important features are unemployment and a class of workers who are shut out of the labor market because they do not have the ability to acquire the skills required for the jobs that are available. We have shown that not only is it possible to build a simple model with such features, but that it is also possible to solve analytically for the adjustment path that connects steady states.[27] This allows us to weigh the short run costs of adjustment against any long run gains that may arise from changes in trade policy. Moreover, the key parameters of our model (labor market turnover rates) are all observable, making it a natural framework for future policy analysis.

Our main goal in this paper was to demonstrate the usefulness of the model in dealing with some basic issues. For example, we began by examining the impact of globalization on unemployment and economic welfare. We show that, in our model, the short-run costs of adjustment cannot outweigh the long-run gains from globalization. However, our numeric exercise suggests that adjustment costs are substantial in the short run (the value of output net of training costs falls by more than 1.5% one year after liberalization), and that they may be large enough to eat away almost all the long run efficiency gains.

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[26] The analysis in this section focuses on the impact of increased turnover on the expected lifetime income of those in training. However, it is clear from (10)-(12) that incomes for employed workers and searchers are positively related to the income of trainers.

[27] There can be no doubt that our model is overly simplistic. It includes only a single factor of production and all of the labor market turnover rates are exogenous. In the future, we hope to find ways to relax these assumptions without sacrificing the tractability of the model.

We also examined the impact of globalization on the distribution of income. When we assume that protection is removed from the low-skill sector, we obtain Stolper-Samuelson type results, even though labor is the only input in our model. In this case, those workers who continue to be attached to the low-skill sector are worse off while those who were initially attached to the high-skill sector benefit. Liberalization causes some workers to switch from the low-skill to the high-skill sector. Of those who shift between sectors, those with higher abilities benefit from the liberalization, while those with lower abilities are harmed.

Our model also provides support for the Bhagwati-Dehijia thesis that if globalization leads to higher job turnover it may lead to a more unequal distribution of income. In particular, we show that an increase in job turnover has a smaller (negative) impact on workers of higher ability compared with lower ability and greater transferability of skills across jobs reduces the sensitivity of lifetime income to changes in job turnover.

In the future, we intend to use our model to address some issues that have received very little attention is the trade literature. Since our model explicitly allows for unemployment and heterogeneity in skills across workers and jobs (which generates a non-trivial income distribution), we can carry out careful policy analysis of a wide variety of labor market policies aimed at helping the jobless and the poor who are adversely affected by changes in the pattern of trade. For example, we could incorporate training subsidies, unemployment compensation, trade adjustment assistance, government sponsored training or job search services and wage subsidies with virtually no change to the underlying structure of the model. We could then choose a target (say, a certain level of income for low-ability workers) and find the policy that achieves the target with the smallest social cost. Moreover, by varying the turnover rates to mimic the structure of the labor markets in different regions of the world, we can investigate how the optimal policy depends on the flexibility of the labor market. After all, there is little reason to believe that policies that may be affective in the United States, where the durations of employment and unemployment are low relative to Europe and Japan, will be equally effective in other parts of the world where turnover rates are vastly different.

Appendix A

We sketch the derivation of closed-form solutions for the adjustment path in this appendix. In addition to the notation introduced in the text, define $S_{12}^{T_2}(t)$ as the measure of workers who switch from sector 1 to sector 2 and are training at time *t*. Similarly define $S_{12}^{S_2}(t)$ as the measure of workers who switch from sector 1 to sector 2 and are searching at time *t*. The system of differential equations then can be written as in (A.1) - (A.4):

$$(A.1) \qquad \dot{S}_{12}^{E_1} = -b_2 S_{12}^{E_1}$$

$$(A.2) \qquad \dot{S}_{12}^{E_2} = e_2 S_{12}^{S_2} - b_2 S_{12}^{E_2}$$

$$(A.3) \qquad \dot{S}_{12}^{S_2} = b_2 f_2 S_{12}^{E_2} + t_2 S_{12}^{T_2} - e_2 S_{12}^{S_2}$$

$$(A.4) \qquad (a_H - a'_H)L = S_{12}^{E_1} + S_{12}^{E_2} + S_{12}^{S_2} + S_{12}^{T_2}$$

where, for notational convenience, we have suppressed the time argument.

Equation (A.4) is a simple differential equation, the solution of which is

$$(A.5) \qquad S_{12}^{E_1}(t) = \frac{t_1}{t_1 + b_1}(a_H - a'_H)Le^{-b_1 t}.$$

In solving (A.1), we make use of the initial condition that $S_{12}^{E_1}(0) = \frac{t_1}{t_1 + b_1}(a_H - a'_H)L$.

To solve (A.2) - (A.4), first substitute (A.5) into (A.4) and then solve for $S_{12}^{T_2}$ in terms of $S_{12}^{E_2}$ and S_{12}^{S}. Substitute the result into (A.3). Then (A.2) and (A.3) form a system of two differential equations which can be written in matrix form:

$$(A.6) \qquad \begin{bmatrix} \dot{S}_{12}^{E_2} \\ \dot{S}_{12}^{S_2} \end{bmatrix} = \begin{bmatrix} -b_2 & e_2 \\ b_2 f_2 - t_2 & -(e_2 + t_2) \end{bmatrix} \begin{bmatrix} S_{12}^{E_2} \\ S_{12}^{S_2} \end{bmatrix} + \begin{bmatrix} 0 \\ h(t) \end{bmatrix}$$

where $h(t) = t_2(a_H - a'_H)L\left(1 - \frac{t_1}{t_1 + b_1}e^{-b_1 t}\right)$. The method for solving a system of this form is provided by Boyce and DiPrima (1977), pp. 329-331. Using the initial conditions that $S_{12}^{E_2}(0) = S_{12}^{S_2}(0) = 0$, the solutions are

$$(A.7)\qquad S_{12}^{E_2}(t)=\frac{e_2t_2(a_H-a'_H)L}{l_1l_2}+\frac{e_2t_1t_2(a_H-a'_H)L}{(t_1+b_1)(l_2-l_1)}\left[\frac{e^{l_1t}}{l_1+b_1}-\frac{e^{l_2t}}{l_2+b_1}\right]-$$

$$\frac{e_2t_2(a_H-a'_H)L}{l_2-l_1}\left[\frac{e^{l_1t}}{l_1}-\frac{e^{l_2t}}{l_2}\right]-\frac{e_2t_1t_2(a_H-a'_H)L}{(t_1+b_1)(l_1+b_1)(l_2+b_1)}e^{-b_1t}$$

$$(A.8)\qquad S_{12}^{S_2}(t)=\frac{t_2b_2(a_H-a'_H)L}{l_1l_2}+\frac{t_1t_2(a_H-a'_H)L}{(t_1+b_1)(l_2-l_1)}\left[\frac{b_2+l_1}{b_1+l_1}e^{l_1t}-\frac{b_2+l_2}{b_1+l_2}e^{l_2t}\right]-$$

$$\frac{t_2(a_H-a'_H)L}{(l_2-l_1)}\left[\frac{b_2+l_1}{l_1}e^{l_1t}-\frac{b_2+l_2}{l_2}e^{l_2t}\right]-$$

$$\frac{t_1t_2(b_2-b_1)(a_H-a'_H)L}{(t_1+b_1)(l_1+b_1)(l_2+b_1)}e^{-b_1t}$$

where l_1 and l_2 are the eigenvalues of the coefficient matrix in (A.6) and are equal to:

$$(A.9.a)\qquad l_1=\frac{-(b_2+e_2+t_2)-\sqrt{(b_2+e_2+t_2)^2-4b_2e_2(1-f_2)}}{2}$$

$$(A.9.b)\qquad l_1=\frac{-(b_2+e_2+t_2)+\sqrt{(b_2+e_2+t_2)^2-4b_2e_2(1-f_2)}}{2}.$$

Appendix B

Our goal is to show that the equilibrium in our model is efficient. To do so, we must calculate the dynamic marginal product of labor in each sector and show that these values are equal in the market equilibrium.

The dynamic marginal product of labor in a sector measures the increase in net output that occurs if the steady state is disturbed by adding an additional worker to that sector taking into account the adjustment path to the new steady state. To calculate the dynamic marginal products we follow the method developed in Diamond (1980).

We begin by defining $c_i(q)$ as the present discounted value of output net of training costs produced in sector i when a (small) measure θ of new workers is added to that sector. These workers are assumed to have ability level a_H. Equilibrium is efficient if $c_1'(q) = c_2'(q)$.

Start with sector 1. We have[28]

$$c_1(q) \equiv \int_0^\infty e^{-rt}\{a_H q_1 q\, I(t) - q\, c_1[1 - I(t)]\}dt$$

where $\dot{q}_1^E = t_1 q - (t_1 + b_1)q_1^E$ and $I(t)$ is an indicator function that takes on the value of 1 when the worker is employed and equals zero at all other times. To find $c_1'(q)$ we start by using the fundamental equation of dynamic programming which states that

$$rc_1(q) = a_H q_1 q\, I(t) - q\, c_1[1 - I(t)] + \frac{\partial c_1}{\partial q_1^E}\dot{q}_1^E$$

Substituting for $\dot{q}_1^E$ from above allows us to write this as

$$(B.1) \qquad rc_1(q) = a_H q_1 q\, I(t) - q\, c_1[1 - I(t)] + \frac{\partial c_1}{\partial q_1^E}\left\{t_1 q - (t_1 + b_1)q_1^E\right\}$$

Differentiating with respect to q yields

$$rc_1'(q) = a_H q_1 I(t) - c_1\{1 - I(t)\} + t_1 \frac{\partial c_1}{\partial q_1^E}$$

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[28] The equation of motion for $\dot{q}_1^E$ is obtained in the following manner. Since search is not required to find employment in sector 1, we have $\dot{q}_1^E = t_1 q_1^T - b_1 q_1^E$. Now, we know that the total measure of trainers (out

but, at the initial moment, none of the new workers are employed. That is, $I(0)=0$, so that we have

$$(B.2) \qquad rc_1^{'}(q) = -c_1 + t_1 \frac{\partial c_1}{\partial q_1^E}$$

To complete our derivation, we must now calculate $\frac{\partial c_1}{\partial q_1^E}$. To do so, we solve (B.1) for $\frac{\partial c_1}{\partial q_1^E}$.

We obtain

$$\frac{\partial c_1}{\partial q_1^E} = \frac{rc_1 - a_H q_1 q\, I(t) + q\, c_1[1-I(t)]}{t_1 q - (t_1+b_1)q_1^E}$$

In the initial steady state, the right-hand side of this equation equals 0/0. Applying L'Hopital's Rule, we have (note that we are differentiating with respect to q_1^E, which is the same as $q\, I(t)$)

$$\frac{\partial c_1}{\partial q_1^E} = \frac{r\frac{\partial c_1}{\partial q_1^E} - a_H q_1 - c_1}{-(t_1+b_1)}$$

or

$$\frac{\partial c_1}{\partial q_1^E} = \frac{a_H q_1 + c_1}{r + t_1 + b_1}$$

We can now substitute this value into (A.2) to obtain the dynamic marginal product of labor in sector 1:

$$(B.3) \qquad rc_1'(q) = \frac{t_1 a_H q_1 - (r+b_1)c_1}{r+t_1+b_1}$$

Note that this dynamic marginal product equals $rV_1^E(a_H)$.

Turn next to sector 2. We have

$$c_2(q) \equiv \int_0^\infty e^{-rt}\{a_H p_2 q_2 q\, I(t) - c_2 p_2 q[1 - I(t) - H(t)]\}dt$$

where $\dot{q}_2^E = eq_2^S - b_2 q_2^E$, $\dot{q}_2^S = t_2 q + (b_2 f - t_2)q_2^E - (t_2+e)q_2^S$, I(t) is an indicator function that equals one when the worker is employed and zero otherwise and H(t) is an indicator function which equals one when the worker is searching and zero otherwise.

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of the q) in sector 1 is equal to the difference between q and the measure of employed workers in that sector. Substituting for q_1^T yields the desired result.

As above, we start by applying the fundamental equation of dynamic programming which implies that

$$rc_2(q) = a_H p_2 q_2 q\, I(t) - c_2 p_2 q\left[1 - I(t) - H(t)\right] + \frac{\partial c_2}{\partial q_2^E}\dot{q}_2^E + \frac{\partial c_2}{\partial q_2^S}\dot{q}_2^S$$

If we now use the equations of motion to substitute for $\dot{q}_2^E$ and $\dot{q}_2^S$ and then differentiate with respect to q we obtain

$$rc_2'(q) = a_H p_2 q_2 I(t) - c_2 p_2\left[1 - I(t) - H(t)\right] + \frac{\partial c_2}{\partial q_2^S} t_2$$

But, in the initial steady state (at $t = 0$), we know that $I(0) = H(0) = 0$; so that

$$(B.4) \qquad rc_2'(q) = -c_2 p_2 + t_2 \frac{\partial c_2}{\partial q_2^S}$$

The final step requires us to solve for $\frac{\partial c_2}{\partial q_2^S}$ and then substitute that value into (B.4). Again following Diamond (1980), we differentiate the fundamental equation of dynamic programming with respect to q_2^E and q_2^S. We obtain

$$\begin{bmatrix} \frac{\partial c_2}{\partial q_2^E} \\ \frac{\partial c_2}{\partial q_2^S} \end{bmatrix} = \left[\left(a_H p_2 q_2 + c_2 p_2\right) \quad c_2\ p_2\right] \begin{bmatrix} r + b_2 & -\left(b_2 f - t_2\right) \\ -e_2 & \left(r + t_2 + e_2\right) \end{bmatrix}^{-1}$$

Solving this system of equations for $\frac{\partial c_2}{\partial q_2^S}$ yields

$$(B.5) \qquad \frac{\partial c_2}{\partial q_2^S} = \frac{p_2\{a_H q_2 e_2 + c_2(e_2 + r + b_2)\}}{(r + b_2)(r + t_2 + e_2) + e_2 t_2 - ef b_2}$$

Substituting (B.5) into (B.4) and collecting terms results in

$$(B.6) \qquad rc_2'(q) = \frac{p_2\left\{a_H q_2 e - \left[(r + b_2)(r + e_2) - e_2 f b_2\right]c_2\right\}}{(r + b_2)(r + t_2 + e) + e_2 t_2 - e_2 f b_2}$$

Note that (B.6) is also equal to $rV_2^T(a_H)$. Thus, since the dynamic marginal products both equal the expected lifetime income for a worker training in that sector, and, since workers are allocated so that the expected lifetime income from training is the same in both sectors, the dynamic marginal products are equal in equilibrium. As a result, equilibrium is efficient.

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Figure 1

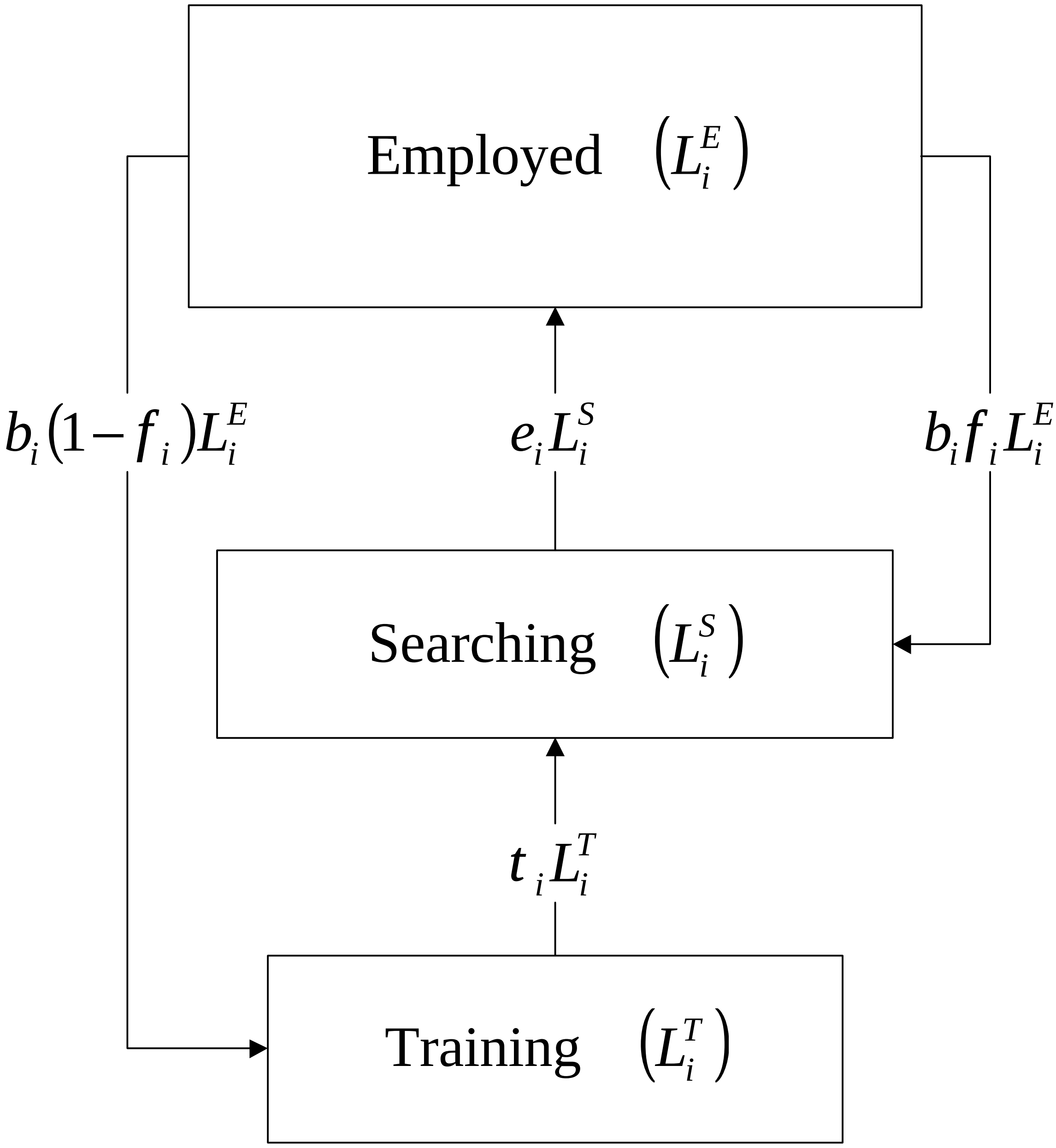

Employed $\left(L_i^E\right)$
$b_i\left(1-f_i\right)L_i^E$
$e_i L_i^S$
$b_i f_i L_i^E$
Searching $\left(L_i^S\right)$
$t_i L_i^T$
Training $\left(L_i^T\right)$

Figure 2

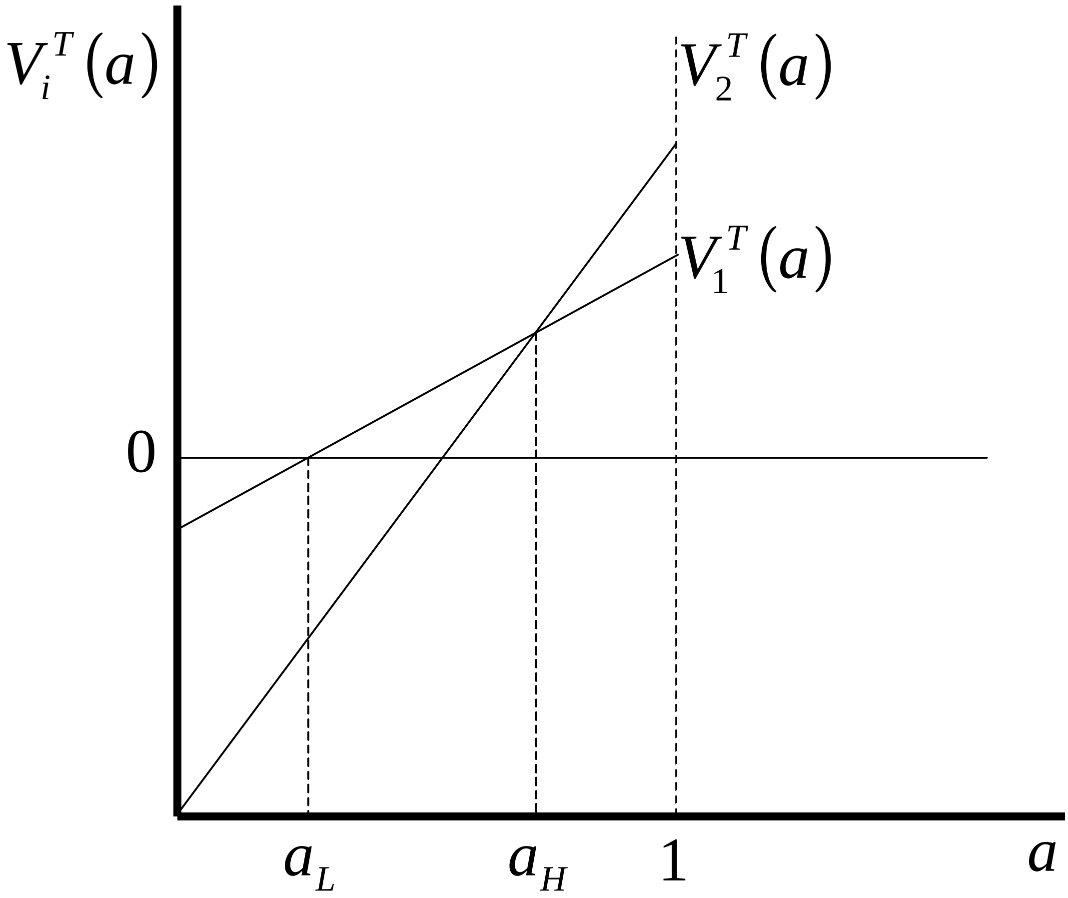


Figure 3

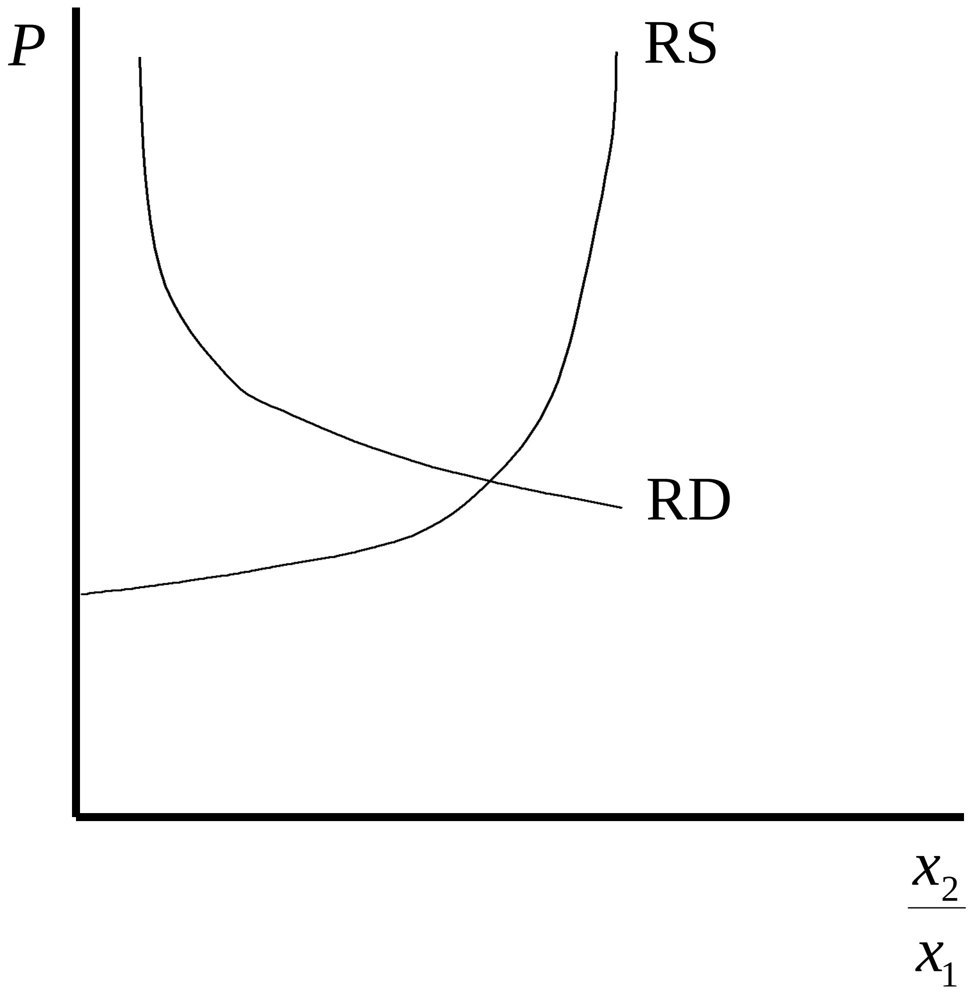


Figure 4

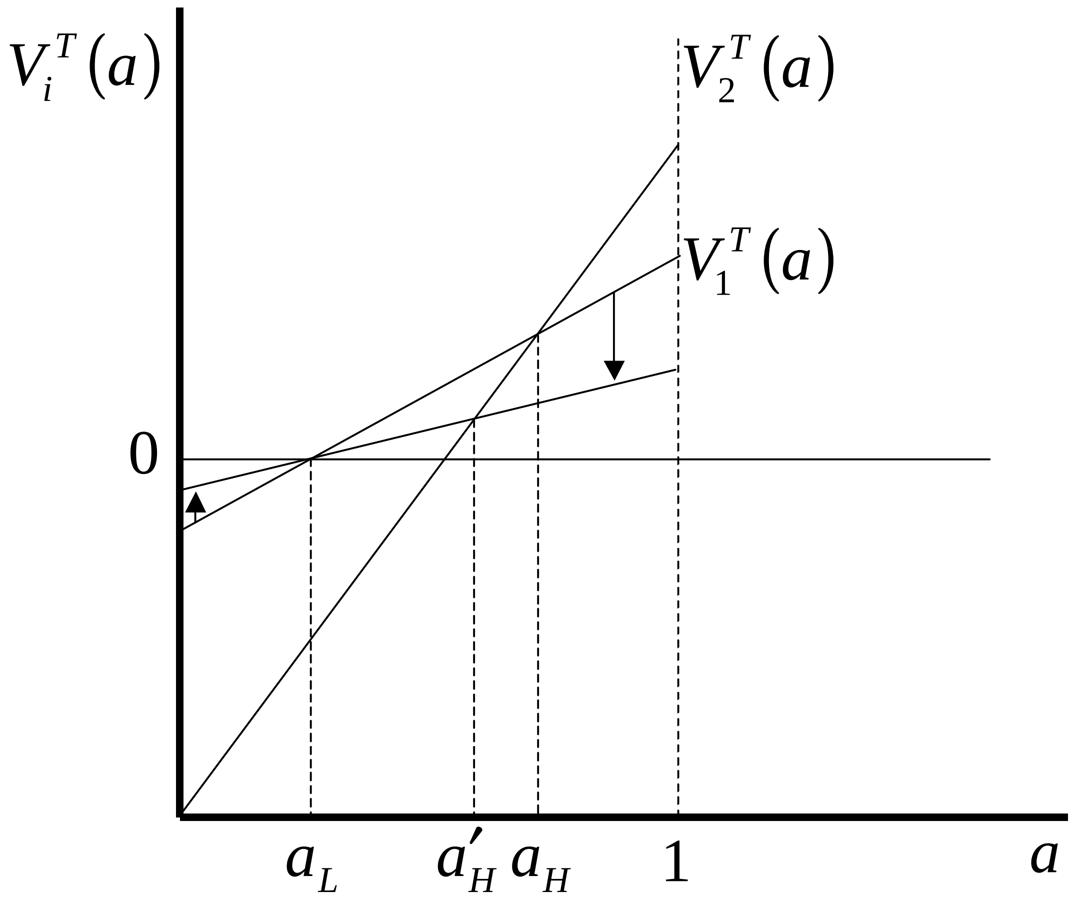


Figure 5

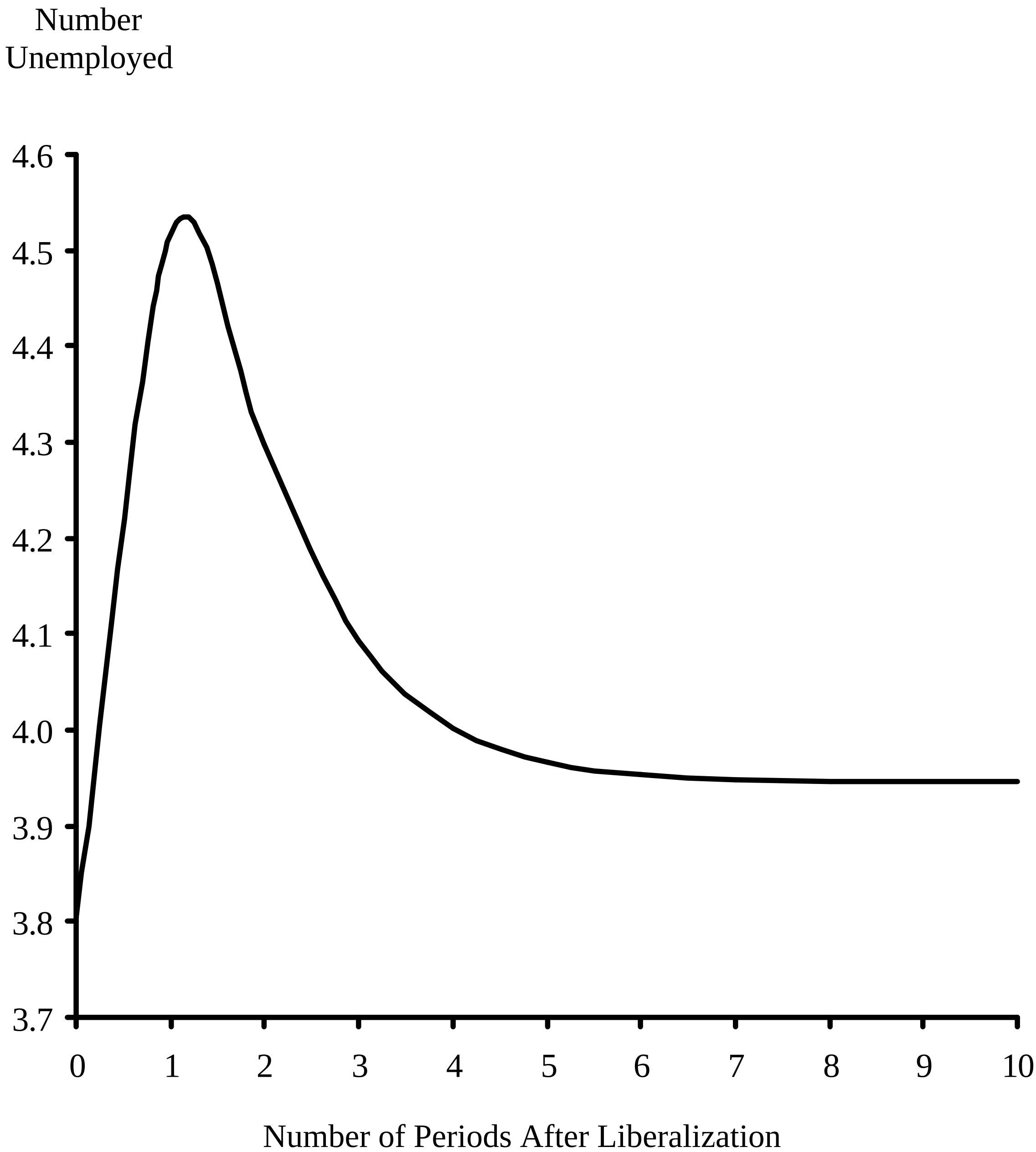

Number
Unemployed
4.6
4.5
4.4
4.3
4.2
4.1
4.0
3.9
3.8
3.7
0
1
2
3
4
5
6
7
8
9
10
Number of Periods After Liberalization

Figure 6

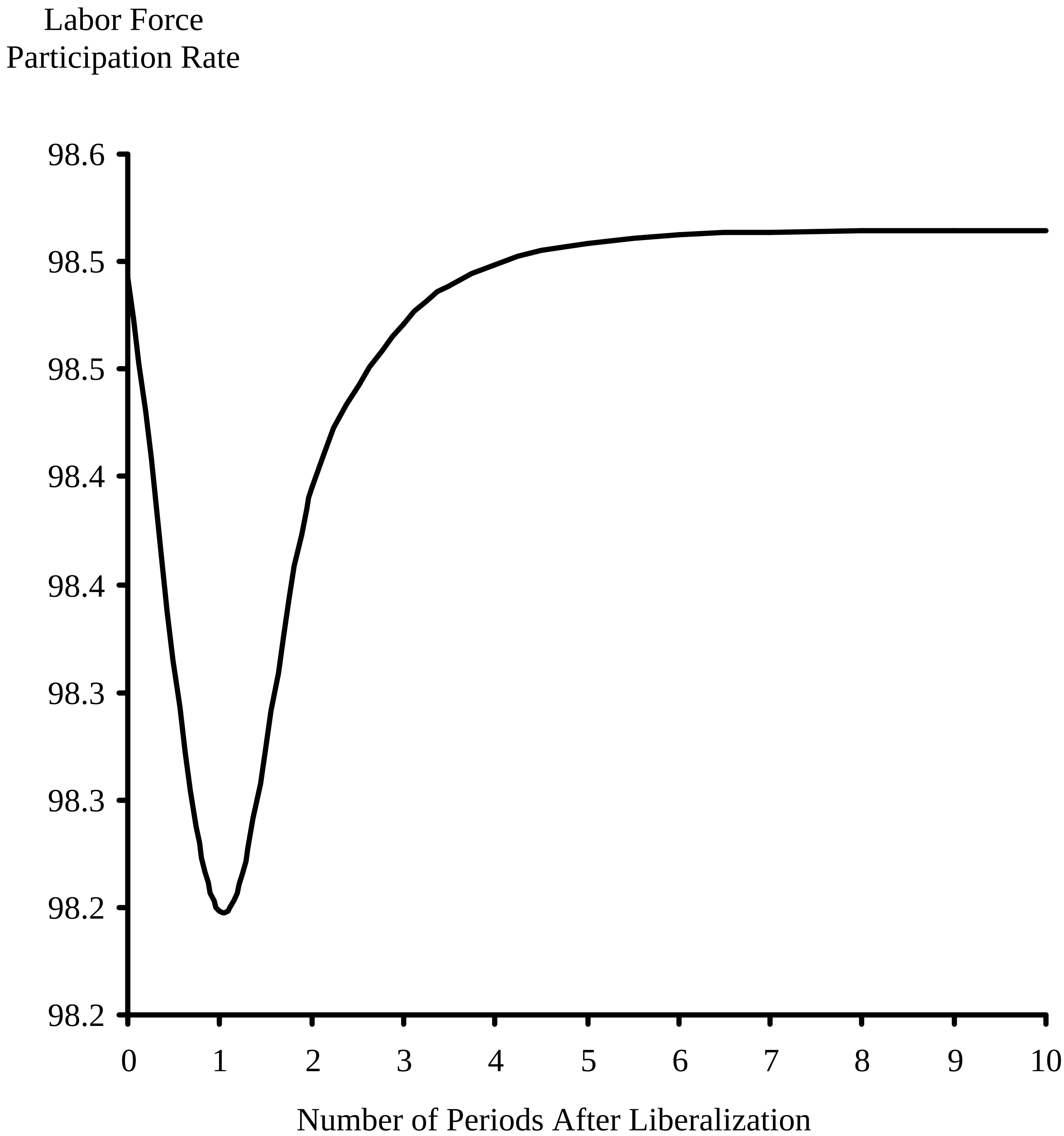

Labor Force
Participation Rate
98.6
98.5
98.5
98.4
98.4
98.3
98.3
98.2
98.2
0
1
2
3
4
5
6
7
8
9
10
Number of Periods After Liberalization

Figure 7

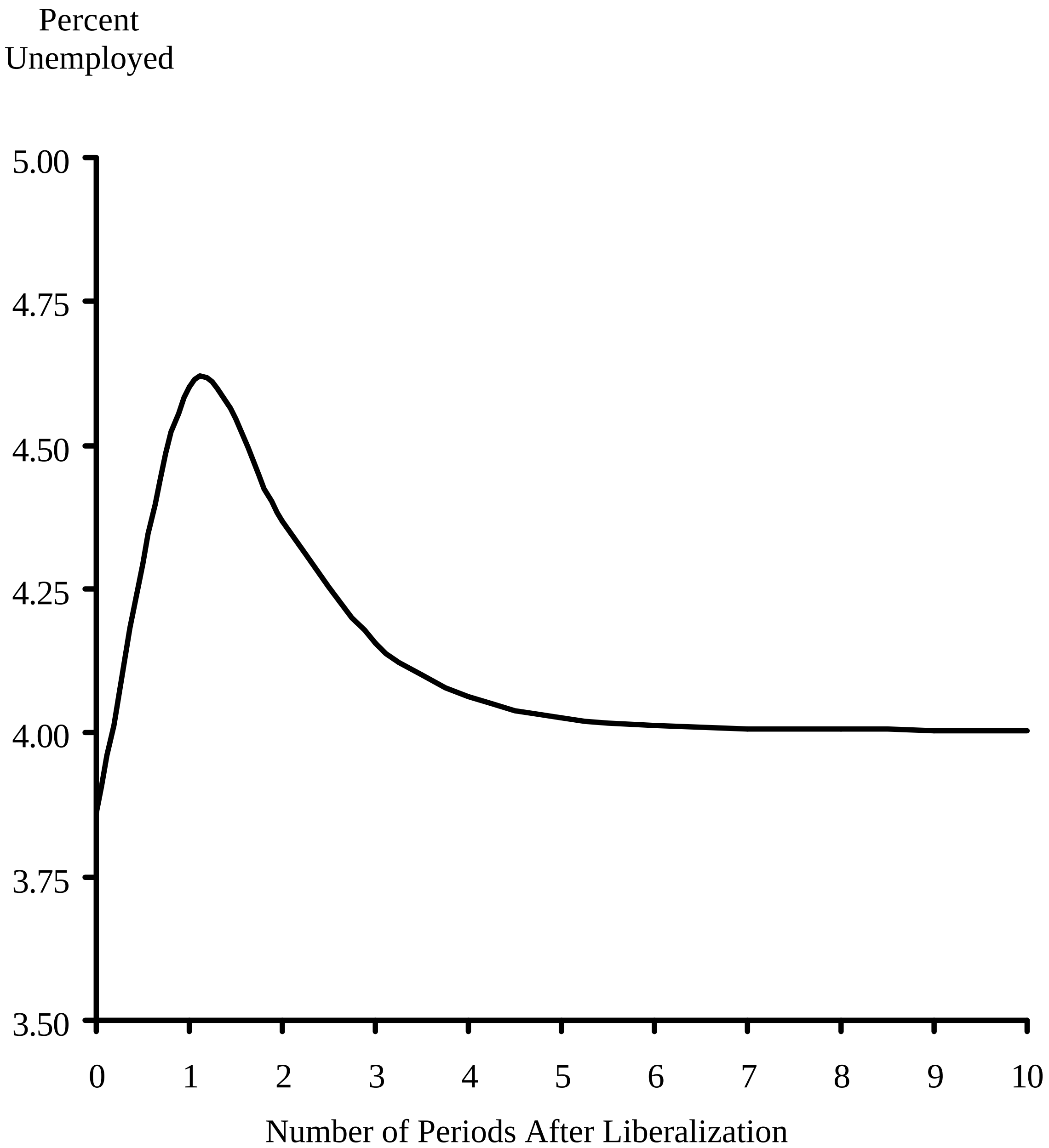
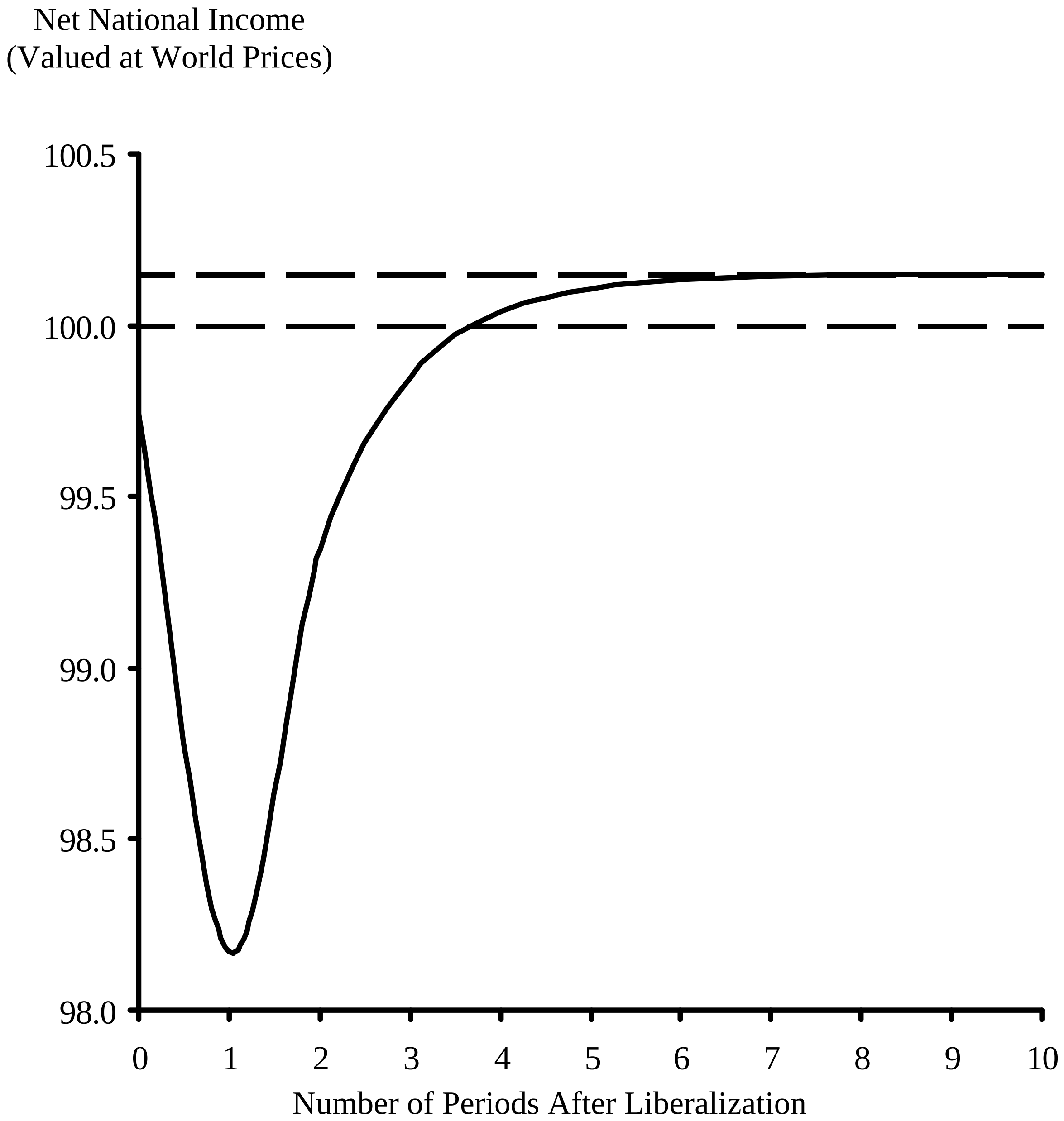

Percent
Unemployed
5.00
4.75
4.50
4.25
4.00
3.75
3.50
0
1
2
3
4
5
6
7
8
9
10
Number of Periods After Liberalization

Figure 8


Net National Income
(Valued at World Prices)
100.5
100.0
99.5
99.0
98.5
98.0
0
1
2
3
4
5
6
7
8
9
10
Number of Periods After Liberalization