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The Le Châtelier Principle in the Theory of International Trade

by

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Abstract

This paper shows in a general framework the broad applicability of the Le Châtelier Principle in international trade theory and policy. By stressing the formal similarity between a small open economy and a price taking multi-product firm, the paper clarifies the links between some earlier applications of the Le Châtelier Principle in the trade literature and derives some new results. In addition, it is shown how some of the Le Châtelier results, which in their established form are locally valid only, can be generalised to finite parameter changes.

JEL classification: F11, F13

Keywords: Le Châtelier Principle, International Trade, Factor Mobility, Small Open Economy

Outline

Non-Technical Summary

Since Paul Samuelson introduced the Le Châtelier Principle (LCP) into the economics discipline in his ``Foundations of Economic Analysis'' in 1947, it has been widely used in different parts of applied microeconomic theory. The standard application in microeconomics is in the case of a profit maximising, competitive multi-product firm where the principle can be invoked to show that (goods) supply and (factor) demand functions are more elastic when quantity constraints are removed. In its general form, the LCP is a purely local result, i.e., it holds for infinitesimal shifts in parameter values only. In the recent microeconomic literature the question has been posed under which conditions the LCP can be shown to hold for finite parameter shifts.

While well known among microeconomists, it would be fair to say that the principle's popularity among trade theorists remained rather limited to date. To be sure, in a series of important papers on trade theory and trade policy issues by Neary (1985, 1988, 1995) and Neary and Ruane (1988) the marginal LCP is mentioned as the driving force behind some of the results. But as the LCP itself is not of central interest to the analysis in these papers, the principle's general applicability in the context of trade theory and policy is not emphasised, and the principle's applicability is not fully exploited. The present paper aims at showing in a common framework the general applicability of the LCP in trade theory and policy. This is accomplished by stressing the formal similarity between a small open economy and a price taking multi-product firm, hence making available Le Châtelier results from microeconomics for economy-wide analysis. The contribution here is mainly to show the common aspects of these results, to make explicit some of them that were implicit in the earlier literature, and to clarify some aspects of previous trade applications of the LCP.

In addition, it is shown under which conditions some of the general Le Châtelier results hold for finite parameter changes, thereby making possible the graphical representation of the results in standard trade theoretic diagrams.

1 Introduction

Since Paul Samuelson introduced the Le Châtelier Principle (LCP) into the economics discipline in his "Foundations of Economic Analysis" in 1947, it has been widely used in different parts of applied microeconomic theory and is given some space in many advanced textbooks.[1] The essence of the LCP is stated in particular clarity by Hatta (1987, p. 155):

> To state this principle explicitly, suppose that a 'just binding' constraint is added to an extremum problem such that the initial solution is on this constraint. [...] The Le Chatelier Principle states that the (compensated) effect of a shift in a parameter upon the solution of a decision variable is smaller with such an additional constraint than without.

In this general form, the LCP is a purely local result, i.e., it holds for infinitesimal shifts in parameter values only. In the below analysis, this variant is called "marginal" LCP. A qualification of this type is indicated because in the recent microeconomic literature the question has been posed under which conditions the LCP can be shown to hold for finite parameter shifts. This "extended" LCP has been analyzed, most notably, by Milgrom and Roberts (1996), using the mathematics of lattice theory, and in a more easily accessible paper by Suen, Silberberg, and Tseng (2000) who employ standard calculus.

While well known among microeconomists, it would be fair to say that the principle's popularity among trade theorists remained rather limited to date. To be sure, in a series of important papers on trade theory and trade policy issues by Neary (1985, 1988, 1995) and Neary and Ruane (1988) the marginal LCP is mentioned as the driving force behind some of the results. But as the LCP itself is not of central interest to the analysis in these papers, the principle's general applicability in the context of trade theory and policy is not emphasized. The fact that a systematic presentation of the principle's possible applications in the theory of international trade is still lacking may be one reason for the widespread neglect

[1]See for example Silberberg (1990) and Takayama (1994).

of the LCP among trade theorists. Arguably as important is, however, that the trade theoretic applications of the LCP have been confined to the marginal variant of the principle. This excludes graphical representations of the results which have traditionally played an important role in trade theory.

The purpose of this paper is twofold: Firstly, with respect to the marginal LCP the paper aims at showing in a common framework the general applicability of the principle in trade theory and policy. This is accomplished by stressing the formal similarity between a small open economy and a price taking multi-product firm, hence making available Le Châtelier results from microeconomics for economy-wide analysis. The contribution here is mainly to show the common aspects of these results, to make explicit some of them that were implicit in the earlier literature, and to clarify some aspects of previous trade applications of the LCP. Secondly, applications of the extended LCP in a trade theoretic context are derived. The plan of the paper is as follows. In section two, the theoretical model is presented. Sections three and four consider comparative static results for the production sector and the whole economy, respectively, which can all be traced back to the LCP. Section five discusses implications of the results for the welfare effects of the policy towards international trade and factor movements. Section six concludes.

2 The Model

The application of the LCP to international trade theory requires first of all – as the principle's definition above makes clear – the formulation of an appropriate extremum problem. This is achieved by the use of duality techniques, i.e., the use of the expenditure function for the demand side and a restricted profit function for the supply side of the economy.

In an economy without distortions and with all firms being price takers it is possible to treat the supply as if it came from a single – possibly vertically integrated – price taking firm (Dixit and Norman 1980, p. 29). In principle, this representative firm's optimization problem is not different from the corresponding problem on a microeconomic level: maximize profits by choosing quantities, taking as given technology and prices. In the standard microeconomic model the profit maximization can take place under restrictions on the input side, or restrictions on the output side, or it can be unrestricted. These three cases are typically called short run profit maximization, cost minimization, and long run profit maximization, respectively. The representative firm in a standard trade theory model faces a restriction that makes its problem formally similar to the case of short run profit maximization described above: it maximizes profits with a given bundle of inputs, namely the economy's factor endowments.

Consider a competitive small open economy, producing only tradable goods. The prices and supply quantities of the goods are denoted by the vectors $\boldsymbol{p}$ and $\boldsymbol{y}$, respectively.[2] The factor prices are given by $\boldsymbol{r} = (r_1, r_2, \boldsymbol{r_3}')'$ where the decomposition of $\boldsymbol{r}$ into the scalars r_1 and r_2 as well as the vector $\boldsymbol{r_3}$ is introduced because different assumptions will be made with respect to the flexibility of the different factor prices. The prices $\boldsymbol{r_3}$ are flexible in all cases, r_1 and r_2 may be fix or flexible. Furthermore, $\boldsymbol{v} = (v_1, v_2, \boldsymbol{v_3}')'$ denotes the vector of factor quantities employed in equilibrium. In the case of flexprice factors, these quantities equal the respective endowments which are supplied inelastically. While the absolute numbers of goods and factors are arbitrary in the framework considered here, a restriction on their relative number has to be imposed, namely, that there are at least as many flexprice factors as tradable goods.[3]

The equilibrium allocation in the economy with all factor prices flexible can be described by the familiar revenue function

$$\pi^0(\boldsymbol{p}, \boldsymbol{v}) = \max_{\boldsymbol{y}} \{\boldsymbol{p}'\boldsymbol{y} \,|\, (\boldsymbol{y}, \boldsymbol{v}) \text{ feasible}\}. \tag{1}$$

Although this procedure is standard in the trade theory literature, one point is worth emphasizing: Revenue maximization (i.e., GDP maximization) is implied

[2]Fat lower-case letters denote column vectors. Their transformation into row vectors is denoted by a prime ($'$).

[3]This assumption ensures that the optimum value functions introduced below are twice differentiable (Neary 1985).

by profit maximization in this special case because the representative firm treats parametrically not only the prices of all factors but also their input quantities. Clearly, the factor prices are not actually fixed but their values are determined on competitive factor markets. What the representative firm – which is a purely notional construct here – takes as given are their equilibrium values. This rather involved reasoning is presented here because it will be helpful for the comparison of situations with different numbers of fixprice factors which is conducted below.

The revenue function is a special case of the so-called "restricted profit function", a term coined by McFadden (1978).[4] This function describes the optimizing behavior by a competitive firm facing binding constraints of some sort – the above mentioned quantity constraints on the input or output side being the most obvious examples (McFadden 1978, pp. 61-2). Hence, the revenue function describes a situation of maximal input restriction.

As is well known, the price derivatives of $\pi^0(\cdot)$ give the vector of goods supplies, in our notation:

$$\pi^0_{\boldsymbol{p}}(\boldsymbol{p}, \boldsymbol{v}) = \boldsymbol{y}^0(\boldsymbol{p}, \boldsymbol{v}) \tag{2}$$

Assume now an economy identical to the former except for the assumption that one of the factors is now supplied infinitely elastic. Let v_1 denote this factor's domestic employment and r_1 denote its price. The infinitely elastic supply may be due to international mobility or fixprice-unemployment of the particular factor (Neary 1985). The supply side of the economy may then be described by

$$\pi^1(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3}) = \max_{\boldsymbol{y}, v_1}\{\boldsymbol{p}'\boldsymbol{y} - r_1 v_1 \,|\, (\boldsymbol{y}, \boldsymbol{v}) \text{ feasible}\} \tag{3}$$

which is another example of a restricted profit function.[5] In addition to the optimal bundle of outputs, the representative firm chooses the optimal input level v_1. Hence,

[4] Alternative names for this function in the literature include "variable profit function" (Diewert 1974), "gross profit function" (Gorman 1968), and "normalized restricted profit function" (Lau 1976).

[5] The resulting allocation is identical to the one described by the "mobile capital GNP-function" which is used, e.g., by Wong (1995, pp. 48-9) and which differs from $\pi^1(\cdot)$ by a constant, namely the income of the fixprice factor's domestic endowment.

profit maximization in this case implies maximization of the flexprice factors' income (as there are no pure profits).[6] The relevant first derivatives of $\pi^1(\cdot)$ are:

$$\begin{aligned}\pi^1_{\boldsymbol{p}}(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3}) &= \boldsymbol{y}^1(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3})\\ \pi^1_{r_1}(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3}) &= -v^1_1(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3})\end{aligned} \tag{4}$$

Due to the assumption that there be at least as many flexprice factors as traded goods (and hence altogether more factors than traded goods), the second derivative $\pi^1_{r_1r_1}$ is positive, showing that the demand curve for factor 1 – as the demand curves for all other factors – is downward sloping. This is clearly different from the case of equal numbers of factors and traded goods (as, e.g., in the Heckscher-Ohlin model) where factor demand in a diversified small open economy is infinitely elastic at the factor price implied by the goods prices.

We have now defined the equilibrium allocations in two economies which are identical except for the assumption with respect to the supply of v_1. It is important to note in which way this assumption is related to the issue of restrictions imposed on the optimizing agents' behavior. At first sight, both situation seem not to be rankable in terms of the numbers of restrictions imposed as a restriction of the quantity of v_1 is replaced by a restriction on the price of v_1. But one has to keep in mind that price taking behavior is assumed throughout, implying that r_1 is treated parametrically by the firm in *both* situations. Hence, in the terminology chosen above the change described has to be interpreted as the removal of the factor input restriction for one of the factors.[7]

[6]When the infinitely elastic supply is due to factor mobility, this is equivalent to maximization of GNP. When it is caused by fixprice-unemployment, GNP (being identical to GDP in this case) is not maximized. This difference is irrelevant for the derivation of the Le Châtelier effects but clearly does matter for the welfare results in the trade policy section below.

[7]In contrast, the seminal contribution by Neary (1985) interprets the imposition of a factor price rigidity on the economy as the addition of a constraint (on factor prices) rather than the removal of a constraint (on inputs). This is not a purely semantic issue because it necessarily leads to a misstatement of the LCP (p. 560): "This result reflects the Le Châtelier-Samuelson principle: as more constraints are imposed on a system, the responsiveness to exogenous shocks of the remaining unconstrained variables is increased."

Clearly, in a completely analogous manner restricted profit functions can be defined for situations where further input restrictions are removed. For notational convenience, only the additional case of $k = 2$ fixprice factors is made explicit in the analysis below, with obvious generalizations to $k > 2$. Let v_2 and r_2 now denote quantity and price of the second fixprice factor, respectively. Call the corresponding restricted profit function π^2. Its first partial derivatives follow from (4) by analogy. Only two of them are of interest below:

$$\begin{aligned} \pi^2_{r_1}(\boldsymbol{p}, r_1, r_2, \boldsymbol{v_3}) &= -v^2_1(\boldsymbol{p}, r_1, r_2, \boldsymbol{v_3}) \\ \pi^2_{r_2}(\boldsymbol{p}, r_1, r_2, \boldsymbol{v_3}) &= -v^2_2(\boldsymbol{p}, r_1, r_2, \boldsymbol{v_3}) \end{aligned} \tag{5}$$

Analogous to the above case of one fixprice factor, the second derivatives $\pi^2_{r_1 r_1}$ and $\pi^2_{r_2 r_2}$ are positive, implying downward sloping demand curves for factors 1 and 2.

3 Comparative Statics of the Production Sector

One requirement for the LCP to apply is fulfilled by the definite ranking of the two situations in terms of restrictions. In addition, one has to ensure that the restriction is 'just binding' in the original equilibrium. In this case the following identities hold, as is clear from the preceding analysis:

$$\boldsymbol{y}^1(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3}) \equiv \boldsymbol{y}^0(\boldsymbol{p}, v_2, \boldsymbol{v_3}, v^1_1(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3})) \tag{6}$$

$$v^2_1(\boldsymbol{p}, r_1, r_2, \boldsymbol{v_3}) \equiv v^1_1(\boldsymbol{p}, r_1, \boldsymbol{v_3}, v^2_2(\boldsymbol{p}, r_1, r_2, \boldsymbol{v_3})) \tag{7}$$

Both identities are central to the Le Châtelier results below. Note what the identities say: The economies to be compared are identical in the sense that in the original equilibrium prices and quantities are the same. The implication for the assumed endowments of the relevant factors – v_1 in equation (6), v_2 in equation (7) – depends on whether the respective factor is assumed to be internationally mobile or unemployed: while in the former case no assumption with respect to the endowment is necessary, in the latter case the endowment of the fixprice economy has to be strictly larger than that in the flexprice economy. Otherwise no infinitely elastic supply response to demand shifts in *both* directions would be possible.

Begin by differentiating (6) for an arbitrary y_i with respect to its own price.[8] This gives the first comparative static result:

$$\frac{\partial y_i^1}{\partial p_i} \equiv \frac{\partial y_i^0}{\partial p_i} + \frac{\partial y_i^0}{\partial v_1}\frac{\partial v_1^1}{\partial p_i} \tag{8}$$

$$= \frac{\partial y_i^0}{\partial p_i} + \frac{\partial y_i^1}{\partial r_1}\left(\frac{\partial v_1^1}{\partial r_1}\right)^{-1}\frac{\partial v_1^1}{\partial p_i} \tag{8'}$$

$$= \frac{\partial y_i^0}{\partial p_i} - \left(\frac{\partial v_1^1}{\partial p_i}\right)^2\left(\frac{\partial v_1^1}{\partial r_1}\right)^{-1} \tag{8''}$$

$$\frac{\partial y_i^1}{\partial p_i} > \frac{\partial y_i^0}{\partial p_i}, \tag{9}$$

using

$$\frac{\partial y_i^1}{\partial r_1} = \frac{\partial y_i^0}{\partial v_1}\frac{\partial v_1^1}{\partial r_1}, \tag{10}$$

which follows from (6), in (8′) and

$$\frac{\partial y_i^1}{\partial r_1} = -\frac{\partial v_1^1}{\partial p_i} \tag{11}$$

in (8″). Equation (11) follows from (4), taking into account the fact that $\pi^1_{p_i r_1}$ and $\pi^1_{r_1 p_i}$ are equal by Young's theorem. The sign of the derivatives in (11) depends on whether the input-output relationship is normal or inferior. Sakai (1974) noted that both cases can be distinguished in two equivalent ways: An input is inferior with respect to output i if the demand for this factor *decreases* with rising goods price p_i and normal otherwise. Alternatively, an output is inferior with respect to input j if this output *increases* with rising factor price r_j and normal otherwise. The equivalence of both definitions is easily seen by inspection of (11).

In words, (9) says that the own price effect on supply is larger in the fixprice economy than in the flexprice economy – irrespective of whether the relationship between the good and factor in question is characterized by normality or inferiority. Clearly, this is a variant of the marginal LCP. The result was first derived by Neary (1985). His proposition 3 reads

Proposition 1. *The imposition of factor-price rigidities leads to an increase in the economy's price-output responsiveness.* (Neary 1985, p. 559)

[8] This follows Pollak (1969) and Silberberg (1990, p. 126f.).

It should be noted that while we concentrate on what Roberts (1999) calls the "scalar LCP", Neary's original analysis is more general. He shows that $\boldsymbol{y}^1_{\boldsymbol{p}} - \boldsymbol{y}^0_{\boldsymbol{p}}$ is a positive semidefinite matrix, which implies restrictions on cross-price effects. This constitutes a valuable gain in generality compared to the scalar case presented here. But it is important to note that the multidimensional version of the LCP does not give any information on how additional restrictions influence *any particular* cross-price effect because the off-diagonal elements of $\boldsymbol{y}^1_{\boldsymbol{p}} - \boldsymbol{y}^0_{\boldsymbol{p}}$ cannot be signed in general. As the scalar LCP thus conveys the principle's essence and is more transparent, this is the version presented throughout this paper. Generalizations that make use of the multidimensional LCP will be mentioned where appropriate.

Turn now to the analysis of identity (7). Differentiating with respect to r_1 yields

$$\begin{aligned}
\frac{\partial v_1^2}{\partial r_1} &\equiv \frac{\partial v_1^1}{\partial r_1} + \frac{\partial v_1^1}{\partial v_2}\frac{\partial v_2^2}{\partial r_1} \\
&= \frac{\partial v_1^1}{\partial r_1} + \frac{\partial v_1^2}{\partial r_2}\left(\frac{\partial v_2^2}{\partial r_2}\right)^{-1}\frac{\partial v_2^2}{\partial r_1} \\
&= \frac{\partial v_1^1}{\partial r_1} + \left(\frac{\partial v_2^2}{\partial r_1}\right)^{2}\left(\frac{\partial v_2^2}{\partial r_2}\right)^{-1} \\
\frac{\partial v_1^2}{\partial r_1} &< \frac{\partial v_1^1}{\partial r_1} \\
\left|\frac{\partial v_1^2}{\partial r_1}\right| &> \left|\frac{\partial v_1^1}{\partial r_1}\right| \qquad (12)
\end{aligned}$$

In words, this result – which is another variant of the marginal LCP – may be stated as follows:

Proposition 2. *The quantity employed by a particular fixprice-factor in equilibrium is more responsive to changes in its own price if there is at least one additional fixprice factor.*

Several interesting applications of this result are possible. For example, changes of the interest rate in a small open economy induce larger capital movements in the presence of minimum wage unemployment than in the full employment case. Conversely, changes in a binding minimum wage have larger employment effects in the presence of international capital mobility. As the above presentation makes clear, results (9) and (12) are perfectly analogous to each other. This is easily explicable by the fact that in the present framework fixprice inputs are just negative outputs and therefore both results are to be understood as an increase in the "netput" supply elasticity due to loosening an input restriction.[9]

Proposition 2 seems to have not been made explicit before, it is however closely related to the following proposition in Neary (1985):

Proposition 2a. *The imposition of factor-price rigidities reduces the responsiveness of the remaining flexible factor prices to changes in endowments.* (Neary 1985, p. 560)

The similarity becomes obvious if this result is stated formally. In our notation, it says

$$\left|\frac{\partial r_2}{\partial v_2^1}\right| < \left|\frac{\partial r_2}{\partial v_2^0}\right| \tag{13}$$

and the comparison with (12) shows that it is what might be called "LCP in reverse".[10] The result highlights again that when employing the LCP one has to distinguish carefully between the exogenous and endogenous variable from the representative firm's point of view. If it is not made clear that both are interchanged in proposition 2a, one is easily puzzled by the fact that in this case the reaction is *smaller* in the *less* restricted case.

Following the derivation of the marginal Le Châtelier results, we turn now to the question under what conditions extended Le Châtelier results, i.e., results for finite price variations, can be derived. Given the above stated formal similarity of the results for goods supply and factor demand, respectively, it is possible to focus on the detailed analysis of one of these cases. We focus on goods supply and infer the factor demand result through a conclusion by analogy. As a first step, we state the extended LCP formally, using the notation established above. Assume the price

[9] Clearly, following the argument just made, an output restriction would have the same effect as an input restriction. However, in a trade theory framework the input restriction is obviously the more relevant case to consider.

[10] Factor prices are not independent from endowments here because – as has been set out above – the factor demand curves are downward sloping.

of good i is changed discretely from p_i' to p_i''. Then, the extended LCP says

$$\left|y_i^1(p_i'') - y_i^1(p_i')\right| > \left|y_i^0(p_i'', v_1') - y_i^0(p_i', v_1')\right| \tag{14}$$

where $v_1' \equiv v_1^1(p_i')$. In words, the supply of good i becomes less responsive to a finite change in its own price if a factor input restriction is added which is just binding in the original equilibrium. Equation (14) can be stated equivalently as

$$\text{sign}\left[y_i^1(p_i'') - y_i^0(p_i'', v_1')\right] = \text{sign}\left[p_i'' - p_i'\right], \tag{15}$$

which states that at the new price the supply is larger in the less restricted situation if and only if the new price is higher than the old one. The equivalence to (14) is obvious. For our purposes it is helpful to rewrite (15) as

$$\text{sign}\left[y_i^0(p_i'', v_1'') - y_i^0(p_i'', v_1')\right] = \text{sign}\left[p_i'' - p_i'\right], \tag{15'}$$

where $v_1'' \equiv v_1^1(p_i'')$. All that has been done here is to replace the less restricted supply function by the more restricted one, where the additional constraint is just binding. Therefore (15′) is another statement of the extended LCP. Now, the marginal LCP from above is slightly rewritten to give, by combining (8) and (9),

$$\frac{\partial y_i^1(p_i')}{\partial p_i} - \frac{\partial y_i^0(p_i', v_1')}{\partial p_i} = \frac{\partial y_i^0(p_i', v_1')}{\partial v_1}\frac{\partial v_1^1(p_i')}{\partial p_i} > 0. \tag{16}$$

It is clear from (16) that at the original equilibrium (p_i', v_1') either both partial derivatives on the right hand side are negative or both are positive. From (8′), the derivatives are positive if and only if good i is normal with respect to factor 1, in which case $\partial v_1^1(p_i')/\partial p_i = -\partial y_i^1(p_i')/\partial r_1 > 0$. As follows from its derivation, (16) is valid only at parameter combinations (p_i, v_1) which make the quantity constraint just binding. Assume however, that the derivatives in (16) keep their sign for independent variations of p_i and v_1 in the intervals $[p_i', p_i'']$ and $[v_1', v_1'']$, respectively – that is, in situations where v_1 may be strictly binding. It is now readily checked that under this condition equation (15′) follows.[11] This gives rise to

[11]For example, assume that both derivatives are positive and the parameter change is a price increase. Clearly, the right hand side of (15′) is positive. Turning to the left hand side, $v'' > v'$ because $\partial v_1^1/\partial p_1 > 0$ and $y_i^0(p_i'', v_1'') > y_i^0(p_i'', v_1')$ because $\partial y_i^0/\partial v_1 > 0$. The validity of the result in the remaining cases can be checked similarly.

Proposition 3. *Assume a finite change in the price of good i. The imposition of a rigidity on the price of factor j increases the induced change in the supply of good i if over the relevant parameter range good i does not switch from being inferior to being normal with respect to factor j (or vice versa).*

An intuitive step-by-step explanation of this strong result may be helpful. The steps that lead to the result are best seen in (8′): Let v_1 – along with v_2 and $\boldsymbol{v_3}$ – be in fixed supply initially. A rising goods price p_i clearly leads to a "first-round" increase in output i for given v_1 ($\partial y_i^0/\partial p_i > 0$). If v_1 is normal with respect to good i, the firm's demand for v_1 increases ($\partial v_1^1/\partial p_i > 0$), leading to a rising shadow factor price for the given input quantity. If the restriction is abolished, the firm increases the amount of v_1 (because the shadow factor price exceeds r_1), inducing a fall in the shadow factor price ($(\partial v_1^1/\partial r_1)^{-1} < 0$). Clearly, the change in the employment of v_1 is identical to the one that would have been *induced* by a fall in r_1 in a – hypothetical – undistorted situation. Hence, because of normality, there is a "second-round" increase in output ($\partial y_i^1/\partial r_1 < 0$). If on the other hand v_1 is inferior with respect to good i, the firm's demand for v_1 decreases, leading to a fall in the shadow factor price below r_1 . With the restriction abolished, the firm reduces the employment of v_1 until the shadow price equals the market price. Given the assumed inferiority, this again yields a "second-round" increase in output i.

Both cases are illustrated for the special case with two goods only in figures 1 and 2, respectively. Good 2 serves as the numéraire, hence p_1 is the relative price of good 1. The notation, which is identical in both figures, is as follows: Prices and quantities in the old equilibrium are denoted by a prime (′), prices and quantities in the new equilibrium by a double prime. The market price for factor 1 is constant throughout at r_1. $T(v_1')$ and $T(v_1'')$ denote the transformation curves for the respective quantities of v_1, holding constant $(v_2, \boldsymbol{v}_3)$. Clearly, given the constancy of r_1, only one point on every transformation curve is an equilibrium point. In the respective left hand panels, $\tilde{r}_1$ is the shadow price of factor one at the new goods price and the old factor input quantity. The resulting output of good 1 is denoted in the respective right

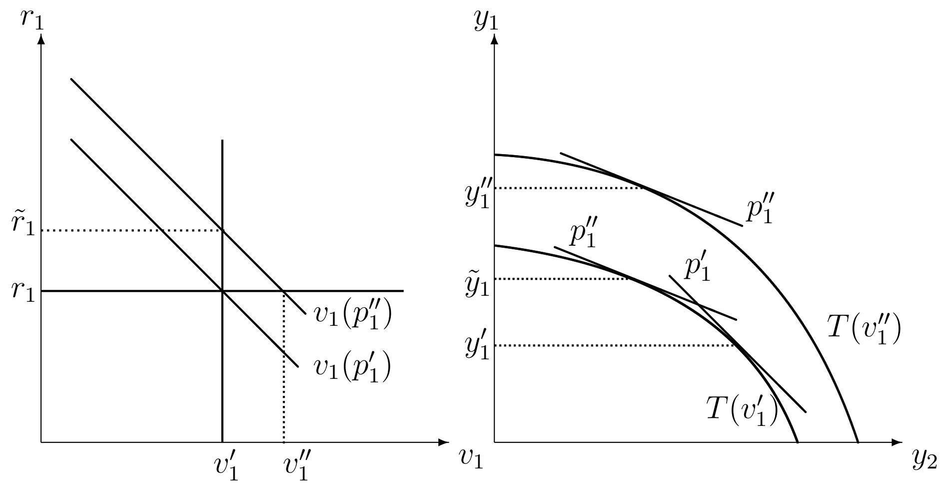

Figure 1: Normal Input-Output-Relation

hand panels by $\tilde{y}_1$.[12]

From the above noted analogy between goods supply and factor demand it follows immediately that proposition 2 can be generalized to discrete parameter changes as well. For the factor demand result, the relevant parameter is the factor price r_1. As the derivation of the marginal Le Châtelier result (12) shows, the condition analogous to the one in the goods supply case is for the derivatives $\partial v_1/\partial r_2$ and $\partial v_2/\partial r_1$ not to change signs over the relevant factor price range. That is, both factors must not switch from being substitutes to being complements. As in the above case, this is a sufficient, but not necessary condition for the extended Le Châtelier result to hold.[13]

[12] Figures 1 and 2 immediately suggests a comparison of the above results with the Rybczynski theorem. After all, the movement from $\tilde{y}_1$ to y_1'' involves a change in the domestic employment of v_1 at constant goods prices and may therefore interpreted as an induced Rybczynski effect. However, given the above assumptions the number of factors necessarily exceeds two, and therefore the Rybczynski theorem does not apply: The effect of the change in employment of v_1 on the output of good 2 in figure 1 is indeterminate while – given the expansion of sector one – sector two necessarily shrinks in the Heckscher-Ohlin case. This is just an illustration of the above statement that the LCP has nothing to say about differences in cross price effects which are due to differing numbers of constraints.

[13] This result has been derived for a profit maximizing firm by Suen et al. (2000).

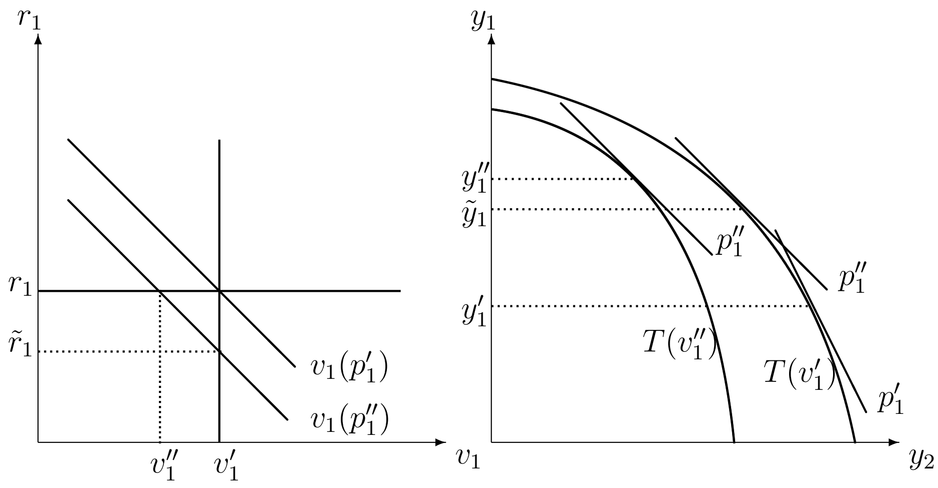

Figure 2: Inferior Input-Output-Relation

4 Comparative Statics of International Goods and Factor Flows

While for the analysis of the production sector above the analogy to the representative firm has proved helpful, this analogy has to be modified once the focus is on cross-border flows of goods and factors. In this case, the demand side has to come into play. In accordance with most of the trade theory literature, a representative consumer is assumed who derives utility exclusively from the consumption of physical goods. Consequently, the demand side can be summarized by the expenditure function

$$e(\boldsymbol{p}, u) = \min_{\boldsymbol{x}} \{\boldsymbol{p}'\boldsymbol{x} \mid f(\boldsymbol{x}) \geq u\} \tag{17}$$

with $f(\cdot)$ denoting the direct utility function, u the utility level, and $\boldsymbol{x}$ goods demand. From Shephard's Lemma, $e_{\boldsymbol{p}}(\boldsymbol{p}, u) = \boldsymbol{x}(\boldsymbol{p}, u)$.

From the properties of the restricted profit functions and the expenditure function, the following equalities hold:

$$\begin{aligned}
\boldsymbol{m}^0(\boldsymbol{p}, \boldsymbol{v}, u) &= e_{\boldsymbol{p}}(\boldsymbol{p}, u) - \pi^0_{\boldsymbol{p}}(\boldsymbol{p}, \boldsymbol{v}) \\
\boldsymbol{m}^1(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3}, u) &= e_{\boldsymbol{p}}(\boldsymbol{p}, u) - \pi^1_{\boldsymbol{p}}(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3}) \\
\boldsymbol{m}^2(\boldsymbol{p}, r_1, r_2, \boldsymbol{v_3}, u) &= e_{\boldsymbol{p}}(\boldsymbol{p}, u) - \pi^2_{\boldsymbol{p}}(\boldsymbol{p}, r_1, r_2, \boldsymbol{v_3})
\end{aligned} \tag{18}$$

where $\boldsymbol{m}^k(\cdot)$ is the net import vector with k fixprice factor(s). Inspection of (18) reveals that the number of fixprice factors influences imports only via its influence on the supply side. Together with (9) this immediately yields

Proposition 4. *An increase in the number of fixprice factors increases the price responsiveness of net import demand.*

Formally, this result can be stated as

$$\left|\frac{\partial m_i^1}{\partial p_i}\right| > \left|\frac{\partial m_i^0}{\partial p_i}\right| \tag{19}$$

The corresponding result due to the "LCP in reverse" is given by

$$\left|\frac{\partial p_i}{\partial m_i^1}\right| < \left|\frac{\partial p_i}{\partial m_i^0}\right| \tag{20}$$

or, in verbal terms:

Proposition 4a. *An increase in the number of fixprice factors reduces the responsiveness of any goods price to an exogenous change in the respective import quantity.*

More interestingly, the analysis of imports instead of supplies allows the consideration of an additional restriction, namely the fixing of import quantities via quotas or voluntary export restraints (VERs).[14] Let m_i^{0r} denote the import quantity of good i in a situation where the imports of good j are restricted by a quota or VER.[15] Then the following identity holds:

$$m_i^0(\boldsymbol{p}, \boldsymbol{v}, u) \equiv m_i^{0r}(\boldsymbol{p}_{\cdot j}, \boldsymbol{v}, u, m_j^0(\boldsymbol{p}, \boldsymbol{v}, u)) \tag{21}$$

with $\boldsymbol{p}_{\cdot j}$ denoting the price vector of all goods except for j. Differentiating with respect to p_i yields

$$\frac{\partial m_i^0}{\partial p_i} \equiv \frac{\partial m_i^{0r}}{\partial p_i} + \frac{\partial m_i^{0r}}{\partial m_j}\frac{\partial m_j^0}{\partial p_i}$$

[14]In the framework employed here, both instruments differ only with respect to the distribution of rents – which accrue to the domestic economy under quotas and to the foreign economy under VERs. The difference is irrelevant for the Le Châtelier effects considered here, but will become important for the welfare analysis in the next chapter.

[15]An identical result clearly holds for $k > 0$ fixprice factors.

$$
\begin{aligned}
&= \frac{\partial m_i^{0r}}{\partial p_i} + \frac{\partial m_i^0}{\partial p_j}\left(\frac{\partial m_j^0}{\partial p_j}\right)^{-1}\frac{\partial m_j^0}{\partial p_i} \\
&= \frac{\partial m_i^{0r}}{\partial p_i} + \left(\frac{\partial m_j^0}{\partial p_i}\right)^{2}\left(\frac{\partial m_j^0}{\partial p_j}\right)^{-1} \\
\frac{\partial m_i^0}{\partial p_i} &< \frac{\partial m_i^{0r}}{\partial p_i} \\
\left|\frac{\partial m_i^0}{\partial p_i}\right| &> \left|\frac{\partial m_i^{0r}}{\partial p_i}\right| \qquad (22)
\end{aligned}
$$

This result, due to Neary (1995), can be stated verbally as

Proposition 5. *The net import demand is more elastic with respect to own price changes if net import quantities of other goods are free to adjust than if they are fixed by quantitative import restrictions.* (Neary 1995, p. 536)

There is a combination of Le Châtelier effects from the supply and demand side that leads to this result (as the import quota can be understood as a combination of output and consumption restriction). On either side of the market, goods can be substitutes or complements. As the argument runs along parallel lines in all four cases, only one of them is considered explicitly, namely the case of complementarity on the supply side. Following Sakai (1974), this means that an increase in p_i increases the supply of good j, and vice versa. Suppose that the output of good j is restricted initially, and that this restriction is abolished. In this case, the increase in p_i induces an increase in the production of good j, leading to a fall in p_j an thus to an increase in the production of i over and above the direct effect. An analogous self-reinforcing process takes place under complementarity on the demand side and under substitutability on either supply or demand side. Therefore, no assumption on the same good being either substitute or complement on both supply and demand side is needed.

Clearly, this Le Châtelier result derived by Neary can be reversed to give

$$
\left|\frac{\partial p_i}{\partial m_i^0}\right| < \left|\frac{\partial p_i}{\partial m_i^{0r}}\right| \qquad (23)
$$

or, in verbal terms,

Proposition 5a. *An exogenous change in the net import quantity of a particular good induces a smaller change in its own price if net import quantities of other goods are free to adjust than if they are fixed by quantitative import constraints.*

A result similar to (22) can be derived for the demand for a fixprice factor which without loss of generality is assumed to be v_1. It is clear from the preceding analysis that the expenditure function and hence demand is independent from r_1. Therefore, the question of how the price elasticity of factor demand is influenced by an import quota m_j^1 is equivalent to the question how it is influenced by an appropriate output target y_j^1. The relevant identity in this context is

$$v_1^1(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3}) \equiv v_1^{1r}(\boldsymbol{p}_{\cdot j}, r_1, v_2, \boldsymbol{v_3}, y_j^1(\boldsymbol{p}, r_1, v_2, \boldsymbol{v_3})). \tag{24}$$

Differentiating with respect to r_1 yields

$$\begin{aligned}
\frac{\partial v_1^1}{\partial r_1} &\equiv \frac{\partial v_1^{1r}}{\partial r_1} + \frac{\partial v_1^{1r}}{\partial y_j}\frac{\partial y_j^1}{\partial r_1} \\
&= \frac{\partial v_1^{1r}}{\partial r_1} + \frac{\partial v_1^1}{\partial p_j}\left(\frac{\partial y_j^1}{\partial p_j}\right)^{-1}\frac{\partial y_j^1}{\partial r_1} \\
&= \frac{\partial v_1^{1r}}{\partial r_1} - \left(\frac{\partial y_j^1}{\partial r_1}\right)^2\left(\frac{\partial y_j^1}{\partial p_j}\right)^{-1} \\
\frac{\partial v_1^1}{\partial r_1} &< \frac{\partial v_1^{1r}}{\partial r_1} \\
\left|\frac{\partial v_1^1}{\partial r_1}\right| &> \left|\frac{\partial v_1^{1r}}{\partial r_1}\right|
\end{aligned} \tag{25}$$

Hence, the following has been shown to be true:

Proposition 6. *The imposition of quantitative import restrictions decreases the responsiveness of the quantity employed by a particular fixprice-factor in equilibrium to changes in its own price.*

Again, this application of the LCP is apparently new, but there exists a closely related result due to Neary (1988) which makes use of the "LCP in reverse":

Proposition 6a. *The effect of an exogenous capital inflow on the domestic rental is larger when the level of imports is constrained by quantitative restrictions than when it is free to adjust.* (Neary 1988, p. 727)

Both results are related in exactly the same way as propositions 5 and 5a. In our notation, proposition 6a reads

$$\left|\frac{\partial r_1}{\partial v_1^0}\right| < \left|\frac{\partial r_1}{\partial v_1^{0r}}\right|, \tag{26}$$

which makes the analogy obvious.

There is a conceptual difficulty with generalizing propositions 4 to 6 to the case of finite parameter changes: In accordance with much of the modern trade literature, the net import functions specified above are derived using the expenditure function, and therefore they are Hicksian demand functions. This entails that with dicrete price variations different compensation levels are needed in the more and less restricted situation, respectively. The issue has been touched upon by Milgrom and Roberts (1996, p. 177), but is not pursued further in this paper.

Further Le Châtelier effects can be derived if one dispenses with the assumption made throughout that consumers derive utility exclusively from consumption of physical goods. Without giving reference to the LCP in this context, Michael (1994) demonstrates that endogenising the labor supply increases the own price effects in net import demand for goods and fixprice factors. The analogy to the analysis in this section is clear if the standard case of exogenous labor supply is interpreted as a situation where the consumption of leisure is restricted to a certain level. Endogenising the labor supply is then appropriately seen as abolishing this restriction.

5 Trade Policy Implications

What the marginal LCP in effect does, is to compare the own price elasticities of supply or demand functions in situations with differing numbers of restrictions. In order to translate these results into statements on the relative size of welfare effects of trade policy one has to make use of the connection between prices, quantities, and welfare. For the welfare analysis in this section it clearly does matter whether the fixprice factors are unemployed or internationally mobile. Until further notice it is assumed that all factors are fully employed. The analysis is restricted to the case of marginal parameter variations in this section because the literature to which we refer here deals with the welfare effects of marginal changes in trade policy.

Tariffs and import quotas have in common that the revenue accrues to the domestic economy. Both regimes give rise to an identical budget constraint which reads for a small open economy with k internationally mobile factors and an arbitrary number of traded goods as follows:

$$e(\boldsymbol{p}, u) = \begin{cases} \pi^k(\cdot) + \boldsymbol{t}'\boldsymbol{m}^k + \sum_{i=1}^{k} \Big[r_i \bar{v}_i + \tau_i (v_i^k - \bar{v}_i) \Big] & \text{for } k \in \{1, 2\} \\ \pi^k(\cdot) + \boldsymbol{t}'\boldsymbol{m}^k & \text{for } k = 0 \end{cases} \tag{27}$$

Here, $\bar{v}_i$ denotes the endowment of factor i, $\boldsymbol{t}$ the vector of import tariffs or quota rents, respectively, and τ_i the tax on net imports of factor i. In order to facilitate the analysis and highlight the Le Châtelier effects it is assumed that at most one element in $\boldsymbol{t}$ and one τ_i, namely t_1 and τ_1, differ from zero. This implies that cross effects between different goods or different factors do not matter.[16] Differentiating (27) totally, holding constant factor endowments, yields

$$e_u du = \begin{cases} t_1 dm_1^j + \tau_1 dv_1^j & \text{for } j \in \{1, 1r, 2, 2r\} \\ t_1 dm_1^j & \text{for } j \in \{0, 0r\} \end{cases} \tag{28}$$

using (4), (5), (18), and the small country assumption. While dv_1^j is clearly endogenous, dm_1^j may be exogenous (in the case of a quota on good one) or endogenous (in the case of a tariff on good one). As (28) shows, welfare effects are only due to changes in net imports of goods or factors in distorted markets.

Turn to the case of distorted goods trade first. Factor mobility is – where present – assumed to be unrestricted, i.e., $\tau_1 = 0$. The implication is that induced factor movements *per se* have no welfare relevance. Then a couple of results, not all of them new, follow straightforwardly from the applications of the LCP spelt out in sections 3 and 4:

[16] Allowing for any number of tariffs or factor import taxes to differ from zero does not alter the results below as long as one restricts the analysis to proportional changes. In this case, the multidimensional instead of the scalar LCP applies.

Proposition 7. *Increasing the number of factors that are internationally mobile increases the welfare cost of tariff protection while leaving the welfare cost of quota protection unaltered.* (Neary and Ruane 1988)

This can be explained as follows: Because of the small country assumption domestic prices change one-to-one with tariffs. The induced change in net imports with and without factor mobility can then be inferred from (19), saying that a decline in imports following a rise in tariffs is more severe with factor mobility than without. And, by inspection of (28), a larger decline in imports implies a larger welfare loss. In contrast, in the case of quotas welfare is influenced by the change in the policy instrument itself, leaving no room for welfare relevant Le Châtelier effects.

Proposition 8. *The welfare cost of tariff protection is decreased by the presence of just binding import quotas on other goods, while the cost of quota protection is unaltered by their presence.*

This proposition, which is apparently new, follows from the LCP in analogy to proposition 7. Here, the difference in the induced import changes for the case of tariffs follows from (22).

Turn now to the case of distorted factor trade. In order to focus on the direct effect of the different regimes, assume that goods trade is either quota restricted ($\boldsymbol{dm} = \mathbf{0}$) or unrestricted ($\boldsymbol{t} = \mathbf{0}$). Then the following results hold:

Proposition 9. *The welfare cost of taxes on factor imports is increased by the presence of mobile factors not subject to import restrictions and decreased by the presence of just binding quotas on goods imports.*

This follows from (12) and (25), respectively: According to these results the own price elasticity of the demand for a particular factor increases with the number of additional fixprice factors and decreases with the number of quantitative import restrictions. As the endowment of the factors are exogenous and supplied inelastically, every change in factor demand translates one-for-one into a change in factor import demand. The size of the induced change in the factor imports in turn determines the size of the welfare effect, as can be seen from (28).

In the case of VERs, the rents due to the restrictions of goods trade accrue to foreigners by assumption. Hence the relevant budget constraint is given by

$$e(\boldsymbol{p}, u) = \begin{cases} \pi^k(\cdot) + \sum_{i=1}^{k} \Big[r_i \bar{v}_i + \tau_i (v_i^k - \bar{v}_i) \Big] & \text{for } k \in \{1, 2\} \\ \pi^k(\cdot) & \text{for } k = 0 \end{cases} \tag{29}$$

In analogy to the tariff and quota case analyzed above, it is assumed that at most one τ_i, namely τ_1, is different from zero and only one import good, namely good 1, is subject to a VER. Differentiating (29) totally, holding constant factor endowments and the prices of all goods except for good 1, yields

$$e_u du = \begin{cases} -m_1^j dp_1 + \tau_1 dv_1^j & \text{for } j \in \{1, 1r, 2, 2r\} \\ -m_1^j dp_1 & \text{for } j \in \{0, 0r\} \end{cases} \tag{30}$$

All p_i except for p_1 are fixed by the small country assumption. Assume again $\tau_1 = 0$. Then, using the LCP, it is straightforward to derive the following:

Proposition 10. *The cost of protection through VERs is decreased by international factor mobility.* (Neary 1988, p. 729)

This is an application of the "LCP in reverse" given in (20): The price effect of a given change in import quantity decreases with additional fixprice factors. As the inspection of (30) shows, this change in the domestic goods price, which is in effect a change in the terms of trade, determines the welfare effect with VERs.

Proposition 11. *The cost of protection through VERs is increased by the presence of just binding quantitative import restrictions on other goods.*

This follows from an application of the "LCP in reverse" in analogy to proposition 10. Here, the relevant difference in the price effects is given in (23). The results of the full employment case are summarized in Table 1. Note that the reference situation does not have to be completely specified. The results hold for reference situations with any number of mobile factors and import quotas.

These results cease to hold if the assumption of full employment is dropped. The see this, assume that v_1 (labor) is paid a binding minimum wage r_1, while v_2 is

Table 1: **Le Châtelier effects on the welfare cost of protection**

		protective instrument			
		tariff	quota	VER	factor import tax
impact of regime change on the welfare cost of protection	additional mobile factor	+	0	-	+
	additional import quota	-	0	+	-

Note: higher/equal/lower cost are indicated by +/0/-.

fully employed. Only the trade of good 1 is restricted, and the restriction takes the form of a tariff or quota, i.e., the revenue accrues to the domestic economy.[17] It is assumed for simplicity that there are no taxes on international factor movements. The budget constraint is in this case given by

$$e(\boldsymbol{p}, u) = \begin{cases} \pi^k(\cdot) + r_1 v_1^k(\cdot) + t_1 m_1^k & \text{for } k = 1 \\ \pi^k(\cdot) + r_1 v_1^k(\cdot) + r_2 \bar{v}_2 + t_1 m_1^k & \text{for } k = 2 \end{cases} \tag{31}$$

Differentiating in the familiar way gives

$$e_u du = t_1 dm_1^j + r_1 dv_1^j \qquad \text{for } j \in \{1, 1r, 2, 2r\} \tag{32}$$

The welfare effect of protection now depends on an additional employment effect. And while it is clear, as spelt out above, that $|dm_1^2| > |dm_1^1|$ for a given tariff change, the LCP gives no guidance whatsoever on the relative size of dv_1^1 and dv_1^2 because these are cross-price effects, not own-price effects. Therefore, in the presence of minimum wages no welfare results can be derived from the LCP alone.

6 Conclusion

In the sections dealing with the marginal Le Châtelier Principle, the paper has shown that the LCP is the unifying principle behind many results in the recent theory of trade and factor movements as well as their applications in the theory of trade

[17]It is easily verified that the following argument applies likewise to the case of VERs.

policy. Many of these results have been attributed to the LCP before. However, it has been shown here that by exploiting the analogy between a small open economy and a representative multi-product firm, the full range of Le Châtelier results from microeconomics is made available for economy-wide analysis. This is used in this paper to derive some additional results in the trade theory as well as the trade policy section. Arguably, the more important contribution of the approach presented here is to increase the transparency of the argument to a considerable degree. As the analysis is guided not by special problems in theory or policy but by an abstract principle, the list of results is furthermore known to be exhaustive: It is clear that no further results are to be expected as long as one sticks to the assumptions of the model. In the sections dealing with the extended LCP, it has been shown that some applications of the principle can be readily generalized in an intuitively plausible way to the case of finite parameter variations and therefore used in standard trade theory diagrams.

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