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Horizontal Mergers with Free Entry

by

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Abstract

We consider the impact of horizontal mergers in the presence of free entry and exit. In contrast to much of the previous literature on horizontal mergers, our model yields predictions that seem intuitively reasonable: with only moderate cost synergies mergers of a small number of industry participants are beneficial (even under quantity competition), there is no "free rider problem" in that insiders always benefit more than outsiders, and quantity-setting and price-setting games yield similar predictions. We also find that all privately beneficial mergers are also socially beneficial.

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Outline

1. *Introduction*
2. *The model*
3. *The Entry and Exit Effects of Mergers*
4. *Welfare*
5. *Conclusion*

Non-Technical Summary

The literature on horizontal mergers has grown at a rapid pace over the last 20 years. Much of this work has been inspired by counter-intuitive results that arise in both quantity-setting and price-setting games. For example, while mergers may not be profitable in quantity-setting games, they are always profitable in price-setting games. Further, mergers benefit the outsiders and they gain more than the insiders. This last result is especially troubling since it makes it difficult to explain how merger activity gets started – each firm would prefer to remain an outsider and free-ride off of the collusive behavior of the insiders.

Almost all of this literature assumes that the number of active firms is fixed exogenously. While this may be appropriate for the short-run, it is clearly inappropriate for long-run industry analysis. In this paper, we consider the impact of horizontal mergers in the presence of free entry and exit. In contrast to much of the previous literature on horizontal mergers, our model yields predictions that seem intuitively reasonable: with only moderate cost synergies mergers of a small number of industry participants are beneficial (even under quantity competition), there is no "free rider problem" in that insiders always benefit more than outsiders, and quantity-setting and price-setting games yield similar predictions. We also find that all privately beneficial mergers are also socially beneficial.

Our analysis is useful for at least one other reason. As already emphasized, courts in the US have occasionally rejected merger challenges based on the logic that entry triggered by the merger would undo any anticompetitive effects that might arise. Our model allows us to examine how the threat of entry affects the profitability of merger and its eventual impact on welfare. It suggests that mergers that are beneficial are likely to be welfare enhancing as well.

1. Introduction

The literature on horizontal mergers has grown at a rapid pace over the last 20 years. Much of this work has been inspired by counter-intuitive results that arise in both quantity-setting and price-setting games. For example, in quantity-setting games without cost synergies, mergers that do not include a vast majority of industry participants are typically harmful for the insiders, whereas *all* mergers benefit the outsiders (Salant, Switzer and Reynolds 1983).[1] In contrast, mergers are always beneficial for the merging parties in price-setting games, although the outsiders gain more than the insiders (Deneckere and Davidson 1985). This last result is especially troubling since it makes it difficult to explain how merger activity gets started – each firm would prefer to remain an outsider and free-ride off of the collusive behavior of the insiders.

Almost all of this literature assumes that the number of active firms is fixed exogenously.[2] While this may be appropriate for the short-run, it is clearly inappropriate for long-run industry analysis. In this paper, we analyze horizontal mergers in a simple model in which free entry and exit shapes the long-run industry structure and show that some of the troubling findings from earlier work disappear in this setting. Moreover, we find that when we allow for free entry, quantity-setting and price-setting games yield qualitatively similar results.

There is at least one other reason to analyze the impact of horizontal mergers is the presence of entry and exit. As emphasized by Werden and Froeb (1998), courts in the US have occasionally rejected merger challenges based on the logic that entry triggered by the merger would undo any anticompetitive effects that might arise. Our model allows us to examine how the threat of entry affects the profitability of merger and its eventual impact on welfare. Our analysis suggests that mergers that are beneficial are likely to be welfare enhancing as well.

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[1] We should note that there is a growing literature devoted to finding ways around this "merger paradox." One typical approach is to assume that the merger changes the rules of the game (for example, Daughety (1990) anlayzes a Stackelberg model with multiple followers and leaders in which a merger between two followers allows the new firm to become a leader). Another approach is to assume that a merger allows the new firm to use strategies that were not available before the merger. Recent contributions to this literature include Creane and Davidson (2004) or Huck, Konrad, and Mueller (2004). These papers all assume that the number of industry participants is fixed.

[2] See Werden and Froeb (1998), Cabral (2003) and Spector (2003) for notable exceptions.

The paper divides into four additional sections. In Section 2, we introduce the basic model and derive two results that illustrate the dramatic way in which allowing free entry and exit can alter the model's predictions. In particular, we show that (a) if the firms compete in quantities and there are no sunk costs of entry, then *any* merger that results in *any* degree of cost synergy is beneficial; and, (b) with free entry there is never a free-rider problem. In the third section we turn to a more general analysis of mergers with free entry and examine conditions under which entry triggered by the merger may moderate its anticompetitive impact. A more complete welfare analysis is offered in Section 4. We offer some concluding thoughts in Section 5.

2. The Model

We assume that there is a large number of firms in the economy and that they each possess the same technology. Each firm faces two costs related to production: a constant marginal cost of $c \geq 0$ and a fixed cost of $F \geq 0$. In addition, entry requires each firm to pay a sunk cost of $S \geq 0$. We assume that the firms produce homogeneous products and compete ala Cournot, although we also discuss the case of Bertrand competition (with the details provided in the Appendix A). We also assume that there is free entry so that in equilibrium the profit of the marginal entrant (net of fixed and sunk costs) is equal to 0.

Denote the inverse market demand for the product by *P(Q)*, where *Q* is aggregate output, and assume that $P'(Q) < 0$ and $P''(Q) \leq 0$ for all *Q*.

We consider the following 3-stage game. In stage 1, firms decide whether to enter the market. To avoid strategic entry, which is not the focus of this paper, we assume that entry occurs sequentially. After the entry decision has been made and the sunk costs have been paid, in stage 2 a subset of *M* firms may (or may not) agree to merge. For simplicity, we assume that this decision is anticipated in stage 1.[3] Finally, in stage 3 the firms compete in output in standard Cournot fashion. In order to avoid integer problems, we treat the number of firms as a continuous variable.

[3] Alternatively, we could assume that the merger is not anticipated and that after the merger takes place there is another stage in which additional firms may enter or existing firms may exit before quantity competition begins. This would make the structure of our game similar to the one considered in Spector (2003). See footnote 4 for a discussion of how this would alter our results.

If a merger occurs in stage 2, we assume that it has two effects. First, the insiders now choose their output to maximize joint profits. Second, the merger may result in cost synergies. This is modeled by assuming that if a merger occurs, the marginal cost of the insiders becomes λc where $\lambda \leq 1$ and that the insiders save by paying the fixed cost of production only once.

We are now in position to present two simple results that will illustrate just how much of a difference allowing for free entry can make for merger analysis. Our first result has to do with the case in which the sunk cost of entry is zero (S = 0). In this case, it is easy to show that if $\lambda < 1$ then *any* merger must be beneficial for the merging parties. To see this, note that if no merger were to occur, every firm would earn zero profits (net of fixed costs). Now, suppose that a merger occurs. Anticipating this, a different number of firms will enter in stage 1. In fact, entry will occur until a typical outsider earns zero net profits. If we let q_O denote the output of a typical outsider, use q_I to represent the output of the merged firm (i.e., the insiders) and let n_O denote the number of outsiders in the free entry equilibrium, then this condition is given by

(1) $$[P(n_O q_O + q_I) - c]q_O - F = 0$$

Now, since the merged firm faces a lower marginal cost (λc) than the outsiders (c) it follows that in the Cournot equilibrium $q_I > q_O$. Thus, we have

(2) $$[P(n_O q_O + q_I) - \lambda c]q_I - F > [P(n_O q_O + q_I) - c]q_O - F = 0.$$

That is, the merger *must* be beneficial. This result is in stark contrast to the results derived by Salant, Switzer and Reynolds (1983) who analyze mergers in the Cournot model with a fixed number of firms. In that setting, *significant* cost synergies are required to make mergers of a small number of firms beneficial.

Our second result has to do with the free-rider problem. This problem arises in both quantity-setting and price-setting games when mergers do not lead to large cost savings. In those settings, the outsiders *always* benefit more from a merger than do the insiders. We note that this problem disappears trivially when we allow for free entry. This is due to the fact that free entry drives the outsiders' net profits to zero both with and without the merger. Thus, the merger has no long run impact on the profitability of the outsiders!

To summarize, we have

Proposition 1: If there are no sunk costs associated with entry, then *any* merger is beneficial for *any* degree of cost synergy.

Proposition 2: Mergers in the presence of free entry have no impact on the equilibrium profits earned by the outsiders.

3. The Entry and Exit Effects of Mergers

In this section, we provide a more general analysis of the impact of mergers on industry structure. Since Proposition 1 takes care of the case in which $S = 0$, we assume that $S > 0$ for the remainder of the paper. For illustrative purposes, we now assume that demand is linear. Accordingly, we assume that $P(Q) = 1 - Q$. It is straightforward to show that with this demand curve, profit and aggregate output in the no merger equilibrium are given by

(3) $$\pi^* = \left[\frac{1-c}{N+1}\right]^2 - F - S$$

(4) $$Q^* = \frac{N(1-c)}{N+1}$$

In Stage 1, firms will enter until π^* is driven to zero. Thus, without a merger the number of active firms in equilibrium is given by

(5) $$N^* = \frac{1-c}{\sqrt{F+S}} - 1$$

Now, suppose instead that firms anticipate that a merger of M firms will occur in Stage 2. Then the Cournot profits for the merged firm and a typical outsider are given by (6) and (7), respectively

(6) $$\pi_I = \left[\frac{1+(N-M)(1-\lambda)c - \lambda c}{N-M+2}\right]^2 - F - MS$$

(7) $$\pi_O = \left[\frac{1-(2-\lambda)c}{N-M+2}\right]^2 - F - S$$

The Cournot output for the merged firm and a typical outsider are given by (8) and (9)

(8) $$q_I = \frac{1+(N-M)c-(N-M+1)\lambda c}{N-M+2}$$

(9) $$q_O = \frac{1-(2-\lambda)c}{N-M+2}$$

Analysis of this case is simplified by first considering how the merger affects industry profits *with N held fixed.* In that case, the merger is beneficial for the insiders if $\pi_I \geq M\pi^*$; it benefits the outsiders if $\pi_O \geq \pi^*$; and, the outsiders benefit more than the insiders if $M\pi_O \geq \pi_I$. Define $\lambda_I(M)$ to be the value of λ such that $\pi_I = M\pi^*$; define $\lambda_O(M)$ to be the value of λ such that $\pi_O = \pi^*$; and, define $\hat{\lambda}(M)$ to be the value of λ such that $M\pi_O = \pi_I$.

Consider first the case in which $F = 0$. Then, from (6) and (7) we have

(10) $$\lambda_I(M) = \frac{(N+1)[1+(N-M)c] - \sqrt{M}(N-M+2)(1-c)}{(N+1)(N-M+1)c}$$

(11) $$\lambda_O(M) = \frac{(N+M)c+1-M}{(N+1)c}$$

(12) $$\hat{\lambda}(M) = \frac{1-\sqrt{M}+(N-M+2\sqrt{M})c}{(N-M+1+\sqrt{M})c}$$

It is straightforward to show that $\lambda_I(M) > \hat{\lambda}(M) > \lambda_O(M)$ for all *N* and *M*.

With the following definition, we are now in position to analyze the impact of mergers with free entry.

Definition: The cost synergies associated with a merger of *M* firms are *modest* if $\lambda \geq \hat{\lambda}(M)$; they are *moderate* if $\lambda \in [\lambda_O(M), \hat{\lambda}(M)]$; and, they are *dramatic* if $\lambda \leq \lambda_O(M)$.

Proposition 3: With free entry and no fixed cost of production, a merger of *M* firms will not take place if cost synergies are modest. If cost synergies are moderate, the merger will occur and it will induce *more* firms to enter in stage 1. If the cost synergies are dramatic, the merger will occur and it will induce *fewer* firms to enter in stage 1.

Proof: Fix *N* at *N** as given in (5) and assume that the *M* firms merge. Then if the cost synergies are modest with $\lambda > \lambda_I(M)$, it follows that $\pi_O > 0 > \pi_I$. This means that in the free entry equilibrium, more than N* firms will enter in Stage 1. These extra outsiders lower π_I further, implying that the merger harms the insiders.

Now, suppose that the cost synergies are modest with $\lambda \le \lambda_I(M)$. Then it follows that when $N = N$*, $\pi_O > \pi_I > 0$. This means that more than N* firms enter in Stage 1 of the free entry equilibrium; in fact, firms enter until $\pi_O = 0 > \pi_I$. Thus, the insiders are harmed by the merger. We conclude that the merger will not occur if cost synergies are modest.

Turn next to the case in which the cost synergies are moderate. Then, when $N = N$*, we have $\pi_I > \pi_O > 0$. This means that more than N* firms enter in Stage 1 of the free entry equilibrium; in fact, firms enter until $\pi_I > \pi_O = 0$. Thus, the insiders benefit from the merger and the anticipation of the merger induces more firms to enter in Stage 1.

Finally, consider the case in which the cost synergies are dramatic. Then, when $N = N$*, we have $\pi_I > 0 > \pi_O$. This means that fewer than N* firms enter in Stage 1 and the smaller number of outsiders pushes π_I up further. As a result, the insiders gain from the merger and the anticipation of the merger results in fewer firms entering in Stage 1. #

Proposition 3 indicates that the Court's presumption that a merger will trigger additional entry that will reverse its anticompetitive effects may be misguided. While it is true that mergers in the presence of moderate cost synergies trigger additional entry, when the cost synergies are dramatic mergers cause fewer firms to enter. This implies that the merger's anticompetitive effects will be magnified by the Stage 1 behavior of the outsiders.[4] Nevertheless, as we show in the next section, such mergers will always be welfare improving because of the large cost savings that they generate.

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[4] With the alternative structure discussed in footnote 3, additional entry would be triggered by a merger in the presence of moderate cost synergies and exit would occur when the merger results in dramatic cost synergies. In the former case, this would result in exactly the same equilibrium outcome as described above with the same number of active firms. In the latter case, the number of firms would be slightly different – since the sunk costs of entry would have already been paid, firms would exit until $\pi_O = -S$ (instead of entering in Stage 1 until $\pi_O = 0$ as above).

Before moving on, it is useful to point out that since $\hat{\lambda}(M) < 1$ for all *M*, there is a simple corollary to Proposition 3 – mergers that purely increase market power without lowering costs are never beneficial.[5]

Corollary: If there are no cost synergies associated with a merger, then it cannot be beneficial for the insiders.

Appendix B shows that the above corollary is a general result and does not depend on the linear demand function.

At this point we can relax some of our assumptions to see if our qualitative results are affected. We begin with the assumption that $F = 0$. So, suppose instead that $F > 0$ so that the merger allows the insiders to lower marginal cost *and* save on fixed costs. This makes it more likely that a merger with *N* held fixed will be beneficial. It also makes it more likely that when *N* is held fixed that the insiders will benefit more from the merger than the outsiders. On the other hand, changes in *F* do not affect the profits earned by the outsiders. It follows that $\lambda_I(M)$ and $\hat{\lambda}(M)$ are increasing functions of *F* whereas $\lambda_O(M)$ is independent of *F*. However, since the ordering of $\lambda_I(M)$, $\hat{\lambda}(M)$ and $\lambda_O(M)$ do not change, the basic message of Proposition 3 is not altered.

Finally, suppose that instead of assuming that the firms compete in quantities, we had assumed that they produce differentiated goods and compete is prices (as in Davidson and Deneckere 1985). Then, as we show in Appendix A, the only fundamental difference would be that with *N* fixed, all mergers would be beneficial to the insiders (i.e., $\lambda_I(M)$ would equal 1 for all *M*). However, for fixed *N,* it would still be the case that when cost synergies are modest the outsiders would earn more than the insiders and when cost synergies rise above a certain level, the insiders' profits would surpass the outsiders. Moreover, there would still be a level of cost synergies that would result in losses for the outsiders. Thus, there would be natural analogs to $\hat{\lambda}(M)$ and $\lambda_O(M)$ and their order would be preserved. It follows that the basic message of Proposition 3 is independent of

[5] Spector (2003) shows that without cost synergies, all beneficial mergers must cause price to rise. Our result indicates that without cost synergies, there are no beneficial mergers – thus, the set of mergers that Spector analyzes is empty in our model. This is due to our assumption that all firms face the same marginal

the type of competition that the firms are engaged in: With modest cost synergies mergers will not benefit the insiders; with moderate cost synergies, insiders will gain and the merger will trigger entry; and, with dramatic cost synergies, the merger will benefit insiders and result in fewer active firms.[6]

This result is surprising for at least two reasons. First, previous work has suggested that price-setting and quantity-setting models yield very different predictions about the impact of horizontal mergers. Our analysis indicates that this is not the case when we allow for free entry. Second, our results indicate that with price competition and free entry not all mergers are beneficial. If cost synergies are modest (i.e., $\lambda > \hat{\lambda}(M)$), then mergers that would be beneficial with N fixed are no longer beneficial. This is due to the fact that such mergers would also benefit the outsiders and would therefore result in more firms entering in Stage 1. In fact, just as in the quantity setting game, firms would enter in Stage 1 until $\pi_O = 0 > \pi_I$; implying that the merger would not occur. This result contrasts sharply with the predictions of Deneckere and Davidson (1985) who studied mergers in price setting games with a fixed number of firms.

4. Welfare

We now consider the impact of the merger on social welfare, which consists of consumer surplus and profits. We begin by noting that in the free entry equilibrium with mergers firms enter in Stage 1 until $\pi_O = 0$. From (7), this implies that

$$(13) \qquad \hat{N} = \frac{1-(2-\lambda)c}{\sqrt{F+S}} + M - 2 .$$

Given our assumption of homogeneous products, after the merger the insiders combine and form just one active firm. Thus, since there are $\hat{N} - M$ outsiders, the total number of active firms becomes $\hat{N} - M + 1$. If we use (7) and (13) we then have (we show in Appendix C that this result does not depend on the linearity of demand):

<u>Proposition 4</u>: A merger of M firms always lowers the number of active firms.

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cost of production. In his model, firms differ in their initial cost structures and thus there are some beneficial mergers even in the absence of cost synergies.

[6] An analysis of bilateral mergers in the Deneckere and Davidson (1985) model is presented in Appendix A to illustrate these points.

Proposition 4 indicates that even when the merger triggers entry, the number of new outsiders that enter is always smaller than $M - 1$. As a result, the merger always reduces the total fixed costs of production incurred by the industry. Moreover, cost synergies allow the merged firm to also lower their marginal costs. This means that society can now produce a given level of output at a lower social cost, suggesting that the merger might be welfare enhancing.

To see if this is the case, we now turn to consumer surplus, which, given our specification of demand, is given by $.5Q^2$. From (8) and (9), aggregate output with the merger is

$$\hat{Q} = (N-M)q_O + q_I = \frac{N-M+1-(N-M)c-\lambda c}{N-M+2} \tag{14}$$

Using (13) to substitute for N then yields

$$\hat{Q} = 1-c-\sqrt{F+S} = Q^* \tag{15}$$

where the last equality follows from substituting (5) into (4). Since the free-entry aggregate output is the same with and without the merger, price and consumer surplus are unaffected by the merger. Thus, we have[7]

Proposition 5: With free entry, all privately beneficial mergers are socially beneficial.

Proof: From (15), consumer surplus is the same with and without the merger. From Proposition 2, all outsiders earn the same with and without the merger (they earn zero). And, since the merger is beneficial, the insiders must be better off with the merger. Hence, the sum of consumer surplus and profits must be higher with the merger. #

Propositions 4 and 5 are strong results. Together they imply that even though profitable mergers always reduce the number of competing firms, such mergers will always raise social welfare because of the cost synergies the generate.

5. Conclusion

[7] The welfare effects of a merger in the price-setting game are more complex due to the assumption of product differentiation. For example, if cost synergies are dramatic, the merger results in fewer firms and less variety for consumers. The impact on welfare then depends on how much the consumers value variety (see the discussion at the end of Appendix A for details).

In this paper we have analyzed a very simple model of horizontal mergers in the presence of free entry and exit. We choose to work with such a simple model in order to make the comparison of our results to others in the merger literature (in particular, Salant, Switzer and Reynolds 1983 and Deneckere and Davidson 1985) as straight forward as possible. And, although we relied on an assumption of linear demand to illustrate some of our basic points, we have provided general proofs of most of our results in Appendices B and C.

Our key result is that when we allow for free entry and exit, most of the counter-intuitive results that have plagued the literature on horizontal mergers disappear. Mergers in quantity-setting games may be beneficial, even in absence of dramatic cost synergies, the free-rider problem that makes it difficult to explain why mergers occur vanishes, and the quantity and price-setting games yield similar predictions about the impact of merger activity on profitability.

The other main result of our paper is that the possibility of entry and exit greatly simplifies the welfare analysis of horizontal mergers. In our model, *all* privately beneficial mergers are also socially beneficial *regardless* of the degree of cost synergies. The fact that we are able to obtain such a strong result may be due to our assumption that all firms are initially symmetric or it may be an artifact of the assumption that demand is linear (of course, it should be clear that our welfare results will generalize to settings in which the demand curve can be closely approximated by a linear function). However, additional research on the welfare effects of horizontal mergers in the presence of free entry is warranted.

Appendix A

An Analysis of Mergers with Bertrand Competition

Our goal is to show that the basic results of Proposition 3 extend to price-setting games. We follow Deneckere and Davidson by assuming that the demand for good *i* is given by $q_i = 1 - p_i + \gamma(\overline{p} - p_i)$, where $\overline{p}$ denotes the average price charged in the industry and $\gamma < 1$ is a parameter that measures product differentiation ($\gamma = 1$ indicates that the goods are perfect substitutes and $\gamma = 0$ indicates that the goods are unrelated). In what follows, we compare the outcome of the 3-stage game in which no merger occurs with the outcome that occurs when firms 1 and 2 plan to merge in Stage 2. The general case in which *M* firms merge in Stage 2 is qualitatively similar but results in more complex expressions.

For notational convenience, we begin by defining $\bar{c} \equiv \frac{1}{N}\sum_j c_j$, $z \equiv \frac{\gamma}{N}$ and $\sigma \equiv 1+\gamma-z$. Then, when no merger occurs, it is straightforward to show that the equilibrium price for each firm is given by $p_i = \frac{1+\sigma c}{1+\sigma}$ and that profit per firm is

(A.1) $\pi^* = \sigma\left[\frac{1-c}{1+\sigma}\right]^2$.

Now, consider the case in which firms 1 and 2 anticipated merging in Stage 2. Then the first order conditions for the two insiders are given by

(A.2) $1+\lambda c(1+\gamma)+\gamma\bar{p}-2p_1(1+\gamma)+z(p_1-\lambda c)+z(p_2-\lambda c)=0$

(A.3) $1+\lambda c(1+\gamma)+\gamma\bar{p}-2p_2(1+\gamma)+z(p_1-\lambda c)+z(p_2-\lambda c)=0$,

whereas the first order condition for each outsider is

(A.4) $1+c(1+\gamma)+\gamma\bar{p}-2p_j(1+\gamma)+z(p_j-c)=0$

Summing over all firms and then dividing by *N* yields:

(A.5) $1+\sigma\bar{c}-(1+\sigma)\bar{p}+\frac{z}{N}(p_1+p_2-2\lambda c)=0$

Summing (A.2) and (A.3) yields

(A.6) $1+\lambda c(\sigma-z)+\gamma\bar{p}-\sigma(p_1+p_2)=0$

We can now solve (A.5) and (A.6) for $\bar{p}$ and (p_1+p_2) . We obtain

(A.7) $$\bar{p} = \frac{(\sigma+\frac{z}{N})+\sigma^2\bar{c}-\lambda c\frac{z}{N}(1+\gamma)}{\sigma(1+\sigma)-z^2}$$

and

(A.8) $$p_1+p_2 = \frac{(1+\gamma+\sigma)+\gamma\sigma\bar{c}+\lambda c[(1+\sigma)(\sigma-z)-2z^2]}{\sigma(1+\sigma)-z^2}.$$

We can now use (A.7) to substitute for $\bar{p}$ in (A.4) and solve for a typical outsider's price:

(A.9) $$p_j = \frac{\sigma(1+\gamma+\sigma)+\gamma\sigma^2 \bar{c}+\sigma[\sigma(1+\sigma)-z^2]c-\lambda c(1+\gamma)z^2}{(1+\gamma+\sigma)[\sigma(1+\sigma)-z^2]}$$

Turn next to profits and start with the insiders. It is straightforward to show that the first order conditions for firms 1 and 2 can be written as:

$$q_1 = \sigma(p_1 - c_1) - z(p_2 - c_2)$$

$$q_2 = \sigma(p_2 - c_2) - z(p_1 - c_1)$$

It follows that the merged firm's profits can be written as

$$\pi_1 + \pi_2 = \sigma[(p_1 - \lambda c)^2 + (p_2 - \lambda c)^2] - 2z(p_1 - \lambda c)(p_2 - \lambda c)$$

With symmetry, we then have

$$\pi_I = .5(\pi_1 + \pi_2) = (\sigma - z)(p_1 - \lambda c)^2$$

Substitution from (A.8) then yields

(A.10) $$\pi_I = (\sigma - z)\left[\frac{1+\gamma+\sigma+\gamma\sigma\bar{c}-\lambda c(1+\sigma)(\sigma+z)}{2[\sigma(1+\sigma)-z^2]}\right]^2$$

For the outsiders, their first order condition is equivalent to $q_i = \sigma(p_i - c)$. So that we may write profits as

$$\pi_i = \sigma(p_i - c)^2$$

Substituting from (A.8) then yields

(A.11) $$\pi_O = \sigma\left[\frac{\sigma(1+\gamma+\sigma)+\gamma\sigma^2\bar{c}-(1+\gamma)[\sigma(1+\sigma)-z^2]c-\lambda c(1+\gamma)z^2}{(1+\gamma+\sigma)[\sigma(1+\sigma)-z^2]}\right]^2$$

In what follows, we make use of the fact that with the merger

(A.12) $$\bar{c} = \frac{(N-2)c+2\lambda c}{N} = \frac{1}{\gamma}[(\gamma-2z)c+2z\lambda c]$$

Now, we know from Deneckere and Davidson (1985) that with N fixed, all mergers are beneficial. Thus, $\lambda_I > 1$. Furthermore, using (A.10)-(A.12) we have:

(A.13) $$\lambda_O = \max\{\frac{c\sigma[2\sigma(1+\sigma)-z\gamma]-z(1+\gamma+\sigma)}{c(1+\sigma)[2\sigma^2-(1+\gamma)z]},0\};$$

and from (A.1) and (A.11)-(A.12) we have

(17) $$\hat{\lambda} = \max\{\frac{\eta_1 - c\eta_2}{c(\eta_3+\eta_4)},0\}$$

where $s \equiv \sqrt{\dfrac{\sigma - z}{4\sigma}}$; $\eta_1 = (1+\gamma+\sigma)[\sigma(1-s) - s(1+\gamma)]$;

$$\eta_2 = (1+\gamma)[\sigma(1+\sigma) - z^2] + (\gamma - 2z)[s\sigma(1+\gamma) - \sigma^2(1-s)];$$

$$\eta_3 = z[(1+\gamma)z - 2\sigma^2]; \text{ and, } \eta_4 = s(1+\gamma+\sigma)[2\sigma z - (1+\sigma)(\sigma+z)].$$

We know that $\lambda_I > 1$. Further, at λ_O, we have $\pi^* = \pi_0$ and $\pi^* < \dfrac{\pi_I}{M}$, since $\lambda_O < 1$. Therefore, $\pi_0 < \dfrac{\pi_I}{M}$. Since, π_I is negatively related to λ and π_0 is positively related to λ, it implies that $\hat{\lambda}$ (where $\pi_0 = \dfrac{\pi_I}{M}$) is greater than λ_O. It can also be checked that $\hat{\lambda} < 1$.

The above analysis shows that the effects of free entry on profits under Bertrand competition are similar to those generated by Cournot competition. Like Cournot competition, a merger creates a trade-off between the cost savings generated by cost synergies and the effect of increased market concentration. However, with Bertrand competition and differentiated goods a new force arises – a merger is likely to alter the amount of variety offered in the market. With product differentiation and Bertrand competition, though the merged firms choose prices to maximize their joint profits, *all* the varieties of the merged firms will be produced in equilibrium. So, if the total number of firms is increased due to merger, then the merger increases the number of varieties produced in the market. In this situation, even though the merger may increase prices due to market concentration, it provides benefits by lowering costs (via cost synergy) and increasing the amount of variety available to consumers. But, if merger reduces the number of firms in the economy, it generates negative effects by increasing market concentration and reducing variety and these must be weighed against the positive benefits triggered by the reduction in costs. Therefore, whether merger is welfare improving under Bertrand competition depends on the relative strengths of the three factors such as market concentration, the degree of cost synergy and the number of product varieties.

Appendix B

A Generalization of the Corollary to Proposition 3

If there is no synergy under merger, the merged firm and the outside firms produce at the marginal cost c. So, the operating profit (i.e., revenue minus total variable cost of production) of the merged firm and an outsider firm is the same. Since the net profit of an outsider is zero in equilibrium and the amount of sunk cost plus the fixed cost incurred by the merged firm is greater compared to a typical outsider firm, the net profit of the merged firm is negative. Therefore, merger is never profitable without any synergy.

Appendix C

A Generalization of Proposition 4

In case of free entry, the net profit of a typical firm under non-cooperation is zero and the zero profit condition determines N^* as the equilibrium number of firms.

If there is a merger of M firms, the net profit of the outsiders becomes zero in the free entry equilibrium. However, under merger, the operating profit of an outsider is lower than that of under non-cooperation whenever there is synergy under merger. Hence, the zero profit condition of the free-entry equilibrium is more binding under merger than non-cooperation, which implies that the total number of active firms is lower under merger than non-cooperation.

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