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The Decision to Export and the Volatility of Sales

By

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Abstract

This paper studies the export decision of risk-averse firms in a model featuring aggregate uncertainty and no capital markets. Firms seeking to enter the foreign market face a sunk cost as well as a fixed participation cost every period they export. Using a calibrated version of the model, I show that firms are more likely to export when the correlation between domestic and foreign aggregate shocks is negative and when their degree of risk-aversion is higher. Counterfactual experiments show that exporting increases the volatility of total sales.

JEL classification: F12, F40.

Keywords: Exports, Aggregate uncertainty, Heterogeneous-firm models of international trade

Outline

Non-Technical Summary

Although the volatility of aggregate output has subsided across the world since the mid 1970s, it is also true that developing countries have faced substantially turbulent conditions, resulting in longer and more frequent recessions as well as larger output declines on average than developed countries. A growing literature has emphasized the role of trade openness as a determinant of volatility both at the aggregate and sectoral levels. However, the relationship between trade openness and firm-level volatility has received considerably less attention.

Although there are several mechanisms through which closer global linkages can influence firm-level volatility, this paper explores the hypothesis that exporting allows firms to hedge against downturns in their domestic market. To do so, this paper presents a dynamic model of a firm's decision to export in an environment characterized by aggregate uncertainty. Firms in the model are assumed to be risk-averse, which means that they prefer to have smoother sales. Additionally, capital markets are assumed to be non-existing, so firms cannot borrow to finance capital investment and they are also unable to sell their capital stock in a secondary market, which means that investment is irreversible. The idea is to reproduce an environment in which the diversification benefits provided by exporting would be highly valuable to firms.

After calibrating the model to match export participation patterns, I find that firms are more likely to participate in foreign markets when the correlation between domestic and foreign aggregate shocks is negative and when firms' degree of risk aversion is high. Moreover, I find that the export participation is more responsive to the correlation of aggregate shocks when risk aversion is higher. These results seem to suggest that exporting provides an avenue to diversify aggregate risk. However, after conducting a counterfactual experiment in which the foreign market is shut down and firms are precluded from exporting, I find that sales volatility is 14 percent lower when firms are restricted to sell in the domestic market alone. The sunk costs associated to start exporting as well as the irreversibility of investment result in a higher volatility of investment for exporting firms, which in turn translates into a higher volatility of sales.

1 Introduction

Developing countries have on average more volatile business cycles and rates of output growth than developed countries. They also suffer from deeper recessions that occur more frequently than in industrial countries (Agenor et. al., 2000; Aguiar and Gopinath, 2007 and Gavin and Hausmann, 1996). Although there are several factors that make developing countries highly volatile, a growing literature has emphasized the role of trade openness as a determinant of volatility both at the aggregate and sectoral levels[1]. However, the relationship between trade openness and volatility at the firm level has received much less attention. The distinction between aggregate and firm-level volatility is an important one since the two can behave in quite different ways, as documented by Comin and Philippon (2005).

There are several ways in which closer global linkages can affect a firm's volatility. International trade can make firms more vulnerable to external shocks[2]. Alternatively, if domestic demand shocks are not perfectly correlated with external shocks, exporters can smooth their revenues by diversifying their sales across markets, thus improving their ability to cope with downturns in the domestic market. However, even if exporting has a positive effect on sales stabilization, it is a hedging mechanism that relatively few firms in an industry can afford to use, given the large sunk and fixed costs of selling in foreign markets documented by Roberts and Tybout (1997), Das et. al. (2007) among many others.

In this paper I ask whether exporting allows firms to stabilize their sales by taking advantage of the imperfect correlation of demand across markets. To do so, I set up and calibrate a model of the decision to export for heterogeneous, risk-averse firms that operate in an environment characterized by aggregate and firm-specific uncertainty and no capital markets. Exporting is costly in the model. Domestic firms that decide to start exporting need to incur an up-front sunk cost as well as a per-period fixed participation cost. In terms of their investment decision, firms can only finance capital accumulation through internal funds. Additionally, investment is assumed to be perfectly irreversible. Hence, the only way to divest capital is by letting it depreciate over time. These assumptions are quite novel to the literature studying the decision to export. Since firms are risk averse, more stable sales have a direct positive impact on firm proprietors' welfare. Similarly, the lack of capital markets intends to reproduce an environment in which the diversification benefits provided by exporting would be highly valuable to firms, since the payoff from this strategy cannot be replicated by a portfolio of securities. In summary, I seek to stack the deck in favor of

[1] Using cross-country data, Easterly et. al. (2001) find that openness increases the volatility of GDP growth. Conversely, Kose et. al. (2006) find that more openness to trade weakens the negative relationship between volatility and growth. Di Giovanni and Levchenko (2009) study the relationship between trade openness and sales volatility at the level of 3-digit industries.

[2] Bergin et. al. (2009) show that Mexican maquiladora plants' employment fluctuations are twice as volatile as those of their U.S. counterparts, which in turn are more volatile than those of Mexican non-maquiladora plants.

exporting as a mechanism to hedge against domestic aggregate shocks.

From a quantitative standpoint the model performs well. Even though the model is calibrated to match export participation rates, it closely approximates investment moments reported in other studies, such as the fraction of episodes of zero investment and the higher frequency of investment spikes among new exporters. The implied costs to start exporting are in line with previous estimates by Alessandria and Choi (2007), although the fixed cost required to remain an exporter is substantially higher than the one they report. The main shortcoming of the calibrated model is the fact that exporters, both new and existing, sell very large shares of their output abroad. Arkolakis (2009) and Eaton et. al. (2008) have shown that firms that start exporting tend to start by selling small quantities, and if successful, subsequently experience very high rates of growth.

After calibrating the model, I find that firms are more likely to participate in foreign markets when the correlation between domestic and foreign aggregate shocks is negative and when firms' degree of risk aversion is high. Moreover, I find that the export participation is more responsive to the correlation of aggregate shocks when risk aversion is higher. These results seem to suggest that exporting provides an avenue to diversify aggregate risk. However, exporting firms are found to have more volatile sales than non-exporters. The reason for this is the substantial difference in the investment patterns of exporters and domestic firms. Even though the volatility of investment rates is slightly lower for exporters, I find that they are more likely to present episodes of zero investment. However, they also present much higher investment rates than domestic firms when either domestic or foreign demand improves. Once firms have begun to export, they have the incentive to remain in the foreign market even when hit by adverse productivity and aggregate demand shocks. This is an hysteresis effect caused by the sunk cost of entry into the foreign market. When favorable foreign demand conditions return, exporters increase their capital stock significantly more than non-exporters, thus experiencing higher volatility of sales.

To gain a better understanding of how exporting affects the volatility of sales, I compare the results from the benchmark calibration to a counterfactual scenario in which firms are not allowed to export. Given the same realizations for the stochastic processes for productivity and the aggregate shocks, I find that the volatility of total sales for firms that would have exported had the foreign market been available is about 14 percent lower when they are only allowed to serve the domestic market. Both the sunk entry cost to export and the irreversibility of investment induce exporting firms to increase their capital stock substantially when either domestic or foreign demand improve. This higher volatility of investment is reflected in a higher volatility of total sales.

This is the first attempt to study the decision to export that takes into account the substantial fixed and sunk costs associated with becoming an exporter while at the same time allowing for firms to be risk-averse, an assumption that seeks to come to grips with the limited alternatives for risk diversification available to firms in developing countries. While some older studies focusing on the effect of exchange rate volatility on the supply of exports (Clark, 1973; Donnenfeld and Zilcha, 1991; Eldor and Zilcha, 1987) allow for firms to be risk-averse, because they do not include exporting costs, the result is that all firms end up exporting, an outcome that is clearly rejected by the empirical evidence. I show that both risk aversion and exporting costs are crucial in order to understand the links between trade openness and firm-level volatility.

Previous research has found that exporting has a stabilizing effect on sales at the firm level. Studying a small sample of firms in Denmark, Israel and The Netherlands, Hirsch and Lev (1971) find a positive correlation between international diversification and total sales stability, even though domestic sales are more stable than export sales. More recently, using plant-level data from the German state of Baden-Wuerttemberg, Buch et. al. (2009) have found that controlling for firm size and productivity, exporters have lower sales volatility than non-exporters. Campa and Shaver (2001), analyzing a panel of Spanish manufacturing firms, find that the investment rates of exporters are more stable than those of non-exporters, and argue that this is because exporting eases liquidity constraints for firms. In a paper closely related to this, Maloney and Azevedo (1995) study the export decision of price-taking, risk-averse firms using firm-level data for Mexico. They find that export supply is an increasing (decreasing) function of the expected return (volatility) in the foreign market relative to that in the domestic market. They also find an ambiguous effect of the covariance of domestic and foreign returns on export sales. However, they ignore the significant costs associated with becoming an exporter, an essential feature of my model. Finally, from a methodological perspective, my paper is closely related to a small but growing literature studying the dynamics of trade models with heterogeneous-firms in environments with aggregate uncertainty (Alessandria and Choi, 2007; Ghironi and Melitz, 2005; Ruhl, 2005 and Utar, 2008).

The paper is organized as follows: section 2 outlines the theoretical model, sections 3 and 4 describe the parameters used in the calibration and the benchmark results respectively. Section 5 presents the results of a counterfactual experiment in which the export market is shut down. Section 6 concludes.

2 Model

Technology

This is a partial equilibrium model of an industry composed of heterogeneous firms that operate in a monopolistic competition environment. There is a fixed number, N, of risk-averse firms, each of which produces a differentiated product, maximizing the expected lifetime utility of profits,

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t u(\pi_t)\right\}, \quad u(\pi) = \frac{\pi^{1-\nu}}{1-\nu}, \tag{1}$$

with $\beta \in (0,1)$, $\nu > 0$ and $u(\pi) = \ln(\pi)$ if $\nu = 1$. This assumption can be rationalized by thinking of firms owned by entrepreneurs who work in their own firms and whose main source of income is the firm's dividends[3]. All firms have access to the same technology, which uses capital as the only input to produce the final good[4]:

$$q = e^{\phi}k^{\alpha}, \quad \alpha \in (0,1). \tag{2}$$

where k is the firm's capital stock and ϕ is a firm-specific productivity index that follows a Markov process $P_{\phi}(\phi'|\phi)$. Capital stock is owned by the firm and is augmented through investment that comes from internal funds. Since capital markets are non-existent, the firm cannot borrow to finance capital investment. A firm's capital stock follows the law of motion,

$$k' = (1-\delta)k + i, \tag{3}$$

where i denotes gross investment, δ is the depreciation rate of capital and $'$'s denote next-period values for the variable of interest. At period t the firm chooses the capital stock that will be available for production in period $t+1$[5]. Furthermore investment is assumed to be perfectly irreversible, which implies that gross investment is constrained to be non-negative. Caballero (1993) notes that because of the smaller size of the manufacturing sector and highly volatile macroeconomic environment, secondary markets for capital goods are particularly thin in developing countries. Gelos and Isgut (2001) provide support for this view and show that irreversibility is a more important component of capital adjustment costs in developing countries like

[3] Maloney and Azevedo (1995) note that in developing countries it is common for firm managers to own large shares of the firms they run. Even in a developed economy like the United States, Moskowitz and Vissing-Jørgensen (2002) find that around 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth.

[4] Alternatively, one could assume a technology that uses both capital and labor, where labor is a fully flexible input. The results of the model would not change dramatically, but adding another factor of production would substantially complicate the computational solution of the model, since it would become necessary to include another stochastic process for the wage.

[5] Thus, investment at t completely determines the production possibilities of the firm at $t+1$. This contrasts with the model of Cooley and Quadrini (2001) where there is a market in which firms can rent capital to/from other firms. In this setting, the amount of output a firm can produce is not constrained by the amount of capital it has accumulated.

Mexico and Colombia than in developed economies such as the United States or Norway. Given that I am restricting the sources of funds available for firms to finance capital accumulation, it makes sense to restrict the secondary market for capital goods as well. The idea behind this characterization of capital markets is to shut down mechanisms other than exporting that firms could potentially use to hedge against domestic aggregate shocks.

Firms' output can potentially be sold in two markets: Home (h) and Foreign (f). The difference between the two is that it is costly for a firm to sell its output in the foreign market. A firm that decides to *start* exporting has to pay a sunk cost s_x. Additionally, and independently of its previous exporting status, a firm that exports in any given period has to pay a fixed participation cost f_x. If firms only had to pay a fixed cost per-period to sell abroad, the only determinant of the decision to export would be current profitability in the foreign market, which means that if current foreign profits were to fall below fixed costs, the firm would stop exporting. The existence of sunk costs of becoming an exporter makes the firm's export decision forward-looking. Current exporters know that if they stop exporting today, they will have to pay s_x again whenever they decide to return. Alternatively, they can choose to weather some periods of low foreign demand and avoid paying the sunk cost. The problem for domestic firms is to determine when to exercise the option to become exporters. If foreign demand (or productivity) is not sufficiently high it might be better to wait until conditions improve. The high turnover rates observed in export markets justify including the fixed per-period cost in addition to the sunk cost to start exporting. Since the Cobb-Douglas production function assumed in the model implies that gross potential export profits are always positive, not including the fixed participation cost would result in firms never exiting the export market. Let y denote the export status of a firm, with $y = 1$ if the firm decides to export and 0 otherwise. Also, let y_{-1} denote the firm's export status in the previous period. Then the cost of exporting for a firm is given by:

$$\text{cost of exporting} = \begin{cases} s_x + f_x, & \text{if } y = 1 \text{ and } y_{-1} = 0, \\ f_x, & \text{if } y = 1 \text{ and } y_{-1} = 1. \end{cases}$$

Finally, conditional on exporting, a firm has to decide what fraction of its capital stock, $\theta \in [0, 1]$, to use to produce for the foreign market. Thus, firms can potentially differ in their capital stock, idiosyncratic productivity, and export status.

Market Structure

In each market $j = h, f$ there is a representative consumer with CES preferences over all varieties available in country j. I assume that the domestic and foreign market are segmented, so firms can charge different prices in each. A firm located in country h faces the following demand curves for its output:

$$q_h = I_h P_h^{\sigma_h - 1} p_h^{-\sigma_h}, \quad q_f = I_f P_f^{\sigma_f - 1} p_f^{-\sigma_h}. \tag{4}$$

where I_j is country j's total expenditure on the industry, σ_j is the elasticity of substitution among varieties available in country j and P_j is the corresponding ideal price index in country j. Assuming that the number of firms located in country h constitute a very small fraction of all the firms in the rest of the world, the pricing decision of Home firms has a negligible effect on P_f. This is not the case in market h, where the domestic price index is given by:

$$P_h = \left(N^{-1} \sum_{i=1}^{N} p_{h,i}^{1-\sigma_h} \right)^{\frac{1}{1-\sigma_h}}. \tag{5}$$

I assume that the aggregate variables determining the demand for Home firms' output, both domestically and abroad, except the price index P_h, evolve stochastically. In particular, letting $\mathbf{Z} \equiv [z_h \;\; z_f]'$, with $z_h = I_h$ and $z_f = I_f P_f^{\sigma_f - 1}$, I assume that aggregate variables follow a joint Markov process $P_Z(\mathbf{Z}'|\mathbf{Z})$. These aggregate shocks can be thought of as country-specific business cycle fluctuations, which might be correlated with each other. Potential revenues in the domestic and foreign market for Home firms are:

$$r_h = \left[z_h P_h^{\sigma_h - 1} \right]^{\frac{1}{\sigma_h}} q_h^{1-1/\sigma_h}, \quad r_f = z_f^{\frac{1}{\sigma_f}} q_f^{1-1/\sigma_f},$$

and total profits are given by:

$$\pi = r_h + y[r_f - f_x - (1 - y_{-1}) s_x] - i. \tag{6}$$

Firm's Problem

An individual firm's state variables can be classified into four categories: 1) *endogenous individual states*, capital stock k and export status y_{-1}; 2) *exogenous individual states*, firm-specific productivity, ϕ; 3) *exogenous aggregate states*, aggregate shocks, $\mathbf{Z}$ and 4) *endogenous aggregate states*, domestic price index, P_h. From the demand functions in equation (4), it can be seen that a firm's pricing decision depends on the domestic price index, P_h, a function of the prices set by all the firms in the industry, which ultimately depends on the distribution of individual firms across capital, export status and productivity denoted by

$\Gamma(k, y_{-1}, \phi)$. In the presence of aggregate shocks, the distribution Γ evolves according to an equilibrium law of motion $\Gamma' = \mathcal{H}(\Gamma, \mathbf{Z}, \mathbf{Z}')$. Individual firms are not able to infer future values of P_h from their own pricing decision alone, and as a consequence, Γ becomes a state variable on the firm's problem. This feature of the model substantially complicates the solution to the firm's problem, since Γ is an infinite-dimensional object, making it impossible to track computationally. Moreover, finding the law of motion $\mathcal{H}$ is non-trivial, since it is a mapping from the set of distribution functions into itself.

In order to circumvent this problem I follow the approach proposed by Krusell and Smith (1998). Their methodology assumes that firms' perceptions of how Γ (and therefore P_h) evolves are boundedly-rational. Individual firms assume that the domestic price index follows a relatively simple law of motion based on a finite number of moments of the underlying distribution Γ. In particular, I consider a log-linear law of motion for the price index of the form,

$$\log P_{h,t+1} = a_{0,z} + a_{1,z} \log P_{h,t}, \quad z = 1, \ldots, N_Z, \tag{7}$$

where N_Z denotes the total number of aggregate states used in the computation of the model. Conditional on this perception, decision rules for capital accumulation, pricing and the decision to export can be computed by replacing the unknown law of motion $\mathcal{H}$ for the distribution of firms with the simplified law of motion (7) in the firm's dynamic problem defined formally in equations (8)-(10). Using the policy rules, I simulate optimal prices for a large panel of firms and compute a time-series of the domestic price index $\{P_h\}$. With this time-series in hand, I estimate the parameters $\{a_{0,z}, a_{1,z}\}$ that determine the law of motion for the price index, updating the previous guess. The algorithm continues in this way until both a fixed point is reached for the parameters and the predictive power of the law of motion for the price index is sufficiently high. The algorithm used to solve the model is described in further detail in the appendix.

The problem of the firm can be partitioned into two subproblems: a dynamic one that involves the decision of whether or not to export and how much capital to use in the next period, and a static one which entails deciding how much output to produce for each market, conditional on the firm deciding to export. The dynamic problem can be represented in recursive form as follows:

$$v(k, y_{-1}, \phi, \mathbf{Z}, P_h) = \max \Big\{ v_n, v_x \Big\}, \tag{8}$$

where v_n denotes the value just servicing the domestic market,

$$v_n \equiv \max_{k' \geq 0} \left\{ u[r_h(k, y_{-1}, \phi, \mathbf{Z}, P_h) - i] + \beta E_{[\phi', \mathbf{Z}', P_h' | \phi, \mathbf{Z}, P_h]} \left[v(k', 0, \phi', \mathbf{Z}', P_h') \right] \right\}, \tag{9}$$

and v_x the value of exporting,

$$v_x \equiv \max_{k' \geq 0} \left\{ u[r_h(k, y_{-1}, \phi, \mathbf{Z}, P_h) + r_f(k, y_{-1}, \phi, \mathbf{Z}, P_h) - i - f_x - (1 - y_{-1}) s_x] + \beta E_{[\phi', \mathbf{Z}', P_h' | \phi, \mathbf{Z}, P_h]} \left[v(k', 1, \phi', \mathbf{Z}', P_h') \right] \right\}, \tag{10}$$

subject to the law of motion for idiosyncratic and aggregate stochastic processes, the law of motion for the domestic price index (7), and the restriction that gross investment has to be non-negative discussed above.

The solution to this problem produces two policy rules, one for next-period's capital g_k, and the other for exporting, $g_y \in \{0, 1\}$. Figure 1 shows the decision rule for capital for both exporters and domestic firms in two different aggregate states for a given level of idiosyncratic productivity. Under risk neutrality and no adjustment costs, next-period period capital would be independent of current capital. In the case of risk aversion, the capital accumulation rule is an increasing function of current capital, and is very similar to the policy rule for a risk-neutral firm with convex adjustment costs. Firms accumulate more capital when demand is high and when they export. It can also be seen that a firm with relatively high current capital chooses not to invest at all in order to adjust towards its desired capital stock.

Figure 1 about here

As noted by Baldwin and Krugman (1988), under the existence of a sunk entry cost to access the export market, a firm needs to take into consideration that in latter periods it can continue exporting without incurring this cost again[6]. This will affect the expected present discounted value of profits, $E_{[\phi', \mathbf{Z}', P_h' | \phi, \mathbf{Z}; P_h]}[v(\cdot)]$, generating hysteresis. So, for instance, if after a positive foreign demand shock a firm decides to pay the sunk cost and starts exporting, after the foreign demand goes back to its pre-shock level, the firm will continue to export. The policy rule for exporting is characterized by two cutoff levels of capital, $\underline{k} < \overline{k}$, such that (conditional on the value of idiosyncratic productivity and demand shocks) a firm with a capital stock above $\overline{k}$ starts exporting, and an exporting firm whose capital stock falls below $\underline{k}$ exits the foreign market. This

[6]Roberts and Tybout (1997) find evidence of non-zero entry costs into foreign markets for manufacturing plants in Colombia. They also find that plants that have not operated in the export market for two years or more face re-entry costs that are not significantly different from the entry costs faced by plants that have not exported before. This is why in the model it is assumed that a firm that stops exporting has to pay the sunk entry cost whenever it decides to start exporting again, regardless of its previous exporting experience. This assumption greatly simplifies the solution of the dynamic problem of the firm.

can be seen in Figure 2, where the value function for the firm is plotted as a function of its capital stock and previous and current exporting status. The hysteresis generated by the the sunk entry cost to start exporting has important implications for the volatility of sales. If sunk costs are sufficiently large, exporters will be reluctant to exit the foreign market when foreign demand is unfavorable and/or productivity falls. Exporters that have experienced prolonged spells of low demand or productivity, whose capital is significantly below their desired level, have a strong incentive to engage in substantial investment when aggregate conditions turn around. These dramatic changes in the level of the capital stock of exporters are reflected on higher sales volatility than that experienced by domestic firms.

Figure 2 about here

The static problem of how much output to export, conditional on the capital stock and the realizations of the stochastic processes, is given by:

$$\max_{\theta \in [0,1]} \left\{ [z_h (P_h)^{\sigma_h - 1}]^{1/\sigma_h} \left[(1-\theta) e^{\phi} k^{\alpha}\right]^{1-1/\sigma_h} + (z_f)^{1/\sigma_f} \left[\theta e^{\phi} k^{\alpha}\right]^{1-1/\sigma_f} \right\}. \tag{11}$$

The key variables that determine the fraction θ of capital used in the production for the foreign market are the relative magnitudes of domestic and foreign demand and the relative elasticities of demand in the two markets. When the size of the foreign market increases relative to that of the domestic market, θ increases. A higher elasticity of foreign demand also increases θ because the optimal quantity to be sold in the foreign market increases. This is the same reason why θ is an increasing and concave function of k as seen in Figure 3. Assuming that $\sigma_h < \sigma_f$, a firm has the incentive to allocate a higher fraction of its capital stock to produce output for the more-elastic foreign market.

Figure 3 about here

Timing

The timing of actions is as follows, and is illustrated in Figure 4:

1. A firm enters period t with a given capital stock k_t and last period's export status $y_{t-1} \in \{0, 1\}$. Demand shocks z_f and z_h and firm-specific productivity ϕ draws are realized at the beginning of the period.

2. The firm decides whether to export or not. If the firm did not export last period, it has to pay a sunk cost s_x to break into the foreign market in period t. Regardless of its previous exporting status, the

firm needs to pay a fixed cost f_x every period it exports. Conditional on deciding to export, a firm chooses the fraction $\theta \in [0,1]$ of its capital stock to devote for production for the foreign market.

3. Finally, the firm chooses its desired capital stock for period $t+1$. Profits for period t are realized.

Figure 4 about here

Since a firm's capital stock in period t is the result of the firm's decision at $t-1$, the timing assumption implies that output is chosen before the resolution of uncertainty, but the allocation of sales is decided ex-post. This implies that a firm has greater flexibility in adjusting the distribution of sales across different markets than it does in changing the total scale of its production. The same timing assumption is used by Eldor and Zilcha (1987) and Donnenfeld and Zilcha (1991).

3 Calibration

Table 1 presents the parameters used in the benchmark solution of the model. The model period is set to one year. The coefficient of relative risk aversion, discount factor, depreciation rate and the technology parameter are standard in the macroeconomics literature. The discount rate $\beta = 0.90$ implies an annual real interest rate of 11%, which is higher than the usual 4% based on the US real interest rate. The depreciation rate is set to 6% annually as in Cooper and Haltiwanger (2006). The coefficient of relative risk aversion of 1 implies a logarithmic utility of profits. Below I show how the decision to export is affected by changes in the firms' degree of risk aversion.

Table 1 about here

The elasticity of substitution among domestic varieties, which under the CES demand assumption is also equal to the price elasticity of demand for individual varieties, is set to 2. Ruhl (2005) argues that a low value of this elasticity, between 1 and 3.5, is more appropriate to match high-frequency fluctuations. For the foreign demand elasticity, I use the Das et. al. (2007) estimate of a foreign elasticity demand premium of 2 based on a constant-elasticity demand structure for knitted fabrics and basic chemical plants in Colombia. This is consistent with foreign markets being characterized by stiffer competition. The calibration of the fixed and sunk costs of exporting seeks to match the observed transition rates into and out of the export market documented by Roberts and Tybout (1997) for Colombia[7]. The results can be seen in Table 2. Given the calibrated values for s_x and f_x, domestic firms breaking into the foreign market would expect to pay

[7]Similar entry and exit rates are reported by Alessandria and Choi (2007) for U.S. manufacturing

on average 12.07 percent of their pre-export sales as an entry cost, and exporters would have to sacrifice on average 7.89 percent of their sales to maintain their presence abroad. The value of the sunk entry cost is close to the 16.5 percent cost calculated by Alessandria and Choi (2007) based on their calibration for U.S. manufacturing plants. The cost to remain an exporter implied by my calibration is substantially higher than the one Alessandria and Choi (2007) report (1.7 percent of total sales). This is due to the assumption of risk aversion. As can be seen in Figure 2, the curvature that risk aversion introduces into the firm's value function increases the hysteresis band relative to the case of risk neutrality. In order to match the conditional probability of exiting the foreign market, a high value of f_x is needed.

Table 2 about here

Aggregate shocks are parametrized as a highly persistent VAR(1) process[8]:

$$\log \mathbf{Z_t} = \begin{bmatrix} 0.90 & a_{12} \\ 0 & 0.90 \end{bmatrix} \log \mathbf{Z_{t-1}} + \varepsilon_t, \tag{12}$$

with

$$\varepsilon_t \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad 0.007^2 \cdot \begin{bmatrix} 2 & \Sigma_{12} \\ \Sigma_{12} & 1 \end{bmatrix} \right),$$

which allows for contemporaneous correlation between innovations in Home and Foreign through the spillover parameter a_{12} and the off-diagonal element of the variance covariance matrix, Σ_{12}. Since Home is assumed to be a small, developing country, aggregate shocks in Home do not generate spillovers in the foreign country. In the benchmark calibration both a_{12} and Σ_{12} are set to zero[9]. The parametrization of the variance covariance matrix of innovations, Σ also follows Kehoe and Perri (2002) but it assumes that the variance of domestic innovations is twice as large as that in the foreign country, based on the findings of Aguiar and Gopinath (2002) that emerging market business cycles are twice as volatile as those in small developed countries. The VAR process is approximated by a 4-state Markov chain with $z_h \in \{\underline{z}_h, \overline{z}_h\}$ and $z_f \in \{\underline{z}_f, \overline{z}_f\}$, using Tauchen's (1986) approximation method. Idiosyncratic productivity is assumed to follow an AR(1) process, independent from the aggregate shocks, and is also approximated using Tauchen's

[8] This parametrization is commonly used in the international real business cycle literature. See Kehoe and Perri (2002) and the references therein.

[9] Kollmann (1996) finds little evidence of spillovers among OECD countries. However, since developing countries' business cycles are documented to be positively correlated with business cycles in industrial countries (Agenor et. al., 2000), I present results showing how the decision to export is affected by the degree of correlation between aggregate shocks.

method with $N_\phi = 15$ grid points.

$$\phi_t = 0.90\phi_{t-1} + \mu_t, \quad \mu_t \sim \mathcal{N}(0, 0.1) \tag{13}$$

4 Results

Table 3 presents summary statistics for the benchmark calibration. In the model, firms accumulate capital when their idiosyncratic productivity increases and when facing high aggregate shocks. As is common in models that feature fixed costs to participate in the export market, firms that start exporting sell a significant fraction of their output in the foreign market. On average, foreign revenues account for 45 percent of total sales for exporters.

Table 3 about here

Table 4 shows that entry into the export market takes place only when foreign demand is high, and conversely, that firms leave the foreign market only during periods of low foreign demand, with almost all exit episodes occurring when domestic demand is also low. During periods of high foreign demand, entry into the export market (extensive margin) accounts for about 40 percent of total exports while exiting exporters account for 24 percent of exports before leaving the foreign market. However, since relatively few firms are close to the thresholds to start/stop exporting, the share of exporting firms is quite stable across aggregate states, even though the number of exporters is slightly higher during periods of low domestic demand and high foreign demand.

At the moment of entry there is a large surge in investment; on average, the investment rate for new exporters is 18 percent, substantially higher than for non-exporters and existing exporters. This pattern is consistent with the findings of Iacovone and Javorcik (2009), who show that Mexican manufacturing plants are significantly more likely to present investment spikes[10] one and two years before starting to export a new product. Upon entry into the export market, firms' total sales increase on average by 29 percent on impact, and although sales fall afterwards, they still remain higher than prior to exporting[11]. Conversely, domestic sales fall as firms channel resources into the foreign market in order to recoup the costs of selling abroad. As a result of the irreversibility of investment, when productivity or aggregate demand fall, firms cannot divest in order to reach a lower capital stock and are therefore forced to let their capital stock depreciate.

[10]Investment rates above 20 percent.

[11]Total sales for firms that have remained in the export market for five years are about 10 percent higher than their pre-export sales.

Because of this asymmetry, about 30 percent of the firms do not invest in any given period. Gelos and Isgut (2001) report rates of zero investment episodes for machinery and equipment of 28 and 30 percent for manufacturing firms in Mexico and Colombia respectively.

Table 4 about here

Table 5 compares the behavior of domestic and exporting firms. As previously noted, exporters sell a substantial amount of output abroad and are therefore 64 percent larger than domestic firms in terms of total sales. Exporters' sales volatility is also 23 percent higher than that of non-exporters. Although mean investment rates do not differ significantly between exporters and domestic firms, the investment behavior for both types of firms is actually quite different. As noted by Caballero (1991), in the presence of irreversibilities, capital stock is more responsive to "good" shocks (realizations in which the capital in place is lower than the desired stock of capital), since downsizing the capital stock is costlier than building it up. For this reason, exporters are more likely to present episodes of zero investment than non-exporters (38 percent, relative to 28 percent for non-exporters), but at the same time present substantially higher investment rates during periods of favorable demand. For instance, when the economy moves from a state with both low domestic and foreign demand to a state with high domestic and foreign demand, the investment rate for exporters is 23 percent compared to 7 percent for non-exporters, more than three times as large. Because exporters have already accumulated substantial capital stocks, irreversibility makes them less likely to continue investing; however, exporters do have the incentive to accumulate capital in response to positive shocks to foreign demand, a stimulus that does not directly affect domestic firms[12].

Table 5 about here

Next, I examine how the degree of correlation between aggregate shocks affects the export participation decision of firms. To do so, I change the parameter Σ_{12} in the aggregate shock process covariance matrix, keeping all other parameters the same as in the benchmark. This affects two elements of the simulation: the grid points $\{\underline{z}_h, \overline{z}_h, \underline{z}_f, \overline{z}_f\}$ for the aggregate shock and the transition matrix $P_Z(\mathbf{Z}'|\mathbf{Z})$. In order to assess the effect of the correlation of demand shocks on export participation while keeping the size of both markets fixed, I use the same grid points as in the benchmark simulation and only allow the transition matrix to change. Figure 5 depicts the results. I find a non-monotonic relationship between the degree of correlation of aggregate shocks and export participation. When the correlation between domestic and foreign demand becomes more negative, a significant increase can be observed in export market participation. Thus, reducing

[12]Although there may be some indirect effects through changes in the price index.

the correlation between aggregate shocks from 0 to -0.6 results in an increase in the share of exporting firms from 21 to 32 percent. At lower levels of correlation there is not a significant difference in the pattern of export participation. However, when the correlation between aggregate shocks increases above 0.4 the number of firms exporting increases relative to the benchmark of zero correlation. This might be due to the higher variance in domestic demand resulting from the higher correlation. Maloney and Azevedo (1995) also find that the covariance between domestic and foreign revenues has an ambiguous effect on the export supply decision of risk-averse firms.

Figure 5 also shows how the decision to export is influenced by a firm's degree of risk aversion. Lower risk aversion increases the optimal scale of firms (reducing ν from 1 to 0.5 increases the average capital stock of firms by 11 percent) which would make it more likely for firms to accumulate enough capital to enter the foreign market. On the other hand, less risk-averse firms would find the diversification benefits provided by the imperfect correlation of shocks across markets less valuable. The results in Figure 5 show that the latter effect dominates. Although the non-monotonic relationship between exporting and the correlation of shocks remains, the share of exporters is consistently lower when risk-aversion falls. This result suggests that risk-averse firms value the diversification advantage provided by the imperfect correlation in demand as a mechanism to smooth their sales revenue.

Figure 5 about here

5 Experiment: Closing Down the Export Market

To quantify the effect that exporting has on the volatility of sales, I take advantage of my calibrated model and conduct a counterfactual experiment in which the foreign market is shut down, so firms do not have the opportunity to export. Using the same realizations for the idiosyncratic and aggregate shocks, I compare the performance of firms that would have become exporters had the foreign market been open with their behavior in the benchmark simulation.

Figure 6 shows the behavior of a representative firm that in the benchmark simulation experiences two long spells of exporting, of 99 and 46 periods respectively. Exporting has a significantly positive effect on the scale of the firm. Total sales are 61 percent higher and 22 percent more volatile in the benchmark simulation during the periods in which the firm would have exported[13]. Table 6 presents the results for all firms. Total sales are 36 percent lower on average and 14.4 percent less volatile for potential exporters

[13]Capital stock is 12.8 percent lower and 45.8 percent less volatile in the counterfactual.

in the counterfactual scenario. Only domestic sales are slightly more volatile when firms are not able to export. This pattern can be better appreciated in the upper panel of Figure 8, which shows the distribution of capital in both scenarios.

Table 6 about here

Figure 6 about here

Figure 7 depicts the evolution of the mean capital stock for firms that are about to become exporters in the benchmark simulation, and compares it with the firms' behavior when the foreign market is closed. The figure shows that potential exporters are firms that have experienced high productivity and demand realizations before starting to export. Therefore, even in the counterfactual scenario, these potential exporters are actively accumulating capital and are significantly larger than the average firm. However, when productivity dwindles, firms in the counterfactual let their capital start to fall relative to the case where they can export. The counterfactual capital stock is 5 percent smaller on average in the first period after potential exporters should have started selling abroad than in the benchmark scenario. This difference increases to 12 percent after 3 periods and 14 percent after 5 periods. Thus, exporting allows firms to sustain higher stocks of capital. Due to the export hysteresis generated by the sunk cost, firms remain in the export market even when foreign conditions are not favorable. Conversely, when demand or productivity improves, exporters substantially increase their investment and sales. In the counterfactual the lack of sunk costs results in firms facing smaller deviations from their desired capital stock. This in turn translates into lower volatility of investment and sales.

Figure 7 about here

The fact that exporting causes higher sales volatility can be seen clearly in the lower panel of Figure 8. Exporters' significant reaction to foreign demand shocks on the intensive margin is missing in the counterfactual scenario. For instance, when the economy moves from a state of low domestic and foreign demand to a state in which only foreign demand improves, there is almost no change on investment by firms in the counterfactual. In the benchmark, however, there is a lot of action taking place in the foreign market, which in turn makes the sales of exporting firms more volatile. Existing exporters increase their investment rate from 2 to 7 percent and adjust the composition of their sales towards the foreign market, increasing the average share of exports on total sales from 42 to 59 percent. There is also significant entry into the export market, which accounts for 11.7 percent of the increase in total exports. As shown in the previous section, entry into the export market is characterized by very high investment rates and large changes in total sales that are reflected in higher sales volatility. Thus, exporters' substantial response to aggregate shocks, especially on the intensive margin, tip the balance towards a net positive effect of exporting on the volatility of sales at the firm level.

Figure 8 about here

6 Concluding Remarks

I set up and calibrate a model in which risk-averse firms accumulate capital and choose whether to sell their output in a foreign market or not. In order to highlight the possible benefits of sales stabilization associated with exporting, I assume that there are no capital markets available for firms, so that the only mechanism available for a firm to hedge against domestic aggregate shocks is exporting. I find that the correlation structure of aggregate shocks is an important factor in determining a firm's decision to become an exporter. Moreover, a lower degree of risk aversion reduces the responsiveness of export participation to the correlation of aggregate shocks. These two findings suggest that exporting could provide a diversification avenue for exporting firms. However, a counterfactual experiment shows that if the foreign market was hypothetically closed, potential exporters would have lower sales volatility than if they were able to export. Large swings in investment for exporters, who due to the sunk entry costs are willing to stay in the export market even during periods of low productivity and/or foreign demand, and the reallocation of sales across markets associated with changes in foreign demand that do not affect firms constrained to sell only in the domestic market, result in a higher volatility of sales for exporters. Thus, I find that taking into consideration the significant costs associated with starting to export and maintaining a presence abroad, two factors that have been ignored by previous research on this topic, is key to understanding how globalization affects firm-level volatility.

Table 1: Baseline Simulation Parameters

Parameter	Description	Value
ν	Relative risk aversion coefficient	1.0
α	Curvature of production function	0.30
N	Number of firms	500
f_x	Per-period fixed cost of exporting	0.189
s_x	Sunk cost to start exporting	0.225
β	Discount factor	0.90
δ	Depreciation rate for capital	0.06
σ_h	Demand elasticity for the domestic market	2.0
σ_f	Demand elasticity for the foreign market	4.0

Table 2: Transition Rates in and out of Export Market

	Year $t+1$ status			
	Roberts & Tybout (1997)		Model	
Year t status	No Exports	Exports	No Exports	Exports
No Exports	0.973	0.027	0.918	0.082
Exports	0.11	0.89	0.125	0.875

Table 3: Summary Statistics: benchmark

Variable	**Mean**	**St. Dev.**	**Min.**	**Max.**
Fraction of Exporters	0.219	0.413	0.000	1.000
Exports/Sales	0.454	0.119	0.194	0.652
Capital Stock	1.578	0.724	0.852	4.733
Domestic Sales	1.425	0.466	0.897	2.512
Foreign Sales	1.086	0.380	0.459	1.975
Total Sales	1.663	0.620	1.112	3.995
Investment rate	0.064	0.101	0.000	0.622

Table 4: Export Market Participation Across Aggregate States

	$z_h = \underline{z}_h$ $z_f = \underline{z}_f$	$z_h = \underline{z}_h$ $z_f = \overline{z}_f$	$z_h = \overline{z}_h$ $z_f = \underline{z}_f$	$z_h = \overline{z}_h$ $z_f = \overline{z}_f$
% Exporters	21.27	21.56	24.16	22.50
% Entry episodes	0.00	55.83	0.00	44.17
% Exit episodes	95.51	0.00	4.49	0.00

Table 5: Statistics by Exporting Status

Variable	**Non-Exporters**	**Exporters**
Mean Capital Stock	1.309	2.541
Mean Investment rate	0.068	0.050
St. Dev. Investment rate	0.104	0.091
Mean Total Sales	1.458	2.393
St. Dev. Total Sales	0.462	0.568

Table 6: Counterfactual Scenario: Shutting Down the Export Market

	Benchmark		**Counterfactual**	
Variable	**Mean**	**St. Dev.**	**Mean**	**St. Dev**
Capital Stock	1.578	0.724	1.376	0.392
Domestic Revenues	1.425	0.466	1.477	0.467
Foreign Revenues	1.086	0.380	n/a	n/a
Total Revenues	1.663	0.620	1.477	0.467

Figure 1: Capital Policy Rule

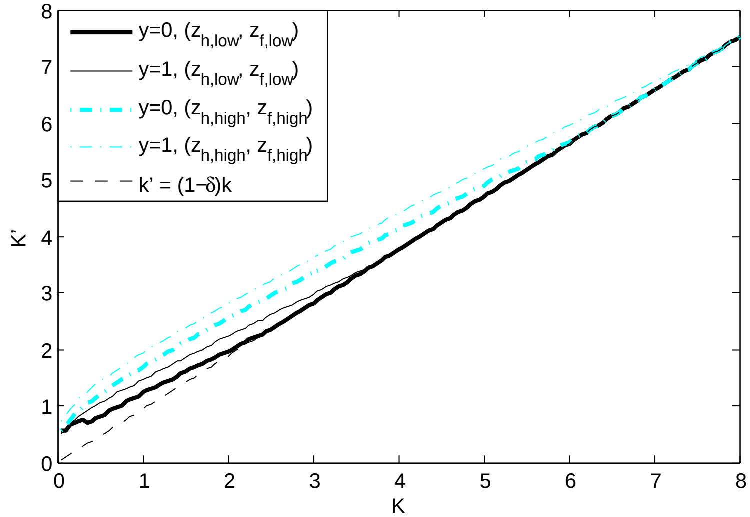
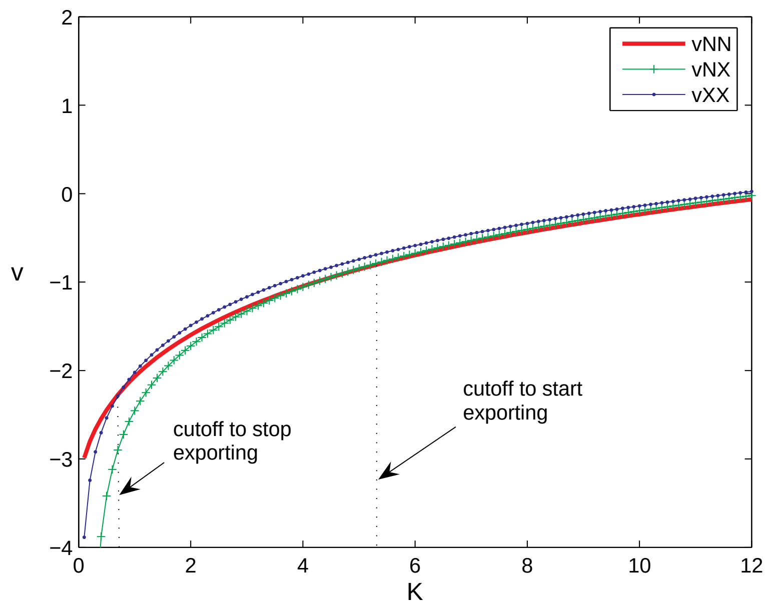


Figure 2: The Decision to Export



v_{NN} denotes the expected present discounted value (PDV) of a firm that does not export (either at $t-1$ or t); v_{NX} is the expected PDV for a firm that did not export at $t-1$ but exports at t; v_{XX} is the value for a firm that exports in both periods.

Figure 3: Export Intensity Decision

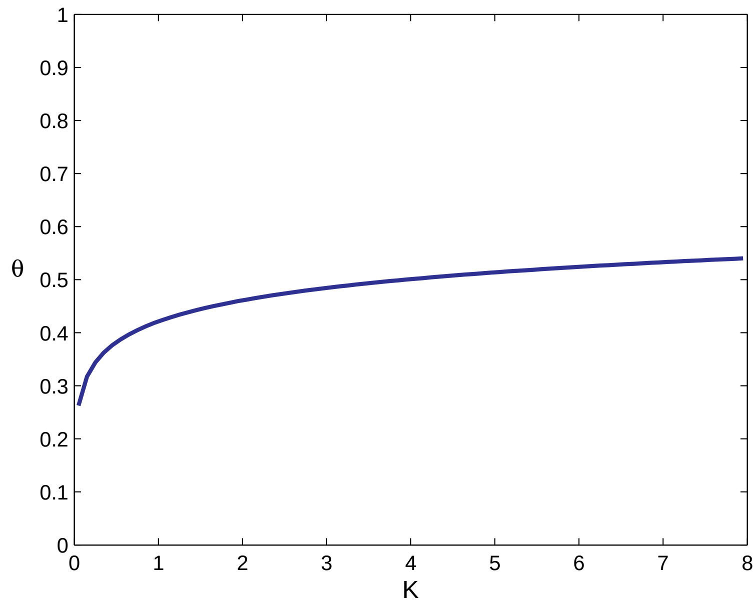


Figure 4: Sequence of Actions

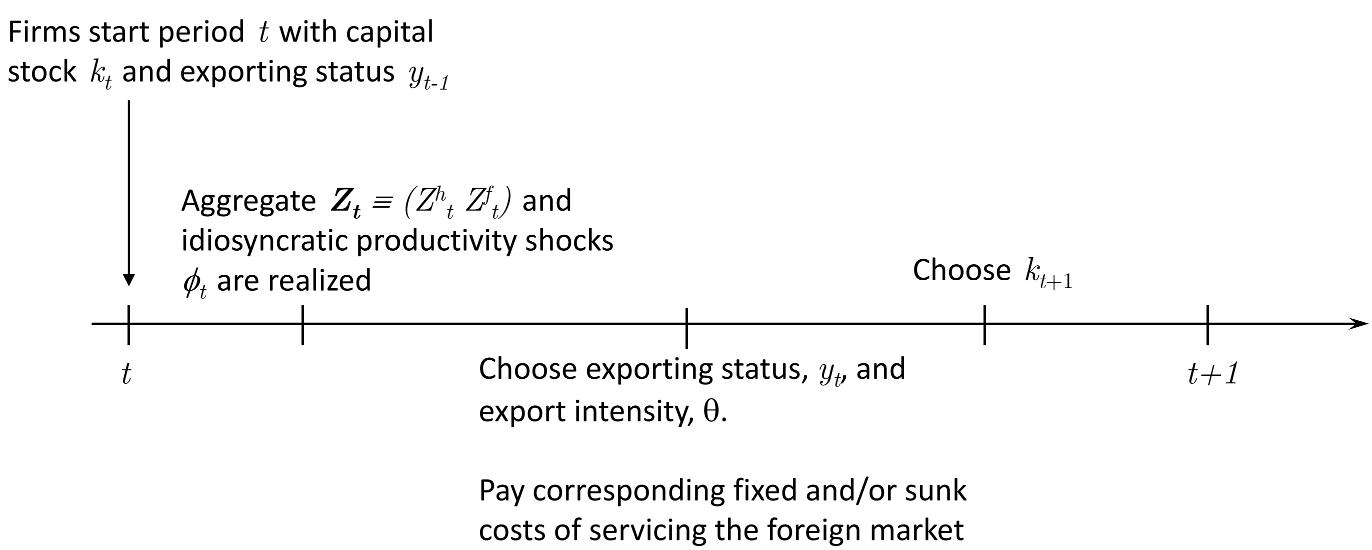


Figure 5: Foreign Market Participation and Demand Correlation

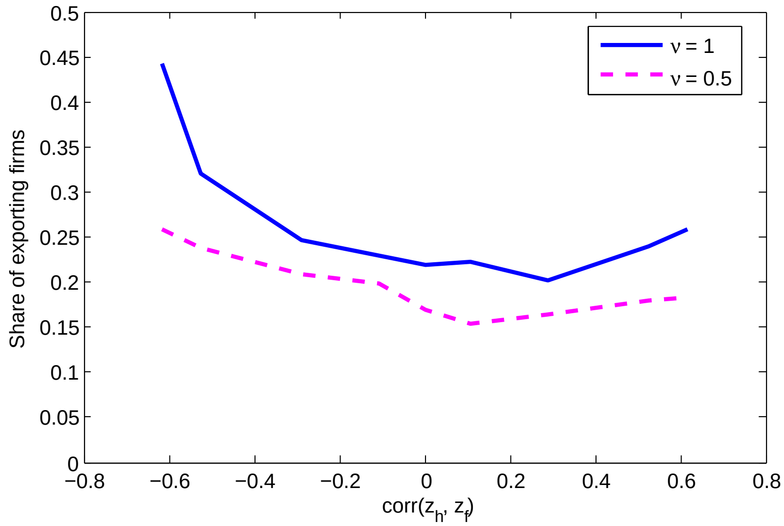


Figure 6: Counterfactual Scenario: Closing Down the Foreign Market

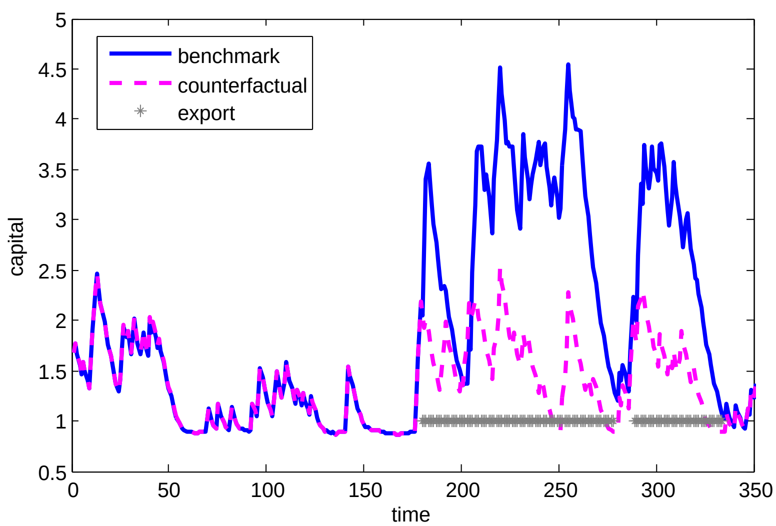


Figure 7: Counterfactual Scenario: Capital Accumulation for Potential Exporters

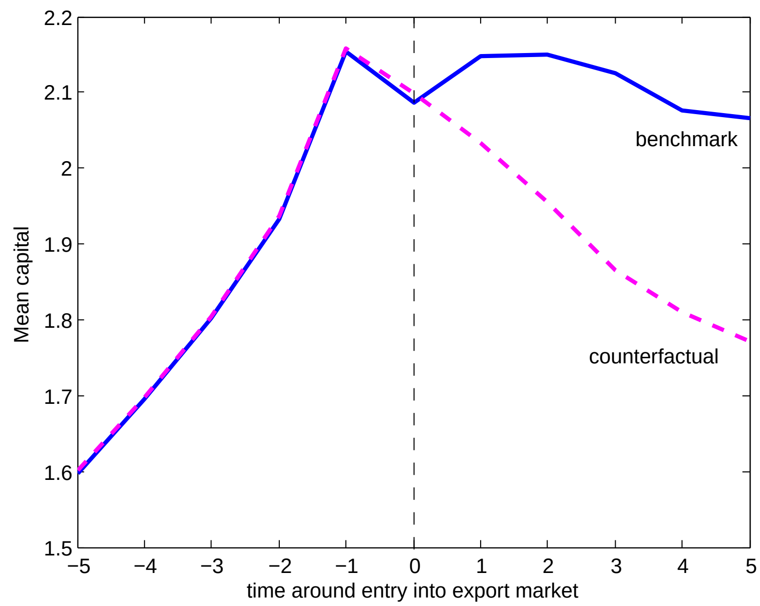
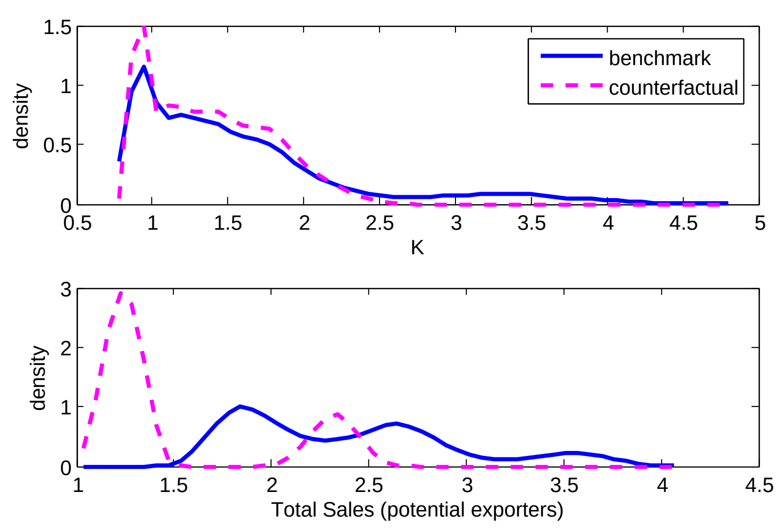


Figure 8: Counterfactual Scenario: Capital Distribution and Total Sales



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A Appendix: Computational Algorithm

1. Create equally-spaced grids for the *endogenous* state variables of the economy: 1) individual capital stock $k \in \mathcal{K} \equiv \{k_1, \ldots, k_{N_k}\}$ with $N_k = 250$, taking care that k_{N_k} is such that the policy rule for capital is always non-binding, and 2) the price index for the domestic market $P_h \in \mathcal{P} \equiv \{P_1, \ldots, P_{Np}\}$ with $N_P = 5$. The price index is defined in equation (5).

2. Create grids for the *exogenous* state variables in the economy: domestic and foreign demand shocks $\{z_h, z_f\}$ and the idiosyncratic productivity shock $\{\phi_i\}_{i=1}^{N_\phi}$ using Tauchen's (1986) method, with $N_\phi = 15$.

3. Initialize the parameters $\{a_{0,z}, a_{1,z}\}$ of the law of motion for the domestic price index (7).

4. Let S denote all the random shocks in the model (both demand shocks and the idiosyncratic productivity shock). Solve the problem described by equations (8), (9) and (10) using a value function iteration procedure with linear interpolation, given the law of motion 7 as follows:

   (i) Solve the first-order condition for θ for all values in the grids of the capital stock, shocks and price index. This is a static problem.

   (ii) Initialize a first guess for the value function, $v(k, y_{-1}, S, P_h)$.

   (iii) given the law of motion for P_h, calculate P'_h for all points on the price index grid. Start iterating over the price index. Call the index of this iteration j.

   (iv) Since P'_h does not necessarily belong to $\mathcal{P}$, linearly interpolate v along the P_h dimension. $v(k', y, S', P'_h)$ is then given by by:

$$v(k', y, S', P'_h) \simeq v(k', y, S', P_{h,i}) + \left[\frac{v(k', y, S', P_{h,j+1}) - v(k', y, S', P_{h,j})}{P_{h,j+1} - P_{h,j}}\right](P'_h - P_{h,j}). \quad \text{(A.1)}$$

   (v) Run over all points in the state space grid $(i_k, y, i_s, j) \in N_k \times 2 \times N_s \times N_p$. And solve:

$$v(k_{i_k}, y_{-1}, S_{i_s}, P_{h,j}) = \max_{k'_j \in \mathcal{K}} \left\{ u[\pi(k_{i_k}, k'_j, y_{-1}, S_{i_s}, P_{h,j})] + \beta E\left[v(k'_j, y, S'_{i_s}, a_{0,Z} + a_{1,Z} \log P_{h,j})\right] \right\}. \quad \text{(A.2)}$$

   (vi) Once that the optimal k' has been determined for all elements of the grid $\mathcal{K}$, the value of k' is determined off the grid points by approximating the value function $v(\cdot, y_{-1}, S, P_{h,j})$ using linear interpolation.

   (vii) Repeat this procedure until convergence of the value function has been achieved. This produces optimal policy rules for the capital stock, g_k, and the export decision g_y.

5. Using the policy rules g_k and g_y simulate an economy with $N = 500$ firms for $T = 2,500$ periods. In each period calculate the optimal price charged by each firm in the domestic market and calculate $P_{h,t} = (N^{-1} \sum_i (p_{h,it})^{1-\sigma_h})^{\frac{1}{1-\sigma_h}}$. To calculate k' off grid points, use bilinear interpolation. For instance, to calculate $k' = g_k(k, y, S_{i_s}, P_h)$ for $k_{i_k} < k < k_{i_k+1}$ and $P_{h,j} < P_h < P_{h,j+1}$, define:

$$h_k \equiv (k - k_{i_k})/(k_{i_k+1} - k_{i_k}), \quad h_P \equiv (P_h - P_{h,j})/(P_{h,j+1} - P_{h,j}). \quad \text{(A.3)}$$

   Then, the optimal next-period capital is given by:

$$\begin{aligned} k'(k, y, S_{i_s}, P_h) = (1 - h_k)(1 - h_P) g_k(k_{i_k}, y, S_{i_s}, P_{h,j}) + h_k(1 - h_P) g_k(k_{i_k+1}, y, S_{i_s}, P_{h,j}) \\ + h_k h_P g_k(k_{i_k+1}, y, S_{i_s}, P_{h,j+1}) + +(1 - h_k) h_P g_k(k_{i_k}, y, S_{i_s}, P_{h,j+1}). \quad \text{(A.4)} \end{aligned}$$

6. Given a time-series for the domestic price index $\{P_{h,t}\}_{t=500}^{2,500}$, estimate the parameters $a_{0,z}$ and $a_{1,z}$ using ordinary least-squares regression. If the parameters are sufficiently close to the initial guess and the fit of the law of motion is good enough (i. e. if the R^2 of the regression is above 0.99), stop. Otherwise, update the law of motion for the price index and return to step # 3. The estimated parameters for the benchmark simulation appear in Table A.1.

Table A.1: Estimated Parameters for the Law of Motion of the Price Index

	Coefficient	**S. E.**
$a_{0,1}$	0.1412	0.0032
$a_{0,2}$	0.1413	0.0038
$a_{0,3}$	0.5128	0.0053
$a_{0,4}$	0.5151	0.0051
$a_{1,1}$	0.2534	0.0139
$a_{1,2}$	0.2178	0.0183
$a_{1,3}$	0.2421	0.0223
$a_{1,4}$	0.2027	0.0224
R^2	0.9904	
Obs.	2,000	