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Strategic Trade Policy with Endogenous Product Differentiation

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Abstract

In this paper I develop a model of international duopoly, where firms invest in product differentiation. I show that firms have an incentive to free-ride on the investment of their rival, due to an externality generated by product differentiation. A further effect of product differentiation is the market-expansion effect, which induces consumers to increase their aggregate spending in the market. Depending on the strength of the two effects, the investments are either strategic substitutes or strategic complements. I link this result to strategic trade policy and show that the optimal policy depends on the strength of the market-expansion effect.

Keywords: R&D; Duopoly; Product differentiation; Industrial Policy; Strategic trade policy;

JEL Classification: F12; F13; F15; L10

Outline

Non-Technical Summary

This paper reconsiders the debate on strategic trade policy in an environment where firms can differentiate the characteristics of their output. When investing in product differentiation firms face two effects. Firstly firms have an incentive to free-ride on the investment of their rival as changing the characteristics does mean higher market power for the other firm as well. Secondly, more differentiated product increase the aggregate demand in the industry. The two effects influence the incentive of the firm in opposing directions: due to the free-riding firms invest less, whereas a larger aggregate demand increases the return on investment. In my work I show that the relative strength of the two effects has important implications for strategic trade policy. For example, in a more mature industry the market-expansion effect is comparatively weak as products are established. I show that in this case an R&D tax is optimal. If the industry is younger the market-expansion effect is strong and dominates the free-riding effect. In this case I show that a subsidy is the optimal R&D strategy.

1 Introduction

In this paper I consider strategic trade policy in an environment where firms invest in horizontally differentiating their products. Due to globalisation domestic firms face fiercer competition on world markets. Firms have a variety of strategies at hand to cope with the increased pressure: invest in cost-reducing R&D, reduce costs by offshoring part of their production. A further strategy is that firms could invest in changing the characteristics of their products to reduce their substitutability. I first investigate the incentives for firms to invest in R&D to differentiate their products. The analysis reveals that the strategic nature of the investment depends on a parameter in the model. I further show that this has important implications for the optimal policy as the latter depends on the parameter as well.

Firms place value in distinguishing their product from the one of their competitor. In a survey of London-based firms being asked to rank competitive strategies, they on average ranked those strategies associated with differentiating their product higher than categories associated with cost reduction (see table 1). Furthermore, firms choose a portfolio of investment in process and product innovation. Evidence in Scherer and Ross (1990) suggests that three quarters of the investment expenditures of US based firms go into product innovation. Out of the countries studied, Japan, where firms invested the lowest share in product innovation, still invested one fifth.[1] This evidence strongly indicates that not only is investing in product differentiation an important strategy for firms, but their investment is biased towards differentiation.

In what way can firms benefit from differentiating their products? Two possible ways are, firstly, firms increase their market power, and secondly that firms try to find new consumers. Consider the computer market and the invention of the netbook as an example. On the one hand the introduction of the netbook influenced consumers who bought a desktop to additionally buy a netbook which is transportable. This increases the overall spending on the computer market. On the other hand, the inventor of the netbook diverts resources from the production of computers to the production of netbook. Thus, the firm forfeits competing on desktops in favour of netbook production where the rival is not producing. Subsequently, both firms have larger market power in their respective segments of the market.

Brander and Spencer (1983) put forward an argument for strategic trade policy

[1] Product innovation is seen as equivalent to differentiating the output of the firm.

Priorities	Mean Value
Quality of Product or Service	4.7
Customer Relations	4.6
Reliability of Product or Service	4.5
Established Reputation	4.5
Knowledgeable Staff	4.4
Speed of Delivery	3.9
Unique Product or Service	**3.9**
Product or Service Range	**3.7**
Design	**3.6**
Low Cost Base	3.5
Marketing	3.3
Low Prices	3.2

Priorities range from 1 to 5

Source: London Annual Business Survey 2006; page 79

Table 1: Strategic Priorities of Firms

which suggests that governments are able to support their domestic firm(s) by announcing a subsidy or tax on the investment in process innovation. By announcing their support for firms, in the form of a tax or subsidy, the government can influence the outcome of the investment game played by the firms. Choosing the policy to be optimal maximizes domestic welfare. The argument in Brander and Spencer (1983) was subsequently debated in the literature considering different environments for firms.[2] The underlying question of the debate is whether there is a unique policy rule. Leahy and Neary (2001) reconsider the arguments put forward and concluded that a case to case approach would be best given the lack of a unifying policy rule. In a more recent contribution Leahy and Neary (2009) analyze multilateral subsidy games in the presence of spillovers. They show that the decision to tax or subsidize R&D depends on the interaction of the friendliness of the investment and the strategic nature of the investments.[3]

[2]For example, Brander and Spencer (1983), Brander and Spencer (1985), Dixit (1984), Bagwell and Staiger (1994), Brander (1995), Eaton and Grossman (1986), Maggi (1996), Leahy and Neary (2000) or Haaland and Kind (2008).

[3]The friendliness is the effect of a rivals strategy on the profits of a firm whereas the strategic nature (substitutes versus complements) is the effect of a rivals action on the marginal return of the firm.

My paper contributes to the literature on strategic trade policy by examining the case of endogenous product differentiation. The somewhat surprising result of my paper is that conditions exists under which an investment tax rather than a subsidy is optimal. The intuition for the result is that by differentiating their product firms are able to reduce the substitutability of the product in the market. The naive intuition suggests that more market power means higher profits in the international market.[4] The result of an R&D tax comes about by the assumption that changing the characteristic of its product benefits the firm's rival by making the product of the rival look more unique as well. This assumption introduces a free-riding effect which the policy makers try to exploit by reducing domestic investment via a tax. The result of an R&D subsidy is owed to the market-expansion effect. By changing the characteristic of the product a firm is able to capture new consumers. This implies an increase in aggregate demand. By subsidizing the domestic firm a government induces a stronger change in characteristics and thus a stronger increase in the market.

Furthermore, I will introduce the concept of the market-expansion effect and show its impact on the optimal policy. This concept suggests that firms might be able to attract new consumers to the market and as such increase aggregate demand. However, this market-expansion effect might vary in its strength. Two different optimal policies can be derived from this variation: if the market-expansion effect is strong, a subsidy is optimal. Whereas if the market-expansion effect is weak the optimal policy is a tax. The interpretation of this result is that the free-riding effect is always present, but is dominated by a strong market-expansion effect.

My paper relates to the literature on international trade and product differentiation as follows. Firstly, the industrial organization literature analyses the topic of endogenous product differentiation. Motta and Polo (1998) analyse the case of endogenous product differentiation with different modes of competition. Lin and Saggi (2002) study the portfolio choice of firm between process and product innovation if firms sequentially invest in both types of R&D. Rosenkranz (2003) and Weiss (2003) investigate the case of simultaneous choice. All papers consider Cournot and Bertrand competition with the exception of Weiss (2003), who develops a robust model with respect to the form of competition. Secondly, in the international trade literature product differentiation has a prominent space with the important example of Krugman (1980). Bernhofen (2001) develops a model of international competi-

[4] The standard assumption in those types of model is to say that domestic consumer surplus does no matter.

tion with product differentiation. Braun (2008) considers the portfolio choice of firms in an international trade context and its impact on the skill premium. I here built on the above papers by considering the choice of the degree of product differentiation where firms compete internationally. However, I allow for differences in the strength of the market-expansion effect and explore the firm's behaviour and its impact on an R&D policy. Furthermore, my paper differs in particular from Leahy and Neary (2009) in the way the externality is modelled.

The paper is organized as follows. In the second section I develop a general model of endogenous product differentiation. In the third section, I numerically solve the model with linear demand functions. In the fourth section, I introduce a government policy and investigate its impact on the investment game.

2 A Model of Product Differentiation

2.1 The General Model

In this section I focus on the behaviour of the firms and will discuss the R&D policy in a later section. Consider two countries $i = A, B$ which each hosts one firm. The two firms compete for a third market, where outputs are horizontally differentiated. The firms compete via quantities in the output market and additionally, the firms are able to invest in differentiating the product they sell. The game is one of complete but imperfect information. The structure of the game and the profit functions of each firm are common knowledge. Further, decisions become common knowledge as soon as they are implemented. At each point in time firms move simultaneously. The timing of the game is as follows: (1) the firms make an investment to differentiate their product; (2) the firms play a Cournot quantity game. The whole game is solved backwards. At each stage of the game, the firms play subgame-perfect strategies. After the firms have chosen their investments, these are treated as fixed costs. I now characterize the solution for each stage of the game.

2.1.1 The Quantity Game

The demand function each firm faces is denoted by $p_i = p_i(q_i, q_j; \theta)$ for $\quad i = A, B$ where q_i is the output of firm i. The usual assumptions on the demand function hold, namely $\partial p_i/\partial q_i < 0$ and $\partial^2 p_i/\partial q_i^2 \leq 0$. The inverse degree of product differentiation is denoted by θ, which is treated as given at this stage. A lower value of θ is associated with a higher degree of product differentiation. Let the quantity stage revenue be denoted by

$$\pi_i = [p_i(q_i, q_j; \theta) - c]q_i \quad i = A, B. \tag{1}$$

Firms are identical with respect to their constant marginal production costs c. The solution is characterized by the standard conditions

$$\frac{\partial \pi_i}{\partial q_i} = \tfrac{\partial p_i}{\partial q_i} q_i + p_i - c \quad = 0 \tag{2}$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = \tfrac{\partial^2 p_i}{\partial q_i^2} q_i + 2\tfrac{\partial p_i}{\partial q_i} \quad < 0, \tag{3}$$

which ensure that the solution is a maximum.

With respect to the dependency of the demand function on the degree of product differentiation I assume the following

Assumption 1. *Product Differentiation and Demand*

$$\frac{\partial p_i}{\partial \theta} < 0 \tag{4}$$

$$\frac{\partial^2 p_i}{\partial \theta \partial q_i} \geq 0. \tag{5}$$

Equation (4) implies that a higher degree of product differentiation (lower θ) increases the demand for a given product. In other words it represents a shift in the demand curve. I will call this the market-expansion effect of product differentiation. Equation (5) is the change of slope of a demand curve with a change in θ which tilts the demand curve, which indicates that the demand curve becomes more inelastic.

What is the interpretation of those two derivatives? The first derivative indicates that consumers value more differentiated products and are willing to spend more on the overall market.[5] Going back to the example in the introduction about the computer industry and the invention of the netbook, the latter increased the overall spending on the computer market. The second derivative captures the idea that with a lower degree of substitution firms have more market power. Again, the inventor of the netbook firm might stop competing on desktops in favour of netbook production. This gives both firms a larger market power in their respective segments of the market.

Determining the sign of the output response of a firm to a change in θ is an interesting comparative static at this point. Totally differentiating the first order condition of the quantity game yields

$$\frac{dq_i}{d\theta} = -\frac{(\partial^2 p_i / \partial q_i \partial \theta) q_i + \partial p_i / \partial \theta}{(\partial^2 p_i / \partial q_i^2) q_i + 2 \partial p_i / \partial q_i} \tag{6}$$

[5] A different way to see this is by interpreting the first line as the change in marginal utility.

Note that the output response to a change in θ is ambiguous and depends on the sign in the numerator.[6] In general, if the market-expansion effect ($\partial p_i/\partial\theta$) is stronger than the slope effect ($\partial^2 p_i/[\partial q_i \partial\theta]$), the output decreases in θ. Conversely, if the slope effect is stronger than market-expansion effect, the output increases in θ. The latter result stems from an increase in market power: the closer firms get to becoming monopolies, the lower the amount they produce and the higher the price they charge. In the case of the strong market-expansion effect, the increased demand dominates the market power effect and firms increase their output.

2.1.2 The Investment Game

At the investment stage firms implicitly choose the degree of product differentiation by choosing their R&D effort. I assume that a direct mapping of the investment into the degree of product differentiation exists.[7] One desirable feature of the mapping is that one firm's investment is non-excludable in the sense that if one firm differentiates itself it implicitly differentiates the other firm as well. Investing in product differentiation therefore exhibits an externality. I denote the mapping by $\theta = \theta(x_A + x_B)$, where the investments enter additively to accommodate the non-excludability. At this point it is worth making assumption on the functional form of the mapping, which become important later in the paper. I assume the following

Assumption 2. *The Investment Mapping*

$$\frac{\partial\theta}{\partial x_i} < 0 \tag{7}$$

$$\frac{\partial^2\theta}{\partial x_i^2} \geq 0 \qquad \frac{\partial^2\theta}{\partial x_i \partial x_j} \geq 0 \tag{8}$$

Equation (7) states the degree of product differentiation increases in the investment. The first derivative in Equation (8) captures that it is getting increasingly harder to change the characteristics of the product of a firm as there are for example technological boundaries. The second derivative builds on the latter idea, and describes that change the characteristics of a firms output is getting harder the more the other firms output is differentiated. I do allow for a linear mapping where those effects are not present, which is the case if the derivatives in the second line hold with equality.

[6]By assumption for the existence of a solution the denominator must be negative.

[7]This is a common assumption in the industrial organization literature of endogenous product differentiation. See for example Lin and Saggi (2002) or Rosenkranz (2003).

Let $\pi_i(q_A^*, q_B^*, \theta) \equiv \pi_i(\theta) = \pi_i(x_A + x_B)$ represent the reduced form second-stage profits. The q_i^* denote the optimal output of the second stage game which itself is a function of θ. I rewrite the maximization problem as

$$\max \Pi_i = \pi_i(\theta) - C(x_i), \tag{9}$$

where $C(x_i)$ denotes the investment costs. The investment costs are assumed to be convex with $\partial C/\partial x_i > 0$ and $\partial^2 C/\partial x_i^2 \geq 0$.[8] For a profit maximum to exist, the first- and second order conditions have to satisfy:

$$\frac{\partial \Pi_i}{\partial x_i} = \frac{\partial \pi_i}{\partial x_i} - \frac{\partial C}{\partial x_i} = 0 \tag{10}$$

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} = \frac{\partial^2 \pi_i}{\partial x_i^2} - \frac{\partial^2 C}{\partial x_i^2} < 0 \tag{11}$$

This characterizes the solution to the investment game. The main interest of this paper is in the strategic nature of the investments. The algebraic definition of the strategic nature is equal to

$$\frac{dx_i}{dx_j} = -\frac{\partial^2 \pi_i/[\partial x_i \partial x_j]}{\partial^2 \Pi_i/\partial x_i^2} \quad i = A, B. \tag{12}$$

The denominator must be negative for a solution to exist.[9] The sign of the nominator is determined by $\partial^2 \pi_i/\partial \theta^2 \times \partial^2 \theta/[\partial x_i \partial x_j]$, where the second derivative in the expression is positive by assumption. This leaves the first term to determine the strategic nature of the investments.

Proposition 1. *Given assumption 2, the strategic nature of the investments is determined by the sign of $\partial^2 \pi_i/\partial \theta^2$. The investments are strategic complements if $\partial^2 \pi_i/\partial \theta^2 \geq 0$ and strategic substitutes if $\partial^2 \pi_i/\partial \theta^2 \leq 0$.*

This proposition summarizes the result of this section: the strategic nature of the investments is ambiguous. The idea here is that although a firms profits always increase in the degree of product differentiation, it is no clear at which rate the profit increase. In the case of negative sign the marginal return from investment decreases whereas in the case of a positive sign marginal profits increase. Again, the interpretation is the strength of the market-expansion effect. I will provide a stronger intuition in the next section where I resort to a linear demand schedule to show in a numerical exercise that with the assumptions made in this section on the demand curve and the investment mapping, the strategic nature of the investments depends on the strength of the market-expansion effect.

[8]Convex investment costs are a standard assumption in the industrial organisation literature.

[9]Note that the slope of the reaction function is derived by totally differentiating the first order condition. The denominator is then second order condition. The numerator in the present case can be simplified as the cost function is independent of the rival's investment.

3 Linear Demand

3.1 Demand

In this subsection, I introduce the underlying utility function and the resulting demand functions of the linear model. I do so to provide a stronger intuition of the demand functions which are non-standard with respect to the market-expansion and slope effect. The utility function I use below is a weighted average of the formulation by Bowley (1924) and Shubik and Levitan (1980). In the former formulation, a higher degree of product differentiation expands the market. In the latter formulation, a higher degree of product differentiation softens competition while the aggregate market remains constant. By weighting both approaches I am able to control for both effects. The utility function takes the form

$$U = a(q_A + q_B) - b(1 + \sigma(1-\theta))\left(\frac{q_A^2}{2} + \frac{q_B^2}{2}\right) - b\theta q_A q_B + m \tag{13}$$

where $\theta \in (-\infty, 1]$ is the degree of product differentiation. The parameter $\sigma \in [0, 1]$ measures the degree of the market-expansion effect, where the upper boundary of σ corresponds to no market-expansion effect at all. The utility function is quasi-linear in m which is chosen as the numeraire. Given the quasi-linear nature of the utility function there are no income effects. Consumers optimize their consumption of good A and B and spend the rest of their income on the numeraire good. The utility function exhibits a taste for variety by the consumer, given by the first two terms on the right-hand side. The third term is a competition term. The more differentiated the products are, the less competition there is amongst the two firms.

The resulting inverse demand function for good $i = A, B$ is

$$p_i = a - \alpha b q_i - b\theta q_j \tag{14}$$

where $\alpha \equiv 1 + \sigma(1-\theta)$. If $\theta \in [0, 1]$ the goods are imperfect substitutes. If θ is close to zero, the goods are highly differentiated and thus the firms are close to being monopolists. The upper bound, θ close to one, implies the goods are closer to homogeneous goods and thus the firms face fiercer competition. If $\theta = 1$ the goods are perfect substitutes. For $\theta < 0$ the goods become complements. Furthermore, note that the sign of the derivatives in Assumption 1 match the ones in the linear model. In particular, the slope effect is zero if $\sigma = 0$ and positive otherwise. In terms of a graphical interpretation of the individual demand curve, if $\sigma = 0$ then the firm's demand curve shifts up, whereas $\sigma > 0$ implies a rotation of the individual demand curve.

To provide a better understanding of the market-expansion effect I aggregate the (direct) demands, which yields

$$Q = q_A + q_B = (1 + \sigma + \theta[1 - \sigma])^{-1} \left(\frac{2a}{b} - \frac{1}{b} p_A - \frac{1}{b} p_B \right) \tag{15}$$

For $\sigma = 1$, the aggregate demand is independent of the degree of product differentiation. Further, if $\sigma = 1$ then the aggregate utility derived from consumption in the market is constant which implies that the consumer has no aggregate benefit from more product differentiation. For $\sigma \in (0, 1)$, the aggregate demand does depend on θ and is decreasing in it. Hence, with a higher degree of product differentiation the aggregate demand for the two goods expands, as the consumer is willing to spend more on the market.

The Second Stage Output Game

In the second stage of the game, each firm maximizes net profits with respect to the output, taking the investments in product differentiation as given. At the unique symmetric Nash equilibrium in the second stage outputs are

$$q_i^* = \frac{a - c}{b(2\alpha + \theta)} \quad i = A, B. \tag{16}$$

Corollary 1. *The optimal output is increasing in the degree of product differentiation if the market expansion effect is strong ($\sigma \leq 1/2$).*

The corollary follows from rearranging the derivative of the optimal output with respect to θ. Note that this corollary follows the idea developed in the general model where I showed that the output increases or decreases depending on whether the market-expansion effect or the slope effect dominates respectively.

Substituting the optimal outputs (16) into the profit function of firm i, I obtain the third stage profits

$$\pi_i^* = \frac{\alpha}{b} \left(\frac{a - c}{2\alpha + \theta} \right)^2 \quad i = A, B. \tag{17}$$

The Investment Game

Having solved for the second stage equilibrium I now consider the actions of the firms in the first stage. Firms maximize their investment subject to the investment mapping. The formal problem is

$$\begin{array}{ll} \max_{x_i} & \Pi_i = \frac{\alpha}{b} \left(\frac{a-c}{2\alpha+\theta} \right)^2 - C(x_i) \\ \text{subject to} & \theta = \theta(x_A + x_B) \end{array}$$

In order to make a statement on the strategic nature of the investment I can appeal to proposition 1.

Proposition 2. *The investments are strategic complements if and only if $\sigma \leq 1/2$. The investments are strategic substitutes if and only if $\sigma \geq 1/2$.*

A proof is relegated to the appendix. The idea of the proof simply is to show how the profits change with respect to θ. Before I discuss the intuition I check the robustness of the proposition by assuming specific functional forms for the mapping and the cost function. Doing so enables me to numerically simulate the reaction functions, which helps in providing the intuition for the result.

I assume that the cost function takes the form $C(x_i) = \gamma x_i^2$, which is convex in the investment. Further, the mapping of the investment into the degree of product differentiation is assumed to take the form $\theta = \max\{0, 1 - x_A - x_B\}$.[10] Below, I show the existence of the optimal investments in a numerical example.

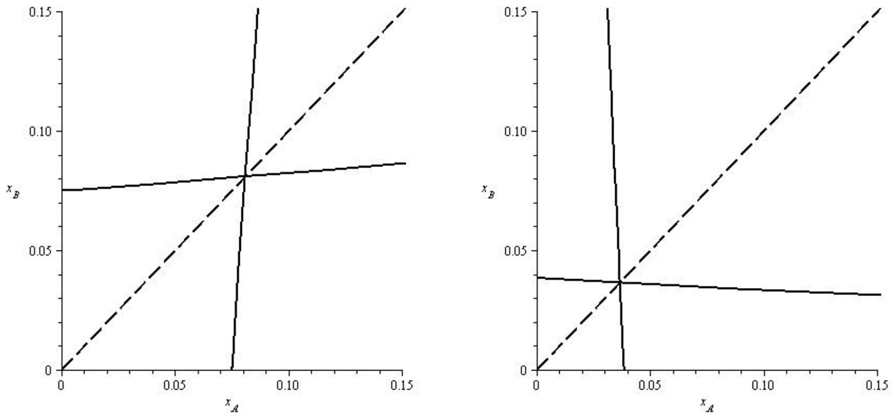
In general, each firm is exposed to two effects which have an opposing effect on the investment incentives. The effects are an incentive to free-ride on the rival's investment and an incentive to increase the size of the market. Due to the spillover of the investments in product differentiation each firm has an incentive to free ride on its rival's investment; if one firm differentiates itself it implicitly differentiates the product of the other firm. Therefore the rival benefits from the differentiation and could even reduce its own investment in order to maintain the level of product differentiation. The market-expansion effect increases the size of the market and thus makes it viable for the firm to invest more.

In figure 1, I numerically simulate the first-order conditions to show the effect of the market-expansion effect on the slope of the reaction function.[11] In the figure, I depict the reaction functions for two different values of σ. In the sequel of the section I discuss, the three resulting reaction functions and the economic intuition of them. With a strong market-expansion effect, $\sigma < \frac{1}{2}$, the reaction function is upwards sloping as depicted in panel 1a.[12] Accordingly, the investments are strategic

[10] A similar mapping is used in Motta and Polo (1998) or Rosenkranz (2003). Furthermore, the functional form on the mapping satisfies the restrictions imposed in the general model. However, the cross derivative of θ with respect to the investments is zero. I will show below that a solution for the investment games exists.

[11] The assumed values for the parameters are: $a = 2$, $b = 1$, $c = 1$ and $\gamma = 0.5$.

[12] The solutions of the numerical simulations are stable. Henriques (1990) raises the point that, depending on the strength of the spillovers, the solution might not be stable due to the wrong slope of the reaction functions. I checked different numerical values specifically of σ which did not change the stability of the equilirbium.



(a) Strategic Complements

(b) Strategic Substitutes

Figure 1: First Order Conditions

complements. With a weak market-expansion effect, $\sigma > \frac{1}{2}$, the reaction function is downwards sloping as illustrated in panel 1b. Accordingly, the investments are strategic substitutes. With strategic substitutes, a higher investment by one firm decreases the marginal return to investment of the other firm and induces the rival to adopt a less aggressive strategy. The reason for this result is the externality of the investment mapping. With a higher investment by one firm the rival experiences the benefit of product differentiation without investing in it. Therefore, the optimal response of the rival is to reduce the investment.

With strategic complements, a higher investment by one firm increases the marginal return to investment of the other firm. In the case under consideration, the market-expansion effect is strong and dominates the free-riding effect. This leads to a mutual reinforcement of the investments.

To complete the discussion, in the case of $\sigma = \frac{1}{2}$, the free-riding incentive and the market-expansion effect are of equal strength and cancel each other out. Accordingly, the investment of one firm is independent of the action of the rival and merely a function of the parameters of the model.

To summarize the findings of this subsection, I showed that the strategic nature of the investment depends on the market-expansion effect. Intuitively, this can be explained as follows. Firms have an incentive to free-ride on the investment of their rival, due to the externality of product differentiation. This effect implies that the investments have the tendency to be strategic substitutes because a higher investment by one firm reduces the incentive to invest of the rival firm. However, the market-expansion effect induces consumers to increase their aggregate spending in the market. The implication is that firms are able increase the aggregate spending and share it with the rival. Thus, the market-expansion effect increases the marginal return to investment for the rival.

Before I investigate the effects of a policy I develop a result needed for the policy games. A strategy of a firm is called friendly if it increases the profits of the other firm. In the model, an increase in the investment by the rival always increases the profit of a firm as a higher degree of product differentiation always means higher profits. Formally this is summarized in the following Lemma.

Lemma 1. *The profits of one firm increase in the investment of the other firm ($\partial\pi_i/\partial x_j \geq 0$). Therefore the investments are friendly.*

Proof. The derivative in the lemma can be rewritten as $\partial\pi_i/\partial x_j = \partial\pi_i/\partial\theta \times \partial\theta/\partial x_j \geq 0$, where the second derivative on the right-hand-side is negative by definition. By deriving $\partial\pi_i/\partial\theta = -2\alpha(1-\sigma) - \sigma\theta \leq 0$ concludes the proof. □

4 The Policy Game

In this section, I investigate how a policy maker can intervene in the investment game to increase domestic welfare. In particular, I look at a R&D policy. The major contribution of this paper is to derive the optimal policy in an environment of product differentiation. Therefore I follow the set-up and derivation of the policy in Brander and Spencer (1983).

In addition to the two stages I introduce a pre-firm stage in which the governments announce their policy to support the respective domestic firm. The policy parameter is denoted by λ_i. The policy is assumed to take the form of a subsidy or a tax of the investment costs of the domestic firm. In the case of $\lambda_i < 0$ the firm would pay a tax, whereas $\lambda_i > 0$ corresponds to a subsidy. The policy is paid per unit of investment. A R&D policy does not directly change the output game; it indirectly influences the output decision by a firm by altering the decision to invest in product differentiation. I assume that a policy maker can credibly announce the policy schedule and therefore rule out any commitment problems on the side of the government. I assume that the policy maker has complete information of the game. The order of the stages is as follows. The government moves first and announces its policy schedule. In the second stage, the firms choose their investment, given the government policy. In the third stage, the firms set quantities, given the investments in product differentiation. In each stage, the players move simultaneously.

I now show that the results on the strategic nature hold, even when the a policy is introduced. For simplicity, I only introduce the policy in the general model, where the specific model yields the same qualitative results. The profit function of a firm changes as follows

$$\Pi_i(x_A, x_B) = \pi_i(\theta) - C_i(x_i) + \lambda_i x_i. \tag{18}$$

The first-order condition is

$$\Pi_i^{i*} = \frac{\partial \pi_i}{\partial \theta}\frac{\partial \theta}{\partial x_i} - \frac{\partial C_i}{\partial x_i} + \lambda_i = 0. \tag{19}$$

The effect the policy schedule has on the reaction function is as follows. Depending on the sign of the policy, the reaction function shifts in or out without changing the slope of the reaction function. For example, a subsidy shifts the reaction function outwards, implying a higher investment in product differentiation. Further, the reason for the constant slope of the reaction function is that the second-order condition and the cross derivative of the firm's profits do not change. Therefore, the results derived in the previous section, especially on the strategic nature of the investments, do not change with the introduction of a policy.

The next step is to derive the optimal policy. I will distinguish between three cases: a unilateral policy, both governments are active and a cooperative policy.

Unilateral Policy Intervention

I start off with the simplest case of a unilateral policy in this subsection. To this end I assume that only country A has an active government such that $\lambda_B = 0$. The policy maker in the active country chooses the policy that maximizes national welfare. Because firms compete for a third market I can neglect consumer surplus in the domestic market. Thus welfare is the profit level of the firm less the total subsidy payments to the firm

$$W_A = \Pi_A(x_A, x_B) - \lambda_A x_A, \tag{20}$$

where the investments are a function of the subsidy, $x_i = x_i(\lambda_A)$ for $i = 1, 2$. The solution for the optimal policy is

$$\lambda_A = \frac{\partial \Pi_A}{\partial x_B}\frac{dx_B}{dx_A}, \tag{21}$$

The optimal schedule depends on the slope of the reaction function and the friendliness of the investments. The interpretation of the optimal policy is that the policy maker has to take into account the response of the foreign firms investment to the policy and the impact of the rivals response on domestic profits.

Lemma 2. *A government has the incentive to set an optimal unilateral policy, which is a subsidy if the investments are strategic complements and a tax if the investments are strategic substitutes.*

Proof. The sign in (21) is determined by lemma 1 and equation (2). □

The intuition for the lemma is as follows. Considering a weak market-expansion effect the optimal policy is a tax. The intuition is that the investments provide an incentive to free ride as they have a positive externality on the other firm. Additionally, the reaction functions are downwards sloping. Therefore, if a government can influence its domestic firm to reduce the investment, the investment of the foreign firm will go up. This in turn implies that the profits of the domestic firm increase because the investments are friendly. Considering a strong market extension effect the optimal policy is to subsidize the investments. The intuition is that the effect of the larger market is stronger than the incentive to free-ride on the other firm's investment. Thus, if a government subsidizes its own firm the other firm increases its investment as well leading to a larger market.

A Nash Subsidy Game

In this section I analyze a policy rivalry between the two countries. Both governments are now able to support their respective firm by announcing a policy schedule for R&D. Each government maximizes its respective domestic welfare function

$$W_i = \Pi_i(x_A, x_B) - \lambda_i x_i \quad i = A, B. \tag{22}$$

The timing of the whole game remains unchanged and both governments announce their policy simultaneously. The optimal subsidy, as in Brander and Spencer (1983), is

$$\lambda_i = \frac{\partial \Pi_i}{\partial x_j}\frac{dx_i}{dx_j}. \tag{23}$$

Note that the structure is the same as for the unilateral subsidy. However, the values of the right hand side are different. The sign of the subsidy is the combined effect of the impact of the foreign investment on the profits and the slope of the reaction function.

Proposition 3. *The optimal Nash policy is a tax if the market-expansion effect is weak and a subsidy if the market-expansion effect is strong.*

The economic interpretation of the proposition is the same as in the case of the unilateral policy. In the model the investments are always friendly because of the externality of product differentiation. The strategic nature depends on the extent of the market-expansion effect: if the market-expansion effect is strong the investments are strategic complements whereas if the market-expansion effect is dominated by the free-riding effect the investments are strategic substitutes.

A subsidy is in line with Brander and Spencer (1983). In their model, a policy maker has an incentive to announce a subsidy to increase the R&D investments of the home firm. I find a similar result for a weak market-expansion effect. However, with a large market-expansion effect I find that a tax is optimal. This is at odds with Brander and Spencer (1983). The difference is explained by the friendliness of the investments and the ambiguity of the strategic nature of the investments due to the market-expansion effect. Furthermore, Leahy and Neary (2001) generalize the conditions under which a subsidy is optimal. They conclude that a subsidy is a robust rule.[13] The results found in this paper show that, in a different set up, this conclusion does not hold due to the ambiguity of the strategic nature of the investments.

The Cooperative Policy

I now analyze what the optimal policy is if both countries collude. I do so by comparing the optimal policy derived previously to a cooperative policy. To this end I assume that both countries coordinate their policy efforts and maximize joint welfare, $W(\lambda_A, \lambda_B) = W_A + W_B$. The optimal policy is

$$\lambda_i = \frac{\partial \Pi_i}{\partial x_j} \quad i = A, B \tag{24}$$

Proposition 4. *The optimal policy under joint welfare maximization is to subsidize the investments in product differentiation.*

This result is obtained regardless of the strategic nature of the investments. Intuitively, the joint government take into account the positive externality of the investment on the each other's profits. By subsidizing the investments each firm increases not only its own profits but the one of the other firm as well. Accordingly welfare increases and governments exploit this externality. In Brander and Spencer (1983) the optimal cooperative policy is a tax. The reason for the difference is that the investments are unfriendly in Brander and Spencer (1983) and thus a higher investment of the foreign firm would reduce domestic profits. In the present case, investments a friendly and governments take that into account when administering a cooperative policy.

[13] With the exception of non-linearities in the demand function.

5 Conclusion

In this paper I introduced a general model in which firms are able to invest in horizontally differentiating their products. The strategic nature, which is of particular interest to my paper, depends on whether the market-expansion effect dominates the free-riding incentive or vice versa. In the former case investments were strategic complements, in the latter strategic substitutes. I subsequently applied this model to a framework of strategic trade policy and showed that the optimal policy depends on the strength of the market-expansion effect.

In markets where investment in product differentiation is important, this model allows policy makers to evaluate the effectiveness of their policy. As I argue in the paper it is crucial to take into account the strength of the market-expansion effect when devising the investment policy. For example, relatively young industries might expand more strongly in response to a change in the degree of product differentiation compared to more mature industries. Therefore, a different policy approach is needed for the young industry compared to a more mature industry.

In this paper I focused on a third market. For future work it might be of interest to include domestic consumer welfare. Domestic producers may indeed benefit from more differentiation as in the case of the market-expansion effect. This may be of consideration in the policy-making process. Additionally, firms might be able to choose between investing in product differentiation and process innovation as discussed at length in the industrial organization literature. A future area of consideration could also be exploring the response of a firm's choice of investment in product differentiation to globalisation.

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Appendix

Proof of Proposition 2

I start out by computing the second order derivative of the net-profits with respect to θ which yields

$$\frac{\partial^2 \pi_i}{\partial \theta^2} = 2\,\frac{(a-c)^2\,(1-2\,\sigma)\left(\sigma - 2\,\sigma^2 + 2\,\sigma^2\theta - \sigma\theta + 3\right)}{(-2-2\,\sigma+2\,\sigma\theta-\theta)^4\,b}$$

This can be simplified to

$$\frac{\partial^2 \pi_i}{\partial \theta^2} = (1-2\sigma)[3+\sigma(1-\theta)(1-2\sigma)]\frac{(a-c)^2}{b}$$

To determine the sign I can neglect the third term in the derivative. The expression in square brackets is always positive. To see this I rewrite the expression as

$$3 \geq -\underbrace{\sigma}_{<1}\underbrace{(1-\theta)}_{\leq 1}\underbrace{(1-2\sigma)}_{\leq 1},$$

where $(1-2\sigma) \in [-1,1]$. Even if $(1-2\sigma)$ is at is lower bound, the inequality holds. This leaves the expression $(1-2\sigma)$ to determine the sign of the derivative. The expression is positive if and only if $\sigma \leq 1/2$, which concludes the proposition as $\sigma \leq 1/2$ corresponds to a strong market expansion effect.

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