The University of Nottingham

research paper series

Globalisation, Productivity and Technology

Research Paper 2014/02

Trade Liberalization, Intermediate Inputs and Firm Efficiency: Direct versus Indirect Modes of Import

By

Michele Imbruno

The Centre acknowledges financial support from The Leverhulme Trust under Programme Grant F/00 114/AM

GEP

Leverhulme Centre for Research on Globalisation and Economic Policy

The Author

Michele Imbruno is a Ph.D. candidate at The Leverhulme Centre for Research on Globalisation and Economic Policy (GEP).

Acknowledgements

The author would like to thank Fabrice Defever, Richard Kneller, Gianmarco I.P. Ottaviano, Alejandro Riaño and Hylke Vandenbussche as well as all participants at Midwest International Trade Meeting (Spring 2012), 5th FIW Research Conference on 'International Economics' (2012), 2nd Brixen Workshop on 'International Trade and Finance' (2011) and ETSG Conference (2011) for their helpful comments and suggestions on an earlier version of this paper circulated with the title 'Trade Liberalization, Intermediate Inputs and Firm Competitiveness: Direct versus Indirect Modes of Import'. All remaining errors remain his own.

Trade Liberalization, Intermediate Inputs and Firm Efficiency:

Direct versus Indirect Modes of Import

by

Michele Imbruno

Abstract

This paper studies the impact of input trade liberalization on firm efficiency, aggregate productivity and welfare. We extend the Melitz (2003)'s framework to incorporate: *a)* trade in both intermediate inputs and final goods between similar countries, *b)* firm's decision to import intermediate inputs in addition to the decision to export its final output. This model shows different effects from reducing input tariffs, according to whether intermediates are assumed to be imported *directly* by final good firms or *indirectly* through an efficient wholesale system.

JEL: F12, F13

Keywords: Heterogeneous firms, Trade liberalization, Intermediate inputs, Productivity, Import-Export behaviour

Outline

Non-technical summary

Does trade openness enhance economic growth and welfare within an economy? Whilst this question ignites an often colourful debate amongst academic researchers and policy-makers, the traditional theories of international trade support the idea that greater international openness leads to efficiency gains amongst firms, and consequently welfare benefits for final consumers.

Thus far most of literature focuses on trade in final goods without paying attention to trade in intermediate goods and the related firm decision to outsource intermediate inputs. The contribution of this paper lies in the study of intermediate inputs. In so doing we build on a long line of theoretical models that have highlighted the important role played by trade in intermediate inputs through several channels, such as learning, variety and quality effects. In these models, firms enhance their performance as they can access a larger number of input varieties and/or a higher quality of intermediate inputs from abroad, or they can learn about new technologies embodied in foreign inputs. Only very recently has an empirical literature emerged which has started to explore this using firm-level data. For instance, using data from Indonesia, Amiti and Konings (2007) were able to disentangle the effect of input trade liberalization between firms directly involved in import activity and other firms. They found that all firms enjoy productivity gains from reducing input tariffs, and that these are larger for importers compared to non-importers. They conclude that while importers gain thanks to increases in variety, quality and learning effects, non-importers also benefit owing to spillover effects from importers. They do not however explore any particular channel through which importers and non-importers improve their performance.

The main purpose of this paper is to study theoretically the impact of trade liberalization in intermediate inputs on firm efficiency and consumer welfare. In particular, we attempt to address the following research questions: *i)* Is there any other mechanism through which importers improve their efficiency? *ii)* Why do non-importers also benefit from reducing input tariffs? *iii)* Do they always gain from input trade liberalization?

Briefly, by allowing for productivity heterogeneity in the intermediate goods sector, we are able to show that importers become more efficient following input tariff cuts because they can replace the most expensive domestic intermediate inputs with the cheaper alternatives from abroad. We label this new mechanism, the *'input switching effect'*. Moreover, we also demonstrate theoretically that non-importers suffer efficiency losses because of decrease in the number of domestic input varieties that are available. Consequently, we argue that a reason why non-importers are found to increase their efficiency in the data, is that some of them are '*invisible*' importers, i.e. they can indirectly access foreign inputs through wholesalers and other trade intermediaries.

1. Introduction

In the last few years, international trade literature has emphasized the important role played by firm heterogeneity in productivity in order to explain the microeconomic relationship between trade openness and economic growth. Most of these studies focus on export behaviour and trade liberalization in final goods[1], without paying any attention to import behaviour and trade liberalization in intermediate inputs.

Amiti and Konings (2007) first investigated empirically the impact of reducing input tariffs on firms, by isolating importers from other firms. They found that all firms enjoy productivity gains from trade liberalization in intermediate inputs, although importers benefit relatively more than non-importers, by arguing that this larger effect for importing firms might be linked to several channels as the theory predicts – such as access to more input varieties, access to higher quality inputs, and learning effects (Ethier 1982; Markusen 1989; Grossman and Helpman 1991). However, they did not attempt to disentangle any single channel and did not mention any peculiar reason why non-importers' performance also enhances[2]. Some other empirical studies explore either the firm-level linkage between imports and productivity (Kasahara and Rodrigue, 2008; Halpern, Koren and Szeidl, 2011; Bas and Strauss-Khan, 2011) or the impact of trade policy reforms on productivity for all firms (Schor, 2004; Goldberg, Khandelwal, Pavcnik and Topalova, 2010; Khandelwal and Topolova, 2011). While the first group of evidences concludes that importers are on average more productive than non-importers because of both self-selection mechanism and post-import effects – i.e. only the most productive firms are able to access foreign inputs, because of sunk fixed costs of importing, and further improve their performance because of these imports – the second group of studies documents that firms increase their efficiency thanks to two main effects: *a)* from increased competition, i.e. tougher competition from

[1] The majority of empirical studies (from Bernard and Jensen, 1995) focus on the export-productivity linkage and show that exporters are actually more productive than non-exporters, because of self-selection mechanism, rather than some post-entry effects – see Wagner (2007) and Greenaway and Kneller (2007) for a survey – as well as productivity gains from trade might arise from resources or market shares reallocation across firms within industry – from the least productive which exit the domestic market (Aw, Chung and Roberts, 2000) and the most productive which also serve international market (Bernard and Jensen, 1999). In light of these facts, Melitz (2003) develops a general equilibrium trade model *à la Krugman (1980)* – i.e. assuming monopolistic competition and increasing returns to scale – with heterogeneous firms, incorporating both self-selection and business reallocation mechanisms, above described.

[2] An increase in nonimporters' productivity has been attributed to some spillover effects, i.e. importers can transfer their benefits to other firms along the vertical production chain through sales of their goods, or alternatively, domestic producers of intermediates can be induced to become more competitive, entailing some indirect benefits for users of such domestic intermediates.

abroad induces firms to become more productive by 'trimming their fat' or innovating their products/processes; and *b)* from the improved access to foreign inputs[3].

The main purpose of this paper is to study the impact of trade liberalization in intermediate inputs on final good firm's efficiency, aggregate productivity and welfare, by developing a theoretical framework *à la Melitz (2003)*, which incorporates: *a)* trade in final goods/intermediate inputs between similar countries, *b)* firm's decisions to import intermediate inputs and to export its final output. In particular, this paper aims at addressing *i)* whether a further mechanism through which importers improve their efficiency exists; as well as *ii)* why and whether non-importers' performance always benefits from input tariff cuts.

To address the first question (*i*), we have to consider that Ethier (1982) demonstrated that trade openness can increase firms' efficiency, because firms can access more differentiated intermediate varieties (*gains from input varieties*). Thus, by considering that firm efficiency is inversely related to the price index of intermediate inputs, which in turn is decreasing in both number and average productivity of input suppliers, he showed that trade liberalization would basically entail an improvement in firm performance thanks to a higher number of input suppliers (or input varieties) available, while the related average productivity remains constant, given that all input suppliers have been assumed to be homogeneous in productivity. Now, through assuming that intermediate good firms are actually heterogeneous in productivity, following Melitz (2003), we can show that trade in intermediates can also determine an increase in average productivity of input suppliers, due to the exit of the least productive domestic firms and the entry of the most productive foreign ones within the intermediate good sector, entailing a further increase in final good firms' efficiency. In other words, final good firms can replace the worst domestic intermediates with the best ones from abroad, becoming more efficient (*gains from input switching*).

With regards to the second research question (*ii*), a reason why non-importers also increase their performance, might be that some of them are '*invisible*' importers. Firms can access some foreign inputs only through *directly* importing them, by incurring an additional fixed

[3] Further empirical works focus on the effective rate of protection (ERP), which incorporates the total effect of both output tariffs and input tariffs, such as Brandt, Van Biesebroeck, Wang and Zhang (2012). The concept of ERP was introduced by Corden (1971), through a simple theoretical framework showing that while lower output tariffs decrease effective protection implying higher import competition, lower input tariffs can lead to opposite effects.

cost (*official or direct importers*). However, firms can also use some foreign inputs by importing them *indirectly* through very efficient wholesalers, so without making any further fixed investment (*invisible or indirect importers*). Thus, some firms can actually use foreign inputs and enjoy efficiency gains, although they look like non-importers in the data. This argumentation is coherent with some recent empirical evidences (Bernard Grazzi and Tomasi, 2010, Bernard, Jensen, Redding and Schott, 2010) and theoretical models (Ahn, Khandelwal and Wei, 2011; Akerman, 2010; Blum, Claro, and Horstmann, 2009; Felbermayr and Jung, 2011; Crozet, Guy and Poncet, 2013) stressing the role of trade intermediaries from export point of view. All these studies show theoretically and/or empirically that the least productive firms serve the domestic market only, the most productive ones also serve the foreign market through *direct exports*, whereas the intermediate productive firms *export indirectly* through wholesalers – by assuming that the direct channel is associated with a higher fixed cost and a lower marginal cost compared to the indirect channel. Bernard, Blanchard, Van Beveren, Vandenbussche (2012) document that even some manufacturing firms operate as trade intermediaries for other firms, since the most productive firms have been found to export some products that they do not produce.

In our framework, we assume to have two similar countries, where any heterogeneous firm enters the home market by paying a fixed cost of entry, with the purpose to produce either a differentiated intermediate good – through using only labour – or a differentiated final good – by combining also all available differentiated intermediates – under increasing returns to scale. In particular, the heterogeneous intermediate inputs enter under CES form within final good firm's production function, implying that the firm's marginal cost is decreasing in the number of intermediate varieties used and in average productivity of input suppliers as in Ethier (1982). Then, two cases have been allowed for: *1) indirect imports case* (i.e. zero fixed cost of importing) and *2) direct imports case* (i.e. high fixed cost of importing).

In the first case (*Indirect imports*), any firm within both sectors can choose to serve the whole foreign market, by paying an additional fixed cost of exporting. Unlike the final good producer, an intermediate good firm has also to face a variable iceberg trade cost, since we assume that trade in intermediate goods is subject to tariff restrictions (i.e. input tariff) while trade in final goods is free (i.e. no output tariff)[4]. Thus, as in Melitz (2003), all

[4] We make these assumptions to make the model as simple as possible in order to study the impact of an input tariff reduction within an economy.

final good exporters serve all the foreign consumers, and similarly, all intermediate good exporters supply all the final good firms abroad ('indirect importers'): i.e. all final good producers within a country can easily access foreign inputs arising from the best suppliers abroad, through a very efficient wholesale system. In this environment, trade liberalization in intermediate inputs implies an increase in average productivity of intermediate good firms through some reallocation effects from the least productive firms (which quit the market) to more productive firms (which export). That in turn entails a uniform increase in final good firms' efficiency, as well as improvements in consumers' welfare, without any particular entry-exit dynamics within the final good sector. A new source of efficiency gains from input trade liberalization can be highlighted: all final good firms become more efficient since they can replace the worst domestic inputs with the best ones from abroad (*gains from input switching*), regardless of the change in the total number of input varieties available (*gains from input variety à la Ethier*).

In the case of '*Direct imports'*, only the most productive final good firms can actually access additional inputs from abroad, by paying a huge fixed cost of importing, which is assumed to be even higher than the fixed cost of exporting ($f_M > f_X$). We make this assumption for several reasons. First, we are interested to see how input trade liberalization can affect export performance according to if exporters can access or not foreign inputs. Second, the 'indirect imports' scenario could be already considered the extreme case where $f_M < f_X$ given that we assume zero-fixed cost of importing. Finally, the assumption made is coherent with several empirical evidences[5]. Consequently, intermediate good firms can serve only a portion of foreign final good producers, by incurring a variable selling cost increasing in the fraction of foreign importers, as in Arkolakis (2010). Hence, all final good exporters serve all the foreign consumers again as in Melitz (2003), whereas all intermediate good exporters supply only the highly-productive final good firms abroad ('direct importers'). In other words, solely the best final good producers within a country can have access to inputs arising from the best input suppliers abroad. In this context,

[5] For example, using Belgian firm-level dataset, Muuls and Pisu (2009) stress that two-way traders outperform importers, who in turn exhibit a higher performance respect to exporters, reaching the conclusion that self-selection would take place in both international activities. The same findings have been achieved by Castellani, Serti and Tomasi (2010), through using Italian firms' data. They certificate more accurately the self-selection hypothesis in import (export) markets, recognizing the existence of an ex-ante productivity premium – i.e. a productivity differential between future importers (future exporters) and permanent pure domestic firms. Altomonte and Bekes (2009) find similar results in Hungary. However, following a deeper exploration about self-selection mechanism across international firms, they realize it actually takes place via importing, rather than via exporting. They argue that the choice of importing might require a more complex organization of production, compared to the choice of exporting.

following an input tariff reduction, while importers would enjoy potential efficiency gains from input switching effects, as described above, non-importers suffer efficiency losses, due mainly to a decrease in availability of domestic inputs. This generates reallocation effects towards more productive firms (import-export firms) within final good sector, at expenses of the least productive firms (quitters) and the least productive exporters (which leave the international market), causing *aggregate efficiency gains* and *final variety losses*. Consumer's welfare appears to increase by considering these positive and negative effects altogether, only if firm heterogeneity is relatively high. Otherwise, it can even decline.

Our theoretical framework is closely related to Kasahara and Lapham (2013)'s study, which also extended the Melitz's model in order to examine the effect of input tariff cuts on firm-level decisions to import and to export simultaneously. However, by assuming that firms are homogeneous within the intermediate input sector, they focus their attention only on the final good sector where firms are assumed to be heterogeneous in productivity. In addition to Melitz's story, they show that trade liberalization in intermediates determines an increase in aggregate productivity and welfare, because of both productivity improvements within importing firms only (which are able to access a larger number of input varieties, by paying a fixed cost of importing) and some reallocation effects from pure domestic firms (exiters) to import-export firms. Unlike their model, our framework pays more attention to the intermediate good sector, where firms are assumed to be heterogeneous in productivity, and consequently the price index of inputs is assumed to be endogenous (i.e. decreasing in both number of input varieties used and average productivity of input suppliers) and different between importers and non-importers. In terms of predictions, our framework is able to show that some gains from input trade liberalization are due to input switching effects, rather than a simple increase in input varieties available, and that these benefits can concern all firms thanks to an efficient wholesale system, or only the most productive firms (direct-importers), given that the least productive ones could actually suffer efficiency losses.

Other recent papers study the impact of declines in input tariffs solely on either firm's decision to import (Gibson and Graciano, 2011; Halpern, Koren and Szeidl, 2011) or firm's decision to export (Bas, 2010; Chevassus-Lozza, Gaigné and Le Mener, 2013). Gibson and Graciano (2011) develop a trade model *à la Melitz (2003)* by assuming two different technologies: the first uses the domestic inputs only, and the second one uses both domestic and foreign inputs. In particular, the latter is associated with higher fixed cost compared to the former. Thus, firms would self-select to import inputs, and trade liberalization (or terms of trade improvement) would cause resources reallocation from the least productive firms (exiters) to the best ones (importers), determining an increase in aggregate productivity and welfare. Halpern, Koren and Szeidl (2011) show more specifically, through a theoretical model and using data from Hungarian firms during 1992-2003 period, that imported inputs generate productivity gains linked to two channels: quality (foreign inputs are better than domestic ones) and complementarity mechanism (gains from intermediates' combination are larger than the sum of the parts). Assuming that all final good firms can access all homogenous intermediate inputs, Bas (2010) extends Melitz-Ottaviano (2008) framework to show that a removal of input import barriers (or simply an increase in input import intensity) within industry would cause an increase in consumers' demand, as well as a proportional enhancement in competitiveness of all domestic final good firms such that both intensive (export volume) and extensive (number of new exporters) margins of exports would rise. In contrast, Chevassus-Lozza et al. (2013) extend Melitz (2003) model to demonstrate that input trade liberalization leads to some foreign market shares reallocation towards the more productive exporters within final good sector, as output price elasticity with respect to a change in input tariffs is increasing in firm productivity, while the effect on probability of exporting is positive only if fixed cost of exporting is sufficiently high, otherwise it can be negative, assuming however an exogenous number of final good firms.

While our theoretical model focuses on tariff liberalization, similar mechanisms could also be generalized to other trade reforms. For instance, it could help to explain the effects of antidumping policy found empirically in France by Konings and Vandenbussche (2008) and Konings and Vandenbussche (2013). The former work documents that the low-productivity firms increase their performance from import antidumping protection, whereas the high-productivity firms lower their efficiency; the latter evidence shows how import protection of intermediate inputs implies a fall in firms' exports.

The rest of the paper is organized as follows. Section 2 introduces the model in both Closed and Open Economy. Section 3 explores the impact of trade liberalization in intermediate inputs within a country. Section 4 concludes. All details about proofs are provided in the Appendix at the end of the paper.

2. Set up of the model

2.1. Closed Economy

The basic framework can be considered an extension of Melitz (2003)'s monopolistic competition model, since an intermediate good (*m*) sector has been added to the final good (*y*) sector which are vertically interrelated to each other, and all firms within both sectors are assumed to be heterogeneous in productivity and to produce differentiated varieties under increasing returns to scale.

2.1.1. Consumers preferences

A country has *L* homogenous final consumers endowed with one unit of labour each. These *L* units of labour are assumed to be inelastically supplied at the common wage rate *w*. A representative consumer has Cobb-Douglas preferences for a composite differentiated good *Y* and a composite homogenous good *H* such that a fraction β of income *wL* is spent on the former and the remaining fraction $(1-\beta)$ on the latter. Moreover, she also exhibits Constant Elasticity of Substitution (CES) preferences over the differentiated varieties $y \in \Omega_y$, i.e. the utility function is $U = Q_h^{1-\beta} Q_y^{\beta}$, where Q_h is the aggregate consumption in homogeneous varieties $h \in \Omega_h$ and Q_y is the aggregate consumption in differentiated varieties $y \in \Omega_y$, which in turn takes the following form:

$$Q_y = \left[\int_{y \in \Omega_y} q_y(y)^{\frac{\sigma-1}{\sigma}} dy \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $q_y(y)$ is the consumption for each variety *y*, and $\sigma = \dfrac{1}{1-\rho} > 1$ denotes the elasticity of substitution between any two products within the set of all final differentiated varieties available Ω_y [6]. Consumer optimization leads to the demand for final homogenous good

[6] In our model, we assume the standard CES function, which implies the exogenous and homogeneous mark-ups across varieties as well as symmetric demand across varieties and countries. Therefore, for the purpose of our analysis, we do not allow for the potential effect of trade liberalization on welfare through the mark-up channel (Melitz and Ottiviano, 2008; Di Comite, Thisse and Vandenbussche, 2013; Konings and Vandenbussche, 2005) and the demand-side drivers of firm performance (Kneller and Yu, 2008; Di Comite, Thisse and Vandenbussche, 2013).

$Q_h = \frac{(1-\beta)wL}{P_h}$, where P_h is the related price, as well as to the following demand for each final differentiated variety y:

$$q_y(y) = \left[\frac{p_y(y)}{P_y}\right]^{-\sigma} \frac{R_y}{P_y} \tag{2}$$

where $R_y = \beta wL$ is the total spending in composite differentiated good which corresponds to the aggregate revenue within the final differentiated good sector, $p_y(y)$ is the price of the variety $y \in \Omega_y$ and P_y is the aggregate price index of all final differentiated varieties available, which is dual to (1):

$$P_y = \left[\int_{y\in\Omega_y} p_y(y)^{1-\sigma} dy\right]^{\frac{1}{1-\sigma}} \tag{3}$$

The homogeneous final good is our *numeraire*, which is assumed to be produced by symmetric firms under perfect competition through a simple linear technology where one unit of output requires one unit of labour only. As result, the common wage rate is normalized to one ($P_h = w = 1$)[7].

2.1.2. Final differentiated good sector

As in Melitz (2003), the production technology in the final differentiated good sector assumes a continuum of firms heterogeneous in productivity φ_y which supply a variety y under monopolistic competition and increasing returns to scale. However, unlike Melitz (2003), each firm's output is produced by combining all intermediate inputs m available arising from heterogeneous firms within intermediate sector through a CES production function *à la Ethier (1982)*:

$$q_y = \varphi_y \left[\int_{m\in\Omega_m} x_m(m)^{\frac{\sigma-1}{\sigma}} dm\right]^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where q_y is the firm-level output, x_m is the quantity used for each differentiated input variety m, and $\sigma = \frac{1}{1-\rho} > 1$ denotes the elasticity of substitution between any two inputs

[7] Similar assumptions have been used by other studies, such as Helpman, Melitz and Yeaple (2004).

within the set of all intermediate differentiated varieties available Ω_m[8]. The resulting firm-level demand for a given intermediate variety *m* is

$$x_m(m) = \left[\frac{p_m(m)}{P_m}\right]^{-\sigma} \frac{q_y}{\varphi_y} \tag{5}$$

where $p_m(m)$ is the price of the input variety *m* and P_m is the aggregate price index of all intermediate differentiated available, which is dual to $\frac{q_y}{\varphi_y}$ in (4):

$$P_m = \left[\int_{m\in\Omega_m} p_m(m)^{1-\sigma} dm\right]^{\frac{1}{1-\sigma}} \tag{6}$$

The production of each variety *y* also requires a labour-intensive fixed cost *f*[9], therefore the final good firm's total cost to serve the whole market is $c_y = f + \frac{P_m}{\varphi_y} q_y$, where the fixed cost *f* is exogenous and the same across firms. Conversely, the marginal cost $mc_y = \frac{P_m}{\varphi_y}$ turns out to be endogenous and different amongst firms, since it is inversely related to the productivity level φ_y – which is exogenous but different across firms – and positively related to the common aggregate price of intermediate inputs P_m – which in turn is endogenous and common to all firms.

By allowing for the residual demand (2), each firm chooses its profit-maximizing price:

$$p_y(\varphi_y) = \frac{P_m}{\rho\varphi_y} \tag{7}$$

by yielding the following profit

$$\pi_y(\varphi_y) = \frac{R_y}{\sigma}\left(\frac{P_y \rho \varphi_y}{P_m}\right)^{\sigma-1} - f \tag{8}$$

[8] The elasticity of input substitution is assumed to be the same as the elasticity of output substitution to save further notation and make the model as simple as possible.

[9] We assume that the fixed cost is labour intensive, rather than intermediate input intensive, in order to have non-homothetic costs, which makes calculations easier. A similar assumption has been made by other previous studies, such as Forslid and Ottaviano (2003).

2.1.3. Intermediate differentiated good sector

If we multiply and divide the equation (5) by P_m, the firm-level demand for a specific input variety $x_m(m)$ can be written as a function of firm-level spending in all intermediate inputs $\frac{P_m}{\varphi_y} q_y$. Then, by aggregating $x_m(m)$ across all final good firms, we can get the following aggregate demand for a specific input variety:

$$q_m(m) = \left[\frac{p_m(m)}{P_m}\right]^{-\sigma} \frac{R_m}{P_m} \tag{9}$$

where R_m is the aggregate spending in all intermediate inputs across all final good firms which corresponds to the aggregate revenue within intermediate good sector.

The intermediate good technology assumes there is a continuum of firms heterogeneous in productivity φ_m which produce a differentiated variety m under monopolistic competition and increasing returns to scale by using only labour. In particular, their linear production function is $q_m = \varphi_m l$, where q_m is firm level output and l denotes the labour units used. By considering that the production of each variety m also requires a labour-intensive fixed cost f, the intermediate good firm's total cost to serve the home market is $c_m = f + \frac{q_m}{\varphi_m}$.

Unlike the final good sector, the marginal cost $mc_m = \frac{1}{\varphi_m}$ is exogenous, but is still different across intermediate good firms, since it is inversely related only to firm productivity φ_m, which in turn has been assumed to be constant but heterogeneous amongst firms. Therefore, the intermediate good sector more closely reflects the single sector' characteristics within Melitz (2003) framework. By facing the residual demand curve (9), each intermediate good firm sets the domestic price

$$p_m(\varphi_m) = \frac{1}{\rho\varphi_m} \tag{10}$$

by yielding the following profit

$$\pi_m(\varphi_m) = \frac{R_m(P_m\rho\varphi_m)^{\sigma-1}}{\sigma} - f \tag{11}$$

2.1.4. Equilibrium

In both differentiated sectors, firms enter the market by paying a fixed cost of entry f_e to draw their productivity φ_j for $j = m, y$ from the Pareto cumulative distribution $G(\varphi_j) = 1 - (\varphi_j)^{-k}$ with $k > 1$, and then decide whether to leave the market or to produce. The shape parameter k denotes the firm heterogeneity degree within sector: higher values of k are associated with lower dispersion in productivity. Any firm will stay in the market as long as its profit is positive. Thus, in each sector, we can define the survival productivity cutoff φ_j^* , i.e. the minimum level of productivity required to survive, through *Zero Profit Condition (ZPC)*: $\pi(\varphi_j^*) = 0$. Furthermore, by considering that in each period there is an exogenous probability of exit δ, a firm will take into consideration the possibility to enter the market only if the present value of expected profits is higher than the sunk fixed cost of entry f_e. Therefore, in each sector, the survival productivity cutoff also arises from the following *Free Entry Condition (FEC)*:

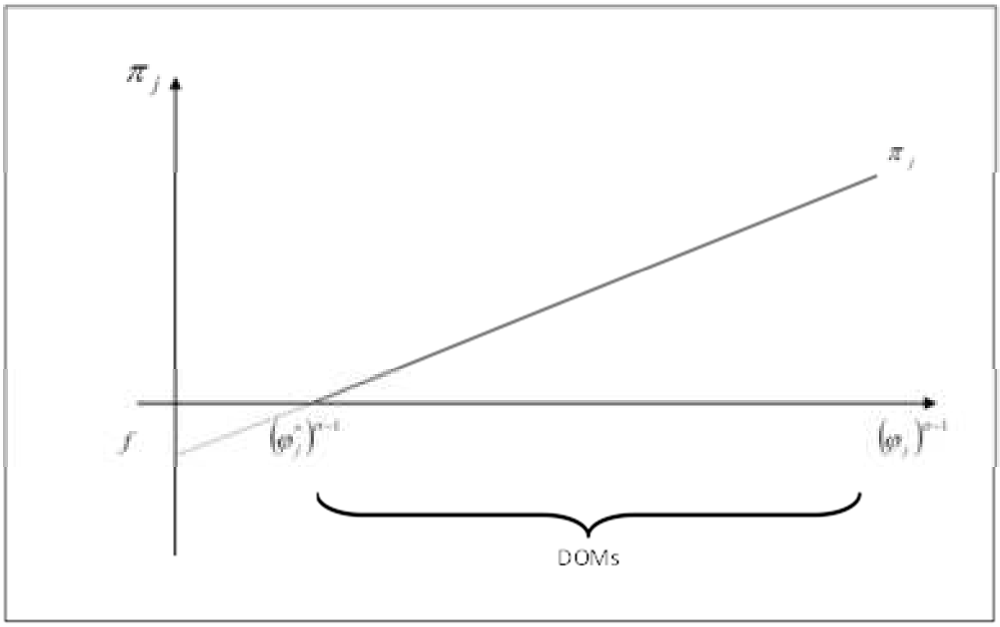
$$[1 - G(\varphi_j^*)]\frac{\tilde{\pi}_j}{\delta} = f_e \tag{12}$$

where $1 - G(\varphi_j^*)$ is the probability of survival and $\tilde{\pi}_j$ is per-period expected profits of surviving firms:

$$\tilde{\pi}_j = \int_{\varphi_j^D}^{\infty} \pi_j(\varphi_j)\frac{g(\varphi_j)}{1 - G(\varphi_j^*)}d\varphi_j$$

By allowing for both conditions above in each differentiated good sector, we can highlight the uniqueness of equilibrium $(\varphi_j^*, \tilde{\pi}_j;\ \text{for}\ j = m, y)$. **Figure 2.1** displays the profit as a function of productivity and shows that only firms whose productivity is high enough ($\varphi_j > \varphi_j^*$) will survive in the market.

Figure 2.1: Closed Economy – Sector *j = m,y*



It is worth noting that the firm-level efficiency (marginal cost) within final good sector is inversely (directly) related to the price index of intermediate inputs available, which – through plugging (10) into (6) – can be written as follows:

$$P_m = \left[\int_{\varphi_m^*}^{\infty} p_m(\varphi_m)^{1-\sigma} M \frac{g(\varphi_m)}{1-G(\varphi_m^*)} d\varphi \right]^{\frac{1}{1-\sigma}} = \frac{M^{\frac{1}{1-\sigma}}}{\rho \tilde{\varphi}_m}$$

where M and $\tilde{\varphi}_m(\varphi_m^*) = \left[\int_{\varphi_m^*}^{\infty} \varphi_m^{\sigma-1} \frac{g(\varphi_m)}{1-G_m(\varphi_m^*)} d\varphi_m \right]^{\frac{1}{\sigma-1}}$ stand for the mass and the weighted average productivity of intermediate good firms respectively.

Proposition 1. *Firm-level efficiency (firm-level marginal cost) within final good sector turns out to be increasing (decreasing) in both number and average productivity of heterogeneous input suppliers.*

The overall welfare is inversely related to the price index of final goods available only, since through considering our utility function, welfare per worker (final consumer) is given by

$$\frac{U}{L} = \left[(1-\beta)^{1-\beta}\beta^{\beta}\right]P_y^{\ -\beta}$$

By plugging (7) into (3), the price index of final goods can be written as

$$P_y = \left[\int_{\varphi_y^*}^{\infty} p_y(\varphi_y)^{1-\sigma} N \frac{g(\varphi_y)}{1-G(\varphi_y^*)} d\varphi_y\right]^{\frac{1}{1-\sigma}} = \frac{(NM)^{\frac{1}{1-\sigma}}}{\rho^2 \tilde{\varphi}_y \tilde{\varphi}_m}$$

where N and $\tilde{\varphi}_y\left(\varphi_y^*\right) = \left[\int_{\varphi_y^*}^{\infty} \varphi_y^{\sigma-1} \frac{g(\varphi_y)}{1-G_y\left(\varphi_y^*\right)} d\varphi_y\right]^{\frac{1}{\sigma-1}}$ are the mass and the weighted average productivity of final good firms respectively.

Proposition 2. *Economy's welfare is increasing in both number and average productivity of input suppliers, in addition to being positively related to both number and average productivity of final good firms as in Melitz (2003).*

Through the equilibrium conditions, both price indexes can be simply seen as function of survival cutoffs:

$$P_m = \left(\frac{R_m}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho \varphi_m^*} \tag{13}$$

$$P_y = \left(\frac{R_y}{\sigma f}\right)^{\frac{1}{1-\sigma}} \left(\frac{R_m}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho^2 \varphi_y^* \varphi_m^*} \tag{14}$$

While the mass of intermediate good firms (i.e. the number of input varieties available) and the mass of final good firms (i.e. the number of final varieties available) are respectively

$$M = \frac{R_m}{\sigma f\left(\frac{k}{1+k-\sigma}\right)} \tag{15}$$

$$N = \frac{R_y}{\sigma f\left(\frac{k}{1+k-\sigma}\right)} \tag{16}$$

Notice that the average efficiency within the final good sector is inversely related to the average marginal cost $amc_y = \frac{P_m}{\widetilde{\varphi}_y}$, which in turn can also be written as a function of both survival cutoffs

$$amc_y = \left[\frac{R_m}{\sigma f} \left(\frac{k}{1+k-\sigma} \right) \right]^{\frac{1}{1-\sigma}} \frac{1}{\rho \varphi_m^* \varphi_y^*} \tag{17}$$

As in Meltiz (2003), we focus on the steady state equilibrium so that $R_y = \beta L$ and $R_m = \rho R_y$. See **Appendix A** for more details.

2.2. Open Economy

This section considers two symmetric countries (i.e. countries with same endowments, wage rate and all the other aggregates) involved in international trade. The first subsection (2.2.1) is related to the case of '*Indirect Imports*', where any firm within both final good and intermediate good sectors can choose to serve the whole foreign market by paying a common additional fixed cost of exporting. Intermediate good firms have also to face per-unit iceberg trade costs, while final good firms have zero variable trade cost[10]. As in Melitz (2003), all final good exporters sell their products to all foreign consumers, and similarly, all intermediate good exporters supply all final good producers abroad ('indirect importers'). In other words, all final good firms within a country can easily access foreign inputs from the best input suppliers abroad, without any particular effort thanks to an efficient wholesale system.

The second subsection (2.2.2) concerns the '*Direct Imports*' case, where only some final good firms can actually import additional inputs by incurring a huge fixed cost of importing (which is assumed to be higher than the fixed cost of exporting). Consequently, intermediate good firms are able to serve only a portion of foreign final good firms, by facing a variable selling cost increasing in the fraction of foreign importers rather than a fixed cost of exporting, as in Arkolakis (2010), in addition to a per-unit iceberg trade cost. Hence, all final good exporters sell their products to all foreign consumers again (as in

[10] We assume free trade in final goods (i.e. zero variable trade cost) because the main focus of this paper is on trade liberalization in intermediate inputs – i.e. to highlight that even when we have free trade in final goods, firm efficiency and consumers welfare can further change through removing trade restrictions to intermediate inputs – and also to make the model as simple as possible.

Melitz, 2003), whereas all intermediate good exporters supply only more productive final good firms abroad ('direct importers'): namely, only the best performing final good producers within a country can have the access to inputs from the best input suppliers abroad, through incurring an additional searching effort and investment in an own distribution network.

2.2.1. Case I: Indirect imports

Final good sector. As in Melitz (2003), a final good firm can choose to export by paying an additional fixed cost f_X which is assumed to be higher than the fixed cost of domestic production f. Unlike Melitz (2003), we assume no per-unit iceberg variable cost to trade final goods (i.e. no *output tariff*). Even though a firm obtains the same revenues from foreign market as the revenues from home market through charging the same price

$$p_y^X(\varphi_y) = p_y(\varphi_y) = \frac{P_m}{\rho\varphi_y} \tag{18}$$

the export profit $\pi_y^X(\varphi_y)$ still turns out to be lower compared to the domestic profit $\pi_y(\varphi_y)$

$$\pi_y^X(\varphi_y) = \frac{R_y}{\sigma}\left(\frac{P_y\rho\varphi_y}{P_m}\right)^{\sigma-1} - f_X$$

Intermediated good sector. Similarly to Melitz (2003), an intermediate good firm within each economy can serve all foreign producers of final goods, by paying an additional fixed cost $f_X > f$ and facing per-unit iceberg variable trade cost $\tau_m > 1$ (i.e. *input tariff*). For this reason, a firm will set a higher export price compared to the domestic one

$$p_m^X(\varphi_m) = \frac{\tau_m}{\rho\varphi_m} = \tau_m p_m(\varphi_m) \tag{19}$$

yielding lower revenues and profits from the international market compared to the home market

$$\pi_m^X(\varphi_m) = \tau_m^{1-\sigma}\frac{R_m(P_m\rho\varphi_m)^{\sigma-1}}{\sigma} - f_X$$

Equilibrium. As in the closed economy case, any firm will stay in the home market as long as its profit is positive. Again, for each sector $j = y,m$ we can define the survival productivity cutoff $\varphi_j{}^D$ through the *Domestic Zero Profit Condition (D-ZPC)*: $\pi(\varphi_j{}^D) = 0$. Moreover, a firm will serve the foreign market only if the export profit is positive. Therefore, we can also determine the export productivity cutoff $\varphi_j{}^X$ in each sector, i.e. the minimum level of productivity required to serve the international market, through the *Export Zero Profit Condition (X-ZPC)*: $\pi_j^X(\varphi_j{}^X) = 0$.

Finally, by allowing for also the *FEC* (12), where the per-period expected profits of surviving firms in the current context is

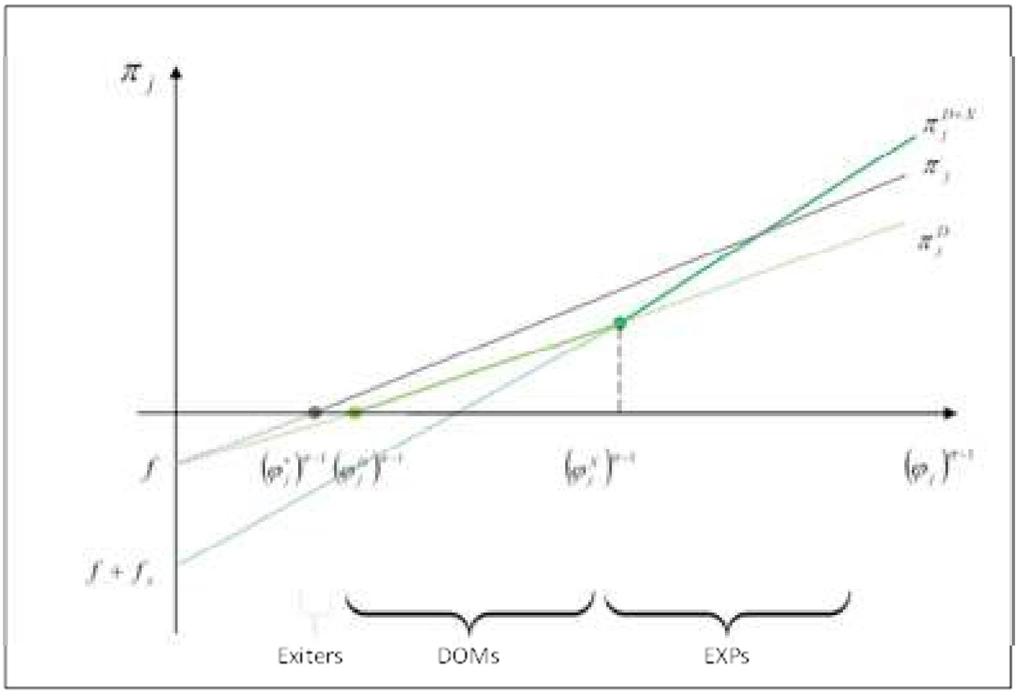
$$\tilde{\pi}_j = \int_{\varphi_j^D}^{\infty} \pi_j(\varphi_j) \frac{g(\varphi_j)}{1 - G(\varphi_j^D)} d\varphi_j + \int_{\varphi_j^X}^{\infty} \pi_j^X(\varphi_j) \frac{g(\varphi_j)}{1 - G(\varphi_j^D)} d\varphi_j \quad .$$

we can highlight the uniqueness of equilibrium $(\varphi_j{}^D, \varphi_j{}^X, \tilde{\pi}_j \text{ for } j = m, y)$. Notice that the export threshold φ_j^X can be written as a function of survival threshold φ_j^D within each differentiated sector:

$$\varphi_m^X = \varphi_m^D \tau_m \left(\frac{f_X}{f} \right)^{\frac{1}{\sigma - 1}}. \qquad \qquad \varphi_y^X = \varphi_y^D \left(\frac{f_X}{f} \right)^{\frac{1}{\sigma - 1}}$$

Thus, $\varphi_j^X > \varphi_j^D$ for $j = m, y$ if trade costs are sufficiently higher than fixed cost of domestic production. From the **Figure 2.2**, we can see as a country opens to trade, the least productive firms will exit the market (i.e. all firms whose productivity φ_j is such that $\varphi_j^* < \varphi_j < \varphi_j^D$ for $j = m, y$), the best ones will also serve the whole market abroad (i.e. all firms whose productivity φ_j is such that $\varphi_j > \varphi_j^X$ for $j = m, y$), and the remaining firms will continue to produce for the home market only (i.e. all firms whose productivity φ_j is such that $\varphi_j^D < \varphi_j < \varphi_j^X$ for $j = m, y$) in both sectors.

Figure 2.2: Open Economy – Indirect Imports case – Sector *j = m,y*



Therefore, some reallocation effects occur across firms within each sector, implying an increase in aggregate productivity. Notice that the aggregate productivity gains within intermediate sector due to reallocation effects entails some uniform efficiency gains within final good firms due to input switching effects.

Indeed, it is worth noting that the price index of intermediates P_m refers to all input varieties available within country – i.e. the set of all domestic intermediate varieties Ω_m and the set of foreign ones Ω_m^X – and is still common amongst all final good firms

$$P_m = \left[\int_{m\in\Omega_m} p_m(m)^{1-\sigma}\,dm + \int_{m\in\Omega_m^X} p_m^X(m)^{1-\sigma}\,dm\right]^{\frac{1}{1-\sigma}} \tag{20}$$

Through plugging (10) and (19) into (20), it is easy to show that this price index turns out to be function of both number and average productivity of all input suppliers competing within country and any change in it will reflect uniformly upon all firms within the final good sector. Similarly, the price index of final goods P_y now refers to all final varieties available within country, i.e. the set of all domestic final varieties Ω_y and the set of foreign ones Ω_y^X

$$P_y = \left[\int_{y \in \Omega_y} p_y(y)^{1-\sigma} dy + \int_{y \in \Omega_y^X} p_y^X(y)^{1-\sigma} dy \right]^{\frac{1}{1-\sigma}} \tag{21}$$

By plugging (18) into (21), it can be written as a function of both number and average productivity of both final good producers and input suppliers competing within country, and any change in it will reflect upon consumer's welfare. Thanks to the equilibrium conditions, both price indexes can be written as functions of survival cutoffs again, i.e. like equations (13) and (14), where closed economy survival cutoffs (φ_j^* for $j = y, m$) are replaced by open economy ones (φ_j^D for $j = y, m$). Therefore, we can notice that both price indexes turn out to be relatively lower in the open economy (given that $\varphi_j^D > \varphi_j^*$ for $j = y, m$), entailing higher firm-level efficiency within final good sector and higher welfare for consumers.

Similarly to the case of closed economy, the mass of both intermediate firms and the mass of final good firms located within a country are given by:

$$M = \frac{R_m}{\sigma\left[f + \psi_m^X f_X\right]\left(\frac{k}{1 + k - \sigma}\right)} \tag{22}$$

$$N = \frac{R_y}{\sigma\left[f + \psi_y^X f_X\right]\left(\frac{k}{1 + k - \sigma}\right)} \tag{23}$$

The mass of input suppliers competing within a country, i.e. the number of intermediate varieties available for final good firms, is $M_T = M + M_X = \left(1 + \psi_m^X\right)M$, where $M_X = \psi_m^X M$ is the mass of intermediate good exporters since $\psi_m^X = \frac{1 - G_m\left(\varphi_m^X\right)}{1 - G_m\left(\varphi_m^D\right)}$ represents the fraction of exporters or the probability of exporting within intermediate good sector. While, the mass of final good firms competing within a country, i.e. the number of varieties available for all final consumers, is $N_T = N + N_X = \left(1 + \psi_y^X\right)N$, where $N_X = \psi_y^X N$ is the mass of final good exporters given that $\psi_y^X = \frac{1 - G_y\left(\varphi_y^X\right)}{1 - G_y\left(\varphi_y^D\right)}$ stands for the portion of exporters or the probability of exporting within final good sector.

Finally, the average marginal cost of final good firms in the current scenario is given by

$$amc_y = \left[\frac{R_m}{\sigma f} \left(\frac{k}{1+k-\sigma} \right) \left(\frac{f + \psi_y^X f_X}{f} \right) \right]^{\frac{1}{1-\sigma}} \frac{1}{\rho \varphi_m^D \varphi_y^D} \tag{24}$$

Through comparing (24) with (17), we can clearly see that it is relatively lower compared to closed economy case. Therefore, the open economy is associated with higher average efficiency within final good sector, due to input switching effects within firm from trade in intermediate inputs, as well as reallocation effects between firms from trade in final goods.

Proposition 3. *When an economy opens to trade, all final good firms are able to replace the worst domestic intermediate inputs with the best ones from abroad, becoming more efficient (**gains from input switching**). Consequently, average efficiency within final good sector as well as overall welfare increase thanks to some homogeneous efficiency gains within final good firms from trade in intermediate inputs, in addition to some aggregate productivity gains from business reallocation across final good firms linked to trade in final goods, as in Melitz (2003).*

More details about 'Indirect imports case' are provided in **Appendix B**.

2.2.2. Case II: Direct imports

Final good sector. A final good firm can still choose to serve the whole foreign market by paying an additional fixed cost $f_X > f$ only, as in the former open economy scenario. In addition, a final good producer can decide to import *directly* foreign intermediates – arising from the most productive foreign intermediate firms (i.e. foreign m-exporters) – to reduce the marginal cost of production, through paying an additional fixed cost of importing f_M, which is assumed to be higher than the fixed cost of exporting f_X.

Therefore, while a firm unable to import keeps similar patterns as in the closed economy case, a firm able to import would exhibit similar patterns as in the former open economy case, so that the resulting firm-level demand for each intermediate input is different between importers and non-importers:

$$x_m^i(m) = \left[\frac{p_m(m)}{P_m^i} \right]^{-\sigma} \frac{q_y^i}{\varphi_y} \quad \text{for } i = D, M \tag{25}$$

where the superscripts D and M denote non-importer and importer status respectively. It is worth noting that P_m^D refers to the price index of all intermediate inputs domestically produced Ω_m and is similar to (6), and P_m^M stands for the price index of all intermediate inputs competing within country $\Omega_m + \Omega_m^X$ and is similar to (20). Moreover, the two price indexes are related to each other and P_m^M turns out to be lower than P_m^D, since the former is associated with a higher number of input varieties as well as a higher average productivity of input suppliers compared to the latter. This can be clearly demonstrated later as the equilibrium is solved, for now we just highlight that $P_m^D = \alpha P_m^M$ where $\alpha > 1$.

The related cost functions are:

$$c_y^D = f + P_m^D \frac{q_y^D}{\varphi_y} \qquad\qquad c_y^M = f + f_M + P_m^M \frac{q_y^M}{\varphi_y}$$

As a consequence, a final good firm able to import charges a relatively lower price in both home and export markets

$$p_y^i(\varphi_y) = p_y^{Xi}(\varphi_y) = \frac{P_m^i}{\rho\varphi_y} \quad \text{for } i = D, M \tag{26}$$

obtaining larger variable profits[11]. In particular, the related profits from domestic market are

$$\pi_y^D(\varphi_y) = \frac{R_y}{\sigma}\left(\frac{P_y \rho \varphi_y}{P_m^D}\right)^{\sigma-1} - f \qquad\qquad \pi_y^M(\varphi_y) = \frac{R_y}{\sigma}\left(\frac{P_y \rho \varphi_y}{P_m^M}\right)^{\sigma-1} - f - f_M$$

while export profits are respectively[12]

$$\pi_y^{Xi}(\varphi_y) = \frac{R_y}{\sigma}\left(\frac{P_y \rho \varphi_y}{P_m^i}\right)^{\sigma-1} - f_X \quad \text{for } i = D, M$$

Intermediate good sector. Differently from the former open economy case, an intermediate good exporter is assumed to serve only a fraction of foreign final good

[11] Unlike in Melitz (2003), the relative variable profit of importer respect to nonimporter is higher than the relative productivity $\left(\frac{\alpha\varphi_y^1}{\varphi_y^2}\right)^{\sigma-1}$, while the relative variable profit of firm respect to another within importer group (or within nonimporter group) is still equivalent to the relative productivity $\left(\frac{\varphi_y^1}{\varphi_y^2}\right)^{\sigma-1}$ as in Melitz (2003).

[12] As in indirect import scenario, despite final good firm-level price is the same across markets, firm-level export profit is still lower than the domestic one because of the additional fixed cost to export.

producers ψ_y^{MX} – i.e. exclusively those final good firms which are productive enough to cover the fixed cost of importing (*direct importers*) – by making additional investments proportional to the share of foreign customers $\psi_y^{MX} f_X$[13] and facing as before a per-unit iceberg *intermediate trade costs* $\tau_m > 1$. By plugging (2) and (26) into (25), we can see that the firm-level demand for each intermediate variety is basically linked to some variables in common across both importers and non-importers. By aggregating it across all final good firms located within country, we obtain the home aggregate demand for each intermediate input:

$$q_m = p_m^{-\sigma} R_m \left(P_m^D\right)^{\sigma-1} \frac{S}{\Delta_y}$$

where $\Delta_y = f + \psi_y^{X+MX} f_X + \psi_y^{MX} f_M$ is the average fixed cost faced by all firms within the final good sector and $S = f + \psi_y^{X+MX} f_X$ is the average fixed cost paid by all final good firms without considering the fixed cost of importing (since $\psi_y^{X+MX} = \frac{1-G_y\left(\varphi_y^X\right)}{1-G_y\left(\varphi_y^D\right)}$ and $\psi_y^{MX} = \frac{1-G_y\left(\varphi_y^{MX}\right)}{1-G_y\left(\varphi_y^D\right)}$ are respectively the portion of all exporters and the portion of import-export firms within the final good sector). It is worth noting that the home aggregate demand for each intermediate variety is decreasing in the fraction of final good importers. Consequently, through charging the profit-maximizing price $p_m(\varphi_m) = \frac{1}{\rho\varphi_m}$, the profit from home market is

$$\pi_m = S\frac{R_m}{\sigma\Delta_y}\left(P_m^D \rho\varphi_m\right)^{\sigma-1} - f$$

In addition, an intermediate exporter will also face the following foreign aggregate demand arising from foreign importers (before input tariff):

[13] The foreign market entry costs are assumed to be increasing in relative foreign market size as in the models of Arkolakis (2010) and Akerman and Forslid (2009): they argue that marketing costs of establishing a new brand would be relatively higher in markets with a higher share of potential buyers. It is worth noting that if all final good firms are able to import directly by paying the fixed cost of importing (i.e. if $\psi_y^{MX} = 1$) implies that intermediate good exporters can serve the whole market abroad, by paying the fixed cost of exporting f_X as in the former scenario.

$$q_m^F = \left[\frac{f_M \psi_y^{MX}}{S\left(\alpha^{\sigma-1}-1\right)}\right] q_m$$

By charging the export price $p_m^X(\varphi_m) = \tau_m p_m(\varphi_m)$, the profit from the foreign market is:

$$\pi_m^X = \tau_m^{1-\sigma} \psi_y^{MX} \frac{f_M}{\left(\alpha^{\sigma-1}-1\right)} \frac{R_m}{\sigma \Delta_y} \left(P_m^D \rho \varphi_m\right)^{\sigma-1} - \psi_y^{MX} f_X$$

Equilibrium. In the current case, final good firms make three decisions: whether to produce for the home market, whether to export their final output and whether to directly import intermediate inputs. Since $f_M > f_X$, all final good importers are assumed to be able to serve international markets, whereas some final good exporters cannot import additional intermediates. A firm will decide to acquire inputs from abroad, only if the related extra-profit from import activity is positive. Thus, in addition to the survival cutoff φ_y^D from *D-ZPC* and the export cutoff φ_y^X from *X-ZPC* as in the former case, we need to define the import-export cutoff φ_y^{MX}, i.e. the minimum level of productivity required to access foreign intermediate inputs, through the *Import-Export Zero Profit Condition (XM-ZPC)*

$$\pi_y^{M+MX}\left(\varphi_y^{MX}\right) - \pi_y^{D+X}\left(\varphi_y^{MX}\right) = 0$$

By considering all these conditions with the *Free Entry Condition* (12) for the final good sector, where the per-period expected profits of surviving firms is now

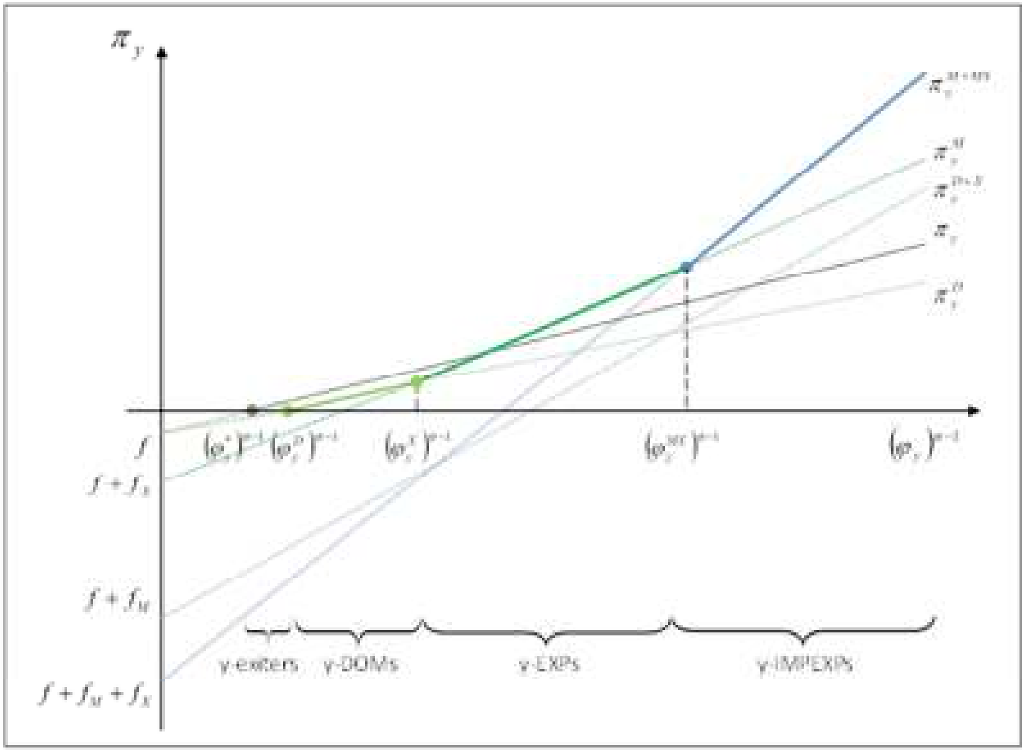
$$\tilde{\pi}_y = \int_{\varphi_y^D}^{\varphi_y^X} \pi_y^D(\varphi_y) \frac{g(\varphi_y)}{1-G(\varphi_y^D)} d\varphi_y + \int_{\varphi_y^X}^{\varphi_y^{MX}} \pi_y^X(\varphi_y) \frac{g(\varphi_y)}{1-G(\varphi_y^D)} d\varphi_y + \int_{\varphi_y^{MX}}^{\infty} \pi_y^{MX}(\varphi_y) \frac{g(\varphi_y)}{1-G(\varphi_y^D)} d\varphi_y$$

we can determine the uniqueness of equilibrium within final good sector – i.e. survival, export, and import-export productivity thresholds, as well as the average profit $\left(\varphi_y{}^D, \varphi_y{}^X, \varphi_y{}^{MX}, \bar{\pi}_y\right)$ – and write the import-export cutoff φ_y^{MX} as a function of export cutoff φ_y^X, which in turn can be expressed as a function of survival cutoff φ_y^D:

$$\varphi_y^{MX} = \left[\frac{f_M}{2 f_X \left(\alpha^{\sigma-1}-1\right)}\right]^{\frac{1}{\sigma-1}} \varphi_y^X \qquad \qquad \varphi_y^X = \left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}} \varphi_y^D$$

Thus, we can notice that $\varphi_y^{MX} > \varphi_y^X > \varphi_y^D$ if and only if the fixed cost of importing is sufficiently higher than the fixed cost of exporting ($f_M > 2 f_X \left(\alpha^{\sigma-1} - 1\right)$)[14], which in turn has to be higher respect to fixed cost of domestic production ($f_X > f$).

Figure 2.3: Open Economy – Direct Imports case – Sector *y*



The **Figure 2.3** shows as trade openness induces the worst firms to leave the market (firms whose productivity φ_y is such that $\varphi_y^* < \varphi_y < \varphi_y^D$) and the best ones to import from the most performing foreign suppliers and to serve all foreign consumers at the same time (i.e. all firms whose productivity φ_y is such that $\varphi_y > \varphi_y^{MX}$). While the remaining firms can be distinguished in two groups: the less productive firms which are able to serve the domestic market only (i.e. all firms whose productivity φ_y is such that $\varphi_y^D < \varphi_y < \varphi_y^X$) and the more productive firms which are also able to export without importing (i.e. all firms whose productivity φ_y is such that $\varphi_y^X < \varphi_y < \varphi_y^{MX}$).

---

[14] After some calculations it becomes simply $f_M > f_X 2^{\frac{k}{\sigma-1}} \tau_m^{-k} \left(\frac{S}{f}\right)^{\frac{\sigma-k-1}{\sigma-1}}$. Therefore, the fixed cost of importing within final good sector should be high enough respect to the fixed cost of exporting. See Appendix C for more details.

Therefore, reallocation effects occur again within the final good sector, implying an increase in aggregate productivity. However, it is worth noting that now only direct importers (import-export firms in our model) can potentially enjoy some efficiency gains from input switching mechanism, whereas the remaining firms (pure domestic firms and only-exporters) basically would suffer some efficiency losses, due mainly to a decrease in intermediate inputs availability.

Intermediate good firms still make two decisions: whether to produce for the home market and whether to serve a portion of a foreign market as well. The uniqueness of equilibrium within sector can be highlighted – i.e. both survival and export productivity thresholds as well as the average profit $\left(\varphi_m{}^D, \varphi_m{}^X, \bar{\pi}_m\right)$ – by allowing for the three conditions similar to the former case (i.e. *D-ZPC, X-ZPC* and *FEC*).

Furthermore, we can notice that φ_m^X can be written as a function of φ_m^D again:

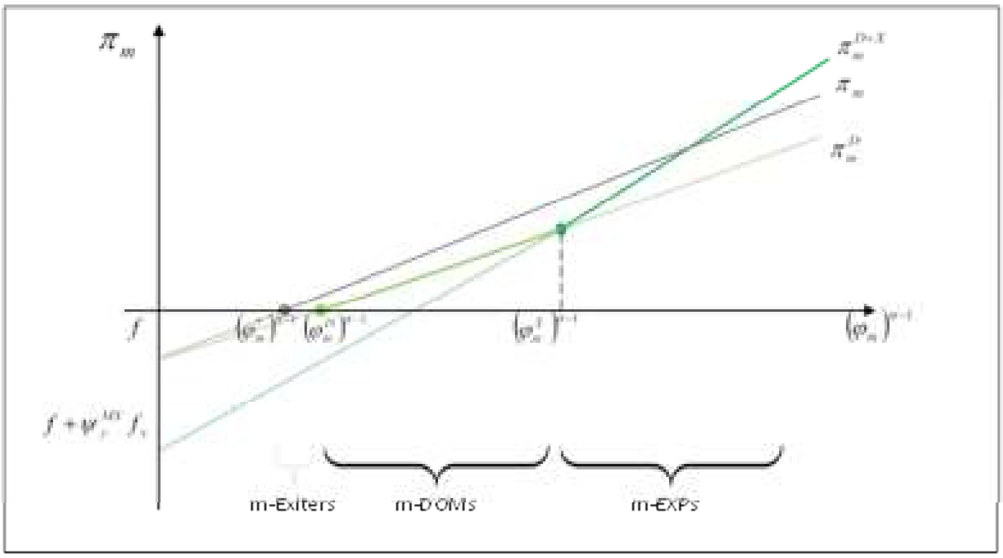
$$\varphi_m^X = \left[\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right]^{\frac{1}{\sigma-1}} \varphi_m^D$$

and realize that $\varphi_m^X > \varphi_m^D$ only if costs of exporting are sufficiently high ($f_X \tau_m^{\sigma-1} > \dfrac{f_M f}{S\left(\alpha^{\sigma-1}-1\right)}$)[15].

Consequently, trade openness leads the least productive intermediate good firms to exit the market (i.e. firms whose productivity φ_m is such that $\varphi_m^* < \varphi_m < \varphi_m^D$), the most productive ones to serve also a fraction of foreign final good producers (i.e. firms whose productivity φ_m is such that $\varphi_m > \varphi_m^X$), and the remaining firms to supply all domestic final good producers only (i.e. firms whose productivity φ_m is such that $\varphi_m^D < \varphi_m < \varphi_m^X$): some reallocation effects occur again within intermediate good sector, entailing an increase in aggregate productivity (see the **Figure 2.4**).

[15] After some calculations, this condition simply becomes $f_X > \dfrac{f_M}{S} f$. Therefore, fixed cost of exporting within intermediate good sector should be high enough respect to the fixed cost of domestic production. See Appendix C for more details.

Figure 2.4: Open Economy – Direct Imports case – Sector *m*



Similarly to the former case, from the average profit level and productivity thresholds in intermediate good sector, the mass of home input suppliers can be derived:

$$M = \frac{R_m}{\sigma\left(\frac{k}{1+k-\sigma}\right)\left[f + \psi_m^X \psi_y^{MX} f_X\right]} \tag{27}$$

which equals the number of intermediate varieties available for final good non-importers, and from *D-ZPC*, the related price index can be written as follows:

$$P_m^D = \left(\frac{R_m}{\sigma f}\frac{S}{\Delta_y}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\varphi_m^D} \tag{28}$$

In contrast, the number of input varieties available for final good importers corresponds to the mass of all input suppliers competing within a country $M_M = M + M_X = \left(1+\psi_m^X\right)M$ and the related price index can be written as follows:

$$P_m^M = \alpha^{-1}P_m^D = \left[1 + \tau_m^{1-\sigma}\left(\frac{f_M f}{S f_X}\right)^{\frac{1+k-\sigma}{k}}\right]^{\frac{1}{1-\sigma}} P_m^D \tag{29}$$

From the average profit level and productivity thresholds in final good sector, the mass of home final good firms can be derived:

$$N = \frac{R_y}{\sigma\left(\frac{k}{1+k-\sigma}\right)\left[f + \psi_y^{X+MX} f_X + \psi_y^{MX} f_M\right]} \tag{30}$$

The mass of final good producers competing within a country is the $N_T = N + N_{X+MX} = \left(1+\psi_y^{X+MX}\right)N$, which in turn equals the number of final good varieties available for all consumers, whose the price index can be written (from *D-ZPC*) as

$$P_y = \left(\frac{R_y}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{P_m^D}{\rho \varphi_y^D} \tag{31}$$

Notice that the price index of final goods is still a function of the marginal cost of the least productive final good firm, which is now related to the price index of intermediate inputs produced domestically P_m^D (and not to the price index of all intermediate inputs available within country as in the former scenario). For this reason, when we move from autarky to open economy, the change in P_y turns out to be ambiguous in the current scenario (given that $P_m^D > P_m$ and $\varphi_y^D > \varphi_y^*$). We are able to show that the impact of trade openness on welfare is certainly positive if firm heterogeneity is relatively high, i.e. if $k < 2(\sigma - 1)$, otherwise it turns out to be ambiguous.

Nevertheless, the 'direct-import' case can still be associated with higher average efficiency within final good sector respect to the autarky case, due to potential input switching effects within firm from trade in intermediate inputs, in addition to reallocation effects between firms from trade in final goods. Indeed, the average marginal cost of final good firms in the current case is given by

$$amc_y = \left[\frac{R_m}{\sigma f}\left(\frac{k}{1+k-\sigma}\right)\left(\frac{f + \psi_y^{X+MX} f_X}{f}\right)\right]^{\frac{1}{1-\sigma}} \frac{1}{\rho \varphi_m^D \varphi_y^D} \tag{32}$$

which is still relatively lower compared to closed economy case (17).

Proposition 4. *When an economy opens to trade, while the most productive final good firms (importers) can potentially enjoy efficiency gains from input switching effects, the other firms (non-importers) suffer efficiency losses due to a decrease in domestic input availability. Therefore, the productivity-enhancing reallocation effects within the final good sector linked to trade in final goods (as in Melitz, 2003) seem to be further boosted through* *these heterogeneous within-firm efficiency effects arising from trade in intermediate inputs. However, while the overall effects on aggregate efficiency within final good sector are positive, the overall effects on welfare seem to be ambiguous: welfare gains (losses) from trade openness would occur if firm heterogeneity is relatively high (low).*

More details about 'Direct imports' case are provided in **Appendix C**.

3. Impact of trade liberalization in intermediate goods

This section aims at studying the impact of trade liberalization in intermediate goods on firm behaviour within both intermediate good sector and final good sector as well as on aggregate efficiency and consumers' welfare, in both cases of Open Economy described in the previous section. The **Appendix D** provides more details.

3.1. Case I: 'Indirect imports'

Following a fall in input tariffs the survival productivity cutoff within intermediate good sector increases $\frac{\partial \varphi_m^D}{\partial \tau_m} < 0$, which is confirmed by a decrease in the fraction of surviving input suppliers (or input supplier's probability of surviving) $\frac{\partial \psi_m^{in}}{\partial \tau_m} > 0$; whereas the export productivity cutoff within the same sector falls $\frac{\partial \varphi_m^X}{\partial \tau_m} > 0$ and the fraction of input suppliers able to export (or input supplier's probability of exporting) increases $\frac{\partial \psi_m^X}{\partial \tau_m} < 0$. Therefore, within intermediate good sector, the least productive firms are forced to exit the home market and more firms can also start exporting, implying some business reallocation effects across firms such that aggregate productivity within sector increases. This aggregate productivity enhancement would entail on average a fall in the price index of intermediates $\frac{\partial P_m}{\partial \tau_m} > 0$, although the change in the total number of intermediates available seems to be ambiguous $\frac{\partial M_T}{\partial \tau_m} = ?$, as in Melitz (2003). Regarding the effects of trade liberalization in intermediate inputs on final good sector, we can notice that only the average marginal cost of final good firms and the price index of final goods decrease $\frac{\partial amc_y}{\partial \tau_m} > 0$; $\frac{\partial P_y}{\partial \tau_m} > 0$.

Proposition 5. *All final good firms uniformly enjoy efficiency gains following input trade liberalization – since they can replace the worst domestic intermediate inputs with better foreign ones (gains from input switching) – which would reflect as whole on consumers' welfare, without any entry-exit of firms within the final good sector.*

3.2. Case II: 'Direct imports'

A reduction in input tariff leads again to an increase in survival productivity cutoff within intermediate good sector $\frac{\partial \varphi_m^D}{\partial \tau_m} < 0$ and a decrease in the fraction of survivors (or input supplier's probability of surviving) $\frac{\partial \psi_m^{in}}{\partial \tau_m} > 0$. Unlike the former case, the export productivity cutoff also increases $\frac{\partial \varphi_m^X}{\partial \tau_m} < 0$, such that the fraction of input suppliers able to export (or input supplier's probability of exporting) remains unchanged $\frac{\partial \psi_m^X}{\partial \tau_m} = 0$, because of a simultaneous increase in foreign market entry cost which is proportional to the share of foreign customers. Namely, a decrease in tariff on intermediate goods would lead to an heterogeneous increase in export variable profit on the one hand, and an homogeneous increase in foreign market entry cost on the other hand within intermediate good sector, such that the least productive exporters would find to serve the foreign market unprofitable[16]. As a result, the least productive firms are forced to exit the home market and the least productive exporters are induced to leave the international market, implying an increase in aggregate productivity within intermediate good sector, due to such reallocation effects towards the more productive firms. In other words, we have two conflicting effects on the final good sector: a decrease in input varieties available and a fall in their average cost.

More specifically, regarding the effects of trade liberalization in intermediates on final good sector, we can notice that both survival and export cutoffs increase $\frac{\partial \varphi_y^D}{\partial \tau_m} < 0$, $\frac{\partial \varphi_y^X}{\partial \tau_m} < 0$;

[16] An increase in variable export profit – due to a common fall in variable trade cost and a common increase in the portion of foreign importers – is heterogeneous across intermediate good firms because is still proportional to firm productivity, whereas the increase in foreign market entry – due to a common increase in share of foreign importers – is homogenous across intermediate good firms.

whereas the import-export cutoff decreases $\frac{\partial \varphi_y^{MX}}{\partial \tau_m} > 0$. Thus, we have two kinds of collateral effect linked to the more intense import activity: the worst final good firms exit the market completely and the least productive exporters leave the international market and decide to focus exclusively on the home market. In fact, we can see clearly that both fractions of survivors and of only-exporters (i.e. both probabilities of surviving and of only-exporting) within final good sector decrease $\frac{\partial \psi_y^{in}}{\partial \tau_m} > 0, \frac{\partial \psi_y^{X}}{\partial \tau_m} > 0$, whereas the fraction of import-export firms (i.e. the probability of importing) increases $\frac{\partial \psi_y^{MX}}{\partial \tau_m} < 0$ [17]. As a consequence, some reallocations effects also take place within final good sector from less productive firms (non-importers) to more productive ones (import-export firms), implying an aggregate productivity improvement, mainly due to some heterogeneous efficiency effects from input trade liberalization. More specifically, the average marginal cost of final good producers located within a country falls $\frac{\partial amc_y}{\partial \tau_m} > 0$, despite non-importer level marginal cost increases $\frac{\partial P_m^D}{\partial \tau_m} < 0$ and the change in importer-level marginal cost becomes ambiguous $\frac{\partial P_m^M}{\partial \tau_m} = ?$. However, we can demonstrate that the input cost gap between importer and non-importer becomes larger, i.e. the relative marginal cost of importer respect to non-importer enhances even if their relative productivity remains constant $\frac{\partial \alpha}{\partial \tau_m} < 0$.

Finally, the impact of input tariff reduction on consumers' welfare (which is inversely related to the price index of final goods P_y) appears to be ambiguous because of some evident losses in final good varieties $\frac{\partial N_T}{\partial \tau_m} > 0$. However, we are able to demonstrate that $\frac{\partial P_y}{\partial \tau_m} > 0$ if firm heterogeneity degree is high enough $k < 2(\sigma - 1)$, and vice versa.

[17] However, the fraction of all exporters (i.e. the probability of exporting) seems to be unaffected $\frac{\partial \psi_y^{X+MX}}{\partial \tau_m} = 0$, since both number of surviving firms and number of all exporters decrease proportionally.

Proposition 6. *Input trade liberalization leads to an increase in final good sector's aggregate efficiency, via both a business reallocation channel (across firms) and an input switching channel (within firm), since while non-importers suffer efficiency losses due to a decrease in domestic input varieties, (new) importers could still gain despite a fall in all input varieties available within country. Overall efficiency gains arising from both reallocation mechanism and input switching effects turn out to be larger than losses from final variety, so that consumers enjoy welfare gains, only if firm heterogeneity (i.e. productivity dispersion within sector) is relatively high, and vice versa.*

4. Conclusion

This paper attempts to study the impact of trade liberalization in intermediate inputs within a general equilibrium framework *à la Melitz (2003),* where all firms are assumed to be heterogeneous in productivity and can produce either intermediate goods or final goods under monopolistic competition.

In particular, our model shows different effects from reducing input tariffs, according to whether all intermediates are assumed to be imported *directly* by final good firms – through incurring additional fixed cost – or *indirectly* – through an efficient wholesale system, without making any further fixed investment.

If foreign intermediates are 'indirectly imported', all final good firms gain uniformly in efficiency from trade liberalization in intermediates, since they are able to substitute the worst domestic inputs with the best foreign ones (*gains from input switching*). These uniform efficiency gains will translate entirely into an increase in consumers' welfare without any entry-exit of firms within final good sector.

If foreign intermediates are 'directly imported', only the most productive firms (importers) will be able to access foreign inputs, and therefore benefit some potential efficiency gains from input trade liberalization. Conversely, the other firms (non-importers) will decline their efficiency, due mainly to a decrease in the availability of domestic inputs. That would force the least productive firms to exit the domestic market and the least productive exporters to leave the international market, causing some market shares reallocation towards the more productive firms (import-export firms), and consequently *aggregate efficiency gains* and some *final variety losses*. Consumer's welfare seems to increase only if firm heterogeneity is relatively high, otherwise it can even decrease. An empirical investigation on main predictions of the current theoretical model is left for future research.

References

Ahn, J., A. K. Khandelwal, and S.-J. Wei (2011). "The Role of Intermediaries in Facilitating Trade." *Journal of International Economics* 84 (1): 73–85.

Akerman, A. (2010). "A Theory on the Role of Wholesalers in International Trade based on Economies of Scope". Research Papers in Economics 2010:1, Stockholm University, Department of Economics.

Akerman, A., and R. Forslid (2009). "Firm Heterogeneity and Country Size Dependent Market Entry Cost". IFN Working Paper 790.

Altomonte, C., and G. Békés (2009). "Trade Complexity and Productivity". Working Papers 2009.62. Fondazione Eni Enrico Mattei.

Amiti, M., and J. Konings (2007). "Trade Liberalization, Intermediate Inputs, and Productivity: Evidence from Indonesia." *American Economic Review* 97 (5): 1611-1638.

Arkolakis, C. (2010). "Market Penetration Costs and the New Consumers Margin in International Trade". *Journal of Political Economy* 118 (6): 1151 - 1199.

Aw, B.Y., S. Chung, and M. J. Roberts (2000). "Productivity and Turnover in the Export Market: Micro-level Evidence from the Republic of Korea and Taiwan (China)". *World Bank Economic Review* 14 (1): 65-90.

Bas, M. (2010). "Trade, Foreign Inputs and Firms' Decisions: Theory and Evidence". CEPII research centre Working Papers 2009-35.

Bas, M., and V. Strauss-Kahn (2011). "Does Importing more Inputs Raise Exports? Firm Level Evidence from France". Working Papers 2011-15, CEPII research center.

Bernard, A. B., and J. B. Jensen (1995). "Exporters, jobs, and wages in U.S. Manufacturing, 1976–1987." Brookings Papers on Economic Activity, Microeconomics, 1995: 67-119.

Bernard, A. B., and J. B. Jensen (1999). "Exporting and Productivity". National Bureau of Economic Research Working Papers 7135.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2010). "Wholesalers and Retailers in US Trade". *American Economic Review* 100 (2): 408-413.

Bernard, A. B., M. Grazzi, and C. Tomasi (2010). "Intermediaries in international trade : Direct versus indirect modes of export". Working Paper Research 199, National Bank of Belgium.

Bernard, A. B., E. J. Blanchard, I. Van Beveren, H. Vandenbussche (2012). "Carry-Along Trade". National Bureau of Economic Research Working Paper 1824.

Blum, B. S., S. Claro, and I. J. Horstmann (2009). "Intermediation and the Nature of Trade Intermediation and the Nature of Trade". University of Toronto. Mimeo.

Brandt, L., J. Van Biesebroeck, L. Wang, and Y. Zhang (2012). "WTO Accession and Performance of Chinese Manufacturing Firms". CEPR Discussion Papers 9166.

Castellani, D., F. Serti, and C. Tomasi (2010). "Firms in International Trade: Importers' and Exporters' Heterogeneity in Italian Manufacturing Industry". *The World Economy* 33 (3): 424-457.

Chevassus-Lozza, E., C. Gaigné, and L.Le Mener (2013). "Does input trade liberalization boost downstream firms' exports? Theory and firm-level evidence". *Journal of International Economics* 90 (2): 391-402.

Corden, M. W. (1971). The Theory of Protection. Oxford: Oxford University Press.

Crozet, M., L. Guy, and S. Poncet (2013). "Wholesalers in international trade". *European Economic Review* 58 (C): 1-17.

Di Comite, F., J.-F. Thisse, and H. Vandenbussche (2013). "Verti-zontal differentiation in export markets". *Journal of International Economics.* Available online 4 February 2014, ISSN 0022-1996, http://dx.doi.org/10.1016/j.jinteco.2014.01.010.

Ethier, W. J. (1982). "National and International Returns to Scale in the Modern Theory of International Trade". *American Economic Review* 72 (3): 389-405.

Felbermayr, G., J., and B. Jung (2011). "Trade Intermediation and the Organization of Exporters". *Review of International Economics* 19 (4): 634-648.

Forslid, R., and G. I. P. Ottaviano (2003). "An analytically solvable core-periphery model". *Journal of Economic Geography* 3 (3): 229-240.

Gibson, M. J., and T. A. Graciano (2011). "The Decision to Import". *American Journal of Agricultural Economics* 93 (2): 444-449.

Greenaway, D., and R. Kneller (2007). "Firm Heterogeneity, Exporting and Foreign Direct Investment: A Survey". *Economic Journal* 117 (517): F134-F161.

Goldberg, P. K., A. K. Khandelwal, N. Pavcnik, and P. Topalova (2010). “Imported Intermediate Inputs and Domestic Product Growth: Evidence from India”. *The Quarterly Journal of Economics* 125 (4): 1727-1767.

Grossman, G. M., and E. Helpman (1991). “Innovation and Growth in the Global Economy”. Cambridge, MA: MIT Press.

Halpern, L., M. Koren, and A. Szeidl (2011). “Imported Inputs and Productivity”. CeFiG Working Papers 8, Center for Firms in the Global Economy.

Helpman, E., M. J. Melitz, and S. R. Yeaple (2004). “Export Versus FDI with Heterogeneous Firms”. *American Economic Review* 94 (1): 300-316.

Kasahara, H., and B. Lapham (2013). “Productivity and the Decision to Import and Export: Theory and Evidence”. *Journal of International Economics* 89 (2): 297–316.

Kasahara, H., and J. Rodrigue (2008). “Does the use of imported intermediates increase productivity? Plant-level evidence”. *Journal of Development Economics* 87 (1): 106-118.

Khandelwal, A. K., and P. Topalova (2011). “Trade Liberalization and Firm Productivity: The Case of India”. *The Review of Economics and Statistics* 93 (3): 995-1009.

Kneller, R., and Z. Yu (2008). “Quality Selection, Chinese Exports and Theories of Heterogeneous Firm Trade”. Discussion Papers 08/44, University of Nottingham, GEP.

Konings, J., and H. Vandenbussche (2005). “Antidumping protection and markups of domestic firms”. *Journal of International Economics* 65 (1): 151-165.

Konings, J., and H. Vandenbussche (2008). “Heterogeneous responses of firms to trade protection”. *Journal of International Economics* 76 (2): 371-383.

Konings, J., and H. Vandenbussche (2013). “Antidumping protection hurts exporters: firm-level evidence”. *Review of World Economics* 149 (2): 295-320.

Krugman, P. (1980). “Scale Economies, Product Differentiation, and the Pattern of Trade”. *American Economic Review* 70 (5): 950-59.

Markusen, J. R. (1989). “Trade in Producer Services and in Other Specialized Intermediate Inputs”. *American Economic Review* 79 (1): 85-95.

Melitz, M. J. (2003). “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity”. *Econometrica* 71 (6): 1695-1725.

Melitz, M. J., and G. I. P. Ottaviano (2008). “Market Size, Trade, and Productivity”. *Review of Economic Studies* 75 (1): 295-316.

Muûls, M., and M. Pisu (2009). “Imports and Exports at the Level of the Firm: Evidence from Belgium”. *The World Economy* 32 (5): 692-734.

Schor, A. (2004). “Heterogeneous productivity response to tariff reduction. Evidence from Brazilian manufacturing firms”. *Journal of Development Economics* 75 (2): 373–396.

Wagner, J. (2007). “Exports and productivity: A survey of the evidence from firm-level data”. *The World Economy* 30 (1): 60–82.

Appendix

A. Closed Economy

A.1. Equilibrium

By considering that the relative variable profits of final good firms are basically equivalent to the relative productivities $\left(\frac{\varphi_y^1}{\varphi_y^2}\right)^{\sigma-1}$ as well as the following conditions within the final good sector:

ZPC: $\pi_y^D\left(\varphi_y^*\right)=0 \Leftrightarrow \frac{R_y}{\sigma}\left(\frac{\rho P_y}{P_m}\right)^{\sigma-1}\left(\varphi_y^*\right)^{\sigma-1}=f$

FEC: $\left[1-G\left(\varphi_y^*\right)\right]\bar{\pi}_y=\delta f_e \Leftrightarrow \left(\varphi_y^*\right)^{-k}\bar{\pi}_y=f_e$

the average profit $\bar{\pi}_y$ can be written as follows

$$\tilde{\pi}_y=\int_{\varphi_y^D}^{\infty}\pi_y\left(\varphi_y\right)\frac{g\left(\varphi_y\right)}{1-G\left(\varphi_y^*\right)}d\varphi=\left(\frac{\sigma-1}{1+k-\sigma}\right)f \qquad \text{with} \quad k>\sigma-1$$

and we can determine the equilibrium survival cutoff φ_y^* and all other variables:

Survival cutoff

$$\varphi_y^*=\left(\frac{\bar{\pi}_y}{\delta f_e}\right)^{\frac{1}{k}}=\left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{f}{\delta f_e}\right]^{\frac{1}{k}}$$

Probability of survival (or portion of y-survivors)

$$\psi_y^{in}=1-G_y\left(\varphi_y^*\right)=\left(\varphi_y^*\right)^{-k}$$

Mass of y-firms at home[18]

$$N=\frac{R_y}{\bar{r}_y}=\frac{R_y}{\sigma f\left(\frac{k}{1+k-\sigma}\right)}$$

[18] By considering that the average profit can be written as the difference between average revenue over elasticity of substitution and average fixed cost, i.e. $\bar{\pi}_y=\frac{\bar{r}_y}{\sigma}-f$, then $N=\frac{R_y}{\bar{r}_y}=\frac{R_y}{\sigma\left(\bar{\pi}_y+f\right)}$.

Price index of final good varieties

$$P_y = \left(\frac{R_y}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{P_m}{\rho \varphi_y^*}$$

Similarly, by bearing in mind that the relative variable profits of intermediate good firms are basically equivalent to the relative productivities $\left(\frac{\varphi_m^1}{\varphi_m^2}\right)^{\sigma-1}$ as well as the following conditions within the intermediate good sector:

ZPC: $\pi_m^D(\varphi_m^*) = 0 \Leftrightarrow \frac{R_m}{\sigma}(\rho P_m)^{\sigma-1}(\varphi_m^*)^{\sigma-1} = f$

FEC: $[1 - G(\varphi_m^*)]\overline{\pi}_m = \delta f_e \Leftrightarrow (\varphi_m^*)^{-k}\,\overline{\pi}_m = \delta f_e$

the average profit $\overline{\pi}_m$ can be written as follows

$$\widetilde{\pi}_m = \int_{\varphi_y^D}^{\infty} \pi_m(\varphi_m) \frac{g(\varphi_y)}{1 - G(\varphi_m^*)} d\varphi = \left(\frac{\sigma - 1}{1 + k - \sigma}\right) f \qquad \text{with} \quad k > \sigma - 1$$

and we are able to highlight the equilibrium survival cutoff φ_m^* and all the other variables:

Survival cutoff

$$\varphi_m^* = \left(\frac{\overline{\pi}_m}{\delta f_e}\right)^{\frac{1}{k}} = \left[\left(\frac{\sigma - 1}{1 + k - \sigma}\right)\frac{f}{\delta f_e}\right]^{\frac{1}{k}}$$

Probability of survival (or portion of m-survivors)

$$\psi_m^{in} = 1 - G_m(\varphi_m^*) = (\varphi_m^*)^{-k}$$

Mass of m-firms at home[19]

$$M = \frac{R_m}{\overline{r}_m} = \frac{R_m}{\sigma f\left(\frac{k}{1 + k - \sigma}\right)}$$

[19] By considering that the average profit can be written as the difference between average revenue over elasticity of substitution and average fixed cost, i.e. $\overline{\pi}_m = \frac{\overline{r}_m}{\sigma} - f$, then $N = \frac{R_m}{\overline{r}_m} = \frac{R_m}{\sigma(\overline{\pi}_m + f)}$.

Price index of intermediate good varieties

$$P_m = \left(\frac{R_m}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\varphi_m^*}$$

As in Meltiz (2003), we focus on the steady state equilibrium so that the mass of new entrants in every period has to replace the mass of exiting firms within each differentiated sector:

$$\left[1 - G\left(\varphi_m^*\right)\right]M_e = \delta M \qquad\qquad \left[1 - G\left(\varphi_y^*\right)\right]N_e = \delta N$$

By combining these two conditions above with respective Free Entry Conditions, we can see clearly that aggregate profits net of entry costs are zero within both differentiated good sectors:

$$\Pi_m = \tilde{\pi}_m M = f_e M_e \qquad (A.1)$$

$$\Pi_y = \tilde{\pi}_y N = f_e N_e \qquad (A.2)$$

Now, the aggregate income ($wL = L$) should reflect the total payments to workers involved in all sectors, through considering that the fixed cost of entry f_e is also labour-intensive. First, the total payments to workers involved in the intermediate differentiated good sector ($wL_m = L_m$) must be equal to the total payments to production workers – given by the difference between aggregate revenue R_m and profit Π_m – and the aggregate entry cost $M_e f_e$: $L_m = (R_m - \Pi_m) + M_e f_e$. This condition reduces to $R_m = L_m$ given the equation (A.1), i.e. the aggregate revenue within intermediate good sector equals the number of workers involved within the same sector. Second, the total payments to workers involved in the final differentiated good sector ($wL_y = L_y$) must be equal to the total payments to production workers – given by aggregate revenue R_y reduced by expenditure in intermediate inputs R_m and profit Π_y – and the aggregate entry cost $N_e f_e$: $L_y = (R_y - R_m - \Pi_y) + N_e f_e$. This condition reduces to $R_y = R_m + L_y = L_m + L_y = \beta L$, given the equation (A.2), i.e. the aggregate revenue within final differentiated good sector equals the number of workers involved within both differentiated good sectors. Finally, the total payments to workers involved in the final homogenous good sector ($wL_h = L_h$) must be equal to the aggregate revenue: $L_h = R_h = (1-\beta)L$, given that $\Pi_h = 0$. Therefore, the total income spent in all final goods corresponds exactly to the number of workers involved in all sectors: $L = L_m + L_y + L_h$.

Notice that while the aggregate revenue within final good sector R_y is clearly exogenous, we are also able to show that the aggregate revenue within intermediate good sector R_m is also exogenous since it is basically equivalent to R_y reduced by the mark-up. Indeed, the aggregate revenues across all intermediate good firms correspond to the aggregate expenditures in all intermediate inputs by all final good firms:

$$R_m = \int_{\varphi_y^*}^{\infty} \frac{P_m}{\varphi_y} q_y \frac{g(\varphi_y)}{1 - G(\varphi_y^*)} N d\varphi$$

Then, by plugging (2), (7), (14) and (16) into the equation above, we can easily end up to $R_m = \rho R_y$.

A.2. Average efficiency within final good sector

The *average efficiency of final good firms* located within a country is inversely related to their average marginal cost amc_y, which in turn corresponds to their average price decreased by the common mark-up:

$$amc_y = \rho \left[\int_{\varphi_y^*}^{\infty} \left[p_y(\varphi_y) \right]^{1-\sigma} \frac{g(\varphi_y)}{1 - G(\varphi_y^*)} d\varphi \right]^{\frac{1}{1-\sigma}} = \left[\int_{\varphi_y^*}^{\infty} \left(\frac{P_m}{\varphi_y} \right)^{1-\sigma} \frac{g(\varphi_y)}{1 - G(\varphi_y^*)} d\varphi \right]^{\frac{1}{1-\sigma}} =$$

$$= P_m \left[\int_{\varphi_y^D}^{\infty} (\varphi_y)^{\sigma-1} \frac{g(\varphi_y)}{1 - G(\varphi_y^*)} d\varphi \right]^{\frac{1}{1-\sigma}} = \frac{P_m}{\varphi_y^*} \left(\frac{k}{1 + k - \sigma} \right)^{\frac{1}{1-\sigma}}$$

By using (13), it can be simply written as (17).

B. Open Economy: Indirect Imports case

B.1. Equilibrium

By considering that the relative variable profits of final good firms are basically equivalent to the relative productivities $\left(\frac{\varphi_y^1}{\varphi_y^2}\right)^{\sigma-1}$ as well as the following conditions within the final good sector:

D-ZPC: $\pi_y^D\left(\varphi_y^D\right)=0 \Leftrightarrow \frac{R_y}{\sigma}\left(\frac{\rho P_y}{P_m}\right)^{\sigma-1}\left(\varphi_y^D\right)^{\sigma-1}=f$

X-ZPC: $\pi_y^X\left(\varphi_y^X\right)=0 \Leftrightarrow \frac{R_y}{\sigma}\left(\frac{\rho P_y}{P_m}\right)^{\sigma-1}\left(\varphi_y^X\right)^{\sigma-1}=f_X$

FEC: $\left[1-G\left(\varphi_y^D\right)\right]\bar{\pi}_y=\delta f_e \Leftrightarrow \left(\varphi_y^D\right)^{-k}\bar{\pi}_y=f_e$

The export cutoff φ_y^X can be written as function of survival cutoff φ_y^D

$$\varphi_y^X=\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}\varphi_y^D$$

and the average profit $\bar{\pi}_y$ can be written as follows

$$\tilde{\pi}_y=\int_{\varphi_y^D}^{\infty}\pi_y\left(\varphi_y\right)\frac{g\left(\varphi_y\right)}{1-G\left(\varphi_y^D\right)}d\varphi+\int_{\varphi_y^X}^{\infty}\pi_y^X\left(\varphi_y\right)\frac{g\left(\varphi_y\right)}{1-G\left(\varphi_y^D\right)}d\varphi=\left(\frac{\sigma-1}{1+k-\sigma}\right)\Delta_y$$

with $k>\sigma-1$

where $\Delta_y=f+\psi_y^X f_X=f+\left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}}f_X$

Therefore, we can highlight the equilibrium survival cutoff φ_y^D and all the other variables:

Survival cutoff

$$\varphi_y^D=\left(\frac{\bar{\pi}_y}{\delta f_e}\right)^{\frac{1}{k}}=\left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{\Delta_y}{\delta f_e}\right]^{\frac{1}{k}}$$

Export cutoff

$$\varphi_y^X = \left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}} \varphi_y^D$$

Probability of survival (or portion of y-survivors)

$$\psi_y^{in} = 1 - G_y\left(\varphi_y^D\right) = \left(\varphi_y^D\right)^{-k}$$

Probability of exporting (or portion of y-exporters)

$$\psi_y^X = \frac{1 - G_y\left(\varphi_y^X\right)}{1 - G_y\left(\varphi_y^D\right)} = \left(\frac{\varphi_y^D}{\varphi_y^X}\right)^{-k} = \left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}}$$

Mass of y-firms located within a country

$$N = \frac{R_y}{\bar{r}_y} = \frac{R_y}{\sigma\Delta_y\left(\frac{k}{1+k-\sigma}\right)}$$

Mass of y-exporters

$$N_X = \psi_y^X N = \left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}} \frac{R_y}{\sigma\Delta_y\left(\frac{k}{1+k-\sigma}\right)}$$

Mass of all y-firms competing within a country (i.e. all y-varieties available within a country)

$$N_T = N + N_X = \left(1+\psi_y^X\right)N = \left[1+\left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}}\right]\frac{R_y}{\sigma\Delta_y\left(\frac{k}{1+k-\sigma}\right)}$$

Price index of final good varieties available within a country

$$P_y = \left(\frac{R_y}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{P_m}{\rho\varphi_y^D}$$

Similarly, by allowing that the relative variable profits of intermediate good firms are basically equivalent to the relative productivities $\left(\frac{\varphi_m^1}{\varphi_m^2}\right)^{\sigma-1}$ as well as the following conditions within the intermediate good sector:

D-ZPC: $\pi_m^D\left(\varphi_m^D\right)=0 \Leftrightarrow \frac{R_m}{\sigma}\left(\rho P_m\right)^{\sigma-1}\left(\varphi_m^D\right)^{\sigma-1}=f$

X-ZPC: $\pi_m^X\left(\varphi_m^X\right)=0 \Leftrightarrow \frac{R_m}{\sigma}\left(\rho P_m\right)^{\sigma-1}\left(\varphi_m^X\right)^{\sigma-1}=\tau_m^{\sigma-1}f_X$

FEC: $\left[1-G\left(\varphi_m^D\right)\right]\overline{\pi}_y=\delta f_e \Leftrightarrow \left(\varphi_m^D\right)^{-k}\overline{\pi}_m=f_e$

The export cutoff φ_m^X can be written as function of survival cutoff φ_m^D

$$\varphi_m^X=\left(\frac{f_X\tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}}\varphi_m^D$$

and the average profit $\overline{\pi}_m$ can be written as follows

$$\tilde{\pi}_m=\int_{\varphi_m^D}^{\infty}\pi_m\left(\varphi_m\right)\frac{g\left(\varphi_m\right)}{1-G\left(\varphi_m^D\right)}d\varphi+\int_{\varphi_m^X}^{\infty}\pi_m^X\left(\varphi_m\right)\frac{g\left(\varphi_m\right)}{1-G\left(\varphi_m^D\right)}d\varphi=\left(\frac{\sigma-1}{1+k-\sigma}\right)\Delta_m \qquad \text{with } k>\sigma-1$$

where $\Delta_m=f+\psi_m^X f_X=f+\left(\frac{f_X\tau_m^{\sigma-1}}{f}\right)^{-\frac{k}{\sigma-1}}f_X$

Therefore, we can highlight the equilibrium survival cutoff φ_m^D and all the other variables:

Survival cutoff

$$\varphi_m^D=\left(\frac{\overline{\pi}_m}{\delta f_e}\right)^{\frac{1}{k}}=\left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{\Delta_m}{\delta f_e}\right]^{\frac{1}{k}}$$

Export cutoff

$$\varphi_m^X=\left(\frac{f_X\tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}}\varphi_m^D$$

Probability of survival (or portion of m-survivors)

$$\psi_m^{in}=1-G_m\left(\varphi_m^D\right)=\left(\varphi_m^D\right)^{-k}$$

Probability of exporting (or portion of m-exporters)

$$\psi_m^X = \frac{1-G_m\left(\varphi_m^X\right)}{1-G_m\left(\varphi_m^D\right)} = \left(\frac{\varphi_m^D}{\varphi_m^X}\right)^{-k} = \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{-\frac{k}{\sigma-1}}$$

Mass of m-firms located at home

$$M = \frac{R_m}{\bar{r}_m} = \frac{R_m}{\sigma\Delta_m\left(\frac{k}{1+k-\sigma}\right)}$$

Mass of m-exporters

$$M_X = \psi_m^X M = \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{-\frac{k}{\sigma-1}} \frac{R_m}{\sigma\Delta_m\left(\frac{k}{1+k-\sigma}\right)}$$

Mass of m-firms competing within a country, i.e. all m-varieties available for y-firms

$$M_T = M + M_X = \left(1+\psi_m^X\right)M = \left[1+\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{-\frac{k}{\sigma-1}}\right] \frac{R_m}{\sigma\Delta_m\left(\frac{k}{1+k-\sigma}\right)}$$

Price index of intermediate good varieties available within a country

$$P_m = \left(\frac{R_m}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\varphi_m^D} \tag{B.1}$$

B.2. Average efficiency within final good sector

The *average efficiency of final good firms* located within a country is inversely related to their average marginal cost amc_y (by considering that some of them serve the foreign market), which in turn corresponds to their average price decreased by the common mark-up:

$$amc_y = \rho\left[\int_{\varphi_y^D}^{\infty}\left[p_y^D\left(\varphi_y\right)\right]^{1-\sigma} \frac{g\left(\varphi_y\right)}{1-G\left(\varphi_y^D\right)} d\varphi + \int_{\varphi_y^X}^{\infty}\left[p_y^X\left(\varphi_y\right)\right]^{1-\sigma} \frac{g\left(\varphi_y\right)}{1-G\left(\varphi_y^D\right)} d\varphi\right]^{\frac{1}{1-\sigma}} =$$

$$=\left[\int_{\varphi_y^D}^{\infty}\left(\frac{P_m}{\varphi_y}\right)^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_y^X}^{\infty}\left(\frac{P_m}{\varphi_y}\right)^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi\right]^{\frac{1}{1-\sigma}}=$$

$$=P_m\left[\int_{\varphi_y^D}^{\infty}\varphi_y{}^{\sigma-1}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_y^X}^{\infty}\varphi_y{}^{\sigma-1}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi\right]^{\frac{1}{1-\sigma}}=$$

$$=P_m\left[\left(\frac{k}{1+k-\sigma}\right)(\varphi_y^D)^{\sigma-1}\frac{\Delta_y}{f}\right]^{\frac{1}{1-\sigma}}=\frac{P_m}{\varphi_y^D}\left[\left(\frac{k}{1+k-\sigma}\right)\frac{\Delta_y}{f}\right]^{\frac{1}{1-\sigma}}=$$

$$=\left(\frac{R_y}{N\sigma f}\right)^{\frac{1}{1-\sigma}}\frac{P_m}{\varphi_y^D}$$

Thus, the average efficiency within final good sector seems to be affected by the input tariff through the common price index of intermediate inputs only (since all the other variables turn out to be independent of input tariff in this scenario). Moreover, by using (B.1) and (23), it can be written as (24).

C. <u>Open Economy: Direct Imports case</u>

C.1. Equilibrium

By considering that the relative variable profit of an importer respect to a non-importer is higher than their relative productivity $\left(\frac{\alpha\varphi_y^1}{\varphi_y^2}\right)^{\sigma-1}$, while the relative variable profit of firm respect to another within importer group (or within nonimporter group) is still equivalent to the relative productivity $\left(\frac{\varphi_y^1}{\varphi_y^2}\right)^{\sigma-1}$, as well as the following conditions within the final good sector:

D-ZPC: $\pi_y^D(\varphi_y^D)=0 \Leftrightarrow \frac{R_y}{\sigma}\left(\frac{\rho P_y}{P_m^D}\right)^{\sigma-1}(\varphi_y^D)^{\sigma-1}=f$

X-ZPC: $\pi_y^X(\varphi_y^X)=0 \Leftrightarrow \frac{R_y}{\sigma}\left(\frac{\rho P_y}{P_m^D}\right)^{\sigma-1}(\varphi_y^X)^{\sigma-1}=f_X$

MX-ZPC:

$$\pi_y^D\left(\varphi_y^{MX}\right)+\pi_y^X\left(\varphi_y^{MX}\right)=\pi_y^M\left(\varphi_y^{MX}\right)+\pi_y^{MX}\left(\varphi_y^{MX}\right)\Leftrightarrow\frac{R_y}{\sigma}\left(\frac{\rho P_y}{P_m^D}\right)^{\sigma-1}\left(\varphi_y^{MX}\right)^{\sigma-1}=\frac{f_M}{2\left(\alpha^{\sigma-1}-1\right)}$$

FEC: $\left[1-G\left(\varphi_y^D\right)\right]\overline{\pi}_y=\delta f_e \Leftrightarrow \left(\varphi_y^D\right)^{-k}\overline{\pi}_y=f_e$

Both export cutoff φ_y^X and import-export cutoff φ_y^{MX} can be written as function of survival cutoff φ_y^D:

$$\varphi_y^X=\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}\varphi_y^D$$

$$\varphi_y^{MX}=\left[\frac{f_M}{2f\left(\alpha^{\sigma-1}-1\right)}\right]^{\frac{1}{\sigma-1}}\varphi_y^D$$

where $\alpha=\frac{P_m^D}{P_m^M}>1$ [20], and the average profit $\widetilde{\pi}_y$ can be written as follows

$$\widetilde{\pi}_y=\int_{\varphi_y^D}^{\varphi_y^X}\pi_y^D\left(\varphi_y\right)\frac{g\left(\varphi_y\right)}{1-G\left(\varphi_y^D\right)}d\varphi+\int_{\varphi_y^X}^{\varphi_y^{MX}}\pi_y^X\left(\varphi_y\right)\frac{g\left(\varphi_y\right)}{1-G\left(\varphi_y^D\right)}d\varphi+\int_{\varphi_y^{MX}}^{\infty}\pi_y^{MX}\left(\varphi_y\right)\frac{g\left(\varphi_y\right)}{1-G\left(\varphi_y^D\right)}d\varphi=\left(\frac{\sigma-1}{1+k-\sigma}\right)\Delta_y$$

with $k>\sigma-1$;

where $\Delta_y=f+\psi_y^{X+MX}f_X+\psi_y^{MX}f_M=f+\left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}}f_X+\left[\frac{f_M}{2f\left(\alpha^{\sigma-1}-1\right)}\right]^{-\frac{k}{\sigma-1}}f_M$

Therefore, we can highlight the equilibrium survival cutoff φ_y^D and write all the other variables as function of the survival cutoff:

Survival cutoff

$$\varphi_y^D=\left(\frac{\overline{\pi}_y}{\delta f_e}\right)^{\frac{1}{k}}=\left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{\Delta_y}{\delta f_e}\right]^{\frac{1}{k}} \tag{C.1}$$

---

[20] It is worth noting that $\varphi_y^{MX}>\varphi_y^X>\varphi_y^D$ only if the fixed cost of importing is sufficiently higher than fixed cost of exporting $f_M>2f_X\left(\alpha^{\sigma-1}-1\right)$.

Export cutoff

$$\varphi_y^X = \left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}} \varphi_y^D$$

Import-Export cutoff

$$\varphi_y^{MX} = \left(\frac{f_M}{2f\left(\alpha^{\sigma-1}-1\right)}\right)^{\frac{1}{\sigma-1}} \varphi_y^D$$

Probability of survival (or portion of y-survivors)

$$\psi_y^{in} = 1 - G_y\left(\varphi_y^D\right) = \left(\varphi_y^D\right)^{-k}$$

Probability of exporting (or portion of y-exporters)

$$\psi_y^X = \frac{1 - G_y\left(\varphi_y^X\right)}{1 - G_y\left(\varphi_y^D\right)} = \left(\frac{\varphi_y^D}{\varphi_y^X}\right)^{-k} = \left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}}$$

Probability of importing (or portion of y-import-export firms)

$$\psi_y^{MX} = \frac{1 - G_y\left(\varphi_y^{MX}\right)}{1 - G_y\left(\varphi_y^D\right)} = \left(\frac{\varphi_y^D}{\varphi_y^{MX}}\right)^{-k} = \left(\frac{f_M}{2f\left(\alpha^{\sigma-1}-1\right)}\right)^{-\frac{k}{\sigma-1}}$$

Probability of exporting only (or portion of only-y-exporters)

$$\psi_y^X = \psi_y^X - \psi_y^{MX} = \left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}} - \left(\frac{f_M}{2f\left(\alpha^{\sigma-1}-1\right)}\right)^{-\frac{k}{\sigma-1}}$$

Mass of y-firms located within a country

$$N = \frac{R_y}{\bar{r}_y} = \frac{R_y}{\sigma\Delta_y\left(\frac{k}{1+k-\sigma}\right)}$$

Mass of y-only-exporters

$$N_X = \psi_y^X N = \left[\left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}} - \left(\frac{f_M}{2f\left(\alpha^{\sigma-1}-1\right)}\right)^{-\frac{k}{\sigma-1}}\right] \frac{R_y}{\sigma\Delta_y\left(\frac{k}{1+k-\sigma}\right)}$$

Mass of import-export y-firms (or importers)

$$N_{MX} = \psi_y^{MX} N = \left(\frac{f_M}{2f\left(\alpha^{\sigma-1} - 1\right)} \right)^{\frac{-k}{\sigma-1}} \frac{R_y}{\sigma \Delta_y \left(\frac{k}{1+k-\sigma} \right)}$$

Mass of y-nonimporters

$$N_D = \left(1 - \psi_y^{MX}\right) N = \left[1 - \left(\frac{f_M}{2f\left(\alpha^{\sigma-1} - 1\right)} \right)^{\frac{-k}{\sigma-1}} \right] \frac{R_y}{\sigma \Delta_y \left(\frac{k}{1+k-\sigma} \right)}$$

Mass of y-exporters

$$N_{X+MX} = \psi_y^{X+MX} N = \left(\frac{f_X}{f} \right)^{\frac{-k}{\sigma-1}} \frac{R_y}{\sigma \Delta_y \left(\frac{k}{1+k-\sigma} \right)}$$

Mass of all y-varieties available within country

$$N_T = N + N_{X+MX} = \left(1 + \psi_y^{X+MX}\right) N = \left[1 + \left(\frac{f_X}{f} \right)^{\frac{-k}{\sigma-1}} \right] \frac{R_y}{\sigma \Delta_y \left(\frac{k}{1+k-\sigma} \right)}$$

Price index of final good varieties available within a country

$$P_y = \left(\frac{R_y}{\sigma f} \right)^{\frac{1}{1-\sigma}} \frac{P_m^D}{\rho \varphi_y^D}$$

Similarly, by bearing in mind that the relative variable profits of intermediate good firms are basically equivalent to the relative productivities $\left(\frac{\varphi_m^1}{\varphi_m^2} \right)^{\sigma-1}$ as well as the following conditions within the intermediate good sector:

D-ZPC: $\pi_m^D\left(\varphi_m^D\right) = 0 \Leftrightarrow \frac{R_m}{\sigma \Delta_y} \left(\rho P_m^D\right)^{\sigma-1} \left(\varphi_m^D\right)^{\sigma-1} = \frac{f}{S}$

X-ZPC: $\pi_m^X\left(\varphi_m^X\right) = 0 \Leftrightarrow \frac{R_m}{\sigma \Delta_y} \left(\rho P_m^D\right)^{\sigma-1} \left(\varphi_m^X\right)^{\sigma-1} = \frac{\tau_m^{\sigma-1} f_X \left(\alpha^{\sigma-1} - 1\right)}{f_M}$

FEC: $\left[1-G\left(\varphi_m^D\right)\right]\overline{\pi}_y = \delta f_e \Leftrightarrow \left(\varphi_m^D\right)^{-k}\overline{\pi}_m = f_e$

The export cutoff φ_m^X can be written as function of survival cutoff φ_m^D

$$\varphi_m^X = \left[\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right]^{\frac{1}{\sigma-1}}\varphi_m^D$$ [21]

and the average profit $\overline{\pi}_m$ can be written as follows

$$\widetilde{\pi}_m = \int_{\varphi_m^D}^{\infty}\pi_m(\varphi_m)\frac{g(\varphi_m)}{1-G\left(\varphi_m^D\right)}d\varphi + \int_{\varphi_m^X}^{\infty}\pi_m^X(\varphi_m)\frac{g(\varphi_m)}{1-G\left(\varphi_m^D\right)}d\varphi = \left(\frac{\sigma-1}{1+k-\sigma}\right)\Delta_m \qquad \text{with } k > \sigma-1$$

where $\Delta_m = f + \psi_m^X\psi_y^{MX} f_X = f + \left[\tau_m^{\sigma-1}\left(\frac{f_X S}{2f^2}\right)\right]^{-\frac{k}{\sigma-1}} f_X$

Therefore, we can highlight the equilibrium survival cutoff φ_m^D and all the other variables:

Survival cutof

$$\varphi_m^D = \left(\frac{\overline{\pi}_m}{\delta f_e}\right)^{\frac{1}{k}} = \left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{\Delta_m}{\delta f_e}\right]^{\frac{1}{k}}$$

Export cutoff

$$\varphi_m^X = \left[\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right]^{\frac{1}{\sigma-1}}\varphi_m^D$$

Probability of survival (or portion of m-survivors)

$$\psi_m^{in} = 1 - G_m\left(\varphi_m^D\right) = \left(\varphi_m^D\right)^{-k}$$

Probability of exporting (or portion of m-exporters)

$$\psi_m^X = \frac{1-G_m\left(\varphi_m^X\right)}{1-G_m\left(\varphi_m^D\right)} = \left(\frac{\varphi_m^D}{\varphi_m^X}\right)^{-k} = \left[\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right]^{-\frac{k}{\sigma-1}}$$

[21] Notice that $\varphi_m^X > \varphi_m^D$ only if costs of exporting are sufficiently high: $f_X\tau_m^{\sigma-1} > \dfrac{f_M f}{S\left(\alpha^{\sigma-1}-1\right)}$.

Mass of m-firms located at home, i.e. all m-varieties available for y-non-importers

$$M = \frac{R_m}{\bar{r}_m} = \frac{R_m}{\sigma\Delta_m\left(\frac{k}{1+k-\sigma}\right)} \tag{C.2}$$

Mass of m-exporters, i.e. additional m-varieties available only for y-importers

$$M_X = \psi_m^X M = \left[\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right]^{-\frac{k}{\sigma-1}} \frac{R_m}{\sigma\Delta_m\left(\frac{k}{1+k-\sigma}\right)}$$

Mass of m-firms competing within a country, i.e. all m-varieties available for y-importers

$$M_M = M + M_X = \left(1+\psi_m^X\right)M = \left[1+\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right]\frac{R_m}{\sigma\Delta_m\left(\frac{k}{1+k-\sigma}\right)}$$

Non-importer's price index of intermediate varieties available[22]

$$P_m^D = \left(\frac{R_y S}{\sigma f \Delta_y}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\varphi_m^D} \tag{C.3}$$

Importer's price index of intermediate varieties available

$$P_m^M = \left[1+\tau_m^{1-\sigma}\left(\frac{f_M f}{S f_X}\right)^{\frac{1+k-\sigma}{k}}\right]^{\frac{1}{1-\sigma}} P_m^D$$

Thus, the *differential between the two price indexes* above is

$$\alpha = \left[1+\tau_m^{1-\sigma}\left(\frac{f_M f}{S f_X}\right)^{\frac{1+k-\sigma}{k}}\right]^{\frac{1}{\sigma-1}} \tag{C.4}$$

given that nonimporters' price index of intermediates is

[22] From domestic zero profit condition within final good sector.

$$\left(P_m^D\right)^{1-\sigma} = \int_{\varphi_m^H}^{\infty} \left[p_m^D(\varphi_m)\right]^{1-\sigma} M \frac{g(\varphi_m)}{1-G(\varphi_m^D)} d\varphi_m = M\rho^{\sigma-1}\left(\frac{k}{1+k-\sigma}\right)\left(\varphi_m^D\right)^{\sigma-1} \tag{C.5}$$

importers' price index of intermediates is

$$\left(P_m^M\right)^{1-\sigma} = \int_{\varphi_m^H}^{\infty} \left[p_m^D(\varphi_m)\right]^{1-\sigma} M \frac{g(\varphi_m)}{1-G(\varphi_m^D)} d\varphi_m + \int_{\varphi_m^X}^{\infty} \left[p_m^X(\varphi_m)\right]^{1-\sigma} M \frac{g(\varphi_m)}{1-G(\varphi_m^D)} d\varphi_m =$$

$$= M\rho^{\sigma-1}\left(\frac{k}{1+k-\sigma}\right)\left(\varphi_m^D\right)^{\sigma-1}\left[1+\left(\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right)^{\frac{\sigma-k-1}{\sigma-1}} \tau_m^{1-\sigma}\right]$$

Therefore,

$$\alpha = \frac{P_m^D}{P_m^M} = \left[1+\left(\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right)^{\frac{\sigma-k-1}{\sigma-1}} \tau_m^{1-\sigma}\right]^{\frac{1}{\sigma-1}} \Leftrightarrow \left(\alpha^{\sigma-1}-1\right) = \left(\tau_m^{\sigma-1}\left(\alpha^{\sigma-1}-1\right)\left(\frac{f_X S}{f_M f}\right)\right)^{\frac{\sigma-k-1}{\sigma-1}} \tau_m^{1-\sigma}$$

$$\Leftrightarrow \left(\alpha^{\sigma-1}-1\right)^{\frac{k}{\sigma-1}} = \left(\frac{f_X S}{f_M f}\right)^{\frac{\sigma-k-1}{\sigma-1}} \tau_m^{-k} \Leftrightarrow \alpha^{\sigma-1}-1 = \tau_m^{1-\sigma}\left(\frac{f_X S}{f_M f}\right)^{\frac{\sigma-k-1}{k}} \Leftrightarrow \alpha = \left[1+\tau_m^{1-\sigma}\left(\frac{f_M f}{S f_X}\right)^{\frac{1+k-\sigma}{k}}\right]^{\frac{1}{\sigma-1}}$$

It is worth noting that $\varphi_y^{MX} > \varphi_y^X > \varphi_y^D$ only if the fixed cost of importing is sufficiently higher than fixed cost of exporting

$$f_M > f_X 2\left(\alpha^{\sigma-1}-1\right) \Leftrightarrow f_M > f_X 2^{\frac{k}{\sigma-1}} \tau_m^{-k}\left(\frac{S}{f}\right)^{\frac{\sigma-k-1}{\sigma-1}}.$$

and $\varphi_m^X > \varphi_m^D$ only if costs of exporting are sufficiently higher than fixed cost of domestic production

$$f_X \tau_m^{\sigma-1} > \frac{f_M f}{S\left(\alpha^{\sigma-1}-1\right)}$$

i.e. if $f_X > \frac{f_M}{S} f$.

Moreover, if we compare (C.3) with (C.5) we can also highlight that the mass of domestic intermediates can be written as

$$
M=\frac{R_m S}{\sigma f \Delta_y\left(\frac{k}{1+k-\sigma}\right)}
$$

By allowing for this equation with (C.2), we can see that Δ_y can be directly related to Δ_m

$$
\Delta_y=\frac{S\Delta_m}{f} \tag{C.6}
$$

C.2. Average efficiency within final good sector and overall welfare

The *average efficiency of final good firms* located within a country is inversely related to their average marginal cost amc_y (by considering that some of them serve the foreign market and some also import), which in turn corresponds to their average price decreased by the common mark-up:

$$
amc_y=\rho\left[\int_{\varphi_y^D}^{\varphi_y^M}\left[p_y^D(\varphi_y)\right]^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_m^M}^{\infty}\left[p_y^M(\varphi_y)\right]^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_y^X}^{\varphi_y^M}\left[p_y^D(\varphi_y)\right]^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_y^M}^{\infty}\left[p_y^M(\varphi_y)\right]^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi\right]^{\frac{1}{1-\sigma}}
$$

$$
=\rho\frac{1}{\rho}\left[\int_{\varphi_y^D}^{\varphi_y^M}\left(\frac{P_m^D}{\varphi_y}\right)^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_m^M}^{\infty}\left(\frac{P_m^M}{\varphi_y}\right)^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_y^X}^{\varphi_y^M}\left(\frac{P_m^D}{\varphi_y}\right)^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_y^M}^{\infty}\left(\frac{P_m^M}{\varphi_y}\right)^{1-\sigma}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi\right]^{\frac{1}{1-\sigma}}
$$

$$
=P_m^D\left[\int_{\varphi_y^D}^{\varphi_y^M}(\varphi_y)^{\sigma-1}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+\int_{\varphi_y^X}^{\varphi_y^M}(\varphi_y)^{\sigma-1}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi+2\alpha^{\sigma-1}\int_{\varphi_y^M}^{\infty}(\varphi_y)^{\sigma-1}\frac{g(\varphi_y)}{1-G(\varphi_y^D)}d\varphi\right]^{\frac{1}{1-\sigma}}
$$

$$
=P_m^D\left[\int_{\varphi_y^D}^{\varphi_y^M}(\varphi_y)^{\sigma-1}\frac{k(\varphi_y)^{-k-1}}{(\varphi_y^D)^{-k}}d\varphi+\int_{\varphi_y^X}^{\varphi_y^M}(\varphi_y)^{\sigma-1}\frac{k(\varphi_y)^{-k-1}}{(\varphi_y^D)^{-k}}d\varphi+2\alpha^{\sigma-1}\int_{\varphi_y^M}^{\infty}(\varphi_y)^{\sigma-1}\frac{k(\varphi_y)^{-k-1}}{(\varphi_y^D)^{-k}}d\varphi\right]^{\frac{1}{1-\sigma}}
$$

$$
=P_m^D\left[\frac{k}{(\varphi_y^D)^{-k}}\left(\int_{\varphi_y^D}^{\varphi_y^M}(\varphi_y)^{\sigma-k-2}d\varphi+\int_{\varphi_y^X}^{\varphi_y^M}(\varphi_y)^{\sigma-k-2}d\varphi+2\alpha^{\sigma-1}\int_{\varphi_y^M}^{\infty}(\varphi_y)^{\sigma-k-2}d\varphi\right)\right]^{\frac{1}{1-\sigma}}
$$

$$
=\frac{P_m^D}{\varphi_y^D}\left[\left(\frac{k}{1+k-\sigma}\right)\frac{\Delta_y}{f}\right]^{\frac{1}{1-\sigma}}=\left(\frac{R_y}{N\sigma f}\right)^{\frac{1}{1-\sigma}}\frac{P_m^D}{\varphi_y^D}
$$

Thus, the average efficiency within final good sector seems to be affected by the input tariff through the price index of domestic intermediate inputs, as well as the number of domestic final good firms and the survival cutoff within final good sector. Moreover, by using (28) and (30), it can be written as (32).

Welfare per worker is inversely related to the price index of final goods (31), which can be written as follows, by using (28):

$$P_y = \left(\frac{R_y}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{P_m^D}{\rho\varphi_y^D} = \left(\frac{R_y}{\sigma f}\right)^{\frac{1}{1-\sigma}} \left(\frac{R_m}{\sigma f}\right)^{\frac{1}{1-\sigma}} \left(\frac{S}{\Delta_y}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho^2 \varphi_y^D \varphi_m^D} \qquad \text{(C.7)}$$

If we compare (C.7) with (14), we can notice that welfare is higher in open economy respect to the closed economy only if

$$\left(\frac{S}{\Delta_y}\right)^{\frac{1}{1-\sigma}} \frac{1}{\varphi_y^D \varphi_m^D} < \frac{1}{\varphi_y^* \varphi_m^*} \Leftrightarrow \left(\frac{\Delta_m}{f}\right)^{\frac{k-2(\sigma-1)}{k(\sigma-1)}} \left(\frac{S}{f}\right)^{-\frac{1}{k}} < 1$$ [23]

and viceversa. Therefore, the impact of trade openness on welfare is certainly positive if firm heterogeneity is relatively high, i.e. if $k < 2(\sigma - 1)$, otherwise it turns out to be ambiguous.

D. Comparative statics

D.1. Case I: Indirect imports

D.1.a. Intermediate good sector

$$\frac{\partial \Delta_m}{\partial \tau_m} = f_X \left(\frac{-k}{\sigma-1}\right) \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}-1} (\sigma-1) \frac{f_X}{f} \tau_m^{\sigma-2} = -k\tau_m^{-1} f_X \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}} < 0$$

$$\frac{\partial \varphi_m^D}{\partial \tau_m} = \left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{1}{\delta f_e}\right]^{\frac{1}{k}} \frac{1}{k} (\Delta_m)^{\frac{1}{k}-1} \frac{\partial \Delta_m}{\partial \tau_m} = \varphi_m^D \frac{1}{k} (\Delta_m)^{-1} \frac{\partial \Delta_m}{\partial \tau_m} =$$

$$= \varphi_m^D \frac{1}{k} (\Delta_m)^{-1} \left(-k\tau_m^{-1} f_X \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}} \right) =$$

$$= -\varphi_m^D (\Delta_m)^{-1} \left(\tau_m^{-1} f_X \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}} \right) < 0$$

---

[23] By considering that equation (C.6) as well as $\varphi_y^D = \varphi_y^* \frac{\Delta_y}{f}$ and $\varphi_m^D = \varphi_m^* \frac{\Delta_m}{f}$.

$$\frac{\partial \varphi_m^X}{\partial \tau_m} = \left(\frac{1}{\sigma-1}\right)\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}-1}(\sigma-1)\frac{f_X}{f}\tau_m^{\sigma-2}\varphi_m^D + \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}}\left(-\varphi_m^D\right)\left(\Delta_m\right)^{-1}\left(\tau_m^{-1} f_X \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}\right) =$$

$$= \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}} \tau_m^{-1}\varphi_m^D + \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}}\left(-\varphi_m^D\right)\left(\Delta_m\right)^{-1}\tau_m^{-1} f_X \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}} =$$

$$= \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}} \tau_m^{-1}\varphi_m^D \left[1 - \frac{f_X \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}}{f + f_X \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}}\right] =$$

$$= \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}} \tau_m^{-1}\varphi_m^D \left[1 - \frac{\Delta_m - f}{\Delta_m}\right] =$$

$$= \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{1}{\sigma-1}} \tau_m^{-1}\varphi_m^D \frac{f}{\Delta_m} > 0$$

$$\frac{\partial \psi_m^{in}}{\partial \tau_m} = -k\left(\varphi_m^D\right)^{-k-1}\frac{\partial \varphi_m^D}{\partial \tau_m} > 0$$

$$\frac{\partial \psi_m^X}{\partial \tau_m} = \frac{-k}{\sigma-1}\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}-1}\frac{f_X}{f}(\sigma-1)\tau_m^{\sigma-2} = -k\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}\tau_m^{-1} < 0$$

$$\frac{\partial M}{\partial \tau_m} = \frac{R_m}{\sigma\left(\frac{k}{1+k-\sigma}\right)}\left(-\Delta_m^{-2}\right)\frac{\partial \Delta_m}{\partial \tau_m} > 0$$

$$\frac{\partial M_X}{\partial \tau_m} = \frac{R_m}{\sigma\left(\frac{k}{1+k-\sigma}\right)}\left[-k\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}\tau_m^{-1}\Delta_m^{-1} + \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}\left(-\Delta_m^{-2}\right)(-k)\tau_m^{-1} f_X\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}\right] =$$

$$= -k\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}\tau_m^{-1}\Delta_m^{-1}\frac{R_m}{\sigma\left(\frac{k}{1+k-\sigma}\right)}\left[1 - \Delta_m^{-1} f_X \left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}\right] =$$

$$= -k\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}}\tau_m^{-1}\Delta_m^{-1}\frac{R_m}{\sigma\left(\frac{k}{1+k-\sigma}\right)}\frac{f}{\Delta_m} < 0$$

$$\frac{\partial M_T}{\partial \tau_m} = ?$$

$$\frac{\partial P_m}{\partial \tau_m} = \left(\frac{R_m}{\sigma f}\right)^{\frac{1}{1-\sigma}}\frac{1}{\rho}(-1)\left(\varphi_m^D\right)^{-2}\frac{\partial \varphi_m^D}{\partial \tau_m} = -P_m\left(\varphi_m^D\right)^{-1}\left[-\varphi_m^D\left(\Delta_m\right)^{-1}\left(\tau_m^{-1}\left(\frac{f_X \tau_m^{\sigma-1}}{f}\right)^{\frac{-k}{\sigma-1}} f_X\right)\right] =$$

$$= P_m \left(\Delta_m\right)^{-1} \left[\tau_m^{-1} \left(\frac{f_X \tau_m^{\sigma-1}}{f} \right)^{\frac{-k}{\sigma-1}} f_X \right] > 0$$

D.1.b. Final good sector

$$\frac{\partial \Delta_y}{\partial \tau_m} = 0$$

$$\frac{\partial \varphi_y^D}{\partial \tau_m} = 0$$

$$\frac{\partial \varphi_y^X}{\partial \tau_m} = 0$$

$$\frac{\partial \psi_y^{in}}{\partial \tau_m} = 0$$

$$\frac{\partial \psi_y^X}{\partial \tau_m} = 0$$

$$\frac{\partial N}{\partial \tau_m} = 0$$

$$\frac{\partial N_X}{\partial \tau_m} = 0$$

$$\frac{\partial N_T}{\partial \tau_m} = 0$$

$$\frac{\partial P_y}{\partial \tau_m} = \left(\frac{R_y}{\sigma f} \right)^{\frac{1}{1-\sigma}} \frac{1}{\rho \varphi_y^D} \frac{\partial P_m}{\partial \tau_m} > 0$$

$$\frac{\partial amc_y}{\partial \tau_m} = \left(\frac{R_y}{N \sigma f} \right)^{\frac{1}{1-\sigma}} \frac{1}{\varphi_y^D} \frac{\partial P_m}{\partial \tau_m} > 0$$

D.2. Case II: Direct imports

D.2.a. Intermediate good sector

$$\frac{\partial \Delta_m}{\partial \tau_m} = -k \tau_m^{-k-1} \left(\frac{f_X S}{2 f^2} \right)^{-\frac{k}{\sigma-1}} f_X < 0$$

$$\frac{\partial \varphi_m^D}{\partial \tau_m} = \left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{1}{\delta f_e}\right]^{\frac{1}{k}} \frac{1}{k}\left(\Delta_m\right)^{\frac{1}{k}-1} \frac{\partial \Delta_m}{\partial \tau_m} = -\varphi_m^D \left(\Delta_m\right)^{-1} \tau_m^{-k-1}\left(\frac{f_X S}{2f^2}\right)^{-\frac{k}{\sigma-1}} f_X < 0$$

$$\frac{\partial \varphi_m^X}{\partial \tau_m} = \left(\frac{f_M f}{S f_X}\right)^{-\frac{1}{k}} \frac{\partial \varphi_m^D}{\partial \tau_m} < 0$$ [24]

$$\frac{\partial \psi_m^{in}}{\partial \tau_m} = -k\left(\varphi_m^D\right)^{-k-1} \frac{\partial \varphi_m^D}{\partial \tau_m} > 0$$

$$\frac{\partial \psi_m^X}{\partial \tau_m} = 0$$ [25]

$$\frac{\partial M}{\partial \tau_m} = \frac{R_m}{\sigma\left(\frac{k}{1+k-\sigma}\right)}\left(-\Delta_m^{-2}\right)\frac{\partial \Delta_m}{\partial \tau_m} > 0$$

$$\frac{\partial M_X}{\partial \tau_m} = \left(\frac{f_M f}{f_X S}\right)\frac{R_m}{\sigma\left(\frac{k}{1+k-\sigma}\right)}\left(-\Delta_m^{-2}\right)\frac{\partial \Delta_m}{\partial \tau_m} > 0$$

$$\frac{\partial M_M}{\partial \tau_m} = \left(1+\frac{f_M f}{f_X S}\right)\frac{R_m}{\sigma\left(\frac{k}{1+k-\sigma}\right)}\left(-\Delta_m^{-2}\right)\frac{\partial \Delta_m}{\partial \tau_m} > 0$$

D.2.b. Intermediate good sector

$$\frac{\partial \Delta_y}{\partial \tau_m} = -k\tau_m^{-k-1}\left[\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right]^{-\frac{k}{\sigma-1}} f_M < 0$$ [26]

$$\frac{\partial \varphi_y^D}{\partial \tau_m} = \left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{1}{\delta f_e}\right]^{\frac{1}{k}} \frac{1}{k}\left(\Delta_y\right)^{\frac{1}{k}-1} \frac{\partial \Delta_y}{\partial \tau_m} = -\varphi_y^D \Delta_y^{-1} \tau_m^{-k-1}\left[\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right]^{-\frac{k}{\sigma-1}} f_M < 0$$

$$\frac{\partial \varphi_y^X}{\partial \tau_m} = \left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}} \frac{\partial \varphi_y^D}{\partial \tau_m} < 0$$

---

[24] By considering (C.4), the export cutoff can be written simply as $\varphi_m^X = \left(\frac{f_M f}{S f_X}\right)^{-\frac{1}{k}} \varphi_m^D$.

[25] By considering (C.4), the probability of exporting can be written simply as $\psi_m^X = \frac{f_M f}{f_X S}$

[26] By considering (C.4): $\Delta_y = f + \left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}} f_X + \left(\tau_m^{\sigma-1}\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right)^{-\frac{k}{\sigma-1}} f_M$.

$$\frac{\partial \varphi_y^{MX}}{\partial \tau_m} = \left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right)^{\frac{1}{\sigma-1}} \left[\varphi_y^D + \tau_m \frac{\partial \varphi_y^D}{\partial \tau_m}\right] =$$

$$= \left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right)^{\frac{1}{\sigma-1}} \left[\varphi_y^D - \tau_m \varphi_y^D \Delta_y^{-1} \tau_m^{-k-1} \left[\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right]^{-\frac{k}{\sigma-1}} f_M\right] =$$

$$= \left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right)^{\frac{1}{\sigma-1}} \varphi_y^D \left[1 - \frac{\Delta_y - f + \left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}} f_X}{\Delta_y}\right] =$$

$$= \left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right)^{\frac{1}{\sigma-1}} \varphi_y^D \left[\frac{f + \left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}} f_X}{\Delta_y}\right] =$$

$$= \left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right)^{\frac{1}{\sigma-1}} \varphi_y^D \frac{S}{\Delta_y} > 0$$ [27]

$$\frac{\partial \psi_y^{in}}{\partial \tau_m} = -k\left(\varphi_y^D\right)^{-k-1} \frac{\partial \varphi_y^D}{\partial \tau_m} > 0$$

$$\frac{\partial \psi_y^{X+XM}}{\partial \tau_m} = 0$$

$$\frac{\partial \psi_y^{MX}}{\partial \tau_m} = -k\left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{\frac{\sigma-k-1}{k}}\right)^{-\frac{k}{\sigma-1}} \tau_m^{-k-1} < 0$$ [28]

---

[27] By considering (C.4), the import-export cutoff can be written simply as $\varphi_y^{MX} = \tau_m \left[\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right]^{\frac{1}{\sigma-1}} \varphi_y^D$.

[28] By considering (C.4), the probability of importing can be written simply as $\psi_y^{MX} = \left[\tau_m^{\sigma-1} \frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{\frac{\sigma-k-1}{k}}\right]^{-\frac{k}{\sigma-1}}$

$$\frac{\partial \psi_y^X}{\partial \tau_m} = k\left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{\frac{\sigma-k-1}{k}}\right)^{-\frac{k}{\sigma-1}} \tau_m^{-k-1} > 0$$ [29]

$$\frac{\partial N}{\partial \tau_m} = \frac{R_y}{\sigma\left(\frac{k}{1+k-\sigma}\right)}\left(-\Delta_y^{-2}\right)\frac{\partial \Delta_y}{\partial \tau_m} = N\Delta_y^{-1} k \tau_m^{-k-1}\left[\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right]^{-\frac{k}{\sigma-1}} f_M > 0$$

$$\frac{\partial N_X}{\partial \tau_m} = \frac{\partial \psi_y^X}{\partial \tau_m} N + \frac{\partial N}{\partial \tau_m}\psi_y^X > 0$$

$$\frac{\partial N_{MX}}{\partial \tau_m} = \frac{\partial \psi_y^{MX}}{\partial \tau_m} N + \frac{\partial N}{\partial \tau_m}\psi_y^{MX} =$$

$$= -k\left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{\frac{\sigma-k-1}{k}}\right)^{-\frac{k}{\sigma-1}} \tau_m^{-k-1} N + N\Delta_y^{-1} k \tau_m^{-k-1}\left[\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{-\frac{\sigma-k-1}{k}}\right]^{-\frac{k}{\sigma-1}} f_M \psi_y^{MX} =$$

$$= k\left(\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{\frac{\sigma-k-1}{k}}\right)^{-\frac{k}{\sigma-1}} \tau_m^{-k-1} N\left[-1 + \frac{f_M \psi_y^{MX}}{\Delta_y}\right] < 0$$

$$\frac{\partial N_{X+MX}}{\partial \tau_m} = \psi_y^{X+MX}\,\frac{\partial N}{\partial \tau_m} > 0$$

$$\frac{\partial N_D}{\partial \tau_m} = \frac{\partial\left(1-\psi_y^{MX}\right)}{\partial \tau_m} N + \frac{\partial N}{\partial \tau_m}\left(1-\psi_y^{MX}\right) > 0$$

$$\frac{\partial N_T}{\partial \tau_m} = \frac{\partial\left(1+\psi_y^{X+MX}\right)}{\partial \tau_m} N + \frac{\partial N}{\partial \tau_m}\left(1+\psi_y^{X+MX}\right) = kN\Delta_y^{-1} f_M C^{\frac{-k}{\sigma-1}} \tau_m^{-1}\left(1 + D^{\frac{-k}{\sigma-1}}\right) > 0$$

$$\frac{\partial P_m^D}{\partial \tau_m} = \left(\frac{R_m}{\sigma}\right)^{\frac{1}{1-\sigma}} \rho^{-1}\left[\left(\frac{\sigma-1}{1+k-\sigma}\right)\frac{1}{\delta f_e}\right]^{-\frac{1}{k}}\left(\frac{1+k-\sigma}{k(\sigma-1)}\right)\Delta_m^{\frac{1+k-\sigma}{k(\sigma-1)}-1}\frac{\partial \Delta_m}{\partial \tau_m} =$$

$$= P_m^D\left(\frac{1+k-\sigma}{k(\sigma-1)}\right)\Delta_m^{-1}\left(-k\tau_m^{-k-1}\left(\frac{f_X S}{2f^2}\right)^{-\frac{k}{\sigma-1}} f_X\right) = -P_m^D\left(\frac{1+k-\sigma}{\sigma-1}\right)\Delta_m^{-1}\left(\tau_m^{-k-1}\left(\frac{f_X S}{2f^2}\right)^{-\frac{k}{\sigma-1}} f_X\right) =$$

---

[29] By considering (C.4), the probability of exporting only can be written simply as $\psi_y^X = \left(\frac{f_X}{f}\right)^{-\frac{k}{\sigma-1}} - \left[\tau_m^{\sigma-1}\frac{f_M}{2f}\left(\frac{f_X S}{f_M f}\right)^{\frac{\sigma-k-1}{k}}\right]^{-\frac{k}{\sigma-1}}$

$$= -P_m^D \tilde{\pi}_m^{-1} \left(\tau_m^{-k-1} \left(\frac{f_X S}{2f^2} \right)^{-\frac{k}{\sigma-1}} f_X \right) < 0$$ [30]

$$\frac{\partial P_y}{\partial \tau_m} = \left(\frac{R_y}{\sigma f} \right)^{\frac{1}{1-\sigma}} \rho^{-1} \left(\frac{\rho^\sigma R_y}{\sigma} \right)^{\frac{1}{1-\sigma}} \left[\left(\frac{\sigma-1}{1+k-\sigma} \right) \frac{1}{\delta f_e} \right]^{-\frac{2}{k}} \left(\frac{S}{f} \right)^{-\frac{1}{k}} \left(\frac{2+k-2\sigma}{k(\sigma-1)} \right) \Delta_m^{\frac{2+k-2\sigma}{k(\sigma-1)}-1} \frac{\partial \Delta_m}{\partial \tau_m} =$$

$$= P_y \left(\frac{2+k-2\sigma}{k(\sigma-1)} \right) \Delta_m^{-1} \frac{\partial \Delta_m}{\partial \tau_m} \Rightarrow$$ [31]

$\Rightarrow \frac{\partial P_y}{\partial \tau_m} > 0$ only iff $k < 2(\sigma-1)$ and viceversa

$$\frac{\partial \alpha}{\partial \tau_m} = \frac{1}{\sigma-1} \left[1 + \tau_m^{1-\sigma} \left(\frac{f_M f}{S f_X} \right)^{\frac{1+k-\sigma}{k}} \right]^{\frac{1}{\sigma-1}-1} \left(\frac{f_M f}{S f_X} \right)^{\frac{1+k-\sigma}{k}} (1-\sigma) \tau_m^{-\sigma} = -\alpha^{2-\sigma} \left(\alpha^{\sigma-1} - 1 \right) \tau_m^{-1} < 0$$

$$\frac{\partial P_m^M}{\partial \tau_m} = ?$$

---

[30] If we plug (C.6) into (2.28): $P_m^D = \left(\frac{R_m}{\sigma} \right)^{\frac{1}{1-\sigma}} \rho^{-1} \left[\left(\frac{\sigma-1}{1+k-\sigma} \right) \frac{1}{\delta f_e} \right]^{-\frac{1}{k}} \Delta_m^{\frac{1+k-\sigma}{k(\sigma-1)}}$.

[31] If we plug (2.28) and (C.1) into (2.31) and use (C.6)(C.6): $P_y = \left(\frac{R_y}{\sigma f} \right)^{\frac{1}{1-\sigma}} \rho^{-2} \left(\frac{R_m}{\sigma} \right)^{\frac{1}{1-\sigma}} \left[\left(\frac{\sigma-1}{1+k-\sigma} \right) \frac{1}{\delta f_e} \right]^{-\frac{2}{k}} \left(\frac{S}{f} \right)^{-\frac{1}{k}} \Delta_m^{\frac{2+k-2\sigma}{k(\sigma-1)}}$.