A Dual Definition for the Factor Content of Trade and its Effect on Factor Rewards in US Manufacturing Sector

Agelos Delis*
University of Nottingham

Theofanis P Mamuneas†
University of Cyprus

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Abstract

In this paper, first we introduce a dual definition of the Factor Content of Trade (*FCT)* using the concept of the equivalent autarky equilibrium. A *FCT* vector is calculated by estimating a symmetric normalized quadratic revenue function for the US manufacturing sector for the period 1965 to 1991. The *FCT* for capital is positive, while the *FCT* for skilled and unskilled labour are both negative, suggesting that the Leontief Paradox was not present for the period of investigation. Following Leamer (1980), capital is revealed by trade to be relatively more abundant compared to either type of labour, while skilled labour is relatively more abundant than unskilled labour. Then using the quadratic approximation lemma, the growth rate of the factor rewards is related to the growth rate of *FCT*, the growth rate of endowments and technological change. We find that technological change is the most important determinant in explaining wage inequality between skilled and unskilled workers in US manufacturing between 1967 to 1991.

Keywords: International trade, relative wages, Factor Content of Trade, skilled and unskilled labour, Leontief Paradox, revenue function.

JEL classification: F11, F16 and J31.

*GEP, School of Economics, University of Nottingham, University Park, Nottingham, NG7 2RD, UK (email: agelos.delis@nottingham.ac.uk)

†Department of Economics, University of Cyprus, Nicosia, CY 1678, Cyprus (email: mamuneas@ucy.ac.cy)

1 Introduction

The possible relationship between international trade and wage inequality in developed countries has been a very important and regularly debated topic for both academics and politicians the last decade. Unskilled workers in many developed countries and especially in US have seen a significant decline in their relative wages, while at the same time international trade increased considerably. Some have argued that the increase of international trade is likely to explain this decline of relative wages. Trade economists have approached this question using the Heckscher-Ohlin model, from two different but equivalent angles. The first is based on the traditional Stolper-Samuelson theorem, where changes in product prices cause changes in factor rewards (Leamer, 1997 and 1994; Harrigan and Balaban, 1999); and the second is based on the Factor Content of Trade (FCT) theorem of Vanek (1968) and the work of Deardorff and Staiger (1988), where changes in the volume of net exports are transformed (via an input-output matrix) into changes in relative factor rewards (Borjas *et al.*, 1992; Katz and Murphy, 1992; Wood, 1995; and Baldwin and Cain, 1997).

The FCT approach has been heavily criticized on the ground that it lacks a solid theoretical foundation and especially that FCT is not related with factor prices. For instance, Panagariya (2000), Leamer and Levinsohn (1995) and Leamer (2000) argue that FCT calculates quantities of indirectly exported and imported factors via international trade but according to the Stolper-Samuelson theorem, it is product prices and not factor quantities, which are related with factor prices. Yet, by introducing the concept of the Equivalent Autarkic Equilibrium (EAE), Deardorff and Staiger (1988) provide the theoretical foundation and show under which assumptions the FCT and relative wages are related (see also, Deardorff, 2000; and Krugman, 2000).

In this paper, we use the concept of EAE to introduce a new dual definition of the FCT to calculate the Factor Content of Net Exports. Our definition of FCT is simply the difference between the endowments at the trade and equivalent autarky equilibria respectively. Then, by using the quadratic approximation lemma (Diewert 1976, 2002) we are able to relate the growth rate of factor rewards of trade equilibria to the growth rate of FCT, the growth rate of endowments and technological change.

In contrast to all previous FCT studies which rely on the use of input-output matrices to calculate the FCT (see Borjas *et al*, 1992; Katz and Murphy, 1992; Wood, 1995; Baldwin and Cain, 1997), we calculate the FCT by directly estimating the endowments required to achieve the EAE.

This is accomplished by estimating a revenue function similar to Harrigan and Balaban (1999) and therefore without relying on the severe restrictions on technology required by the input-output matrices. We find that the FCT for capital is positive, the FCT for skilled labour is negative but quite close to zero, while the FCT of unskilled labour is negative and big in magnitude Hence, there is no Leontief Paradox for the period 1965-1991 We assume the revenue function to be of the Symmetric Normalized Quadratic functional form, discussed in Kohli (1991, 1993) which is more attractive because it has the important property of flexibility when convexity and concavity are imposed. In accordance with most studies of both approaches with the exception of Wood (1995) and Leamer (1997) we also find that technological change is the most important determinant for the decline in relative factor rewards for unskilled workers in the US from 1967 to 1991.

The partition of labour into skilled and unskilled, is consistent with some of the early explanations in the literature about the Leontief Paradox (Kenen, 1965; Baldwin, 1971 and Winston, 1979) and could be a possible explanation for the absence of the Leontief Paradox in our data for the period 1967-1991. The FCT for capital is positive, the FCT for skilled labour is negative but quite close to zero, while the FCT of unskilled labour is negative and big in magnitude[1].

The rest of the paper is organized into four sections. Section 2 develops the theoretical model and provides a dual definition of the factor content of trade. Section 3 contains a discussion of the empirical specification and estimation of *FCT*. Section 4 provides a decomposition of the changes of factor rewards using the quadratic approximation lemma. The last section of the paper contains the conclusions.

[1]This result is similar to the one found by Bowen et al (1987), with the only difference the sign of FCT for professional/technical workers. Possible explanations for this could be the use of an input-output matrix by Bowen et al (1987) to calculate the FCT, the different categories of labour used, the inclusion of other inputs and the implicit assumption of non-jointness in output quantities.

2 The Model

In this section we develop a general equilibrium model for a trading economy using duality. The production side of the economy is described by a revenue function while the consumption side by an expenditure function. The use of duality, and more specifically the implementation of a revenue function, is preferred because it complies with the standard assumptions made in international trade theory that product prices and endowments are given while factor prices and outputs are the endogenous variables to be determined.

Let $F(y, v, t) = 0$ be a transformation function for an economy with a linearly homogeneous technology, which produces $y = (y_1, ...y_n)$ goods with the use of $v = (v_1, ...v_m)$ inputs $(n \leq m)$ in a perfect competitive environment where t is a time index that captures technological change. Then, at given international prices $p = (p_1, ...p_n)$ and domestic inputs v, there exists a competitive production equilibrium. In such equilibrium we can think of the economy as one that maximizes the value of total output subject to the technological and endowment constraints. In other words there is a revenue or Gross Domestic Product (GDP) function such that:

$$R(p, v, t) = \max_y \{py : F(y, v, t) = 0\} \tag{1}$$

The revenue function has the usual properties, i.e., it is increasing, linearly homogeneous and concave in v and non-decreasing, linearly homogeneous and convex in p. In addition if $R(p, v, t)$ is differentiable then from Hotteling's Lemma (Diewert 1974) the equilibrium output and factor rewards are:

$$y(p, v, t) = R_p(p, v, t) \tag{2}$$

$$w(p, v, t) = R_v(p, v, t) \tag{3}$$

where R_p and R_v are the vectors of first partial derivative of the revenue function with respect to product prices and endowments, respectively.

On the consumption side the economy's preferences defined over the n goods are represented by an expenditure function, which is continuous and twice differentiable on prices:

$$E(p, u) = \min_x \{px : u(x) \geq u\} \tag{4}$$

where u is the level of utility and $x = (x_1, ...x_n)$ is the consumption bundle. The expenditure function is non-decreasing, linear homogenous and concave in prices and increasing in u. From Shepherd's Lemma (Diewert 1974) the consumption vector of the economy is:

$$x(p, u) = E_p(p, u) \tag{5}$$

where E_p is the vector of first partial derivative of the expenditure function with respect to product prices.

The trade equilibrium is defined as

$$R(p, v, t) = E(p, u) \tag{6a}$$

$$T = R_p(p, v, t) - E_p(p, u) \tag{6b}$$

that is the total value of production should be equal to the total expenditure for the economy, which implies trade balance and the difference between production and consumption gives the economy's vector of net exports, T.

Consider now a hypothetical equilibrium, the equivalent autarky equilibrium introduced by Deardorff and Staiger (1988), where production equals consumption, at the same product prices and at the same utility level as in the trading equilibrium. This equilibrium can be achieved by changing the initial endowment of the economy such that the economy is producing what it desires to consume, having no incentive to trade with other countries. Hence, the vector of net exports is going to be a vector of zeros and trade is by definition balanced

$$R(p, v^e, t) = E(p, u) \tag{7a}$$

$$R_p(p, v^e, t) = E_p(p, u) \tag{7b}$$

where v^e is the equivalent autarky equilibrium endowments vector and p, u the price vector and utility level respectively as in the trade equilibrium.

In Figure 1, following Krugman (2000), we depict the trading and equivalent autarky equilibria. In the Trade Equilibrium, the economy is producing where the production possibilities frontier DE is tangent to the relative product prices line AB, at P, while the economy is consuming at C where the relative product prices line is tangent to the indifference curve u. The economy is exporting $y_1 - x_1$ units of good 1 and imports $x_2 - y_2$ units of good 2. The equivalent autarky equilibrium is depicted at C. There, the economy is endowed with the necessary inputs that allow the production of its consumption bundle at the trade relative product prices AB. At the EAE, the production possibilities frontier is FG, and both consumption and production takes place at C and therefore the trade volume is zero. Note that at the trading equilibrium P and at the EAE C preferences are the same and because product prices are also unchanged the vector of consumption is unaltered. Under the assumption of balanced trade, GDP and the economy's total expenditure would be identical in both equilibria.

Since consumption is the same in both equilibria then from (6b) and (7b) we have

$$R_p(p, v^e, t) = R_p(p, v, t) - T \tag{8}$$

and therefore we can explicitly solve from (8) for the EAE endowments vector v^e by knowing the net exports and the revenue function of the economy. Assuming that the implicit function theorem holds, $|R_{pv}(p, v^e, t)| \neq 0$[2], we can solve for the EAE endowment vector $v^e(p, v, t; T)$ which is going to depend on the trade equilibrium prices, initial endowment, technology and the net export vector. Then, the factor content of trade is defined as the difference between the actual endowments in a trading equilibrium and the endowments at the equivalent autarky equilibrium,

$$f = v - v^e(p, v, t; T) \tag{9}$$

In the literature, the usual definition of FCT is just the product of an input requirement matrix, Γ, times the trade vector T (see for example Deardorff and Staiger, 1988). Harrigan (2001) has shown that if there is non-jointness in output quantities, the input requirement matrix Γ is equal to R_{pv}^{-1} and therefore the factor content of trade will be equal to $R_{pv}^{-1}T$. It is not difficult to show that our definition of FCT is identical to $R_{pv}^{-1}T$ under the non-jointness assumption. Under this assumption a revenue function can be written as $R(p, v, t) = r(p, t)v$, then the vector of outputs is $R_p = r_p v$, where r_p is the vector of partial derivatives of $r(p, t)$ with respect to product prices and $R_{pv} = r_p$ which is independent of the endowment vector. From (8) we have that $T = R_p(p, v, t) - R_p(p, v^e, t) =$

[2] The determinant of matrix R_{pv} is different from zero, where R_{pv} is the matrix of the second partial derivatives of the revenue function with respect to product prices and endowments.

$r_p v - r_p v^e = r_p(v - v^e) = R_{pv} f$, and therefore $f = R_{pv}^{-1} T$.

Consider now the more general case, and note that from the linear homogeneity of the revenue function in v we have

$$R_{pv} v = R_p(p, v, t) \tag{10}$$

at the trade equilibrium. Substituting (10) in (6b) we have

$$R_{pv} v = T + E_p(p, u) \tag{11}$$

Assuming that $|R_{pv}| \neq 0$ and that the R_{pv} is locally independent of v, we can solve (11) for the vector of endowments which supports the trade equilibrium, i.e.,

$$v = R_{pv}^{-1} T + R_{pv}^{-1} E_p(p, u) \tag{12}$$

Similarly, using the linear homogeneity property of the revenue function with respect to endowments vector at the EAE and (7b) we have

$$R_{pv} v^e = R_p(p, v^e, t) = E_p(p, u)$$

and the EAE endowment vector will be given by

$$v^e = R_{pv}^{-1} E_p(p, u) \tag{13}$$

Substituting (12) and (13) in (9) we have

$$\begin{aligned} f &= R_{pv}^{-1} T + R_{pv}^{-1} E_p(p, u) - R_{pv}^{-1} E_p(p, u) \\ &= R_{pv}^{-1} T \end{aligned} \tag{14}$$

Equation (14) shows that our definition of FCT given by (9) is equivalent to the usual definition appearing in the literature under the above assumptions. Our definition is however a generalization to wider technologies even in cases where jointness in output quantity is present.

The factor content of trade, f, can be easily computed from (9) if we know the equivalent autarky equilibrium endowments v^e. This in turn can be estimated if we have knowledge of the revenue function and net trade vector of the economy. In the next section we specify a revenue function in order to estimate the factor content of trade for capital, skilled labour and unskilled labour in the US for the period 1965 to 1991.

3 Econometric Specification and Estimation

The revenue function is assumed to have the symmetric normalized quadratic functional form as discussed in Kohli (1991, 1993):

$$
\begin{aligned}
R(p,v,t) = {} & \frac{1}{2}\left(\sum_{j=1}^{M}\psi_j v_j\right)\left(\sum_{i=1}^{N}\sum_{h=1}^{N} a_{ih}p_i p_h\right)\left(\sum_{i=1}^{N}\theta_i p_i\right)^{-1} \\
& + \sum_{i=1}^{N}\sum_{j=1}^{M}(c_{ij}p_i v_j) + \frac{1}{2}\left(\sum_{i=1}^{N}\theta_i p_i\right)\left(\sum_{j=1}^{M}\sum_{k=1}^{M} b_{jk}v_j v_k\right)\left(\sum_{j=1}^{M}\psi_j v_j\right)^{-1} \\
& + \left(\sum_{j=1}^{M}\psi_j v_j\right)\left(\sum_{i=1}^{N} d_i p_i\right)t + \left(\sum_{i=1}^{N}\theta_i p_i\right)\left(\sum_{j=1}^{M} e_j v_j\right)t \\
& + \left(\sum_{i=1}^{N}\theta_i p_i\right)\left(\sum_{j=1}^{M}\psi_j v_j\right)\left(\frac{1}{2}h_{tt}t^2 + h_t t\right)
\end{aligned}
\tag{15}
$$

where p, and v, are the product prices and input endowment vectors respectively and t is an index of exogenous technological change. There are $N(N-1) + M(M-1) + (N \times M) + 2$ unknown parameters a_{ih}, b_{jk}, c_{ij}, d_i, e_j, h_t and h_{tt}, where $i, h = 1, ...N$ and $j, k = 1, ..M$. There are also $N + M$ predetermined parameters θ_i and ψ_j. In particular, θ_i and ψ_j are set equal to the share value of each product and input respectively at the base year. Symmetry conditions are imposed $a_{ih} = a_{hi}$; $b_{jk} = b_{kj}$ and the assumptions of linear homogeneity in p and v require some additional restrictions:

$$
\sum_{i=1}^{N}\theta_i = \sum_{j=1}^{M}\psi_j = 1, \text{ and } \sum_{h=1}^{N} a_h = \sum_{k=1}^{M} b_{jk} = \sum_{i=1}^{N} d_i = \sum_{j=1}^{M} e_j = 0 \tag{16}
$$

This functional form is attractive because it is a flexible functional form that retains its flexibility under the imposition of convexity and concavity in prices and endowments respectively. The necessary and sufficient condition for global concavity in inputs is that the matrix $B = [b_{jk}]$ is negative semi-definite and for global convexity that the matrix $A = [a_{ih}]$ is positive semi-definite. If these are not satisfied then they are imposed following Diewert and Wales (1987) without removing the flexibility properties of the revenue function.

Based on (15) the reward of the jth factor becomes:

$$\begin{aligned}
w_j &= \frac{1}{2}\psi_j \left(\sum_{i=1}^{N}\sum_{h=1}^{N}\alpha_{ih}p_ip_h\right)\left(\sum_{i=1}^{N}\theta_ip_i\right)^{-1} + \left(\sum_{i=1}^{N}\theta_ip_i\right)\left(\sum_{k=1}^{M}b_{jk}v_k\right)\left(\sum_{j=1}^{M}\psi_jv_j\right)^{-1} \\
&\quad -\frac{1}{2}\psi_j\left(\sum_{i=1}^{N}\theta_ip_i\right)\left(\sum_{j=1}^{M}\sum_{k=1}^{M}b_{jk}v_jv_k\right)\left(\sum_{j=1}^{M}\psi_jv_j\right)^{-2} + \sum_{i=1}^{N}c_{ij}p_i + \psi_j\left(\sum_{i=1}^{N}d_ip_i\right)t \\
&\quad +e_j\left(\sum_{i=1}^{N}\theta_ip_i\right)t + \psi_j\left(\sum_{i=1}^{N}\theta_ip_i\right)h_tt + \frac{1}{2}\psi_j\left(\sum_{i=1}^{N}\theta_ip_i\right)h_{tt}t^2 \qquad (17)
\end{aligned}$$

Similarly the output supply of good ith becomes:

$$\begin{aligned}
y_i &= \frac{1}{2}\theta_i\left(\sum_{j=1}^{M}\sum_{k=1}^{M}b_{jk}v_jv_k\right)\left(\sum_{j=1}^{M}\psi_jv_j\right)^{-1} + \left(\sum_{j=1}^{M}\psi_jv_j\right)\left(\sum_{h=1}^{N}\alpha_{ih}p_h\right)\left(\sum_{i=1}^{N}\theta_ip_i\right)^{-1} \\
&\quad -\frac{1}{2}\theta_i\left(\sum_{j=1}^{M}\psi_jv_j\right)\left(\sum_{i=1}^{N}\sum_{h=1}^{N}\alpha_{ih}p_ip_h\right)\left(\sum_{i=1}^{N}\theta_ip_i\right)^{-2} + \sum_{j=1}^{M}c_{ij}v_j + d_i\left(\sum_{j=1}^{M}\psi_jv_j\right)t \\
&\quad +\theta_i\left(\sum_{j=1}^{M}e_jv_j\right)t + \theta_i\left(\sum_{j=1}^{M}\psi_jv_j\right)h_tt + \frac{1}{2}\theta_i\left(\sum_{j=1}^{M}\psi_jv_j\right)h_{tt}t^2 \qquad (18)
\end{aligned}$$

The estimating model is the equation sets (17) and (18) together with the parameter restrictions (16). The errors related to equations (17) and (18) are assumed to be identically, and independently distributed with zero expected value and a positive definite covariance matrix. These equations are jointly estimated by the iterative three stages least square estimator applied to data for the US manufacturing sector over the period from 1965 to 1991. There are six equations, three relating to outputs and three relating to factor rewards. The goods are exportables, importables and non-tradeable and the three factors of production are capital, skilled and unskilled labor. In appendix A we provide a detailed construction and sources of the data.

Table 2 shows the estimated parameters and the R^2 for the system of the six equations. The revenue function is linearly homogeneous in prices and inputs, but initially convexity in prices and concavity in inputs were not satisfied. Following the method proposed by Diewert and Wales (1987) we impose convexity for product prices and concavity for input quantities. The hypothesis of convexity and concavity cannot be rejected at a 5% level of significance (Wald test statistic(4)=32.7). The joint null hypothesis of non-jointness in output quantities is rejected at a 5% level of significance

(Wald test statistic(2)=29.1), which is in accordance with the more general technology used above. In addition, the hypothesis of non technological change is rejected (Wald test statistic(2)=98).

The price elasticities of output supply (ε_{ih}) are presented in Table 3. All own price elasticities are well below unity, suggesting that the output supplies are inelastic. An increase in the price of exportables reduces the quantity of both importable and non-tradable goods. While an increase of the price of importables increases the output of non-tradable goods. The quantity elasticities of inverse input demand $\left(\varepsilon_{w_j v_k}\right)$ are reported in Table 4. Capital is the most elastic input compared to skilled and unskilled labor. Capital is a gross-substitute with skilled and unskilled labor while skilled and unskilled labor are gross-complements.

In Tables 5 and 6 we present the quantity and technological change elasticities of inverse input demand $\left(\varepsilon^e_{w_j v_k}, \varepsilon^e_{w_j t}\right)$ and the price elasticities of inverse input demand $\left(\varepsilon^e_{w_j p_i}\right)$ in the Equivalent Autarky Equilibrium. We see that the quantity elasticities of inverse input demand have the same signs as in the Trade Equilibrium, but are much smaller in magnitude suggesting that factor demands are less elastic in the hypothetical EAE. Regarding the technological change elasticities of inverse input demand, capital's and skilled labour's rewards gain from technological change, but unskilled labour is hurt. From the price elasticities of inverse input demand we see that an increase in the price of the exportable raises the factor reward to the capital and unskilled labour and reduce the factor reward to the skilled labour. While a rise in the price of the importable and non-tradables lead to higher rewards for capital and skilled labour and a lower reward for unskilled labour.

The estimated parameters of the revenue function are used in order to calculate the FCT for each input. In particular, solving equation (8) for v^e and then using equation (9), allow us to obtain the factor content of trade, f_j, for each input for the period 1965 to 1991. The FCT for all three factors are plotted in Figure 2. We observe that FCT of capital, f_K, was positive and generally increasing throughout our sample period. The FCT of both skilled, f_S, and unskilled, f_U, labor was negative and declining till 1986 and then increased till 1991, with the FCT of skilled labour having a relatively smaller magnitude. Hence, the US economy was exporting the services of capital and importing the services of both types of labour for all the years between 1965 to 1991. The net exports of capital services in 1965 were 16.34 billion USD[3], reached a maximum of 62 billion USD in 1986 and fell to 54.30 billion USD in 1991. While the net imports of skilled labour services rose

[3] All net trade services of factors are measured in prices of the year 1970 and is assumed that the economy is in a balaced trade equilibrium (see more in the Appendix A).

from 9.89 billion USD in the first year of the period to 44.04 billion in 1986 and then were reduced to 32.50 billion USD in 1991. Similarly, the net imports of unskilled labour increased from 20.45 billion in 1965 to 96.88 billion in 1986 and then decreased to 68.48 billion USD in the last year of the sample.

It is evident that for the period 1965-1991 in our analysis there is no Leontief Paradox in the US economy, since the FCT for capital is positive. Our result is consistent with the analysis of Leamer (1980), because the FCT that we calculate is by definition the factor content of net trade. Leamer also showed that in a multi-factor, multi-product H-O-V environment, a country is revealed by trade to be relatively abundant in a particular factor compared to any other factor, if the FCT of this factor is positive and the FCT of the other is negative. Hence, capital is revealed by trade to be relatively abundant compared to either type of labour in the US economy for the period 1965-1991. In addition, Leamer (1980) showed that within the same setup, a country with any pair of factors j and k with positive (negative) FCT is revealed by trade to be relatively abundant in factor j compared to factor k, if the ratio of the FCT of factor j to the FCT of factor k is greater (smaller) than the ratio of factor j to factor k in the production. and with positive (negative) FC of net exports for both capital and labour is revealed by trade to be relatively capital abundant, if and only if the ratio of the FC of net exports for capital to the FC of net exports for labour is greater (smaller) than the ratio of capital to labour in the production. In our case, we find that the share of skilled labour imported is less than the share of unskilled labour imported and trade reveals that skilled labour is relatively abundant to unskilled labour in the US economy between 1965 to 1991.

For all of the years in the sample period more unskilled and skilled labor would have been employed in a hypothetical EAE relative to capital, but more unskilled labor would have been employed relative to skilled labor. Therefore in the US manufacturing sector there is a clear ordering of factor abundance revealed by trade. Capital is the most abundant factor relative to both types of labour, while skilled labour is relatively more abundant when compared with unskilled labour between 1965 to 1991.

4 Factor Rewards Decomposition

So far we have discussed the definition of the Equivalent Autarky Equilibrium, the calculation of the FCT using duality in the case of jointness in output quantities and the estimation of the revenue function for US. In this section our goal is to establish a general relationship between changes in factor prices in one side and changes of endowments, FCT and technology in the other. But first we show how the difference between the factor rewards in the two equilibria can be approximated. We have information on the factor rewards at the Trade Equilibrium and we need to calculate the factor rewards in the EAE in order to obtain the difference between the two of them. With the use of EAE and equation (8) we are able to calculate the factor rewards that would have been obtained assuming the economy was endowed with the appropriate allocation of inputs to produce its consumption bundle at the observed product prices. The difference in factor rewards between a trade equilibrium and EAE for time period s can be approximated by using the quadratic approximation lemma (Diewert, 1976, 2002) that is

$$w^s - w^{es} = \overline{R^s_{vv}}\left(v^s - v^{es}\right) = \overline{R^s_{vv}} f^s \tag{19}$$

where matrix $\overline{R^s_{vv}} = \frac{1}{2}(R^s_{vv} + R^{es}_{vv})$ has a typical entry $\overline{r^s_{v_j v_k}}$ that is the mean effect of a change in the kth endowment on the reward of the jth factor evaluated at the trade and equivalent autarky equilibrium at period s[4]. Then we evaluate (19) at two different time periods t and s ($t \geq s$), take their difference and after rearranging we get:

$$w^t - w^s = \overline{R^t_{vv}} f^t - \overline{R^s_{vv}} f^s + w^{et} - w^{es} \tag{20}$$

an expression that relates the change of factor rewards $\left(w^t - w^s\right)$ to the difference of the FCT $\left(\overline{R^t_{vv}} f^t - \overline{R^s_{vv}} f^s\right)$ and the change of the factor rewards at the EAE $\left(w^{et} - w^{es}\right)$.

Similarly, we know that the factor reward at the EAE at time period t is $w^{et} = R^t_v\left(p, v^e, t\right)$. We also know that in the EAE product prices are not exogenous and depend on endowments and technological change[5] and that the equilibrium product price (p) is a function of EAE endowments (v^e)

[4] Notice that when there is non-jointness in output quantities, $R_{vv} = 0$, and therefore $w^s = w^{es}$.

[5] Leamer (1997) states the importance of interdependence between product prices and technical change. He seperates the changes of product prices into two components. The first arising from international markets (globalisation) and the second from technological change. Although such a decomposition seems reasonable, it is not consistent with the assumption of price exogeneity in the H-O model.

and technology (t), $p(v^e, t)$. Hence factor rewards at EAE can be written as $w^{et} = R^t_v\left(p\left(v^e, t\right), v^e, t\right)$. Using the quadratic approximation lemma for the expression of w^{et} for two different time periods t and s and rearranging we get:

$$\begin{aligned} w^{et} - w^{es} &= \frac{1}{2}\left(R^{et}_{vp}\frac{\partial p^t}{\partial v^{et}} + R^{es}_{vp}\frac{\partial p^s}{\partial v^{es}} + R^{et}_{vv} + R^{es}_{vv}\right)\left(v^{et} - v^{es}\right) \\ &+\frac{1}{2}\left(R^{et}_{vp}\frac{\partial p^t}{\partial t} + R^{es}_{vp}\frac{\partial p^s}{\partial s} + R^{et}_{vt} + R^{es}_{vt}\right) \end{aligned} \tag{21}$$

a relationship between the change of factor rewards at the EAE $\left(w^{et} - w^{es}\right)$, the change of inputs at the EAE $\left(v^{et} - v^{es}\right)$ and the effect of technological change on factor rewards in the EAE. $R^e_{vp} = \frac{\partial w^e}{\partial p}$ is the matrix of first partial derivatives of factor rewards with respect to prices in EAE, $R^e_{vv} = \frac{\partial w^e}{\partial v^e}$ is the matrix of partial derivatives of factor rewards with respect to endowments in EAE and $R^e_{vt} = \frac{\partial w^e}{\partial t}$ is the vector of partial derivatives of factor rewards with respect to technological change evaluated at the EAE.

From the definition of the FCT (9) we get that $v^e = v - f$, the endowments in EAE are equal to the endowments in the Trade Equilibrium less the FCT. Taking the difference of this expression between two points in time t and s we have

$$v^{et} - v^{es} = v^t - v^s - \left(f^t - f^s\right) \tag{22}$$

Substituting (21) and (22) in (20) we get that:

$$\begin{aligned} w^t - w^s &= \overline{R^t_{vv}}f^t - \overline{R^s_{vv}}f^s + \frac{1}{2}\left(R^{et}_{vp}\frac{\partial p^t}{\partial v^{et}} + R^{es}_{vp}\frac{\partial p^s}{\partial v^{es}} + R^{et}_{vv} + R^{es}_{vv}\right)\left(v^t - v^s\right) \\ &-\frac{1}{2}\left(R^{et}_{vp}\frac{\partial p^t}{\partial v^{et}} + R^{es}_{vp}\frac{\partial p^s}{\partial v^{es}} + R^{et}_{vv} + R^{es}_{vv}\right)\left(f^t - f^s\right) \\ &+\frac{1}{2}\left(R^{et}_{vp}\frac{\partial p^t}{\partial t} + R^{es}_{vp}\frac{\partial p^s}{\partial s} + R^{et}_{vt} + R^{es}_{vt}\right) \end{aligned} \tag{23}$$

Expression (23) relates changes in the observed factor rewards at trade equilibrium to changes in actual endowments, changes in FCT and technology. It is a generalization of the results of Deardorff and Staiger (1988) and also of Leamer (1997). If we assume no technological change and that the endowments remain constant, the change in factor rewards would have been just a function of the change of the FCT. In addition, if there is non-jointness in output quantities or R_{pv}

is locally independent of v, factor rewards and consequently their changes between the trade and the equivalent autarky equilibrium are the same. Then the effect of FCT on the changes of factor rewards collapses to

$$w^t - w^s = -\frac{1}{2}\left(R_{vp}^{et}\frac{\partial p^t}{\partial v^{et}} + R_{vp}^{es}\frac{\partial p^s}{\partial v^{es}}\right)\left(f^t - f^s\right) \tag{24}$$

(24) states that the change of factor rewards in the Trade Equilibrium is directly linked to changes of the FCT in the special case that there is neither jointness in output quantities nor changes on endowments nor in technology. Hence, we are able to establish a relationship between changes in factor rewards and changes in *FCT*, similar to the one of Deardorff and Stager (1988) under the same set of assumptions, while we use a different (dual) definition of FCT.

An unfortunate fact of this decomposition is that it still depends on the demand side of the economy. From (7b), the matrix of first partial derivatives of product prices with respect to EAE endowments is $\frac{\partial p}{\partial v^e} = -\left(R_{pp} - E_{pp}\right)^{-1} R_{pv}$ and the vector of first partial derivatives of product prices with respect to time is $\frac{\partial p}{\partial t} = -\left(R_{pp} - E_{pp}\right)^{-1} R_{pt}$. To compute (23) we need information on the second derivatives of the expenditure function with respect to prices. Instead we estimate directly $\frac{\partial p}{\partial v^e}$ and $\frac{\partial p}{\partial t}$ by using SURE and assuming a linear relationship between the growth rate of prices, the growth rate of EAE endowments and technological change $\widehat{p_i} = a_i + \sum \beta_{ij}\widehat{v_j^e}$ [6]; $i = E, I, N$ and $j = K, S, U$.

In Table 8 we present the results of the above decomposition (23) in growth rates (see Appendix B) for the period 1967-1991. For both types of labour the *FCT Effect* $\left(\overline{R_{vv}^t}f^t - \overline{R_{vv}^s}f^s - \frac{1}{2}\left(R_{vp}^{et}\frac{\partial p^t}{\partial v^{et}} + R_{vp}^{es}\frac{\partial p^s}{\partial v^{es}} + R_{vv}^{et} + R_{vv}^{es}\right)\left(f^t - f^s\right)\right)$, third column in Table 8, has a positive impact on the growth of their factor rewards. The FCT Effect raised by 2.75% the growth of skilled labour's reward and by 4.47% the growth of unskilled labour' reward, respectively for the period 1967-1981. For the same period, the FCT effect on the growth of the reward of capital was negative, -4.84%. For the rest of the time period, the FCT effect is positive for all three factors of production. The highest magnitude is observed for the reward to capital, 2.79%, while the lowest is for unskilled labour's reward, 1.62%.

Bearing in mind that capital is the only factor that had experienced positive and high magnitudes of FCT, it strikes as confusing the result that the *FCT effect* had a negative impact on the growth of capital's reward. There are two explanations for this surprising result and are discussed bellow.

[6] Results are presented in Table 7

The first is that the *FCT Effect* consists of an expression that is linear on the FCT growth of each factor of production and also depends on the several inverse input demand elasticities with respect to price, quantity and time and also on product price elasticities with respect to endowments. The sign of most of these elasticities is not a priori determined, because of the more general technology used for the estimation (absence of non-jointness in output quantities). As a consequence, there is not a clear theoretic prediction as in the simple 2x2 Heckscher-Ohlin model. Here, the sign of the effect depends on the sign and also magnitude of all the elasticities mentioned above. And since the sign of these elasticities is an open empirical question all possible outcomes could occur.

The second explanation rests on the fact that we can further decompose the FCT effect in (23) into two components as shown in Table 9. One arising from the more general technology, is called *From Jointness* and is equal to $\overline{R^t_{vv}}f^t - \overline{R^s_{vv}}f^s - \frac{1}{2}\left(R^{et}_{vv} + R^{es}_{vv}\right)\left(f^t - f^s\right)$. The second originates from the relationship between product prices and endowments in EAE, $-\frac{1}{2}\left(R^{et}_{vp}\frac{\partial p^t}{\partial v^{et}} + R^{es}_{vp}\frac{\partial p^s}{\partial v^{es}}\right)\left(f^t - f^s\right)$ and is termed *From Product Prices*. The latter is the effect that is usually attributed to international trade in the literature and is given by (24). From Table 9 is obvious that the *From Product Prices* is positive for all three factors and also that there is a clear ranking of the magnitudes for all periods with the reward to capital having the highest gains, 1.28%, the reward to skilled labour the second highest, 0.47%, and unskilled labour the least gains, 0.19% for the whole period. At the same time, the *From Jointness* was negative on average for the reward to capital, -3.07% and positive for the reward of both skilled, 2.07%, and unskilled labour, 3.14%, respectively. Hence, it is evident that the reward of capital had the biggest gains from international trade if a no-joint technology is assumed, followed by skilled labour and finally the factor with the lowest gains was unskilled labour. From the estimation it is clear that the quantity elasticity of inverse input demand for capital was negative and large in magnitude relative to the price and technology elasticity of inverse input demands and also the product price elasticities in EAE and as a consequence the *From Jointness Effect* dominated the *From Product Prices Effect* leading to a negative *FCT Effect* for the case of capital.

Next we see that the *Endowments Effect* $\left(\frac{1}{2}\left(R^{et}_{vp}\frac{\partial p^t}{\partial v^{et}} + R^{es}_{vp}\frac{\partial p^s}{\partial v^{es}} + R^{et}_{vv} + R^{es}_{vv}\right)\left(v^t - v^s\right)\right)$, fourth column in Table 8, was negative for the growth rate of capital's and skilled labour's reward, -13.35% and -1.27% respectively for the whole period and positive for the growth rate of unskilled labour's reward, 2.10% over the same period. The signs and the magnitudes of such effects are the expected ones. Capital was the factor that experienced the highest growth in its endowments, followed by skilled labour and naturally this growth had affected adversely the reward for each factor. On the opposite side, unskilled labour endowments have declined over the period of investigation and such decline in the supply of unskilled labour has caused, ceteris paribus, an increase on the reward of unskilled labour.

The last column of Table 8 presents the total *Technology Effect* $\left(\frac{1}{2}\left(R_{vp}^{et}\frac{\partial p^t}{\partial t}+R_{vp}^{es}\frac{\partial p^s}{\partial s}+R_{vt}^{et}+R_{vt}^{es}\right)\right)$. This effect is positive on average for the growth rate of factor rewards for all three inputs over the period 1967-1991. The technological effect on the growth of capital's reward was the highest in magnitude, an average of 17.52% for the whole period, followed by skilled labour's growth, 5.68%. For the same period the total *Technology Effect* on the growth of unskilled labour's reward was slightly above zero, 0.50%. It altered signs from positive, 1.51%, in the first sub-period to negative for the second, -1.03%, while it was positive for all subperiods for the other two factors. So it is clear from Table 8 that all factors gained from technological change, with capital experiencing the largest gain and unskilled labour the smallest one in terms of the growth of their reward.

But this *Technology Effect* can be further decomposed into *Endogenous Technological Change Effect* $\left(\frac{1}{2}\left(R_{vp}^{et}\frac{\partial p^t}{\partial t}+R_{vp}^{es}\frac{\partial p^s}{\partial s}\right)\right)$ and *Exogenous Technological Change Effect*$\left(\frac{1}{2}\left(R_{vt}^{et}+R_{vt}^{es}\right)\right)$ as it is shown in Table 10. The *Endogenous Technological Change Effect* arises from the fact that product prices in the EAE are endogenous and are affected by technological change. While the *Exogenous Technological Change* is similar to a shift of the production possibilities frontier. From Table 10, we see that the *Endogenous Technological Change Effect* was positive for all three factors and time periods, with the reward to capital to have again the highest gains, 13.02%, and unskilled labour to gain the least, 2.25% from 1967 to 1991. On the other hand the *Exogenous Technological Change Effect* effect was positive for the growth of the reward to capital, 4.5%, and skilled labour, 0.17%, but negative for the growth of unskilled labour's reward, -1.75%. Hence, we find that *Exogenous Technological Change* has hurt unskilled labour and has benefited capital and skilled labour. This indicates that the small positive effect of overall technological change on the growth of unskilled labour's reward is mainly due to a persistent negative exogenous technical change.

Finally, from Table 8 we see that the difference between the rewards of capital and the two types of labour has narrowed, since the annual growth over the whole period for capital, 2.38%, was much smaller than the annual growth for skilled labour, 6.95% and unskilled labour, 5.93% respectively. But at the same time, the inequality between workers has increased at a rate of slightly above 1% for every year. This seems to be attributed to technological change that has favoured considerably much more skilled labour relative to unskilled labour. The total *FCT effect* is higher for the unskilled labour and so does the *Endowment Effect*, in fact this effect is negative for skilled labour. Consequently, the increasing wage inequality between skilled and unskilled workers seems to be due to the *Technology Effect*. In particular the *Technology Effect* was 6.53% for skilled labour's reward and only 1.51% for the reward of unskilled labour over the period 1967-1981. While for the last period the difference of *Technology Effect* became even bigger between the reward of the two types of labour. It was 4.39% for skilled labour and -1.03% for unskilled. Hence, the widening on relative wages between skilled and skilled workers seems to be the result of technological change that is biased towards skilled labour.

5 Conclusion

In this paper, we provide a dual definition for the factor content of trade based on the equivalent autarky equilibrium introduced by Deardorff and Staiger (1988). This new definition of FCT allows for a more general technology that permits the existence of jointness in output quantities. By estimating a symmetric normalized quadratic revenue function we calculate the FCT of capital, skilled and unskilled labour for the US manufacturing sector for the period 1965 to 1991. Moreover by applying the quadratic approximation lemma to the difference of factor rewards between the trading equilibrium and EAE, we are able to link the observed growth of factor rewards to the growth of FCT, endowments and technological change for 1967-1991.

We find that the FCT of capital is positive while the FCT of skilled and unskilled labor are negative. Hence, for the period of investigation and under the technological specification of our model, it appears that there is no Leontief Paradox. This suggests that if the economy was at EAE less capital would have been employed relative to skilled and unskilled labor. The positive sign of capital's FCT and the negative sign of the FCT of both types of labour implies that US manufacturing sector was a net exporter of goods that are more capital intensive between 1965 to 1991 and that capital was revealed by trade to be relatively more abundant to the two types of labour. In addition, following Leamer (1980) we show that skilled labour is revealed by trade to be relatively more abundant to unskilled labour, since the ratio of factor content of skilled labour to factor content of unskilled labour is smaller than the ratio of skilled to unskilled labour used in the production.

Overall factor rewards between the two types of labour and capital have narrowed but within labour wage inequality has increased. We find that the *FCT Effect* on factor rewards, for the period considered, is positive for the two types of labour and negative for capital. This is probably the result of the more general technology used in the analysis as the decomposition of the FCT Effect indicates in Table 8. From the estimation we found that the sign and magnitude of all the elasticities involved on the calculation of the *FCT Effect* for capital were such that the *From Jointness Effect* was negative and dominated the positive *From Product Prices Effect.* If a non-joint technology was assumed then capital's reward would have experienced the highest *FCT Effect*, implying that the reward to capital has gained the most because of international trade. The *Endowments Effect* is negative for the growth of capital's and skilled labour's reward and positive for unskilled labour.

Suggesting that the increasing endowments of capital and skilled labour have suppressed their rewards, ceteris paribus, while the opposite happened for unskilled labour. Technological change has benefited mainly the reward to capital, but also skilled labour's reward to a smaller magnitude. On the contrary, the reward to unskilled labour had almost no gains arising from technological innovation. Finally, the increasing inequality between skilled and unskilled labour's reward seems to be the cause of technological change, both exogenous and endogenous, that was biased in favour of skilled labour's reward.

Appendix A

There are three inputs in our model, capital, v_K, skilled labour, v_S, and unskilled labour,v_U. Data for the value and price of capital and aggregate labour, at a 2-digit SIC87 analysis are obtained from Dale's Jorgenson database for the period 1963-1991[7]. We construct the value added for capital and aggregate labour and also the price of capital and labour. In particular, the price of inputs is a weighted average of their prices in each 2-digit industry with weights the share of each input in every 2-digit industry. We get the quantity of capital and aggregate labour by dividing their value added by their price, respectively.

The division of aggregate labour into skilled and unskilled labour is implemented by using data from the NBER collection of Mare-Winship Data, 1963 1991. We get data on educational levels, weekly wages, status and weeks worked for full time workers in 2-digit SIC industries. We divide workers into skilled and unskilled following Katz and Murphy (1992), a worker is treated as skilled if he or she spent at least twelve years in education. Our sample contains only full time workers, aged 16-45, that have completed their educational grade and are working in the private sector. First, we calculate the total number of weeks worked per year and also the annual wages and salaries for skilled and unskilled workers[8]. Then we divide the annual value of wages and salaries by the corresponding total weeks worked in order to calculate the full time weekly wage for each group respectively. After that we calculate the share of weeks worked for skilled and unskilled workers relative to the total hours worked of all workers. Similarly, we find the shares of wages for each occupational group in the sample. Finally, these shares are multiplied with the total quantity and total wages of aggregate labour, respectively, obtained from Jorgenson's data set in order to get the quantity and wages for skilled and unskilled workers in US.

In our model there are three aggregate products, exportable, y_E, importable, y_I, and non tradable, y_N. Initially the products are divided into tradeable and non-tradeables. A 2-digit industry is termed tradable if the ratio of its exports plus imports divided by its revenue is above 10%, otherwise it is termed as non-tradable[9]. Then tradable industries are grouped to exportables and importables depending on whether their net exports are positive or negative, respectively.

[7] http://post.economics.harvard.edu/faculty/jorgenson/data/35klem.html

[8] Following Katz, L. and Murphy, K. (1992) we include only full time workers that have worked more than 39 weeks in that year. Also, top code wage and salaries were multiplied by 1.45

[9] Trade data at a 2-digit SIC87 level were obtained online from the Centre for International Data at the University of California Davis.

http://data.econ.ucdavis.edu/international/index.html

For the calculation of value added of the three aggregate products we again use Jorgenson's data set. While data for output deflators are obtained from the Bureau of Economic Analysis at a 2-digit SIC level. Since these are available from 1977 onwards, the values of output deflators for years before 1977 are obtained by interpolation assuming a constant growth rate equal to the growth rate between 1977 and 1978. The aggregation of the three goods is achieved in three stages[10]. First, we calculate the value added for each aggregate good, then an aggregate price is constructed for each of them. This aggregate price is a weighted average of the prices of all 2-digit industries that belong to an aggregate good, with weights the share of each 2-digit industry. The aggregate quantity of output is calculated by dividing the value of each aggregate good by its aggregate price. Similarly, the volume of net exports is calculated by dividing the value of net exports for each aggregate good by its corresponding aggregate price.

The assumption of balanced trade is not satisfied by the data. For that reason, the actual trade volumes for each good are adjusted according to the share of output relative to total revenue in the economy in order to guarantee balanced trade. We define the volume of net exports T as:

$$T = y - c, \tag{A1}$$

where y and c is the vector of production and consumption respectively. We assume trade balance, which implies that economy's volume of net exports priced at the exogenous given international prices p is zero:

$$pT = p\,(y - c) = 0 \tag{A2}$$

From the data we get that (A2) is not satisfied, instead we get a trade deficit or surplus ($IMBAL$) varying from year to year:

$$pT = py - pc = IMBAL \tag{A3}$$

In order to impose trade balance, we subtract from the volume of net exports for every good i a fraction of $IMBAL$ equal to each good's share of total value of output in the economy:

$$ADJT_i = T_i - \left(\frac{y_i}{\sum_{i=1}^{N} y_i p_i}\right) IMBAL \tag{A4}$$

[10] Table 1 shows the SIC categories that are included in each aggregate good.

and from $(A1)$ and $(A4)$ we calculate the new consumption vector x which is equal to the production of the economy:

$$x_i = y_i - ADJT_i = y_i - T_i + \left(\frac{y_i}{\sum_{i=1}^{N} y_i p_i} \right) IMBAL \tag{A5}$$

Appendix B

Note that $\widehat{x} = \frac{x^t - x^s}{\left(\frac{x^t + x^s}{2}\right)} = \frac{x^t - x^s}{x^m}$ indicates the growth rate of variable x between time periods t and s and $x^m = \frac{x^t + x^s}{2}$ is the average of variable x for these time periods, $\varepsilon^{it}_{xy} = \frac{\partial x^{it}}{\partial y^{it}} \frac{y^{it}}{x^{it}}$ indicates the elasticity of x with respect to y in equilibrium i at time t and $R^{it}_{vz} = \frac{\partial^2 R^{it}}{\partial v \partial z} = \frac{\partial w^{it}}{\partial z}$ is the matrix of second partial derivatives of the revenue function with respect to v and z in equilibrium i at time t. (20) can be written in a growth format as

$$\begin{aligned}
\frac{w^t - w^s}{(w^m)} &= \frac{1}{(w^m)}\left[\overline{R^t_{vv}} f^t - \overline{R^s_{vv}} f^s + \frac{w^{et} - w^{es}}{(w^{em})}\right] \\
\widehat{w} &= \widetilde{R^t_{vv}} f^t - \widetilde{R^s_{vv}} f^s + \widehat{w^e}\left(\frac{w^{em}}{w^m}\right) \qquad \text{(B1)}
\end{aligned}$$

the growth rate of factor rewards in the Trade Equilibrium ($\widehat{w}$) depends on the difference of FCT between the two time periods and the growth of factor rewards in the EAE $\left(\widehat{w^e}\right)$. In the case that non-jointness in output quantities is assumed ($R_{vv} = 0$), then ($B1$) states that the factor rewards and consequently their growth rates will be the same in both the Trade and the EAE.

Similarly, rearranging (21)

$$\begin{aligned}
\frac{w^{et} - w^{es}}{w^{em}} &= \frac{1}{2}\left[\left(\frac{\partial w^{et}}{\partial p^t}\frac{p^t}{w^{et}}\right)\left(\frac{\partial p^t}{\partial v^{et}}\frac{v^{et}}{p^t}\right)\frac{w^{et}}{v^{et}} + \left(\frac{\partial w^{es}}{\partial p^s}\frac{p^s}{w^{es}}\right)\left(\frac{\partial p^s}{\partial v^{es}}\frac{v^{es}}{p^s}\right)\frac{w^{es}}{v^{es}}\right]\left(\frac{v^{et} - v^{es}}{v^{em}}\right)\left(\frac{v^{em}}{w^{em}}\right) \\
&+ \frac{1}{2}\left[\left(\frac{\partial w^{et}}{\partial v^{et}}\frac{v^{et}}{w^{et}}\right)\frac{w^{et}}{v^{et}} + \left(\frac{\partial w^{es}}{\partial v^{es}}\frac{v^{es}}{w^{es}}\right)\frac{w^{es}}{v^{es}}\right]\left(\frac{v^{et} - v^{es}}{v^{em}}\right)\left(\frac{v^{em}}{w^{em}}\right) \\
&+ \frac{1}{2}\left[\left(\frac{\partial w^{et}}{\partial p^t}\frac{p^t}{w^{et}}\right)\left(\frac{\partial p^t}{\partial t}\frac{1}{p^t}\right)w^{et} + \left(\frac{\partial w^{es}}{\partial p^s}\frac{p^s}{w^{es}}\right)\left(\frac{\partial p^s}{\partial s}\frac{1}{p^s}\right)w^{es}\right]\left(\frac{1}{w^{em}}\right) \\
&+ \frac{1}{2}\left[\left(\frac{\partial w^{et}}{\partial t}\frac{1}{w^{et}}\right)w^{et} + \left(\frac{\partial w^{es}}{\partial s}\frac{1}{w^{es}}\right)w^{es}\right]\left(\frac{1}{w^{em}}\right) \\
\widehat{w^e} &= \frac{1}{2}\left[\left(\varepsilon^{et}_{wp}\varepsilon^{et}_{pv} + \varepsilon^{et}_{wv}\right)\frac{\left(\frac{w^{et}}{w^{em}}\right)}{\left(\frac{v^{et}}{v^{em}}\right)} + \left(\varepsilon^{es}_{wp}\varepsilon^{es}_{pv} + \varepsilon^{es}_{wv}\right)\frac{\left(\frac{w^{es}}{w^{em}}\right)}{\left(\frac{v^{es}}{v^{em}}\right)}\right]\widehat{v^e} \\
&+ \frac{1}{2}\left[\left(\varepsilon^{et}_{wp}\varepsilon^{et}_{pt} + \varepsilon^{et}_{wt}\right)\left(\frac{w^{et}}{w^{em}}\right) + \left(\varepsilon^{es}_{wp}\varepsilon^{es}_{ps} + \varepsilon^{es}_{wt}\right)\left(\frac{w^{es}}{w^{em}}\right)\right] \qquad \text{(B2)}
\end{aligned}$$

we get a relationship between the growth rate of factor rewards in EAE $\left(\widehat{w^e}\right)$, the growth rate of endowments in EAE $\left(\widehat{v^e}\right)$, the price, quantity and time elasticities of inverse input demand at EAE and the elasticities of product prices with respect to endowments and technical change at EAE[11].

[11] Table 6 reports the mean values for ε^e_{wp}, while Table 5 reports the mean values for ε^e_{wv} and ε^e_{wt}.

Applying (22) for another time period s and subtracting from (22) we get

$$\begin{aligned}
\frac{v^{et}-v^{es}}{v^{em}} &= \frac{v^{t}-v^{s}}{v^{m}}\left(\frac{v^{m}}{v^{em}}\right)-\frac{f^{t}-f^{s}}{f^{m}}\left(\frac{f^{m}}{v^{em}}\right) \\
\widehat{v^{e}} &= \widehat{v}\left(\frac{v^{m}}{v^{em}}\right)-\widehat{f}\left(\frac{f^{m}}{v^{em}}\right) \qquad \text{(B3)}
\end{aligned}$$

the growth of endowments in the EAE $\left(\widehat{v^e}\right)$ is positively related to the growth of endowments in the Trade Equilibrium $(\widehat{v})$ and inversely related to the growth of the FCT $\left(\widehat{f}\right)$.

Substituting $(B2)$ and $(B3)$ into $(B1)$ and rearranging we get

$$\begin{aligned}
\widehat{w} &= \widetilde{R^{t}_{vv}}f^{t}-\widetilde{R^{s}_{vv}}f^{s} \\
&+\frac{1}{2}\left[\left(\varepsilon^{et}_{wp}\varepsilon^{et}_{pv}+\varepsilon^{et}_{wv}\right)\frac{\left(\frac{w^{et}}{w^{em}}\right)}{\left(\frac{v^{et}}{v^{em}}\right)}+\left(\varepsilon^{es}_{wp}\varepsilon^{es}_{pv}+\varepsilon^{es}_{wv}\right)\frac{\left(\frac{w^{es}}{w^{em}}\right)}{\left(\frac{v^{es}}{v^{em}}\right)}\right]\widehat{v} \\
&-\frac{1}{2}\left[\left(\varepsilon^{et}_{wp}\varepsilon^{et}_{pv}+\varepsilon^{et}_{wv}\right)\frac{\left(\frac{w^{et}}{w^{em}}\right)}{\left(\frac{v^{et}}{f^{m}}\right)}+\left(\varepsilon^{es}_{wp}\varepsilon^{es}_{pv}+\varepsilon^{es}_{wv}\right)\frac{\left(\frac{w^{es}}{w^{em}}\right)}{\left(\frac{v^{es}}{f^{m}}\right)}\right]\widehat{f} \\
&+\frac{1}{2}\left[\left(\varepsilon^{et}_{wp}\varepsilon^{et}_{pt}+\varepsilon^{et}_{wt}\right)\left(\frac{w^{et}}{w^{em}}\right)+\left(\varepsilon^{es}_{wp}\varepsilon^{es}_{ps}+\varepsilon^{es}_{wt}\right)\left(\frac{w^{es}}{w^{em}}\right)\right] \qquad \text{(B4)}
\end{aligned}$$

this is the equivalent of (23) in growth format. It involves the growth rate of TE endowments, the growth rate of FCT and the effect of technological change. Assume now the case that technology is non-joint in output quantities ($\varepsilon_{wv}=\varepsilon^{e}_{wv}=0$), then factor rewards in both equilibria are going to be the same ($w=w^{e}$) and consequently their growth rates are going to be the same $\left(\widehat{w}=\widehat{w^{e}}\right)$. If in addition there is neither growth in TE endowments ($\widehat{v}=0$) nor technological change $\left(\varepsilon^{e}_{wt}=\varepsilon^{e}_{pt}\right)$, the growth of factor rewards in both equilibria is going to be equal to the following expression

$$\widehat{w}=\widehat{w^{e}}=-\frac{1}{2}\left[\varepsilon^{et}_{wp}\varepsilon^{et}_{pv}\frac{\left(\frac{w^{et}}{w^{em}}\right)}{\left(\frac{v^{et}}{f^{m}}\right)}+\varepsilon^{es}_{wp}\varepsilon^{es}_{pv}\frac{\left(\frac{w^{es}}{w^{em}}\right)}{\left(\frac{v^{es}}{f^{m}}\right)}\right]\widehat{f} \qquad \text{(B5)}$$

This is the *FCT Effect From Prices* in growth format. Under all the above assumptions, the growth rate of factor rewards in TE depends on the growth rate and level of FCT, the level of EAE endowments, the elasticity of factor rewards with respect to product prices and the elasticity of product prices with respect to EAE endowments.

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Table 1: SIC Codes for Aggregate Goods

Aggregate Good	**SIC Code Category**
Exportable	Food & Kindred Products (SIC 20)
	Chemicals & Allied Products (SIC 28)
	Industrial & Commerce Machinery & Computer Equipment (SIC 35)
	Electronic & Other Electric Equipment (SIC 36)
	Transportation Equipment (SIC 37)
	Instruments, Photographic, Medical & Optical Goods (SIC 38)
Importable	Textile Mill Products (SIC 22)
	Apparel & Other Finished Products (SIC 23)
	Lumber & Wood Products (SIC 24)
	Paper & Allied Products (SIC 26)
	Petroleum Refining & Related Industries (SIC 29)
	Leather & Leather Products (SIC 31)
	Primary Metal Industries (SIC 33)
	Miscellaneous Manufacturing Industries (SIC 39)
Non-tradable	Tobacco Products (SIC 21)
	Furniture & Fixtures (SIC 25)
	Printing, Publishing & Allied Industries (SIC 27)
	Rubber & Miscellaneous Plastic Products (SIC 30)
	Stone, Clay, Glass & Concrete Products (SIC 32)
	Fabricated Metal Products, Except Machinery (SIC 34)

Table 2: Parameter Estimates for Revenue Function

Parameter	**Estimate**	**t-stat**	**Parameter**	**Estimate**	**t-stat**
a_{EE}	47085.9	0.286	b_{KK}	-68690.5	-2.394
a_{EI}	-31871.6	-0.394	b_{KS}	29583.7	2.294
a_{EN}	-15214.3	-0.171	b_{KU}	39106.7	1.779
a_{II}	21573.3	0.521	b_{SS}	-12741.2	-1.515
a_{IN}	10298.3	0.213	b_{SU}	-16842.6	-2.523
a_{NN}	4916	0.120	b_{UU}	-22264.2	-1.303
e_K	2184.5	1.333	c_{NK}	-2048	-0.851
e_S	-620.7	-1.003	c_{NS}	61639.2	4.617
c_{EK}	64498	2.044	c_{NU}	-3075.6	-0.243
c_{ES}	-11935.4	-0.420	d_E	1557.5	0.607
c_{EU}	64737.3	3.018	d_I	-948.9	-0.639
c_{IK}	-13286.6	-0.607	h_t	1146.6	0.808
c_{IS}	72514	3.714	h_{tt}	42.2	0.386
c_{IU}	6805.5	0.428	*Syst.* $\mathbf{R}^2$	0.980	
Hypothesis Testing		**Test Statistic**		$\chi^2_{0.5}$	
No convexity & concavity		Wald(4)=32.7		9.488	
Non-jointness		Wald(2)=29.1		5.991	
No technological change		Wald(2)=98		5.991	

Table 3: PRICE ELASTICITIES OF OUTPUT SUPPLY (ϵ_{ih})
(Mean values, Std. Error in parenthesis)

	Price		
	Exportable	*Importable*	*Non-tradable*
Quantity	ϵ_{iE}	ϵ_{iI}	ϵ_{iN}
Exportable	0.316	-0.217	-0.010
	(1.103)	(0.543)	(0.598)
Importable	-0.515	0.353	0.162
	(1.291)	(0.669)	(0.780)
Non-tradable	-0.241	0.165	0.076
	(1.449)	(0.796)	(0.659)

Table 4: QUANTITY ELASTICITIES OF INVERSE INPUT DEMAND ($\varepsilon_{w_j v_k}$)
(Mean values, Std. Error in parenthesis)

	Input		
	Capital	*Skilled Labor*	*Unskilled Labor*
Factor Reward	$\varepsilon_{w_j v_K}$	$\varepsilon_{w_j v_S}$	$\varepsilon_{w_j v_U}$
Capital	-1.180	0.705	0.475
	(0.482)	(0.277)	(0.254)
Skilled Labor	0.331	-0.197	-0.133
	(0.130)	(0.088)	(0.058)
Unskilled Labor	0.766	-0.457	-0.309
	(0.411)	(0.201)	(0.224)

Table 5: QUANTITY & TECHN. CHANGE ELASTICITIES OF INVERSE INPUTS DEMANDS IN THE EAE ($\varepsilon^e_{w_j v_k}$) & ($\varepsilon^e_{w_j t}$)
(Mean values, Std. Error in parenthesis)

	Input			Technical Change
	Capital	*Skilled Labor*	*Unskilled Labor*	
Factor Reward	$\varepsilon^e_{w_j v_K}$	$\varepsilon^e_{w_j v_S}$	$\varepsilon^e_{w_j v_U}$	$\varepsilon^e_{w_j t}$
Capital	-0.397	0.062	0.334	0.019
	(0.051)	(0.043)	(0.045)	(0.001)
Skilled Labor	0.039	-0.009	-0.031	0.002
	(0.029)	(0.010)	(0.020)	(0.001)
Unskilled Labor	0.831	-0.150	-0.681	-0.049
	(0.407)	(0.139)	(0.295)	(0.017)

Table 6: PRICE ELASTICITIES OF INVERSE INPUT DEMAND IN THE EAE ($\varepsilon^e_{w_j p_i}$)
(Mean values, Std. Error in parenthesis)

	Price		
	Exportable	*Importable*	*Non-tradable*
Factor Reward	$\varepsilon^e_{w_j p_E}$	$\varepsilon^e_{w_j p_I}$	$\varepsilon^e_{w_j p_N}$
Capital	0.841	0.042	0.116
	(0.030)	(0.018)	(0.012)
Skilled Labor	-0.056	0.569	0.487
	(0.058)	(0.040)	(0.023)
Unskilled Labor	1.997	-0.329	-0.668
	(0.698)	(0.279)	(0.420)

Table 7: PARAMETER ESTIMATES FROM GROWTH EQUATIONS

Parameter	**Estimate**	**t-stat**	**Parameter**	**Estimate**	**t-stat**
a_E	0.069	6.607	β_{IS}	0.446	-2.260
β_{EK}	-0.208	-1.743	β_{IU}	-0.308	-1.473
β_{ES}	0.288	2.108	a_N	0.071	6.670
β_{EU}	-0.274	-1.889	β_{NK}	-0.191	-1.562
a_I	0.076	5.011	β_{NS}	0.269	1.919
β_{IK}	-0.227	-1.312	β_{NU}	-0.238	-1.601
Syst. $\mathbf{R}^2$		0.99			

Table 8: FACTOR REWARDS DECOMPOSITION
(Annual growth rates)

Period	Fitted Growth	FCT Effect	Endowment Effect	Technology Effect
Capital				
1967-1981	**3.63**	**-4.84**	**-10.01**	**18.48**
1982-1991	**0.53**	**2.79**	**-18.34**	**16.08**
1967-1991	**2.38**	**-1.79**	**-13.35**	**17.52**
Skilled Labor				
1967-1981	**9.17**	**2.75**	**-0.11**	**6.53**
1982-1991	**3.62**	**2.23**	**-3.00**	**4.39**
1967-1991	**6.95**	**2.54**	**-1.27**	**5.68**
Unskilled Labor				
1967-1981	**8.44**	**4.47**	**2.46**	**1.51**
1982-1991	**2.16**	**1.62**	**1.57**	**-1.03**
1967-1991	**5.93**	**3.33**	**2.10**	**0.50**

Table 9: FCT Effect Further Decomposition
(Annual growth rates)

Period	FCT Effect	From Jointness	From Product Prices
Capital			
1967-1981	**-4.84**	-5.79	0.95
1982-1991	**2.79**	1.01	1.78
1967-1991	**-1.79**	-3.07	1.28
Skilled Labor			
1967-1981	**2.75**	2.33	0.42
1982-1991	**2.23**	1.67	0.56
1967-1991	**2.54**	2.07	0.47
Unskilled Labor			
1967-1981	**4.47**	4.28	0.19
1982-1991	**1.62**	1.44	0.18
1967-1991	**3.33**	3.14	0.19

Table 10: Technology Effect Further Decomposition
(Annual growth rates)

Period	Technology Effect	Endogenous	Exogenous
Capital			
1967-1981	**18.48**	14.30	4.18
1982-1991	**16.08**	11.10	4.98
1967-1991	**17.52**	13.02	4.50
Skilled Labor			
1967-1981	**6.53**	6.40	0.13
1982-1991	**4.39**	4.18	0.21
1967-1991	**5.68**	5.51	0.17
Unskilled Labor			
1967-1981	**1.51**	3.42	-1.91
1982-1991	**-1.03**	0.47	-1.50
1967-1991	**0.50**	2.25	-1.75

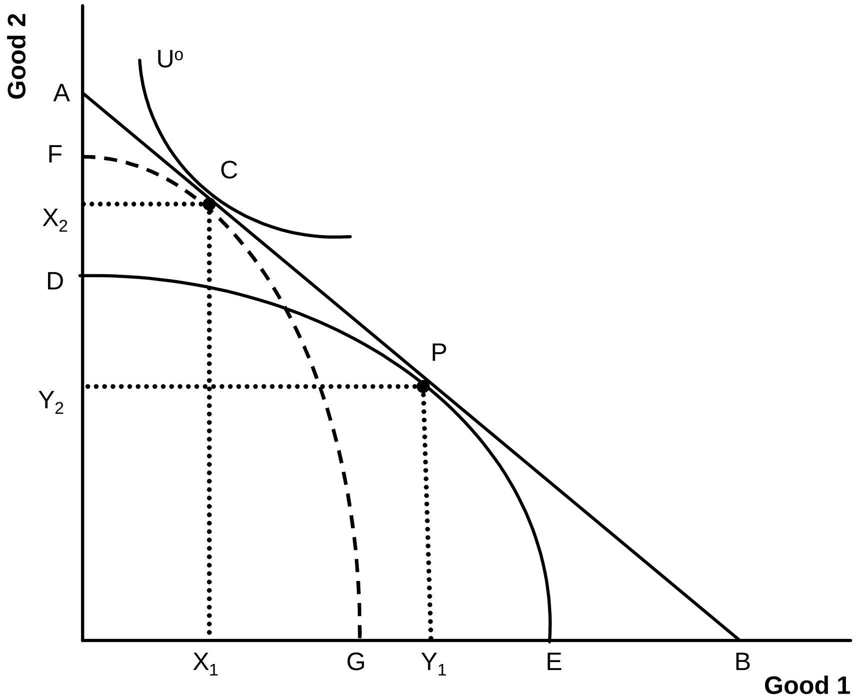

Figure 1: Trade and Equivalent Equilibria

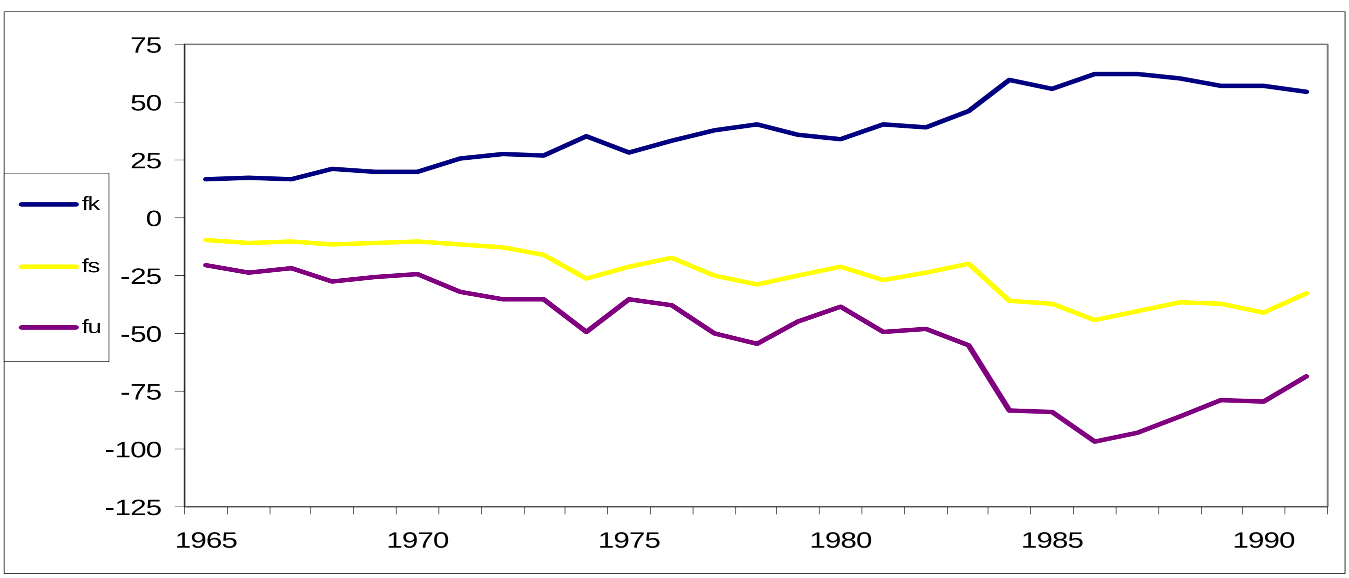

Figure 2: Factor Content of Capital (f_K), Factor Content of Skilled Labour (f_s) and Factor Content of Unskilled Labour (fu) in billions of 1970 USD.