Aggregate Productivity, Human–Capital Investments, and Trade

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1 Introduction

Recent literature on the effects of trade liberalization emphasizes aggregate productivity gains caused by selection effects in a setting with heterogeneous firms. These analyses are guided by results from empirical research that reveals (i) persistent productivity differences among firms within narrowly defined industries and (ii) that firms engaging in international trade are more productive than non–exporters. Manasse and Turrini (2001) and Melitz (2003) have developed models with firm–specific productivities where the existence of a fixed–cost component in trade costs implies that only the most productive firms find it profitable to export. As a result, large highly productive firms acting on large world markets coexist with smaller less productive firms that constrain their activities to national markets. In such a framework trade liberalization due to declining trade costs generates further expansion of high–productive firms relative to low–productive ones—in Melitz's approach some low–productive firms even exit the market—thus raising an economy's aggregate productivity. Although trade is costly, the compositional change within the pool of firms generated by trade liberalization is shown to enhance aggregate social welfare. That positive welfare effect typically

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is enhanced by access to a greater spectrum of product varieties induced by trade liberalization.

The papers mentioned above differ within the respect how differences in firm productivity are introduced. Manasse and Turrini (2001) attribute firm heterogeneity to exogenously given differences in the abilities of entrepreneurs that determine firms' productivities. With the number of entrepreneurs/firms given exogenously, trade liberalization draws resources to the high–productive exporting firms thus raising aggregate productivity. Melitz (2003) assumes that *ex ante* identical firms have to spend a fixed investment in order to discover their productivity. Specifically, firms draw their productivities from an exogenously given pool of productivities in the process of developing their differentiated products. Consequently, firms become heterogeneous *ex post* and self–select into exporters, nonexporters, and those firms exiting the market. Again, the outcome of this selection process depends on the extent to which an economy is exposed to trade.[1]

The present paper combines both approaches. Alike Manasse and Turrini (2001) we attribute differences in the productivities of firms to differences in the skills of entrepreneurs operating that firms. Rather than taking the distribution of skills as given, however, we extend their analysis by allowing individuals to react to changes in the rentals of acquiring education thus endogenizing the equilibrium mass of active firms and the supply of workers. As a result, we arrive at basically the same endogenous selection mechanism as Melitz (2003). The advantage of our approach is that we substitute for Melitz's abstract lottery with firms drawing their productivities randomly from an arbitrarily specified distribution of productivities by an economically intuitive explanation of the emergence of persistent productivity differentials. It is the human–capital investments of heterogeneous individuals that determines the distribution of productivities in an economy. Consequently, our approach allows for an analysis of policies manip-

[1] A related paper is Yeaple (2005) who allows firms to employ different technologies and different types of workers. Highly skilled labor is assumed to have a comparative advantage in technologies with low unit costs but high fixed costs. Depending on the availability of skills and on the costs of trade, firms select into the group of exporters by employing highly skilled labor together with low–unit–cost technologies or into the group of nonexporters by employing less skilled labor together with high–unit–cost technologies. Trade liberalization then causes more firms to employ low–unit–cost technologies. However, Yeaple does not emphasize productivity differentials between firms and aggregate prooductivity effects. Furthermore, he does not analyze effects on aggregate welfare.

ulating the distribution of productivities by educational policies, such as educational subsidies or improvements in the educational technology. Eventually, we can apply our approach to explain the empirically observed distribution of firm productivities to plausible distributions of individual abilities.

The paper is organized as follows. Section 2 presents the basic model. In section 3 we characterize the closed–economy equilibrium. In section 4 we characterize the open–economy equilibrium, and we derive the implications of trade liberalization. Section 5 then concludes.

2 The Model

2.1 Demand

We assume that consumers have preferences over product varieties according to the CES–aggregate

$$U = \left[\int_{j \in J} c(j)^{\rho} dj \right]^{1/\rho}, \qquad \rho \in (0,1)\,, \quad \sigma \equiv \frac{1}{1-\rho} > 1\,,$$

where $c(j)$ denotes consumption of product variety j, and the measure of set J represents the mass of available goods; σ denotes the onstant elasticity of substitution between any two goods.

Due to the homotheticity of the utility function, we can derive aggregate demand from the problem of a representative consumer. With E denoting aggregate consumer income, demand for variety i is given by

$$c(i) = Ep(i)^{-\sigma} P^{\sigma-1}\,,$$

where $p(i)$ is the price of variety i and P is the price index defined over prices of varieties

$$P := \left[\int_{j \in J} p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}\,.$$

2.2 Production

Production requires two factors of production: skills and raw labor. We assume that each skilled worker can employ her skills in the production of at most one variety. The size of the firm is normalized such that one firm employs the skills of one entrepreneur. We follow Manasse and Turrini (2001) and assume that the skills of the entrepreneur determines the productivity of the firm. Technology is represented by a cost function with constant marginal costs. The demand for raw labor l is linear in output x: $l = x/q$. Firms differ in their productivity levels q. Following Melitz (2003), higher productivity is modelled as producing a symmetric product variety at lower cost.

Each firm faces a residual demand curve with constant elasticity σ, and a wage rate w. Profit maximization yields the markup–pricing rule

$$p = \frac{w}{\rho q}\,.$$

2.3 Human–Capital Investment and Labor Supply

The economy is populated by a continuum of individuals of mass L. Individuals are heterogeneous with respect to their innate abilities a. Abilities are distributed according to some continuous and differentiable density function $g(a)$ with support $[0, \infty)$; the respective distribution function is denoted by $G(a)$.[2]

An individual with ability a can choose to enter the labor force and supply on unit of raw labor at the wage w. Alternatively, an individual can choose to acquire education and become a skilled entrepreneur. In that case her income is her firm's profit. To simplify, we abstract from any direct cost of education. Of course, the wage income of unskilled labor is the opportunity cost of education.

Education is assumed to raise individual abilities according to some function $h(a)$ with $h'(a) > 0$. We assume that the firms productivity and the entrepreneurs human capital are related by

$$q = h(a)\,, \quad h'(a) > 0\,.$$

The self selection of individuals endogenizes the economy's supply of raw labor. In case of $a \in [t, \infty)$ individuals acquiring education, aggregate labor supply is given by

$$L^S(t) = G(t)L\,.$$

Consequently, the mass of active firms iis given by

$$M(t) = [1 - g(t)]L\,.$$

[2] This means that the support can be quite large but is finite at some upper bound.

3 The Closed–Economy Equilibrium

We begin the analysis of the closed–economy equilibrium by deriving the threshold ability t. Individuals with abilities $a \geq t$ will invest in education and establish a firm. Therefore, we solve the individuals' decisions about aquiring education. Those with abilities $a < t$ will not educate but supply raw labor.

In order to prove this assertion, we look at the profit of a firm with productitity $q = h(a)$. With markup pricing according to

$$p(a) = \frac{w}{\rho h(a)} \tag{1}$$

profits are given by

$$\pi = (1 - \rho)\frac{w}{\rho}\frac{x(a)}{h(a)} \,. \tag{2}$$

An individual with ability a invests in education long as her profits from entrepreneurial activities exceed the wage rate. This requires

$$\frac{x(a)}{h(a)} \geq \sigma - 1 \,. \tag{3}$$

Making use of the clearing of product markets

$$x(a) = E[p(a)]^{-\sigma} P^{\sigma - 1} \,, \tag{4}$$

and applying the markup pricing according to (1), condition (3) can be written as

$$h(a) \geq \left(\frac{\sigma - 1}{E}\right)^{\frac{1}{\sigma - 1}} \left(\frac{w}{\rho}\right)^{\frac{\sigma}{\sigma - 1}} P^{-1} \,. \tag{5}$$

Since $h'(a) > 0$, there exists a unique threshold $t = a(w, E, P)$ given by

$$h(t) = \left(\frac{\sigma - 1}{E}\right)^{\frac{1}{\sigma - 1}} \left(\frac{w}{\rho}\right)^{\frac{\sigma}{\sigma - 1}} P^{-1} \,. \tag{6}$$

As a result, the individuals' decision problem about aquiring education has a unique solution for given macroeconomic variables (w, E, P).

In order to determine the general equilibrium we solve for the equilibrium values of the macroeconomic variables. For any threshold $t = a(w, E, P)$, the price index is[3]

$$P(t) = \left[L \int_t^\infty p(a)^{1-\sigma} dG(a) \right]^{\frac{1}{1-\sigma}} = \frac{w}{\rho} \frac{L^{\frac{1}{1-\sigma}}}{Q(t)}, \tag{7}$$

where

$$Q(t) := \left[\int_t^\infty h(a)^{\sigma-1} dG(a) \right]^{\frac{1}{\sigma-1}}, \qquad Q'(t) < 0$$

is a measure of the per–capita aggregate stock of human capital. Note that Q is negatively correlated to the aggregate productivity

$$\tilde{Q}(t) := \left[\frac{1}{1-G(t)} \int_t^\infty h(a)^{\sigma-1} dG(a) \right]^{\frac{1}{\sigma-1}} = [1-G(t)]^{\frac{1}{1-\sigma}} Q(t).$$

Substituting in (6) for $P(t)$, we get

$$h(t) = \left(\frac{\sigma-1}{\rho} \frac{wL}{E} \right)^{\frac{1}{\sigma-1}} Q(t). \tag{8}$$

We finally have to solve for w/E. Aggregate income E is made up from aggregate profits and aggregate wages. Profits of a firm a can be calculated by substituting in (2) for $x(a)$ by (4) making use of our solution for $P(t)$:

$$\pi(a,t,w,E,L) = (1-\rho)\frac{E}{L} \left[\frac{h(a)}{Q(t)} \right]^{\sigma-1}.$$

Aggregate profits then are

$$\Pi(t,w,E,L) = L \int_t^\infty \pi(a,t,E,w,L) dG(a) = (1-\rho)E.$$

Adding aggregate wage income $wG(t)L$, we get

$$E = \Pi(t,w,E,L) + wG(t)L = (1-\rho)E + wG(t)L$$

or

$$\frac{E}{wL} = \frac{G(t)}{\rho}.$$

[3] In contrast to Melitz (2003), we do not formulate the model in terms of the conditional distribution of productivities in equilibrium but in terms of the distribution of productivities resp. abilities in general. As a result, the price level is written as depending on aggregate human capital Q rather than on aggregate productivity Q. Alternatively, we could write the price index as $P(t) = \frac{w}{\rho} \frac{M(t)}{\tilde{Q}(t)}$.

By substituting for E/wL in (8), the threshold is determined as the solution of

$$h(t) = \phi(t) := (\sigma - 1)^{\frac{1}{\sigma-1}} G(t)^{\frac{1}{1-\sigma}} Q(t) \,. \tag{9}$$

From $h'(t) > 0$, $\phi'(t) < 0$, and $\lim_{t\to 0} \phi(t) = \infty$, $\lim_{t\to\infty} \phi(t) = 0$, (9) determines a unique solution $t \in (0, \infty)$. Note that the equilibrium value of t is independent from the size of the economy. As a result, a country's aggregate productivity is independent from its size.

Once we have solved for the equilibrium threshold t, all endogenous variables can be calculated as functions of t. For normative analysis, we calculate the welfare per capita $W = U/L$. Substituting for the demand values $c(j)$ and using our definition of the price index, per–capita welfare is given by E/PL. In equilibrium, welfare per capita, given by

$$W(t) = G(t)Q(t)L^{\frac{1}{\sigma-1}} \,, \qquad W'(t) > 0 \,. \tag{10}$$

In order to provide a better interpretation for this welfare index, we decompose it by rewriting (10) as follows:

$$W(t) = G(t)\tilde{Q}(t) \left\{ [1 - G(t)] \, L \right\}^{\frac{1}{\sigma-1}} \,.$$

Welfare is made up of two components: the aggregate quantity of production per capita and the number of differentiated products.[4] With respect to the first component, note that the equilibrium labor employment is given by $G(t)L$ while $\tilde{Q}$ denotes the average firm productivity (average output per unit of labor input). It is intuitively clear that aggregate production—and hence welfare per capita—is the higher, the higher either the average productivity of firms given the number of workers using this productivity or the higher the number of workers employed given that productivity. The term $\{[1 - G(t)] \, L\}^{\frac{1}{\sigma-1}}$ measures the impact of product variety on welfare: in equilibrium, $[1 - G(t)] \, L$ different varieties are available to consumers. This is to be expected from related models of product differentiation (cf. Krugman, 1980; Melitz, 2003), where per–capita welfare depends on country size L in exactly the same way.

[4] Some authors (cf., e.g., Egger and Kreickemeier, 2006) apply the Blanchard–Giavazzi (2003) specification of the CES utility index that multiplies the utility integral by $M^{(\rho-1)/\rho} = M^{1/(1-\sigma)}$, where M is a measure of the available product varieties. In that framework, the measure of product differentiation component vanishes in the welfare index. The same applies in the present case with $M(t) = [1 - G(t)]L$. The advantage of this formulation is to abstract from possibly negative welfare impacts of trade liberalization.

4 The Open–Economy Equilibrium

We analyze the interdependence of trade and education in a two–country model. For the self–selection of firms to occur, we assume that trade is associated with fixed costs $f_X > 0$ of exporting. These costs measure exporters' costs to set up and maintain distributional channels in foreign market and take the form of output that has to be produced but cannot be sold. Additionally we assume that export incurs additional variable costs taking the form of iceberg transportation costs so that for one unit of an export good to arrive, $\tau > 1$ units have to be produced and shipped. To simpliy the analysis we concentrate on the case of two symmetrical countries. Symmetry of both countries allows us to consider the equilibrium allocation and prices in one country.[5]

4.1 Trade Equilibrium

Allowing for trade increases the set of possibilities for all active firms. Because of the existence of the sufficiently high fixed costs of exporting,[6] however, only a subset of firms will find it profitable to export. Thus, we establish a second threshold $s > t$ such that only firms with $a \geq s$ engage in trade.

The indifference condition (6) for acquiring education is not affected by the additional options provided by trade; only the equilibrium values of the macroeconomic variables (w, E, P) are affected by trade. To determine the correspondig indifference condition for exporting, we calculate the profits from exports. Let $x_X(a)$ denote the export quantity of firm a sold at export price $p_X(a)$. Profit maximization for the export activity yields the markup–pricing rule for exports as

$$p_X(a) = \tau p(a)\,. \tag{11}$$

The corresponding export profits can be written as:

$$\pi_X = \tau(1-\rho)\frac{w}{\rho}\frac{x_X(a)}{h(a)} - wf_X\,. \tag{12}$$

[5]Note that symmetry does not imply that countries are completely identical. Since some firms will only serve their home market while each firm of both countries produces a different variety of the differentiated good, the varieties available to consumers differ between countries. The equilibrium number of available product varieties, however, will be identical in both countries as well as the prices for the differentiated products consumed.

[6]The corresponding condition for f_X will be derived below.

A firm will now engage in trade if the respective profit is non–negative:

$$\frac{x(a)}{h(a)} \geq \frac{(\sigma-1)f_X}{\tau}\,. \tag{13}$$

Applying the market–clearing condition $c(a) = x_X(a)$ and the pricing rule (11), we get

$$h(a) \geq \left(\frac{(\sigma-1)f_X}{E}\right)^{\frac{1}{\sigma-1}} \left(\frac{w}{\rho}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau}{P}\,.$$

Since $h'(a) > 0$, there exists a unique threshold for exporting firms $s = a(\tau, w, E, P)$ according to

$$h(s) = \left(\frac{(\sigma-1)f_X}{E}\right)^{\frac{1}{\sigma-1}} \left(\frac{w}{\rho}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau}{P}\,. \tag{14}$$

Given the macroeconomic variables (w, E, P), the firms' decision problem about exporting has a unique solution. Comparing the two threshold conditions (6) and (14) indicates that $s > t$ requires $\tau^{\sigma-1} f_X > 1$. We assume this condition to hold since otherwise all active firms would engage in export activities in equilibrium.[7]

In order to solve for the equilibirum, we determine the macroeconomic variables (w, E, P). The mass of domestically produced varieties is given by $M(t) = [1-G(t)]L$ while that of imported varieties amounts to $M(s) = [1-G(s)]L$. Thus, the total numer of varieties available to consumers is given by $[2-G(t)-G(s)]L$. For given threshold values (s, t), the price index P containing domestic varieties as well as imported varieties is

$$\begin{aligned} P(s,t,\tau) &= L^{\frac{1}{1-\sigma}} \left[\int_t^\infty p(a)^{1-\sigma} dG(a) + \tau^{1-\sigma} \int_s^\infty p(a)^{1-\sigma} dG(a)\right]^{\frac{1}{1-\sigma}} \\ &= \frac{w}{\rho} \frac{L^{\frac{1}{1-\sigma}}}{Z(s,t,\tau)}\,, \end{aligned} \tag{15}$$

where

$$Z(s,t,\tau) := \left[\int_t^\infty h(a)^{\sigma-1} dG(a) + \tau^{1-\sigma} \int_s^\infty h(a)^{\sigma-1} dG(a)\right]^{\frac{1}{\sigma-1}}$$

now measures the aggregate stock of human capital, with $\partial Z(.)/\partial s < 0$, $\partial Z(.)/\partial t < 0$ and $\partial Z(.)/\partial \tau < 0$ (cf. the appendix). Note that τ negatively affects aggregate

[7] Melitz applies a similar regularity condition to ensure self–selection of firms. If the above condition is not satisfied, all firms export and the threshold for acquiring education is determined by $[p(a) - w/h(a)][x(a) + \tau x_X(a)] = w(1 + f_X)$.

human capital because part of the productivity must be foregone to cover transport costs in case of exporting.

Substituting in (6) and (14) by (15) gives

$$h(t) = \left(\frac{\sigma-1}{\rho}\frac{wL}{E}\right)^{\frac{1}{\sigma-1}} Z(s,t,\tau) \tag{16}$$

$$h(s) = \tau\left(\frac{(\sigma-1)f_X}{\rho}\frac{wL}{E}\right)^{\frac{1}{\sigma-1}} Z(s,t,\tau)\,. \tag{17}$$

To derive the wL/E we again calculate aggregate profits and wage income as in the closed–economy model. Aggregate profits are now comprised by profits from domestic sales and from exports. Applying our definitions of the different profits, aggregate profits are given by

$$\Pi(a,w,\tau,E,P) = (1-\rho)E - wf_X[1-G(s)]L$$

Adding the wage income of workers $wG(t)L$ and rearranging terms we get wL/E as

$$\frac{E}{wL} = \frac{G(t)-[1-G(s)]f_X}{\rho}\,. \tag{18}$$

Susbstitution for this term in (16) bzw. (17) gives

$$Y_1(s,t,\tau) := h(t) - (\sigma-1)^{\frac{1}{\sigma-1}}\left[G(t)-[1-G(s)]f_X\right]^{\frac{1}{1-\sigma}} Z(s,t,\tau) = 0 \tag{19}$$

$$Y_2(s,t,\tau) := h(s) - \tau\left[(\sigma-1)f_X\right]^{\frac{1}{\sigma-1}}\left[G(t)-[1-G(s)]f_X\right]^{\frac{1}{1-\sigma}} Z(s,t,\tau) = 0. \tag{20}$$

These two conditions determine both thresholds (s,t) as functions of the parameters (τ, f_X, σ)—again, country size is irrelevant for the threshold values—as well as of the underlying density function $g(a)$, and the human–capital function $h(a)$.[8] As shown in the appendix and illustrated in figure 1, both (19) and (20) define a declining curve in (s,t)–space. Since at any point of intersection the slope of the $Y_1 = 0$–curve must be less than the slope of the $Y_2 = 0$–curve, the solution for the threshold values is unique.

Eventually, per–capita welfare $U = E/LP$ is given by

$$W(s,t) = \left[G(t)-[1-G(s)]f_X\right] Z(s,t,\tau) L^{\frac{1}{\sigma-1}}\,. \tag{21}$$

[8]Manasse and Turrini (2001) can be interpreted as a special case of our model where t is given exogenously. Put formally: (19) is substituted by the condition $t = \bar{t}$.

Following the same procedure as in the closed–economy model and defining aggregate productivity of the economy in the presence of trade by

$$\tilde{Z}(s,t,\tau) := [2 - G(t) - G(s)]^{\frac{1}{1-\sigma}} Z(s,t,\tau)\,,$$

we can write the welfare index in an analogous form:

$$W(s,t) = [G(t) - [1 - G(s)]f_X]\,\tilde{Z}(s,t,\tau)\,\{[2 - G(t) - G(s)]L\}^{\frac{1}{\sigma-1}}\ .$$

This again allows a decomposition of per–capita welfare into aggregate production quantities and the number of available products. In equilibrium, $G(t)L$ workers produce at average productivity $\tilde{Z}$. Consequently, $G(t)\tilde{Z}$ can be interpreted as gross aggregate production of the economy. However, $[1 - G(s)]f_X$ units of production are used to cover the fixed costs of exporting goods. This amount of production does not generate welfare, only net aggregate production enters the welfare index.

4.2 Impact of Trade Liberalization

Trade liberalization is interpreted as increased exposure to trade (symmetrical for both countries) and will be modeled as a decline in transportation costs τ. As shown by the comparative–static analysis of the model in the appendix, a decline in τ raises t and reduces s. As with trade liberalization in the Melitz model, the number of active firms decreases while the number of exporters rises. However, the interpretation of results differ slightly from Melitz. In the present model, fewer individuals invest in education thereby reducing the number of firms while at the same time aggregate labor employment rises. The intuition for this result is straightforeward. As transportation costs decline, the demand for exports rises due to the change in relative prices of the differentiated products. Hence exporting becomes profitable for some firms that only served home markets before, and these firms expand their production. Additionally, incumbent exporters face higher demand and also expand their export production. The resulting additional demand for labor is in parts met by the increase in labor supply as fewer individuals invest in education. Additionally, firms serving only the home markets reduce their production and compensate for the rise in labor demand.

With respect to economic welfare we get different counteracting partial effects. First, consider the effects on net aggregate production of the economy. The increase in t caused by trade liberalization raises the economy's aggregate net production by raising both labor employment and the aggregate productivity. On the other hand, the corresponding decline in s raises the amount of production required to cover the fixed costs of exports (thus reducing net production) and it reduces average productivity. Additionally, the reduction of τ raises aggregate net production by driving up aggregate productivity. In sum, the reaction of aggregate net production is ambiguous. Second, the changes in t and s also generate counteracting effects on our measure of available product varieties. As a result of these multiple counteracting effects, the change in aggregate welfare seems to be ambiguous.[9] However, we could not find a proof that ambiguous results are indeed possible. Furthermore, numerical simulations of the model did not yet deliver negative welfare effects of trade liberalization. Maybe we just have not yet found the proof of unambiguous welfare reactions.

Let us relate our results to the welfare implications of trade liberalization derived by Melitz (2003). The expansion of highly–productive firms and the shrinking (or vanishing) of low–producitve firms also cause opposite partial welfare impacts in his model. In his model, however, aggregate welfare can be shown to depend exclusively on the productivity of the least–productive firms.[10] This result depends crucially on the fact that the modelling of the market–entry game ensures average profits of all firms participating in the game (i.e., expected profits before the uncertainty about productivity is resolved) to be zero. As a result of this specific feature, the change in aggregate productivity can be shown to dominate all compensating effects with respect to impacts on aggregate welfare.

5 Conclusions

The present paper has developed an alternative mechanism of explaining the distribution of productivities across firms and their self–selection with respect to export activities. The mechanism is based on the decisions about acquiring education made by heterogeneous individuals. Thus, our apporoach integrates the models developed by Manasse and Turrini (2001) and by Melitz (2003). Most importantly, the impact of trade liberalization on the self–selection of firms is

[9]Even when applying the Blanchard–Giavazzi specification of the CES utility index the problem remains although the measure of product differentiation does not affect welfare.

[10]This would correspond to $h(t)$ in the present context.

preserved by our model. However, the results about the welfare effects of trade liberalization could not (yet) be shown to carry over to our model.

A nice feature of our model is that it provides an explanation of the observed distribution of productivities. Empirical studies (cf., e.g., Del Gatto, Mion and Ottaviano, 2006) find that the distribution of firms' productivities can be reasonably well approximated by a Pareto distribution. Therefore, many extensions or applications of the Melitz model (cf. Helpman, 2006, for an overview) simply postulate productivities to be distributed that way. In our model, a reasonable approximation of the distribution of producitvities can be traced back to the distribution of the individuals' innate abilities. The literature on psychology established that the distribution of inherent abilities can be well approximated by a normal distribution (cf. Wechsler, 1936). Applying that argument to the present model implies the following. As long as the threshold t is sufficiently high, and provided that education transforms abilities into productivities according to a monotonically increasing function, then the normal distribution of abilities generates a good approximation to the observed distribution of productivities. Our approach allows to substitute for the ad–hoc assumption of Pareto–distributed productivities in the market–entry game by an empirically well approved assumption about the distribution of abilities.

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Appendix

The appendix contains details of the analysis of the two–country trade model.

Equilibrium

The conditions for the threshold values for acquiring education and exporting are

$$Y_1(s,t,\tau) \;:=\; h(t) - (\sigma-1)^{\frac{1}{\sigma-1}} A(s,t) Z(s,t,\tau) = 0 \qquad \text{(A.1)}$$

$$Y_2(s,t,\tau) \;:=\; h(s) - \tau\left[(\sigma-1)\, f_X\right]^{\frac{1}{\sigma-1}} A(s,t) Z(s,t,\tau) = 0\,, \qquad \text{(A.2)}$$

where

$$A(s,t) := [G(t) - [1-G(s)] f_X]^{\frac{1}{1-\sigma}}$$

and

$$Z(s,t,\tau) := \left[\int_t^\infty h(a)^{\sigma-1} dG(a) + \tau^{1-\sigma} \int_s^\infty h(a)^{\sigma-1} dG(a)\right]^{\frac{1}{\sigma-1}}.$$

The partial derivatives of the functions A and Z are given by:

$$\begin{aligned}
\frac{\partial A}{\partial t} &= \frac{A^\sigma}{1-\sigma} g(t) < 0 \\
\frac{\partial A}{\partial s} &= \frac{A^\sigma}{1-\sigma} g(s) f_X < 0 \\
\frac{\partial Z}{\partial t} &= -\frac{Z^{2-\sigma}}{\sigma-1} h(t)^{\sigma-1} g(t) < 0 \\
\frac{\partial Z}{\partial s} &= -\frac{Z^{2-\sigma}}{\sigma-1} h(s)^{\sigma-1} g(s) < 0 \\
\frac{\partial Z}{\partial \tau} &= -Z^{2-\sigma} \tau^{-\sigma} \int_s^\infty h(a)^{\sigma-1} dG(a) < 0\,.
\end{aligned}$$

Since all partial derivatives of A and Z with respect to the thresholds (s,t) are negative, the derivatives of the product of both functions must be negative as well. This implies

$$\begin{aligned}
\frac{\partial Y_1}{\partial s} &= -h(t)\left[\frac{\partial A}{\partial s}\frac{1}{A} + \frac{\partial Z}{\partial s}\frac{1}{Z}\right] > 0 \\
\frac{\partial Y_1}{\partial t} &= h'(t) - h(t)\left[\frac{\partial A}{\partial t}\frac{1}{A} + \frac{\partial Z}{\partial t}\frac{1}{Z}\right] > 0 \\
\frac{\partial Y_2}{\partial s} &= h'(s) - h(s)\left[\frac{\partial A}{\partial s}\frac{1}{A} + \frac{\partial Z}{\partial s}\frac{1}{Z}\right] > 0 \\
\frac{\partial Y_2}{\partial t} &= -h(s)\left[\frac{\partial A}{\partial t}\frac{1}{A} + \frac{\partial Z}{\partial t}\frac{1}{Z}\right] > 0\,.
\end{aligned}$$

We can now derive the solpes of the equilibrium curves Y_1 and Y_2 as follows:

$$\left.\frac{ds}{dt}\right|_{Y_1=0} = -\frac{h'(t)/h(t) - \left(\frac{\partial A}{\partial t}\frac{1}{A} + \frac{\partial Z}{\partial t}\frac{1}{Z}\right)}{-\left(\frac{\partial A}{\partial s}\frac{1}{A} + \frac{\partial Z}{\partial s}\frac{1}{Z}\right)} < 0 \tag{A.3}$$

$$\left.\frac{ds}{dt}\right|_{Y_2=0} = -\frac{-\left(\frac{\partial A}{\partial t}\frac{1}{A} + \frac{\partial Z}{\partial t}\frac{1}{Z}\right)}{h'(s)/h(s) - \left(\frac{\partial A}{\partial s}\frac{1}{A} + \frac{\partial Z}{\partial s}\frac{1}{Z}\right)} < 0\,. \tag{A.4}$$

Since the numerator of (A.3) is less than the numerator of (A.4) while the denominator of (A.3) exceeds that of (A.4), we get for the slopes at points of intersection of both curves:

$$\left.\frac{ds}{dt}\right|_{Y_1=0} < \left.\frac{ds}{dt}\right|_{Y_2=0}\,.$$

With both curves having negative slopes, this implies that the $Y_1 = 0$–curve is steeper at any common point than the $Y_2 = 0$–curve. As a result, the equilibrium is unique.

Comparative–Static Effects

The partial derivatives of the functions Y_1 and Y_2 with respect to τ are:

$$\frac{\partial Y_1}{\partial \tau} = -(\sigma-1)^{\frac{1}{\sigma-1}} A \frac{\partial Z}{\partial \tau} > 0 \tag{A.5}$$

$$\frac{\partial Y_2}{\partial \tau} = \frac{h(s)}{\tau}\left[\tau^{1-\sigma} Z^{1-\sigma}\int_s^\infty h(a)^{\sigma-1}dG(a) - 1\right] < 0\,. \tag{A.6}$$

Note that the sign of (A.6) follows from

$$\tau^{1-\sigma}\int_s^\infty h(a)^{\sigma-1}dG(a) < Z^{\sigma-1}\,.$$

The Jacobian J of the system (A.1) and (A.2) is given by:

$$J = \begin{bmatrix} \frac{\partial Y_1}{\partial s} & \frac{\partial Y_1}{\partial t} \\ \frac{\partial Y_2}{\partial s} & \frac{\partial Y_2}{\partial t} \end{bmatrix}.$$

Calculating the determinant of the Jacobian yields:

$$|J| = \frac{\partial Y_1}{\partial s}\frac{\partial Y_2}{\partial t} - \frac{\partial Y_2}{\partial s}\frac{\partial Y_1}{\partial t} < 0\,,$$

where the sign is determined by the slope condition at the point of intersection.

The impact of a change in τ on the equilibrium values of t and s can now be calculated as

$$\begin{aligned}
\left.\frac{ds}{d\tau}\right|_{Y_1=Y_2=0} &= -\frac{\frac{\partial Y_1}{\partial \tau}\frac{\partial Y_2}{\partial t} - \frac{\partial Y_2}{\partial \tau}\frac{\partial Y_1}{\partial t}}{|J|} > 0 \\
\left.\frac{dt}{d\tau}\right|_{Y_1=Y_2=0} &= -\frac{\frac{\partial Y_1}{\partial s}\frac{\partial Y_2}{\partial \tau} - \frac{\partial Y_2}{\partial s}\frac{\partial Y_1}{\partial \tau}}{|J|} < 0\,.
\end{aligned}$$

As indicated by (A.5) and (A.6), an increase in τ shifts down the $Y_1 = 0$–curve while shifting up the $Y_2 = 0$–curve. Since the $Y_1 = 0$–curve is steeper than the $Y_2 = 0$–curve at the initial equilibrium, s rises and t falls.

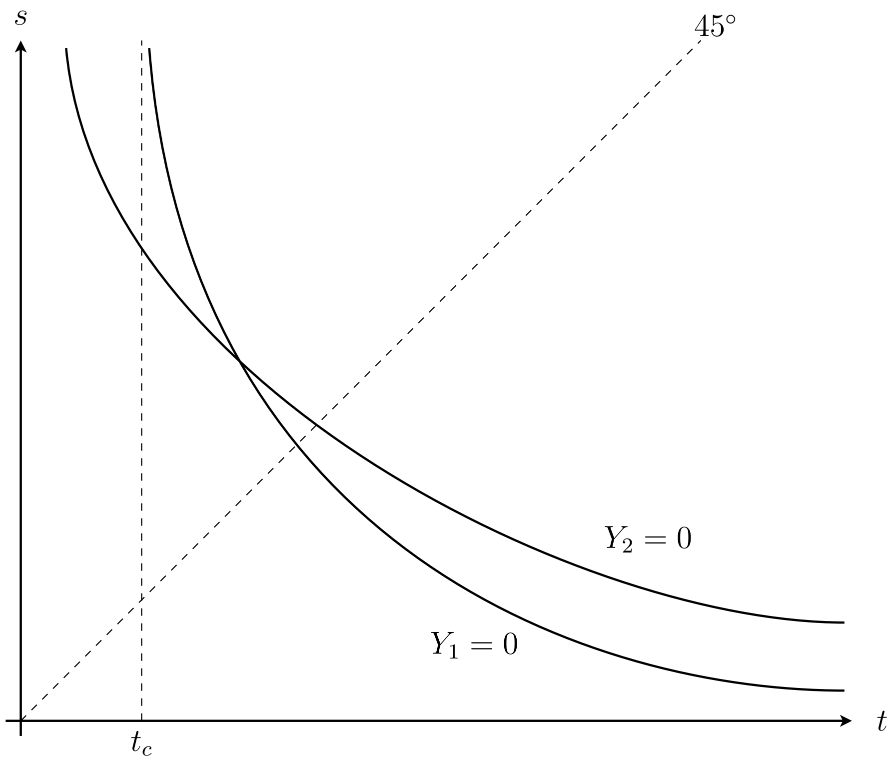

Figure 1: Determination of equilibrium threshold values