

Optimal Tariffs, Retaliation and the Welfare Loss from Tariff Wars in the Melitz Model*

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Abstract

This paper characterizes analytically the optimal tariff of a large one-sector economy with monopolistic competition and firm heterogeneity in general equilibrium, thereby extending the small-country results of Demidova and Rodriguez-Clare (JIE, 2009) and the homogeneous firms framework of Gros (JIE, 1987). The optimal tariff internalizes a markup distortion and a terms of trade externality. It is larger the higher the dispersion of firm-level productivities, and the bigger the country's relative size or relative average productivity. Furthermore, in the two-country Nash equilibrium, tariffs turn out to be strategic substitutes. Small or poor economies set lower Nash tariffs than large or rich ones. Lower transportation costs or smaller fixed market entry costs induce higher equilibrium tariffs and larger welfare losses relative to the case of zero tariffs. Similarly, cross-country productivity or size convergence increases the global welfare loss due to non-cooperative tariff policies. These results suggest that post WWII trends have increased the relative merits of the WTO.

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1 Introduction

So far the literature has analyzed optimal tariffs in abridged versions of the Melitz (2003) trade model with heterogeneous firms. Demidova and Rodriguez-Clare (2009) have provided analytical results for the small economy case; Cole and Davis (2011) derive optimal tariffs in a model with quasi-linear preferences. While these papers have considerably enhanced our understanding of policy in new trade models, their assumptions preclude modeling non-cooperative Nash equilibria between two large economies or rule out general equilibrium effects. In this paper, we provide an analytical characterization of non-cooperative tariff policy in an asymmetric one-sector two-country Melitz (2003) model.

Gros (1987) has studied optimal tariffs and the two-country Nash equilibrium for the Krugman (1980) model of monopolistic competition and trade in differentiated goods. This paper extends Gros (1987) to the case of firms differing with respect to productivity. While *markup pricing* provides a rationale for import tariffs beyond the conventional *terms-of-trade* argument (Johnson, 1953), firm-level productivity heterogeneity may work against tariffs. The reason is that tariffs protect inefficient firms which would otherwise not survive international competition. This leads to a lower level of productivity of the average domestic firm. Allowing for firm heterogeneity therefore has a qualitative and quantitative bearing on the analysis of optimal tariffs and the two-country Nash equilibrium. Demidova and Rodriguez-Clare (2009) show for the small economy case that the markup distortion dominates, ensuring the existence of an optimal tariff. They do not study how this effect interacts with the terms-of-trade channel and what it implies for the two-country Nash equilibrium.

Understanding the incentives of governments to use commercial policy is important for any assessment of the potential welfare gains due to an institution such as the World Trade Organization (WTO). With this objective, the present paper studies non-cooperative tariff policy, retaliation, and welfare in a heterogeneous firms trade model of the Melitz (2003) type where trade is due to product differentiation, producers operate under conditions of monopolistic competition and increasing returns to scale, and international trade is subject to transportation costs. This setup enjoys massive empirical support on the micro level (the market entry decision

of heterogeneous firms) and on the macro level (aggregate trade flows).¹

We present the following results. (i) In a large country, holding market shares fixed, the optimal tariff rises in the elasticity of substitution across varieties and falls in the degree of productivity dispersion; it increases in the relative size of the country and in the freeness of trade (variable and fixed trade costs). Moreover, it can be analytically bounded below and above. Quantitatively, the terms-of-trade externality has similar influence on the size of the optimal tariff than the markup and consumer surplus distortions. (ii) Countries' reaction functions are negatively sloped, i.e., tariffs are strategic substitutes. Retaliation leads to a new equilibrium tariff that is lower than the optimal tariff of a country in the non-retaliation case. (iii) Lower variable trade costs and lower fixed costs of foreign market access lead to higher tariffs in the Nash-equilibrium, while the convergence of country sizes and average productivities lead to higher tariff-induced world welfare losses relative to free trade. Hence, the Melitz (2003) framework suggests that a multilateral trade agreement such as the World Trade Organization (WTO) has become more important in avoiding the welfare damages due to tariff wars as the world has become more symmetric and natural trade barriers have fallen. Our results on import tariffs carry over to policy measures such as the provision of subsidies on the consumption of domestic varieties or ad valorem export taxes. The first policy measure is hard to implement in practice, and the second is rarely observed. Given the overwhelming empirical relevance of import tariffs, we focus on them in the subsequent analysis.²

Our research is related to at least two important strands of literature. The first deals with the endogenous determination of trade policy and the role of the WTO. The literature distinguishes between two general motives for commercial policy: to protect the interests of special lobbying groups (owners of specific assets, trade unions), see Grossman and Helpman (1994), or to simply maximize national welfare. Following Gros (1987) and the ensuing literature, in the present paper, we choose the second option and characterize the ad valorem tariff that maximizes Home's

¹See Bernard et al. (2007a) for a survey on firm-level evidence and Helpman, Melitz and Rubinstein (2008) for evidence on aggregate trade flows.

²Details of the derivation of optimal consumption subsidies and export taxes are available upon request. Recent literature also considers optimal fixed cost subsidies. Pflüger and Südekum (2009) focus on optimal entry fixed subsidies in a model with two large countries and two sectors. Jung (2011) derives optimal entry and operating fixed cost subsidies in a small open economy setting with a single differentiated good sector.

welfare. Maggi and Goldberg (1999) find that the weight of welfare in the government's objective function is many times more important than the weight of special interests, so that our approach seems sensible. It is also consistent with the empirical evidence presented by Broda et al. (2008) who show that countries use tariffs to exploit their market power on international markets.³ Recently, Ossa (2011) has shown a novel motivation for import tariffs when a homogeneous firms, differentiated goods sector is complemented by a numeraire sector with costless transportation of goods, perfect competition and linear technology. In such a framework, wage rates are fixed by technology. There is a new and interesting rationale for import tariffs, as these allow the country to attract additional firms into the sector afflicted by trade costs. If this tariff-induced *delocation effect* dominates the direct import price effect of the tariff on the ideal price index, consumers benefit.⁴ However, in our single-sector setup, additional entry of firms bids up the wage rate, counteracting the delocation effect. Our paper relates to research on the WTO since it sheds light on the role of exogenous trends (country size and productivity convergence, declining transportation costs) in the shaping relative welfare losses due to non-cooperation.⁵ We simulate simple scenarios that are motivated by real-world trends such as the convergence of GDPs across countries and the fall in transportation costs in order to understand how those trends affect countries' incentives to use tariffs.

Second, our paper relates to research on asymmetric versions of the Melitz (2003) model. Falvey et al. (2006) as well as Pflüger and Russek (2010) are examples of papers that derive analytical results under the presence of a numeraire good and (in the latter case) quasi-linear preferences. Our paper is also related to Arkolakis et al. (2011) who work with a more standard version of the core model. None of the mentioned papers investigates optimal trade policy. We appear to be the first to provide an analysis of endogenous import tariffs in full-fledged asymmetric Melitz (2003) model.

The remainder of the paper is structured as follows. Section 2 introduces the model -

³Recent literature also addresses different incentives for government interventions. Antras and Staiger (2011) focus on international cost-shifting incentives in a framework with offshoring and contractual imperfections. Mrazova (2011) considers profit shifting in a world with oligopolistic competition.

⁴See Bagwell and Staiger (2009) for a more general discussion of the delocation argument.

⁵See Rose (2004), Subramanian and Wei (2007) and Tomz et al. (2007). Bagwell and Staiger (2010) survey recent theoretical and empirical literature on the functioning of the WTO.

essentially a version of the Melitz (2003) model with two asymmetric countries and Pareto-distributed firm level productivities. Section 3 studies the effects of a given tariff on model outcomes. Section 4 characterizes the optimal tariff given the other country's tariff rates, and section 5 analyzes the outcome of a non-cooperative Nash game between tariff-setting countries. Section 6 contains our quantitative analysis and section 7 concludes. Analytical details are relegated to an Appendix.

2 Model setup

We consider a world with two countries that differ with respect to their labor forces and average productivities but are otherwise structurally identical. Each worker supplies one unit of labor inelastically and spends income on domestic and imported varieties of a differentiated good. Preferences are given by

$$U_i = \left(\int_{\omega \in \Omega_i} q[\omega]^\rho d\omega \right)^{1/\rho}, \quad 0 < \rho < 1, \quad i \in \{H, F\}, \quad (1)$$

where Ω_i is the set of varieties available in country i , $q[\omega]$ is the quantity of variety ω consumed, $\sigma = 1/(1 - \rho) > 1$ is the elasticity of substitution, H denotes Home and F Foreign. The price index dual to (1) is given by $P_i^{1-\sigma} = \int_{\omega \in \Omega_i} p[\omega]^{1-\sigma} d\omega$. Then, demand for any variety is

$$q[\omega] = R_i P_i^{\sigma-1} p[\omega]^{-\sigma}, \quad (2)$$

where R_i denotes aggregate expenditure.

Labor is the only factor of production. There is a continuum of monopolistically competitive firms, who hire workers on a competitive labor market at wage w_i . Firms pay fixed setup costs $w_i f^e$; thereafter they obtain information about their productivity level φ which is sampled from a Pareto distribution $G_i[\varphi] = 1 - (b_i/\varphi)^\beta$ with b_i the lowest possible productivity draw and $\beta > 2$ the shape parameter. A higher value of b_i is associated to a higher mean, but leaves the coefficient of variation constant.⁶ A higher value of β comes with a lower coefficient of variation.

⁶The mean is $b_i\beta/(\beta - 1)$, the standard deviation is $b_i\sqrt{\beta}/((\beta - 1)\sqrt{\beta - 2})$ and the coefficient of variation

A firm from country i pays fixed market access costs $w_i f_{ij}$ to access consumers in the other market. The marginal costs of a producer with productivity φ are w_i/φ . Given the demand function (2), the price charged (at the factory gates) by that firm is $w_i/(\rho\varphi)$. As usual, we assume that there are symmetric iceberg trade costs $\tau_{ij} = \tau_{ji} \geq 1$, where $\tau_{ii} = 1$. Moreover, country j imposes a tariff on imports from country i t_{ji} , where $t_{ii} = 1$. Operating profits of a firm from country i on market j are $R_j P_j^{\sigma-1} t_{ji}^{-\sigma} (\rho\varphi/\tau_{ij}w_i)^{\sigma-1} / \sigma - w_i f_{ij}$. We denote by φ_{ij}^* the productivity of the country i firm which makes zero profits by entering market j

$$R_j P_j^{\sigma-1} t_{ji}^{-\sigma} \left(\frac{\rho\varphi_{ij}^*}{\tau_{ij}w_i} \right)^{\sigma-1} = \sigma w_i f_{ij}, \quad i \in \{H, F\}, \quad j \in \{H, F\}. \quad (3)$$

This is the **zero cutoff profit condition (ZCP)**. As is customary in this literature, we restrict exogenous parameters such that $\varphi_{ij}^* > \varphi_{ii}^*$ for all i and j . Then, only the most productive firms sell to the foreign markets. Moreover, firms with $\varphi < \varphi_{ii}^*$ do not even sell on the domestic market and remain inactive. Hence, if M_i^e is the mass of entrants in country i , then $M_i = (1 - G[\varphi_{ii}^*]) M_i^e$ denotes the mass of active firms. The mass of exporters is then simply $M_{ij} = m_{ij} M_i$ where the export participation rate $m_{ij} \equiv (1 - G[\varphi_{ij}^*]) / (1 - G[\varphi_{ii}^*]) = (\varphi_{ii}^*/\varphi_{ij}^*)^\beta$ is independent from the scale parameter b_i .

These considerations allow to write the price level P_i as (see Appendix 8.1)

$$P_i^{1-\sigma} = \theta \sum_{j \in \{H, F\}} m_{ji} M_j \left(\frac{\rho\varphi_{ji}^*}{w_j \tau_{ji} t_{ij}} \right)^{\sigma-1}, \quad i \in \{H, F\}, \quad (4)$$

where parameters are restricted such that $\theta \equiv \beta / (\beta - (\sigma - 1))$ is strictly positive. Note that the scale parameter affects price levels only indirectly through endogenous variables m_{ji}, M_j and φ_{ji}^* .

Expected profit from entering is given by $\bar{\pi}_i \equiv E[\pi(\varphi)] = (\theta - 1) w_i \sum_j m_{ij} f_{ij}$ (see Appendix 8.3). Free entry requires that expected profits are equal to entry costs discounted by the probability of successful entry, i.e., $\bar{\pi}_i = w_i f_i^e / (1 - G[\varphi_{ii}^*])$. Using the Pareto distribution and

$1/\sqrt{\beta(\beta-2)}$.

substituting for $\bar{\pi}$, we obtain

$$(\theta - 1) (\varphi_{ii}^*)^{-\beta} \sum_{j \in \{H, F\}} m_{ij} f_{ij} = f_i^e b_i^{-\beta}, \quad i \in \{H, F\}. \quad (5)$$

This is the **free entry condition (FEC)**. Since entry costs and expected profits are both proportional to w_i , wages drop out from this equation. Expected entry costs depend negatively on b_i but expected profits do not depend on b_i . So, a higher value of, for instance, b_H would require either or both cutoffs $\varphi_{HH}^*, \varphi_{HF}^*$ to increase.

Using the **labor market clearing condition (LMC)**, we can express the mass of firms as (see Appendix 8.4)

$$M_i = \frac{L_i}{\sigma \theta \sum_{j \in \{H, F\}} m_{ij} f_{ij}} = \frac{(\theta - 1) b_i^\beta L_i}{\sigma \theta f_i^e} (\varphi_{ii}^*)^{-\beta}, \quad i \in \{H, F\}, \quad (6)$$

where (5) has been used to substitute out the term $\sum_j m_{ij} f_{ij}$.

The **trade balance condition (TBC)** requires that Home's aggregate imports from Foreign are equal to its aggregate exports to Foreign. Average sales \bar{r}_{ij} of a firm located in i from selling to j is given by $\bar{r}_{ij} \equiv E[r(\varphi)] = \sigma \theta w_i m_{ij} f_{ij}$ (see Appendix 8.2). Assuming $f_{HF} = f_{FH} = f^x$, $f_{HH} = f_{FF} = f^d$, $f_H^e = f_F^e = f^e$ and using the wage in Foreign, w_F , as the numeraire, one may write $w_H M_{HF} f^x = M_{FH} f^x$, or, equivalently, $w_H m_{HF} M_H = m_{FH} M_F$. Using (6) and substituting $m_{HF} = (\varphi_{HF}^* / \varphi_{HH}^*)^{-\beta}$, this yields

$$w_H b_H^\beta L_H (\varphi_{HF}^*)^{-\beta} = L_F b_F^\beta (\varphi_{FH}^*)^{-\beta}. \quad (7)$$

Tariff revenue is redistributed in a lump sum fashion to consumers.⁷ The **balanced budget condition** implies that aggregate expenditure R_i is the sum of expenditure spent on domestic varieties and on imported varieties

$$R_i = \sum_{j \in \{H, F\}} t_{ij} M_j \bar{r}_{ji} = \sigma \theta M_i w_i \sum_{j \in \{H, F\}} t_{ij} m_{ij} f_{ij}. \quad (8)$$

⁷In the homogeneous firms model, Ossa (2011) shows that accounting for tariff income is important to guarantee finite levels of optimal tariffs. Schröder and Sørensen (2011) parameterize the *effectiveness* of redistribution.

The second equality follows from using the balanced trade condition and inserting the expression for average sales.

Equilibrium is determined by 4 zero cutoff profit conditions, 2 free entry conditions, 2 labor market clearing conditions, 2 balanced budget conditions, and the balanced trade condition.

3 Effects of a given tariff

Before turning to the welfare-maximizing tariff policy of a country, we consider the impact of exogenous changes in a given import tariff on equilibrium.

Domestic expenditure and revenue shares. For the subsequent analysis, it turns out useful to define the share of revenues earned domestically as

$$\alpha_i \equiv \frac{M_i \bar{r}_{ii}}{M_i \bar{r}_{ii} + M_i \bar{r}_{ij}} = \frac{1}{1 + m_{ij}(f^x/f^d)}. \quad (9)$$

We obtain the second term from using the expressions for average sales and the labor market clearing conditions. The share of domestic revenues is larger, the smaller the export participation rate m_{ij} . Similarly, we write the share of expenditure spent on domestic varieties as

$$\tilde{\alpha}_i \equiv \frac{M_i \bar{r}_{ii}}{M_i \bar{r}_{ii} + t_{ij} M_j \bar{r}_{ji}} = \frac{1}{1 + t_{ij} m_{ij}(f^x/f^d)}, \quad (10)$$

where we have used $\bar{r}_{ji} = \sigma \theta w_i m_{ij} f_{ij}$ and the balanced trade condition $w_i m_{ij} M_i = w_j m_{ji} M_j$ to substitute for w_j/w_i .

Comparing equations (9) and (10) shows that for a country that does not impose an import tariff, the revenue and expenditure shares coincide. The reason is that Home's imports equal Foreign's exports due to the balanced trade condition. A positive tariff $t_{ij} > 1$ drives a wedge between Home's expenditure on imports and Foreign export sales: One fraction of Home's expenditure on imports goes to Foreign firms, the other fraction is Home's tariff revenue. The balanced trade condition, however, links the imports of a country evaluated at *ex-factory prices* to its export sales. Thus, a positive tariff results in a domestic expenditure share that is *smaller*

than the domestic revenue share, i.e., $\tilde{\alpha}_i \leq \alpha_i$. The inequality holds strictly for $t_{ij} > 1$.

Within-industry reallocation and the relative wage. Holding Home's aggregate expenditure and its price index fixed, an increase in Home's import tariff raises the import cutoff productivity level φ_{FH}^* ; see the zero cutoff profit condition (3).⁸ A tariff makes imported varieties more expensive, which leads to decline in demand and lower export sales for Foreign firms. Thus, the least productive Foreign exporters become purely domestic firms.

The import-selection effect also occurs if we take into account general equilibrium adjustments. In order to see this, we write Home's import cutoff condition given by equation (3) relative to its domestic cutoff condition in totally differentiated form (holding trade and fixed costs constant)

$$\rho(\hat{\varphi}_{FH}^* - \hat{\varphi}_{HH}^*) + \hat{w}_H - \hat{t}_{HF} = 0, \quad (11)$$

where $\hat{x} = dx/x$ denotes a percentage change of variable x . If we had perfectly elastic labor demand as in Ossa (2011), i.e., $\hat{w}_H = 0$, the relative import cutoff had to carry the full burden of adjustment to an import tariff increase. In the absence of perfectly elastic labor demand, an increase in Home's relative wage dampens the effect on the relative import cutoff. With perfect substitutes, i.e., $\rho \rightarrow 1$, the change in the relative import cutoff will be smaller. In the absence of firm heterogeneity as in Gros (1987) or with perfect complements, i.e., $\rho \rightarrow 0$, we would only see an adjustment of the wage rate.

Home's domestic entry φ_{HH}^* cutoff is negatively linked to Home's export cutoff φ_{HF}^* through the free entry condition (5). For the average firm with a fixed domestic cutoff, a lower export cutoff increases the probability of exporting and therefore raises expected export sales. In order to restore a zero net value of entry, the probability of successful entry must decline, which implies that the domestic entry cutoff goes up. Formally, we have

$$\hat{\varphi}_{ii}^* = -\frac{1 - \alpha_i}{\alpha_i} \hat{\varphi}_{ij}^* = -m_{ij} \frac{f^x}{f^d} \hat{\varphi}_{ij}^*, \quad (12)$$

where the share of domestic revenues α_i is defined as in equation (9).

⁸Recall that Foreign's wage w_F is chosen as the numeraire.

Home's export cutoff φ_{HF}^* is linked to its relative wage and its import cutoff through the balanced trade condition (7)

$$\hat{\varphi}_{HF}^* = \hat{\varphi}_{FH}^* + \frac{\hat{w}_H}{\beta}. \quad (13)$$

With perfectly elastic labor supply, there is a one-to-one match of changes in Home's import and its export cutoff. Inelastic labor supply results in *asymmetric* adjustment of these two cutoffs.

In changes, Home's export cutoff condition given in equation (3) reads

$$\hat{\varphi}_{HF}^* + \frac{\hat{R}_F}{\sigma - 1} + \hat{P}_F = \frac{1}{\rho} \hat{w}_H, \quad (14)$$

where we assume that Home takes Foreign's tariff t_F as given ($\hat{t}_F = 0$). With fixed Foreign aggregate expenditure and price index, there is a positive link between Home's export cutoff and its wage. In general equilibrium, however, we have to take into account changes in Foreign's aggregate expenditure and Foreign's price index.

Totally differentiating equation (8), we obtain

$$\hat{R}_F = \hat{M}_F + \hat{w}_F + (1 - \tilde{\alpha}_F) (\hat{t}_F + \hat{m}_{FH}) = -\beta \hat{\varphi}_{FF}^* + (1 - \tilde{\alpha}_F) \hat{m}_{FH},$$

where the second equality follows from (i) the choice of numeraire ($\hat{w}_F = 0$), and (ii) the labor market clearing condition (6). Using equation (12) to substitute for $\hat{\varphi}_{FF}^*$ and $\hat{m}_{FH} = -\beta \hat{\varphi}_{FH}^* + \beta \hat{\varphi}_{FF}^*$, we are left with

$$\hat{R}_F = -\beta \frac{\alpha_F - \tilde{\alpha}_F}{\alpha_F} \hat{\varphi}_{FH}^*. \quad (15)$$

Equation (15) implies that Foreign's aggregate expenditure R_F remains constant when Foreign does not impose an import tariff ($t_F = 1$) since then Foreign's domestic revenue and expenditure shares coincide; Foreign's per-capita wage income being fixed by choice of numeraire. In the more general case $t_F > 1$, Foreign's aggregate expenditure is affected through changes in the import volume even if its tariff is hold fixed. A decline in Foreign's import volume reduces its tariff revenue, resulting in less aggregate expenditure.

Totally differentiating the expression for Foreign's price index given in equation (4), we

obtain (see Appendix 8.5)

$$\hat{P}_F = \frac{\tilde{\alpha}_F}{\theta - 1} \hat{\varphi}_{FF}^* + \frac{1 - \tilde{\alpha}_F}{\theta - 1} \hat{\varphi}_{HF}^* + (1 - \tilde{\alpha}_F) \hat{w}_H \quad (16)$$

$$= \frac{1 - \tilde{\alpha}_F}{\theta - 1 + \tilde{\alpha}_F} \hat{\varphi}_{HF}^* - \frac{\tilde{\alpha}_F}{\theta - 1 + \tilde{\alpha}_F} \frac{\hat{R}_F}{\sigma - 1} + \frac{(1 - \tilde{\alpha}_F)(\theta - 1)}{\theta - 1 + \tilde{\alpha}_F} \hat{w}_H, \quad (17)$$

where the second line follows from substituting for $\hat{\varphi}_{FF}^*$ by means of Foreign's domestic entry cutoff condition

$$\hat{P}_F = -\hat{\varphi}_{FF}^* - \frac{\hat{R}_F}{\sigma - 1}. \quad (18)$$

Using equation (15) and (17) to substitute for respectively \hat{R}_F and \hat{P}_F in Home's export cutoff condition (14), employing (13) and remembering that $\theta = \beta/(\beta - \sigma + 1)$ we obtain

$$\hat{w}_H = \xi_H \hat{\varphi}_{HF}^*. \quad (19)$$

The elasticity ξ_H is given as $\rho \leq \xi_H \equiv \beta\rho/(\rho + \alpha_F(\beta - \rho)) < \beta$. If Foreign is large, i.e., the domestic revenue share α_F approaches unity, the elasticity ξ_F asymptotically reaches ρ , which is exactly the elasticity if we fixed Foreign's aggregate expenditure and price level. In the other extreme, $\alpha_F = 0$, we have $\xi_F = \beta$.

The elasticity ξ_H is decreasing in the share of Foreign's domestic revenues α_F , and therefore increasing in Foreign's export participation rate m_{FH} ; see equation (9). Using $\hat{w}_H = \xi_F \hat{\varphi}_{HF}^*$, we can rewrite the balanced trade condition as

$$\hat{\varphi}_{HF}^* = \frac{\beta}{\beta - \xi_H} \hat{\varphi}_{FH}^*. \quad (20)$$

Given that $\xi_H < \beta$, there is a positive relationship between Home's export and import cutoff. Moreover, the percentage change in the import cutoff is translated into a more than proportional change in Home's export cutoff since $\beta/(\beta - \xi_H) > 1$.

Using equations (12), (19), and (20) to substitute for respectively the changes in Home's domestic cutoff, the wage, and Home's export cutoff, we can rewrite equation (11) as

$$\hat{\varphi}_{FH}^* = \left(\frac{\beta\rho}{\beta - \xi_H} \left(\frac{\xi_H}{\rho} + \frac{1 - \alpha_H}{\alpha_H} \right) + \rho \right)^{-1} \hat{t}_H.$$

Since the term in brackets is positive, an increase in Home's import tariff raises its import cutoff in general equilibrium.

Let us summarize the within-industry reallocation effects of a small import tariff. Two observations stand out. First, there is reallocation of resources from more productive to less productive firms. Second, the percentage increase in the export cutoff is larger in Home than in Foreign. This can easily be seen from equation (20). Starting from a symmetric situation with $m_{HF} = m_{FH}$, the larger percentage change in the export cutoff unambiguously translates into a change of the domestic entry cutoff that is larger in Home than in Foreign. This can be seen from equation (12).

The average firm reallocates resources from export to domestic activity. This has important implications for aggregate productivity, which is given by total output (inclusive of loss in transit) per worker. Following Demidova and Rodriguez-Clare (2009, equation (19)), we write industry productivity as $\lambda \varphi_{ii}^{\beta+1} \left[f^d + f^x \left(\varphi_{ii}^*/\varphi_{ij}^* \right)^{\beta-1} \right]$, where λ is a positive constant and $\beta > 1$. Since the domestic entry cutoff and the export participation rate fall in response to Home's import tariff, aggregate productivity falls in both countries. Starting from a symmetric situation, the effect is unambiguously larger in Home than in Foreign.

Product variety. A change in the domestic entry cutoff translates into a change of domestic product variety through the labor market clearing condition given in equation (6). A decline in the domestic entry cutoff reduces the productivity of the average firm, resulting in a higher average price, lower demand, and lower labor input. Full employment then implies that the mass of domestic varieties rises in both countries ($\hat{M}_i = -\beta \hat{\varphi}_{ii}^*$). Starting from a symmetric situation, the effect is unambiguously larger in Home than in Foreign.

The change in import variety $m_{ji}M_j$ is given by $-\beta \hat{\varphi}_{ji}^*$. Thus, import variety falls in both countries in response to an increase in Home's import tariff. The effect of an import tariff on total product variety is *a priori* ambiguous, since domestic variety rises, while import variety falls.

Trade volume. The change in Home's imports, evaluated at ex-factory prices, is given by $\hat{M}_F + \hat{m}_{FH} = -\beta \hat{\varphi}_{FH}^* < 0$. The inequality follows directly from the fact that the variety effect

dominates the export selection effect.

All these results are summarized in the following Proposition:

Proposition 1 (*Effects of a given import tariff*) *An increase in Home's import tariff has the following effects:*

a) *Home's relative wage rises.*

b) *The trade volume, evaluated at ex-factory prices, declines.*

c) *The gap between Home's domestic expenditure and revenue share is widened.*

d) *In both countries, Home and Foreign, the shares of revenues earned on the domestic market go up, the domestic productivity cutoffs decline (anti-selection effect), the export productivity cutoff levels go up (export selection effect), the aggregate productivities fall, domestic product varieties rise, and import varieties fall. The export selection effect is stronger in Home than in Foreign. Starting from a symmetric situation, the domestic anti-selection and the domestic variety effect is stronger in Home than in Foreign.*

Proof. In the text. ■

4 The optimal tariff

In this section, we show that a small tariff raises Home's welfare to the detriment of Foreign. Moreover, we characterize the tariff that maximizes Home's welfare if it takes Foreign's tariff as given.

Welfare can be written as a function of productivity cutoff levels and firm masses (see Appendix 8.6)

$$W_i \equiv U_i^\rho = \theta (\sigma - 1)^\rho \left(M_i \left(f^d \varphi_{ii}^* \right)^\rho + M_{ji} \left(\frac{f^x}{\tau_{ji}} \varphi_{ji}^* \right)^\rho \right), \quad (21)$$

Not surprisingly, welfare is larger the more varieties are available to the consumer (M_i, M_{ji}) and the cheaper the goods are (higher cutoffs φ_{ii}^* and φ_{ji}^* imply lower prices).

Welfare directly depends on variable trade costs τ , but only indirectly on tariffs. The reason is that variable trade costs generate loss in transit, which results in an adjustment for the productivity level of exporters. Tariffs, on the other hand, are a mark-up on the price of imports which is redistributed to consumers.

In terms of percentage changes, we have (see Appendix 8.7)

$$\hat{W}_i = \tilde{\alpha}_i \left(\rho \hat{\varphi}_{ii}^* + \hat{M}_i \right) + (1 - \tilde{\alpha}_i) \left(\rho \hat{\varphi}_{ji}^* + \hat{M}_{ji} \right) = \tilde{\alpha}_i (\rho - \beta) \hat{\varphi}_{ii}^* + (1 - \tilde{\alpha}_i) (\rho - \beta) \hat{\varphi}_{ji}^*, \quad (22)$$

where we have used the labor market clearing conditions to substitute for \hat{M}_i . Since $\beta > 1 > \rho$, the variety effect always dominates the selection effect.

Given that an increase in Home's import tariff lowers the domestic entry cutoffs and increase the import cutoffs in both countries, utility achieved from consumption of domestic varieties increases, whereas utility obtained from consumption of imported varieties goes down.

Welfare effects of a tariff. Using equations (12) and (20) to substitute for respectively $\hat{\varphi}_{HH}^*$ and $\hat{\varphi}_{FH}^*$, we obtain

$$\hat{W}_H = \frac{\beta - \rho}{\beta} \left[-\beta \frac{\alpha_H - \tilde{\alpha}_H}{\alpha_H} + \xi_H (1 - \tilde{\alpha}_H) \right] \hat{\varphi}_{HF}^*. \quad (23)$$

For a zero tariff rate, i.e., $t_H = 1$, the first term vanishes because in this case the share of expenditure on domestic varieties $\tilde{\alpha}_H$ equals the share of domestic revenues α_H . The second term is always positive since $\tilde{\alpha}_H < 1$. Thus, a small import tariff unambiguously raises Home's welfare. The welfare effect of a further increase is *a priori* ambiguous because then the first term becomes negative.

In a similar vein, we can compute the change in Foreign's welfare as

$$\hat{W}_F = \frac{\beta - \rho}{\beta} \left[-\beta \frac{\alpha_F - \tilde{\alpha}_F}{\alpha_F} - \frac{\tilde{\alpha}_F}{\alpha_F} \xi_F (1 - \alpha_F) \right] \hat{\varphi}_{HF}^* < 0.$$

The strict inequality follows from the observations that $\alpha_F \geq \tilde{\alpha}_F$ and Home's export cutoff increases in response to Home's tariff. Thus, Home's welfare gain comes at the expense of Foreign.

Characterization of the optimal tariff. The first order condition of Home's welfare maximization problem is given by $dW_H/dt_H = 0$. Since Home's export cutoff monotonically increases in its tariff, equation (23) implies that Home's optimal tariff T_H is determined by $\beta(\tilde{\alpha}_H - \alpha_H) + \alpha_H \xi_H(1 - \tilde{\alpha}_H) = 0$. Substituting for α_H and $\tilde{\alpha}_H$ by means of equations (9) and (10), we obtain (see Appendix 8.8)

$$T_H = \frac{\beta}{\beta - \xi_H} = 1 + \frac{\rho}{\alpha_F(\beta - \rho)} > 1. \quad (24)$$

As with homogeneous firms, we cannot solve for the optimal tariff in closed form; see Gros (1987). We can nevertheless discuss the following characteristics of the optimal tariff.

The optimal tariff is decreasing in Foreign's share of revenues earned on its domestic market, α_F . The limiting case $\alpha_F \rightarrow 1$ implies a lower bound $\underline{T}_H = \beta/(\beta - \rho)$, which yields the optimal tariff a small open economy imposes.

Recall that $\alpha_F = (1 + m_{FH}(f^x/f^d))^{-1}$. Setting Foreign's export participation rate to unity and substituting the resulting term back into the the optimal tariff formula, we obtain $\bar{T}_H = \frac{\beta + \rho f^x/f^d}{\beta - \rho}$ as the upper bound of the optimal tariff.⁹

Foreign's export participation rate $m_{FH} = (\varphi_{FH}^*/\varphi_{FF}^*)^{-\beta}$ can be rewritten as

$$m_{FH} = w_H^{\frac{\rho - \beta}{\rho}} t_F^{\frac{-\beta}{\rho}} \tau^{-\beta} (F^x)^{\frac{\beta}{1 - \sigma}} LB^\beta, \quad (25)$$

where we have used the trade balance condition given in equation (7) to substitute for φ_{FH}^* , the ratio of the Foreign domestic entry condition and Home's export cutoff condition as given by equation (3), and where $F^x \equiv f^x/f^d$ denotes export fixed costs relative to domestic fixed costs, $L \equiv L_H/L_F$ is relative country size, and $B \equiv (b_H/b_F)^\beta$ is relative productivity.

With a fixed wage rate, Home's optimal tariff decreases in (symmetric) variable trade costs τ and in Foreign's tariff t_F and increases in its relative country size L . We show in Appendix 8.9 that these direct effects also dominate if we take the general equilibrium adjustment of the wage rate into account.

⁹For reasonable parametrization of the elasticity of substitution and Pareto shape parameter ($\sigma = 3.8$ and $\beta = 4$) and with $f^x/f^d = 1.6$, we have $(\underline{T}_H - 1) \times 100 \approx 22.6\%$ and $(\bar{T}_H - 1) \times 100 \approx 58.7\%$.

Proposition 2 (Welfare effects of a tariff and characteristics of the optimal tariff)

- a) A small tariff raises Home's welfare at the detriment of Foreign.
- b) The welfare-maximizing tariff is finite and unique.
- c) The optimal tariff is bounded from above and below by respectively $\bar{T}_H = \frac{\beta + \rho f^x / f^d}{\beta - \rho}$ and $\underline{T}_H = \frac{\beta}{\beta - \rho}$. It is decreasing in variable trade cost τ and increasing in its relative country size $L \equiv L_H / L_F$.
- d) The best response function is downward sloping.

Proof. In the text. ■

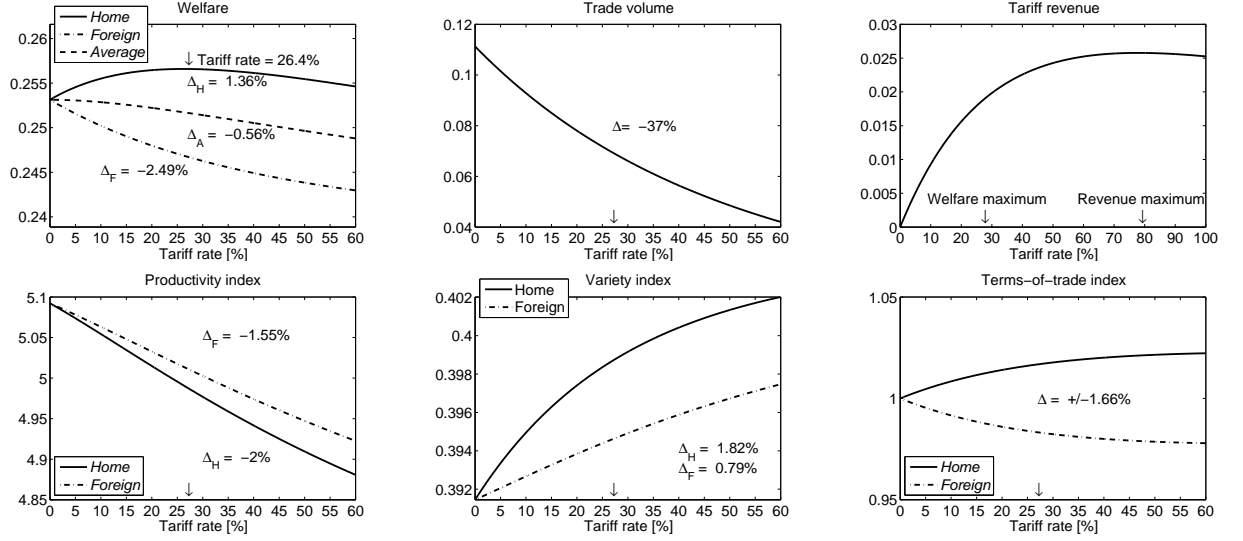
Quantitative illustration. To gain a sense of the magnitudes involved, Figure 1 illustrates the impact of a tariff on both countries quantitatively. Using a standard parameterization of the model following Bernard et al. (2007b), we analyze how Home's import tariff affects otherwise identical countries. The following observations stand out. Home's welfare-maximizing tariff rate is 26.4%, a sensible magnitude.¹⁰ Foreign's utility unambiguously falls in Home's tariff. Average welfare unambiguously falls. By imposing the optimal tariff, Home can increase its welfare level by 1.36% relative to free trade. At the same time, Foreign's welfare loss amounts to 2.49%. Home's gain cannot compensate Foreign's loss, such that average welfare falls by 0.56%.

Home's imports, evaluated at ex-factory prices, fall by 37% if Home goes from free trade to its optimal tariff. By balanced trade, Home's exports decline by the same amount. Home's tariff revenue follows the standard Laffer curve logic. Notice, however, that the horizontal axis is rescaled in order to include the downward sloping part of the tariff revenue curve. The revenue-maximizing tariff rate is close to 80%. Foreign is assumed to allow for free trade, raising no tariff revenue. The drop in aggregate productivity is larger in Home (2%) than in Foreign (1.55%). The rise in product variety is also stronger in Home (1.82%) than in Foreign (0.79%). Home's terms of trade improve, while Foreign's terms of trade deteriorate.¹¹

¹⁰In all graphs, the optimal tariff rate is indicated by an arrow.

¹¹Terms of trade are defined as ratio of the price of exports to the price of imports, weighted by the export share in production over the import share in consumption; see Demidova and Rodriguez-Clare (2009).

Figure 1: Quantitative illustration



Notes: Countries differ in their tariff rates, but are otherwise identical with $\beta = 4$, $\sigma = 3.8$, $\tau = 1.3$, and $f^x/f^d = 1.6$. Foreign allows for free trade.

5 Non-cooperative tariff policy

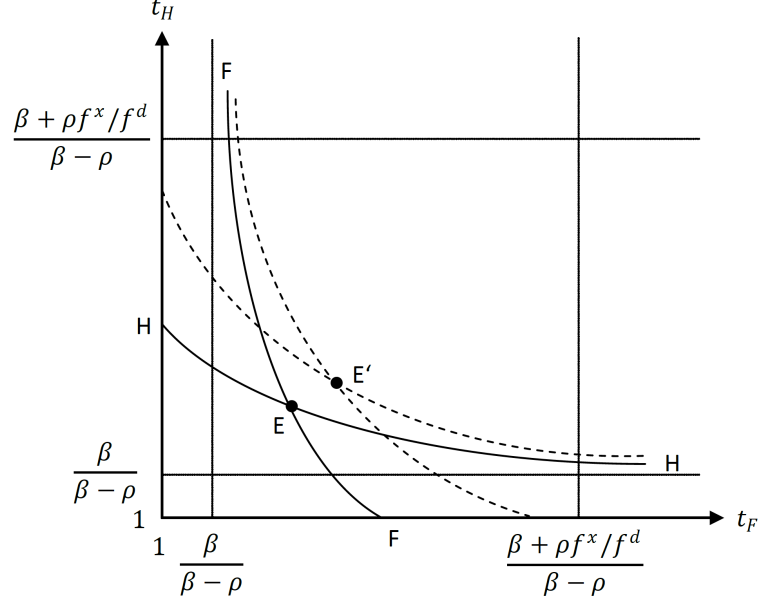
In the previous section, we have assumed Foreign's tariff on imports from Home to be fixed. Now we allow Foreign to choose an optimal tariff, too. Then the setting of tariffs becomes a two-player game. The Nash equilibrium in the uncoordinated game is achieved if no player has an incentive to deviate from the optimal tariff. In this section, we show that Nash equilibrium tariff rates exist and are unique. Moreover, we discuss the characteristics of the Nash equilibrium tariffs.

Existence and uniqueness. Tariffs are strategic substitutes. This observations directly follows from the fact that the best response functions derived above are downward sloping. They asymptotically reach $\underline{T}_H = \beta/(\beta - \rho)$ as the other country's tariff goes to infinity. Moreover, they are bound from above by $\bar{T}_H = (\beta + \rho f^x/f^d)/(\beta - \rho)$. Given these observations, the best response functions must be strictly convex and must have a unique intersection point. The non-cooperative Nash equilibrium is found at this intersection point.

Figure 2 illustrates the best response functions. The HH curve represents Home's best response to Foreign's tariff t_F , whereas the FF curve depicts Foreign's best response function.

The unique Nash equilibrium is reached at point E . The figure also depicts the asymptotes and the upper bounds of the best response functions.

Figure 2: The effect of a decline in natural trade barriers on Nash tariffs



Further characteristics of the Nash equilibrium. Assume for a moment that the two countries are symmetric. Both countries choose their optimal tariff non-cooperatively. In the resulting Nash equilibrium both countries impose the same optimal tariff, i.e., $T_H = T_F \equiv T$, and wages are equalized. The expression for the (symmetric) export participation rate reads $m^x = t^{-\frac{\beta}{\rho}} \tau^{-\beta} (F^x)^{-\frac{\beta}{\sigma-1}}$, and the share of revenue earned on the domestic market is given by $\alpha = \left(1 + t^{-\frac{\beta}{\rho}} \tau^{-\beta} (F^x)^{-\frac{1}{\theta-1}}\right)^{-1}$. Then, the Nash tariffs are implicitly defined by

$$T = 1 + \frac{\rho \left(1 + T^{-\frac{\beta}{\rho}} \tau^{-\beta} (F^x)^{-\frac{1}{\theta-1}}\right)}{\beta - \rho}, \quad (26)$$

where $\theta \equiv \beta / (\beta - \sigma + 1) > 1$. The following observations stand out from equation (26). First, it is easy to check that the Nash tariffs decrease in variable trade cost τ and in export fixed costs relative to domestic fixed costs $F^x \equiv f^x / f^d$ (see Appendix 8.10). Hence, lower natural trade barriers and fixed cost harmonization lead to higher Nash tariffs. Figure 2 also illustrates these

situations. The new best response functions are represented by the dotted lines. The resulting Nash equilibrium is E' .

Second, the Nash tariffs are affected neither by a change in relative country size L nor by a change in relative productivity B . Thus, symmetrically scaling up the world or making it more productive has no impact on the Nash tariffs. The welfare level, on the other hand, positively depends on world population and productivity. The reason is that product variety and average productivity increase in country size and the productivity bound, respectively. The welfare loss in percentage terms of non-cooperative trade policy relative to free trade is unaffected by the size of world population and productivity.

All results are summarized in the following Proposition:

Proposition 3 (*Non-cooperative tariff policy*)

- a) *The Nash equilibrium exists and is unique.*
- b) *A drop in variable trade costs shifts both best response functions upwards. With symmetric countries, both Nash tariffs rise.*
- c) *With symmetric countries, an equiproportional change in population sizes or in productivities does not affect the Nash equilibrium tariffs.*
- d) *With symmetric countries, the percentage welfare loss from non-cooperative tariff policy relative to free trade is not affected by equiproportional changes in country size and productivity.*

Proof. In the text. ■

6 The relative merits of the WTO in the post WWII world

What is the WTO worth in terms of world welfare gains relative to a situation of non-cooperative tariff policy? And how have these gains evolved given three important stylized facts: (i) trade costs unrelated to tariffs (such as transportation or communication costs) have fallen; (ii) countries have reduced fixed regulatory costs on domestic producers more quickly than fixed costs of

foreign market access; (iii) formerly poor Southern countries have emerged on the world economy and have caught up with Northern countries in market sizes (overall GDPs) and productivities (per capita incomes). In this section, we relate these observed trends in exogenous variables to the Nash equilibrium tariff levels predicted by the model. In a nutshell, we find that all these trends increase the welfare gains of outlawing non-cooperative tariff policies. Hence, the WTO is more important than before.

In a first scenario, we analyze how a decline in ad valorem transportation costs (natural trade barriers) from 60% to 30% affects the Nash tariffs and welfare. Hummels (2007) reports that air and ocean transport price indices declined by at least 50% from 1970 to 2004. Table 1 reports the results for two *symmetric* countries with reasonable parametrization of the elasticity of substitution and Pareto shape parameter ($\sigma = 3.8$ and $\beta = 4$). Moreover, fixed foreign market access relative to domestic entry fixed costs are assumed to be $f^x/f^d = 1.6$. The following observations stand out. First, the Nash tariff set by symmetric countries as predicted by our model are sizable, but their magnitude seems reasonable; see the discussion in Ossa (2011) on historic tariff levels. Second, a decline in variable trade costs, other things equal, results in higher Nash tariff rates. When natural protection disappears, countries substitute for it using artificial trade barriers. However, the substitution is not one-for-one and the effective ad-valorem trade barriers (the sum of the Nash tariff plus transportation costs) falls from 83.5% to 27.7% when transportation costs fall from 60% to zero. Third, the welfare loss from non-cooperative tariff policy becomes larger relative to the free trade (WTO) case when natural trade barriers fall. This last observation implies that in a world with shrinking natural trade barriers, the WTO is relatively more important.

Table 1: Decline in natural trade barriers

	Natural trade barriers		
	60%	30%	0%
Welfare change relative to 60% benchmark	n.a.	3.4%	12.7%
Nash tariff rate	23.5%	24.6%	27.7%
Welfare change relative to zero tariff equilibrium	-0.9%	-1.6%	-3.4%

Notes: Natural trade barriers represent the ad valorem equivalent of variable trade costs. Two symmetric countries with $\beta = 4$, $\sigma = 3.8$, and $f^x/f^d = 1.6$.

In a second scenario, we consider different types of fixed cost deregulation. Stylized facts discussed in Felbermayr and Jung (2011) suggest that domestic deregulation has proceeded faster than deregulation of fixed market entry costs. Deregulation of entry fixed costs does not affect optimal tariff levels. The reason is that a decline in entry fixed costs does not affect the profitability of exporting relative to domestic activity. For the same reason, optimal tariffs do not change if domestic and foreign market access costs fall in tandem. Lower market access costs for domestic firms, holding foreign market access costs constant, result in lower optimal tariff levels. The intuition is that reducing domestic market access while leaving foreign market access unaffected, acts as a protective policy. Similarly, deregulating foreign market access, holding domestic market access costs constant, results in higher optimal tariff rates. Table 2 quantifies the effect of harmonization in market access costs.¹² Again, non-cooperative trade policy leads to a welfare loss relative to free trade. Nash tariffs are increasing in liberalization of regulatory barriers. Moreover, in a more harmonized world the welfare loss from tariff wars are larger.

Table 2: Fixed cost deregulation

	Regulatory barriers		
	200%	150%	100%
Welfare change relative to 200% benchmark	n.a.	0.7%	1.7%
Nash tariff rate	24.5%	24.6%	24.9%
Welfare change relative to zero tariff equilibrium	-1.4%	-1.6%	-1.8%

Notes: Regulatory barriers are measured as fixed foreign relative to fixed domestic market access costs, f^x/f^d . Two symmetric countries with $\beta = 4$, $\sigma = 3.8$, and $\tau = 1.3$.

One key trend of the last decades is the emergence of new players in the world economy. Using data from the Penn World Tables 7.0, from 1970 to 2007, the share of emerging countries (all Non-OECD countries except Mexico, Chile, Turkey and those newly created after 1970) in world GDP has increased from less than 30% in 1970 to almost 50% in 2007. This convergence trend has accelerated around 1990 and has remained fairly strong since then. In our model, this fact can be either reproduced by an increase in the relative populations of our two economies, or by productivity convergence, or by a mixture of the two.

¹²Exporters must customize their goods to meet the importing country's technical norms, its health, safety, or environmental norms. Firms undergo costly product labeling and conformity assessment procedures. Harmonization means that discrimination of foreign firms is lowered.

We have shown above that rescaling country size proportionally does affect neither the optimal tariff nor the welfare loss. Table 3 reports the effects of convergence in country size, holding world population constant. Obviously, the small country's share of world real per-capita income increases if the size distribution becomes less skewed. Moreover, the Nash tariff rates converge. The large country's Nash tariff falls from 29.5% to 24.6%, while the small country's Nash tariff only slightly increases from 23.0% to 24.6%. A tariff war harms the relatively small country much more than the relatively large country. As country sizes converge, the welfare loss of the world economy becomes more important. Thus, from a world perspective, size convergence makes the welfare loss of non-cooperative trade policy more severe.

Table 3: Equalization of country size distribution

	Small country's population share				
	10%	20%	30%	40%	50%
<i>Small country's income share</i>					
Free trade	5.8%	14.6%	25.3%	37.3%	50.0%
<i>Nash tariff rates</i>					
Small country	23.0%	23.4%	23.7%	24.1%	24.6%
Large country	29.5%	27.0%	25.8%	25.1%	24.6%
<i>Welfare change relative to zero tariff equilibrium</i>					
Small country	-7.5%	-4.4%	-3.0%	-2.2%	-1.6%
Large country	-0.1%	-0.4%	-0.7%	-1.1%	-1.6%
World level	-0.5%	-1.0%	-1.3%	-1.5%	-1.6%

Notes: The two countries differ in population size, but are otherwise identical with $\beta = 4, \sigma = 3.8, \tau = 1.3$, and $f^x/f^d = 1.6$. World population is normalized to 1.

Finally, we consider catching up and convergence in productivities. Table 4 refers to a situation in which the poor country catches up to the frontier. Catching up is modeled as an increase in the lower bound of the Pareto location parameter. The poor country's share of real per-capita income increases. Moreover, Nash tariffs converge. A tariff war harms the poor country much more than the rich country. Again, the poor country becomes more important as it catches up, which implies that the welfare loss from a world perspective rises.¹³

¹³A limitation of the analysis is that the world becomes richer on average. In Table 5 (Appendix 8.11), we choose the location parameters of the Pareto distribution such that world real per-capita income is constant, evaluated in the free trade equilibrium. It turns out that the conclusions drawn from Table 4 carry over to this scenario.

Table 4: Catching up

	Laggard country's relative productivity				
	20%	40%	60%	80%	100%
<i>Laggard country's country's income share</i>					
Free trade	18.0%	29.7%	38.2%	44.6%	50.0%
<i>Nash tariff rates</i>					
Laggard country	23.5%	23.9%	24.1%	24.4%	24.6%
Leading country	26.0%	25.5%	25.0%	24.7%	24.6%
<i>Welfare change relative to zero tariff equilibrium</i>					
Laggard country	-3.8%	-2.7%	-2.1%	-1.8%	-1.6%
Leading country	-0.5%	-0.9%	-1.1%	-1.4%	-1.6%
World level	-1.1%	-1.4%	-1.5%	-1.5%	-1.6%

Notes: Relative productivity represents the percentage difference between the Pareto location parameter of the laggard and the leading country. The two countries are otherwise identical with $\beta = 4$, $\sigma = 3.8$, $\tau = 1.3$, and $f^x/f^d = 1.6$. World population is normalized to 1.

7 Conclusions

To the best of our knowledge, this paper is the first to address tariff wars between non-cooperative welfare-maximizing governments in the plain-vanilla heterogeneous firms Melitz model.¹⁴ The framework features a mark-up and an entry distortion as well as a conventional terms-of-trade effect. These distortions rationalize strictly positive tariff rates beyond terms of trade considerations from the perspective of a single country. The framework is tractable enough to allow for a number of analytical results. We generalize Gros (1987) to the heterogeneous firms case and Demidova and Rogriguez-Clare (2009) to the case of two large economies. The model provides an understanding of Nash tariff policies when selection effects due to heterogeneous firms and a rather rich description of non-policy related trade costs are present. Computing counterfactual non-cooperative tariffs and the associated welfare effects provides a better quantitative understanding of the beneficial role that the World Trade Organization (WTO) has played. We discuss the role of big trends, such as falling transportation costs or convergence of market sizes and conclude that the benefits of the WTO might have increased over time.

More specifically, we report the following results. First, the optimal tariff is bounded from

¹⁴That is, without making additional structural assumptions (e.g., assuming an outside sector) or ruling out the income effect (by choosing quasi-linear preferences).

below and above. Holding fixed market shares, it is decreasing in the mark-up and in the degree of productivity dispersion. Thus, optimal tariffs are smaller in a world with heterogeneous than with homogeneous firms. Second, changes in variable trade costs and in fixed costs of foreign market access affect the optimal tariff in a similar way. Hence, the optimal tariff increases in the freeness of trade. Third, relative country size and relative productivity have similar effects on the optimal tariff. Thus, a rich country imposes a higher tariff than a poor country. Fourth, tariffs are strategic substitutes. Thus, retaliation leads to lower optimal tariffs than in the non-retaliation case. Welfare, however, is lower in the non-cooperative Nash equilibrium than with zero tariff rates for a reasonable parametrization of the model. Finally, higher freeness of trade leads to higher tariffs in both countries in the Nash equilibrium, while the convergence of country sizes results in convergence of tariffs. In all scenarios, however, the damages to total welfare on world level due to tariff wars increase. Thus, in a world with declining trade cost, harmonization of market entry regulation, and country size convergence, the WTO has become more important.

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8 Appendix

8.1 The price level

We can rewrite the price level P_i as

$$\begin{aligned}
P_i^{1-\sigma} &= \int_{\varphi_{ii}^*}^{\infty} \left(p_i^{fob}[\varphi] \right)^{1-\sigma} M_i \frac{dG[\varphi]}{1-G[\varphi_{ii}^*]} + \sum_{j \neq i} \int_{\varphi_{ji}^*}^{\infty} \left(p_{ji}^{cif}[\varphi] \right)^{1-\sigma} M_j m_{ji} \frac{dG[\varphi]}{1-G[\varphi_{jj}^*]} \\
&= \left(\frac{w_i}{\rho} \right)^{1-\sigma} M_i \int_{\varphi_{ii}^*}^{\infty} \varphi^{\sigma-1} \frac{dG[\varphi]}{1-G[\varphi_{ii}^*]} + \sum_{j \neq i} \left(t_{ij} \tau_{ji} \frac{w_j}{\rho} \right)^{1-\sigma} M_j m_{ji} \int_{\varphi_{ji}^*}^{\infty} \varphi^{\sigma-1} \frac{dG[\varphi]}{1-G[\varphi_{ji}^*]} \\
&= \left(\frac{w_i}{\rho} \right)^{1-\sigma} M_i \theta (\varphi_{ii}^*)^{\sigma-1} + \sum_{j \neq i} \left(t_{ij} \tau_{ji} \frac{w_j}{\rho} \right)^{1-\sigma} M_j m_{ji} \theta (\varphi_{ji}^*)^{\sigma-1} \\
&= \theta \sum_{j \in \{H, F\}} m_{ji} M_j \left(\frac{\rho \varphi_{ji}^*}{w_j \tau_{ji} t_{ij}} \right)^{\sigma-1},
\end{aligned}$$

where $p_i^{fob}[\varphi] = \frac{w_i}{\rho \varphi}$, $p_{ji}^{cif}[\varphi] = t_{ij} \tau_{ji} \frac{w_j}{\rho \varphi}$ and $\theta \equiv \beta / (\beta - \sigma + 1)$.

8.2 Average sales of a firm

Average sales of a firm located in country i from activity in country j \bar{r}_{ij} can be written as

$$\begin{aligned}
\bar{r}_{ij} &= \int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi) \frac{dG[\varphi]}{1-G[\varphi_{ii}^*]} \\
&= R_j P_j^{\sigma-1} \left(\frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} t_{ji}^{-\sigma} m_{ij} \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \frac{dG[\varphi]}{1-G[\varphi_{ij}^*]} \\
&= R_j P_j^{\sigma-1} \left(\frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} t_{ji}^{-\sigma} m_{ij} \theta (\varphi_{ij}^*)^{\sigma-1} \\
&= R_j P_j^{\sigma-1} \left(\frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} t_{ji}^{-\sigma} m_{ij} \theta \sigma w_i f_{ij} R_j^{-1} P_j^{1-\sigma} \tau_{ji}^{\sigma} \left(\frac{t_{ij} w_i}{\rho} \right)^{\sigma-1} \\
&= \theta \sigma m_{ij} w_i f_{ij},
\end{aligned}$$

where we used the zero cutoff profit condition (3) to replace $(\varphi_{ij}^*)^{\sigma-1}$.

8.3 Average profits of a firm

Similarly, expected average profits of a firm in country i from activity in country j $\bar{\pi}_{ij}$ are given by

$$\begin{aligned}
 \bar{\pi}_{ij} &= \int_{\varphi_{ij}^*}^{\infty} \pi_{ij}(\varphi) \frac{dG[\varphi]}{1 - G[\varphi_{ii}^*]} \\
 &= m_{ij} \int_{\varphi_{ij}^*}^{\infty} \left(\frac{r_{ij}(\varphi)}{\sigma} - w_i f_{ij} \right) \frac{dG[\varphi]}{1 - G[\varphi_{ij}^*]} \\
 &= m_{ij} (\theta w_i f_{ij} - w_i f_{ij}) \\
 &= m_{ij} w_i f_{ij} (\theta - 1).
 \end{aligned}$$

Expected profit from entering is given by $\bar{\pi}_i \equiv E[\pi[\varphi]] = (\theta - 1) w_i \sum_j m_{ij} f_{ij}$.

8.4 Mass of a firms

The mass of firms can be found by noting that labor supply L_i must equal labor demand for product development, $M_i^e f_i^e$, market access, $M_i \sum_j m_{ij} f_{ij}$, and the production activity, $M_i \sum_j \int_{\varphi_{ij}^*}^{\infty} \frac{\tau_{ij} q_{ij}(\varphi)}{\varphi} \frac{dG[\varphi]}{1 - G[\varphi_{ij}^*]}$. Using $M_i = (1 - G[\varphi_{ii}^*]) M_i^e$, optimal demand from equation (2), $\bar{\pi}_i = w_i f_i^e / (1 - G[\varphi_{ii}^*])$, the zero cutoff profit condition (3) and $\bar{\pi}_i = (\theta - 1) w_i \sum_j m_{ij} f_{ij}$ gives

us

$$\begin{aligned}
L_i &= M_i^e f_i^e + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j \int_{\varphi_{ij}^*}^{\infty} \frac{\tau_{ij} q_{ij}[\varphi]}{\varphi} \frac{dG[\varphi]}{1 - G[\varphi_{ij}^*]} \\
&= \frac{M_i}{1 - G[\varphi_{ii}^*]} f_i^e + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j m_{ij} R_j P_j^{\sigma-1} \tau_{ij}^{1-\sigma} \left(t_{ji} \frac{w_i}{\rho} \right)^{-\sigma} \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \frac{dG[\varphi]}{1 - G[\varphi_{ij}^*]} \\
&= M_i \frac{\bar{\pi}_i}{w_i} + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j m_{ij} R_j P_j^{\sigma-1} \tau_{ij}^{1-\sigma} \left(t_{ji} \frac{w_i}{\rho} \right)^{-\sigma} \theta (\varphi_{ij}^*)^{\sigma-1} \\
&= M_i \frac{\bar{\pi}_i}{w_i} + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j m_{ij} R_j P_j^{\sigma-1} \tau_{ij}^{1-\sigma} \left(t_{ji} \frac{w_i}{\rho} \right)^{-\sigma} \theta (\tau_{ij} w_i)^{\sigma-1} \sigma w_i f_{ij} R_j^{-1} P_j^{1-\sigma} t_{ji}^{\sigma} \rho^{1-\sigma} \\
&= M_i (\theta - 1) \sum_j m_{ij} f_{ij} + M_i \sum_j m_{ij} f_{ij} + \theta M_i (\sigma - 1) \sum_j m_{ij} f_{ij} \\
&= M_i \theta \sum_j m_{ij} f_{ij} + \theta M_i (\sigma - 1) \sum_j m_{ij} f_{ij} \\
&= M_i \theta \sigma \sum_j m_{ij} f_{ij}.
\end{aligned}$$

Solving for M_i and using the free entry condition yields

$$M_i = \frac{L_i}{\sigma \theta \sum_j m_{ij} f_{ij}} = \frac{(\theta - 1) b^\beta L_i}{\sigma \theta f_i^e} (\varphi_{ii}^*)^{-\beta}. \quad (27)$$

8.5 Derivation of the Foreign's price index in percentage changes

Taking the definition of the price index given in equation (4), using equation (6) and assuming $b_F = b_H$, we can write:

$$P_F^{1-\sigma} = \frac{\theta(\theta - 1) b^\beta \rho^{\sigma-1}}{\sigma \theta f^e} \left[L_H w_H^{1-\sigma} (\varphi_{HF}^*)^{\sigma-1-\beta} \tau_{HF}^{1-\sigma} t_{FH}^{1-\sigma} + L_F (\varphi_{FF}^*)^{\sigma-1-\beta} \right]. \quad (28)$$

In changes, this expression reads as follows

$$\begin{aligned}
\hat{P}_F^{1-\sigma} &= \frac{w_H^{1-\sigma} L_H (\varphi_{HF}^*)^{\sigma-1-\beta} \tau_{HF}^{1-\sigma} t_{FH}^{1-\sigma} \hat{w}_H + (\sigma - 1 - \beta)/(1 - \sigma) w_H^{1-\sigma} L_H (\varphi_{HF}^*)^{\sigma-1-\beta} \tau_{HF}^{1-\sigma} t_{FH}^{1-\sigma} \hat{\varphi}_{HF}^*}{L_H w_H^{1-\sigma} (\varphi_{HF}^*)^{\sigma-1-\beta} \tau_{HF}^{1-\sigma} t_{FH}^{1-\sigma} + L_F (\varphi_{FF}^*)^{\sigma-1-\beta}} \\
&\quad + \frac{(\sigma - 1 - \beta)/(1 - \sigma) L_F \hat{\varphi}_{FF}^*}{L_H w_H^{1-\sigma} (\varphi_{HF}^*)^{\sigma-1-\beta} \tau_{HF}^{1-\sigma} t_{FH}^{1-\sigma} + L_F (\varphi_{FF}^*)^{\sigma-1-\beta}}.
\end{aligned}$$

Now using ratios of equation (3) to substitute $f^x/f^d = t_{FH}^{-\sigma} \left(\frac{\varphi_{HF}^*}{\varphi_{FF}^* \tau_{HF} w_H} \right)^{\sigma-1} w_H^{-1}$, equation (7) to substitute $\varphi_{FH}^\beta = \frac{L_F}{L_H} \frac{1}{w_H} (\varphi_{HF}^*)^\beta$, and employing the definition of $\tilde{\alpha}_F$, we end up with (16).

8.6 Derivation of welfare expression

Using the expressions for optimal demand (2) and the cutoff conditions (3), we can rewrite utility as (1)

$$\begin{aligned}
U^\rho &= \sum_j M_j \int_{\varphi_{ji}^*}^{\infty} (q[\varphi])^\rho \frac{dG[\varphi]}{1 - G[\varphi_{jj}^*]} \\
&= \sum_j m_{ji} M_j R_i^\rho P_i^{(\sigma-1)\rho} \left(t_{ij} \tau_{ji} \frac{w_j}{\rho} \right)^{1-\sigma} \int_{\varphi_{ji}^*}^{\infty} \varphi^{\sigma-1} \frac{dG[\varphi]}{1 - G[\varphi_{ji}^*]} \\
&= \theta \sum_j m_{ji} M_j R_i^\rho P_i^{(\sigma-1)\rho} \left(t_{ij} \tau_{ji} \frac{w_j}{\rho} \right)^{1-\sigma} (\varphi_{ji}^*)^{\sigma-1} \\
&= \theta \sum_j m_{ji} M_j (\sigma w_j f_{ji})^\rho t_{ij}^{\sigma-1} \left(\frac{\tau_{ji} w_j}{\rho} \right)^{(\sigma-1)\rho} (\varphi_{ji}^*)^{(1-\sigma)\rho} \left(t_{ij} \tau_{ji} \frac{w_j}{\rho} \right)^{1-\sigma} (\varphi_{ji}^*)^{\sigma-1} \\
&= \theta (\sigma-1)^\rho \sum_j m_{ji} M_j \left(\frac{f_{ji}}{\tau_{ji}} \varphi_{ji}^* \right)^\rho.
\end{aligned}$$

8.7 Derivation of welfare expression in percentage changes

Expressing welfare as given in equation (21) in percentage changes leads to:

$$\hat{W}_i = \frac{M_i (f^d \varphi_{ii}^*)^\rho}{M_i (f^d \varphi_{ii}^*)^\rho + M_{ji} \left(\frac{f^x}{\tau_{ji}} \varphi_{ji}^* \right)^\rho} \left(\rho \hat{\varphi}_{ii}^* + \hat{M}_i \right) + \frac{M_{ji} \left(\frac{f^x}{\tau_{ji}} \varphi_{ji}^* \right)^\rho}{M_i (f^d \varphi_{ii}^*)^\rho + M_{ji} \left(\frac{f^x}{\tau_{ji}} \varphi_{ji}^* \right)^\rho} \left(\rho \hat{\varphi}_{ji}^* + \hat{M}_{ji} \right)$$

Using ratios of (3), $t_{ij}^{-\sigma} \left(\frac{\varphi_{ji}^* w_i}{\varphi_{ii}^* \tau_{ji} w_j} \right)^{\sigma-1} = \frac{w_j f^x}{w_i f^d}$, the balanced trade condition $w_i m_{ij} M_i = w_j M_{ji}$ and the definition of $\tilde{\alpha}_i$ leads to (22).

8.8 Derivation of the optimal tariff

By rearranging the terms in brackets in equation (23), we obtain

$$\left(1 - \frac{\tilde{\alpha}_H}{a_H} \right) (1 - \tilde{\alpha}_H)^{-1} = \frac{\xi_H}{\beta}.$$

Substituting for α_H and $\tilde{\alpha}_H$ by means of equations (9) and (10), we obtain

$$\frac{T_H m_{HF}(f^x/f^d) - m_{HF}(f^x/f^d)}{T_H m_{HF}(f^x/f^d)} = \frac{\xi_H}{\beta} \Leftrightarrow 1 - T_H^{-1} = \frac{\xi_H}{\beta} \Leftrightarrow T_H = \frac{\beta}{\beta - \xi_H}.$$

8.9 Characteristics of the optimal tariff in general equilibrium

We characterize the optimal tariff by adding the optimal tariff formula (24) to the system of equilibrium conditions. As in section 3, we log-linearize the system, but **now** take exogenous changes in variable trade costs and country size, into account.

Home's import cutoff relative to its domestic cutoff. The ratio of Home's domestic and import cutoff conditions (11) generalizes to

$$\begin{aligned} \rho(\hat{\varphi}_{FH}^* - \hat{\varphi}_{HH}^*) + \hat{w}_H - \hat{t}_H - \rho\hat{\tau} &= 0 \\ \Leftrightarrow \rho\hat{\varphi}_{FH}^* + \rho\frac{1-\alpha_H}{\alpha_H}\hat{\varphi}_{HF}^* + \hat{w}_H - \hat{t}_H - \rho\hat{\tau} &= 0. \end{aligned} \quad (11')$$

Notice that we have used the free entry condition (12).

Balanced trade condition. In changes, the balanced trade condition (13) extends to

$$\hat{\varphi}_{HF}^* = \hat{\varphi}_{FH}^* + \frac{\hat{w}_H}{\beta} + \frac{\hat{L}}{\beta}, \quad (13')$$

where $\hat{L} = \hat{L}_H - \hat{L}_F$ is the change in relative country size.

Home's export cutoff condition. Home's export cutoff condition (14) generalizes to

$$\hat{P}_F + \frac{\hat{R}_F}{\sigma-1} + \hat{\varphi}_{HF}^* = \frac{\hat{w}_H}{\rho} + \hat{\tau} + \frac{\hat{t}_F}{\rho}. \quad (14')$$

Home's optimal tariff. Totally differentiating the optimal tariff formula (24), we obtain

$$\hat{T}_H = -\frac{\xi_H}{\beta}\hat{\alpha}_F = \xi_H(1-\alpha_F)(\hat{\varphi}_{FF}^* - \hat{\varphi}_{FH}^*) = -\xi_H\frac{1-\alpha_F}{\alpha_F}\hat{\varphi}_{FH}^*, \quad (30)$$

where we have used $\hat{\alpha}_F = -(1 - \alpha_F)\hat{m}_{FH}$ and the free entry condition (12). Home's optimal tariff unambiguously declines in Foreign's export cutoff.

Balanced trade condition, continued. Using Home's import cutoff relative to its domestic cutoff condition (11') to substitute for \hat{w}_H in the balanced trade condition (13'), we obtain

$$\left(1 + \frac{\rho}{\beta} \frac{1 - \alpha_H}{\alpha_H}\right) \hat{\varphi}_{HF}^* = \frac{\beta - \rho}{\beta} \hat{\varphi}_{FH}^* + \frac{\hat{t}_H}{\beta} + \frac{\rho}{\beta} \hat{\tau} + \frac{\hat{L}}{\beta}.$$

Noting that

$$1 + \frac{\rho}{\beta} \frac{1 - \alpha_H}{\alpha_H} = \frac{\beta - \rho}{\beta} T_F,$$

we have

$$\hat{\varphi}_{HF}^* = \frac{\hat{\varphi}_{FH}^*}{T_F} + \frac{\hat{t}_H}{(\beta - \rho) T_F} + \frac{\rho \hat{\tau}}{(\beta - \rho) T_F} + \frac{\hat{L}}{(\beta - \rho) T_F}.$$

Moreover, we know that $\hat{t}_H = \hat{T}_H$. Hence, using equation (30) to substitute for \hat{t}_H we obtain

$$\hat{\varphi}_{HF}^* = \left(1 - \frac{\xi_H(1 - \alpha_F)}{\alpha_F(\beta - \rho)}\right) \frac{\hat{\varphi}_{FH}^*}{T_F} + \frac{\rho \hat{\tau}}{(\beta - \rho) T_F} + \frac{\hat{L}}{(\beta - \rho) T_F}. \quad (31)$$

Home's export cutoff condition, continued. Using Foreign's domestic entry condition given in equation (18) and the trade balance condition (13') to substitute for respectively \hat{P}_F and \hat{w}_H in Home's export cutoff condition (14'), we get

$$-\hat{\varphi}_{FF}^* - \frac{\beta - \rho}{\rho} \hat{\varphi}_{HF}^* = -\frac{\beta}{\rho} \hat{\varphi}_{FH}^* - \frac{\hat{L}}{\rho} + \hat{\tau} + \frac{\hat{t}_F}{\rho}.$$

Substituting for $\hat{\varphi}_{FF}^*$ by means of the free entry condition given in equation (12) and multiplying both sides with $\rho/(\beta - \rho)$, we are left with

$$T_H \hat{\varphi}_{FH}^* - \hat{\varphi}_{HF}^* = -\frac{\hat{L}}{\beta - \rho} + \frac{\rho}{\beta - \rho} \hat{\tau} + \frac{\hat{t}_F}{\beta - \rho}, \quad (32)$$

where we have used $T_H = \left(\frac{1 - \alpha_F}{\alpha_F} + \frac{\beta}{\rho}\right) \frac{\rho}{\beta - \rho}$.

Characteristics of the optimal tariff. Equations (31) and (32) relate changes in both countries export cutoffs to changes in exogenous variables. Using equation (31) to substitute for

$\hat{\varphi}_{HF}^*$ in equation (32), we obtain

$$\left(T_H T_F - 1 + \frac{\xi_H (1 - \alpha_F)}{\alpha_F (\beta - \rho)} \right) \hat{\varphi}_{FH}^* = -\frac{T_F - 1}{\beta - \rho} \hat{L} + \frac{(T_F + 1) \rho}{\beta - \rho} \hat{\tau} + \frac{T_F}{\beta - \rho} \hat{t}_F.$$

The terms in brackets on the left hand side is positive since $T_H T_F - 1 \geq 0$. The following results start out. First, an equiproportional increase in country size does not affect Foreign's export cutoff and thus leaves the optimal tariff unchanged. An increase in Home's relative country size lowers Foreign's export cutoff and therefore raises Home's optimal tariff if Foreign's tariff rate is positive. Second, a decline in variable trade costs reduces Foreign's export cutoff and raises Home's optimal tariff. Third, an exogenous increase in Foreign's tariff rate increases Foreign's export cutoff and reduces Home's optimal tariff. Hence, Home's best response function is downward sloping.

8.10 Characteristics of Nash tariffs

First reformulate equation (26) by multiplying the left and right hand side with $\beta - \rho$

$$(\beta - \rho) T = \beta + \rho T^{-\frac{\beta}{\rho}} \tau^{-\beta} (F^x)^{-\frac{1}{\theta-1}}$$

Taking the total differential with respect to T and t leads to

$$(\beta - \rho) dT = -\beta T^{-\frac{\beta}{\rho}-1} \tau^{-\beta} (F^x)^{-\frac{1}{\theta-1}} dT - \beta \rho T^{-\frac{\beta}{\rho}} \tau^{-\beta-1} (F^x)^{-\frac{1}{\theta-1}} d\tau$$

$$\frac{dT}{d\tau} = \frac{-\beta \rho T^{-\frac{\beta}{\rho}} \tau^{-\beta-1} (F^x)^{-\frac{1}{\theta-1}}}{\beta - \rho + \beta T^{-\frac{\beta}{\rho}-1} \tau^{-\beta} (F^x)^{-\frac{1}{\theta-1}}} < 0.$$

Similarly

$$\frac{dT}{dF^x} = \frac{-1/(\theta - 1) \rho T^{-\frac{\beta}{\rho}} \tau^{-\beta} (F^x)^{-\frac{1}{\theta-1}-1}}{\beta - \rho + \beta T^{-\frac{\beta}{\rho}-1} \tau^{-\beta} (F^x)^{-\frac{1}{\theta-1}}} < 0.$$

8.11 Additional numerical results

Table 5: Productivity convergence with fixed world income

	Laggard country's relative productivity		
	4.1%	64.5%	100%
<i>Laggard country's income share</i>			
Free trade	4.9%	26.8%	50.0%
<i>Nash tariff rates</i>			
Laggard country	23.0%	23.8%	24.6%
Leading country	30.1%	25.7%	24.6%
<i>Welfare change relative to zero tariff equilibrium</i>			
Laggard country	-8.6%	-2.8%	-1.6%
Leading country	-0.05%	-0.8%	-1.6%
World level	-0.5%	-1.3%	-1.6%
<i>Memo: Pareto location parameter</i>			
Laggard country	0.082	0.517	1
Leading country	2	1.5	1

Notes: World income is the same across all specifications. The two countries differ in the location parameter of the productivity distribution, but are otherwise identical with $\beta = 4$, $\sigma = 3.8$, $\tau = 1.3$, and $f^x/f^d = 1.6$. World population is normalized to 1.