Trade Policy:
Home Market Effect versus Terms-of-Trade Externality

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Abstract

We study trade policy in a two-sector Krugman (1980) trade model, allowing alternatively for production subsidies, import tariffs or export subsidies. For each instrument, we consider the unilateral trade policy without retaliation, the Nash solution and the cooperative solution and contrast those with the efficient allocation. While previous studies have identified the home market externality, which gives incentives to agglomerate firms in the domestic economy, as the driving force behind non-cooperative trade policy in this model, we show that this, in fact, is never the case. Instead, the prevailing incentives for a non-cooperative trade policy arise from the desire to eliminate monopolistic distortions and to improve domestic terms of trade. As a consequence, uncoordinated trade policies are not necessarily protectionist and allowing countries to set production or export subsidies strategically can be welfare improving compared to the free trade equilibrium. The implications are relevant: the Krugman (1980) model provides no rationale for prohibiting production and export subsidies.

Keywords: Home Market Effect, Terms of Trade, Tariffs and Subsidies
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1 Introduction

The aim of this paper is to study optimal trade policy in the canonical two-sector Krugman (1980) model, where one sector is characterized by monopolistic competition, increasing returns and iceberg trade costs, while the other features perfect competition and constant returns. Within this framework we study cooperative, unilateral and strategic (Nash) production subsidies/taxes, import subsidies/tariffs as well as export subsidies/taxes.

The common wisdom of the literature[1] (Venables (1987), Helpman and Krugman (1989), Ossa (2011)) is that in this model unilateral trade policy is set so as to agglomerate firms in the domestic economy. As first shown by Venables (1987), in the presence of transport costs an import tariff can lower the domestic price index, thereby increasing domestic welfare. An import tariff makes foreign differentiated goods more expensive relative to domestic ones so that domestic consumers shift expenditure towards domestic differentiated goods. As a consequence, domestic firms sell more thus making profits and foreign firms sell less thus making losses. This triggers entry into the domestic differentiated sector and exit out of the foreign differentiated sector, thereby reducing the domestic price index – since now less of the domestically consumed goods are subject to transport costs – and increasing the foreign one. Similarly, a production or an export subsidy also renders the domestic market a more attractive location and reduces the domestic price index at the expense of increasing the foreign one. According to the literature, this home market externality (also called production relocation externality) provides a reason for protectionist and ultimately welfare detrimental unilateral trade policy in the Krugman (1980) model and, as argued by Ossa (2011), gives an alternative theoretical justification to the neoclassical terms-of-trade externality explanation (Johnson (1953-1954), Grossman and Helpman (1985) and Bagwell and Staiger (1999)) for the World Trade Organization (WTO)'s rules on tariffs. More importantly, the same mechanism also provides a theoretical justification for the WTO's limitation of production and export subsidies,[2] which cannot be explained within the neoclassical framework.[3]

[1] A detailed review of the literature is provided in the next section.

[2] See, e.g., WTO (2006). GATT Article XVI and the Uruguay Round Subsidies Code prohibit the use of export subsidies, while the second also establishes that countervailing duties can be imposed on countries using production subsidies subject to an injury test.

[3] Production and export subsidies are puzzling within the neoclassical framework because they increase foreign welfare at the expense of domestic welfare.

Our contribution is twofold. First, we show that, in contrast to the previous literature, the home market externality in fact cannot help explain unilateral or strategic trade policy in the Krugman (1980) model. Instead, trade policy outcomes are driven by the domestic policy maker's attempt to alleviate the underprovision of varieties in the differentiated sector due to monopolistic distortions, and to improve domestic terms of trade. Second, the implications of this result are very relevant: Although the Krugman (1980) model indeed provides a justification for the prohibition of import tariffs, it cannot help justify the prohibition of production and export subsidies, as unilateral and Nash-equilibrium subsidies may be welfare increasing relative to the free trade equilibrium.[4]

To give a concrete example, consider production subsidies. Given the monopolistic distortions, the price level in the free trade equilibrium is inefficiently high.[5] A unilateral production subsidy reduces it through three channels. First, the direct impact of the subsidy is to reduce the price of domestically produced varieties. Second, by increasing entry in the monopolistically competitive sector, the world number of varieties rises (love for variety effect). Third, the subsidy leads to an agglomeration of firms in the domestic economy, reducing transport costs for domestic consumers (home market effect). However, this comes at the cost of a lower income due to a negative terms-of-trade effect. We show that the balance always tips in favor of the terms-of-trade effect before monopolistic distortions are completely eliminated: while optimal (unilateral and Nash) production subsidies are positive, they are always lower than the cooperatively set ones. Thus, the home market externality cannot explain production subsidies. Moreover, the equilibrium with non-cooperative trade policy entails welfare gains compared to the free trade equilibrium because it mitigates underprovision of variety.

The result on production subsidies makes it clear that policy makers' incentives are, to a large extent, driven by the desire to eliminate monopolistic distortions, and that deviations from the free trade allocation which increase variety are welfare improving. We show that these findings also apply to the cases of import tariffs and export subsidies. The difficulty in disentangling incentives arises because both an increase in the world number of varieties and

[4]More precisely, this is always the case for production subsidies and can be also true for export subsidies.

[5]The inefficiency arises because there are two sectors in the model, so that monopolistic markups lead to a too low provision of variety in the monopolistically competitive sector. In their seminal paper, Dixit and Stiglitz (1977) show that the market solution is not first-best Pareto optimal in such a model, and that subsidies on fixed costs and on marginal costs are required to implement it.

the home market effect can reduce the domestic price level. Although policy makers sometimes exploit the home market effect to reduce the domestic price level towards the efficient one, it is the means to reduce the domestic price level rather than the cause. To tackle this problem, we start by considering two different policy scenarios with the purpose of disentangling efficiency considerations from other incentives to set trade policies. Once those have been clarified, we study the welfare properties of the Nash solution.

In the first experiment we follow the literature and consider trade policies starting from the (inefficient) free trade allocation. Not surprisingly, we find that the optimal unilateral policy is to set import tariffs or export subsidies. These results have previously been interpreted in the light of the home market effect, i.e.: policymakers use trade policy in a protectionist fashion to attract firms to the domestic economy. However, we show that this outcome is instead driven by the desire to reduce the domestic price level in order to correct for the monopolistic distortion. We do so by considering a second experiment, where distortions have been eliminated by appropriate production subsidies so that the market allocation is first-best efficient. In this case, the optimal unilateral trade policy entails import subsidies or export taxes, both of which relocate firms to the Foreign economy and improve domestic terms of trade.[6] Intuitively, the reason for this switch in policy is that at the free trade allocation gains from reducing the price level are large, while the income cost of worsened terms of trade is relatively small. This is so because the number of varieties is inefficiently low, which implies a high price level and a small volume of intra-industry trade. Thus, the choice of an import tariff/export subsidy. However, at the first-best allocation the price level is efficient, thus gains from further reducing it are small, while the terms-of-trade effect gains importance due to the higher volume of trade, and this explains the switch to an import subsidy/export tax.

Finally, we analyze welfare under the Nash solution. For import tariffs/subsidies we find that, independently on whether the initial allocation is the free trade or the efficient one, the Nash solution always leads to welfare losses for both countries. This confirms Ossa (2011)'s result that Nash tariffs are inefficient compared to the free trade allocation. We now turn to export taxes/subsidies. When the initial allocation is efficient, the Nash solution, which entails export

[6]Note that even import tariffs have terms-of-trade effects. Even though they cannot influence international prices of individual varieties, they affect the number of Home and Foreign varieties and thus change aggregate price indices of imports and exports, which are relevant for policy makers' decisions.

taxes aiming at improving terms of trade, is welfare reducing. However, when starting from the free trade allocation, Nash export subsidies can deliver higher welfare than free trade. The reason is that Nash export subsidies mitigate the underprovision of variety.

Note that we consider the scenario where the initial allocation is efficient with the sole purpose of clarifying the incentives behind policymakers' actions. We believe that the interesting case is in fact the one where monopolistic distortions are present. However, the conclusions drawn from the counterfactual scenario become particularly relevant when considering the welfare effects of production or export subsidies without the correction for distortions, since the model provides no rationale for prohibiting production or export subsidies in that case.

1.1 Related Literature

Our results differ markedly from those of the previous literature on trade policy in the two-sector Krugman (1980) model (Venables (1987), Helpman and Krugman (1989) chapter 7 and Ossa (2011)). All these contributions find that in this model trade policy is driven by home market effects, leading to aggregate inefficiencies. In particular, Venables (1987) studies unilateral incentives to set tariffs, production and export subsidies and shows that any of those can improve domestic welfare due to the home market effect. However, he does not study the welfare consequences of a strategic game. Helpman and Krugman (1989) limit their discussion to unilaterally set tariffs, while Ossa (2011) considers a tariff game, where positive tariffs are set in equilibrium due to the home market effect. The main reason why we obtain different results from the aforementioned contributions is that they have neglected the fact that monopolistic distortions render the free trade equilibrium inefficient, thus affecting policy makers' incentives. As a consequence, they have overlooked that deviations from free trade, first, are not driven by home market effects[7] and, second, need not necessarily be welfare reducing in a Nash equilibrium. Our analysis is also more general than the previous contributions because: we study all the main trade policy instruments (production subsidies, import tariffs and export subsidies);

[7]To increase analytical tractability, Venables (1987), Helpman and Krugman (1989) and Ossa (2011) (see section 2) do not allow for lump-sum taxes/transfers to consumers and thus require tariffs to be non-negative. As a result, the best achievable allocation under cooperation is the free trade equilibrium, while without such a restriction cooperative policy makers would actually use import subsidies in order to improve the efficiency of the allocation by increasing the number of firms in the differentiated sector. Hence, a seemingly innocuous assumption has led those contributions to overlook monopolistic distortions and to choose a misleading reference point. This has led to a misinterpretation of the tariff result as being due to the home market effect.

we allow for revenue generating trade policies; and we compare cooperative, unilateral and strategic policies. In particular, we are the first comparing welfare of the Nash solution with the free trade allocation for production and export subsidies.

Flam and Helpman (1987) and Helpman and Krugman (1989), chapter 7 discuss the production efficiency effect of trade policy, which is related to our monopolistic distortion effect. Since with imperfect competition prices are set above marginal costs, domestic consumption of each variety is too low. Thus, an import tariff (or a production or export subsidy), which shift demand towards domestic varieties, can improve efficiency. However, this effect refers to changes in average cost induced by changes in firm size and not to changes in variety. Since firm size is optimal in the Krugman (1980) model, there is no room for a production efficiency effect in the narrow sense.

Also closely related to our paper is Bagwell and Staiger (2009), who consider a two-sector Krugman (1980) model with quasi-linear preferences. Differently from our work, they allow policy makers to simultaneously choose import tariffs and export taxes. They show that in this case Nash-equilibrium policy choices are explained exclusively by the terms-of-trade effects and not by the home market externality, because import-tariff-induced home market effects are counterbalanced by export-subsidy induced home market effects. We differ from their work along two dimensions. We show that monopolistic distortions are the main driver for unilateral trade policy, whereas home market effects are not, even in the case when only one policy instrument is available. Moreover, they do not study the welfare effects of deviating from the free trade allocation. We instead show that the Nash solution can differ from the cooperative (optimal) outcome, and still be preferable to the free trade solution, i.e., showing that the Nash and the cooperative solution do not coincide is not sufficient to justify trade agreements, that limit the use of production or export subsidies.

Other related work is Gros (1987), who studies an import tariff game in the one-sector variant of the Krugman (1980) model. In that version of the model agglomeration effects are absent and the free trade allocation is Pareto-optimal. He finds that in the Nash equilibrium policy makers set import tariffs which aim at increasing domestic wages due to terms-of-trade effects.

The paper proceeds as follows. In the next section we set up the model. In section 3 we compare the market allocation with the planner solution and discuss cooperative and non-cooperative policy makers' problems and incentives. The sections 4, 5 and 6 are dedicated to the study of individual policy instruments: production taxes/subsidies, import tariffs/subsidies and export taxes/subsidies. The last section presents our conclusions.

2 The Model

The setup is exactly as in Venables (1987) or Ossa (2011). The only difference is that we allow for transfers. The world economy consists of two countries: Home and Foreign. Each country produces a homogeneous good and a continuum of differentiated goods. All goods are tradable but only the differentiated goods are subject to transport costs. The differentiated goods sector is characterized by monopolistic competition, while there is perfect competition in the homogeneous good sector. Both countries are identical in terms of preferences, production technology, market structure and size. In what follows Foreign variables will be denoted by a (*).

2.1 Households

Households' utility function in the Home country is given by:

$$U(C,Z) \equiv \; C^{\alpha} Z^{1-\alpha}, \tag{1}$$

where C aggregates over the varieties of differentiated goods, Z represents the homogeneous good and α is the expenditure share of the differentiated bundle in the aggregate consumption basket. While the homogeneous good is identical across countries, each country produces a different subset of differentiated goods. In particular, N varieties are produced in the Home country while N^* are produced by Foreign. The differentiated varieties produced in the two countries are aggregated with a CES function:[8]

$$C = \left[C_H^{\frac{\varepsilon-1}{\varepsilon}} + C_F^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \varepsilon > 1 \tag{2}$$

[8]Note that our definitions for C_H and C_F imply $C = \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh + \int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}$, i.e., the model is the standard one considered in this literature. However, as will become clear soon, it is useful to define optimal consumption indices in order to understand the trade policy makers' incentives.

$$C_H = \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad C_F = \left[\int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

Given the Dixit-Stiglitz structure of preferences, the households' maximization problem can be solved in three stages. At the first two stages households choose how much to consume of each domestic and foreign variety and how to allocate consumption between the domestic and foreign bundle. The optimality conditions imply the following demand functions and domestic price indices:

$$c(h) = \left[\frac{p(h)}{P_H}\right]^{-\varepsilon} C_H \qquad C_H = \left[\frac{P_H}{P}\right]^{-\varepsilon} C \tag{4}$$

$$c(f) = \left[\frac{p(f)}{P_F}\right]^{-\varepsilon} C_F \qquad C_F = \left[\frac{P_F}{P}\right]^{-\varepsilon} C \tag{5}$$

$$P = \left[P_H^{1-\varepsilon} + P_F^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{6}$$

$$P_H = \left[\int_0^N p(h)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}} \qquad P_F = \left[\int_0^{N^*} p(f)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} \tag{7}$$

Finally, households choose how to allocate income between the homogeneous good and the differentiated bundle. Thus, they maximize (1) subject to the following budget constraint:

$$PC + p_Z Z = I, \tag{8}$$

where $I = WL + T$, W is the wage, p_Z is the price of the homogeneous good, P is the price of the differentiated bundle and T is a lump sum tax/transfer which depends on the tariff/subsidy scheme adopted by the domestic government. The solution to the domestic consumer problem implies that the marginal rate of substitution between the homogeneous good and the differentiated bundle equals their relative price:

$$\frac{\alpha}{1-\alpha}\frac{Z}{C} = \frac{P}{p_Z} \tag{9}$$

Foreign households solve a symmetric problem.

2.2 Government

The government of each country disposes of three fiscal instruments. A production tax/subsidy (τ_C) on firms' fixed and marginal costs, a tariff/subsidy on imports (τ_I) and a tax/subsidy on exports (τ_X).[9] All government revenues are redistributed to consumers through a lump sum transfer T. The government is assumed to run a balanced budget. Hence, the government's budget constraint is given by:

$$(\tau_I - 1)\tau_X^* \tau P_H^* C_F + (\tau_X - 1)\tau P_H C_F^* + (\tau_C - 1)\int_0^N (y(h) + f)dh = T \tag{10}$$

Government income consists of import revenues charged on imports of differentiated goods gross of transport costs and Foreign export taxes; export taxes charged on exports gross of transport costs; and production tax revenues.

2.3 Firms

Firms in the differentiated sector operate under monopolistic competition. They pay a fixed cost in terms of labor, f, and then produce with linear technology:

$$y(h) = L_C(h) - f, \tag{11}$$

where $L_C(h)$ is the amount of labor allocated to the production of the differentiated good h. Goods sold in the Foreign market are subject to an iceberg transport cost $\tau \geq 1$. Given the constant price elasticity of demand, optimal prices charged by Home firms in the domestic market are a fixed markup over their perceived marginal cost $\tau_C W$ and optimal prices paid by

[9] In general, τ_i indicates a gross subsidy/tax for $i \in \{C, I, X\}$ i.e., $\tau_i < 1$ indicates a subsidy and $\tau_i > 1$ indicates a tax. We assume that subsidies (taxes) are received (paid) directly by the firms. Equivalently, we could have consumers receiving (paying) them from (to) the government.

Foreign consumers equal domestic prices augmented by transport costs and tariffs:[10]

$$p(h) = \tau_C \frac{\varepsilon}{\varepsilon - 1} W \qquad\qquad p^*(h) = \tau_I^* \tau_X \tau p(h) \tag{12}$$

Foreign firms adopt a symmetric optimal pricing rule.

The homogenous good is produced in both countries with identical production technology:

$$Q_Z = L_Z, \tag{13}$$

where L_Z is the amount of labor allocated to producing the homogeneous good. Since the good is sold in a perfectly competitive market without trade costs, price equals marginal cost and is the same in both countries. We assume that the homogeneous good is produced in both countries in equilibrium. Given the production technology, this implies factor price equalization:

$$p_Z = p_Z^* = W = W^* \tag{14}$$

For convenience, we normalize $p_Z = 1$.

2.4 Market Clearing Conditions

The good market clearing condition for each differentiated variety produced at Home is given by:

$$y(h) = c(h) + \tau c^*(h). \tag{15}$$

A similar condition holds for Foreign varieties. Free entry in the differentiated sector implies that monopolistic producers make zero profit in equilibrium[11] and that production of each differentiated variety is fixed: $y(h) = y^*(h) = (\varepsilon - 1) f$. Moreover, given that firms share the same production technology, the equilibrium is symmetric: all firms in the differentiated sector of a given country charge the same price and produce the same quantity. Hence, in equilibrium

[10] Following the previous literature (Venables (1987), Ossa (2011)), we assume that tariffs and export taxes are charged ad valorem on the factory gate price augmented by transport costs. This implies that transport services are taxed.

[11] $\Pi(h) = c(h) \left[p(h) - \tau_C\right] + c^*(h) \left[\tau p(h) - \tau \tau_C\right] - f \tau_C = 0.$

$\frac{p(h)}{P_H} = N^{\frac{1}{\varepsilon-1}}$ and $P_F = \tau_I \tau_X^* \tau P_H^*$. Using these price relations, the demand functions (4) and (5) and the fact that the production of each variety is equal to $(\varepsilon - 1)f$, we can rewrite the good market clearing condition (15) as:

$$(\varepsilon - 1)f = N^{\frac{\varepsilon}{1-\varepsilon}} P_H^{-\varepsilon} \left[P^\varepsilon C + \tau^{1-\varepsilon}(\tau_I^* \tau_X)^{-\varepsilon}(P^*)^\varepsilon C^*\right] \tag{16}$$

Using the demand functions, the market clearing condition for the homogeneous good – $Q_Z + Q_Z^* = Z + Z^*$ – can be written as:

$$Q_Z + Q_Z^* = \frac{(1-\alpha)}{\alpha}\left[PC + P^* C^*\right] \tag{17}$$

Finally, equilibrium in the labor market implies that $L = L_C + L_Z$ with $L_C = N L_C(h)$. Making use of (11) and (13), labor market clearing can be written as:

$$Q_Z = L - N\varepsilon f \tag{18}$$

2.5 Balanced Trade Condition

We assume that there is no trade in financial assets, so trade is balanced. The balanced trade condition is defined as:[12]

$$\tau\tau_X P_H C_F^* + (Q_Z - Z) = \tau\tau_X^* P_H^* C_F \tag{19}$$

The left hand side of (19) is the sum of the net export value of the homogeneous goods and the value of exports of differentiated varieties, while the right hand side is the value of imports of differentiated varieties. Equation (19) can be rewritten as:

$$\tau C_F^* + (\tau_X P_H)^{-1}(Q_Z - Z) = \frac{\tau_X^* P_H^*}{\tau_X P_H}\tau C_F \tag{20}$$

This condition implicitly defines the terms of trade as $\left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)$, the relative international price of imports of the Foreign differentiated bundle in terms of exports of the Home differentiated bundle[13] and $(\tau_X P_H)^{-1}$, the relative international price of imports of the homogeneous good in

[12] Import tariffs/subsidies are collected directly by the governments at the border so they do not enter into this condition.

[13] Defining terms of trade as the aggregate price of imports relattive to exports follows the convention of the

terms of exports of the Home differentiated bundle.

Using the definition of the price indices, we can write terms of trade as $\left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right) = \left(\frac{N}{N^*}\right)^{\frac{1}{\varepsilon-1}} \frac{\tau_X^* \tau_C^*}{\tau_X \tau_C}$. Thus, terms of trade of the differentiated bundle depend directly on Home and Foreign production taxes τ_C, τ_C^* and export taxes τ_X, τ_X^* through the impact of those taxes on the international prices of individual varieties. In particular, a domestic production or export tax reduces the relative price of imports and improves domestic terms of trade. Moreover, terms of trade improve whenever the relative number of varieties produced in Foreign increases. Intuitively, an increase in the relative number of Foreign varieties implies that domestic consumers obtain larger amount of the Foreign consumption bundle – which includes relatively more varieties and therefore is relatively more valuable for consumers – for each unit of the domestic consumption bundle.[14]

2.6 Equilibrium

The optimal pricing rules (12), the good market clearing condition (16), the labor market clearing condition (18) and the corresponding conditions for Foreign, the market clearing condition for the homogeneous good (17) and the balanced trade condition (19), together with the expressions for the price indices, fully characterize the equilibrium of the economy.

It is possible to solve this system explicitly for N and N^* as functions of the trade policy instruments:

$$N = \frac{L(A_2 - A_1^*)}{A_2^* A_2 - A_1 A_1^*} \qquad N^* = \frac{L(A_2^* - A_1)}{A_2^* A_2 - A_1 A_1^*}, \tag{21}$$

where A_1, A_2, A_1^* and A_2^* are non-linear functions of τ_C, τ_C^*, τ_I, τ_I^*, τ_X and τ_X^*. The expressions for these coefficients, as well as the derivation of the equilibrium allocation, can be found in

international macroeconomics literature.

[14]The use of optimal price indices for defining terms of trade is standard in the macroeconomic literature (see, for example, Corsetti and Pesenti (2001) and Epifani and Gancia (2009)). Note that previous literature (Venables (1987), Helpman and Krugman (1989), Ossa (2011), Bagwell and Staiger (2009)) has defined terms of trade as relative prices of individual varieties in international markets without considering that terms of trade are also affected by the relative number of varieties produced in the other country. This more restrictive definition of terms of trade has the advantage of having a more obvious empirical counterpart, but departs from the microeconomics of optimal aggregation and is thus not very helpful for studying welfare effects of price changes.

Appendix A.

Let the superscript FT denote the market solution in the absence of trade policies (free trade allocation). Then (21) simplifies to $N^{FT} = (N^*)^{FT} = \frac{\alpha L}{\varepsilon f}$. In this case the marginal rate of substitution between Z and C (equation (9)) becomes:

$$\frac{\alpha}{1-\alpha}\frac{Z^{FT}}{C^{FT}} = P^{FT} = \frac{\varepsilon}{\varepsilon - 1}(N^{FT})^{\frac{1}{1-\varepsilon}}[1+\tau^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{22}$$

In the next section we compare the free trade solution with the first-best allocation. We then lay out the general structure of the policy makers' problem and discuss the incentives that determine their trade policy choices.

3 Trade Policy

3.1 The First-Best Allocation

The first-best allocation constitutes the natural benchmark to which one can compare the outcomes of the different policy games. The social planner chooses an allocation that maximizes total world welfare subject to the technology constraints and full employment in each country.

$$\max_{C,C^*,Z,Z^*} C^\alpha Z^{(1-\alpha)} + (C^*)^\alpha (Z^*)^{1-\alpha} \tag{23}$$

subject to (11), (13), (15), $Q_Z + Q_Z^* = Z + Z^*$, $L = L_C + L_Z$, the definitions of consumption indices and the corresponding constraints for Foreign.

Proposition 1 presents the solution to this problem and compares it to the free trade solution:[15]

Proposition 1: First-Best Allocation. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then, at the first-best allocation, the marginal rate of substitution (MRS) between the differentiated bundle and the homogeneous good equals their marginal rate of transformation (MRT). The free trade allocation entails the same firm size as the one chosen by the planner. However, due to the presence of monopolistic competition in the differentiated sector, the MRS is higher than the MRT. In particular, the market price level is higher than the one required to implement the*

[15]All proofs can be found in the Appendix.

first-best allocation. As a result, the free trade allocation provides too little variety. Formally,

(1) $\frac{\alpha}{1-\alpha}\frac{Z^{FB}}{C^{FB}} = (N^{FB})^{\frac{1}{1-\varepsilon}}[1+\tau^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \equiv P^{FB}$, implying $y^{FB} = f(\varepsilon-1)$ and $N^{FB} = \frac{\alpha L}{(\varepsilon-1+\alpha)f}$

(2) $\frac{\alpha}{1-\alpha}\frac{Z^{FT}}{C^{FT}} = \frac{\varepsilon}{\varepsilon-1}(N^{FT})^{\frac{1}{1-\varepsilon}}[1+\tau^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = P^{FT}$, implying $y^{FT} = f(\varepsilon-1)$ and $N^{FT} = \frac{\alpha L}{\varepsilon f}$

This result just replicates Dixit and Stiglitz (1977)'s finding that the market provides optimal scale but too little variety. Because of monopolistic competition in the differentiated sector, individual prices are too high and there is too little variety. The price level P is inefficiently high and that creates a wedge between the marginal rate of substitution and the marginal rate of transformation between the sectors. In particular, the marginal rate of substitution between C and Z under free trade is higher than what would be efficient, i.e. households would be better off by consuming less of Z and more of C. In order to correct for this inefficiency, policy makers have an incentive to use their policy instruments to reduce P.

3.2 Optimal Policy Problems

We consider three policy instruments: production subsidies/taxes, tariffs/import subsidies and export subsidies/taxes. In each case, we assume that policy makers choose only one policy instrument at a time and we study cooperative and non-cooperative (Nash) policies.

Note that given Cobb-Douglas utility, welfare, represented by the indirect utility function, can be written as:

$$V(P(\tau_i,\tau_i^*), I(\tau_i,\tau_i^*)) = -\alpha \log\left(P(\tau_i,\tau_i^*)\right) + \log\left(I(\tau_i,\tau_i^*)\right) \tag{24}$$

where P and I are functions of the policy instruments $\tau_i \in \{\tau_C, \tau_I, \tau_X\}$ only.

The *cooperative policy maker* chooses Home and Foreign trade policy instruments in order to maximize total world welfare, which is given by the sum of Home and Foreign indirect utility:

$$\max_{\tau_i,\tau_i^*} V(P(\tau_i,\tau_i^*), I(\tau_i,\tau_i^*)) + V(P^*(\tau_i,\tau_i^*), I^*(\tau_i,\tau_i^*)) \tag{25}$$

Differently, the *single-country policy maker* chooses the domestic trade policy instrument $\tau_i \in \{\tau_C, \tau_I, \tau_X\}$ in order to maximize Home welfare, given the level of the Foreign trade policy instrument:

$$\max_{\tau_i} V(P(\tau_i), I(\tau_i)) \tag{26}$$

In both cases, trade policy affects indirect utility through two channels. On the one hand, through its effect on the relative price of the differentiated good and, on the other hand, through the impact of trade policy on income.

3.3 The Price and the Income Channels

In order to better understand the incentives of the single-country and the cooperative policy makers, we further decompose the price and the income channels as follows.

The effects of changes in trade policy on the price of the differentiated bundle can be disentangled by rewriting P as:

$$P = (N + N^*)^{\frac{1}{1-\varepsilon}} \left[s \left(\frac{\varepsilon}{\varepsilon - 1} \tau_C \right)^{1-\varepsilon} + (1 - s) \left(\frac{\varepsilon}{\varepsilon - 1} \tau_C^* \tau_I \tau_X^* \tau \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{27}$$

where $s \equiv \frac{N}{N+N^*}$ is the share of differentiated varieties produced at Home. First, trade policy changes P by altering the total amount of varieties available at the world level, a *love for variety effect* – due to Dixit-Stiglitz preferences, everything else being equal, more varieties lower the expenditure needed to achieve one unit of utility. Second, it has a direct impact on the prices of individual varieties by changing τ_C, τ_I, τ_X or their Foreign counterparts. More specifically τ_C, τ_I, τ_X affect directly the wedge between the individual price and the marginal cost of producing in the Home and in the Foreign market. Finally, unilateral trade policy influences the price index by changing s. An increase in s benefits domestic consumers via a larger fraction of goods on which they pay no transport cost. This is the *home market effect.* To study the effect of trade policy on income, it is instructive to look at income as implicitly defined by the trade balance:

$$(\tau_X P_H)^{-1} [(L - \varepsilon f N) - (1 - \alpha) I] + \tau C_F^* - \left(\frac{\tau_X^* P_H^*}{\tau_X P_H} \right) \tau C_F = 0 \tag{28}$$

Implicitly differentiating this equation with respect to the generic trade policy instrument

$\tau_i \in \{\tau_C, \tau_I, \tau_X\}$ and rearranging, we obtain:

$$\frac{(1-\alpha)}{(\tau_X P_H)}\frac{\partial I}{\partial \tau_i} =$$
$$\underbrace{-\frac{\varepsilon f}{\tau_X P_H}\frac{\partial N}{\partial \tau_i}}_{BT_1} \underbrace{- \left([(1-\alpha)I - (L - \varepsilon f N)]\frac{\partial (\tau_X P_H)^{-1}}{\partial \tau_i} + \tau C_F \frac{\partial \left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)}{\partial \tau_i} \right)}_{BT_2} \underbrace{+\tau \frac{\partial C_F^*}{\partial \tau_i}}_{BT_3} \underbrace{- \left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)\tau \frac{\partial C_F}{\partial \tau_i}}_{BT_4} \tag{29}$$

Here, BT_1 is the *opportunity cost effect* in terms of production of the homogeneous good – a change in the trade policy instrument varies income by modifying the production in the homogeneous sector which is linked to the differentiated sector via the labor market clearing condition. BT_2 is the import weighted *terms-of-trade effect*: a unilateral change in the trade policy instrument shifts domestic terms of trade by changing the relative prices of imports. The term BT_3 is the change in Foreign imports of differentiated goods induced by the variation in trade policy. Similarly, BT_4 is the change in domestic imports of differentiated goods induced by the variation in trade policy.[16] In the next section we use this decomposition to understand the incentives behind each policy choice.

3.4 Trade Policy Makers' Incentives

When trade policy moves cooperatively and the initial allocation is symmetric, *terms of trade effects* are absent, since $\frac{\partial \left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)}{\partial \tau_i}$ is zero and the homogeneous good is not traded. Moreover, $BT_3 + BT_4 = 0$, since those effects are identical and of opposite signs. Thus, the only effect on income is the *opportunity cost effect*: $\frac{\partial I}{\partial \tau_i} = -\frac{\varepsilon f}{1-\alpha}\frac{\partial N}{\partial \tau_i}$. Similarly, under cooperation the *home market effect* is absent because $N = N^*$. Therefore, the only relevant effects of cooperative trade policy on the aggregate price index are the *love for variety effect* and its direct effect on individual prices. As shown in Proposition 1, monopolistic competition in the differentiated sector induces a wedge between the marginal rate of substitution and the marginal rate of transformation of C and Z – the prices of individual varieties are too high and the number of varieties is too low. The cooperative policy maker optimally sets subsidies on production or

[16] As shown in the Appendix, the term BT_4 can be further decomposed into income and substitution effects in order to derive an explicit expression for the change in income induced by the trade policy instrument.

on imports or on exports in order to reduce the price index and she is willing to give up some income in order to do so.

Differently, the single-country policy maker – beside the desire to move the domestic price index towards the efficient level – has two additional and opposing motives to set trade policy. First, the *home market effect* induces her to use the policy instrument to relocate firms to the domestic economy. This reduces the domestic price index by increasing s. Second, a unilateral change in trade policy has a *terms-of-trade effect*. Since an improvement in domestic terms of trade requires a relocation of firms from Home to Foreign, this effect goes in the opposite direction to the home market effect.

As we will show in the next section, the prevailing motives for unilateral and non-cooperative trade policies are, on the one hand, the desire to move the domestic price index towards the efficient level and, on the other hand, the incentives to improve domestic terms of trade. It crucially depends on the (in)efficiency of the initial allocation whether the balance tips towards the first or the second effect. The *home market effect*, though present, is always dominated by either one or the other.

4 Production Subsidies

In this section we study cooperative and non-cooperative production subsidies. We have already investigated how the *cooperative policy maker* trades off the incentive to reduce the price index by increasing the total number of varieties (*love for variety effect*), and by lowering the price of individual varieties against the income cost of such a policy in terms of the reduced production of the homogeneous good (*opportunity cost effect*). The optimal policy in such a case is summarized in Proposition 2.

Proposition 2: Cooperative Production Subsidy. *Let* $\tau > 1$, $\varepsilon > 1$ *and* $0 < \alpha < 1$. *Then the optimal cooperative production subsidy is set to exactly offset the price markup generated by monopolistic competition. This subsidy implements an equilibrium with the first-best number of varieties and the first-best price level. Formally,*

(1) $\tau_C^{Coop} = \frac{\varepsilon - 1}{\varepsilon}$

(2) $N_C^{Coop} = N^{FB}$

(3) $P_C^{Coop} = P^{FB}$

Proposition 2 implies that the cooperative production subsidy is determined solely by the desire to reach efficiency. Such a subsidy lowers the price of differentiated varieties and closes the wedge between prices and marginal costs. Welfare is maximized, because with this subsidy the marginal rate of transformation between C and Z equals their marginal rate of substitution (see Proposition 1).

Single-country policy makers' actions are instead driven by two additional effects. On the one hand, a unilateral increase of production subsidies reduces the price index through the *home market effect*. On the other hand, it lowers income through a deterioration of the *terms of trade*. This is shown in Lemma 1.

Lemma 1 *Let* $\tau > 1$, $\varepsilon > 1$ *and* $0 < \alpha < 1$. *If* $\tau_C = \tau_C^* \leq 1$ *then a unilateral increase in the production tax increases the domestic price index (*$\frac{\partial P}{\partial \tau_C} > 0$*) both directly through an increase in the individual price of domestic varieties, and indirectly through a reduction in the total number of varieties (*$\frac{\partial (N+N^*)}{\partial \tau_C} < 0$*) and in the share of varieties produced domestically (*$\frac{\partial s}{\partial \tau_C} < 0$*). Income increases (*$\frac{\partial I}{\partial \tau_C} > 0$*) due to terms-of-trade and opportunity cost effects.*

Whether the price channel or the effect on income predominates, crucially depends on the (in)efficiency of the initial allocation, as shown in Proposition 3 for unilateral deviations without retaliation.

Proposition 3: Unilaterally Set Production Subsidies. *Let* $\tau > 1$, $\varepsilon > 1$ *and* $0 < \alpha < 1$. *Then the optimal unilateral production subsidy is positive but strictly smaller than the efficient subsidy. Formally,*

(1) $\left.\frac{\partial V(P(\tau_C), I(\tau_C))}{\partial \tau_C}\right|_{\tau_C = \tau_C^* = 1} < 0$

(2) $\left.\frac{\partial V(P(\tau_C), I(\tau_C))}{\partial \tau_C}\right|_{\tau_C = \tau_C^* = \frac{\varepsilon - 1}{\varepsilon}} > 0$

The intuition behind Proposition 3 is as follows: at the free trade allocation, the domestic price level is inefficiently high and thus single-country policy makers try to reduce it by exploiting the *love for variety effect*, the *home market effect* and by reducing the wedge between price and marginal cost at the level of individual domestic varieties. At the same time, the effect on income is small. The reason is twofold. First, the homogeneous good is inefficiently abundant, thus *opportunity cost effects* of reducing its production are small. Second, the volume of intra-industry trade is inefficiently low and therefore *terms-of-trade effects* are relatively weak. Differently, at the first-best allocation, the price level is efficient and thus gains from further reducing it are small, while the effect on income becomes more important. This is so, since there is an efficient provision of the homogeneous good, so that the *opportunity cost effect* of a further reduction in production is large. Moreover, the volume of intra-industry trade is larger, strengthening *terms-of-trade effects*.

Lemma 1 and Proposition 3 provide the crucial insight regarding the incentives behind unilateral trade policies. Proposition 3(1) confirms Venables (1987)'s finding: single-country policy makers have an incentive to deviate from the free trade allocation by introducing a production subsidy. But why do they do so? Because they want to mitigate the under-provision of differentiated goods due to monopolistic competition. In fact, Lemma 1 shows that although a production subsidy clearly worsens domestic *terms of trade*, it increases welfare by moving the price index towards the efficient level, thanks to both *love for variety* and *home market effects*. However, according to Proposition 3(2), once we remove efficiency considerations by starting from the first-best allocation, the *terms-of-trade effect* predominates i.e., the trade-off is between efficiency and terms-of-trade. Proposition 4 proves that these results carry over to a situation where both countries set production subsidies strategically.

Proposition 4: Nash-Equilibrium Production Subsidies. *Let* $\tau > 1$, $\varepsilon > 1$ *and* $0 < \alpha < 1$. *Then the Nash solution of the trade policy game between the two countries implies a production subsidy strictly smaller than the one needed to implement the first-best allocation. The total number of varieties is larger than in the free trade allocation, but remains lower than the first-best level. At the same time, the price level is lower than in the free trade allocation but higher than the first-best level. Formally,*

(1) $1 > \tau_C^{Nash} > \tau_C^{FB}$.

(2) $N^{FT} < N_C^{Nash} < N^{FB}$.

(3) $P^{FB} < P_C^{Nash} < P^{FT}$

Thus, *single-country policy makers* never over-subsidize domestic production, as would be required if the *home market effect* were the dominating incentive for non-cooperative policy choice. Instead, *terms-of-trade effects* lead policy makers to choose an inefficiently low production subsidy. We therefore conclude that the *terms-of-trade effect* and the desire to reach efficiency outweighs the other effects in the non-cooperative choice of production subsidies. This is an important result, because it contradicts the standard wisdom that in the two-sector Krugman model countries have an incentive to over-subsidize production in order to attract more firms (Venables (1987)). The policy implication of this result is crucial: the Nash equilibrium of this game entails welfare gains for both countries compared to the free trade allocation. Indeed, the aggregate number of varieties moves towards efficiency even though it does not go all the way to the first-best allocation. Thus, according to this model production subsidies should not be banned since they are welfare improving even when set in a strategic way.

5 Tariffs

Here, we suppose that the only strategic trade policy instrument available is an import tariff/subsidy. Given the results of the previous section, where we pointed out the importance of the (in)efficiency of the allocation, we study cooperative and non-cooperative tariffs under two scenarios. In the first scenario monopolistic distortions are present (i.e., $\tau_C = \tau_C^* = 1$), while in the second scenario a production subsidy has already been set in a non-strategic fashion such as to eliminate monopolistic distortions and to implement the first-best allocation (i.e., $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$). Let us first study the cooperative policy maker's problem.

Proposition 5: Cooperative Import Subsidy. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then, if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker refrains from using subsidies/tariffs on imports. If $\tau_C = \tau_C^* = 1$, the cooperative policy maker finds it optimal to subsidize imports. The total number of varieties is larger than in the free trade allocation, but remains lower than the first-best level. At the same time, the price level is lower than in the free trade allocation but higher than the first-best level. Formally,*

(1) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then, $\tau_I^{Coop} = 1$, $N_I^{Coop} = N^{FB}$ and $P_I^{Coop} = P_I^{FB}$

(2) If $\tau_C = \tau_C^* = 1$ then, $\tau_I^{Coop} < 1$, $N^{FT} < N_I^{Coop} < N^{FB}$ and $P^{FB} < P_I^{Coop} < P^{FT}$

Proposition 5 is the counterpart to Proposition 2. As for the cooperative production subsidy, the *cooperative policy maker's* decisions are driven by the sole desire to improve efficiency. If the inefficiencies due to monopolistic distortions have already been offset by production subsidies, the equilibrium allocation coincides with the first-best and there is no need for further policy interventions. Differently, if there is no correction for the monopolistic distortions, the cooperative policy maker subsidizes imports. This is the same outcome as in the case of cooperative production subsidies. However, import subsidies cannot close the wedge between market price and marginal cost in the domestic market for domestically produced varieties. Thus, the price level remains too high and the wedge between the marginal rate of substitution and the marginal rate of transformation cannot be fully eliminated.

Finally, Proposition 5 implies that the free trade allocation is not the constrained efficient allocation and thus cannot be taken as the reference point to which to compare the non-cooperative outcome.[17]

Once we move to the case of non-cooperation, both the *home market effect* and the *terms-of-trade effect* become crucial to understand trade policy interventions, as shown in Lemma 2.

Lemma 2: *Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ and $\tau_I = \tau_I^* = 1$. Then,*

(1) *If $\tau_C = \tau_C^* = 1$ or $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ and $\varepsilon - 1 > \alpha$[18] a unilateral increase in the import tariff increases the price of imported varieties, decreases the total number of varieties ($\frac{\partial(N+N^*)}{\partial \tau_I} < 0$) and increases the share of domestically produced varieties ($\frac{\partial s}{\partial \tau_I} > 0$). Overall, the domestic price index decreases ($\frac{\partial P}{\partial \tau_I} < 0$).*

(2) *If $\tau_C = \tau_C^* = 1$, then a unilateral increase in the import tariff increases income ($\frac{\partial I}{\partial \tau_I} > 0$) while if $\tau_C = \tau_C^* = \frac{(\varepsilon-1)}{\varepsilon}$, then the effect on income is negative ($\frac{\partial I}{\partial \tau_I} < 0$). Moreover, terms-of-trade and opportunity costs are always negative, while other effects on income are always positive.*

[17] Note that Ossa (2011) constrains tariffs to be non-negative and does not allow for transfers. Thus, zero tariffs is the constrained efficient allocation in his model. Those assumptions are not innocuous, as will become clear in Proposition 6 and 7.

[18] Note that the condition $\varepsilon - 1 > \alpha$ is needed only for $\frac{\partial P}{\partial \tau_I}|_{\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}} < 0$.

According to the first part of Lemma 2, a unilateral tariff has three effects on the price index. First, by shifting demand towards domestic varieties, a tariff induces a relocation of firms to the domestic economy, thereby increasing s and reducing transport costs for domestic consumers (*home market effect*). This has a negative effect on the domestic price index. Second, it reduces the world number of varieties (*love for variety effect*) and third, it introduces a wedge between prices and marginal costs for imported varieties. This has a positive effect on the domestic price index. Independently of the (in) efficiency of the initial allocation, the *home market effect* is the strongest of the three. This implies that the only way to reduce the domestic price index towards the efficient level is to set an import tariff. The second part of Lemma 2 says that when starting from the free trade allocation, the effect of a tariff on income is also positive. However, this result turns around when we consider an initial allocation that is first-best efficient. In this case, the effect of a tariff on income is negative. The switch in the sign of the derivative of income can be explained as follows. Intuitively, when starting from the first-best allocation, the volume of trade is larger than when starting from the free trade allocation, because the total number of varieties in the differentiated sector is higher. Therefore, the income loss due to a terms of trade worsening is larger as well. Similarly, the opportunity cost of increasing the domestic production in the differentiated sector in terms of homogeneous good is higher because the homogeneous good is no longer inefficiently abundant. Next, we show that this difference in incentives results in differences in import policy choices.

Proposition 6: Unilaterally Set Import Tariffs/Subsidies. *Let* $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ *and* $\tau_I = \tau_I^* = 1$. *The optimal unilateral import policy is to set an import tariff when starting from the free trade allocation and to set an import subsidy when starting from the first-best allocation implemented by a production subsidy. Formally:*

(1) If $\tau_C = \tau_C^* = 1$, then $\frac{\partial V(P(\tau_I), I(\tau_I))}{\partial \tau_I} > 0$

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon - 1}{\varepsilon}$ then $\frac{\partial V(P(\tau_I), I(\tau_I))}{\partial \tau_I} < 0$

Proposition 6 proves that when starting from the free trade allocation, single-country policy makers set an import tariff in order to move the domestic price level towards efficiency. Differently, when the production subsidy implements the first-best allocation, policy makers actually set an import subsidy which induces the exit of firms from the domestic market, increases the domestic price level and improves domestic terms of trade. Proposition 6 allows to interpret existing results in a new light since we show that, again, the real trade-off is between efficiency and *terms-of-trade* considerations. The reason for setting a tariff when starting from the free trade allocation is that it is the only way to move the domestic price level towards efficiency. However, policy makers refrain from trying to reduce the price level by agglomerating firms in the domestic economy, when the price level without trade policy intervention equals the first best. This implies that in the case of import tariffs too, the dominating incentives are the desire to eliminate monopolistic distortions and *terms of trade effects*.

Our findings are in line with Venables (1987)'s and Ossa (2011)'s results. What is critically different is the interpretation. According to their interpretation, the *home market effect* is the only incentive driving unilateral trade policy. This is so because they take the free trade allocation as the reference point. As a consequence, they overlook that the desire to set unilateral import tariffs is crucially driven by the incentive to eliminate monopolistic distortions in order to get closer to the efficient allocation. Instead, we show that even if import tariffs reduce the domestic price index through the *home market effect*, the home market effect is not sufficient to induce policymakers to set import tariffs. What is required is an inefficiency of the initial allocation i.e., the home market is the means to reduce the domestic price level rather than the cause. Proposition 7 presents the outcome of the Nash policy game where both countries set import tariffs simultaneously.

Proposition 7: Nash-Equilibrium Import Tariffs/Subsidies. *Let* $\tau > 1$, $\varepsilon > 1$ *and* $0 < \alpha < 1$. *When starting from the free trade allocation, the Nash-equilibrium tariff is positive implying less varieties and higher price level than the free trade allocation. Differently, when starting from the first-best allocation, the Nash-equilibrium policy consists of an import subsidy implying more varieties and lower price level than the first-best allocation. Formally:*

(1) If $\tau_C = \tau_C^* = 1$ then, there exists a $\tau_I^{Nash} > 1$ such that $N_I^{Nash} < N^{FT} < N^{FB}$ and $P_I^{Nash} > P^{FT} > P^{FB}$.

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then, $\tau_I^{Nash} < 1$ and $N_I^{Nash} > N^{FB} > N^{FT}$ and $P^{FT} > P^{FB} > P_I^{Nash}$.

According to the first part of Proposition 7, when there is no correction of the monopolistic distortion, non-cooperative trade policy brings about a positive tariff. The incentive to set a positive tariff is the same as in Proposition 6, however, in the Nash equilibrium no country manages to relocate firms to its domestic market thereby failing to reduce the price index. Instead, tariffs reduce the world equilibrium number of varieties and increase the price level thus being welfare detrimental compared to the free trade allocation.

When starting from the first-best allocation, the optimal non-cooperative policy is an import subsidy[19] which aims at improving domestic terms of trade. Yet, in equilibrium no country reaches this aim and, again, the Nash equilibrium is welfare detrimental compared to the initial allocation. Thus, for import tariffs we confirm the policy implications of existing literature: the Krugman (1980) model provides a rationale for international agreements prohibiting the use of import tariffs because uncoordinated policies are welfare detrimental compared to free trade.

6 Export Subsidies

As a last experiment, we consider export taxes/subsidies as the only strategic trade policy instrument available. In line with the previous analysis, we study cooperative and Nash policies under two scenarios. In the first one monopolistic distortions have not been corrected (i.e., $\tau_C = \tau_C^* = 1$), while in the second production subsidies have been set such as to implement the first-best allocation (i.e., $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$).

Proposition 8: Cooperative Export Subsidy. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then, if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker refrains from using subsidies/tariffs on exports and the number of varieties and the price level equal the first-best ones. If $\tau_C = \tau_C^* = 1$, the cooperative policy maker finds it optimal to subsidize exports. The total number of varieties increases compared to the free trade allocation, but remains lower than the first-best level, while the price level is lower than in the free trade allocation, but higher than the first-best one. Formally,*

(1) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then, $\tau_X^{Coop} = 1$, $N_X^{Coop} = N^{FB}$ and $P_X^{Coop} = P^{FB}$

(2) If $\tau_C = \tau_C^* = 1$ then, $\tau_X^{Coop} < 1$, $N^{FT} < N_X^{Coop} < N^{FB}$ and $P^{FB} < P_X^{Coop} < P^{FT}$

[19] In the proof of Proposition 7 we also show that $\varepsilon > 2$ is a sufficient, though not necessary, condition for the existence of a Nash equilibrium when the initial allocation is efficient.

Again, the cooperative policy makers' decisions are driven by the desire to improve efficiency. When the equilibrium allocation coincides with the first-best, there is no need for further policy intervention. However, when this is not the case, the cooperative policy maker subsidizes exports. This lowers the price of the exported bundle and reduces the wedge between the price and the marginal cost for exported varieties. Like in the case of cooperative import subsidies, it is not possible to correct the monopolistic distortion for those varieties produced and consumed within the same country. As a consequence, the first-best allocation is not achievable.

In the case of non-cooperation, the *home market effect* and the *terms-of-trade effect* also play a role in trade policy choices.

Lemma 3: *Let* $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ *and* $\tau_X = \tau_X^* = 1$. *Then,*

(1) *If* $\tau_C = \tau_C^* = 1$ *or* $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$*, a unilateral increase in the export tax decreases the total number of varieties (*$\frac{\partial(N+N^*)}{\partial \tau_X} < 0$*) and reduces the share of domestically produced varieties (*$\frac{\partial s}{\partial \tau_X} < 0$*). As a result, the domestic price index increases (*$\frac{\partial P}{\partial \tau_X} > 0$*).*

(2) *If* $\tau_C = \tau_C^* = 1$ *or* $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$*, a unilateral increase in the export tax increases domestic income (*$\frac{\partial I}{\partial \tau_X} > 0$*). Moreover, terms-of-trade and opportunity costs are always negative, while other effects on income are always positive.*

According to Lemma 3, a unilateral increase in the export tax increases the domestic price index because of a negative *home market effect* and a decrease in the world number of varieties (*love of variety effect*). Income always increases as a result of a unilateral increase in the domestic export tax due to a positive *opportunity cost effect* and a positive *terms-of-trade effect*. Thus, like in the case of production subsidies, the *non-cooperative policy maker* always faces a trade-off between reducing the price index and increasing the level of income.

Proposition 9: Unilaterally Set Export Taxes/Subsidies. *Let* $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ *and* $\tau_X = \tau_X^* = 1$. *The optimal unilateral export policy entails a positive export subsidy when starting from the free trade allocation, and an export tax when starting from the first-best allocation implemented by a production subsidy. Formally:*

(1) If $\tau_C = \tau_C^* = 1$, then $\frac{\partial V(P(\tau_X), I(\tau_X))}{\partial \tau_X} < 0$

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then $\frac{\partial V(P(\tau_X), I(\tau_X))}{\partial \tau_X} > 0$

Like in the case of production subsidies and import tariffs, the differences in outcome are driven by the (in)efficiency of the initial allocation. The mechanisms for export taxes are exactly the same as the ones outlined for production subsidies in section 4. At the free trade allocation the domestic price level is inefficiently high, while income effects (given by *terms-of-trade effects* and *opportunity cost effects*) are weak, and thus policy makers try to reduce the price level by setting export subsidies, exploiting the *love for variety effect* and the *home market effect*. Differently, when starting from the first-best allocation, gains from further reducing the price level are low, while income effects are large. This is so because *terms-of-trade effects* and *opportunity cost effects* are strong, since the volume of intra-industry trade is larger than at the free trade allocation and the homogeneous good is no longer inefficiently abundant. Proposition 10 extends the result of Proposition 9 to a trade policy game, where both countries set export taxes simultaneously.

Proposition 10: Nash-Equilibrium Export Taxes/Subsidies. *Let* $\tau > 1$, $\varepsilon > 1$ *and* $0 < \alpha < 1$. *When starting from the free trade allocation, the Nash-equilibrium policy consists of an export subsidy implying more varieties and lower price level than the free trade allocation. Differently, when starting from the first-best allocation, the Nash-equilibrium policy consists of an export tax implying less varieties and higher price level than the first-best allocation. Formally:*

(1) If $\tau_C = \tau_C^* = 1$, then $\tau_X^{Nash} < 1$, $N^{FB} > N_X^{Nash} > N^{FT}$ and $P^{FB} < P_X^{Nash} < P^{FT}$.

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_X^{Nash} > 1$ $N_X^{Nash} < N^{FB}$ and $P_X^{Nash} > P^{FB}$.

Proposition 10 makes it clear that in this case, too, the outcome of the policy game depends crucially on whether the initial allocation is (in)efficient. While when starting from the free trade allocation the optimal Nash policy is an export subsidy, when starting from the first-best allocation the optimal non-cooperative policy turns out to be an export tax. It can be shown that in the first case, in which export subsidies are set in order to increase domestic efficiency, the Nash outcome can be welfare improving compared to the free trade allocation.[20] Therefore the

[20]A supplementary appendix on this result is available upon request.

Krugman model does not provide a theoretical justification to ban export subsidies. Conversely, in the second case, where the decisions to set export taxes are driven by terms of trade effects, the Nash solution is welfare detrimental.

7 Conclusions

In this paper we have studied unilateral, strategic and cooperative trade policies in a two-sector Krugman (1980) model of intra-industry trade, considering production taxes, import tariffs and export taxes as alternative trade policy instruments. It is common wisdom that in this model non-cooperative trade policies are set in order to try to agglomerate firms in the domestic economy, which reduces transport costs for domestic consumers and thus the domestic price level (*home market effect*).

Contrary to the results of the previous literature, we show that in this model non-cooperative trade policies are actually never determined by the home market effect. Instead, they are driven by the incentive to reduce the domestic price level towards the efficient one on the one hand, and by *terms-of-trade effects* on the other. Indeed, due to monopolistic competition, the domestic price level is too high in the free trade equilibrium and this affects policy makers' incentives in a crucial way. Thus, when production taxes/subsidies are available, non-cooperative policy makers reduce the price level by setting production subsidies and manage to increase welfare compared to the free trade allocation. However, due to terms-of-trade effects these subsidies are lower than the cooperatively set ones. When import tariffs or export subsidies are available, non-cooperative policy makers use these instruments to reduce the price level towards efficiency, as long as the allocation without trade policy is inefficient. However, once monopolistic distortions have been offset by appropriate production subsidies, policy makers set import subsidies or export taxes, which improve domestic terms of trade and increase the domestic price level.

Our analysis has important policy implications: non-cooperative trade policies are not always protectionist and can improve welfare compared to the free trade allocation. As a result, although the Krugman (1980) model justifies the ban on import tariffs, it does not provide a rationale for prohibiting production or export subsidies.

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APPENDIX

A Equilibrium

A.1 Equilibrium Allocation and Prices

Substituting the optimal pricing rules (12) and $p(f) = \tau_I \tau_X^* \tau p^*(f)$ into the definition of Home (6) (and Foreign) aggregate price indices we obtain:

$$P = \frac{\varepsilon}{\varepsilon - 1}\left[N\tau_C^{1-\varepsilon} + N^*\left(\tau_I \tau_X^* \tau \tau_C^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \qquad P^* = \frac{\varepsilon}{\varepsilon - 1}\left[N^*(\tau_C^*)^{1-\varepsilon} + N\left(\tau_I^* \tau_X \tau \tau_C\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{30}$$

Combining the zero profit condition (16) with the analogous one for Foreign and substituting out the expressions for the prices (30), gives:

$$C = \frac{fP^{-\varepsilon}(\varepsilon-1)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon} \tau^{\varepsilon}(-\tau(\tau_C^*)^{\varepsilon} + (\tau\tau_C\tau_I^*\tau_X)^{\varepsilon})(\tau_I\tau_X^*)^{\varepsilon}}{\tau^{2\varepsilon}(\tau_I^*\tau_X\tau_I\tau_X^*)^{\varepsilon} - \tau^2} \tag{31}$$

$$C^* = \frac{f(P^*)^{-\varepsilon}(\varepsilon-1)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon} \tau^{\varepsilon}(-\tau(\tau_C)^{\varepsilon} + (\tau\tau_C^*\tau_I\tau_X^*)^{\varepsilon})(\tau_I^*\tau_X)^{\varepsilon}}{\tau^{2\varepsilon}(\tau_I^*\tau_X\tau_I\tau_X^*)^{\varepsilon} - \tau^2} \tag{32}$$

Using the trade balance condition (19), the labor market clearing condition (18), the equivalent equations for Foreign, and the expressions for C, C^*, P and P^* just derived, we have the following system of equations in N and N^*:

$$A_1 N + A_2 N^* - L = 0 \tag{33}$$

$$A_2^* N + A_1^* N^* - L = 0 \tag{34}$$

the solution of which is:

$$N = \frac{L(A_2 - A_1^*)}{A_2^* A_2 - A_1 A_1^*} \qquad N^* = \frac{L(A_2^* - A_1)}{A_2^* A_2 - A_1 A_1^*} \tag{35}$$

where:

$$A_1 = \frac{f\varepsilon\tau_C^{-\varepsilon}(\tau^{2\varepsilon}(\tau_C\tau_I\tau_I^*\tau_X\tau_X^*)^{\varepsilon}(\alpha + (1-\alpha)\tau_C) + \tau(\alpha\tau\tau_C^{\varepsilon}(\tau_C\tau_X - 1) - \tau_C(\tau\tau_C^*\tau_I\tau_X^*)^{\varepsilon}(1 - \alpha + \alpha\tau_X)))}{\alpha(\tau^{2\varepsilon}(\tau_I\tau_I^*\tau_X\tau_X^*)^{\varepsilon} - \tau^2)} \tag{36}$$

$$A_2 = \frac{f\varepsilon\tau(\tau_C^*)^{1-\varepsilon}(-\alpha - (1-\alpha)\tau_I)\tau_X^*(\tau(\tau_C^*)^{\varepsilon} - (\tau\tau_C\tau_I^*\tau_X)^{\varepsilon})}{\alpha(\tau^{2\varepsilon}(\tau_I\tau_I^*\tau_X\tau_X^*)^{\varepsilon} - \tau^2)} \tag{37}$$

$$A_1^* = \frac{f\varepsilon(\tau_C^*)^{-\varepsilon}(\tau^{2\varepsilon}(\tau_C^*\tau_I\tau_I^*\tau_X\tau_X^*)^{\varepsilon}(\alpha+(1-\alpha)\tau_C^*)+\tau(\alpha\tau(\tau_C^*)^{\varepsilon}(\tau_C^*\tau_X^*-1)-\tau_C^*(\tau\tau_C\tau_I^*\tau_X)^{\varepsilon}(1-\alpha+\alpha\tau_X^*)))}{\alpha(\tau^{2\varepsilon}(\tau_I\tau_I^*\tau_X\tau_X^*)^{\varepsilon}-\tau^2)} \tag{38}$$

$$A_2^* = \frac{f\varepsilon\tau\tau_C^{1-\varepsilon}(-\alpha-(1-\alpha)\tau_I^*)\tau_X(\tau\tau_C^{\varepsilon}-(\tau\tau_C^*\tau_I\tau_X^*)^{\varepsilon})}{\alpha(\tau^{2\varepsilon}(\tau_I\tau_I^*\tau_X\tau_X^*)^{\varepsilon}-\tau^2)} \tag{39}$$

A.2 Free Trade Allocation

Let $\tau_C = \tau_C^* = \tau_I = \tau_I^* = \tau_X = \tau_X^* = 1$. Then, (30) becomes

$$P = P^* = \frac{\varepsilon}{\varepsilon-1} N^{\frac{1}{1-\varepsilon}} \left[1+\tau^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \equiv P^{FT} \tag{40}$$

and (35) simplifies to:

$$N = N^* = \frac{\alpha L}{\varepsilon f} \equiv N^{FT} \tag{41}$$

B The Planner's Problem

Proposition 1: First-Best Allocation. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then, at the first-best allocation, the marginal rate of substitution (MRS) between the differentiated bundle and the homogeneous good equals their marginal rate of transformation (MRT). The free trade allocation entails the same firm size as the one chosen by the planner. However, due to the presence of monopolistic competition in the differentiated sector, the MRS is higher than the MRT. In particular, the market price level is higher than the one required to implement the first-best allocation. As a result, the free trade allocation provides too little variety. Formally,*

(1) $\frac{\alpha}{1-\alpha}\frac{Z^{FB}}{C^{FB}} = (N^{FB})^{\frac{1}{1-\varepsilon}}[1+\tau^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \equiv P^{FB}$, implying $y^{FB} = f(\varepsilon-1)$ and $N^{FB} = \frac{\alpha L}{(\varepsilon-1+\alpha)f}$

(2) $\frac{\alpha}{1-\alpha}\frac{Z^{FT}}{C^{FT}} = \frac{\varepsilon}{\varepsilon-1}(N^{FT})^{\frac{1}{1-\varepsilon}}[1+\tau^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = P^{FT}$, implying $y^{FT} = f(\varepsilon-1)$ and $N^{FT} = \frac{\alpha L}{\varepsilon f}$

Proof of Proposition 1.

(1) First-Best Allocation

The Lagrangian for the planner's problem is:

$$\mathcal{L} = \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}}dh + \int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}}df\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} Z^{1-\alpha} + \left[\int_0^N c^*(h)^{\frac{\varepsilon-1}{\varepsilon}}dh + \int_0^{N^*} c^*(f)^{\frac{\varepsilon-1}{\varepsilon}}df\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} (Z^*)^{1-\alpha}$$
$$+ \int_0^N \lambda_1(h)[L_c(h) - f - c(h) - \tau c^*(h)]dh + \int_0^{N^*} \lambda_2(f)[L_c^*(f) - f - c^*(f) - \tau c(f)]df$$
$$+ \lambda_3[L + L^* - \int_0^N L_c(h)dh - \int_0^{N^*} L_c(f)df - Z - Z^*]$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c(h)} = 0 : \alpha C^{\alpha} \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh + \int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{-1} Z^{1-\alpha} c(h)^{\frac{-1}{\varepsilon}} = \lambda_1(h) \tag{42}$$

$$\frac{\partial \mathcal{L}}{\partial c(f)} = 0 : \alpha C^{\alpha} \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh + \int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{-1} Z^{1-\alpha} c(f)^{\frac{-1}{\varepsilon}} = \tau \lambda_2(f) \tag{43}$$

$$\frac{\partial \mathcal{L}}{\partial Z} = 0 : (1-\alpha) C^{\alpha} Z^{-\alpha} = \lambda_3 \tag{44}$$

$$\frac{\partial \mathcal{L}}{\partial L_c(h)} = 0 : \lambda_1(h) = \lambda_3 \tag{45}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial N} = 0 : \alpha \frac{\varepsilon}{\varepsilon-1} \Bigg\{ & C^{\alpha} Z^{1-\alpha} \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh + \int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{-1} c(N)^{\frac{\varepsilon-1}{\varepsilon}} + \\ & (C^*)^{\alpha} (Z^*)^{1-\alpha} \left[\int_0^N c^*(h)^{\frac{\varepsilon-1}{\varepsilon}} dh + \int_0^{N^*} c^*(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{-1} c^*(N)^{\frac{\varepsilon-1}{\varepsilon}} \Bigg\} = \lambda_3 L_c(N), \end{aligned} \tag{46}$$

where in the last condition we have already used the fact that $\lambda_1(N)[L_c(N) - f - c(N) - \tau c^*(N)] = 0$.

The first-order conditions with respect to Foreign variables are completely symmetric and are thus omitted for the sake of space. By imposing symmetry we find $\lambda_1(h) = \lambda_2(f)$. Combining (42) and (43) we obtain:

$$c(f) = c(h) \tau^{-\varepsilon} \tag{47}$$

Combining (42), (45) and (46) we get that:

$$\frac{\varepsilon}{\varepsilon-1} [c(h)^{\frac{\varepsilon-1}{\varepsilon}} + c(f)^{\frac{\varepsilon-1}{\varepsilon}}] = L_c(h) c(h)^{\frac{1}{\varepsilon}} \tag{48}$$

Combining (47) and (48), we obtain:

$$c(h) = \frac{\varepsilon}{\varepsilon-1} [1 + \tau^{1-\varepsilon}]^{-1} \tag{49}$$

Substituting the expression for $c(h)$ and $c(f)$ into the resource condition for domestic varieties $L_c(h) = f + c(h) + \tau c(f)$, we get $L_c(h) = \varepsilon f$ and using the production function

$y(h) = L_c(h) - f$ we obtain $y^{FB} = (\varepsilon - 1)f$. Moreover, $c^{FB}(h) = (\varepsilon - 1)f[1 + \tau^{1-\varepsilon}]^{-1}$ and $c^{FB}(f) = (\varepsilon - 1)f\tau^{-\varepsilon}[1 + \tau^{1-\varepsilon}]^{-1}$.

Using the resource condition for Z, we get $Z = L - N\varepsilon f$.

Finally, combining (42), (44) and (45)

$$(1-\alpha)C^{\frac{\varepsilon-1}{\varepsilon}} = \alpha Z c(h)^{-\frac{1}{\varepsilon}} \tag{50}$$

Substituting the expressions for Z, C, $c^{FB}(h)$ and $c^{FB}(f)$ into (50), we can solve for $N^{FB} = \frac{\alpha L}{f(\varepsilon+\alpha-1)}$.

Finally, from (50):

$$\frac{\alpha}{1-\alpha}\frac{Z}{C} = C^{-\frac{1}{\varepsilon}}c(h)^{\frac{1}{\varepsilon}} \tag{51}$$

Using $N = N^*$, (47) and (49) into the definition of C we obtain:

$$\begin{aligned} C &= \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}}dh + \int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}}df\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= \left[Nc(h)^{\frac{\varepsilon-1}{\varepsilon}} + Nc(h)^{\frac{\varepsilon-1}{\varepsilon}}\tau^{1-\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= N^{\frac{\varepsilon}{\varepsilon-1}}c(h)\left[1+\tau^{1-\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned} \tag{52}$$

Therefore,

$$(C^{FB})^{-\frac{1}{\varepsilon}} = (N^{FB})^{\frac{1}{1-\varepsilon}}c(h)^{-\frac{1}{\varepsilon}}\left[1+\tau^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{53}$$

So that the first-best allocation can be written as:

$$\frac{\alpha}{1-\alpha}\frac{Z^{FB}}{C^{FB}} = (N^{FB})^{\frac{1}{1-\varepsilon}}\left[1+\tau^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{54}$$

(2) Free Trade Allocation

$$\frac{\alpha}{1-\alpha}\frac{Z^{FT}}{C^{FT}} = P^{FT} \tag{55}$$

with $P^{FT} = \frac{\varepsilon}{\varepsilon-1}(N^{FT})^{\frac{1}{1-\varepsilon}}\left[1+\tau^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, therefore:

$$\frac{\alpha}{1-\alpha}\frac{Z^{FT}}{C^{FT}} = \frac{\varepsilon}{\varepsilon-1}(N^{FT})^{\frac{1}{1-\varepsilon}}\left[1+\tau^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{56}$$

Because of the monopolistic distortion the price level is too high ($\frac{\varepsilon}{\varepsilon-1} > 1$), thus the free trade allocation in inefficient.

■

C The Effect on Income of a Change in the Trade Policy Instrument

In this section we recover an explicit expression for the effect on income induced by a change in a trade policy instrument as function of the corresponding changes in the different components of the trade balance. This decomposition of the effect on income allows to assess whether the change in income due to a specific trade policy can be exclusively attributed to the *terms of trade effect* and the *opportunity cost effect*. In section 3.3 we obtain condition (29) by implicitly differentiated the trade balance, that is:

$$\frac{(1-\alpha)}{(\tau_X P_H)}\frac{\partial I}{\partial \tau_i} = \underbrace{-\frac{\varepsilon f}{\tau_X P_H}\frac{\partial N}{\partial \tau_i}}_{BT_1} \underbrace{-\left([(1-\alpha)I-(L-\varepsilon f N)]\frac{\partial(\tau_X P_H)^{-1}}{\partial \tau_i}+\tau C_F\frac{\partial\left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)}{\partial \tau_i}\right)}_{BT_2} \underbrace{+\tau\frac{\partial C_F^*}{\partial \tau_i}}_{BT_3} \underbrace{-\left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)\tau\frac{\partial C_F}{\partial \tau_i}}_{BT_4} \tag{57}$$

In this condition the term $\tau\frac{\partial C_F}{\partial \tau_i}$ can be further decomposed into income and substitution effects as follows:

$$\tau\frac{\partial C_F}{\partial \tau_i} = \underbrace{\tau\frac{\partial\left[\left(\frac{P_H}{P_F}\right)^{1-\varepsilon}+1\right]^{\frac{\varepsilon}{1-\varepsilon}} C}{\partial \tau_i}}_{BT_{41}\quad Substitution\quad Effect} + \underbrace{\underbrace{\tau\left[\left(\frac{P_H}{P_F}\right)^{1-\varepsilon}+1\right]^{\frac{\varepsilon}{1-\varepsilon}}\alpha\frac{\partial P^{-1}}{\partial \tau_i}I}_{BT_{42}} + \underbrace{\tau\left[\left(\frac{P_H}{P_F}\right)^{1-\varepsilon}+1\right]^{\frac{\varepsilon}{1-\varepsilon}}}_{BT_{43}}\frac{\alpha}{P}\frac{\partial I}{\partial \tau_i}}_{Income\quad Effect} \tag{58}$$

Then by substituting (58) into (57), the total effect of trade policy on domestic income can be explicitly expressed as:

$$\frac{\partial I}{\partial \tau_i} = \left[\frac{(1-\alpha)}{(\tau_X P_H)}+\left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)\frac{\alpha\tau BT_{43}}{P}\right]^{-1}\left[BT_1+BT_2-\left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)(BT_{41}+BT_{42})+BT_3\right] \tag{59}$$

Similarly, $BT_3 = \tau\frac{\partial C_F^*}{\partial \tau_i}$ can also be further decomposed into income and substitution effects:

$$\tau\frac{\partial C_F^*}{\partial \tau_i} =$$

$$\underbrace{\tau\frac{\partial\left[\left(\frac{P_H^*}{P_F^*}\right)^{1-\varepsilon}+1\right]^{\frac{\varepsilon}{1-\varepsilon}} C^*}{\partial \tau_i}}_{BT_{31}\quad Substitution\quad Effect} + \underbrace{\underbrace{\tau\left[\left(\frac{P_H^*}{P_F^*}\right)^{1-\varepsilon}+1\right]^{\frac{\varepsilon}{1-\varepsilon}}\alpha\frac{\partial (P^*)^{-1}}{\partial \tau_i}I^*}_{BT_{32}} + \underbrace{\tau\left[\left(\frac{P_H^*}{P_F^*}\right)^{1-\varepsilon}+1\right]^{\frac{\varepsilon}{1-\varepsilon}}}_{BT_{33}}\frac{\alpha}{P^*}\frac{\partial I^*}{\partial \tau_i}}_{Income\quad Effect} \tag{60}$$

D Production Subsidies

In this section we set $\tau_I = \tau_I^* = \tau_X = \tau_X^* = 1$ and we prove the propositions and the lemmata of section 4.

Proposition 2: Cooperative Production Subsidy. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then the optimal cooperative production subsidy is set to exactly offset the price markup generated by monopolistic competition. This subsidy implements an equilibrium with the first-best number of varieties and the first-best price level. Formally,*

(1) $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$

(2) $N_C^{Coop} = N^{FB}$

(3) $P_C^{Coop} = P^{FB}$

Proof of Proposition 2. By setting $\tau_C = \frac{\varepsilon-1}{\varepsilon}$ in both countries, the cooperative policymaker exactly eliminates the price markup charged by the monopolistic firms in the differentiated sector. Indeed, from equation (12) we see that individual domestic varieties are now priced at their marginal costs i.e. $p(h) = W$ and $p^*(h) = \tau W$ and the same holds for the foreign country. Substituting $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ into equation (35) we obtain $N = N^* = \frac{\alpha L}{f(\varepsilon-1+\alpha)} \equiv N^{Coop}$. This coincides with the N^{FB} of Proposition 1. From equation (30) it follows that $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ implies $P_C^{Coop} = (N^{FB})^{\frac{1}{1-\varepsilon}}[1+\tau^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \equiv P^{FB}$. ■

Lemma 1 *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* \leq 1$ then a unilateral increase in the production tax increases the domestic price index ($\frac{\partial P}{\partial \tau_C} > 0$) both directly through an increase in the individual price of domestic varieties, and indirectly through a reduction in the total number of varieties ($\frac{\partial(N+N^*)}{\partial \tau_C} < 0$) and in the share of varieties produced domestically ($\frac{\partial s}{\partial \tau_C} < 0$). Income increases ($\frac{\partial I}{\partial \tau_C} > 0$) due to terms-of-trade and opportunity cost effects.*

Proof of Lemma 1.

(1) First, we prove that $\frac{\partial N}{\partial \tau_C} < 0$, $\frac{\partial N^*}{\partial \tau_C} > 0$ and $|\frac{\partial N^*}{\partial \tau_C}| - |\frac{\partial N}{\partial \tau_C}| < 0$.

$$\left.\frac{\partial N}{\partial \tau_C}\right|_{\tau_C=\tau_C^*} =$$

$$\frac{L\alpha\left[\tau^2\left(\alpha^2+(1-\alpha^2)\tau_C\right)+\tau^{\varepsilon+1}\left(2(1-\alpha)(\varepsilon-1)\tau_C+\alpha(2\varepsilon-1)\right)+(1-\alpha)\tau^{2\varepsilon}\left((1-\alpha)\tau_C+\alpha\right)\right]}{f\varepsilon\left(\tau^\varepsilon-\tau\right)\left[\alpha-(\alpha-1)\tau_C\right]^2\left[\alpha(\tau+\tau^\varepsilon)(\tau_C-1)-\tau_C(\tau^\varepsilon-\tau)\right]}$$

$$\left.\frac{\partial N^*}{\partial \tau_C}\right|_{\tau_C=\tau_C^*} = \frac{L\alpha\tau\left[\alpha(\tau^\varepsilon-\tau)+\tau^\varepsilon\left(2(\alpha-1)\varepsilon\tau_C-2\alpha\varepsilon\right)\right]}{f\varepsilon\left(\tau^\varepsilon-\tau\right)\left[\alpha-(\alpha-1)\tau_C\right]^2\left[\alpha(\tau+\tau^\varepsilon)(\tau_C-1)-\tau_C(\tau^\varepsilon-\tau)\right]}$$

The denominator of both expressions is negative whenever $\tau_C \leq 1$. The numerator of the first expression is always positive being the sum of only positive terms. For the numerator of the second expression to be positive we would need $\tau_C < \alpha\frac{1-\tau^{1-\varepsilon}-2\varepsilon}{2(1-\alpha)\varepsilon}$, which is not possible given that $\tau_C \geq 0$ by definition. Finally, $|\frac{\partial N^*}{\partial \tau_C}| - |\frac{\partial N}{\partial \tau_C}| = \frac{\partial N^*}{\partial \tau_C} + \frac{\partial N}{\partial \tau_C} = -\frac{L(1-\alpha)\alpha}{f\varepsilon[\alpha-(\alpha-1)\tau_C]^2} < 0$.

(2) Second, it follows from (1) that $\frac{\partial (N+N^*)}{\partial \tau_C} < 0$. Also,

$$\frac{\partial s}{\partial \tau_C} = -\frac{(1+\alpha)\tau^2 + (1-\alpha)\tau^{2\varepsilon} + (4\varepsilon-2)\tau^{1+\varepsilon}}{4(\tau^\varepsilon - \tau)[\alpha(\tau+\tau^\varepsilon)(1-\tau_C) + (\tau^\varepsilon - \tau)\tau_C]} < 0$$

(3) Second, we prove that $\frac{\partial P}{\partial \tau_C} > 0$.

$$\left.\frac{\partial P}{\partial \tau_C}\right|_{\tau_C = \tau_C^*} = \frac{1}{\varepsilon-1}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{(1-\varepsilon)} P^\varepsilon \left[\tau_C^{-\varepsilon}(\varepsilon-1)N - \tau_C^{1-\varepsilon}\left(\frac{\partial N}{\partial \tau_C} + \frac{1}{\tau^{\varepsilon-1}}\frac{\partial N^*}{\partial \tau_C}\right)\right]$$

which is positive given that
$\frac{\partial N}{\partial \tau_C} + \frac{1}{\tau^{\varepsilon-1}}\frac{\partial N^*}{\partial \tau_C} = L\alpha\tau^{-\varepsilon}\left[\frac{\alpha\tau^2+(1-\alpha)\tau^{2\varepsilon}(\alpha+(1-\alpha)\tau_C)+\tau^{1+\varepsilon}(\alpha(2\varepsilon-\alpha)+(1-\alpha)(2\varepsilon-1-\alpha)\tau_C}{f\varepsilon[\alpha-(\alpha-1)\tau_C]^2[\alpha(\tau+\tau^\varepsilon)(\tau_C-1)-\tau_C(\tau^\varepsilon-\tau)]}\right] < 0$, since the numerator is a sum of positive terms while the denominator is negative for $\tau_C < 1$.

(4) Next we prove that $\frac{\partial I}{\partial \tau_C} > 0$.
For the derivative of income it is enough to remember that $I = L + N\varepsilon f(\tau_C - 1)$ and that $\frac{\partial N}{\partial \tau_C} < 0$.

(5) Finally, we prove that income increases due to terms of trade ($BT_2 > 0$) and opportunity cost effects ($BT_1 > 0$) since all the other effects on domestic income are negative ($BT_{41} + BT_{42} < 0$ and $BT_3 < 0$).

(a) $BT_1 > 0$: This is so given that $BT_1 = \left[-\varepsilon f \frac{1}{P_H}\frac{\partial N}{\partial \tau_C}\right]$ and $\frac{\partial N}{\partial \tau_C} < 0$.

(b) $BT_2 > 0$: When $\tau_C = \tau_C^*$ the equilibrium is symmetric thus there is no trade in the homogeneous good and $BT_2 = -\tau\alpha\left(\frac{P_F}{P}\right)^{-\varepsilon}\frac{I}{P}\frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial \tau_C}$. Note that $\frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial \tau_C} = \frac{\frac{\partial P_H^*}{\partial \tau_C}P_H - \frac{\partial P_H}{\partial \tau_C}P_H^*}{P_H^2} < 0$ given that $\frac{\partial P_H^*}{\partial \tau_C} = \frac{\varepsilon}{\varepsilon-1}N^{*\left(\frac{1}{1-\varepsilon}\right)}\left(-\frac{\tau_C^*}{(\varepsilon-1)N^*}\frac{\partial N^*}{\partial \tau_C}\right) < 0$ and $\frac{\partial P_H}{\partial \tau_C} = \frac{\varepsilon}{\varepsilon-1}N^{\left(\frac{1}{1-\varepsilon}\right)}\left(1 - \frac{\tau_C}{(\varepsilon-1)N}\frac{\partial N}{\partial \tau_C}\right) > 0$.

(c) $BT_3 < 0$:

- $BT_{31} + BT_{32} < 0$: This is so given that
$BT_{31} + BT_{32} = \tau\alpha\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon} I^*\left(\frac{1}{P^*}\right)^2\left[-\varepsilon\frac{P^*}{P_F^*}\frac{\partial P_F^*}{\partial \tau_C} + (\varepsilon-1)\frac{\partial P^*}{\partial \tau_C}\right]$,
$\frac{\partial P_F^*}{\partial \tau_C} = \tau\frac{\varepsilon}{\varepsilon-1}N^{\left(\frac{1}{1-\varepsilon}\right)}\left(1 - \frac{\tau_C}{(\varepsilon-1)N}\frac{\partial N}{\partial \tau_C}\right) > 0$, $\frac{\partial P^*}{\partial \tau_C}|_{\tau_C=\tau_C^*=1} = -P^*\frac{\varepsilon\tau}{(\varepsilon-1)(\tau^\varepsilon-\tau)} < 0$ and $\frac{\partial P^*}{\partial \tau_C}|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = -P^*\frac{\varepsilon^2\tau}{(\alpha+\varepsilon-1)((\alpha+\varepsilon-1)\tau^\varepsilon+(\alpha-\varepsilon+1)\tau)} < 0$.
- $BT_{33} \leq 0$: This is so given that $BT_{33} = \left[\tau\alpha\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon}\left(\frac{1}{P^*}\right)\frac{\partial I^*}{\partial \tau_i}\right]$, $I^* = L + (\tau_C^* - 1)\varepsilon f N^*$ and $\frac{\partial I^*}{\partial \tau_C} = (\tau_C^* - 1)\varepsilon f\frac{\partial N^*}{\partial \tau_C} \leq 0$

(d) $-(BT_{41}+BT_{42})<0$: Note that since $\left[\left(\frac{P_H}{P_F}\right)^{\frac{\varepsilon}{\varepsilon-1}}+1\right]^{\frac{\varepsilon}{1-\varepsilon}}=\left(\frac{P_F}{P}\right)^{-\varepsilon}$ we have that
$-(BT_{41}+BT_{42})=\tau\alpha\left(\frac{P_F}{P}\right)^{-\varepsilon}\frac{I}{P}\left[\varepsilon\frac{P}{P_F}\left(\frac{1}{P}\right)^2\left(\frac{\partial P_F}{\partial\tau_C}P-\frac{\partial P}{\partial\tau_C}P_F\right)+\frac{\partial P}{\partial\tau_C}\frac{1}{P}\right]=$
$\tau\alpha\left(\frac{P_F}{P}\right)^{-\varepsilon}\frac{I}{P}\left[\varepsilon\left(P_F\right)^{-1}\frac{\partial P_F}{\partial\tau_C}-(\varepsilon-1)\frac{\partial P}{\partial\tau_C}\frac{1}{P}\right]<0$
given that $\frac{\partial P_F}{\partial\tau_C}=\tau\frac{\varepsilon}{\varepsilon-1}N^{*\left(\frac{1}{1-\varepsilon}\right)}\left(-\frac{\tau_C^*}{(\varepsilon-1)N^*}\frac{\partial N^*}{\partial\tau_C}\right)<0$ and $\frac{\partial P}{\partial\tau_C}>0$.

■

Proposition 3: Unilaterally Set Production Subsidies. *Let $\tau>1$, $\varepsilon>1$ and $0<\alpha<1$. Then the optimal unilateral production subsidy is positive but strictly smaller than the efficient subsidy. Formally,*

(1) $\left.\frac{\partial V(P(\tau_C),I(\tau_C))}{\partial\tau_C}\right|_{\tau_C=\tau_C^*=1}<0$

(2) $\left.\frac{\partial V(P(\tau_C),I(\tau_C))}{\partial\tau_C}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}}>0$

Proof of Proposition 3.

(1) $\left.\frac{\partial V(P(\tau_C),I(\tau_C))}{\partial\tau_C}\right|_{\tau_C=\tau_C^*=1}=-\frac{\alpha((1-\alpha)\tau^{\varepsilon}+\tau(\alpha+\varepsilon-1))}{(\varepsilon-1)(\tau^{\varepsilon}-\tau)}<0$

(2) $\left.\frac{\partial V(P(\tau_C),I(\tau_C))}{\partial\tau_C}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}}=\frac{\alpha\varepsilon^2\tau(\tau^{\varepsilon}+\tau)}{(\varepsilon-1)(\tau^{\varepsilon}-\tau)(\alpha(\tau+\tau^{\varepsilon})+(\varepsilon-1)(\tau^{\varepsilon}-\tau))}>0$

■

Proposition 4: Nash-Equilibrium Production Subsidies. *Let $\tau>1$, $\varepsilon>1$ and $0<\alpha<1$. Then the Nash solution of the trade policy game between the two countries implies a production subsidy strictly smaller than the one needed to implement the first-best allocation. The total number of varieties is larger than in the free trade allocation, but remains lower than the first-best level. At the same time, the price level is lower than in the free trade allocation but higher than the first-best level. Formally,*

(1) $1>\tau_C^{Nash}>\tau_C^{FB}$.

(2) $N^{FT}<N_C^{Nash}<N^{FB}$.

(3) $P^{FB}<P_C^{Nash}<P^{FT}$

Proof of Proposition 4. The Nash solution of this game will be symmetric due to the symmetric assumption across the two countries. Therefore, to derive τ_C^{Nash} it is enough to compute the best reply of Home, $\frac{\partial V(P(\tau_C,\tau_C^*),I(\tau_C,\tau_C^*))}{\partial\tau_C}=0$, and then impose symmetry, $\tau_C=\tau_C^*$. Here, $P(\tau_C,\tau_C^*)$ is given by equation (30), which is implied by the equilibrium expressions for $N(\tau_C,\tau_C^*)$

and $N^*(\tau_C, \tau_C^*)$, equation (35). Moreover, $I(\tau_C, \tau_C^*)$ is given by $L+(\tau_C-1)\varepsilon f N(\tau_X, \tau_X^*)$. When doing so we obtain a quadratic expression in τ_C^{Nash}:

$$a(\tau_C^{Nash})^2 + b\tau_C^{Nash} + c = 0 \tag{61}$$

where $a \equiv \alpha(1-\alpha)\varepsilon\tau^\varepsilon[(3-2\varepsilon-\alpha)\tau-(1-\alpha)\tau^\varepsilon]$, $b \equiv \alpha[(\varepsilon-1+\alpha)\tau^2+(1-\alpha)(\varepsilon-1-\alpha(2\varepsilon-1))\tau^{2\varepsilon}+(2\varepsilon-2+\alpha)(\varepsilon-1-\alpha(2\varepsilon-1))\tau^{1+\varepsilon}]$ and $c \equiv \alpha^2(\varepsilon-1)\tau^\varepsilon((2\varepsilon-1+\alpha)\tau+(1-\alpha)\tau^\varepsilon)$. Note that $a < 0$ and $c > 0$. To prove that $a < 0$ it suffices to see that:

(i) $\tau^\varepsilon > \tau \ \forall\varepsilon > 1$ and $\forall\tau > 1$;

(ii) $1-\alpha > 3-2\varepsilon-\alpha \ \forall\varepsilon > 1$.

This implies that $\forall\varepsilon > 1$, $\alpha \in (0,1)$ and $\tau > 1$, (61) has two real solutions, one positive and one negative. Given that $\tau_C \in [0,\infty)$, this implies that the Nash solution always exists and is unique.

(1) (i) At $\tau_C = 1$ we have: $a\tau_C^2+b\tau_C+c = -\alpha(\tau^\varepsilon-\tau)[(\varepsilon+\alpha-1)\tau+(1-\alpha)\tau^\varepsilon] < 0$ implying that $\tau_C^{Nash} < 1$ since $a < 0$.

(ii) At $\tau_C^{FB} = \frac{\varepsilon-1}{\varepsilon}$ we have: $a\tau_C^2+b\tau_C+c = \frac{\alpha(\varepsilon-1)(\varepsilon+\alpha-1)\tau(\tau+\tau^\varepsilon)}{\varepsilon} > 0$ implying $\tau_C^{FB} < \tau_C^{Nash}$.

(2) Follows from (1) and Lemma 1 where we proved that $\frac{\partial(N+N^*)}{\partial\tau_C} < 0 \ \forall\tau_C = \tau_C^* \leq 1$.

(3) Follows from (1) and Lemma 1 where we proved that $\frac{\partial P}{\partial\tau_C} > 0 \ \forall\tau_C = \tau_C^* \leq 1$.

■

E Tariffs

In this section while retaining the assumption $\tau_X = \tau_X^* = 1$, we prove the propositions and the lemmata of section 5 where we allow for the use of an import tariff as main policy instrument.

Proposition 5: Cooperative Import Subsidy. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then, if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker refrains from using subsidies/tariffs on imports. If $\tau_C = \tau_C^* = 1$, the cooperative policy maker finds it optimal to subsidize imports. The total number of varieties is larger than in the free trade allocation, but remains lower than the first-best level. At the same time, the price level is lower than in the free trade allocation but higher than the first-best level. Formally,*

(1) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then, $\tau_I^{Coop} = 1$, $N_I^{Coop} = N^{FB}$ and $P_I^{Coop} = P_I^{FB}$

(2) If $\tau_C = \tau_C^* = 1$ then, $\tau_I^{Coop} < 1$, $N^{FT} < N_I^{Coop} < N^{FB}$ and $P^{FB} < P_I^{Coop} < P^{FT}$

Proof of Proposition 5. In the case of tariffs, the cooperative policy maker maximizes:

$$\max_{\tau_I,\tau_I^*} V(P(\tau_I,\tau_I^*), I(\tau_I,\tau_I^*)) + V(P^*(\tau_I,\tau_I^*), I^*(\tau_I,\tau_I^*)) \tag{62}$$

where $P(\tau_I, \tau_I^*)$ is given by equation (30) once we substitute in $N(\tau_I, \tau_I^*)$ as implicitly determined by equation (21). $I(\tau_I, \tau_I^*)$ is equal to $L + (\tau_I - 1)\tau_X^* \tau P_H^*(\tau_I, \tau_I^*) C_F(\tau_I, \tau_I^*) + (\tau_C - 1) N(\tau_I, \tau_I^*)\varepsilon f$ where $P_H^*(\tau_I, \tau_I^*) = \frac{\varepsilon}{\varepsilon-1}\tau_C^*(N^*(\tau_I, \tau_I^*))^{\frac{1}{1-\varepsilon}}$, $C_F(\tau_I, \tau_I^*) = P_F(\tau_I, \tau_I^*)^{-\varepsilon} P(\tau_I, \tau_I^*)^{\varepsilon} C(\tau_I, \tau_I^*)$, $P_F(\tau_I, \tau_I^*) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_X\tau_C^*(N^*(\tau_I, \tau_I^*))^{\frac{1}{1-\varepsilon}}$ and finally $C(\tau_I, \tau_I^*)$ is given by its equilibrium values in equation (31). Symmetric conditions apply to foreign variables.

(1) To prove the first part of the proposition it is sufficient to show that if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ the cooperative policy maker finds it optimal to set $\tau_I = \tau_I^* = 1$.

By taking the derivative of (62) with respect to τ_I and imposing symmetry, the first order condition can be written as:

$$-\frac{\alpha\varepsilon\tau\,(\tau_I - 1)\left((\alpha+\varepsilon-1)\tau^{2\varepsilon}\tau_I^{2\varepsilon} + (1-\alpha)\tau^2\tau_I + \varepsilon\tau^{\varepsilon+1}\tau_I^{\varepsilon}\right)}{\tau_I\left(\tau^{\varepsilon}\tau_I^{\varepsilon} + \tau\right)\left(\tau^{\varepsilon}\tau_I^{\varepsilon} + \tau\tau_I\right)\left((\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_I^{\varepsilon} + (1-\alpha)(\varepsilon-1)\tau\tau_I + \alpha\varepsilon\tau\right)} = 0$$

It is easy then to verify that this condition is satisfied iff $\tau_I = 1$.

(2) To prove the second part of the proposition we follow three steps: (i) first we show if $\tau_C = \tau_C^* = 1$, no cooperative solution exists for $\tau_I > 1$; (ii) second we show that if $\tau_C = \tau_C^* = 1$, there exists a solution for $\tau_I < 1$; (iii) third we prove that if $\tau_C = \tau_C^* = 1$ $\tau_I^{Coop} < 1$, $N^{FT} < N_I^{Coop} < N^{FB}$ and $P^{FB} < P_I^{Coop} < P^{FT}$. If $\tau_C = \tau_C^* = 1$ then, by taking the derivative of (62) with respect to τ_I and imposing symmetry, the first order condition can be written as:

$$\frac{A_I^{Coop}(\tau_I)}{B_I^{Coop}(\tau_I)} = 0$$

where:

$$\begin{aligned} A_I^{Coop}(\tau_I) &\equiv \alpha\tau(\tau^{\varepsilon+1}\tau_I^{\varepsilon}\left(\tau_I\left(2\alpha - \varepsilon^2 + \varepsilon - 2\right) + (\varepsilon-1)\varepsilon\right) \\ &\quad + \tau^{2\varepsilon}\tau_I^{2\varepsilon}\left(\varepsilon(\alpha+\varepsilon-2) - (\varepsilon-1)\tau_I(\alpha+\varepsilon-1)\right) + (\alpha-1)\tau^2\tau_I\left(\varepsilon\tau_I - \varepsilon + 1\right)) \\ B_I^{Coop}(\tau_I) &\equiv (\varepsilon-1)\tau_I\left(\tau^{\varepsilon}\tau_I^{\varepsilon} + \tau\right)\left(\tau^{\varepsilon}\tau_I^{\varepsilon} + \tau\tau_I\right)\left(\tau_I(\tau - \alpha\tau) + \tau^{\varepsilon}\tau_I^{\varepsilon} + \alpha\tau\right) \end{aligned}$$

(i) At the optimum, it must be that $A_I^{Coop}(\tau_I) = 0$. To prove that no cooperative solution exists for $\tau_I > 1$ it suffices to notice that all terms in $A_I^{Coop}(\tau_I)$ are strictly negative for $\tau_I > 1$ and thus $A_I^{Coop}(\tau_I)$ has no zeros for $\tau_I > 1$.

(ii) In order to prove that a cooperative solution exists for $\tau_I < 1$, consider that: (a) $A_I^{Coop}(\tau_I)$ is a continuous function in τ_I; (b) $A_I^{Coop}|_{\tau_I=1} = -(1-\alpha)\alpha\tau\left(\tau^{\varepsilon} + \tau\right)^2 < 0$; (c) $A_I^{Coop}|_{\tau_I=0} = 0$ and $(\partial A_I^{Coop}/\partial\tau_I)|_{\tau_I=0} = (1-\alpha)\alpha(\varepsilon-1)\tau^3 > 0$. Then by the fix point theorem there exists a value $\tau_I \in (0,1)$ such that $A_I^{Coop}(\tau_I) = 0$.

(iii) Finally we need to prove that if $\tau_C = \tau_C^* = 1$ $\tau_I^{Coop} < 1$, $N^{FT} < N_I^{Coop} < N^{FB}$ and $P^{FB} < P_I^{Coop} < P^{FT}$. We do this in several steps.

(a) If $0 < \tau_I < 1$, by symmetrically increasing the subsidy on imports in both countries the cooperative policy maker increases the total number of varieties and reduces the price level. Indeed totally differentiating N at $\tau_C = \tau_C^* = 1$, we obtain:

$$dN = \frac{\partial N}{\partial \tau_I} d\tau_I + \frac{\partial N}{\partial \tau_I^*} d\tau_I^*$$

which after imposing symmetry can be read as:

$$\frac{dN}{d\tau_I} = \frac{\partial(N+N^*)}{\partial \tau_I}\Bigg|_{\tau_I=\tau_I^*} = -\frac{L(1-\alpha)\alpha\tau\left(\tau^\varepsilon\left(\varepsilon-(\varepsilon-1)\tau_I\right)\tau_I^\varepsilon+\tau\tau_I\right)}{f\varepsilon\tau_I\left(\tau_I\tau(1-\alpha)+\tau^\varepsilon\tau_I^\varepsilon+\alpha\tau\right)^2} < 0$$

At the same time when totally differentiating P we find:

$$\begin{aligned} dP &= -\frac{P^\varepsilon}{\varepsilon-1}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{(1-\varepsilon)} \\ &\left[\frac{\partial N}{\partial\tau_I}d\tau_I + \frac{\partial N}{\partial\tau_I^*}d\tau_I^* + (\tau\tau_I)^{1-\varepsilon}\left(\frac{\partial N^*}{\partial\tau_I^*}d\tau_I^* + \frac{\partial N^*}{\partial\tau_I}d\tau_I\right) - (\varepsilon-1)\tau^{1-\varepsilon}\tau_I^{-\varepsilon}N^*d\tau_I\right]\end{aligned}$$

which once we impose symmetry can be written as:

$$\frac{dP}{d\tau_I} = -\frac{P^\varepsilon}{\varepsilon-1}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{(1-\varepsilon)}\left[\left(1+(\tau\tau_I)^{1-\varepsilon}\right)\frac{\partial(N+N^*)}{\partial\tau_I} - (\varepsilon-1)\tau^{1-\varepsilon}\tau_I^{-\varepsilon}\right]\Bigg|_{\tau_I=\tau_I^*} > 0$$

First note that $\frac{dN}{d\tau_I} < 0$ and $\frac{dP}{d\tau_I} > 0$, together with point (ii) proving that the cooperative solution entails $\tau_I < 1$, imply that $N^{FT} < N^{Coop}$ and $P^{FT} > P^{Coop}$. Thus, we are left with the comparison between the cooperative solution and the first-best. Imposing $\tau_C = \tau_C^* = \tau_X = \tau_X^* = 1$ and $\tau_I = \tau_I^*$ into (35) we compute the number of varieties produced in each country in the symmetric equilibrium: $N(\tau_I) = \frac{L\alpha(\tau+(\tau\tau_I)^\varepsilon)}{f\varepsilon(\alpha\tau+\tau(1-\alpha)\tau_I+(\tau\tau_I)^\varepsilon)}$. Thus, $\lim_{\tau_I\to 0} N(\tau_I) = \frac{L}{f\varepsilon}$. Note that $\frac{L}{f\varepsilon} > \frac{L\alpha}{f(\varepsilon+\alpha-1)} = N^{FB}$ for $\varepsilon > 1$ and $0 < \alpha < 1$. This implies that there exists a τ_I small enough so that the cooperative policy maker can implement the first-best number of varieties. The question is whether he wants to do so.

(b) Let $\tau_I^{Coop} = f(\alpha,\varepsilon,\tau)$ and $\tau_I^{FB} = g(\alpha,\varepsilon,\tau)$ be, respectively, the solution to the cooperative problem and the subsidy implementing the first-best level of number of varieties. Thus, τ_I^{Coop} is such that $A_I^{Coop}|_{\tau_I=\tau_I^{Coop}} = 0$ while τ_I^{FB} is such that $N(\tau_I^{FB}) = \frac{L\alpha(\tau+(\tau\tau_I^{FB})^\varepsilon)}{f\varepsilon(\alpha\tau+\tau(1-\alpha)\tau_I^{FB}+(\tau\tau_I^{FB})^\varepsilon)} = \frac{L\alpha}{f(\varepsilon+\alpha-1)} = N^{FB}$. Though it is not possible to find an explicit solution for τ_I^{FB}, the condition $N(\tau_I^{FB}) = N^{FB}$ simplifies to $(\tau\tau_I^{FB})^\varepsilon = -\varepsilon\tau\tau_I^{FB} + \tau(\varepsilon-1)$. If we substitute this condition into $A_I^{Coop}(\tau_I) = 0$ we obtain a cubic expression in τ_I. The solutions are $\tau_I = \{\frac{\varepsilon-1}{\varepsilon}, 1, 1\}$. However, none of these solves $(\tau\tau_I)^\varepsilon = -\varepsilon\tau\tau_I + \tau(\varepsilon-1)$. More precisely, they all imply a level of subsidy on imports smaller than what needed to implement the first-best level of varieties. Thus, we conclude that there is no intersection between the set of τ_I^{Coop} and the set of τ_I^{FB}.

(c) The last step is to prove that $\tau_I^{FB} < \tau_I^{Coop}$ always. From (i) it will then follow that $N_I^{Coop} < N_{FB}$ and $P_I^{Coop} > P^{FB}$. To this purpose, note that f and g are two continuous functions in the space $\{0 < \alpha < 1, \tau > 1, \varepsilon > 1\}$. This is so since, by the implicit function theorem, we can compute the derivatives of τ_I^{FB} and τ_I^{Coop} with respect to the three parameters and the derivative always exists in such parametric space. In point (b) we proved that there is no intersection between g and f. It must then be that one always lies on top of the other i.e. it is either always $\tau_I^{FB} < \tau_I^{Coop}$ or the other way around. We evaluate both functions at $\{\alpha = 0.5, \varepsilon = 2, \tau = 1.5\}$ and find $\tau_I^{FB} = 0.39 < 0.63 = \tau_I^{Coop}$.[21] Thus, the cooperative import subsidy is always smaller than the one needed to implement the first best number of varieties.

■

Lemma 2: *Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ and $\tau_I = \tau_I^* = 1$. Then,*

(1) *If $\tau_C = \tau_C^* = 1$ or $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ and $\varepsilon - 1 > \alpha$ a unilateral increase in the import tariff increases the price of imported varieties, decreases the total number of varieties ($\frac{\partial(N+N^*)}{\partial\tau_I} < 0$) and increases the share of domestically produced varieties ($\frac{\partial s}{\partial\tau_I} > 0$). Overall, the domestic price index decreases ($\frac{\partial P}{\partial\tau_I} < 0$).*

(2) *If $\tau_C = \tau_C^* = 1$, then a unilateral increase in the import tariff increases income ($\frac{\partial I}{\partial\tau_I} > 0$) while if $\tau_C = \tau_C^* = \frac{(\varepsilon-1)}{\varepsilon}$, then the effect on income is negative ($\frac{\partial I}{\partial\tau_I} < 0$). Moreover, terms-of-trade and opportunity costs are always negative, while other effects on income are always positive.*

Proof of Lemma 2.

(1) In order to prove the first part of Lemma 2 we need to show that if $\tau_C = \tau_C^* = 1$ or $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ and $\varepsilon - 1 > \alpha$, then:

(i) $\frac{\partial(N+N^*)}{\partial\tau_I} < 0$. This follows from:

$$\left.\frac{\partial N}{\partial\tau_I}\right|_{\tau_C=\tau_C^*=1} = \frac{L\alpha\tau^{\varepsilon+1}\left[(1+\varepsilon-\alpha)\tau + (\alpha+\varepsilon-1)\tau^{\varepsilon}\right]}{f\varepsilon(\tau-\tau^{\varepsilon})^2(\tau+\tau^{\varepsilon})} > 0$$

$$\left.\frac{\partial N^*}{\partial\tau_I}\right|_{\tau_C=\tau_C^*=1} = -\frac{L\alpha\tau\left[(1-\alpha)\tau^2 + \varepsilon\tau^{2\varepsilon} + (\alpha+\varepsilon-1)\tau^{\varepsilon+1}\right]}{f\varepsilon(\tau-\tau^{\varepsilon})^2(\tau+\tau^{\varepsilon})} < 0$$

Summing the previous derivatives we obtain:

$$\left.\frac{\partial(N+N^*)}{\partial\tau_I}\right|_{\tau_C=\tau_C^*=1} = -\frac{L(1-\alpha)\alpha\tau}{f\varepsilon(\tau+\tau^{\varepsilon})} < 0$$

Moreover:

$$\left.\frac{\partial N}{\partial\tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = \frac{L\alpha\tau(\varepsilon-1)\left[(1-\alpha)\alpha\tau^2 + (\alpha+\varepsilon-1)^2\tau^{2\varepsilon} + (\varepsilon^2+\alpha-1)\tau^{1+\varepsilon}\right]}{f(\alpha+\varepsilon-1)^2\left[\alpha(\tau+\tau^{\varepsilon}) + (\varepsilon-1)(\tau^{\varepsilon}-\tau)\right](\tau^{2\varepsilon}-\tau^2)} > 0$$

[21] The other solutions are either negative or zero, thus we exclude them since $\tau_I > 0$.

$$\left.\frac{\partial N^*}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} =$$

$$-\frac{L\alpha\tau(\varepsilon-1)\left[(\varepsilon-1)(1-\alpha)\tau^2+\varepsilon(\alpha+\varepsilon-1)\tau^{2\varepsilon}+((\varepsilon-1)^2+\alpha(2\varepsilon-1))\tau^{\varepsilon+1}\right]}{f(\alpha+\varepsilon-1)^2\left[\alpha(\tau+\tau^\varepsilon)+(\varepsilon-1)(\tau^\varepsilon-\tau)\right](\tau^{2\varepsilon}-\tau^2)} < 0$$

Again summing the previous derivatives one can show that:

$$\left.\frac{\partial (N+N^*)}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = -\frac{L(1-\alpha)\alpha\tau(\varepsilon-1)}{f(\alpha+\varepsilon-1)^2(\tau+\tau^\varepsilon)} < 0$$

(ii) $\frac{\partial s}{\partial \tau_I} > 0$. This can be seen from:

$$\left.\frac{\partial s}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=1} = \frac{\tau\left[(\alpha+2\varepsilon-1)\tau^\varepsilon+(1-\alpha)\tau\right]}{4\left(\tau-\tau^\varepsilon\right)^2} > 0$$

$$\left.\frac{\partial s}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = \frac{(\varepsilon-1)\tau\left[(\alpha+2\varepsilon-1)\tau^\varepsilon+(1-\alpha)\tau\right]}{4\left(\tau^\varepsilon-\tau\right)\left[\alpha(\tau^\varepsilon-\tau)+(\varepsilon-1)(\tau^\varepsilon-\tau)\right]} > 0$$

(iii) $\frac{\partial P}{\partial \tau_I} < 0$. Indeed:

$$\left.\frac{\partial P}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=1} = -P\,\frac{\tau\left[\varepsilon\tau+\alpha\left(\tau^\varepsilon-\tau\right)\right]}{(\varepsilon-1)\left(\tau^{2\varepsilon}-\tau^2\right)} < 0$$

$$\left.\frac{\partial P}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = -\frac{P\left[\alpha(1-\alpha)+(\varepsilon-1-\alpha)\varepsilon\right]\tau^2}{(\alpha+\varepsilon-1)\left(\tau^\varepsilon+\tau\right)\left[\alpha(\tau+\tau^\varepsilon)+(\varepsilon-1)(\tau^\varepsilon-\tau)\right]} < 0 \quad \text{if } \varepsilon-1>\alpha$$

(2) In order to prove the second part of Lemma 2 we need to show that

(i) If $\tau_C=\tau_C^*=1$, then $\frac{\partial I}{\partial \tau_I} > 0$. Indeed:

$$\left.\frac{\partial I}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=1} = \frac{L\alpha\tau}{\tau^\varepsilon+\tau} > 0$$

Conversely, if $\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}$, then $\frac{\partial I}{\partial \tau_I} < 0$. In fact:

$$\left.\frac{\partial I}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = -\frac{L\alpha(\varepsilon-1)\tau^2\left[\left((\varepsilon-1)^2+2\varepsilon(\varepsilon-1)+\alpha(2\varepsilon-1)\right)\tau^\varepsilon-\left((\varepsilon-1)^2-\alpha\right)\tau\right]}{(\alpha+\varepsilon-1)^2\left(\tau^\varepsilon-\tau\right)\left(\tau^\varepsilon+\tau\right)\left[\alpha(\tau+\tau^\varepsilon)+(\varepsilon-1)(\tau^\varepsilon-\tau)\right]} < 0$$

(ii) If $\tau_C=\tau_C^*=1$ or $\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}$ then a unilateral increase in the domestic tariff has a negative opportunity cost effect ($BT_1<0$), a negative terms of trade effect ($BT_2<0$), a positive Foreign substitution and income effect ($BT_3<0$) and a positive domestic substitution and income effect ($-(BT_{41}+BT_{42})<0$).

(a) $BT_1<0$:

This is so given that $BT_1=\left[-\varepsilon f\left(\frac{1}{P_H}\right)\frac{\partial N}{\partial \tau_I}\right]$ and $\frac{\partial N}{\partial \tau_I}>0$.

(b) $BT_2 < 0$:

Since we start from a symmetric equilibrium, $BT_2 = -\tau\alpha \left(\frac{P_F}{P}\right)^{-\varepsilon} \frac{I}{P} \frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial \tau_I}$. Note that $\frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial \tau_I} = \frac{\frac{\partial P_H^*}{\partial \tau_I} P_H - \frac{\partial P_H}{\partial \tau_I} P_H^*}{P_H^2} > 0$ given that $\frac{\partial P_H^*}{\partial \tau_I} = \frac{\varepsilon}{\varepsilon-1} N^{*\left(\frac{1}{1-\varepsilon}\right)} \left(-\frac{\tau_C^*}{(\varepsilon-1)N^*} \frac{\partial N^*}{\partial \tau_I}\right) > 0$ and $\frac{\partial P_H}{\partial \tau_I} = \frac{\varepsilon}{\varepsilon-1} N^{\left(\frac{1}{1-\varepsilon}\right)} \left(-\frac{\tau_C}{(\varepsilon-1)N} \frac{\partial N}{\partial \tau_I}\right) < 0$.

(c) $BT_3 < 0$: Remember that $BT_3 = BT_{31} + BT_{32} + BT_{33}$.

· $BT_{31} + BT_{32} < 0$: This is so given that $BT_{31} + BT_{32} = \tau\alpha \left(\frac{P_F^*}{P^*}\right)^{-\varepsilon} I^* \left(\frac{1}{P^*}\right)^2 \left[-\varepsilon \frac{P^*}{P_F^*} \frac{\partial P_F^*}{\partial \tau_C} + (\varepsilon - 1) \frac{\partial P^*}{\partial \tau_C}\right]$, $\frac{\partial P_F^*}{\partial \tau_C} = \tau \frac{\varepsilon}{\varepsilon-1} N^{\left(\frac{1}{1-\varepsilon}\right)} \left(1 - \frac{\tau_C}{(\varepsilon-1)N} \frac{\partial N}{\partial \tau_C}\right) > 0$, $\frac{\partial P^*}{\partial \tau_C}|_{\tau_C=\tau_C^*=1} = -P^* \frac{\varepsilon\tau}{(\varepsilon-1)(\tau^\varepsilon-\tau)} < 0$ and $\frac{\partial P^*}{\partial \tau_C}|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = -P^* \frac{\varepsilon^2\tau}{(\alpha+\varepsilon-1)((\alpha+\varepsilon-1)\tau^\varepsilon+(\alpha-\varepsilon+1)\tau)} < 0$.

· $BT_{33} \leq 0$: This is so given that $BT_{33} = \left[\tau\alpha \left(\frac{P_F^*}{P^*}\right)^{-\varepsilon} \left(\frac{1}{P^*}\right) \frac{\partial I^*}{\partial \tau_i}\right]$, $I^* = L + (\tau_C^* - 1)\varepsilon f N^*$ and $\frac{\partial I^*}{\partial \tau_C} = (\tau_C^* - 1)\varepsilon f \frac{\partial N^*}{\partial \tau_C} \leq 0$

(d) $-(BT_{41} + BT_{42}) > 0$:

Note that $-(BT_{41}+BT_{42}) = \tau\alpha \left(\frac{P_F}{P}\right)^{-\varepsilon} \frac{I}{P} \left[\varepsilon \frac{P}{P_F} \left(\frac{1}{P}\right)^2 \left(\frac{\partial P_F}{\partial \tau_I} P - \frac{\partial P}{\partial \tau_I} P_F\right) + \frac{\partial P}{\partial \tau_I} \frac{1}{P}\right] =$

$\tau\alpha \left(\frac{P_F}{P}\right)^{-\varepsilon} \frac{I}{P} \left[\varepsilon (P_F)^{-1} \frac{\partial P_F}{\partial \tau_I} - (\varepsilon - 1) \frac{\partial P}{\partial \tau_I} \frac{1}{P}\right] > 0$

given that $\frac{\partial P_F}{\partial \tau_I} = \tau\tau_C^* \frac{\varepsilon}{\varepsilon-1} N^{*\left(\frac{1}{1-\varepsilon}\right)} \left(1 - \frac{\tau_I}{(\varepsilon-1)N^*} \frac{\partial N^*}{\partial \tau_I}\right)$ and

$\frac{\partial P}{\partial \tau_I} = \frac{1}{\varepsilon-1} \left(\frac{\varepsilon}{\varepsilon-1}\right)^{(1-\varepsilon)} P^\varepsilon \tau_C^{1-\varepsilon} \left[(\varepsilon - 1)\tau^{1-\varepsilon}\tau_I^{-\varepsilon} N^* - \left(\frac{\partial N}{\partial \tau_I} + (\tau_I\tau)^{1-\varepsilon} \frac{\partial N^*}{\partial \tau_I}\right)\right] < 0$.

■

Proposition 6: Unilaterally Set Import Tariffs/Subsidies. *Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ and $\tau_I = \tau_I^* = 1$. The optimal unilateral import policy is to set an import tariff when starting from the free trade allocation and to set an import subsidy when starting from the first-best allocation implemented by a production subsidy. Formally:*

(1) If $\tau_C = \tau_C^* = 1$, then $\frac{\partial V(P(\tau_I),I(\tau_I))}{\partial \tau_I} > 0$

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, then $\frac{\partial V(P(\tau_I),I(\tau_I))}{\partial \tau_I} < 0$

Proof of Proposition 6.

(1) If $\tau_C = \tau_C^* = 1$, it is easy to show that:

$$\left.\frac{\partial V(P(\tau_I), I(\tau_I))}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=1} = \frac{\alpha\tau\left((\alpha + \varepsilon - 1)\tau^\varepsilon + (1 - \alpha)\tau\right)}{(\varepsilon - 1)\left(\tau^{2\varepsilon} - \tau^2\right)} > 0.$$

(2) If $\tau_C = \tau_C^* = 1$, it is easy to show that:

$$\left.\frac{\partial V(P(\tau_I), I(\tau_I))}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = -\frac{\alpha\tau^2\left((\alpha + 2\varepsilon - 1)\tau^\varepsilon + (1 - \alpha)\tau\right)}{\left((\alpha(\tau^\varepsilon + \tau) + (\varepsilon - 1)(\tau^\varepsilon - \tau)\right)\left(\tau^{2\varepsilon} - \tau^2\right)} < 0.$$

■

Proposition 7: Nash-Equilibrium Import Tariffs/Subsidies. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. When starting from the free trade allocation, the Nash-equilibrium tariff is positive implying less varieties and higher price level than the free trade allocation. Differently, when starting from the first-best allocation, the Nash-equilibrium policy consists of an import subsidy implying more varieties and lower price level than the first-best allocation. Formally:*

(1) If $\tau_C = \tau_C^* = 1$ then, there exists a $\tau_I^{Nash} > 1$ such that $N_I^{Nash} < N^{FT} < N^{FB}$ and $P_I^{Nash} > P^{FT} > P^{FB}$.

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then, $\tau_I^{Nash} < 1$ and $N_I^{Nash} > N^{FB} > N^{FT}$ and $P^{FT} > P^{FB} > P_I^{Nash}$.

Proof of Proposition 7. In the case of tariffs, the non-cooperative policy maker maximizes:

$$\max_{\tau_I} V(P(\tau_I, \tau_I^*), I(\tau_I, \tau_I^*)) \tag{63}$$

where $P(\tau_I, \tau_I^*)$ is given by equation (30) once we substitute in $N(\tau_I, \tau_I^*)$ as implicitly determined by equation (21). $I(\tau_I, \tau_I^*)$ is equal to $L + (\tau_I - 1)\tau_X^* \tau P_H^*(\tau_I, \tau_I^*) C_F(\tau_I, \tau_I^*) + (\tau_C - 1) N(\tau_I, \tau_I^*)\varepsilon f$ where $P_H^*(\tau_I, \tau_I^*) = \frac{\varepsilon}{\varepsilon-1}\tau_C^*(N^*(\tau_I, \tau_I^*))^{\frac{1}{1-\varepsilon}}$, $C_F(\tau_I, \tau_I^*) = P_F(\tau_I, \tau_I^*)^{-\varepsilon} P(\tau_I, \tau_I^*)^{\varepsilon} C(\tau_I, \tau_I^*)$, $P_F(\tau_I, \tau_I^*) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_X\tau_C^*(N^*(\tau_I, \tau_I^*))^{\frac{1}{1-\varepsilon}}$ and finally $C(\tau_I, \tau_I^*)$ is given by its equilibrium values in equation (31).

(1) By taking the derivative of (63) with respect to τ_I and imposing symmetry, the first order condition evaluated at $\tau_C = \tau_C^* = 1$ can be written as:

$$\frac{A_I^{Nash}(\tau_I)}{B_I^{Nash}(\tau_I)} = 0$$

where

$$\begin{aligned}
A_I^{Nash}(\tau_I) \equiv\ & \alpha(\tau^{2\varepsilon+3}\tau_I^{2\varepsilon}(\tau_I\left((\alpha-1)(\varepsilon+1)\tau_I(\alpha+\varepsilon-1) - \alpha^2(2\varepsilon+1) - 2\alpha(\varepsilon-1)\varepsilon + (\varepsilon-1)\varepsilon + 1\right) \\
& + \alpha\varepsilon(\alpha+\varepsilon-1)) + \varepsilon\tau^{\varepsilon+4}\left((\alpha-1)\tau_I - \alpha\right)(\varepsilon\tau_I - \varepsilon + 1)\tau_I^{\varepsilon} \\
& - \varepsilon\tau^{3\varepsilon+2}\tau_I^{3\varepsilon}\left(\tau_I(\alpha+\varepsilon-1) - \alpha - \varepsilon\right) \\
& + (-\alpha-\varepsilon+1)\tau^{4\varepsilon+1}\left((\varepsilon-1)\tau_I - \varepsilon\right)\tau_I^{4\varepsilon} - (\alpha-1)\tau^5\tau_I\left((\alpha-1)\tau_I - \alpha\right)(\varepsilon\tau_I - \varepsilon + 1)) \\
B_I^{Nash}(\tau_I) \equiv\ & (\varepsilon-1)\tau_I\left(\tau^{\varepsilon}\tau_I^{\varepsilon} + \tau\tau_I\right)\left(\tau^{2\varepsilon}\tau_I^{2\varepsilon} - \tau^2\right)\left((\alpha-1)\tau\tau_I + \tau^{\varepsilon}\tau_I^{\varepsilon} - \alpha\tau\right)\left(\tau_I(\tau-\alpha\tau) + \tau^{\varepsilon}\tau_I^{\varepsilon} + \alpha\tau\right)
\end{aligned}$$

To prove the first part of proposition 7 we need to show that: (i)there exist at least one Nash equilibrium of the policy game for which $\tau_I^{Nash} > 1$; (ii) for such a $\tau_I^{Nash} > 1$, we have $N_I^{Nash} < N^{FT} < N^{FB}$ and $P_I^{Nash} > P^{FT} > P^{FB}$

(i) To show this point consider that:

(a) $A_I^{Nash}(\tau_I)$ is a continuous function of τ_I;

(b) $A_I^{Nash}|_{\tau_I=1} = \tau\left(\tau^{\varepsilon} - \tau\right)\left(\tau^{\varepsilon} + \tau\right)^2\left[(\alpha+\varepsilon-1)\tau^{\varepsilon} - \alpha\tau + \tau\right] > 0$;

(c)

$$A_I^{Nash}|_{\tau_I=\frac{\varepsilon}{\varepsilon-1}} = -\frac{\varepsilon\tau^2}{(\varepsilon-1)^3}\Big((\varepsilon-1)\tau\left(\alpha(\varepsilon-\alpha)+\alpha(1-\alpha)+(2\varepsilon^2-3\varepsilon+1)\right)\left(\frac{\varepsilon\tau}{\varepsilon-1}\right)^{2\varepsilon}$$
$$+(1-\alpha)(2\varepsilon-1)\tau^3(\varepsilon-\alpha)+(2\varepsilon-1)(\varepsilon-1)\tau^2(\varepsilon-\alpha)\left(\frac{\varepsilon\tau}{\varepsilon-1}\right)^{\varepsilon}+\alpha(\varepsilon-1)^{2-3\varepsilon}(\varepsilon\tau)^{3\varepsilon}\Big)$$
$$< 0$$

where the last inequality follows from the fact that $f(\varepsilon) \equiv (2\varepsilon^2 - 3\varepsilon + 1) > 0$ when $\varepsilon > 1$ because $f(\varepsilon)|_{\varepsilon=0} = 1$ and $f'(\varepsilon) = 4\varepsilon - 3 > 0$.

Therefore, by the fix point theorem there exists a $1 < \tau_I < \frac{\varepsilon}{\varepsilon-1}$ such that $A_I^{Nash}(\tau_I) = 0$.

(ii) To prove this statement we recall that by point (2)-(iii) of proposition 5 the total differential of N and P evaluated at the symmetric equilibrium ($\tau_I = \tau_I^*$), satisfies the following conditions:

$$\tau_I < \frac{\varepsilon}{\varepsilon-1} \quad\Longrightarrow\quad \frac{dN}{d\tau_I} < 0 \qquad \frac{dP}{d\tau_I} > 0 \tag{64}$$

Hence, from (i) and (64) we can be sure that there exists a solution $1 < \tau_I^{Nash} < \frac{\varepsilon}{\varepsilon-1}$ such that $N_I^{Nash} < N^{FT} < N^{FB}$ and $P_I^{Nash} > P^{FT} > P^{FB}$.

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_I = \tau_I^*$, the first order condition of (63) with respect to τ_I can be read as:

$$\frac{A_I^{Nash}(\tau_I)}{B_I^{Nash}(\tau_I)} = 0$$

where

$$\begin{aligned}
A_I^{Nash}(\tau_I) \;\equiv\;& \alpha(\varepsilon-2)\varepsilon^2\tau^{\varepsilon+3}\tau_I^{\varepsilon}+(\alpha-1)\varepsilon\left(\alpha+\varepsilon^2-1\right)\tau^{\varepsilon+3}\tau_I^{\varepsilon+2}\\
&+(\alpha-1)\left(\varepsilon^2+\varepsilon-1\right)(\alpha+\varepsilon-1)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon+2}\\
&-(\alpha+\varepsilon-1)\left(\alpha\varepsilon+\alpha+\varepsilon^2+\varepsilon-1\right)\tau^{3\varepsilon+1}\tau_I^{3\varepsilon+1}\\
&+\left((1-2\alpha)\varepsilon^3+2(\alpha-1)\varepsilon^2-(\alpha-1)\alpha\varepsilon+(\alpha-1)^2\right)\tau^{\varepsilon+3}\tau_I^{\varepsilon+1}\\
&+\varepsilon(\alpha(\varepsilon-1)-1)(\alpha+\varepsilon-1)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\\
&+\varepsilon(\alpha+\varepsilon-1)^2\tau^{3\varepsilon+1}\tau_I^{3\varepsilon}-\varepsilon(\alpha+\varepsilon-1)((2\alpha-1)\varepsilon+2)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon+1}\\
&+\varepsilon(\alpha+\varepsilon-1)^2\tau^{4\varepsilon}\tau_I^{4\varepsilon}-\varepsilon(\alpha+\varepsilon-1)^2\tau^{4\varepsilon}\tau_I^{4\varepsilon+1}-(\alpha-1)^2(\varepsilon-1)\varepsilon\tau^4\tau_I^3\\
&+(\alpha-1)(\varepsilon-1)\tau^4\tau_I^2(\alpha(2\varepsilon-1)-\varepsilon+1)+(1-\alpha)\alpha(\varepsilon-2)\varepsilon\tau^4\tau_I\\
B_I^{Nash}(\tau_I) \;\equiv\;& \tau_I\left(\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I\right)\left(\tau^{2\varepsilon}\tau_I^{2\varepsilon}-\tau^2\right)\left((\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_I^{\varepsilon}+(\alpha-1)(\varepsilon-1)\tau\tau_I-\alpha(\varepsilon-2)\tau\right)\\
&\left((\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I(-\alpha\varepsilon+\alpha+\varepsilon-1)+\alpha\varepsilon\tau\right)
\end{aligned}$$

The second part of Proposition 7 is proved by showing that if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$: (i) there is no solution of the Nash equilibrium of the non-cooperative policy game for $\tau_I > 1$; (ii) there exists a solution of the non-cooperative policy game for $\tau_I < 1$; (iii) $N_I^{Nash} > N^{FB} > N^{FT}$ and $P^{FT} > P^{FB} > P_I^{Nash}$.

(i) In order to show that no Nash equilibrium exists we need to prove that there are no zeros of $A_I^{Nash}(\tau_I)$ for $\tau_I > 1$. This is so because: (a) A_I^{Nash} is a second order polynomial in α; (b) if $\alpha = 0$ or $\alpha = 1$, $A_I^{Nash} < 0$; (c) $\frac{\partial A_I^{Nash}}{\partial \tau_I}|_{\alpha=0} < 0$.

(a) It is straightforward to see that A_I^{Nash} is quadratic in α.

(b) If $\alpha = 0$ and $\tau_I > 1$

$$\begin{aligned} A_I^{Nash} \equiv\ & -(\varepsilon-1)(\tau^{\varepsilon+3}\tau_I^{\varepsilon+1}\left((\varepsilon+1)\varepsilon\tau_I - \varepsilon(\varepsilon-1)+1\right) \\ & \tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\left(\left(\varepsilon^2+\varepsilon-1\right)\tau_I^2 - (\varepsilon-2)\varepsilon\tau_I + \varepsilon\right) \\ & +\tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\left(\left(\varepsilon^2+\varepsilon-1\right)\tau_I - (\varepsilon-1)\varepsilon\right) \\ & +\tau^{4\varepsilon}\tau_I^{4\varepsilon}(\varepsilon-1)\varepsilon\left(\tau_I-1\right) \\ & +\tau^4\tau_I^2\left(\varepsilon\left(\tau_I-1\right)+1\right)) < 0 \end{aligned}$$

If $\alpha = 1$ and $\tau_I > 1$

$$A_I^{Nash} \equiv -\varepsilon^2\tau^\varepsilon\tau_I^\varepsilon\left(\tau^\varepsilon\tau_I^\varepsilon+\tau\right)\left(2\tau^{\varepsilon+1}\tau_I^{\varepsilon+1} + \varepsilon\left(\tau_I-1\right)\tau^{2\varepsilon}\tau_I^{2\varepsilon} + \tau^2\left(\varepsilon\tau_I - \varepsilon + 2\right)\right) < 0$$

(c) To see why $\partial A_I^{Nash}/\partial\tau_I < 0$ first consider that if $\alpha = 0$

$$\frac{\partial A_I^{Nash}}{\partial \tau_I} = \tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\kappa_3 + \tau^{\varepsilon+3}\tau_I^{\varepsilon}\kappa_4 + \tau^4\tau_I\kappa_5$$

where:

$$\begin{aligned} \kappa_1 &\equiv -2(\varepsilon-1)\varepsilon\left(\tau_I-1\right) \\ \kappa_2 &\equiv -\left(\left(2\varepsilon^2+\varepsilon-2\right)\tau_I - 2(\varepsilon-1)\varepsilon\right) \\ \kappa_3 &\equiv (\varepsilon-2)\left(\varepsilon^2+\varepsilon-1\right)\tau_I^2 + ((3-2\varepsilon)\varepsilon-2)\varepsilon\tau_I + (\varepsilon-2)\varepsilon^2 \\ \kappa_4 &\equiv \tau_I\left[\left(\varepsilon^2-2\right)\varepsilon\tau_I - 2(\varepsilon-1)\varepsilon^2 + \varepsilon - 2\right] + (\varepsilon-2)\varepsilon^2 \\ \kappa_5 &\equiv (\varepsilon-1)\tau_I\left(2\varepsilon\tau_I - 3\varepsilon + 2\right) + (\varepsilon-2)\varepsilon \end{aligned}$$

First, we show that $\partial A_I^{Nash}/\partial\tau_I < 0$ for $\varepsilon < 2$. Under this assumption $\kappa_1 < 0$, $\kappa_2 < 0$, $\kappa_3 < 0$ and $\kappa_3 - \kappa_4 < 0$. In this case it is sufficient to show that $\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^4\tau_I\kappa_5 < 0$. Note that $\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^4\tau_I\kappa_5 < \delta(\tau_I)$ where $\delta(\tau_I) \equiv (\kappa_1+\kappa_2)\tau_I^{2\varepsilon} + \kappa_5$. It can be shown that $\delta'(\tau_I) < 0$. It follows then from $\delta(\tau_I) = -2\varepsilon$ at $\tau_I = 1$ that $\delta(\tau_I) < 0$.

Second, we show that $\partial A_I^{Nash}/\partial\tau_I < 0$ for $\varepsilon > 2$. Under this assumption $\kappa_1 < 0$, $\kappa_2 < 0$ and $\kappa_5 < 0$. Therefore in this case it suffices to show that$\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\kappa_3 < 0$ and $\tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^{\varepsilon+3}\tau_I^{\varepsilon}\kappa_4 + \tau^4\tau_I\kappa_5 < 0$ or alternatively that $\delta_1(\tau_I) \equiv \kappa_1\tau_I^{2\varepsilon} + \kappa_3 < 0$ and $\delta_2(\tau_I) \equiv \kappa_2\tau_I^{2\varepsilon} + \kappa_4 + \kappa_5 < 0$. These last conditions are always satisfied because at $\tau_I = 1$, $\delta_1(\tau_I) = 2 - 5\varepsilon$ and $\delta_2(\tau_I) = -2 - 3\varepsilon$ and it can be proved that $\delta_2'(\tau_I) < 0$ and $\delta_1'(\tau_I) < 0$.

(ii) This is equivalent to show that there is at least one zero of $A_I^{Nash}(\tau_I)$ for $\tau_I < 1$. A sufficient condition for the existence of a Nash solution is $\varepsilon > 2$. To see why this is the case, consider that: a) A_I^{Nash} is a continuous function in τ_I; b) $A_I^{Nash}|_{\tau_I=1} = -\tau(\alpha+\varepsilon-1)\left(\tau^\varepsilon+\tau\right)^2\left((1-\alpha)\tau + (\alpha+2\varepsilon-1)\tau^\varepsilon\right) < 0$; c) $A_I^{Nash}|_{\tau_I=0} = 0$ and $(\partial A_I^{Nash}/\partial\tau_I)|_{\tau_I=0} = (1-\alpha)\alpha(\varepsilon-2)\varepsilon\tau^4 > 0$ for $\varepsilon > 2$. Then by the fix point theorem there exists a value $\tau_I \in (0,1)$ such that $A_I^{Nash}(\tau_I) = 0$.

(iii) We follow the same line of reasoning at point (1),(ii). To prove the statement we show that if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then $dN/d\tau_I < 0$ and $dP/d\tau_I > 0$ for $\tau_I < 1$. Totally differentiating N, imposing symmetry and assuming that $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ lead us to recover the following condition:

$$\frac{dN}{d\tau_I} = \frac{\partial(N+N^*)}{\partial\tau_I}\bigg|_{\tau_I=\tau_I^*} = -\frac{L(1-\alpha)\alpha(\varepsilon-1)\tau\left(\tau^\varepsilon\left(\varepsilon-(\varepsilon-1)\tau_I\right)\tau_I^\varepsilon+\tau\tau_I\right)}{f\tau_I\left((\alpha+\varepsilon-1)\tau^\varepsilon\tau_I^\varepsilon+\tau\tau_I(1-\alpha)(\varepsilon-1)+\alpha\varepsilon\tau\right)^2} < 0$$

for all $\tau_I < 1$. This implies also that $dP/d\tau_I > 0$ since:

$$\frac{dP}{d\tau_I} = -\frac{P^\varepsilon}{\varepsilon-1}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{(1-\varepsilon)}\left[\left(1+\tau^{1-\varepsilon}\tau_I^{1-\varepsilon}\right)\frac{\partial(N+N^*)}{\partial\tau_I} - (\varepsilon-1)\tau^{1-\varepsilon}\tau_I^{-\varepsilon}N^*\right]\bigg|_{\tau_I=\tau_I^*}$$

■

F Export Subsidies

In this section while retaining the assumption $\tau_I = \tau_I^* = 1$, we prove the propositions and the lemmata of section 6 where we allow for the use of export subsidies as main policy instrument.

Proposition 8: Cooperative Export Subsidy. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then, if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker refrains from using subsidies/tariffs on exports and the number of varieties and the price level equal the first-best ones. If $\tau_C = \tau_C^* = 1$, the cooperative policy maker finds it optimal to subsidize exports. The total number of varieties increases compared to the free trade allocation, but remains lower than the first-best level, while the price level is lower than in the free trade allocation, but higher than the first-best one. Formally,*

(1) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then, $\tau_X^{Coop} = 1$, $N_X^{Coop} = N^{FB}$ and $P_X^{Coop} = P^{FB}$

(2) If $\tau_C = \tau_C^* = 1$ then, $\tau_X^{Coop} < 1$, $N^{FM} < N_X^{Coop} < N^{FB}$ and $P^{FB} < P_X^{Coop} < P^{FT}$

Proof of Proposition 8.

(1) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker solves:

$$\max_{\tau_X,\tau_X^*} V(P(\tau_X,\tau_X^*), I(\tau_X,\tau_X^*)) + V(P^*(\tau_X,\tau_X^*), I^*(\tau_X,\tau_X^*)) \tag{65}$$

Here, $P(\tau_X,\tau_X^*)$ and $P^*(\tau_X,\tau_X^*)$ are given by equation (30), which is implied by the equilibrium expressions for $N(\tau_X,\tau_X^*)$ and $N^*(\tau_X,\tau_X^*)$, equation (35). $I(\tau_X,\tau_X^*)$, $I^*(\tau_X,\tau_X^*)$ are given by $L + (\tau_X - 1)\tau P_H(\tau_X,\tau_X^*)C_F^*(\tau_X,\tau_X^*) + (\tau_C - 1)N(\tau_X,\tau_X^*)\varepsilon f$ and $L^* + (\tau_X^* - 1)\tau P_H^*(\tau_X,\tau_X^*)C_F(\tau_X,\tau_X^*) + (\tau_C^* - 1)N^*(\tau_X,\tau_X^*)\varepsilon f$, where $P_H(\tau_X,\tau_X^*) = \frac{\varepsilon}{\varepsilon-1}\tau_C N(\tau_X,\tau_X^*)^{\frac{1}{1-\varepsilon}}$, $P_H^*(\tau_X,\tau_X^*) = \frac{\varepsilon}{\varepsilon-1}\tau_C^* N^*(\tau_X,\tau_X^*)^{\frac{1}{1-\varepsilon}}$, $C_F(\tau_X,\tau_X^*) = P_F(\tau_X,\tau_X^*)^{-\varepsilon}P(\tau_X,\tau_X^*)^\varepsilon C(\tau_X,\tau_X^*)$, $C_F^*(\tau_X,\tau_X^*) = P_F^*(\tau_X,\tau_X^*)^{-\varepsilon}P^*(\tau_X,\tau_X^*)^\varepsilon C^*$, $P_F(\tau_X,\tau_X^*) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_X\tau_C^* N^*(\tau_X,\tau_X^*)^{\frac{1}{1-\varepsilon}}$, and $P_F^*(\tau_X,\tau_X^*) =$

$\frac{\varepsilon}{\varepsilon-1}\tau\tau_X^*\tau_C N(\tau_X,\tau_X^*)^{\frac{1}{1-\varepsilon}}$. Finally, $C(\tau_X,\tau_X^*)$, $C^*(\tau_X,\tau_X^*)$ are given by their equilibrium values in equation (31).

Taking derivatives with respect to τ_X and τ_X^* and imposing symmetry, the first-order condition can be written as

$$\frac{\alpha\varepsilon\tau(\tau_X-1)[(\alpha+\varepsilon-1)\tau^{2\varepsilon}\tau_X^{2\varepsilon}+(1-\alpha)\tau^2\tau_X+\varepsilon\tau^{\varepsilon+1}\tau_X^{\varepsilon}]}{\tau_X(\tau^\varepsilon\tau_X^\varepsilon+\tau)(\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X)[-(\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon+(\alpha-1)(\varepsilon-1)\tau\tau_X-\alpha\varepsilon\tau]}=0$$

It is straightforward to see that $\tau_X=1$ is the unique solution to this equation. We have already shown that $\tau_c=\tau_c^*=\frac{\varepsilon-1}{\varepsilon}$, $\tau_X=\tau_X^*=1$ implements the first-best allocation. Thus, $N_X^{Coop}=N^{FB}$ and $P_X^{Coop}=P^{FB}$.

(2) If $\tau_C=\tau_C^*=1$, the cooperative policy maker solves the same problem as in (1) but income is now given by $I(\tau_X,\tau_X^*)=L+(\tau_X-1)\tau P_H C_F^*$ and $I^*(\tau_X,\tau_X^*)=L^*+(\tau_X^*-1)\tau P_H^* C_F$.

Taking derivatives with respect to τ_X and τ_X^* and imposing symmetry, the first-order condition can now be written as

$$\frac{A_X^{Coop}(\tau_X)}{B_X^{Coop}(\tau_X)}=0$$

where

$$\begin{aligned}A_X^{Coop}(\tau_X) &\equiv \alpha\tau[\tau^{\varepsilon+1}\tau_X^\varepsilon[\tau_X(2\alpha-\varepsilon^2+\varepsilon-2)+(\varepsilon-1)\varepsilon]+\\ &\quad \tau^{2\varepsilon}\tau_X^{2\varepsilon}[\varepsilon(\alpha+\varepsilon-2)-(\varepsilon-1)\tau_X(\alpha+\varepsilon-1)]+(\alpha-1)\tau^2\tau_X(\varepsilon\tau_X-\varepsilon+1)]\\ B_X^{Coop}(\tau_X) &\equiv (\varepsilon-1)\tau_X(\tau^\varepsilon\tau_X^\varepsilon+\tau)(\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X)(\tau_X(\tau-\alpha\tau)+\tau^\varepsilon\tau_X^\varepsilon+\alpha\tau)\end{aligned}$$

Note that $A_X^{Coop}(\tau_X)=A_I^{Coop}(\tau_I)$ and $B_X^{Coop}(\tau_X)=B_I^{Coop}(\tau_I)$ with the only difference that they are functions of τ_X instead of τ_I. Thus, the proof is the same as the one for the cooperative import subsidy of Proposition 5. It also implies that the cooperative policymaker implements the same equilibrium allocation independently on whether he is using import or export subsidies.

■

Lemma 3: *Let $\tau>1$, $\varepsilon>1$, $0<\alpha<1$ and $\tau_X=\tau_X^*=1$. Then,*

(1) *If $\tau_C=\tau_C^*=1$ or $\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}$, a unilateral increase in the export tax decreases the total number of varieties ($\frac{\partial(N+N^*)}{\partial\tau_X}<0$) and reduces the share of domestically produced varieties ($\frac{\partial s}{\partial\tau_X}<0$). As a result, the domestic price index increases ($\frac{\partial P}{\partial\tau_X}>0$).*

(2) *If $\tau_C=\tau_C^*=1$ or $\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}$, a unilateral increase in the export tax increases domestic income ($\frac{\partial I}{\partial\tau_X}>0$). Moreover, terms-of-trade and opportunity costs are always negative, while other effects on income are always positive.*

Proof of Lemma 3.

(1) We first consider the case $\tau_C = \tau_C^* = 1$:

$$\frac{\partial N}{\partial \tau_X} = -\frac{L\alpha\tau\left((\alpha+\varepsilon-1)\tau^{\varepsilon+1}+(\varepsilon-\alpha)\tau^{2\varepsilon}+\tau^2\right)}{f\varepsilon\left(\tau-\tau^{\varepsilon}\right)^2\left(\tau^{\varepsilon}+\tau\right)} < 0$$

and that

$$\frac{\partial N^*}{\partial \tau_X} = \frac{L\alpha\tau\left((-\alpha+\varepsilon+1)\tau^{\varepsilon+1}+\alpha\tau^2+(\varepsilon-1)\tau^{2\varepsilon}\right)}{f\varepsilon\left(\tau-\tau^{\varepsilon}\right)^2\left(\tau^{\varepsilon}+\tau\right)} > 0$$

Moreover, taking differences of the above derivatives, it is straightforward to show that

$$\left[\frac{\partial N}{\partial \tau_X} + \frac{\partial N^*}{\partial \tau_X}\right] = -\frac{L(1-\alpha)\alpha\tau}{f\varepsilon\left(\tau^{\varepsilon}+\tau\right)} < 0$$

Also

$$\frac{\partial s}{\partial \tau_X} = -\frac{\tau\left((2\varepsilon-1-\alpha)\tau^{\varepsilon}+(1+\alpha)\tau\right)}{4\left(\tau-\tau^{\varepsilon}\right)^2} < 0$$

and

$$\frac{\partial P}{\partial \tau_X} = P\frac{\tau(\alpha\tau+(\varepsilon-\alpha)\tau^{\varepsilon})}{(\varepsilon-1)(\tau^{2\varepsilon}-\tau^2)} > 0$$

and

$$\frac{\partial I}{\partial \tau_X} = \frac{L\alpha\tau}{\tau^{\varepsilon}+\tau} > 0$$

(2) We now consider the case $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$

$$\frac{\partial N}{\partial \tau_X} = -\frac{L\alpha(\varepsilon-1)\tau\left(\tau^2\left(\alpha^2+\varepsilon-1\right)+\left(\alpha(2\varepsilon-1)+(\varepsilon-1)^2\right)\tau^{\varepsilon+1}+(\varepsilon-\alpha)(\alpha+\varepsilon-1)\tau^{2\varepsilon}\right)}{f(\alpha+\varepsilon-1)^2\left(\tau^{2\varepsilon}-\tau^2\right)\left(\alpha(\tau^{\varepsilon}+\tau)+(\varepsilon-1)(\tau^{\varepsilon}-\tau)\right)} < 0$$

$$\frac{\partial N^*}{\partial \tau_X} = \frac{L\alpha(\varepsilon-1)\tau\left(\left(\alpha+\varepsilon^2-1\right)\tau^{\varepsilon+1}+(\varepsilon-1)(\alpha+\varepsilon-1)\tau^{2\varepsilon}+\alpha\varepsilon\tau^2\right)}{f(\alpha+\varepsilon-1)^2\left(\tau^{2\varepsilon}-\tau^2\right)\left(\alpha(\tau^{\varepsilon}+\tau)+(\varepsilon-1)(\tau^{\varepsilon}-\tau)\right)} > 0$$

$$\left[\frac{\partial N}{\partial \tau_X} + \frac{\partial N^*}{\partial \tau_X}\right] = -\frac{L(1-\alpha)\alpha(\varepsilon-1)\tau}{f(\alpha+\varepsilon-1)^2\left(\tau^{\varepsilon}+\tau\right)} < 0$$

$$\frac{\partial s}{\partial \tau_X} = -\frac{(\varepsilon-1)\tau\left((2\varepsilon-\alpha-1)\tau^{\varepsilon}+(\alpha+1)\tau\right)}{4\left(\tau^{\varepsilon}-\tau\right)\left(\alpha(\tau^{\varepsilon}+\tau)+(\varepsilon-1)(\tau^{\varepsilon}-\tau)\right)} < 0$$

$$\frac{\partial P}{\partial \tau_X} = P\frac{\tau(\alpha\varepsilon\tau+(\varepsilon-\alpha)(\varepsilon+\alpha-1)\tau^{\varepsilon})}{(\alpha+\varepsilon-1)(\tau+\tau^{\varepsilon})(\alpha(\tau+\tau^{\varepsilon})+(\varepsilon-1)(\tau^{\varepsilon}-\tau))} > 0$$

$$\frac{\partial I}{\partial \tau_X} = \frac{L\alpha(\varepsilon-1)\tau\left(\left(\alpha-(\varepsilon-1)^2\right)\tau^{\varepsilon+1}+(2\varepsilon-1)(\alpha+\varepsilon-1)\tau^{2\varepsilon}+(\varepsilon-1)\varepsilon\tau^2\right)}{(\alpha+\varepsilon-1)^2\left(\tau^{2\varepsilon}-\tau^2\right)\left(\alpha(\tau+\tau^{\varepsilon})+(\varepsilon-1)(\tau^{\varepsilon}-\tau)\right)} > 0$$

where the last inequality follows from the fact that $\tau^{2\varepsilon} > \tau^{\varepsilon+1}$ and $|(2\varepsilon-1)(\alpha+\varepsilon-1)| > |\alpha-(\varepsilon-1)^2|$.

(3) Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* = 1$ or $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then a unilateral increase in the domestic export tax has a positive opportunity cost effect ($BT_1 > 0$), a positive terms of trade effect ($BT_2 > 0$), a negative Foreign substitution and income effect ($BT_3 < 0$) and a negative domestic substitution and income effect ($-(BT_{41} + BT_{42}) < 0$)

(i) $BT_1 > 0$: This is so given that $BT_1 = \left[-\varepsilon f\left(\frac{1}{P_H}\right)\frac{\partial N}{\partial \tau_X}\right]$ and $\frac{\partial N}{\partial \tau_X} < 0$.

(ii) $BT_2 > 0$: Since we start from a symmetric equilibrium, $BT_2 = -\tau\alpha\left(\frac{P_F}{P}\right)^{-\varepsilon}\frac{I}{P}\frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial \tau_X}$.
Note that $\frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial \tau_X} = \frac{\frac{\partial P_H^*}{\partial \tau_X}P_H - \frac{\partial P_H}{\partial \tau_X}P_H^*}{P_H^2} < 0$ given that $\frac{\partial P_H^*}{\partial \tau_X} = \frac{\varepsilon}{\varepsilon-1}N^{*\left(\frac{1}{1-\varepsilon}\right)}\left(-\frac{\tau_C^*}{(\varepsilon-1)N^*}\frac{\partial N^*}{\partial \tau_X}\right) < 0$ and $\frac{\partial P_H}{\partial \tau_X} = \frac{\varepsilon}{\varepsilon-1}N^{\left(\frac{1}{1-\varepsilon}\right)}\left(-\frac{\tau_C}{(\varepsilon-1)N}\frac{\partial N}{\partial \tau_X}\right) > 0$.

(iii) If $\tau_C = \tau_C^* = 1$ then $BT_3 \equiv BT_{31} + BT_{32} + BT_{33} < 0$: This is so given that $BT_{31} + BT_{32} =$
$\alpha\tau\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon} I^*\left(\frac{1}{P^*}\right)^2\left[-\varepsilon\frac{P^*}{P_F^*}\frac{\partial P_F^*}{\partial \tau_X} + (\varepsilon-1)\frac{\partial P^*}{\partial \tau_X}\right]$, $\frac{\partial P_F^*}{\partial \tau_X} = \tau\tau_C\frac{\varepsilon}{\varepsilon-1}N^{\left(\frac{1}{1-\varepsilon}\right)}\left(1-\frac{1}{\varepsilon-1}\frac{1}{N}\frac{\partial N}{\partial \tau_X}\right) > 0$ and
$\frac{\partial P^*}{\partial \tau_X}|_{\tau_C=\tau_C^*=1} = -\frac{[\tau(\tau^2+(-\alpha+\varepsilon)\tau^{2\varepsilon}+(-1+\alpha+\varepsilon)\tau^{1+\varepsilon})]}{((\tau-\tau^{\varepsilon})^2(\tau+\tau^{\varepsilon}))} < 0$

(iv) Also, if $\tau_C = \tau_C^* = 1$ then $BT_{31} + BT_{32} < 0$ This is so given that $BT_{31} + BT_{32}$ can also be expressed as $\tau\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon} C^*\left[-\varepsilon\left(\frac{P_F^*}{P^*}\right)^{-1}\frac{\partial\left(\frac{P_F^*}{P^*}\right)}{\tau_X} - \frac{\partial P^*}{\partial \tau_X}\frac{1}{P^*}\right]$,
where the term in square brackets is given by:
$\frac{\varepsilon\tau^3\left(-\alpha^2+\varepsilon^2-3\varepsilon+2\right)-\left[\alpha^2(\varepsilon-1)+\alpha(3(\varepsilon-1)\varepsilon+1)+2(\varepsilon-1)^2\varepsilon\right]\tau^{\varepsilon+2}+(\alpha+\varepsilon-1)\left(2\alpha\varepsilon-\alpha-\varepsilon^2\right)\tau^{2\varepsilon+1}}{(\alpha+\varepsilon-1)(\tau^{2\varepsilon}-\tau^2)[(\alpha+\varepsilon-1)\tau^{\varepsilon}+\tau(\alpha-\varepsilon+1)]} < 0$
The negative sign follows since the first term in the numerator is strictly dominated by the second term and also the third term is negative, while the denominator is positive.
Moreover, $BT_{33} \leq 0$: This is so given that $BT_{33} = \left[\tau\alpha\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon}\left(\frac{1}{P^*}\right)\frac{\partial I^*}{\partial \tau_X}\right]$, $I^* = L + (\tau_C^*-1)\varepsilon f N^*$ and $\frac{\partial I^*}{\partial \tau_X} = (\tau_C^*-1)\varepsilon f\frac{\partial N^*}{\partial \tau_X} \leq 0$.

(v) $-(BT_{41} + BT_{42}) < 0$:
Note that $-(BT_{41} + BT_{42}) = \tau\alpha\left(\frac{P_F}{P}\right)^{-\varepsilon}\frac{I}{P}\left[\varepsilon\frac{P}{P_F}\left(\frac{1}{P}\right)^2\left(\frac{\partial P_F}{\partial \tau_X}P - \frac{\partial P}{\partial \tau_X}P_F\right) + \frac{\partial P}{\partial \tau_X}\frac{1}{P}\right] =$
$\tau\alpha\left(\frac{P_F}{P}\right)^{-\varepsilon}\frac{I}{P}\left[\varepsilon(P_F)^{-1}\frac{\partial P_F}{\partial \tau_X} - (\varepsilon-1)\frac{\partial P}{\partial \tau_X}\frac{1}{P}\right] < 0$
given that $\frac{\partial P_F}{\partial \tau_X} = \tau\frac{\varepsilon}{\varepsilon-1}N^{*\left(\frac{1}{1-\varepsilon}\right)}\left(-\frac{\tau_X^*}{(\varepsilon-1)N^*}\frac{\partial N^*}{\partial \tau_X}\right) < 0$ and $\frac{\partial P}{\partial \tau_X} > 0$.

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Proposition 9: Unilaterally Set Export Taxes/Subsidies. *Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ and $\tau_X = \tau_X^* = 1$. The optimal unilateral export policy entails a positive export subsidy when starting from the free trade allocation, and an export tax when starting from the first-best allocation implemented by a production subsidy. Formally:*

(1) If $\tau_C = \tau_C^* = 1$, then $\frac{\partial V(P(\tau_X),I(\tau_X))}{\partial \tau_X} < 0$

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then $\frac{\partial V(P(\tau_X),I(\tau_X))}{\partial \tau_X} > 0$

Proof of Proposition 9.

(1)

$$\left.\frac{\partial V(P(\tau_X),I(\tau_X))}{\partial \tau_X}\right|_{\tau_C=\tau_C^*=1} = -\frac{\alpha\tau\left(\tau(\alpha+\varepsilon-1)+(1-\alpha)\tau^{\varepsilon}\right)}{(\varepsilon-1)\left(\tau^{2\varepsilon}-\tau^2\right)} < 0 \tag{66}$$

(2)

$$\left.\frac{\partial V(P(\tau_X),I(\tau_X))}{\partial \tau_X}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = \frac{\alpha\tau\left((1-\alpha)\tau^{\varepsilon+1}+(\alpha+\varepsilon-1)\tau^{2\varepsilon}+\varepsilon\tau^2\right)}{\left(\tau^{2\varepsilon}-\tau^2\right)\left(\alpha(\tau^{\varepsilon}+\tau)+(\varepsilon-1)(\tau^{\varepsilon}-\tau)\right)} > 0 \tag{67}$$

■

Proposition 10: Nash-Equilibrium Export Taxes/Subsidies. *Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. When starting from the free trade allocation, the Nash-equilibrium policy consists of an export subsidy implying more varieties and lower price level than the free trade allocation. Differently, when starting from the first-best allocation, the Nash-equilibrium policy consists of an export tax implying less varieties and higher price level than the first-best allocation. Formally:*

(1) If $\tau_C = \tau_C^* = 1$, then $\tau_X^{Nash} < 1$, $N^{FB} > N_X^{Nash} > N^{FT}$ and $P^{FB} < P_X^{Nash} < P^{FT}$.

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_X^{Nash} > 1$, $N_X^{Nash} < N^{FB}$ and $P_X^{Nash} > P^{FB}$.

Proof of Proposition 10.

(1) If $\tau_C = \tau_C^* = 1$, the Nash policy maker solves:

$$\max_{\tau_X} V(P(\tau_X,\tau_X^*), I(\tau_X,\tau_X^*)) \tag{68}$$

Here, $P(\tau_X,\tau_X^*)$ is given by equation (30), which is implied by the equilibrium expressions for $N(\tau_X,\tau_X^*)$ and $N^*(\tau_X,\tau_X^*)$, equation (35). Moreover, $I(\tau_X,\tau_X^*)$ is given by $L + (\tau_X - 1)\tau P_H(\tau_X,\tau_X^*)C_F^*(\tau_X,\tau_X^*)$, where $P_H(\tau_X,\tau_X^*) = \frac{\varepsilon}{\varepsilon-1}\tau_C N(\tau_X,\tau_X^*)^{\frac{1}{1-\varepsilon}}$, $C_F = P_F(\tau_X,\tau_X^*)^{-\varepsilon}P(\tau_X,\tau_X^*)^{\varepsilon}C(\tau_X,\tau_X^*)$, $P_F(\tau_X,\tau_X^*) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_X\tau_C^*N^*(\tau_X,\tau_X^*)^{\frac{1}{1-\varepsilon}}$, and finally $C(\tau_X,\tau_X^*)$, is given by its equilibrium value in equation (31).

Taking derivatives with respect to τ_X and τ_X^* and then imposing symmetry, the first-order conditions at the symmetric Nash equilibrium can be written as

$$\frac{A_X^{Nash}(\tau_X)}{B_X^{Nash}(\tau_X)} = 0 \tag{69}$$

with

$$\begin{aligned}
A_X^{Nash}(\tau_X) \quad \equiv \quad & \alpha\{\tau^{\varepsilon+4}\tau_X^{\varepsilon+1}[\tau_X(\tau_X(\varepsilon-\alpha^2\varepsilon)+2\alpha^2\varepsilon+(\alpha-1)\alpha-\varepsilon^2+\varepsilon)-\alpha^2(\varepsilon+1)+\alpha+(\varepsilon-1)\varepsilon]+ \\
& \tau^{2\varepsilon+3}\tau_X^{2\varepsilon}[\tau_X(\tau_X((\alpha-1)\varepsilon^2-\alpha+\varepsilon+1)+\alpha(-2\varepsilon^2+\varepsilon-1)+(\varepsilon-1)^2)+\alpha(\varepsilon-1)\varepsilon]+ \\
& \tau^{3\varepsilon+2}\tau_X^{3\varepsilon}[\tau_X(\alpha(\varepsilon-1)\tau_X(\alpha+\varepsilon-1)-(2\alpha+1)\varepsilon^2-2(\alpha-2)\alpha\varepsilon+(\alpha-1)\alpha)+ \\
& \varepsilon(\alpha(\alpha+\varepsilon-2)+\varepsilon-1)]+ \\
& \tau^{4\varepsilon+1}\tau_X^{4\varepsilon}(\varepsilon(\alpha+\varepsilon-2)-(\varepsilon-1)\tau_X(\alpha+\varepsilon-1))+\tau^5\tau_X^2(\alpha+\varepsilon-1)\} \\
B_X^{Nash}(\tau_X) \quad \equiv \quad & (\varepsilon-1)\tau_X(\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X)(\tau^{2\varepsilon}\tau_X^{2\varepsilon}-\tau^2)(-(\alpha+1)\tau\tau_X+\tau^\varepsilon\tau_X^\varepsilon+\alpha\tau) \\
& (\tau_X(\tau-\alpha\tau)+\tau^\varepsilon\tau_X^\varepsilon+\alpha\tau)
\end{aligned}$$

(i) In order to show that there exists a solution with $\tau_X < 1$, we first show that $A_X^{Nash}(\tau_X = 1)$ is negative. This is so given that $A_X^{Nash}(\tau_X = 1) = \tau(\tau^\varepsilon - \tau)(\tau^\varepsilon + \tau)^2[(\alpha-1)\tau^\varepsilon + \tau(-\alpha-\varepsilon+1)] < 0$

(ii) Next, we show that for $\varepsilon > 2$ there exists a $\tau_X \in \{0, 1\}$ with $A_X^{Nash}(\tau_X) > 0$. By continuity of $A_X^{Nash}(\tau_X)$ this is enough to guarantee existence of a solution. Consider $\tau_X = \frac{\varepsilon-2}{\varepsilon}$. Then, $A_X^{Nash}(\tau_X = \frac{\varepsilon-2}{\varepsilon}) = \frac{\tau}{\varepsilon^2}[(\varepsilon-2)^2\tau^4(\alpha+\varepsilon-1)+(\varepsilon-2)^{4\varepsilon}\varepsilon^{1-4\varepsilon}(2+2\varepsilon^2-5\varepsilon+3\alpha\varepsilon-2\alpha)\tau^{4\varepsilon}+(\varepsilon-2)^{1+\varepsilon}\varepsilon^{-\varepsilon}(4+2\alpha-6\alpha^2+3(\varepsilon-2)\varepsilon)\tau^{3+\varepsilon}+\left(\frac{\varepsilon-2}{\varepsilon}\right)^{2\varepsilon}(\alpha(6\varepsilon-4)+(\varepsilon-2)(\varepsilon^2-2))\tau^{2+2\varepsilon}+\tau\left(\alpha^2(6\varepsilon-4)+2\alpha(\varepsilon-2)(2\varepsilon-1)+\varepsilon^3\right)\left(\frac{(\varepsilon-2)\tau}{\varepsilon}\right)^{3\varepsilon}] > 0$ since each of the coefficients is positive for $\varepsilon > 2$. This proves that a solution with $\tau_X < 1$ exists.

(iii) Finally we show that $N_X^{Nash} < N_{FB}$ and $P_X^{Nash} > P^{FB}$. We follow the same line of reasoning we used in Proposition 5.

(a) Let $\tau_X^{Nash} = f(\alpha, \varepsilon, \tau)$ and $\tau_X^{FB} = g(\alpha, \varepsilon, \tau)$ be, respectively, the Nash equilibrium export subsidy and the export subsidy that implements the first-best number of varieties. Hence, if $\tau_X = \tau_X^{Nash}$, $A_X^{Nash}(\tau_X^{Nash}) = 0$. At the same time τ_X^{FB} is such $N_X = \frac{L\alpha(\tau+(\tau\tau_X)^\varepsilon)}{f\varepsilon(\alpha\tau+\tau(1-\alpha)\tau_X+(\tau\tau_X)^\varepsilon)} = \frac{L\alpha}{f(\varepsilon+\alpha-1)} = N^{FB}$. This last condition can be rewritten as $(\tau\tau_X)^\varepsilon = -\varepsilon\tau\tau_X + \tau(\varepsilon-1)$. Note that when combined, this two conditions are a system of two equations in τ_X. We now investigate if there exists a τ_X such that both conditions are satisfied simultaneously. Once we substitute the above condition into A_X^{Nash} we obtain a fifth-order polynomial in τ_X which can be factorized into two polynomials. The first polynomial is $-\varepsilon\tau^5(\tau_X-1)^2(\alpha+\varepsilon-1)$, with solutions $\tau_X = \{1, 1\}$. None of these solutions solves $(\tau\tau_X)^\varepsilon = -\varepsilon\tau\tau_X + \tau(\varepsilon-1)$. The second polynomial is cubic and we call it ${A_X}_{mod}^{Nash}$. It can be shown that there exist at most one real solution of ${A_X}_{mod}^{Nash}$. However, evaluating ${A_X}_{mod}^{Nash}$ at $\tau_X = 1$ and $\tau_X = 0$ we find that both ${A_X}_{mod}^{Nash}(\tau_X = 1) < 0$ and ${A_X}_{mod}^{Nash}(\tau_X = 0) < 0$. Thus, by continuity of ${A_X}_{mod}^{Nash}$, either there exists no real solution or there are at least two zeros of ${A_X}_{mod}^{Nash} = 0$ that are real. Since we already showed that there exists at most one real solution of ${A_X}_{mod}^{Nash} = 0$ in $[0, 1]$, we can conclude that there is no intersection between the set of τ_X^{Nash} and the set of τ_X^{FB} in the interval $[0, 1]$.

(b) The second step is to show that $\tau_X^{FB} < \tau_X^{Nash}$ in the interval $[0, 1]$ for any $\{\alpha \in (0, 1), \tau > 1, \varepsilon > 1\}$. To this end, recall that f and g are two continuous functions in the space $\{0 < \alpha < 1, \tau > 1, \varepsilon > 1\}$, given that the derivatives of τ_X^{FB} and τ_X^{Nash} with respect to the three parameters always exists in the permitted parameter space. In point (a) we proved that there is no intersection between g and f. As a consequence, we either have $\tau_X^{FB} < \tau_X^{Nash}$ for any $\{\alpha \in (0,1), \tau > 1, \varepsilon > 1\}$ or the other way around. We evaluate both functions at $\{\alpha = 0.5, \varepsilon = 2, \tau = 1.5\}$ and find $\tau_X^{FB} = 0.39 < 0.82 = \tau_X^{Nash}$. Thus, the non-cooperative export subsidy is always smaller than the one needed to implement the first-best number of varieties.

(c) Finally note that $\frac{dN}{d\tau_X}(\tau_X) = \frac{dN}{d\tau_I}(\tau_I)$ and $\frac{dP}{d\tau_X}(\tau_X) = \frac{dP}{d\tau_I}(\tau_I)$ with the only difference that they are functions of τ_X instead of τ_I. Thus, from Proposition 5 we know that by symmetrically increasing the export subsidy in both countries policy makers increase the number of varieties and reduce the price level, i.e. at the symmetric equilibrium:

$$\frac{dN}{d\tau_X} < 0 \qquad \frac{dP}{d\tau_X} > 0$$

It then follows that $N_X^{Nash} < N_{FB}$ and $P_X^{Nash} > P^{FB}$.

(2) If $\tau_C = \tau_C^* = \frac{\varepsilon - 1}{\varepsilon}$, the Nash policy maker solves the same problem as in (1) but income is now given by $I(\tau_X, \tau_X^*) = L + (\tau_X - 1)\tau P_H(\tau_X, \tau_X^*) C_F^*(\tau_X, \tau_X^*) + \tau_C N(\tau_X, \tau_X^*)\varepsilon f$

Taking derivatives with respect to τ_X and τ_X^* and then imposing symmetry, the first-order conditions at the symmetric Nash equilibrium can be written as

$$\frac{A_X^{Nash}(\tau_X)}{B_X^{Nash}(\tau_X)} = 0 \tag{70}$$

where

$$\begin{aligned} A_X^{Nash}(\tau_X) \equiv\ & \alpha\tau\{-\tau^{\varepsilon+3}\tau_X^{\varepsilon+1}[\tau_X((\alpha^2-1)(\varepsilon-1)\varepsilon\tau_X + (-2\alpha^2+\alpha-2)\varepsilon^2 + (\alpha-1)^2 + \varepsilon^3) + \\ & \varepsilon((\alpha-1)\alpha\varepsilon + (\alpha-1)\alpha - \varepsilon^2 + \varepsilon)] + \\ & \tau^{2\varepsilon+2}\tau_X^{2\varepsilon}[\tau_X(\varepsilon\tau_X(\alpha^2 + \alpha(\varepsilon-1)^2 - (\varepsilon-2)(\varepsilon-1)) - \\ & \alpha^2(\varepsilon+1) + 2\alpha(-\varepsilon^3+\varepsilon^2+1) + (\varepsilon-1)(\varepsilon^2+1)) + \alpha\varepsilon^3] + \\ & \tau^{3\varepsilon+1}\tau_X^{3\varepsilon}(\alpha+\varepsilon-1)[(\varepsilon-1)\tau_X\,(\tau_X(\alpha(\varepsilon-1)+1) - (2\alpha+1)\varepsilon) + (\alpha+1)\varepsilon^2] - \\ & \tau_X^{4\varepsilon}\tau^{4\varepsilon}(\alpha+\varepsilon-1)^2[(\varepsilon-1)\tau_X - \varepsilon] + \tau^4\tau_X^2\varepsilon(\alpha+\varepsilon-1)\} \\ B_X^{Nash}(\tau_X) \equiv\ & \tau_X(\tau^\varepsilon\tau_X^\varepsilon + \tau\tau_X)(\tau^{2\varepsilon}\tau_X^{2\varepsilon} - \tau^2)[(\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon - (\alpha+1)(\varepsilon-1)\tau\tau_X + \alpha\varepsilon\tau] \\ & [(\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon + \tau\tau_X(-\alpha\varepsilon+\alpha+\varepsilon-1) + \alpha\varepsilon\tau] \end{aligned}$$

(i) We first show that no solution with $\tau_X < 1$ exists. Focusing on the numerator of the first-order condition, this is so since all terms of $A_X^{Nash}(\tau_X)$ are positive for $\tau_X < 1$.

(ii) Next, we show that there exists at least one solution with $\tau_X > 1$. Note that $A_X^{Nash}(\tau_X = 1) = (\varepsilon + \alpha - 1)(\tau^\varepsilon + \tau)^2[(1-\alpha)\tau^\varepsilon\tau + \varepsilon\tau^2 + \tau^{2\varepsilon}(\varepsilon - 1 + \alpha)] > 0$. Thus, for a Nash solution with $\tau_X > 1$ to exist, by continuity of $A_X^{Nash}(\tau_X)$ it is enough to find a $\tau_X > 1$ such that $A_X^{Nash}(\tau_X) < 0$. It is straightforward to show that $\lim_{\tau_X \to \infty} A_X^{Nash}(\tau_X) = -\infty$. Therefore, there exists a solution with $\tau_X^{Nash} > 1$.

(iii) It remains to show that if $\tau_X^{Nash} > 1$, then $N_X^{Nash} < N^{FB}$ and $P_X^{Nash} > P^{FB}$. In a way similar to what we did in Proposition 5, we first look at $\frac{dN}{d\tau_X}$ evaluated at $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$:

$$\frac{dN}{d\tau_X} = \frac{\partial(N+N^*)}{\partial\tau_X}\Bigg|_{\tau_X=\tau_X^*} = \frac{L(1-\alpha)\alpha(\varepsilon-1)\tau\left[\tau^\varepsilon\tau_X^\varepsilon\left((\varepsilon-1)\tau_X-\varepsilon\right)-\tau\tau_X\right]}{f\tau_X\left[(\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X(1-\alpha)(\varepsilon-1)+\alpha\varepsilon\tau\right]^2}$$

Note that $\frac{dN}{d\tau_X} \geq 0 \iff \tau^\varepsilon\tau_X^\varepsilon\left[(\varepsilon-1)\tau_X-\varepsilon\right]-\tau\tau_X \geq 0$. Let us define the following two continuous and monotonic functions $f(\tau_X) \equiv (\varepsilon-1)\tau^\varepsilon\tau_X^{\varepsilon+1}$ and $g(\tau_X) \equiv \varepsilon\tau^\varepsilon\tau_X^\varepsilon + \tau\tau_X$ with $f'(\tau_X) > 0$, $f''(\tau_X) > 0$, $g'(\tau_X) > 0$ and $g''(\tau_X) > 0$. Note that $f(1)-g(1) < 0$ implying $\frac{dN}{d\tau_X} < 0$. By continuity and monotonicity of the two functions, only two cases are possible. They either never cross, in which case $\frac{dN}{d\tau_X} < 0 \forall\tau_X \in [1,\infty)$ and consequently $N_X^{Nash} < N^{FB}$. Or, they cross only once. That implies that $\exists\bar{\tau}_X > 1$ such that $f(\tau_X) \geq g(\tau_X)$, $\forall\tau_X \geq \bar{\tau}_X$ implying $\frac{dN}{d\tau_X} > 0 \iff \tau_X \in (\bar{\tau}_X,\infty)$. However note that:

$$\lim_{\tau_X->\infty} N = \lim_{\tau_X->\infty} \frac{L\alpha\left(\tau^\varepsilon\tau_X^\varepsilon+\tau\right)}{f\left((\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X(\alpha(-\varepsilon)+\alpha+\varepsilon-1)+\alpha\varepsilon\tau\right)} = N^{FB}$$

implying that also in this case $N_X^{Nash} < N_X^{FB}$.
Finally, $P_X^{Nash} > P^{FB}$ follows from $N_X^{Nash} < N_X^{FB}$, $\tau_X^{Nash} > 1$ and the fact that $P_X^{Nash} = \left(N_X^{Nash}\right)^{\frac{1}{1-\varepsilon}}\left[1+\left(\tau\tau_X^{Nash}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$ while $P^{FB} = \left(N^{FB}\right)^{\frac{1}{1-\varepsilon}}\left[1+\tau^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$.

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