Name:

PGCE Mathematics

Didactics

Workbook

**Mathematical Knowledge for teaching**

This booklet is designed to help you to enhance your knowledge of the 11 to 18 curriculum and your thinking about teaching and learning mathematics. It is linked to the requirements of Key Stages 3 and 4 of the National Curriculum for schools, the AS/A level subject criteria and the general Standards for Teachers. It is primarily targeted at developing different strands of knowledge you will need as a teacher so there are obvious links to aspects of lesson planning, assessment and ICT.

**You should aim to complete this workbook by the start of the PGCE course. However, you will be expected to add to it throughout the course as you develop your understanding of how to approach teaching different areas of maths.**

The points below indicate the wide range of aspects of subject knowledge that are essential to a good teacher of mathematics.

• developing mathematical fluency and understanding

• keeping things simple

• exploring alternative approaches

• recognising common errors and misconceptions and responding to them

• clarifying key mathematical ideas

• making links and connections

• explaining common algorithms

• ensuring precise and correct use of mathematical language

• improving problem solving and proving skills

• responding to pupils' questions

• using ICT, including calculators of all kinds, to provoke questions and to encourage productive discussion

• accessing research evidence to inform and improve teaching

• assessing pupils' work and planning appropriate follow up

• understanding progression, particularly between KS2 and KS3 and between KS4 and post-16

• extending 'general knowledge' about mathematics, including historical and multi-cultural aspects

One of the most demanding requirements in qualifying as a teacher is to demonstrate that you can cope securely with subject-related questions which pupils raise. The ability to cope with pupils' questions is a major test of the skills of a mathematics teacher, and involves a deep and detailed knowledge of mathematics and of the way pupils make sense of it. This booklet offers you some starting points in a process of learning which should continue throughout your career.

In working through this booklet, you are certain to spot areas where your knowledge and understanding is not as sound as you would wish. Some suggestions for strategies to adopt and resources to employ in deepening your understanding in these areas are as follows:

* Key Ideas in Mathematics Teaching is a website and book which aims to look deeply into key maths concepts:

<http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics>

* There are several sets of video clips on youtube.com aimed at self-teaching mathematics topics, eg Khan Academy <http://www.youtube.com/user/khanacademy>
* There are many printed and online revision guides for GCSE and A level mathematics – read, discuss with a peer, try examples, practice explanations, invent problems.
* Text books have their place – many contain explanations of ideas and examples of related problems. It is important to remember the aim of **delving deeper than the ability to answer typical exam questions**: What questions might a pupil ask and how would I answer them?
* GCSE and A level specifications are available online from the major examination boards.
* The NCETM has a subject knowledge auditing tool on its website – the examples section can be especially useful. <https://www.ncetm.org.uk/>
* The support of a peer or tutor in discussing the ideas cannot be over-valued. Do you know anyone else who is aiming to become a teacher?

**Acknowledgement:**

This workbook was developed by the late Doug French at the University of Hull.

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**Number**

**Mental Calculation - Whole Numbers and Money**

Do the following mentally and, in each case, show briefly how you did each calculation.



**Multiplication Tables**

You can instantly and correctly recall that 7 times 8 is 56, but for a pupil who cannot recall the answer there is a problem to be solved. The most elementary strategy is to count up in eights: 8, 16, 24, 32, 40, 48, **56** or in sevens: 7, 14, 21, 28, 35, 42, 49, **56**.

At a slightly more sophisticated level a range of possibilities open up:

• Counting back from : 80, 72, 64, **56**, or, from : 70, 63, **56**.

• Starting from a known fact: I know that , so  is 7 more, or, I know that , so  is 8 less.

• Multiplying by 8 is 'double, double, double', so ,  and .

• Using addition of known results:  and  and therefore .

• Doubling and halving:  is the same as , which is the same as , which is 56.

Give several alternative approaches that pupils might use to work out:





**Errors with Whole Numbers and Money**

Look at these examples of pupils' errors. In each case indicate what the pupil has done, how you would get them to see that it was wrong and how you would help them towards a correct method of doing the calculation.













**Understanding Fractions**

Discuss the misconceptions that give rise to .

Give several alternative methods (numerical, pictorial, algebraic) to show that: 

**Fractions and Division**

Suggest various ways of helping pupils understand that:





**Some Sequences with Decimals**

Explain briefly how sequences like these can be generated on a graphical calculator and on a spreadsheet.

|  |  |  |  |
| --- | --- | --- | --- |
| Sequence A | Sequence B | Sequence C | Sequence D |

Indicate some of the key ideas and misconceptions that can be discussed with pupils in relation to each sequence.

**Fractions, Decimals and the Number Line**

Label each number line from 0 to 1 at intervals of 0.1. Show the fractions indicated, and their decimal equivalents, on the number line. Indicate briefly some of the key ideas that can be discussed with pupils arising from such examples.

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**Errors with Decimal Fractions**

Look at these examples of pupils' errors. In each case indicate what the pupil has done, how you would get them to see that it was wrong and how you would help them towards a correct method of doing the calculation.









**Percentages**

Give some alternative non-calculator methods that pupils could use to work out:









**Percentages with a Calculator**

Give appropriate ways of doing these calculations with a calculator:









**Powers**

Pupils understand simple powers like .

Suggest ways of making the definitions of ,  and  make sense.

How do you explain that ?

What is ?

**Difference of Two Squares**

**1 3 6 10 15 21 28 36**

• The first 8 triangle numbers are shown above.

• Explain why the th triangle number is given by .

• Explain why the th triangle number is given by .

• Investigate numerically the difference between the squares of consecutive triangle numbers. Explain your findings algebraically.

**Understanding Integers**

How would you help pupils make sense of these calculations?





**Algebra**

**The Meaning of Letters in Algebra**

Criticise this statement:

A week is 7 days, so , where  stands for week and  stands for day.

Comment on the extract from a text book shown below.

Find a green rod.

Make the same length with three white rods

You can write

One green rod equals 3 white rods

In algebra

***g = 3w*** (notice that you do not write a 1 in front of the ***g***)

**Misconceptions in Algebra**

Comment on the misconceptions revealed by these examples of pupils' responses:

Simplify : 

Simplify : 

Evaluate  when : 

Solve : 

**Mathematical Language**

Distinguish between the words in each pair, giving examples to illustrate their meanings.

**Expression and Equation**

**Equation and Identity**

**Sequence and Series**

**Implication and Equivalence**

Discuss whether each of these statements is true or false and, where false, indicate how the statement can be modified to make it true.







If  is a prime number greater than 3, then  is of the form , where  is a natural number.

|  |  |
| --- | --- |
|  | Put a circle round all the prime numbers in the array on the left. How would you prove the statement in the box above to a year 9 class? |

How could the discussion be extended to draw attention to the *converse* statement?

**Conjectures**

Each conjecture is to be investigated and either proved or refuted by finding a counter example. Comment on this pupil's responses.

The square of every even number is a multiple of 4.



The square of every odd number is odd.



A square number with a units digit of 1 is the square of a number with a units digit of 1.



121 is the only palindromic square number with 3 digits.



**Solving Quadratic Equations**

Annotate your solution to each part to show how you would explain each step to pupils.

Solve the equation  by completing the square.

Solve the equation  by completing the square.

Use the previous result to derive the formula for solving .

**Functions and Graphs**

Suggest possible functions for each of these graphs, giving **alternative forms** where possible. Assume units of one on both axes.

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**Algebraic Fractions**



Discuss each result, highlighting the errors that pupils may make in attempting to simplify the algebraic fraction without a calculator. Suggest ways to enhance their understanding of the results.

**An Optimisation Problem**

Give a model solution to this question for A level students and **annotate** your solution to indicate possible sources of difficulty and error.

Find the dimensions of a cylinder with a volume of  to give the minimum surface area.

**Functions and Graphs with Asymptotes**

Suggest possible functions for each of these graphs, explaining your reasoning carefully. Assume units of one on both axes.

|  |  |
| --- | --- |
|  |  |
|  | The line  is an asymptote. |

**Differentiation from First Principles**

Superimpose a sketch graph of the gradient function of the function whose graph is shown below. Explain briefly the steps in your reasoning.



Differentiate the function  from first principles. Note clearly the key points of potential difficulty for students in understanding the argument.

### *Challenging Algebra questions*

|  |  |
| --- | --- |
| 1. | How would you respond to an A-level student who says “I think proof by induction is a fiddle – you are assuming what you want to prove” Your discussion should cover at least the following (a) can we *prove* that proof by induction is valid, or is it just an assumption? (b) …suppose that it is true for *n = k*, then we can deduce that it is true for *n = k+*1 – discuss the roles of *k* and *n*. Does *n = k* and *n = k+*1 imply that *k = k*+1? |
| 2. | Let  Now  so  However  and so  because for any . How would you explain this apparent paradox to an A-level student? |
| 3. | My dad told me that  because  But my sister says that  because  Who is right? |
| 4. | Do the following division:  Wait for it – the numbers are in base SIX. Describe and justify each step in the process you use. |
| 5. | Find the first five digits in the “decimal” expansion of  in base SIX. |
| 6. | Why is it that so many people think that ? Can you find some circumstances in which this is actually correct – for numbers. matrices, vectors, anything else? |
| 7. | 1/4 = 0.25  3/7 = 0.428571428571428571… Explain why the decimal representation of a fraction either terminates or recurs.  What kind of explanation would you give to a group of year 9 pupils who have been investigating fractions and decimals on their calculators?  What kind of explanation would you give to a fellow mathematics teacher who asked you about this? |
| 8. | Why is  How would you justify this to a year 9 pupil? |

### *Calculus questions*

|  |  |
| --- | --- |
| 1. | Can this function be differentiated at  What about |
| 2. | The graph of a function  is shown above.   1. Where is the function *f* increasing most rapidly? 2. Where is it decreasing least rapidly? 3. Sketch the derivative of *f*. 4. Where is the second derivative largest? 5. Sketch a graph of the function  given by |
| 3. | Consider the function defined as follows    Prove from the definition of the derivative that this function is differentiable at  What is the value of  Use this to write down the equation of the tangent at  Use this to discuss possible geometrical definitions of a tangent to a graph. |
| 4. | The function  has the limit *L* as *x* tends to *a* if the value of gets closer and closer to *L* as *x* gets closer to *a*.  Discuss this verbal definition of a limit using the following examples, taking  (i)  (ii) . |
| 5. | What is the derivative with respect to *x* of  when *x* is measured in degrees? |

**Ratio, Proportion and Rates of change**

**Proportionality**

What else can you work out if you know that 5 miles is about 8 kilometres?

In what different forms can you express the map scale 1:25000?

A 75cl bottle of wine costs £2.35 and a litre bottle of the same wine costs £3.15. Give some alternative ways of determining the best buy.

**Proportionality and Volume**

Give brief model solutions to these question (not using ideas beyond GCSE level) and **annotate** your solutions to show clearly how you would explain your reasoning to pupils.

A cube has edges of length 1 metre. Find the length of the edges of another cube which has twice the volume.

A container in the form of an inverted cone is half full of water. Find the depth of the water in terms of the height,, of the cone.

**A Problem with Speed**

Give some alternative model solutions to this question (not using ideas beyond GCSE level) and **annotate** your solutions to indicate possible sources of difficulty and error for pupils.

A car travelling at 60mph joins the M62. Another car is a quarter of a mile ahead travelling at 50mph. How long does it take the first car to catch up with the second and how far does it travel in doing so? How does the time vary for different speeds and distances?

**Velocity Time Graphs**

|  |  |
| --- | --- |
|  | The graph represents the motion of a body moving with constant acceleration.  Explain how these three formulae may be derived directly from the graph and discuss the key ideas involved in each case: |

Show how to find the time taken to reach the ground when a stone is dropped from the top of a 50 metre high building.

Derive the first two formulae by solving the differential equation .

**Geometry and Measure**

**Angle Measurement**

Comment briefly on the errors revealed by the responses to the item below.

Question – ‘Which is the largest angle?’

The percentages are the proportion of pupils who gave each response.



List the errors that pupils commonly make in using a semi-circular protractor to measure angles and suggest how they can be helped to avoid them.



**Angles of Polygons**

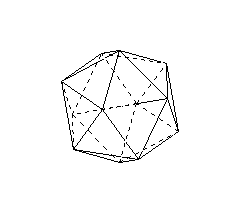
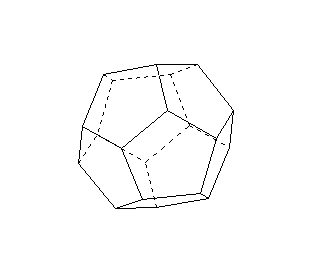
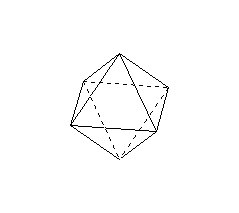
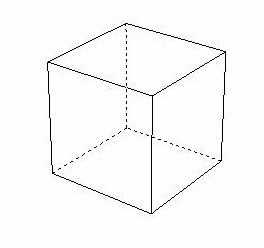
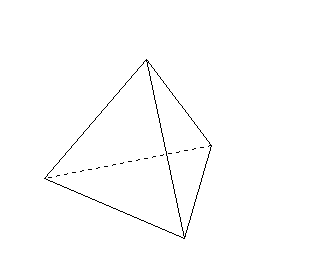
Give a proof that the angle sum of a triangle is  based on this diagram.



Use each of these diagrams to suggest a formula for the angle sum of an -sided polygon and show how the two formulae are linked.

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**Polyhedra**



Complete the table for the five regular polyhedra pictured here.

|  |  |  |  |
| --- | --- | --- | --- |
| Name | No. of vertices | No. of edges | No. of faces |
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What is Euler’s Formula? Does it apply to all polyhedra?

Use some congruent equilateral triangles to make four regular tetrahedra and one regular octahedron. Fit the five polyhedra together to make a larger tetrahedron.

What is the relationship between the volume of the larger tetrahedron and one of the small tetrahedra?

What is the relationship between the volume of the octahedron and one of the small tetrahedra?

**Constructing Triangles**

Construct **all** the possible triangles which have sides of length 5cm and 8cm and an angle of 30˚ using ruler and compasses only.

**A Tessellation of Quadrilaterals**

Create a tessellation using this quadrilateral.

• What difficulties might pupils have with this task?

• What suggestions would you make to help them?

• What properties of a quadrilateral can be discussed with pupils in relation to this tessellation?



**Properties of Quadrilaterals**

Complete this table giving the key properties of different types of quadrilateral.

|  |  |  |  |
| --- | --- | --- | --- |
| Name | Sides | Angles | Diagonals |
| SQUARE  RHOMBUS  RECTANGLE  PARALLELOGRAM  KITE  ISOSCELES  TRAPEZIUM  TRAPEZIUM | All sides equal.  Opposite sides parallel. | Opposite angles  equal. | Not equal in length.  Perpendicular.  Both diagonals  bisected. |

**Mathematical Words**

What is the plural of each of these mathematical words?

|  |  |  |  |
| --- | --- | --- | --- |
| **index** |  | **trapezium** |  |
| **polyhedron** |  | **axis** |  |
| **polygon** |  | **locus** |  |
| **formula** |  | **rhombus** |  |
| **radius** |  | **vertex** |  |
| **series** |  | **Focus** |  |

**Interesting Facts about Pi**

Give a wide variety of interesting historical and mathematical facts about π suitable for adding interest to lessons at different levels.

**Perimeters and Areas of Circles**

Find the areas and perimeters of these shapes in terms of π.





**Angles in Circles**

Give a proof that the angle in a semi-circle is a right angle based on this diagram.



Give a proof that the angle at the centre of a circle is twice the angle at the circumference standing on the same arc based on this diagram. What other cases would need to be considered?



**Explaining and Proving in Geometry**

Explain how the angles of triangle ABC can be calculated, indicating how each step in the argument could be made clear to pupils.



Two circles intersect at P and Q. PA and PB are diameters. Prove that the points A, Q and B lie in a straight line.



**The Medians of a Triangle**



Find two alternative proofs (not using vectors) that the medians of a triangle meet in a point which divides each median in the ratio 2 to 1.

**The puzzling triangle**

3

7

4

10

Calculate the total area of this shape in two different ways. Do you notice a problem? If so, how can you explain it? How could you use this as a basis for a lesson on gradient?

**Proving Pythagoras’ Theorem**

Give an algebraic proof of Pythagoras’ theorem based on this diagram, where four congruent right angled triangles are placed in a square.



Explain why each of the shaded areas are equal. What else is needed to complete a proof of Pythagoras’ theorem.

Give below another proof of Pythagoras' theorem.

**Pythagoras’ Theorem**

Give a model solution to this question (not using ideas beyond GCSE level) and **annotate** your solution to indicate sources of potential difficulty and error for pupils.

Calculate the distance between the two points (-2, -3) and (3, 5).

**Trigonometry**

On each diagram write expressions for the lengths of the unknown sides. Use surd form where appropriate, but **do not evaluate** your expression when numerical values are given.



What form does Pythagoras' theorem take for the last two triangles above?

**Distance and Gradient**

Give a model solution to this question and **annotate** your solution to indicate possible sources of difficulty and error for pupils.

The map references and heights of Summit House and Westfield Farm are 913538 and 132 metres and 931526 and 59 metres respectively. Calculate the distance and the gradient of the road between the two places. Give the gradient in the form  and as a percentage, and calculate the angle of slope of the road.

**Some Properties of Sine and Cosine**



Explain each result in more than one way if possible.

**Calculating an Angle**

Give several alternative methods of calculating the angle  in this square.



**Addition Formulae**

By annotating this diagram suitably show how it can be used to prove the formulae for  and .

1

θ

φ

φ

Cos θ

Sin θ

Show below how the formula for  can be derived from the formulae for  and .

**Two Problems**

Give model solutions to these two problems for A level students and **annotate** your solutions to indicate possible sources of difficulty and error.

Find the area of a segment of height 3cm taken from a 5cm radius circle.

Find the volume of a cap of height 3cm taken from a 5cm radius sphere.

**Projectiles**

A body is projected with velocity  at an angle  to the horizontal. Explain using the formula  why the position of the body is given by the two parametric equations:

 and 

Show how to find the Cartesian equation of the path of the body.

Find an expression for the horizontal range and explain how to determine the angle at which it is at its maximum.

### Challenging *Geometry questions*

|  |  |  |  |
| --- | --- | --- | --- |
| 1. |  | The diagram shows a square on whose base stands an equilateral triangle. Calculate the marked angle, explaining your reasoning. | |
| 2. |  | Which of the following statements is true?    Give an explanation suitable for a year 9 pupil. | |
| 3. | Given a line and a point not on the line there is a unique perpendicular from the point to the line. Discuss this in various geometries. | | |
| 4. | When does a pair of linear equations not have a solution? Interpret this geometrically – in the plane, in three dimensions, … | | |
| 5. |  | | The internal bisector of the right angle of a right-angled triangle bisects the square on the hypotenuse of the triangle.  Discuss whether you think the diagram constitutes an adequate proof of this statement. |

### *Challenging Trigonometry questions*

|  |  |
| --- | --- |
| 1. | On one diagram, sketch the graphs of      for  Label them clearly.  Hence sketch the following graphs, creating separate diagrams for each one and giving a full and clear explanation of your reasoning in each case:      What important aspects of trigonometric functions do these questions illustrate? |
| 2. | True or false? |
| 3. | Use trigonometry to analyse geometrical relationships involved in the Egyptian pyramids, both their external and internal features. |
| 4. | Prove that the shortest distance between two points on a sphere is along the great circle joining the two points. |
| 5. | Starting with the addition formula  derive as many other trigonometrical identities as you can. If you now use the identity  see how many more identities you can derive. |

**Probability**

**A Probability Problem**

Give a model solution to this question (not using ideas beyond GCSE level) and **annotate** your solution to indicate possible sources of difficulty and error for pupils.

A domino is selected at random from a full set of dominoes. What is the probability that it displays at least one six? If two dominoes are selected without replacement, what is the probability that they both display at least one six?

### *Probability Questions*

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| 1. | Suppose we have 100 red counters with numbers on them whose average is 5, and 100 blue counters with numbers on them whose average is 50. We pick a red one and a blue one at random. There is a high probability that the number on the blue one will be larger than the number on the red one. True or false? |
| 2. | A coin is tossed 7 times, and it lands heads each time. What do you think will happen on the next toss? What odds would you give on a head showing on the next toss? |

**Statistics**

**Mean and Standard Deviation**

Prove the equivalence of these two forms of the formula for standard deviation:

Find the mean and standard deviation of the set of numbers 1, 2, 3, 4, 5, leaving the latter answer in surd form.

Explain how you can write down directly, using the results above, the mean and standard deviation for each of these sets of numbers:

2, 4, 6, 8, 10

4, 5, 6, 7, 8

11, 12, 13, 14, 15

1, 1, 2, 2, 3, 3, 4, 4, 5, 5

0.1, 0.2, 0.3, 0.4, 0.5

1, 3, 5, 7, 9

**Using a Spreadsheet to Represent Data**



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Comment on the advantages and disadvantages of these two ways of representing this data.

***Statistics Questions***

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| 1. What statistical information would we need to obtain to predict how many mathematics teachers will be needed in 10 years’ time. How would you convince the government of the accuracy of your predictions in order to bid for the necessary finance to train and pay the teachers needed? |
| 2. A survey measures the heights of all the pupils in two classes, 4A and 4B. Five children are then moved from class 4A to class 4B. As a result the average height for both classes increases, and the standard deviation for both classes decreases. How is this possible? |

**Other A level Maths Topics**

**Mechanics Questions**

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| 1. | A driver leaves home at 8.00 a.m. and turns right onto a main road. He accelerates smoothly until he reaches 30 mph. He drives at this speed for 5 minutes before realising that he has left his briefcase behind, and at the next roundabout, two minutes later, he does a U-turn. He drives home at 40 mph except for a two-minute break half way home where he is held up by a red traffic light. Sketch a graph of his velocity and displacement as functions of time. |
| 2. | a) Imagine that you have a metre rule resting horizontally, supported only by resting it on a forefinger at each end. Write down what you would expect to happen if you move your fingers slowly together, keeping the rule horizontal. Do it and write down what actually happens. Explain why this happens.  b) Imagine that you have a metre rule resting horizontally, supported only by two forefingers placed next to each other as close as possible to the centre of the rule. Write down what you would expect to happen if you move your fingers slowly apart, keeping the rule horizontal. Do it and write down what actually happens. Explain why this happens. |
| 3. | Two toy cars leave the edge of a table at the same time. The red one is travelling twice as fast, and is twice as heavy, as the blue one. Discuss what you think the subsequent motion of the cars will be. Discuss how you think that a class of year 7 students would respond to this question. Analyse it using the equations of motion, and then consider what kind of explanations for the motion you would be able to offer to year 7 pupils. |
| 4. | A heavy object is being whirled round in a horizontal plane above your head, on a piece of string. Many people think that if the string were to suddenly break the object would fly out along the radius. How would you explain that the laws of motion suggest that it should continue in the direction of the tangent to its original circular path? |
| 5. | If you sit in a sailing boat and blow into the sail what will happen? |
| 6. | If you throw a ball off a stationary railway carriage what will happen? |
| 7. | If you balance a fixed wheel bicycle and push one of the pedals (a) forward and (b) backwards which way will the bike move, (if at all)? |

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### Discrete mathematics

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| 1. AD25 | Here are four cards with a number on one side and a letter on the other side.  Which cards would you turn over to check whether this statement was true?  *“If there is an even number on one side there is a vowel on the other.”*  Construct some more problems like this and explain how your choices would relate to a study of mathematical reasoning for GCSE pupils. |
| 2. | In my Discrete Mathematics class there are 32 boys. Each boy knows five of the girls in the class and each girl knows eight of the boys. How many girls are there in the class? |
| 3. | Prove that the set of all positive integers whose digits are all different is finite. How many such integers are there? |
| 4. | An equilateral triangle of side length 1cm contains five points A,B,C,D,E. Show that there must be at least two of these points whose distance apart is less than 0.5cm |