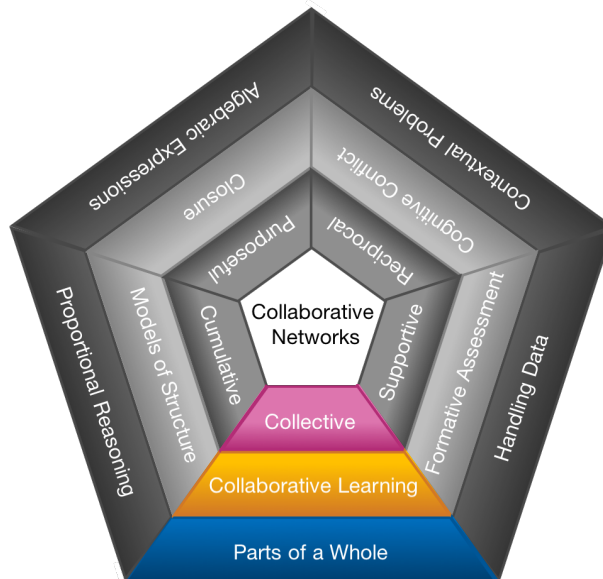


Overview

The focus in the lesson, **Parts of a Whole**, is on how to develop **collaborative learning** and to see how this contributes towards creating a **collective** classroom.



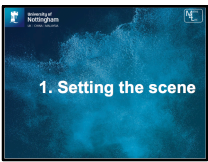

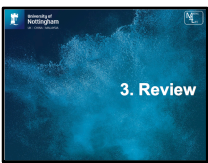
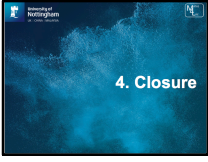

Collaborative learning is when students work together in both pairs and small groups, and as a whole class, towards common goals.

A **collective** classroom is one in which both the students and the teacher see their lessons as being based around joint learning and enquiry.

Research Question

How does **collaborative learning** (through the design of resources and the actions of the teacher) promote **collective** endeavour?

Lesson Summary

Phase	Timings (minutes)	Notes
 1. Setting the scene	5	The initial problem is explained, clarifying why Ali and Blair split the payment in the given ratio.
 2. Cards	5 - 10	Students match the fractions cards to the ratios on the grid. <i>It is important that students are given the time to follow their thinking through without teacher correction.</i>
	10 - 15	Students work with the representation cards. <i>Students resolve misconceptions and cognitive conflict through the insight that the representations provide.</i>
	10 - 15	Students work with the word description cards. <i>These cards allow students to make links between fractions, ratios and the ways that they are often described in exam questions.</i>
 3. Review	5 - 10	Check understanding of the activity using the review slides in the electronic presentation.
 4. Closure	10 - 15	Return to the original Ali and Blair problem. <i>Ensure a common understanding of the misconception and its resolution.</i> <i>Examine how given amounts can be shared using fractions and ratios.</i>
 5. Extension		Extension questions used if appropriate. <i>These slides allow formative assessment with a variety of possible questions and answers.</i>

Mathematical goals

To help students:

- understand ways of mathematically describing a part-part relationship;
- understand ways of mathematically describing a part-whole relationship;
- understand how to use representations to give insight into solving problems.

Starting points

There is often confusion about the connections and differences between fractions and ratio. Many students assume that the ratio 1:2 is equivalent to the fraction $\frac{1}{2}$.

Diagrams can be a powerful representation that allow students to understand the relationship between ratio and fraction.

Materials required

For each group of students, you will need:

- L1.2 Cards (fraction cards, diagram cards and word cards all separate);
- L1.3 Template **enlarged** onto A3 paper;
- glue sticks.

L1.4 PowerPoint Presentation

Time needed

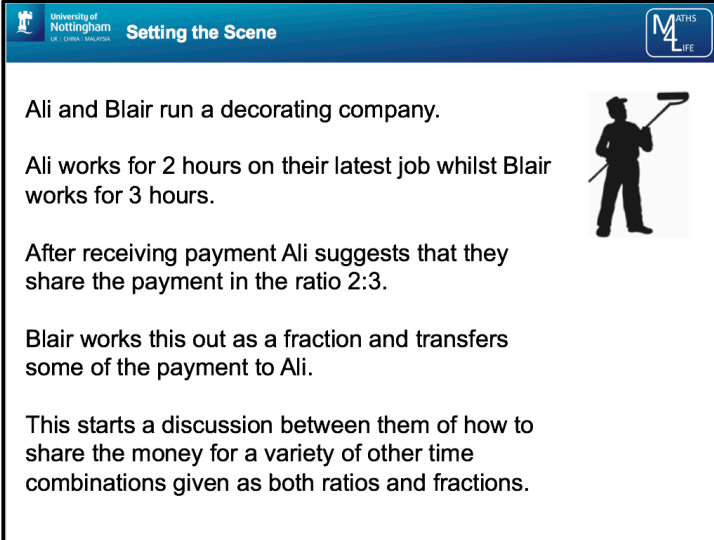
Approximately 1 to 1½ hours.

Lesson Structure

Setting the Scene

Introduce the problem using the PowerPoint presentation. This allows students to understand why there is the suggestion to share the money in the ratio 2:3.

1. What assumption must be made for the money to be shared in the ratio 2:3?



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Setting the Scene

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Ali and Blair run a decorating company.

Ali works for 2 hours on their latest job whilst Blair works for 3 hours.

After receiving payment Ali suggests that they share the payment in the ratio 2:3.

Blair works this out as a fraction and transfers some of the payment to Ali.

This starts a discussion between them of how to share the money for a variety of other time combinations given as both ratios and fractions.



Collaborative Learning using Cards

2. Why is it important to hold on to the representation cards when the students first tackle the question?

Arrange students into pairs. Explain to the students that each pair will be given a template and a set of fraction matching cards. (You must only give out the fraction cards at this point. Save the cards with representations on them for later in to the activity).

Students must take turns to match a pair, and to explain their reasons for putting them together. The other person in the pair must agree to the match, disagree or ask for further explanation.

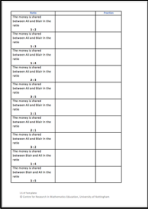
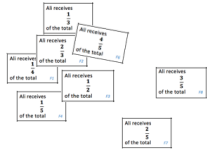
3. What mistakes and difficulties do you expect?

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Cards

MATHS
LIFE

In your pairs you will be given a set of matching cards and an A3 template.

Take **turns** to place the cards on the template.

For each match the first person must **explain** why they're placing it there and the second person must agree/disagree/ask for further explanation.

“Don't intervene too early”

4. When is the optimum time for handing out the diagram cards?

Once students are part way through the task of matching, give them the representation cards. They should be asked to see whether the diagrams change their thinking or support it.

“Use mistakes and misconceptions to encourage dialogue”

Remind students to complete the blank cards and to explain to one another their reasoning.

5. Which word cards will be particularly important to highlight in the whole group learning phase?

Once students have completed the ratio-diagram-fraction matchings they can be given the set of word cards. Note that these cards will have to be placed on the edge of the template. Also note that some rows have more than one word card and some rows don't have any. Students are not expected to create cards for rows that have no word cards.

To extend the task further, groups of students could be asked to calculate how much Ali and Blair receive in each situation if the total sum to be shared is £120. (Their answers could be written on the picture cards).

Review

The presentation includes slides to show how the cards should have been matched.

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Review – part 3

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Ratio		Fraction		
The money is shared between Ali and Blair in the ratio 1 : 2		All receives $\frac{1}{3}$ of the total	Blair receives double the amount that Ali receives	All receives half the amount Blair receives
The money is shared between Ali and Blair in the ratio 1 : 3		All receives $\frac{1}{4}$ of the total	Blair receives three quarters of the total	Blair receives three times the amount Ali receives
The money is shared between Ali and Blair in the ratio 1 : 4		All receives $\frac{1}{5}$ of the total	All receives one quarter of the amount Blair receives	
The money is shared between Ali and Blair in the ratio 2 : 3		All receives $\frac{2}{5}$ of the total	All receives $\frac{4}{10}$ of the total	
The money is shared between Ali and Blair in the ratio 3 : 5				

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Review – part 4

MATHS LIFE

The money is shared between Ali and Blair in the ratio 1 : 1		All receives $\frac{1}{2}$ of the total	All and Blair both receive the same amount
The money is shared between Ali and Blair in the ratio 2 : 1		All receives $\frac{2}{3}$ of the total	All receives double the amount that Blair receives
The money is shared between Ali and Blair in the ratio 3 : 2		All receives $\frac{3}{5}$ of the total	
The money is shared between Blair and Ali in the ratio 1 : 4			
The money is shared between Blair and Ali in the ratio 1 : 5		All receives $\frac{4}{5}$ of the total	

Before proceeding with the PowerPoint presentation examine any interesting mistakes or misconceptions that have taken place in the classroom.

All the time, it is important to keep re-phrasing student explanations using the language of part to part and part to whole.

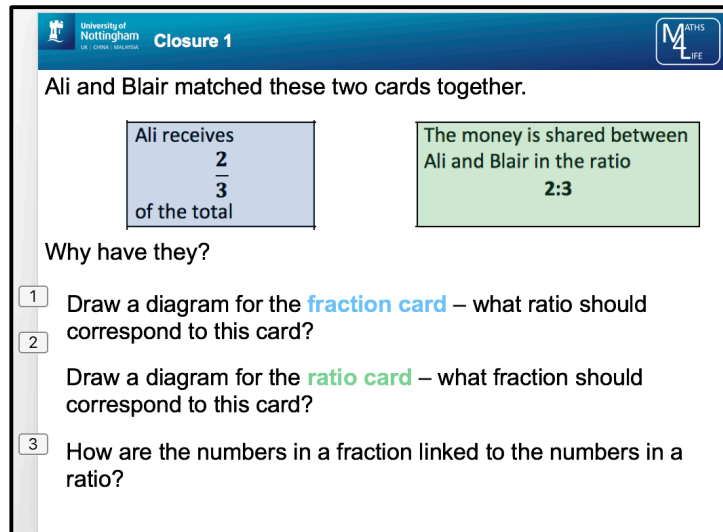
For example,

*Ali receives 1 part for every 2 parts that Blair receives.
Ali receives 1 part out of 3 parts of the whole.*

Closure

Allow approximately 15 minutes for this section of the lesson. Using the PowerPoint presentation as a guide, bring closure to the lesson by asking students to work in a pair to answer the questions on the next slide.

“Students need closure in a lesson”



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Closure 1

Ali and Blair matched these two cards together.

Ali receives $\frac{2}{3}$ of the total

The money is shared between Ali and Blair in the ratio 2:3

Why have they?

- 1 Draw a diagram for the **fraction card** – what ratio should correspond to this card?
- 2 Draw a diagram for the **ratio card** – what fraction should correspond to this card?
- 3 How are the numbers in a fraction linked to the numbers in a ratio?

6. How can the link between fractions and ratios be explained succinctly? What do you expect students to say?

Ask students to identify the link between information provided as a fraction and information provided as a ratio. Re-phrase student explanations to draw out an understanding of ratio showing part to part whilst fractions show a part of a whole.

Now ask students to calculate how much each person would receive if the total money paid was £30. Doing this for both the ratio 2:3 and the fraction $\frac{2}{3}$ emphasises the importance of understanding these calculations.

7. Which features of the representation cards are important to highlight to students?

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Going Deeper

M4 MATHS LIFE

If the total payment received for the job is £30, how much would Ali and Blair each receive?

Ali receives
 $\frac{2}{3}$
 of the total

The money is shared between
 Ali and Blair in the ratio
2:3

A	A	B
---	---	---

← £30 →

A	A	B	B	B
---	---	---	---	---

← £30 →

Extension

The next slides ask open-ended questions that give the opportunity for formative assessment to take place.

8. Which of these ways are most important to stress with your class?

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Extension 1

M4 MATHS LIFE

Draw a diagram to illustrate Ali and Blair sharing money in the ratio 3:1

1	A	A	A	B
---	---	---	---	---

How many different ways can you describe how the money is shared?

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Extension 2

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LIFE

A	A	A	B
---	---	---	---

What questions could be asked if the monetary amount involved is £36?

9. What are the key messages from the lesson that you expect to draw out?

“It’s about what we do and what we don’t do”

“We can affect the way students work together through the way we structure a task”

“We want to see students developing both questions and ideas together”