Maths-for-Life

Welcome to the Maths-for-Life resources for Lead Teachers. The materials here are intended to provide a full programme for Lead Teachers to work with when supporting a group of Maths-for-Life teachers working to implement a diagnostic learning approach to teaching GCSE resit students. All materials for teachers, videos and other materials can be found at: (https://www.nottingham.ac.uk/maths-for-life/).

The Maths-for-Life programme has been developed by the Centre for Research in Mathematics Education at the University of Nottingham and funded by the Education Endowment Foundation as part of a joint initiative with JP Morgan. Its design was informed by a range of research that indicates that the programme's approach to teaching will support GCSE resit students in revisiting their learning of important concepts that are fundamental to attaining a good grade in GCSE exams.

The programme has been designed to be conducted by a collaborative network of teachers. Core to Maths-for-Life is the dialogic approach that sees students talking about mathematical ideas together and with their teachers. Five lessons exemplify the approach and the resources here include both a description of each of these and questions on the key pedagogical issues they require.

Importantly, the programme has been designed to be conducted within a collaborative network of fellow professionals. The resources here are designed to support the Lead Teacher working with such a group: they provide insight into the sorts of professional and pedagogical questions that would support the group in their "lesson-study" approach to exploring the lessons and the different aspects of teaching that can best support dialogic learning.

We hope you have a great experience and profitable time working as a Maths-for-Life Lead Teacher.

Our thanks go to EEF and J P Morgan who funded the project and the Behavioural Insights Team who evaluated it.

Additionally, we thank all the Lead Teachers and students who took part in the Maths-for-Life research project (2017-18). All these groups of participants provided valuable insights and feedback which helped us improve and refine the resources. Further to this, we also thank the Lead Teachers, teachers and students who supported the research that was carried out 2018 – 2020.

Finally, the research team extend their heartfelt thanks to their colleagues in the research support team led by Kanchana Minson that made sure that the Maths-for-Life project was delivered on time and to a high level.

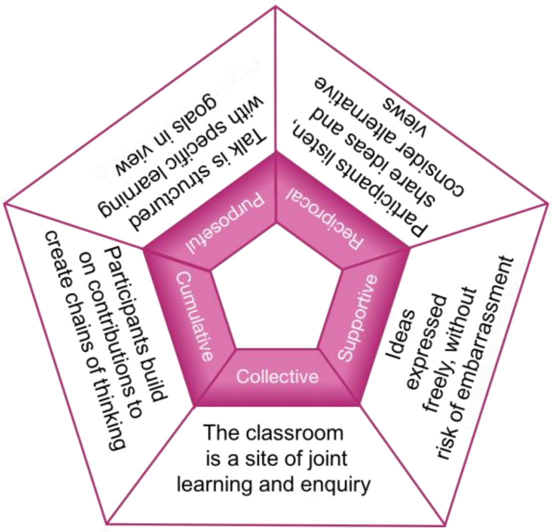
Geoff Wake, Matt Woodford, Sheila Evans, Michael Adkins and Marie Joubert

Centre for Research in Mathematics Education, University of Nottingham

Underpinning Principles

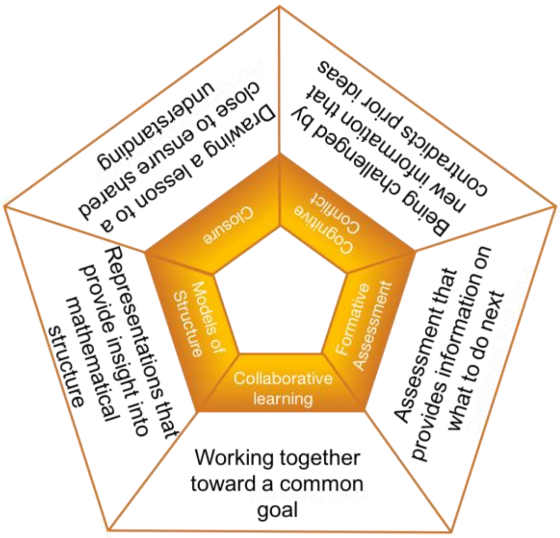
Dialogic Learning

Fundamental to these lessons is the belief that dialogic learning is essential to improving students’ confidence and outcomes. The pink pentagon in Maths-for-Life summarises the five principles of dialogic learning that we are seeking to develop in classrooms. These define the behaviours that we would expect to see develop over the year in all members of the classroom – including teachers, teaching assistants and students.



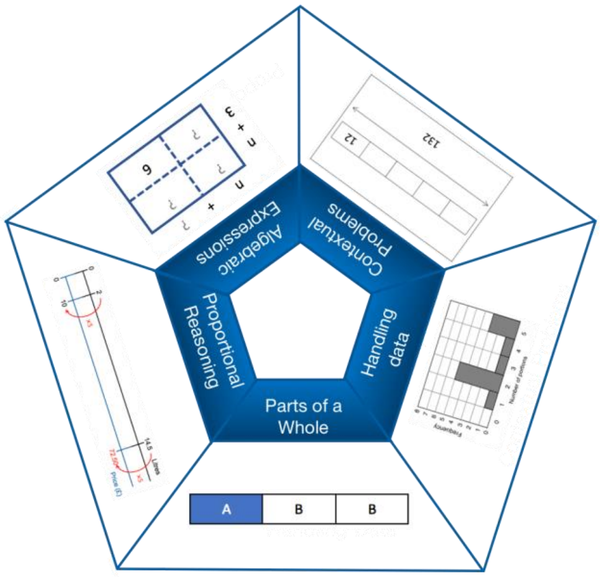
Pedagogies

Five key pedagogical ideas underpin the design of each of the five lessons. Each pedagogy will be studied in turn and the effect that it has on one of the principles of dialogic learning examined. These pedagogies are seen throughout all lessons in the actions of teachers and are supported by the design of the resources.



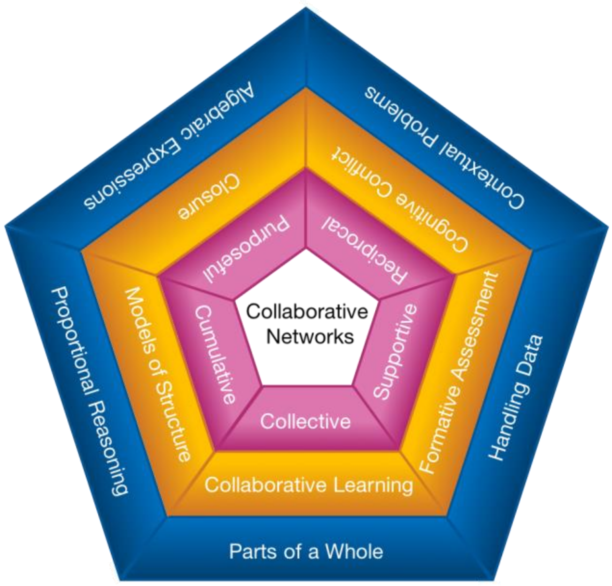
Lessons

The Maths-for-Life lessons have evolved from resources and principles used in the Improving Learning in Mathematics (Standards Unit) box. These new resources have been designed and trialled so that teachers are able to focus on exploiting anticipated learning opportunities.



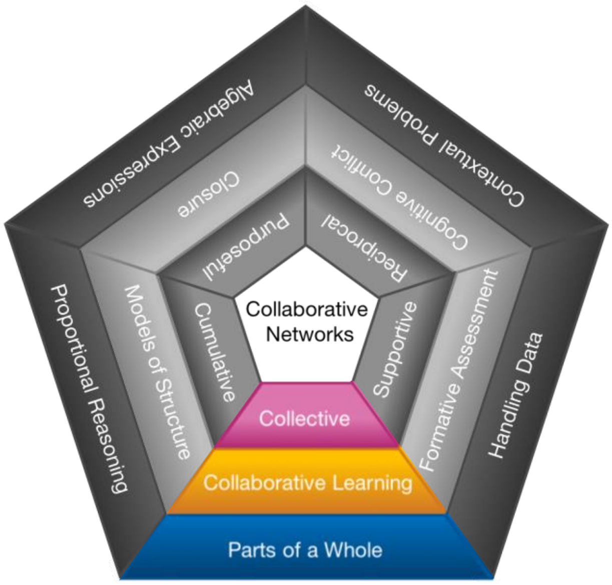
The Maths-for-Life Pentagon

Together, the three individual pentagons combine to give the Maths-for-Life Pentagon. Each section could be rotated to line up with any other section. However, the orientations have been chosen by the designers working with the project's Lead Teachers as a best fit for the focus of each research lesson.



Overview

The focus in the lesson, **Parts of a Whole**, is on how to develop **collaborative learning** and to see how this contributes towards creating a **collective** classroom.



Collaborative learning is when students work together in both pairs or small groups, and as a whole class, toward a common goal.

A **collective** classroom is one in which both the students and the teacher see their lessons as being based around joint learning and enquiry.

Aims of professional development session 1

1. Enable teachers to successfully teach the lesson, **Parts of a Whole**.
2. Explore how actions affect **collaborative learning**.
3. Explore how the design of resources affects **collaborative learning**.
4. Develop an understanding of what a **collective** ethos looks like in the classroom.

Checklist for session 1

☐	Prior to session, ask teachers to review the documents: • **L1.1 Lesson Outline** (in the resource file) • **PD1.3 Collaborative Learning** (see teacher handouts)
☐	At the *planning session*, work through **PD1 Professional Development** to understand the resources, pedagogy and aspect of dialogic learning focused on in the lesson, Parts of a Whole.
☐	At the *planning session*, ensure that everyone is clear how a research lesson operates including use of the **Collaborative Lesson Research Form**
☐	At the *planning session*, schedule the *research lesson*. Agreed date of research lesson: ☐
☐	Carry out the *research lesson*.
☐	At the *research lesson*, discuss the learning that has taken place in the lesson alongside a focus on the research question.
☐	At the *research lesson*, (if necessary) schedule the date for the next planning session (for lesson 2). Agreed date: ☐

1. Enable teachers to successfully teach the lesson, *Parts of a Whole*.

Teachers should use the summary **L1.0 Introductory Overview** (in their folders) to understand this lesson. **L1.1 Lesson Outline** (also in their folders) provides more detail.

Lead teachers are additionally supported by **PD1.0 Lesson Pedagogy**.

Note that **PD1.2(LT) Guided Discussions of Videos** may also be helpful in supporting you in your role as lead teacher, in understanding the purpose of the video clips. You may also decide to give teachers a copy of the transcripts to read through (**PD1.2(T) Transcript of selected video clips)**.

"Don't intervene too early"

Before the teachers start matching the fraction cards, show them a copy of **PD1.1 Cognitive Conflict**, and ask them to complete the match based on this misconception.

Now hand out the representation cards so that teachers can correct their matches.

Use the video clip **V1.1 Allowing cognitive conflict to occur** to show that both Teachers and Teaching Assistants risk sabotaging the task if they intervene too early.

"Use mistakes and misconceptions to encourage dialogue"

Use the recorded clip **V1.2 Supporting resolution of cognitive conflict** to show that teachers should watch out for students matching the representations to either the fractions or the ratios without ensuring consistency across the entire row. A good question to ask students would be "explain to me how that representation relates to both the ratio and the fraction".

"Students need closure in a lesson"

Use the recorded clip **V1.4 Closure** to show one possible way of drawing a class together to create a shared understanding.

Focus on the closure slides provided in the electronic presentation (**L1.4**). Note how they provide resolution of the original problem. Emphasise the importance of correct payments.

2. Explore how teacher behaviour and actions affect *collaborative learning*.

"It's about what we do and what we don't do"

Use the recorded clip **V1.5 Teacher Actions** to identify strategies that the teacher uses to encourage collaborative learning in the classroom.

Look at the handout **PD1.3 Collaborative Learning**

Ask teachers to consider ways in which they will *mainly*, *occasionally* and *never* act. Emphasise the importance of affecting student behaviour through the 'ground rules for students'.

3. **Explore how the design of resources affects *collaborative learning*.**

"We can affect the way students work together through the way we structure a task"
Hand out all the cards from **PD1.4 Original Task** in one go. By identifying changes from this version to the latest version show that the potential for ***collaborative learning*** is improved through the design of resources.

Important changes include:

i. the use of a template that both helps the teacher and makes decisions 'public'.
ii. the position of representations on the template encourages resolution.
iii. the gradual hand out of cards stops a task from becoming overwhelming.

4. **Develop an understanding of what a *collective ethos* looks like in the classroom.**

"We want to see students developing both questions and ideas together"

Use the recorded clip **V1.3 Students' dialogic talk** to illustrate dialogic learning. In particular, look for behaviour that is **collective** (manifested as enquiry and joint learning)?

Introduce the research question for this lesson:

How does collaborative learning (through the design of resources and the actions of the teacher) promote collective endeavour?

Mathematical goals

To help students:

- understand ways of mathematically describing a part-part relationship;
- understand ways of mathematically describing a part-whole relationship;
- understand how to model and solve problems.

Engaging with the lesson outline

Engage Question	Prompts
1. What assumption must be made for the money to be shared in the ratio 2:3?	Some students may want to clarify that both Ali and Blair worked at an equally hard rate during their 2 and 3 hours.
2. Why is it important to hold on to the representation cards when the students first tackle the question?	It is important that students are allowed to explore incorrect thinking if necessary. The representation cards provide a way for students to then discuss and correct their own reasoning.
3. What mistakes and difficulties do you expect?	Most students will simply match the numbers that they see on the template with the card. For example, 1:2 will be matched with ½. However, students will start to see this strategy failing and will start questioning their thinking. After the representation cards have been given out, watch out for groups that only match the diagram with either the ratio or the fraction (but not both). A good question to ask students at this point is “explain to me how the representation matches to both the fraction and the ratio”.
4. When is the optimum time for handing out the diagram cards?	This is a critical, but difficult to manage, point in the lesson. You are aiming to get cards out as students realise that their matching number strategy does not make sense because the cards they have fail to match. There may be many groups in the classroom, so don’t dwell on any one pair. Instead, as you hand the cards out, simply ask each pair to see if these representation cards help.

5. Which word cards will be particularly important to highlight in the whole group learning phase?	This will depend on the problems experienced by your class, however there are excellent opportunities for discussion around the following cards: • Cards where the name Blair comes before the name Ali. • The fraction card that requires simplifying.
6. How can the link between fractions and ratios be explained succinctly? What do you expect students to say?	A fraction expresses a part out of a whole, a ratio expresses a part to part relationship*. The whole (the denominator) can be found by adding the parts. * Note that there could be occasions when the ratio is specified as part : whole.
7. Which features of the representation cards are important to highlight to students?	Some students will be unfamiliar with the diagrammatic representation, and it may not be obvious to them why you divide by 3 (or 5). Help them to note that each section is of equal size.
8. Which of these ways are most important to stress with your class?	Possible ways of describing the same diagram include: 1. Ali and Blair share in the ratio 3:1. 2. Ali receives three times the amount Blair receives. 3. Blair receives a third of the amount Ali receives. 4. Ali receives ¾ of the total. 5. Bair receives ¼ of the total. The second approach in the list may be a useful one to highlight to students.
9. What are the key messages from the lesson that you expect to draw out?	It is vital that thinking is brought together at the end of the lesson. Some of what you share should be based on good thinking, or difficulties, that you saw in the classroom. Key points should include the easy mistake of simply matching the numbers in a ratio with the numbers in a fraction. One of the most powerful ways of helping make sense of this situation is to draw a diagram.

To support comprehension, there are pauses in some of the video clips. This means the pace of the dialogue is slower than it actually was in real-time.

V1.1 Video Clip: Allowing cognitive conflict to occur

The resources have been carefully designed to provide opportunities for misconceptions to emerge as students work on the task. Confronted by cognitive conflict, students may revise their old understandings. Allowing students time to acknowledge and then resolve a misconception independent of teacher scaffolding, ultimately results in meaningful learning. It also ensures students maintain ownership of the mathematics. The video clip highlights a misconception in students' understanding. Unfortunately, this is addressed by the teaching assistant providing a substantial 'hint' to students that there is an error in their thinking.

Introductory video captions

Three students and a teaching assistant are working together to place the fraction cards on the ratio template.

At this stage, they are discussing where to place the fraction cards on the grid.

In addition to the students there is also a teaching assistant supporting one of the students, who also contributes.

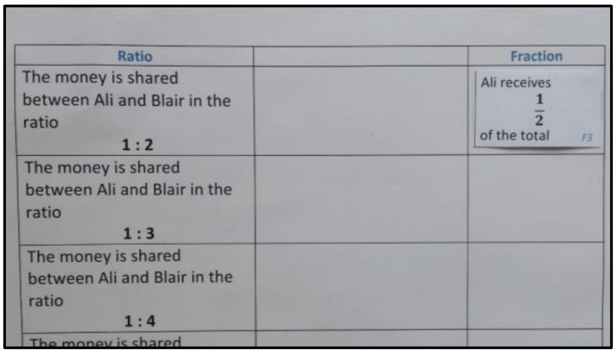
The clip begins as they discuss how to place the fraction card " ½".

Transcript of dialogue

Student 2: That goes there for sure.

Oh, this is a fraction though isn't it? I'll move it here.

Student 2 moves the card to the fraction column.



Student 1: So why do you want to be having this one?

Student 2: Because one is one over two so it's like … two…

Student 1: …halves…

Student 2: that yeah…two and a half.

Next.

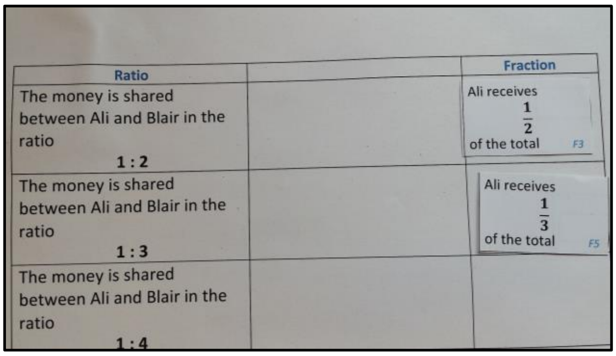
Student 1: Oh, yeah, we've done the ratio I think…the ratio to me…

Student 2: But it's the same fraction.

Assistant: If he's getting half it will be shared in the ratio one to one – don't you think?

Assistant questions the placement of the $\frac{1}{2}$ card in the 1:2 row.

Student 2 ignores the question and places the $\frac{1}{3}$ card in the 1:3 row.



Student 3: Are you sure?

Student 2: Yeah. It looks…it looks like it should though.

Post video guided discussion

Discuss with the teachers:

- The error in student thinking, and how the teacher assistant 'helps' students overcome this error.
  Teachers may suggest that the students are simply converting the figures in the ratio to fractions, with little understanding of the fraction Ali receives of the total amount. Although the teaching assistant does not explicitly correct the students mistake, her 'hint' takes away the opportunity for the students to work out the error in their thinking for themselves.
- Whether, and in what ways, the teaching assistant could scaffold student thinking. In doing so, encourage teachers to draw on their experience of teaching assistants in the classroom.
  Teachers may reflect on whether the teaching assistant should do anything at all. Providing time for students to resolve the issue for themselves can sometimes be the better strategy. Alternatively, the teaching assistant could ask questions such as 'why have you placed that there?' Clearly, it is important that other adults in the room are briefed about the importance of dialogic learning.

V1.2 Video Clip: Supporting resolution of cognitive conflict

The video clip highlights how the introduction of the third set of cards, the diagrammatic representations, can provide a stimulus for students who are stuck, or their thinking is incomplete. The clip also illustrates how students can, at least initially, focus their attention on ensuring there is a correct match between just two cards, rather than all three.

Introductory video captions

These students have had a long discussion about how to place the fraction cards onto the 'ratio template' – without resolution.

Aware of this, the teacher now provides them with the representation cards.

How do these representation cards help, and what issues remain?

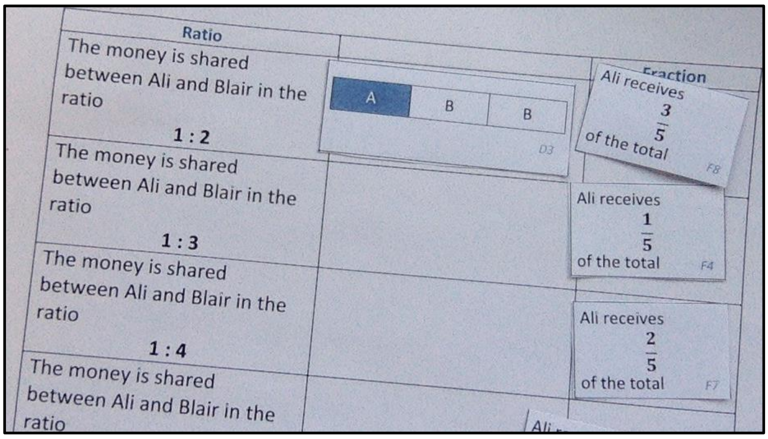
Transcript of dialogue

Teacher: Do you think you could think about where these cards go? That might help you with your fractions.

Teacher passes out the representation cards.

Student 4: Oh wait, hold on I get it, so look there's three portions, so then one is divided already, so that'll be one: two, so that'll be this one here, because one is already taken out from Ali yeah, and Blair is left with two.

Student 4 places the 1:2 representation card on the 1:2 ratio row of the template.



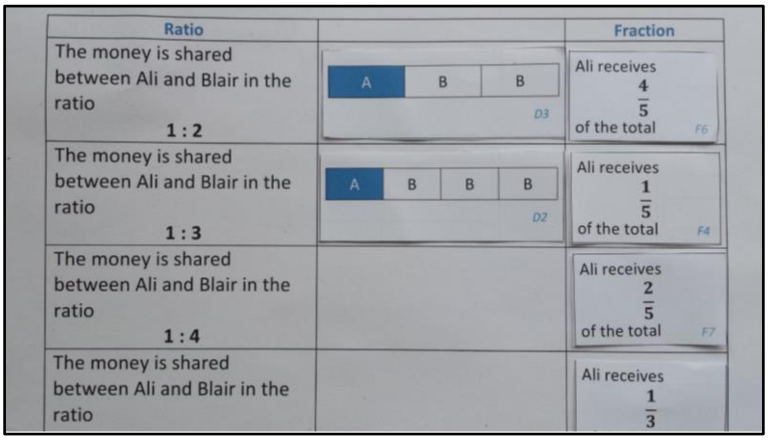
Do you get it?

Student 5: Ohhh

Student 4: Do you get it? I get it.

Student 5: So this is going to be the one for there

Student 5 places the 1:3 representation card on the 1:3 ratio row of the template.



Student 4 then realises that 1:3 does not match the fraction card that they had previously placed there.

Student 4: No, no, no, no, wait, wait, wait. One, two, three, four that doesn't make sense….If we put one fifth here…

Student 4 wants to place the representation card showing $\frac{1}{5}$ (or 1:4) as it would match with the fraction card $\frac{1}{5}$.

Student 5: But this can be wrong also, because we also have this wrong so you don't know if you can have that here.

Post video guided discussion

Discuss with teachers:

- Whether they think it is good practice to simply hand out the representation cards without any explanation.
  Teachers may recognise there was no need for an explanation, and adding one could have been too directive.
- The role of the representation cards played in resolving mistakes in student thinking.
  Teachers may reflect on how students often match the representation cards by matching it to just one of either the ratio or the fraction cards. In such a situation, students could be asked, for example 'can you explain how the representation matches everything in that row?'.

V1.3 Video Clip: Students' dialogic talk

The purpose of the video clip is to provide a concrete example of what joint collaborative enquiry looks and sounds like. Make sure the teachers are familiar with the five broad aspects of dialogic learning. Reflecting on the video may help teachers achieve similar interactions with students in their own classes.

Introductory video captions

Two students are checking how they have placed the cards on the 'ratio template'.

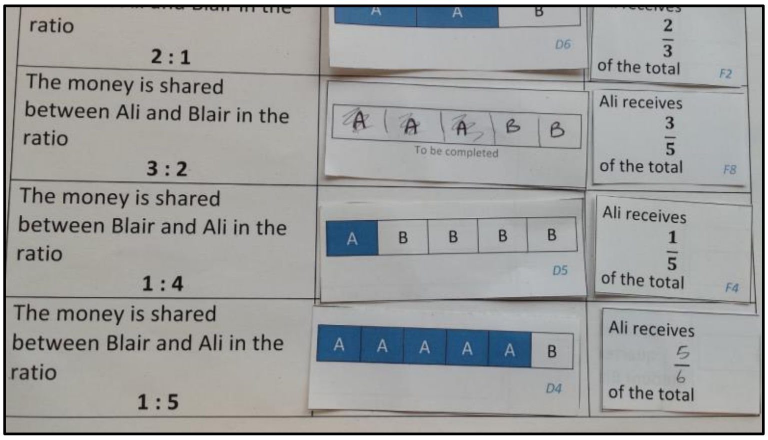
The clip begins after they review the row "the money is shared between Blair and Ali in the ratio 1:4".

At this stage we see the representation they had previously inserted.

For the moment it is Ali and Blair in the ratio 1:4

In the discussion that follows, notice the ways the students exhibit the five aspects of 'dialogic learning'

Student matches



Transcript of dialogue

Student 6: Blair gets how much? Blair gets one.

Student 7: Blair gets one, yeah, yeah, yeah.

Student 6: So this is wrong.

Student 7: Well yeah it is.

Student 6: So let's skip this…

Student 7: That, what, wait, if Blair gets one.

Student 6: I want to do something, yeah, so, let's, so these are all right so far, it means we have to draw this one. I think we have to draw this one.

Student 7: This one has to be moved innit, yeah.

Student 6: Are you sure moved, or does someone just has to redraw?

Student 7: Yeah, if this, and that'll be … Blair…that, what, wait...so this must be..

Students realise that the 1:4 representation replaces one that they had already (mistakenly) drawn.

Student 6: Ali one, yeah lol, lol.

Students then check the final row containing the ratio 1:5

So let's check this one again. Blair gets one and Ali gets five, and that's it okay, so that's fine. So let's just redraw this.

It always starts with Ali doesn't it? That's why we got confused. How much does Ali get? Four?

Student 7: Four, yeah, four

Student 6: One, two, one, four, Ali, Ali, Ali,

Then how much does Blair get?

Student 7: He gets one.

Student 6: So there we go. So that's five.

Student 7: One out of five that one yeah.

Student 6: Ali receives one out of five.

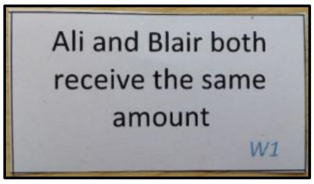
Student 7: And he was four out of five on that one…

Student 6: …and then Ali receives four out of five.

Done though.

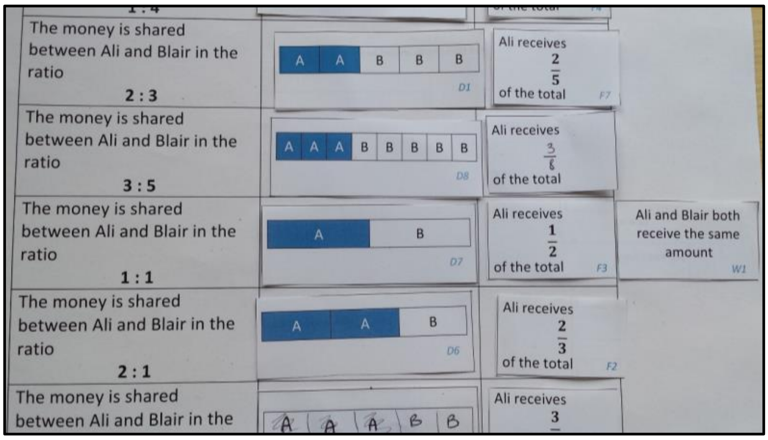
Okay

Student 6 picks up the following card



She places it on the row containing the ratio 1:1.

Same amount



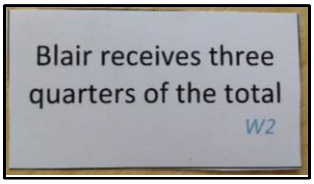
Student 7: Both receive, no, wait, …

Student 6: …the same amount

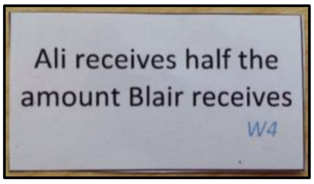
Student 7: …oh yeah the same amount

Student 6: Done

Student 6 picks the card



at the same time as student 7 picks up



Student 6: Three quarters is three out of four innit? Because a quarter's a fourth.

Student 7: I'm looking for the bit where they had a half.

Student 6: Three quarters…what do you mean?

Student 7 leaves her card (and thinking) to discuss student 7's card.

Student 7: Cause, it's like…

Student 6: I don't see it's…

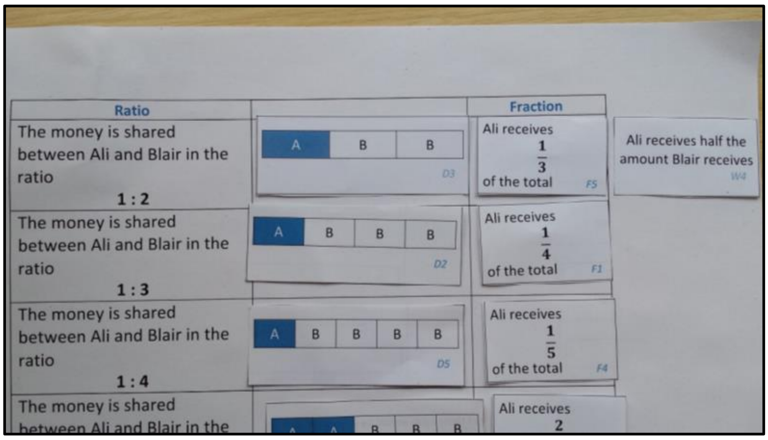
Student 7: Oh, he means that half the amount that receives.

Student 6: Oh, so it's like he got six and he only got…

Student 7: …three…

Student 6: Yeah, so that could go,…half…so two : one. He receives half.

Student 7: Oh yeah, yeah, yeah,



Student 6: I'm starting to like it you know.

Post video guided discussion

Discuss with the teachers any evidence they have seen in the students' interactions of joint learning.
Teachers may suggest there was evidence of students listening to each other's comments. On some occasions students built on each other's contributions, and at other times they queried them. There was a sense of equity in students' interactions. Progress was made not by one student dominating the conversation, but by establishing a joint, and evolving understanding of ratio.

V1.4 Video Clip: Closure

The clip highlights how whole class discussions can provide an opportunity for the teacher to check and review students' understanding, and also deepen students understanding of the topic.

Introductory video captions

Notice, in this clip, how the teacher tries to ensure the class share a common understanding. In this case, the common understanding concerns the link between the ratios, the fractions and their representations.

Transcript of dialogue
The transcript is not included here because the talk on the video is clear.

Post video guided discussion

Discuss with the teachers the purpose of the whole class discussion.
Teachers may reflect on how linking the three types of cards will help students deepen their understanding of the underlying concept. The discussion can also be an opportunity for the teacher to formatively assess students' thinking.

V1.5 Teacher Actions

The purpose of the clip is to provoke teachers into thinking about how they can deliberately, through their own actions, encourage students to collaborate

Introductory video captions

> *Notice in this clip: the things the teacher does to encourage collaborative learning -*
>
> *and the things he doesn't do.*

Transcript of dialogue
The transcript is not included here because the talk on the video is clear.

Post video guided discussion

Discuss with the teachers

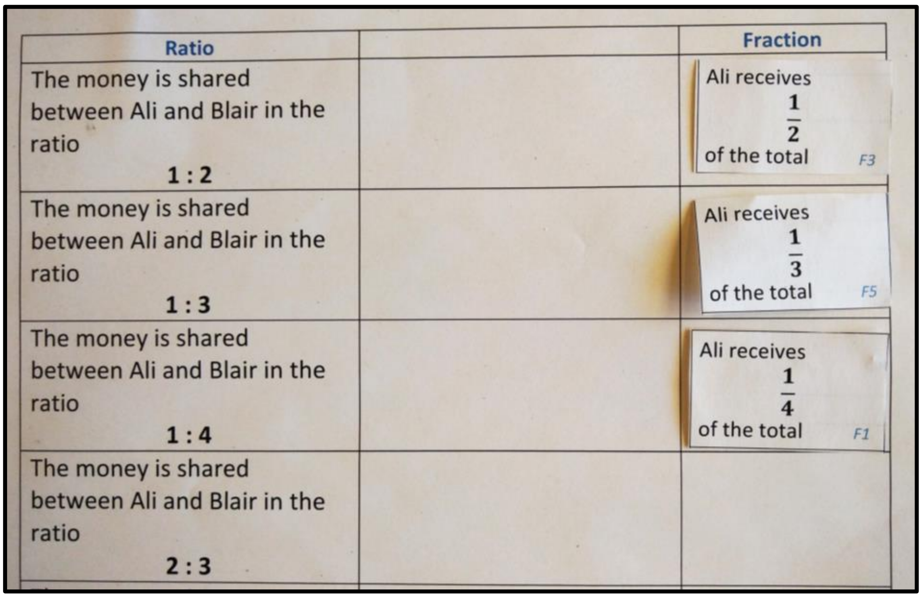
- How the teacher encouraged collaboration.
  Teachers may suggest that by the teacher explaining how talking to others can help their learning, may motivate students to discuss. By the teacher in the clip, adding the specificity to how students should behave, teachers may propose this can help those who are not used to talking in maths lessons. Teachers may also reflect on how the teacher asked students to explain the match they were happiest with. This can encourage those that lack confidence to talk out loud. The teachers may also notice that the teacher highlights a 'good moment'. Doing so may encourage students to make further progress.
- Other practices, that teachers have used in their own classes, that can encourage collaboration.

Handouts for teachers session 1

PD1.1 Cognitive Conflict

Understanding Student Thinking

The vast majority of groups observed in the trial lessons created initial matches along the lines shown in the diagram below.



- Follow this way of thinking through to see how far students can get.
- What is the optimum window for when the teacher should introduce the representation cards?

Teachers and/or Teaching Assistants should not interfere too early in an attempt to ease cognitive conflict.

PD1.2(T) Transcript of selected video clips

To support comprehension, there are pauses in some of the video clips. This means the pace of the dialogue is slower than it actually was in real-time.

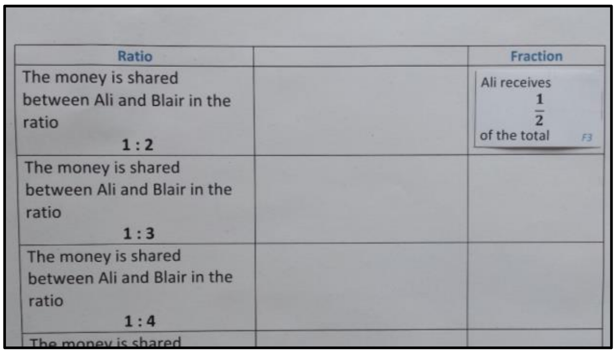
V1.1 Video Clip: Allowing cognitive conflict to occur

Transcript of dialogue

Student 2: That goes there for sure.

Oh, this is a fraction though isn't it? I'll move it here.

Student 2 moves the card to the fraction column.



Ratio		Fraction
The money is shared between Ali and Blair in the ratio 1 : 2		Ali receives $\frac{1}{2}$ of the total F3
The money is shared between Ali and Blair in the ratio 1 : 3		
The money is shared between Ali and Blair in the ratio 1 : 4		
The money is shared		

Student 1: So why do you want to be having this one?

Student 2: Because one is one over two so it's like … two…

Student 1: …halves…

Student 2: that yeah…two and a half.

Next.

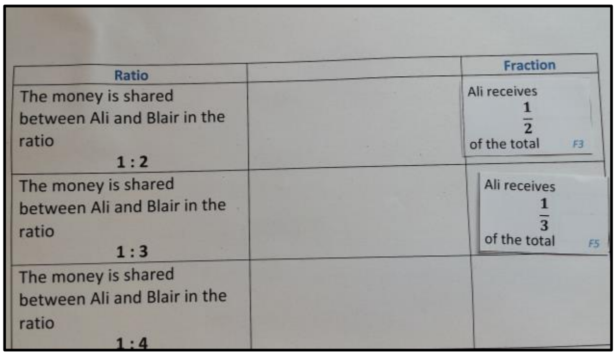
Student 1: Oh, yeah, we've done the ratio I think…the ratio to me…

Student 2: But it's the same fraction.

Assistant: If he's getting half it will be shared in the ratio one to one – don't you think?

Assistant questions the placement of the $\frac{1}{2}$ card in the 1:2 row.

Student 2 ignores the question and places the $\frac{1}{3}$ card in the 1:3 row.



Student 3: Are you sure?

Student 2: Yeah. It looks…it looks like it should though.

V1.2 Video Clip: Supporting resolution of cognitive conflict

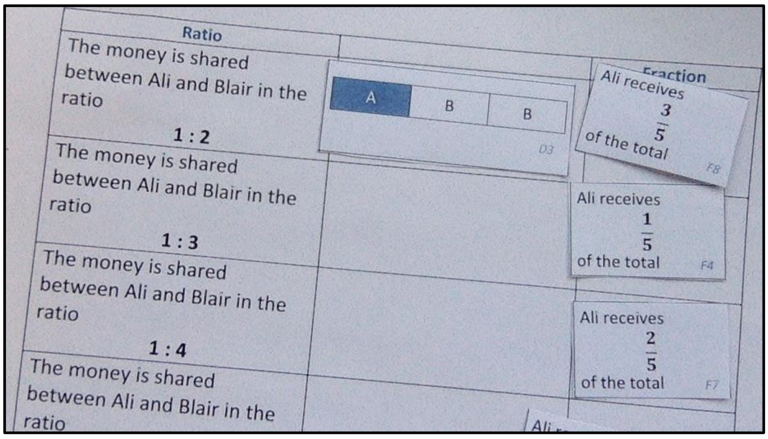
Transcript of dialogue

Teacher: Do you think you could think about where these cards go? That might help you with your fractions.

Teacher passes out the representation cards.

Student 4: Oh wait, hold on I get it, so look there's three portions, so then one is divided already, so that'll be one: two, so that'll be this one here, because one is already taken out from Ali yeah, and Blair is left with two.

Student 4 places the 1:2 representation card on the 1:2 ratio row of the template.



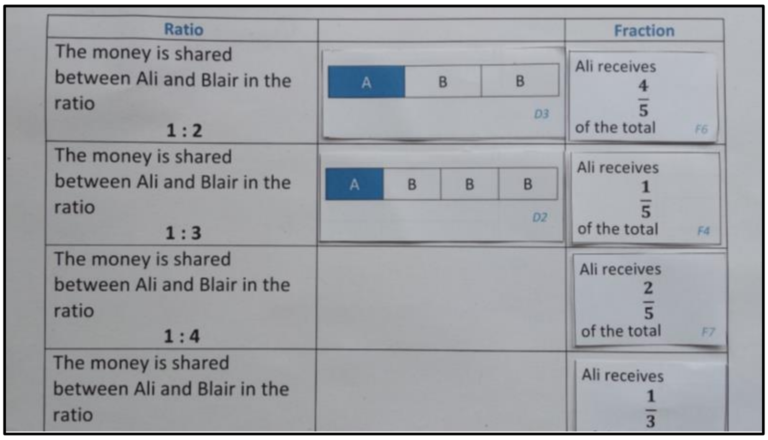
Do you get it?

Student 5: Ohhh

Student 4: Do you get it? I get it.

Student 5: So this is going to be the one for there

Student 5 places the 1:3 representation card on the 1:3 ratio row of the template.



Student 4 then realises that 1:3 does not match the fraction card that they had previously placed there.

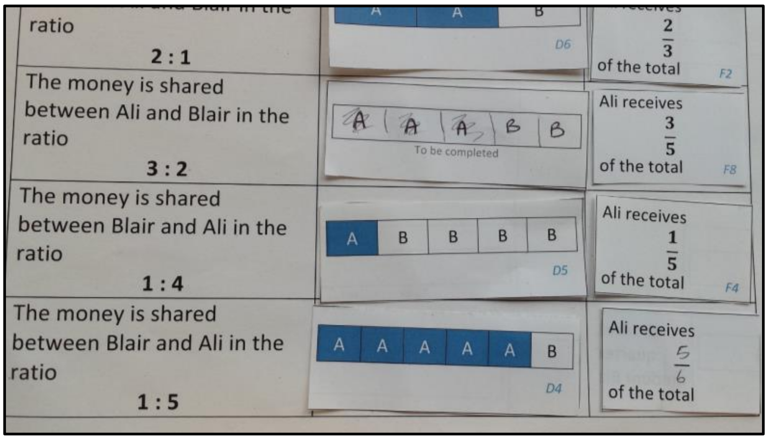
Student 4: No, no, no, no, wait, wait, wait. One, two, three, four that doesn't make sense….If we put one fifth here…

Student 4 wants to place the representation card showing $\frac{1}{5}$ (or 1:4) as it would match with the fraction card $\frac{1}{5}$.

Student 5: But this can be wrong also, because we also have this wrong so you don't know if you can have that here.

V1.3 Video Clip: Students’ dialogic talk

Student matches



Transcript of dialogue

Student 6: Blair gets how much? Blair gets one.

Student 7: Blair gets one, yeah, yeah, yeah.

Student 6: So this is wrong.

Student 7: Well yeah it is.

Student 6: So let’s skip this…

Student 7: That, what, wait, if Blair gets one.

Student 6: I want to do something, yeah, so, let’s, so these are all right so far, it means we have to draw this one. I think we have to draw this one.

Student 7: This one has to be moved innit, yeah.

Student 6: Are you sure moved, or does someone just has to redraw?

Student 7: Yeah, if this, and that’ll be … Blair…that, what, wait...so this must be..

Students realise that the 1:4 representation replaces one that they had already (mistakenly) drawn.

Student 6: Ali one, yeah lol, lol.

Students then check the final row containing the ratio 1:5

So let’s check this one again. Blair gets one and Ali gets five, and that’s it okay, so that’s fine. So let’s just redraw this.

It always starts with Ali doesn’t it? That’s why we got confused. How much does Ali get? Four?

Student 7: Four, yeah, four

Student 6: One, two, one, four, Ali, Ali, Ali,

Then how much does Blair get?

Student 7: He gets one.

Student 6: So there we go. So that's five.

Student 7: One out of five that one yeah.

Student 6: Ali receives one out of five.

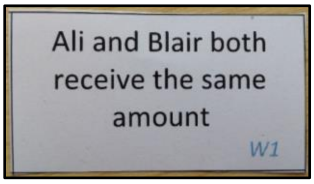
Student 7: And he was four out of five on that one…

Student 6: …and then Ali receives four out of five.

Done though.

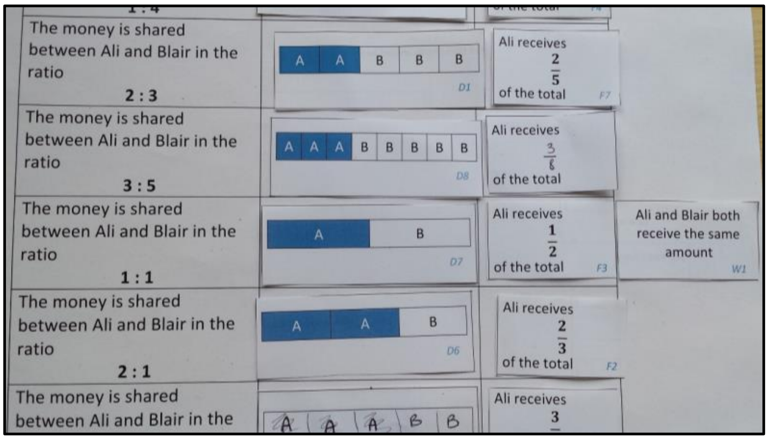
Okay

Student 6 picks up the following card



She places it on the row containing the ratio 1:1.

Same amount



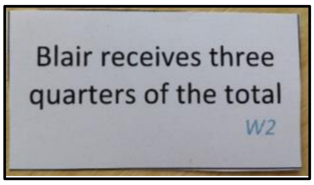
Student 7: Both receive, no, wait, …

Student 6: …the same amount

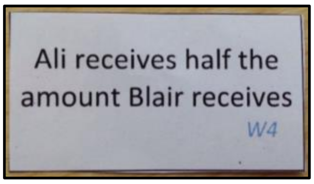
Student 7: …oh yeah the same amount

Student 6: Done

Student 6 picks the card



at the same time as student 7 picks up



Student 6: Three quarters is three out of four innit? Because a quarter's a fourth.

Student 7: I'm looking for the bit where they had a half.

Student 6: Three quarters…what do you mean?

Student 7 leaves her card (and thinking) to discuss student 7's card.

Student 7: Cause, it's like…

Student 6: I don't see it's…

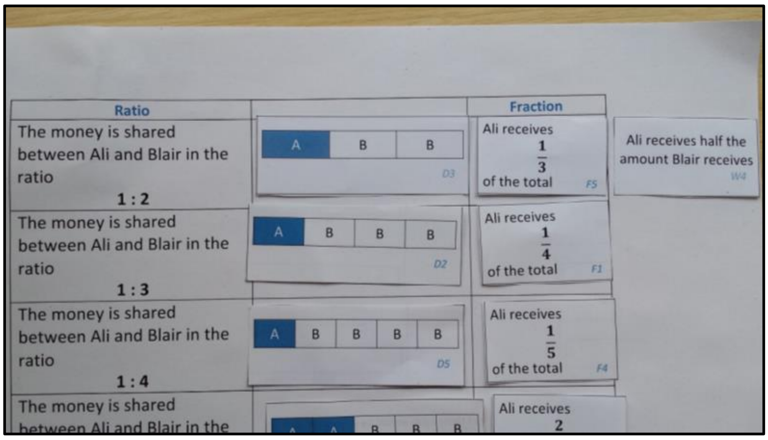
Student 7: Oh, he means that half the amount that receives.

Student 6: Oh, so it's like he got six and he only got…

Student 7: …three…

Student 6: Yeah, so that could go,…half…so two : one. He receives half.

Student 7: Oh yeah, yeah, yeah,



Student 6: I'm starting to like it you know.

PD1.3 Collaborative Learning

Collaborative learning requires us to develop certain ways of working in the classroom that are often different when compared to students' previous experiences. They will probably have experienced maths lessons in which the teacher knows the right answer and views of other students are not shared. In Maths-for-Life lessons it is recognised that students often have different ways of seeing and working mathematically –ways that often lead to the right answer. Our intention is that we use a dialogic approach to collaborate and build on each other's knowledge and understanding so that we all master the topic.

The teacher's role in collaborative learning is to…

A. **Mainly** be a 'facilitator' who:

1. Directs the flow of the discussion to allow everyone to participate.	"Listen to what Jane is saying."
2. Does not interrupt or allow others to interrupt the speaker.	"Thanks Harpreet, but keep listening to Hannah a little longer"
3. Values everyone's opinion and does not push his or her point of view.	"How do you react to that Tom?"
4. Helps learners to clarify their own ideas in their own words.	"So are you saying…

B. **Occasionally** be a 'challenger' who:

1. Introduces a new idea when the discussion is flagging.	"What would happen if…?"
2. Follows up a point of view.	"Can we just pick up on what Ali said…"
3. Plays devil's advocate.	"Surely x multiplied by x is 2x?"
4. Focuses on an important concept.	"Can you explain how you factorise an expression – rather than multiplying out the answer."
5. Asks provocative questions, but not 'leading', or 'closed' questions.	"When is the rate method a better way to tackle questions than the unitary method?"

C. **Never** be an 'evaluator' who:

1. Assesses every response with a 'yes', 'good' or 'interesting', etc.	"That's not quite what I had in mind." "Yes, that's right."
2. Sums up prematurely.	"No, you should have said…"

The ground rules for students during collaborative learning should be…

1. Give everyone in your group a chance to speak	"Let's take it in turns to say what we think." "Claire, you haven't said anything yet."
2. Listen to what people say	"Don't interrupt - let Sam finish." "I think Sam means that ...."
3. Check that everyone else listens	"What did Sue just say?" "I just made a deliberate mistake - did you spot it?"
4. Try to understand what is said	"I don't understand. Can you repeat that?" "Can you *show* me what you mean?"
5. Build on what others have said	"I agree with that because ..." "Yes and I also think that ...."
6. Demand good explanations	"Why do you say that?" "Go on ... convince me."
7. Challenge what is said	"That cannot be right, because..." "This explanation isn't good enough yet."
8. Treat opinions with respect	"That is an interesting point." "We all make mistakes!"
9. Share responsibility	"Let's make sure that we are all able to report this back to the whole class."
10. Reach agreement	"We've got the general idea, but we need to agree on how we will present it."

PD1.4 Original Task

These cards are from the first design of the lesson, Parts of a Whole.

They are included in this pack to illustrate how dialogic learning can be affected by the design of resources.

Ali receives $\frac{1}{2}$ of the total *F1*	Ali receives $\frac{8}{10}$ of the total *F6*
Ali receives $\frac{1}{3}$ of the total *F2*	Ali receives $\frac{3}{5}$ of the total *F7*
Ali receives $\frac{1}{4}$ of the total *F3*	Ali receives $\frac{2}{5}$ of the total *F8*
Ali receives $\frac{2}{3}$ of the total *F4*	Ali receives $\frac{3}{8}$ of the total *F9*
Ali receives $\frac{1}{5}$ of the total *F5*	Ali receives $\frac{5}{6}$ of the total *F10*

R1 The money is shared between Ali and Blair in the ratio **1:3**	*R6* The money is shared between Ali and Blair in the ratio **4:1**
R2 The money is shared between Ali and Blair in the ratio **1:1**	*R7* The money is shared between Ali and Blair in the ratio **3:2**
R3 The money is shared between Ali and Blair in the ratio **2:1**	*R8* The money is shared between Ali and Blair in the ratio **3:5**
R4 The money is shared between Ali and Blair in the ratio **1:2**	*R9* The money is shared between Ali and Blair in the ratio **2:3**
R5 The money is shared between Blair and Ali in the ratio **4:1**	*R10* The money is shared between Blair and Ali in the ratio **1:5**

P1 A | B	*P6* A | A | A | B | B
P2 A | A | B	*P7*
P3 A | B | B	*P8*
P4 A | B | B | B	*P9* A | A | A | B | B | B | B | B
P5 A | B | B | B | B	*P10* A | A | A | A | A | B

W1 Ali and Blair both receive the same amount	*W6* Ali receives one quarter of the total
W2 Blair receives double the amount that Ali receives	*W7* Ali receives half the amount Blair receives
W3 Ali receives double the amount that Blair receives	*W8* Blair receives four fifths of the total
W4 Blair receives three times the amount Ali receives	*W9* Ali receives 4 times the amount that Blair receives
W5 Ali receives a quarter of the amount Blair receives	*W10* Ali receives five times the amount that Blair receives

Ali receives $\frac{1}{4}$ of the total *F1*	A A B B B *D1*	Ali and Blair both receive the same amount *W1*
Ali receives $\frac{2}{3}$ of the total *F2*	A B B B *D2*	Blair receives three quarters of the total *W2*
Ali receives $\frac{1}{2}$ of the total *F3*	A B B *D3*	Blair receives double the amount that Ali receives *W3*
Ali receives $\frac{1}{5}$ of the total *F4*	A A A A A B *D4*	Ali receives half the amount Blair receives *W4*
Ali receives $\frac{1}{3}$ of the total *F5*	A B B B B *D5*	Ali receives double the amount that Blair receives *W5*
Ali receives $\frac{4}{5}$ of the total *F6*	A A B *D6*	Blair receives three times the amount Ali receives *W6*
Ali receives $\frac{2}{5}$ of the total *F7*	A B *D7*	Ali receives one quarter of the amount Blair receives *W7*
Ali receives $\frac{3}{5}$ of the total *F8*	A A A B B B B B *D8*	Ali receives $\frac{4}{10}$ of the total *W8*

One set of these cards should be printed for each group

Ali receives — of the total	To be completed
Ali receives — of the total	To be completed

One set of these cards should be printed for each group

Ali receives — of the total	To be completed
Ali receives — of the total	To be completed

One set of these cards should be printed for each group

Ali receives — of the total	To be completed
Ali receives — of the total	To be completed

PD1.5 Lesson 1 Research Form

Observe a pair/group of students working during the lesson.

Note down the key moments and development in their mathematical thinking.

Note down examples of the Maths-for-Life pedagogies seen during this lesson.

Collaborative Learning

Working together toward a common goal

Models of Structure

Representations that provide insight into mathematical structure

Closure

Drawing a lesson to a close to ensure shared understanding

Cognitive Conflict

Being challenged by new information that contradicts prior ideas

Formative Assessment

Assessment that provides information on what to do next

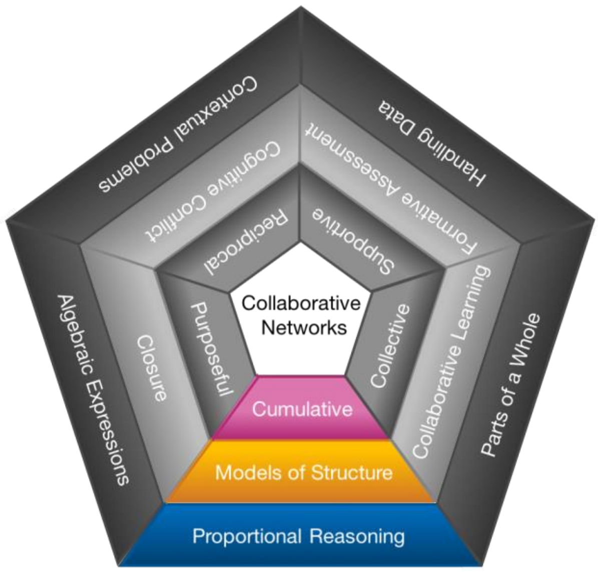
Research Question

How does collaborative learning (through the design of resources and the actions of the teacher) promote collective endeavour?

PD Session 2: Proportional Reasoning

Overview

The focus in the lesson, **Proportional Reasoning**, is on how **models of structure** can support **cumulative** dialogue in the classroom.



Models of Structure are representations that provide insight into mathematical structure.

Cumulative dialogue is seen when both students and teachers build on each other's contributions to create chains of thinking.

Aims of Professional Development Session 2

1. Understand how a **model of structure** provides insight into the mathematics.
2. Enable teachers to successfully teach the lesson, **Proportional Reasoning**.
3. Explore how a **model of structure** can create a shared understanding of mathematical structure.
4. Develop an understanding of what **cumulative** dialogue looks like in the classroom.

Checklist for Session 2

☐	Prior to the meeting, ask teachers to look at the documents • **L2.1 Lesson Outline** (in the resource file) • **PD2.3 Models of Structure** (see teacher handouts)
☐	At the *planning session*, work through **PD2 Professional Development** file to understand the resources, pedagogy and aspect of dialogic learning focused on in the lesson, Proportional Reasoning.
☐	At the *planning session*, ensure that everyone is clear how a research lesson operates including use of the **Collaborative Lesson Research Form**.
☐	At the *planning session*, schedule the *research lesson*. Agreed date of research lesson: ☐
☐	Carry out the *research lesson*.
☐	At the *research lesson*, discuss the learning that has taken place in the lesson alongside a focus on the research question.
☐	At the research lesson, (if necessary) schedule the date for the next planning session (for lesson 3). Agreed date: ☐

1. **Understand how a model of structure provides insight into the mathematics.**

"The double number can show different ways of mathematical thinking"
Ask teachers to attempt the rope problem in **PD2.0 Proportional Problems**.

Capture teachers' methods on your own hand-drawn double number lines, highlight key features and ensure the four main ways of thinking are identified.

Note that **PD2.1 Guided Discussion of Videos** will support your understanding of the purpose of the video clips. You may also decide to give teachers a copy of the transcripts to read through (**PD2.1 Guided Discussion of Videos_Teachers)**.

Use video clip **V2.1 Different Methods** to illustrate the importance of building on current student thinking rather than trying to replace it.

Refer to the handout **PD2.3 Models of Structure** for a summary of the key concept.

2. **Equip teachers to successfully teach the lesson, Proportional Reasoning.**

Teachers should use the summary **L2.1 Lesson Outline** to understand this lesson.

Lead teachers are additionally supported by the summary in the centre of this page and **PD2.2 Lesson Pedagogy**.

"Encourage discussion – rather than try to fix"
After teachers have looked at the paint problem ask them to consider their actions whilst their students are attempting the task. Use video clip **V2.2 Encouraging Discussion** to highlight how their role is to encourage students to talk to one another, not to tell them what to do.

"Use the double number line to illustrate student thinking"
After students have tackled the paint problem (see **L2.1**), teachers will be expected to capture student methods on a double number line in the lesson.
Using **PD2.4 Paint Problem**, ask teachers to illustrate each of these ways of thinking on a separate double number line.

Phase	Approximate Timings (minutes)	Notes
 1. Setting the scene	10	The initial problem is explained. *Ensure students have enough time to consider the strategy that could be used to find the cost of using 4.54 litres.*
	10	Review methods. *Review strategies used by the class and then discuss the methods shown in the PowerPoint presentation.*
 2. Cards	20 - 30	Students work with the problem cards. *Support group work – this may include asking students to explain their thinking on a double number line.* *An extension question is available in the PowerPoint presentation.*
 3. Review	15 - 20	Check understanding using the review slides in the PowerPoint presentation. *Encourage students to share alternative ways of thinking and illustrate them on double number lines.*
 4. Closure	10 - 15	Students work on the final problem. *Students work out which decorator charges the least.* *Ensure clarity of what has been covered in this lesson is shared.*
 5. Extension		Extension questions used if appropriate.

Use video clip **V2.3 Compare and Contrast** to show how the teacher uses the pre-prepared PowerPoint solutions to ensure understanding.

"Predict what students might do and how to make the most of their thinking"
Work through all the tasks in the lesson (all in **L2.1**), noting mistakes that may get made and methods that students may use. Remind teachers of the importance of listening to students and using their ideas to encourage dialogue.

3. **Explore how a model of structure can create a shared understanding of mathematical structure.**

"The double number line gives students something to structure their conversations around"
Use the video clip **V2.4 Using the Double Number Line** to illustrate that when students share a representation they are in a better position to follow one another's thinking, to share ideas and to work together.

4. **Develop an understanding of what cumulative dialogue looks like in the classroom.**

"The aim is for students to build on one another's ideas"

It takes time to build a classroom culture that displays cumulative dialogue. Such a classroom would have students creating chains of reasoning and building on one another's ideas. Use video clip **V2.5 Developing Cumulative Dialogue** to show how one teacher seeks to encourage such behaviour in his classroom.

Introduce the research question for this lesson:

How do models of structure help to facilitate cumulative dialogue and insight into mathematical structure?

V2.1 Video Clip: Different Methods

Students will have been taught a range approaches to solving proportional reasoning problems. Several students may have also created their own methods. Some of these approaches may be inefficient, novel, or incorrect, others may be more powerful and correct. To resolve these differences teachers may assert their own way of solving a problem. This will guarantee students are using a correct approach. Teachers may also argue that ensuring all students in the class are using the same method can make teaching more manageable. However, there are clear disadvantages to this pedagogical strategy. For example, it inhibits student ownership of the mathematics, and can limit discussions - there will be few opportunities to compare and contrast methods. The purpose of video clip is to provoke a discussion with teachers of these issues.

Introductory video captions

As teachers, we often want to tell students the method we prefer.
Here one GCSE student shares her experience of this.

Transcript of dialogue

The transcript is not included here because the talk on the video is clear.

Post video guided discussion

Discuss with the teacher, the advantages and disadvantages of students' using their own method. Teachers may want to consider this from both the teacher's and students' perspectives.

The teachers may suggest the advantages of students using their own methods are:

- *Students have ownership of the mathematics.*
  A method students understand, that works, however inefficient, is better than one that they do not fully understand. If students are simply following a procedure in a rote-like manner, then they are likely to find it difficult to adapt this procedure to a range of problems
- *Having a range of methods in one class allows students and teacher to compare and contrast methods. This can help students deepen their understanding of the underlying structure of a problem situation. It can also provide an opportunity for students to revise their own approach to these problems.*

Teacher may reflect on the disadvantages of students using their own method:

- *Teachers may find it challenging to understand and work with several different methods. Some methods may be new to the teacher. It can be difficult in-the-moment of the lesson, to grasp new approaches, and then know how best to support students' learning.*
- *Some methods may not be correct.*

You may also want to reflect with the teachers on how dialogic learning provides opportunities to overcome these disadvantages.

V2.2 Video Clip: Encouraging Discussion

The purpose of the video clip is to highlight specific teaching strategies that focus on students' current understanding rather than what students should do next. The prompts and questions the teacher asks not only helps him make sense of students' thinking, but helps students secure their own understandings. The teacher prompts may even provoke students to reflect on, and review their approach.

Introductory video captions

In this video, two students have been working together to solve the Paint Prices problem.

As you watch, notice the way that the teacher makes sense of what has happened and sets up the student-to-student discussion.

Transcript of dialogue of the first part of the video

Teacher: What do you mean it's in a mixed up order then?

Student 1: It's like, it's not … erm ... how it's laid out, so …

Teacher: And … and ... err … why is that then?

Discussion prompt for the first part of the video clip

Briefly discuss with the teachers the question the teacher asks.

Teachers may notice how the teacher in the video carefully listens and tries to make sense of what the students have done, as opposed to speaking over them with a pre-planned explanation.

Transcript of dialogue of the second part of the video

Three teachers now discuss their experience of the double number line –
And how it allows students' different methods.

Student 1: Because I went from one, to another one, then I came back to the other one.

Teacher: Which one did you go to? When you say you went from one to another, which one did you go to sort of first?

Student 1: I done the 5 litres and then … erm … from the 5 litres I went to the 14 and then I came back to the 11.

Teacher: Hang on, you went, okay, so you went 5 …

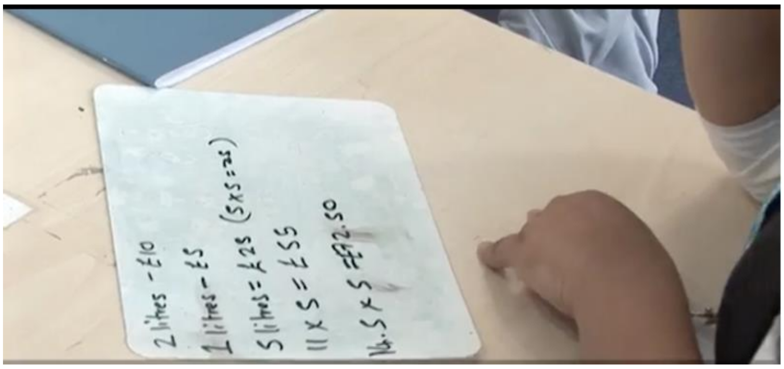
Student 1: … to the 14 and then came back to 11.

Teacher: Did you? And you did the same?

Student 1: Yeah, we was working together, so …

Teacher: How did you go from 5 to 11?

Student 1: So we worked out what 1 litre was, and then, erm, so that was £5, and then obviously from, then we timesed it by 5 because 1 litre is £5, so we did 5 times 5 which gave us 25, so it was £25.



Teacher: Okay.

Student 1: And then it was kind of obvious for 11 litres because you just have to times it by 11, 5 times 11.

Student 2: Which gives you 55.

Student 1: Yeah. And then you do the same for … ohh, then what we done is find half of 1 litre.

Student 2: Which is £2 and 5.

Student 1: 2 point 5, so it equals £2.50.

Student 2: So …

Student 1: So we found out what 14 litres was, and then you just added two point, err, two fifty on.

Teacher: So just to get that clear, you found out what half of one litre was ...

Discussion prompt for the second part of the video clip

Ask the teachers whether they have noticed any specific teaching strategies the teacher used*:*

Teachers may reflect on how the teacher echoes the students' thinking. This strategy supports both the teacher and other students understanding of the student's approach. Teachers may also notice how again the focus is on clarifying current understanding, rather than progressing the solution.

Transcript of dialogue of the third part of the video

Student 2: … yeah – which is £2.50…

Teacher: …and you added that to 14…

Student 2 … yeah, 14 times 5 which gives you 70.

Teacher: But also, look what's written there.

Student 2: Oh yeah, you wrote it wrong…

Student 1: Did I write it wrong?

Teacher: Is that the same as what you've just said to me?

Is that the same?

Student 2: No.

Teacher: You said you did half of one litre, and then you added it to fourteen, and you agreed?

Student 2: Yeah…

Teacher: … but then I notice this … 14.5 times 5 …

Student 2: Yeah.

Teacher: I wonder if, do you both agree? You seem not to agree?

Student 2: Yeah because first of all we said that we…

Teacher: Can I leave you, can I leave you to discuss whether this is also okay and why? Is that okay? Can you try and explain?

Video pause

The teacher has got to a point where he can encourage dialogue between the students rather than it being directed towards him. By highlighting the different approaches the two students have used, the teacher has created a reason for the ensuing student-student discussion.

Student 1: Because erm… 14 I think, … I don't know. I think it is right I don't think it is wrong.

Student 2: No because we did, we did 14 times 5 …. which gives 70 …

V2.3 Video Clip: Compare and Contrast

The clip shows various approaches to the Paint Prices problem represented on the double number line. The purpose of the clip is to highlight to teachers how the double number line can be used as a pedagogical tool, in a whole class discussion, to represent and also contrast methods. It clearly illustrated, for example, how some methods are more efficient than others.

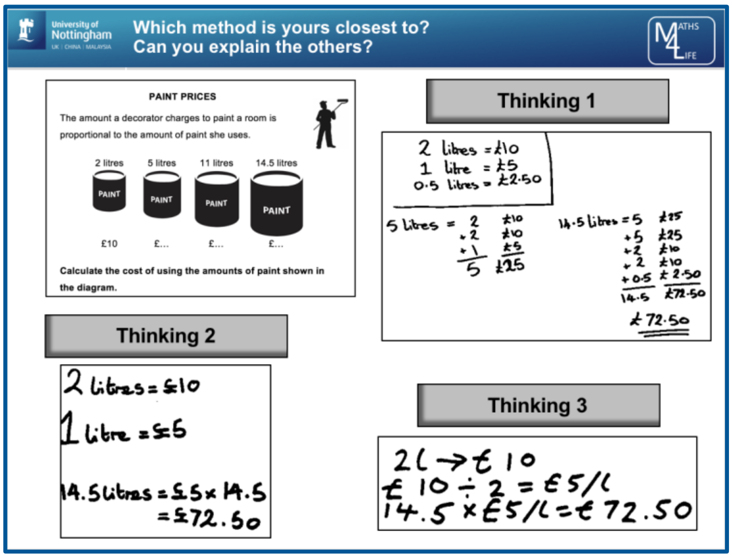
Introductory video captions

The students have tackled the Paint Prices problem and now the teacher introduces worked solutions on the board for comparison.

Whilst watching the clip notice how the teacher explores a variety of methods without telling students to adopt a particular one.

Problem tackled and worked out solutions to the problem

Note that the PowerPoint slides being worked with are from earlier trials of the lesson. Unlike the final release, the slide used in the trial focuses on the cost of 14.5 litres and shows 3 rather than 4 methods. The slide is shown below.



Transcript of dialogue

The transcript is not included here because the talk on the video is clear.

Post video guided discussion

Discuss with the teachers:

- The advantages of using an animation to represent a method on the double number line.
  Teachers may suggest the step by step nature of the animation helps students recognise the processes involved in the various methods. It may also help them recognise the comparative efficiency of methods.

- The ways the double number line can enhance students' understanding.
  Teachers may suggest that contrasting methods in this way highlights the structure of the problem situation, including the relationship between variables. It also demonstrates the efficiency of each solution. Teachers may also recognise the issue of students mixing up the multiple measures can be overcome when using the double number line as it physically separates them.

- Why the teacher chose not to recommend one method over the others.
  Teachers may reflect on the importance of students' 'owning' their method.

V2.4 Video Clip: Using the Double Number Line

Students can represent their method on the double number line. This can help them make sense of a problem, and support others' understanding of their method. The clip demonstrates students' effectively using the double number line.

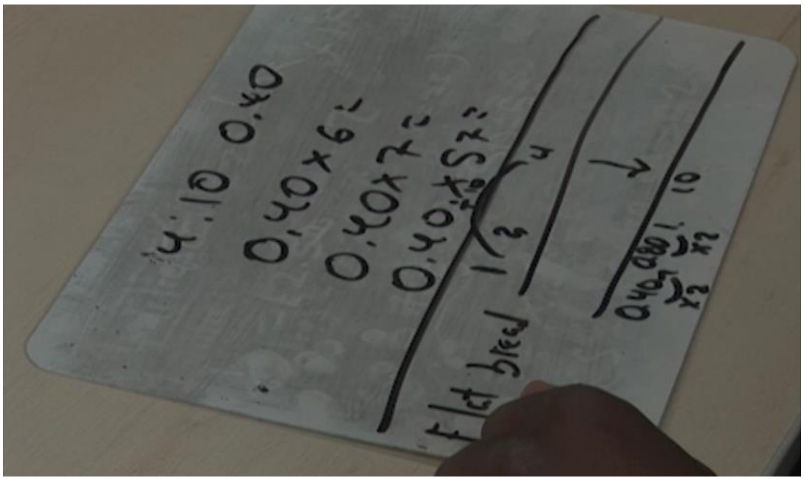
Introductory video captions

Two students are working together to find the number of spoons of flour needed for 6 flatbreads if 10 spoons are needed for 4.

Student 2 has taken the lead and used the double number line to show his thinking. However, he has used the fact that 1 spoon of flour makes 0.4 flatbreads rather than the fact that 1 flatbread requires 2.5 spoons.

Notice that the double number line provides common ground for the students to make sense of the structure of the mathematics.

Transcript of dialogue



Student 2: Oh, I'm so dumb.

Student 1: What?

Student 2: I'm so dumb.

You need to do the flatbreads divided by 4.

Student 1: 10 divided by 4?

Student 2: I'm so dumb.

That's right.

Flatbreads… flatbreads…and spoons here…..

Four, no, so do we need to find …

We need to find how many spoons of flour do we need to make …

We need to do the four divided by…

Student 1: I think we need to do 10 divided by 4.

Video pause
Notice how Student 1 has been able to follow what Student 2 has been thinking and that the double number line provides clarity on the mathematical calculation that needs to take place.

Student 2: Yeah.

That'll give you 2.5, you know what I mean?

If you do divided by 4.

If you do it … divide by 4.

We get 2.5 … which is 1.

Yeah?

Post video guided discussion
Discuss with the teachers the purpose of students using the double number line.
Some teachers may view the double number line as another method for students to use, however, this is not the case. Rather, the double number line is a tool to represent various methods. Using it can help students clarify their method, and also support other's understanding of the method.

V2.5 Video Clip: Developing Cumulative Dialogue
Students find it difficult to engage in each others' thinking, particularly if their own understanding of a problem is fragile. The purpose of the video clip is to highlight this difficulty and exemplify how students can be encouraged to engage in their peers approaches through teacher's questioning and prompts.

Introductory video captions

Two students are looking at finding the cost of using 4.54 litres of paint.
Student 2 has attempted previous questions based around the unitary method.
Student 1 on the other hand is trying to use an 'additive method'.
The teacher notices that Student 2 is trying to find the cost of using 0.4 litres (rather than 0.04 litres).

As you watch this clip, look for how the teacher explores their different ideas with the intention of reconnecting them to each other to build cumulative dialogue.

Transcript of dialogue

Teacher: Zero point zero four …

Student 1: … and if zero point one is 50p, then this probably will be …

Teacher: …oh yeah … and zero point two will be …

Student 1: 0.2 that will be £1.

Teacher: Do you agree?

Student 2: Yeah.

Teacher: Your workings that you've done before that though…

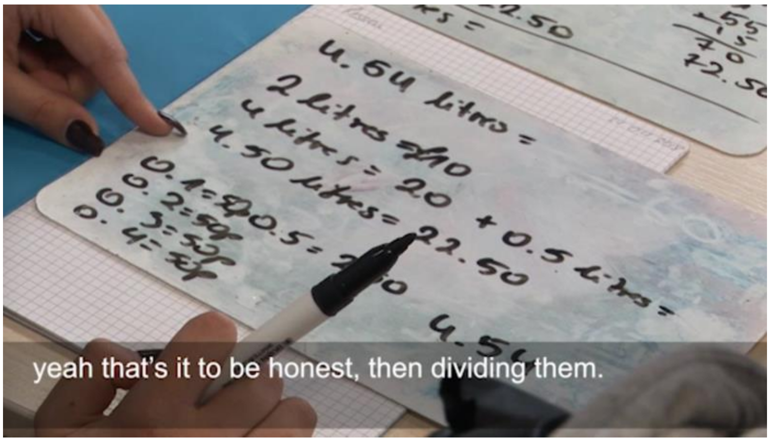
Student 1: Sorry?

Teacher: What have you found out before this question?

Before you did the 4.54 litres what did you find out?

Using, you know, the 2 litres and the 11 litres and the 14.5 litres. How did you do those questions?

Student 1: I just calculated them with … adding up all of them… and then…yeah that's it to be honest, then dividing them.



[Number covered by pen is 2.50]

Teacher: How did you do it Sahar?

Explain what you've written down here?

Have you seen this Student 1?

Video pause

The teacher has identified the opening that will create student discussion.

Student 1: Yeah.

Teacher: So 2 litres …

Student 1: … is £10 …

Teacher: … and then what happened next for you?

Student 2: Then I halved that, to check how much is one litre … and then …

Teacher: … why did you do that?

Student 1: Because then we have to find out …

Student 2: It'll make it easier for me…

Teacher: Why?

Student 1: Because it's quicker when you divide it…

Teacher: What…?

Student 1: the 2 litres…

Student 2: So even if you have to times it by any number I know that 1 litre is 5, so …

Teacher: … listen, did you hear that?

Video pause

The students are starting to display cumulative dialogue but are finding it difficult as they still don't quite understand what the other is doing.

Teacher: Okay.

Student 2: Yeah, so to, to find ten litres do five times ten equals fifty pounds, and then five times eleven equals fifty-five pounds.

Student 1: Because we add up the one litre, which we calculated before …

Teacher: Oh, Student 1, so are you doing this plus this?

Student 1: Yeah.

Teacher: How are you doing it?

Student 2: Timesing it

Teacher: How did Sahar do it?

Student 1: She timesed it

Teacher: She timesed what?

Student 1: She timesed … this … no … yeah … she timesed fifty by… no … yeah she timesed … no I don't know

Teacher: Then you need to ask her then, okay?

Can you discuss it? Because I think if you can work out how she's got from here to here and here to here then it might make your 4.54 question easier. But it's also important for you to understand how Student 1 is doing it. Because she seems to be doing some adding.

Student 2: Yeah

Teacher: Is that ok?

Try and understand how Sahar has got this number here, because we know it's correct.

Student 1: How did you do it?

Student 1: Okay, so obviously…

Transcript of post lesson discussion

The transcript is not included here because the teachers' talk is clear.

Post video guided discussion

Discuss with the teachers:

- Why they think students initially found it difficult to build on each other's mathematics. *Teachers may suggest it is hard to take on board another person's perspective of how to solve a problem, particularly when their own understanding is insecure. Students are likely to (unwittingly) respond their partner's contributions to the conversation in terms of their own approach to the problem*
- How the teacher in the video encourages cumulative dialogue, and also other, more general teacher practices, that can help or hinder cumulative dialogue.
  The teachers may suggest:
  - *It is important to encourage students to listen to each other.*
  - *As in the video, the teacher should try to ensure the discussions is purposeful.*
  - *In the video, the teacher stated the answer was correct. Students knowing this can help shift the focus of their discussion from performance to understanding.*

  Teachers may also reflect on student contributions to a conversation in which they simply repeat their partner explanation, or simply respond with 'yeah' or 'I get it'. Such responses are not necessarily a guarantee of understanding. Teachers may need to probe further to ensure learning is more than surface deep.

PD2.2 Lesson Pedagogy

Mathematical goals

To help students:

- reflect on the reasoning they currently use when solving proportion problems;
- understand the power and efficiency of a multiplicative structure through the concept of rate.

Engaging with the lesson outline

Engage Question	Prompts
1. What strategies do you expect students to use?	The most common strategies adopted by students is an additive method. For example, the cost of 5 litres is given by adding the costs of 2 litres, 2 litres and 1 litre. This strategy works when the numbers are relatively neat but is not always efficient and can lead to arithmetic mistakes – particularly for 14.5 litres and 4.54 litres. Other ways of thinking include unitary, rate and scale factor. Whilst students are working on the problem, the teacher should try not to interfere or guide them. Instead, they should note the strategies that they use so that these can be built on.
2. Why has 11 litres been chosen to demonstrate students thinking rather than 5 litres or 14.5 litres?	Finding the cost of 14.5 litres highlights the inefficiency of the additive method, but the awkward numbers make understanding the methods less accessible. Finding the cost of 5 litres is accessible but doesn't highlight the inefficiency of the additive method as well as 14.5 litres. Finding the cost of 11 litres is both accessible to students and highlights relative efficiency of the ways of thinking.
3. Is it essential that every way of thinking is gone through with students?	No – not necessarily. It is likely that some variation of additive will already have been suggested by the class. So there is little need to dwell on it. However, to be able to attempt questions such as finding the cost of 4.54 litres efficiently another way of thinking is helpful. Both the unitary and the rate ways of thinking can be of help.

	The scale factor way of thinking is less common and is still relatively inefficient for many problems. The balance must be found between overloading a class and helping them to see alternative (powerful) ways of thinking.
4. What are the features of the thinking that should be brought out to students?	The additive thinking works, but is not very efficient, and could result in calculation mistakes.
5. What are the features of the thinking that should be brought out to students?	The unitary thinking relies on students being able to find the related quantity of 1 unit. They then scale up the number of units and scale up the corresponding quantity. This is why, in this example, the 11 is written first – as in '11 lots of £5'. Once the cost for 1 litre has been found then the cost for any quantity to be found relatively easily.
6. What are the features of the thinking that should be brought out to students?	The rate thinking requires students to divide the price by the quantity to give a £/litre rate. The number of litres is then multiplied by £/litre rate. Once the rate price per litre has been found then the cost for any quantity to be found relatively easily.
7. What are the features of the thinking that should be brought out to students?	The scale factor thinking can be straightforward to see for some numbers, but more difficult for others. For example, finding the charge for using 1.5 litres is a scale factor of 0.75). The scale factor way of thinking is less efficient as it would require a new scale factor to be worked out in each new example.
8. Which of the ways of thinking are most helpful in this example?	Unitary and rate thinking are more efficient. Additive and scale factor are more difficult. However, all ways that get students to the correct answer can be celebrated.
9. Why is it important to set out expectations for how students will attempt the questions?	Some students will tend to dominate discussions in pairs. If you refer back to these expectations during the paired work you will be able to encourage quieter students to talk. The use of the double number line to illustrate thinking encourages reasoning and facilitates extension to other methods.
10. How do you expect students to tackle these questions?	To find the number of spoons of flour for 4 flatbreads, it is quite reasonable to use the additive method. To find the number of spoons of flour for 7 flatbreads students using the additive method will have to find the number of spoons for 1 flatbread. This then lends itself to students being able to use the unitary or rate ways of thinking.

11. Why have the numbers 6, 7, 57 and 117 been chosen?	The amount of flour for 6 flatbreads is relatively straight forward to find using a variety of methods. However, once students have to find the flour for 7 flatbreads they have to find the flour for 1 flatbread. This opens up a variety of options to students for finding flour for 57 flatbreads. There will be a mix of approaches for finding the amount of yoghurt for 117 flatbreads. As the number isn't neatly related to 4 flatbreads the most efficient will be the unitary or rate ways of thinking. By gentle discussion with pairs help them to see the power of the rate thinking (vertical relationship) on a double number line.
12. How do you expect students to tackle these questions?	These questions are difficult to approach using the additive method. For converting pounds to dollars, both the unitary method and the rate method are efficient. For converting dollars to pounds the rate method may be easier for students to understand.
13. What do you anticipate will cause difficulties?	Students may find using their understanding within the context of a wordy question difficult, but they should be encouraged to persevere.
14. What are the key messages that you expect to draw out?	It is vital that thinking is brought together at the end of the lesson. Some of what is shared should be based on good thinking, or difficulties, seen in the classroom. Try to encourage students to use the most appropriate way of thinking but note that some ways are more powerful and efficient.
15. What is the purpose of the double number line?	Many students will want to know 'a method' and will want to know whether you are telling them to use a double number line. The double number line is not necessarily a method – but a way to illustrate thinking.
16. How does the double number line help students to understand this question?	Use the double number line to show the two rates of £0.50/\$ (multiply by 0.5) and \$2/£ (multiply by 2). These two actions can be reversed by dividing by 0.5 and dividing by 2 respectively.

Handouts for teachers session 2

Rope Problem

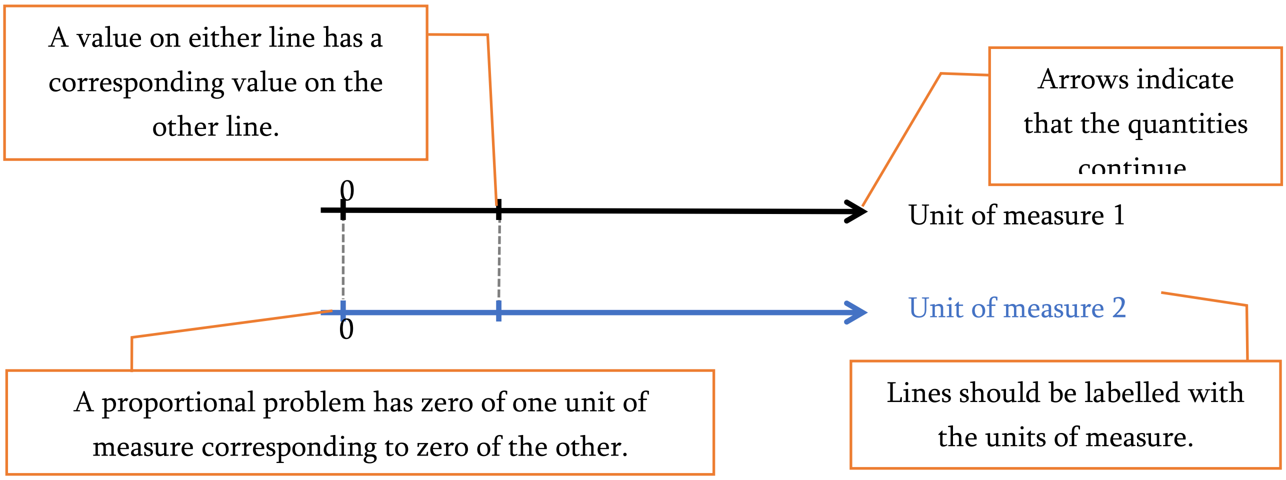
The mass of rope is in proportion to the length of rope.

If a 4 metre length of rope has a mass of 18kg, find the mass of 10 metres of the same rope.

How would you solve this problem?
Write down some alternative methods that you might expect students to use.

The double number line

As teachers share their methods for the rope problem capture their thinking on a double number line and note the general features of a double number line shown below.

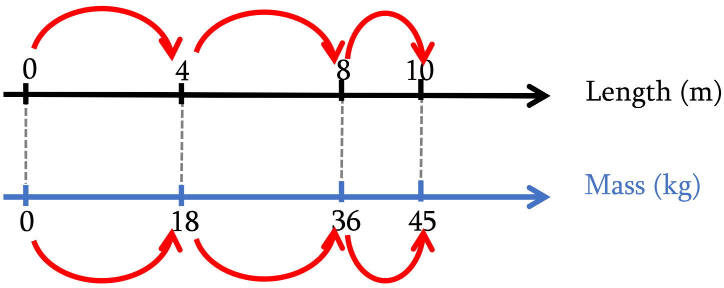


There are four distinct (though linked) ways to solve proportional problems. To give clarity, each should be illustrated on a separate double number line. Note that it is better to draw the double number lines yourself rather than to simply show the images below, as it will help teachers understand how to construct them, and provides a dynamism to the activity.

The double number lines are being used to gain insight into different mathematical thinking – they are **not** being used to prescribe a method.

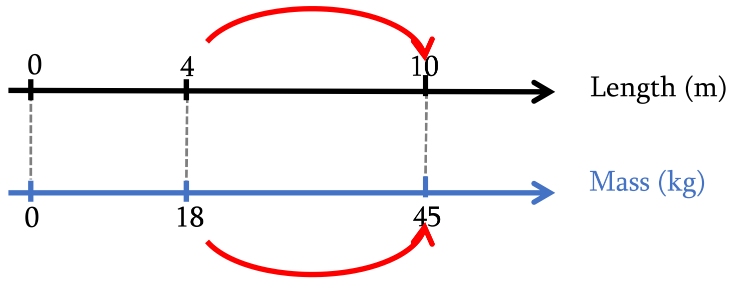
1. An additive method

Typically, this method is described using words such as “4 metres, another 4 metres and half as much again). It can be represented on the double number line as many horizontal steps.



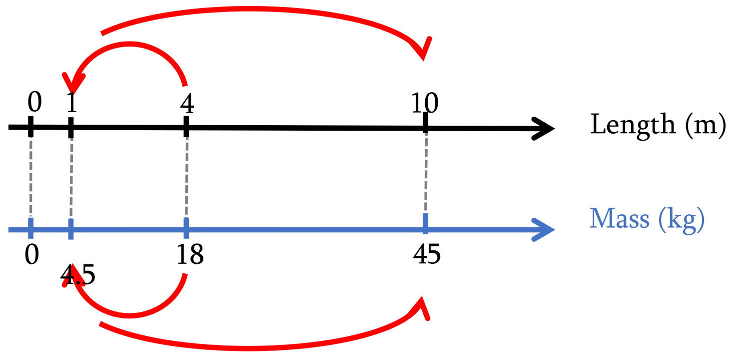
2. *The scale factor method*

Typically, this method is seen by noting that 10m of rope is 2.5 times longer than 4m of rope, so should weigh 2.5 times as much. This can be illustrated on the double number line as one horizontal step.



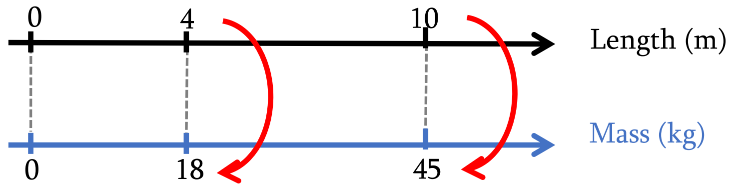
3. *The unitary method*

Typically, this method is understood as finding the mass of 1m then multiplying by 10 to get the mass of 10m. It can be represented on the double number line as two horizontal steps.



4. *The rate method*

Typically, this method is understood as finding a rate of 2.5kg/m. It can be represented on the double number line as one vertical step.



PD2.1(T) Transcript of selected video clips

V2.2 Video Clip: Encouraging Discussion

Transcript of dialogue of the first part of the video

Teacher: What do you mean it's in a mixed up order then?

Student 1: It's like, it's not … erm ... how it's laid out, so …

Teacher: And … and ... err … why is that then?

Transcript of dialogue of the second part of the video

Student 1: Because I went from one, to another one, then I came back to the other one.

Teacher: Which one did you go to? When you say you went from one to another, which one did you go to sort of first?

Student 1: I done the 5 litres and then … erm … from the 5 litres I went to the 14 and then I came back to the 11.

Teacher: Hang on, you went, okay, so you went 5 …

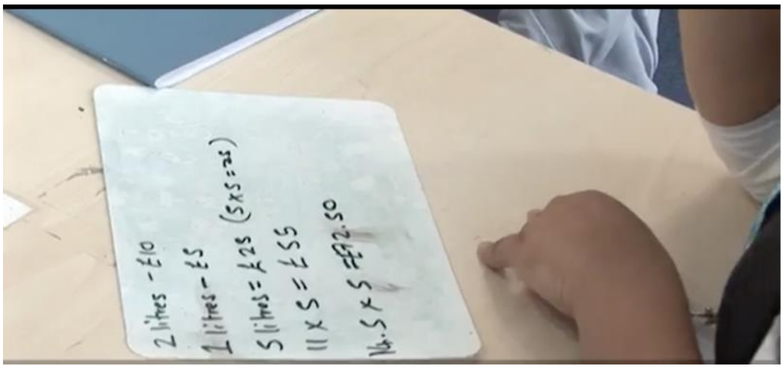
Student 1: … to the 14 and then came back to 11.

Teacher: Did you? And you did the same?

Student 1: Yeah, we was working together, so …

Teacher: How did you go from 5 to 11?

Student 1: So we worked out what 1 litre was, and then, erm, so that was £5,
and then obviously from, then we timesed it by 5
because 1 litre is £5, so we did 5 times 5 which gave us 25, so it was £25.



Teacher: Okay.

Student 1: And then it was kind of obvious for 11 litres because you just have to times it by 11, 5 times 11.

Student 2: Which gives you 55.

Student 1: Yeah. And then you do the same for … ohh, then what we done is find half of 1 litre.

Student 2: Which is £2 and 5.

Student 1: 2 point 5, so it equals £2.50.

Student 2: So …

Student 1: So we found out what 14 litres was, and then you just added two point, err, two fifty on.

Teacher: So just to get that clear, you found out what half of one litre was ...

Transcript of dialogue of the third part of the video

Student 2: … yeah – which is £2.50…

Teacher: …and you added that to 14…

Student 2 … yeah, 14 times 5 which gives you 70.

Teacher: But also, look what's written there.

Student 2: Oh yeah, you wrote it wrong…

Student 1: Did I write it wrong?

Teacher: Is that the same as what you've just said to me?

Is that the same?

Student 2: No.

Teacher: You said you did half of one litre, and then you added it to fourteen, and you agreed?

Student 2: Yeah…

Teacher: … but then I notice this … 14.5 times 5 …

Student 2: Yeah.

Teacher: I wonder if, do you both agree? You seem not to agree?

Student 2: Yeah because first of all we said that we…

Teacher: Can I leave you, can I leave you to discuss whether this is also okay and why? Is that okay? Can you try and explain?

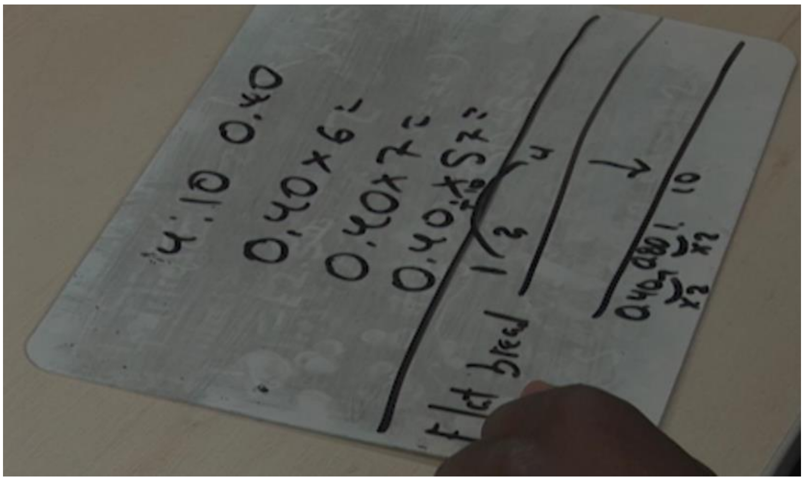
Video pause

Student 1: Because erm… 14 I think, … I don't know. I think it is right I don't think it is wrong.

Student 2: No because we did, we did 14 times 5 …. which gives 70 …

V2.4 Using the Double Number Line

Transcript of dialogue



Student 2: Oh, I'm so dumb.

Student 1: What?

Student 2: I'm so dumb.

You need to do the flatbreads divided by 4.

Student 1: 10 divided by 4?

Student 2: I'm so dumb.

That's right.

Flatbreads… flatbreads…and spoons here…..

Four, no, so do we need to find …

We need to find how many spoons of flour do we need to make …

We need to do the four divided by…

Student 1: I think we need to do 10 divided by 4.

Video pause

Student 2: Yeah.

That'll give you 2.5, you know what I mean?

If you do divided by 4.

If you do it … divide by 4.

We get 2.5 … which is 1.

Yeah?

V2.5 Video Clip: Developing Cumulative Dialogue

Transcript of dialogue

Teacher: Zero point zero four …

Student 1: … and if zero point one is 50p, then this probably will be …

Teacher: …oh yeah … and zero point two will be …

Student 1: 0.2 that will be £1.

Teacher: Do you agree?

Student 2: Yeah.

Teacher: Your workings that you've done before that though…

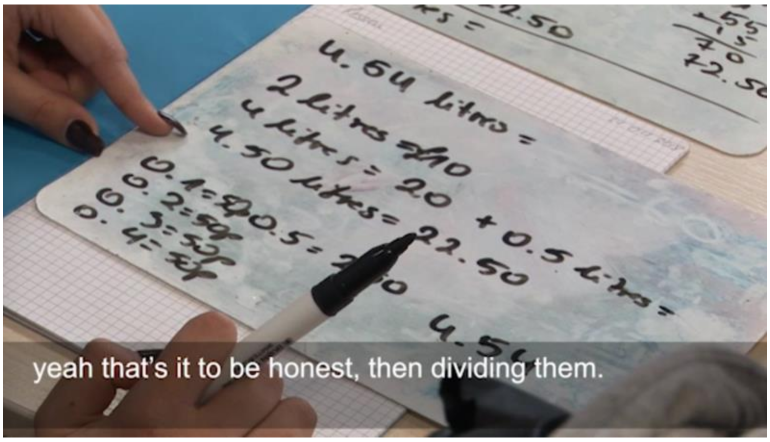
Student 1: Sorry?

Teacher: What have you found out before this question?

Before you did the 4.54 litres what did you find out?

Using, you know, the 2 litres and the 11 litres and the 14.5 litres. How did you do those questions?

Student 1: I just calculated them with … adding up all of them… and then…yeah that's it to be honest, then dividing them.



[Number covered by pen is 2.50]

Teacher: How did you do it Sahar?

Explain what you've written down here?

Have you seen this Student 1?

Video pause

Student 1: Yeah.

Teacher: So 2 litres …

Student 1: … is £10 …

Teacher: … and then what happened next for you?

Student 2: Then I halved that, to check how much is one litre … and then …

Teacher: … why did you do that?

Student 1: Because then we have to find out …

Student 2: It'll make it easier for me…

Teacher: Why?

Student 1: Because it's quicker when you divide it…

Teacher: What…?

Student 1: the 2 litres…

Student 2: So even if you have to times it by any number I know that 1 litre is 5, so …

Teacher: … listen, did you hear that?

Video pause

Teacher: Okay.

Student 2: Yeah, so to, to find ten litres do five times ten equals fifty pounds, and then five times eleven equals fifty-five pounds.

Student 1: Because we add up the one litre, which we calculated before …

Teacher: Oh, Student 1, so are you doing this plus this?

Student 1: Yeah.

Teacher: How are you doing it?

Student 2: Timesing it

Teacher: How did Sahar do it?

Student 1: She timesed it

Teacher: She timesed what?

Student 1: She timesed … this … no … yeah … she timesed fifty by… no … yeah she timesed … no I don't know

Teacher: Then you need to ask her then, okay?

Can you discuss it? Because I think if you can work out how she's got from here to here and here to here then it might make your 4.54 question easier. But it's also important for you to understand how Student 1 is doing it. Because she seems to be doing some adding.

Student 2: Yeah

Teacher: Is that ok?

Try and understand how Sahar has got this number here, because we know it's correct.

Student 1: How did you do it?

Student 1: Okay, so obviously…

PD2.3 Models of Structure

A model of mathematical structure is a representation that allows students to make sense of the mathematics taking place.

For example, to solve the proportional problem ‘calculate the charge for using 5 litres of paint if 2 litres is charged at £10’, one common representation used is a rate (or ratio) table.

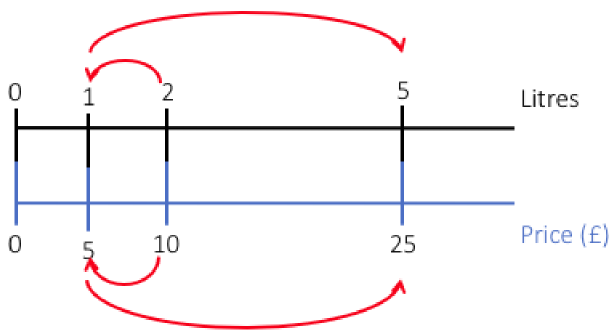
2	10
5	?

→ $10 \div 2 \times 5 = 25$

This method is not necessarily a bad method – but is not a model of structure. It does little to help make sense of the mathematics taking place in each step of the calculation. It is not obvious how this representation can be used to understand why one should divide by two (and then multiply by five). Is a student doing this seeing it as finding the cost of 1 litre? As finding the rate £5/litre? As finding a multiplier? Or simply as a procedure that they follow with no meaning?

In contrast a double number line (which would be a model of structure) can help students to make sense of the mathematics taking place.

For instance, the unitary method can be explained in the following diagram:



This leads to two important points regarding models of structure:

1. The model of structure does not need to be taught as a new procedure. Some students may like to use the representation to help them solve problems, others may prefer to retain another representation. However, all students should be able to make sense of their method using the model of structure.
2. Once students have understood their method on a model of structure they are then better able to comprehend alternative structures using the same model.

PD2.4 Paint Problem

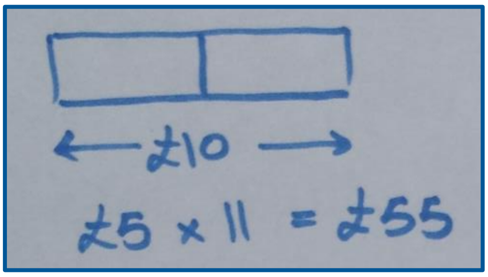
The methods shown on this page have all been witnessed during the Proportional Reasoning Maths-for-Life lesson.

They have been recreated here to illustrate the varied methods that students use to tackle the proportional problem of working out how much a decorator would charge to use 11 litres of paint if they would charge £10 to use 2 litres.

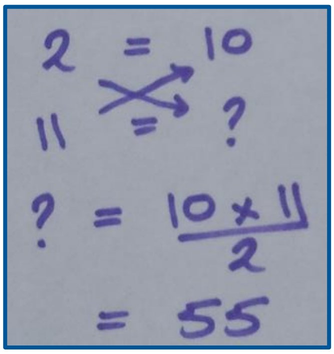
As you look at each method:

- Try to understand what the student has done.
- Represent the thinking on a double number line.
- Attempt to classify it as additive, scale factor, unitary or rate.

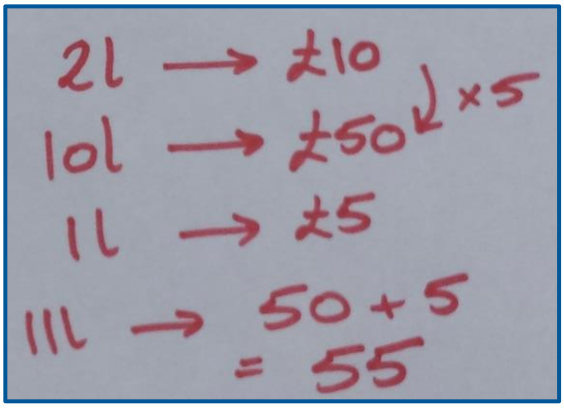
1.



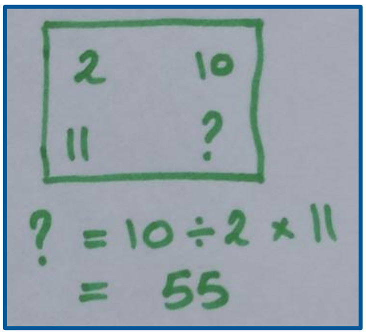
2.



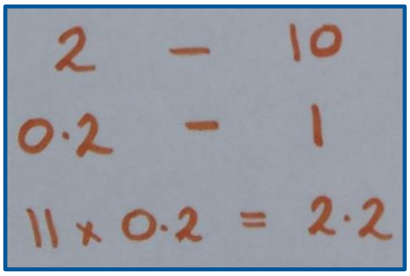
3.



4.



5.



Comments on student methods

1. This student has found the cost of 1 litre using a variation of the strip diagram. It would be important not to destroy their thinking, but to re-enforce it using the double number line.

2. Many students who use this method have little understanding of what they have done. It has often been taught to them as a process that must be followed. It could be explained on the double number line as a scale factor method. Multiplying by 11 gives the cost for 22 litres, then dividing by 2 gives the cost for 11 litres.

3. Students will often use a mix of thinking. They may use a multiplier method to get close and then use an additive method to add on 1 litre.

4. Many students who use this method have little understanding of what they have done. It has often been taught to them as a process that must be followed. It could be explained on the double number line using the unitary method – finding the cost of 1 litre then multiplying by 11.

5. Students may drop units from their calculations and so lose sight of what they are trying to find. Illustrating this (unitary) method on the double number line helps them to see the mistake that has been made.

PD2.5 Lesson 2 Research Form

Observe a pair/group of students working during the lesson.

Note down the key moments and development in their mathematical thinking.

Note down examples of the Maths-for-Life pedagogies seen during this lesson.

Collaborative Learning

Working together toward a common goal

Models of Structure

Representations that provide insight into mathematical structure

Closure

Drawing a lesson to a close to ensure shared understanding

Cognitive Conflict

Being challenged by new information that contradicts prior ideas

Formative Assessment

Assessment that provides information on what to do next

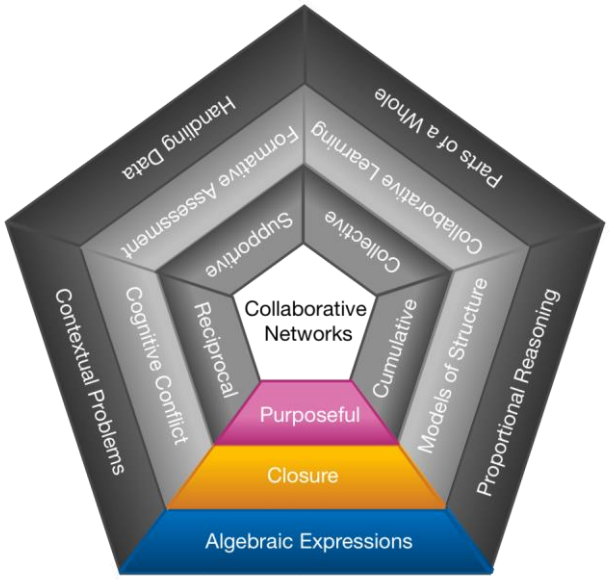
Research Question

How do models of structure help to facilitate cumulative dialogue and insight into mathematical structure?

PD3 Session 3: Algebraic Expressions

Overview

The focus in the lesson, **Algebraic Expressions**, is on how **closure** can help to develop **purposeful** dialogue in the classroom.



Closure is when part of a lesson is drawn together to ensure a shared understanding.

Purposeful dialogue is when talk is structured with specific learning goals in view.

Aims of Professional Development Session 3

1. Enable teachers to successfully teach the lesson, **Algebraic Expressions**.
2. Explore what level of mathematical thinking contributes to effective **closure**.
3. Explore how teacher actions and behaviour contribute to effective **closure**.
4. Develop an understanding of what **purposeful** dialogue looks like in the classroom.

Checklist for Session 3

☐	Prior to the meeting, ask teachers to look at the documents • **L3.1 Lesson Outline** (in the resource file) • **PD3.5 Closure** (see teacher handouts)
☐	At the *planning session*, work through **PD3 Professional Development**. This will help teachers understand the resources, pedagogy and aspect of dialogic learning focused on in the lesson, Algebraic Expressions.
☐	At the *planning session*, ensure that everyone is clear how a research lesson operates including use of the **Collaborative Lesson Research Form**.
☐	At the *planning session*, schedule the *research lesson*. Agreed date of research lesson: ☐
☐	Carry out the *research lesson*.
☐	At the *research lesson*, discuss the learning that has taken place in the lesson alongside a focus on the research question.
☐	At the research lesson, (if necessary) schedule the date for the next planning session (for lesson 4). Agreed date: ☐

1. **Enable teachers to successfully teach the lesson, Algebraic Expressions.**

Teachers should use the summary **L3.1 Lesson Outline** to understand this lesson.

Lead teachers are additionally supported by the summary bekow and **PD3.0 Lesson Pedagogy**.

Note also, that **PD3.3 Guided Discussions of Videos** may be helpful in supporting lead teachers understanding and discussion of the video clips.

You may also decide to give teachers a copy of the transcripts to read through (**PD3. Guided Discussion of Videos_Teachers**).

Before looking at the lesson, ask teachers to complete the matching cards found in **PD3.1 Letters in Algebra**. Establish that students will learn how to view 'letters as variable' at the start of this lesson.

"The opening task has been carefully crafted"
Work with the teachers through **PD3.2 Opening Task** to understand some of the principles used in the design of the task. The previous versions of the task help to illustrate that careful design is required to facilitate student learning. As teachers work through the final version of the opening task in the lesson PowerPoint (**L3.5**) reflect on how the principles provided are met.

"Use the task to encourage discussion"
Ask the teachers to, in pairs, place the cards on the template (**L3.2 and L3.3**). As they do this encourage the teachers to think about the difficulties students will face and the mistakes they are likely to make.

Watch video clip **V3.1 Working Through the Task.** This shows a research group noticing some of the key features of the main card matching activity. Consider alternative factorisations of $2n^2 + 4n$ and how the corresponding representation would look.

2. **Explore how synthesising understanding contributes to effective closure.**

"Avoid closing a lesson by turning what has been covered into a procedure"
Watch video clip **V3.2 Synthesising** to see how a teacher focuses on bringing together the different approaches used in the lesson. This encourages a comparison of different approaches and the mathematical structure to be discussed.

Work through **PD3.4 Algebraic Expressions Closure** to highlight the differences between focusing students on a procedure and synthesising understanding.

3. Explore how teacher actions and behaviour contribute to effective closure.

"Closure is more effective when the students shape the thinking"
Look at the section entitled 'Teacher Actions' in the handout **PD3 Closure**. Ask teachers to consider which of the actions they tend to use with their classes and which they will aim to develop in this lesson.

Use the recorded clip **V3.3 Actions During Closure**
to identify examples of the teacher actions listed in the **Closure** handout.

4. Develop an understanding of what purposeful dialogue looks like in the classroom.

"Look for discussion that contributes to the aims of the lesson"
Purposeful dialogue may take place at any point during the lesson – and may be between students or between students and a teacher. For this lesson look for examples of purposeful dialogue during the closure phase.

Introduce the research question for this lesson:

How does purposeful dialogue contribute to student understanding during the closure phase of the lesson?

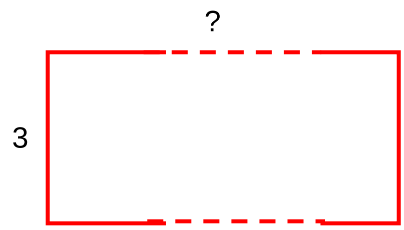
PD3.0 Lesson Pedagogy

Mathematical goals

To help students:

- understand that *n* can represent a variable;
- understand multiplicative algebraic structure using an area representation;
- create algebraic expressions from area representations.

Engaging with the lesson outline

Engage Question	Prompts
1. What diagrams (and dimensions) might students draw?	Students may draw something like the following diagram:  Use of a question mark would indicate that students see this as a fixed (unknown) value. Only once this is known can the area be found. We would like students to use a letter (such as *n*) to indicate that the length is a variable. This means that the area can then be written as 3*n*. Note that one reason letters are used instead of question marks is for situations with multiple variables. Also note that question marks are used in the lesson to find values/expressions. Use the idea of dotted lines to link to the animated PowerPoint showing the rectangle varying in length. Note that as a convention we draw a fixed looking rectangle, but the implication is the same as drawing dotted lines or animating the length.
2. What difficulties might students have with this question?	Firstly, there are a lot of words on the PowerPoint slide, so it may be helpful to read the situation out to students. Encourage students to draw a diagram (similar to the last question) and to mark any information on that they can.
3. Why is it important to share this slide with students?	This slide establishes the conventions to be adopted for the rest of the lesson. It creates the conditions to allow: • *n* to be understood as a variable.

	• Student thinking to move from the concrete to more abstract ideas. • Students to see question marks as something they need to find the value of.
4. What is the purpose of giving the students a speaking frame?	Students may need a framework to give them the confidence to speak to one another about maths. By encouraging them to say whether they agree/disagree or need more information we are seeking to stop one of the students becoming passive.
5. Where do you expect difficulties in this task?	Encourage students to write on the template – particularly to find the values represented by the grey question marks. Many students will think that n x n = 2 n. What questions can help them make sense of this? Students have a preference for multiplying out brackets rather than factorising. To reveal this it may be appropriate to cover up the card with (say) $2n(n + 2)$ and ask them how to factorise the expression $2n^2+4n$. The extension questions are particularly powerful in helping students understand why we factorise and expand quadratic expressions in the way that we do.
6. Ensure your questions check student understanding, not act to correct their understanding.	Think carefully about how to operate whilst students are working on the task. It can be tempting to explain to students what they are doing is wrong and correct their thinking. These resources are more powerful if you ask questions that prompt students to discuss their thinking.
7. Which of these rows do you expect to emphasise with your class?	It will be important to pick up on understanding why n x n = $2n$ is not correct. It may be helpful to explore any alternative factorisations for $2n^2 + 4n$.
8. What are the key messages from the lesson that you expect to draw out?	It is vital that thinking is brought together at the end of the lesson. Some of what is shared should be based on card matches students found straightforward and those they had difficulty with. Points to highlight include helping students see that they may have to create algebraic expressions from area representations, and that area representations can help us to make sense of some of the algebra that we do.
9. What misconception does this explanation risk creating?	This approach only works because both diagrams (and expressions) shared a common length of $(n + 2)$. This approach would not appropriate if the question were (for example) $2(n + 2) + 3(n + 1)$.

PD3.1 Letters in Algebra

Letters are used in a number of different ways in mathematics.

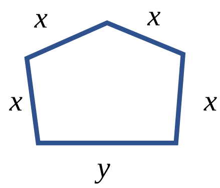
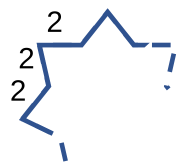
Use the matching cards to link an example question to the description of how students are expected to understand the letters in the question.[1]

Once correct:

1. Identify which ways of understanding you would expect to see needed in a GCSE Foundation exam.

2. Order the ways of understanding from most common to least common in GCSE re-sit students.

[1] To find out more about how students view letters, see Teaching and Learning Algebra by Doug French.

Letters as a specific unknown	$x + 5 = 13$
Letters as an object	If $x + y = 10$ what is $x + y + 2$ equal to?
Letters evaluated	 P = ________
Letters ignored	There are x sides, each of length 2.  P = ________
Letters as a variable	What can you say about x if $x + y = 10$ and $x < y$?
Letters as a generalised number	Which is larger, $2x$ or $x + 2$?

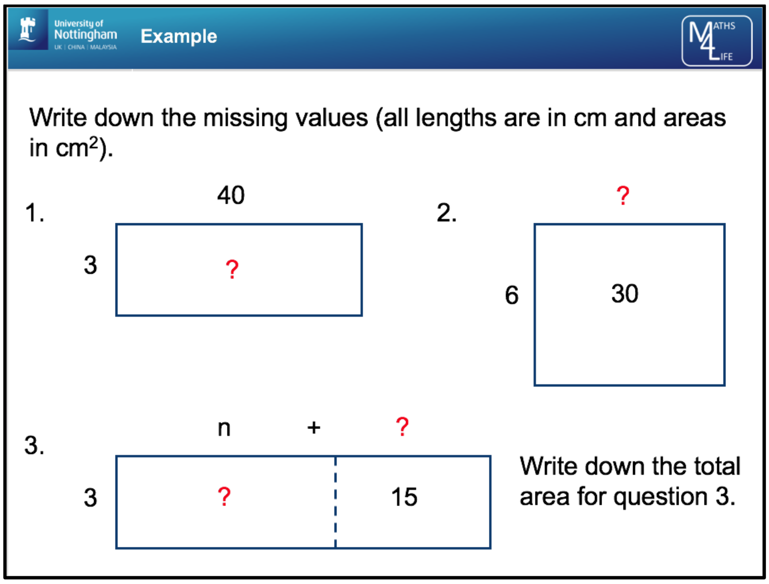
PD3.2 Opening Task

In Maths-for-Life the opening tasks have been designed with four key principles in mind. The task should:

1. Facilitate students to access the main lesson activity.
2. Create opportunities for dialogue.
3. Help students to see the relevance of mathematics by being set in a realisable[2] context.
4. Lead to a model of structure that can help make sense of the mathematics.

Version 1

The original design of the opening task is shown below along with a summary of how the opening task principles were addressed.



Principle	Intentions when the task was designed
1. Facilitate students to access the main lesson activity.	Students would attempt questions very similar to those shown in example number 3.
2. Create opportunities for dialogue.	The question marks indicate where students may need to discuss their understanding as well as indicating missing values to be found.

[2] The word “realisable” has been used as opposed to “real-world”. This is used in the same sense that Hans Freudenthal (Realistic Mathematics Education) uses the term. That is a student can understand and relate to the situation, and the underlying structure, even though the task may be imaginary rather than real-world.

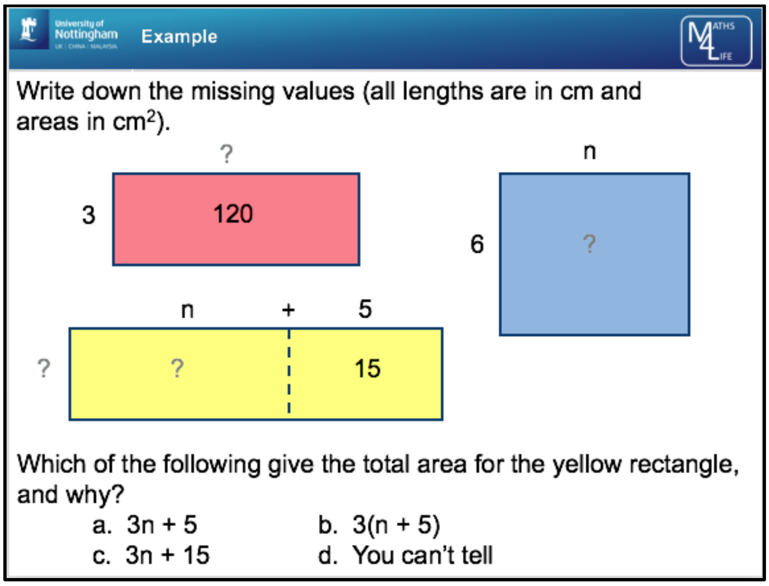
3. Help students to see the relevance of mathematics by being set in a realisable context.	The context chosen was to find expressions for lengths and areas using rectangles.
4. Lead to a model of structure that can help make sense of the mathematics.	The area of rectangles provides a model of structure by which multiplying out of brackets (single and double) can be understood.

Work through version 1 of the initial task.

- What are the strengths of the design?
- What are the weaknesses of the design?

Version 2

Version 2 of the opening task addressed a number of the practical and surface issues found with version 1.

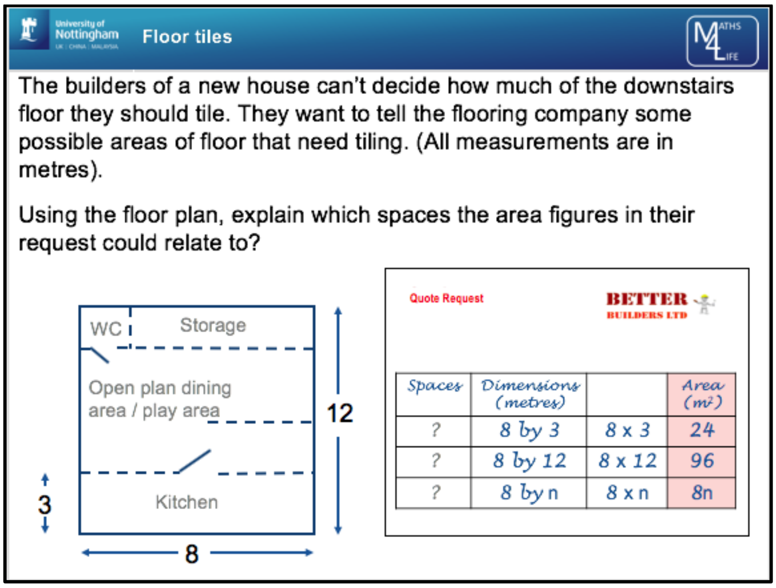


- Identify the changes that were made, and suggest reasons for those changes?
- What issues regarding the real-life context remain (principle 3)?
- What issues regarding how students understand the meaning of *n* remain (principle 4)?

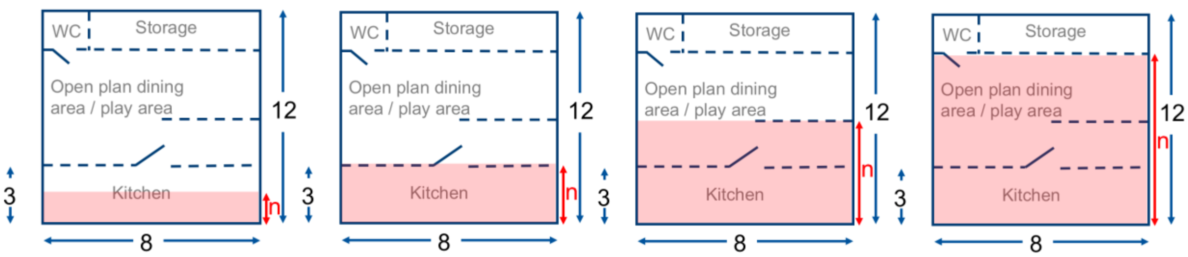
Version 3

Version 2 still did not provide a good real-life context to help students see the relevance of mathematics. Furthermore, it failed to address the issue that students were struggling to understand that *n* represents a variable and hence that an area could be written as 6*n*.

Version 3 attempted to address these two issues.

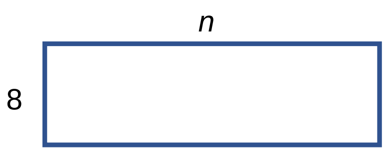


As part of the PowerPoint the following areas were then shown as possible spaces represented by 8*n*.



This version of the task was rejected before being trialed with a class.

- What issues remain if this task is meant to provide a real-world context?
- Does the way that *n* is handled in this task help students to see that *n* represents a variable in the following diagram?



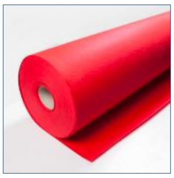
Now work through the opening task in lesson 3 and discuss how the four principles for opening tasks are addressed.

Floors-R-Us

A new trainee is about to start work at Floors-R-Us selling carpets.

A roll of red carpet is 3 metres wide, but the customer may want any length

Draw a diagram (with values marked on) to help the manager show the trainee how to find the area.



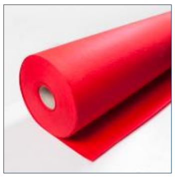
Floors-R-Us Extension

The manager usually recommends that an extra 2 metres (to allow for mistakes in carpet fitting) is bought **in addition** to whatever amount the customer needs.

The manager says that this is the same as doing the calculation $(n + 2) \times 3$

The assistant manager disagrees and says the calculation should be $6 + 3n$

Draw a diagram to help explain how each of them is thinking.



Additional notes for lead teachers

In trialling version 1 of this task, some of the issues included:

- The numbering of the questions (as 1, 2, and 3) sometimes caused confusion with the numbers representing lengths.
- Some students did need a reminder of how to find area, but question 1 was often too simple.
- Writing an area as 3*n* in question 3 was a problem for many students, and consequently few could answer the final question of ‘write down the total area for question 3’.

Version 2 of the task addressed a number of surface issues, but the real-life context is inadequate and it fails to address the issue that many students saw *n* as representing an unknown. Even when the teacher explained that *n* represents a variable this is difficult to believe on a diagram with a fixed looking length.

Version 3 of the task, whilst providing some real-life context, risks causing further confusion.

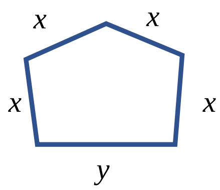
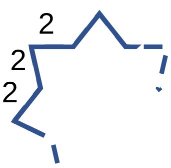
For example, in terms of a real-life context there are many values of *n* that would never be used.

In terms of the inadequacy of the mathematical model this illustration implies that there are specific unknown values (part way through the rectangle) that could be called *n*. What we actually want the students to see is that *n* on a diagram represents a value that can vary.

For the final version of the opening task given in the PowerPoint the principles are addressed in the following ways:

Principle	Intentions when the task was designed
1. Facilitate students to access the main lesson activity.	Students would attempt questions very similar to those shown in the task.
2. Create opportunities for dialogue.	Students will have to discuss drawing diagrams of varying lengths and interpret the yellow carpet question.
3. Help students to see the relevance of mathematics by being set in a realisable context.	The context of any length of carpet being ordered is believable.
4. Lead to a model of structure that can help make sense of the mathematics.	The animation helps students to see that *n* represents a variable length on a fixed looking length. Factorised expressions and expanded expressions can be seen as multiplying two lengths together and adding two areas together respectively.

Answers

Understanding	Example	Explanation
Letters evaluated	$x + 5 = 13$	x can be evaluated, or assigned a value, immediately.
Letters ignored	If $x + y = 10$ what is $x + y + 2$ equal to?	$x + y$ can be ignored to find $x + y + 2$.
Letters as an object	 $P =$ ________	x and y are seen as names or labels for the sides (rather than as numbers).
Letters as a specific unknown	There are x sides, each of length 2.  $P =$ ________	x stands for an unknown number which cannot be evaluated.
Letters as a generalised number	What can you say about x if $x + y = 10$ and $x < y$?	x represents a set of numbers rather than just one value.
Letters as a variable	Which is larger, $2x$ or $x + 2$?	A relationship needs to be understood as x varies.

PD3.3(LT) Guided discussions: Video Clips

V3.1 Video Clip: Working Through the Task

The teachers in the video clip are working through a task in order to identify the most likely mistakes, misconceptions, difficulties and strategies that may take place. The purpose of the clip is to provoke further discussion on potential issues students may face and the sorts of questions that could encourage dialogue and understanding.

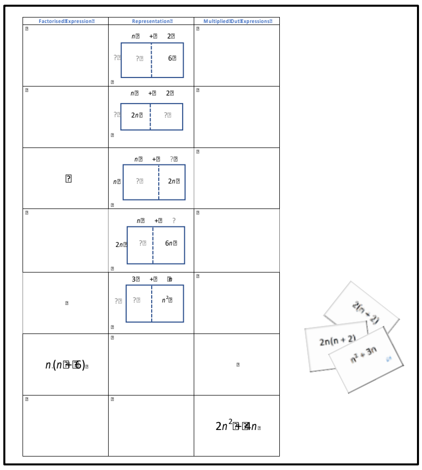
Introductory video captions

In this clip, teachers, from one of the research groups, are working through the main task together.

In doing this, they share observations about the design of the task.

As you watch, think about their observations and how you might use the features of the task with your students to provoke dialogue and understanding.

Task teachers are tackling in the video



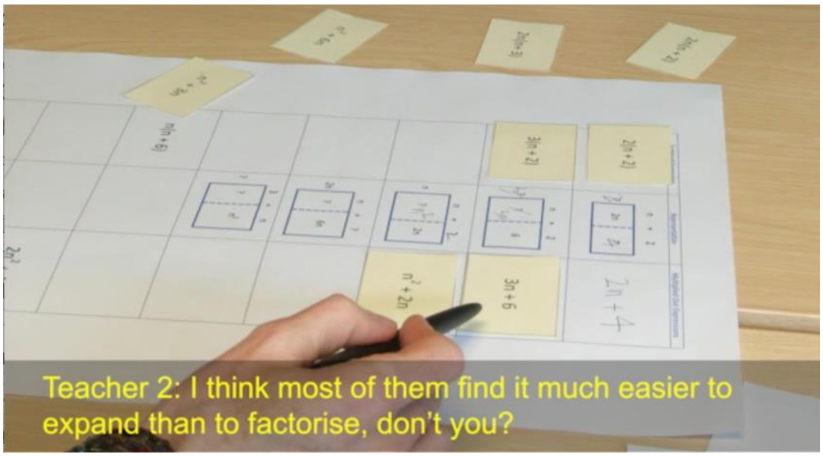
Transcript of dialogue

Teacher 1: Lots of question marks. Oh I like that, the fact that they've added ..

Teacher 2: Yes that's nice, a quadratic. Because we asked about that last time, I think. Here we go. What do we know? …

Teacher 1: That was the hard bit, actually, going back that way for them. So that way is an easier entry point into the activity.

Teacher 2: I think most of them find it much easier to expand than factorise, don't you?



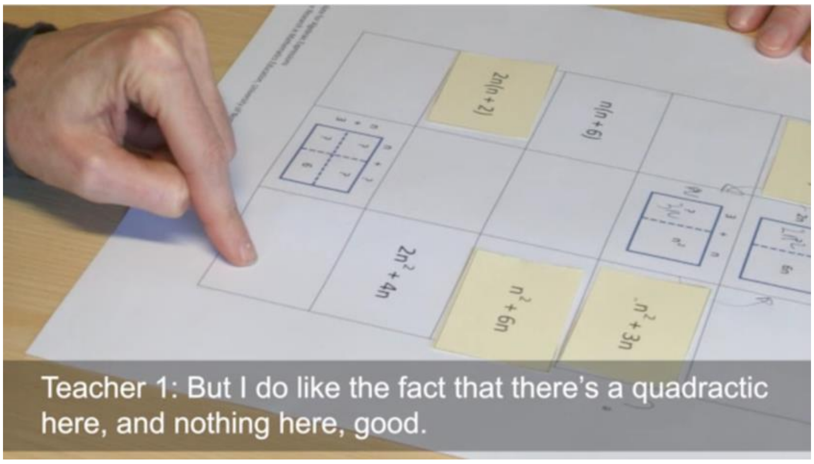
Teacher 1: So now, the next one here would be … so they've got to multiply these by two …

So now we're taught to use the *n* squared notation.

So two n squared, and here … err…

So in a way, same again. This one here is easier because what makes n square is n

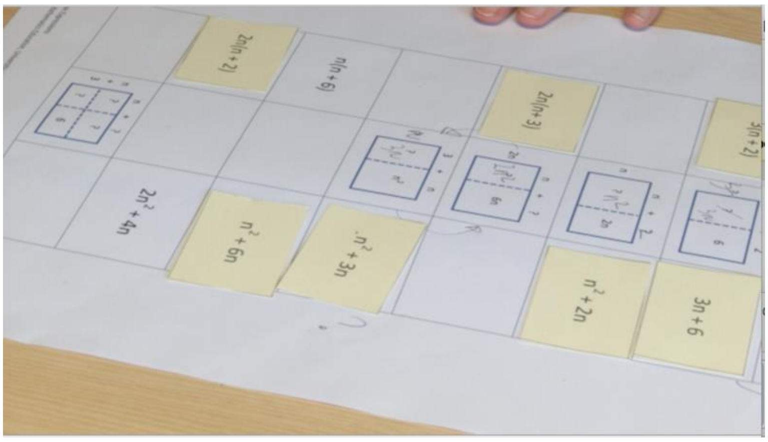
Teacher 1: But I do like the fact that there's a quadratic here, and nothing here, good.



I like the fact that here there's a … erm … factorisation to work out. And here the same

Teacher 2: Yes. So we've got two, two expands to identify, two factorised and two pictures and then …

I think the number of cards is actually okay.



Teacher 3:	They’ve got something to refer back to.
	It adds a little bit more confidence.
Teacher 2:	They’ve got an anchor, haven’t they?
Teacher 3:	Because we’re talking about some students who haven’t a clue with this have they really …
Teacher 2:	No.
Teacher 3:	… initially?

Post video guided discussion

Discuss with the teachers the concerns of the teachers in the video clip expressed, and any other challenges students may encounter when tackling the task. You may also want to consider how teachers could support students to overcome these challenges, whilst resisting telling them what to do.

Teachers may suggest, for example:

1. *Many students think that n x n is 2n.*
2. *Many students find it easier to explain factorisation by explaining how to multiply out the expression. To help with this, cover up the factorised card on a completed row and ask students to explain (with the help of the representation) how to factorise.*
3. *Ensure that students can relate all three expressions/representations to each other.*

V3.2 Video Clip: Synthesising

The video clip first shows students explaining, within a whole class discussion, different ways to solve a problem. The purpose of this part of the clip is to illustrate teacher pedagogy that can both encourage a collaborative classroom, and help progress the collective thinking of the class. The second part of the video introduces pre-designed student solutions. The purpose here, is to prompt a post-video discussion on the pros and cons of using these solutions in class.

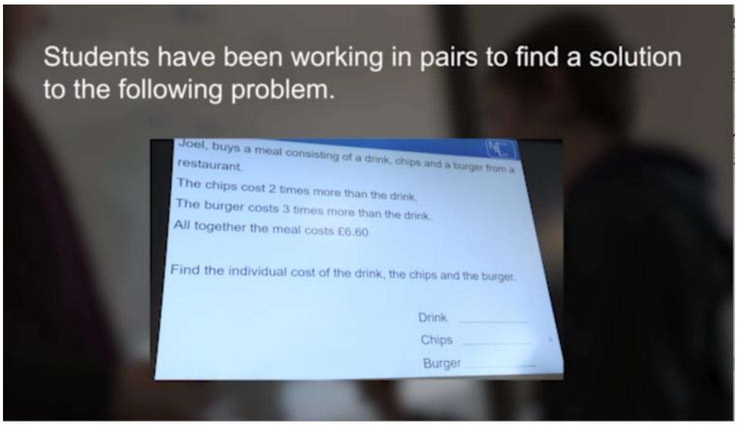
Introductory captions for first part of video clip

Students have been working in pairs to find a solution to the following problem

The teacher leads a whole-class discussion in which he asks different pairs to share their solutions.

Think about how the teacher attempts to draw out students' thinking about what they have done.

Notice how he encourages a collaborative classroom.



Introductory captions for second part of video clip

The teacher now asks students to explore two pre-designed solutions.

Think about the pros and cons of doing this.

Transcript of dialogue

The transcript is not included here because the talk on the video is clear.

Post-video guided discussion

Discuss with the teachers:

- How the teacher ensures the whole class discussion is more than simply ‘show and tell’.
  Teachers may suggest that through the teacher’s questioning and prompts, he encourages classmates to participate in the discussion.
- The pros and cons of students reviewing pre-designed solutions to the same problem.
  The solutions ensure students are working with anonymous, correct and coherent solutions (in this case). This can make it easier for students to critique these solutions - they do not need to be sensitive to the anonymous student’s feelings. They also ensure the students are addressing the intended learning goals of the lesson. It also can ensure students work with more powerful methods than they would have thought up on their own. Moreover, the solutions can shift the focus of the activity from performance to understanding.

V3.3 Video Clip: Actions during closure

The video shows students within a whole class discussion, talking about different ways to solve a problem. The purpose of the clip is to expose teachers to further ways to encourage a productive discussion that not only highlights the mathematical structure of the problem, but promotes collaborative dialogue. A whole class discussion is also a useful opportunity to also model how students should talk when working in groups.

Introductory video captions

The students have tackled the Paint Prices problem and the Flatbread mix problem.

The teacher has observed that students have adopted different methods.

As you watch, try to identify examples that help illustrate the following:

- *Focusing on mathematical structure*
- *Joining in not judging*
- *Ensuring students do the thinking.*

Transcript of dialogue

The transcript is not included here because most of the talk on the video is clear.

Post video guided discussion

Discuss with the teachers:

- How the teacher attends to the mathematical structure.
  Teachers may recognise how the double number line helps to raise students' awareness of how variables are related. Using this pedagogical tool to compare methods can help students to recognise the contrasting relationships that occur within the two methods. Teachers may also suggest that the teacher in the video highlights how the structure of a problem may prompt different methods.

How the teacher encourages the students to be responsible for their own learning. Teachers may suggest that the teacher asks students to explain their work, rather than explaining it for them. He asks the class whether they understand the explained work. He also does not judge their work, simply repeats their explanation. In doing so he provides some extra time for the class to absorb what has been said. Teachers may also suggest that asking students to instruct them encourages a supportive dialogue.

PD3.4 Algebraic Expressions Closure

The three mathematical goals of the Algebraic Expression lesson are to help students:

- understand that *n* can represent a variable;
- understand multiplicative algebraic structure using an area representation;
- create algebraic expressions from area representations.

The following two teacher plans cover their ideas about how they would seek to bring closure around the second bullet point – "understand multiplicative algebraic structure using an area representation".

Look at the two plans to identify the differences in approach and to consider the positives and drawbacks.

- Teacher 1 is aiming to proceduralise. This can mean that students can only solve problems given in a certain way.
- Teacher 2 is aiming to synthesise. This should lead to students understanding the mathematical structure, and ideas, of what they're doing.

Ask teachers to create closure sessions similar to teacher 2 for the other two mathematical goals:

- understand that *n* can represent a variable;
- create algebraic expressions from area representations.

(Some ideas are given later in this document).

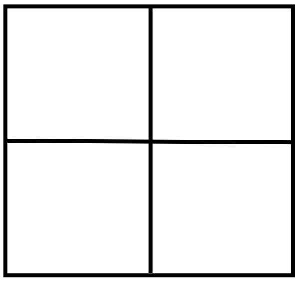
Teacher 1

One of the things we've seen today is that it helps us to understand multiplying algebraic terms together by thinking about an area representation.

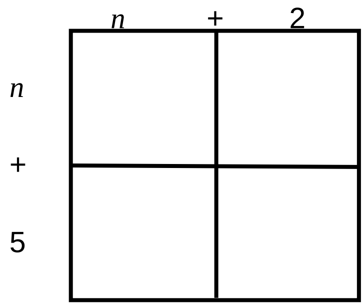
It's important that you remember that n times n is n^2 for you to get this right.

So, if we're given a question such as (n + 2)(n + 5) then we need to do the following steps:

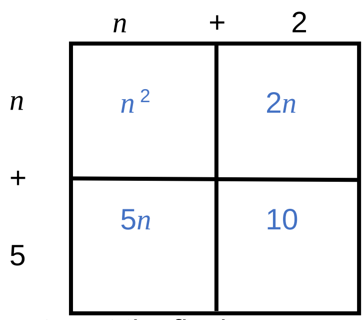
1. *Draw a two by two grid.*



2. *Write the first n at the top, followed by the number, then the second n n followed by the number down the side.*



3. *Fill in the boxes working left to right and top to bottom … n times n is n^2, 2 times n is 2n, 5 times n is 5n and 5 times 2 is 10.*



4. *Add up all of the boxes to get the final answer … $n^2 + 7n + 10$*

This should be clearer than methods you've been shown before.

Teacher 2

One of the things we've seen today is that it helps us to understand multiplying algebraic terms together by thinking about an area representation.

Remember, we saw that a width (of 3) x length (of n) gives us a variable area of 3n.

With that in mind let's look at how we've tried to make sense of multiplying out double brackets.

As a class, we've probably seen loads of different ways to expand an expression such as (n + 2)(n + 5).

Let's list some….

For each one, see how they relate to an area representation …

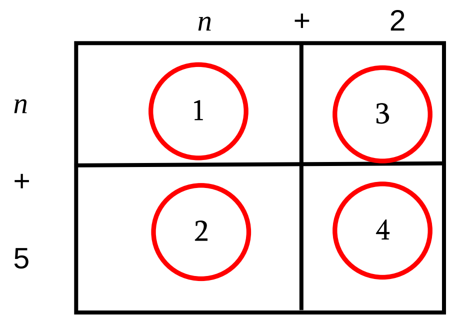
(As the area representation is drawn remind students that n indicates a variable length, the numbers a fixed length).

1. ***FOIL and the claw***

   First Outside Inside Last and the claw can both be illustrated as

$$(n + 2)(n + 5) = n^2 + 5n + 2n + 10$$

   Note that this would just be the addition of the four areas in the order given…

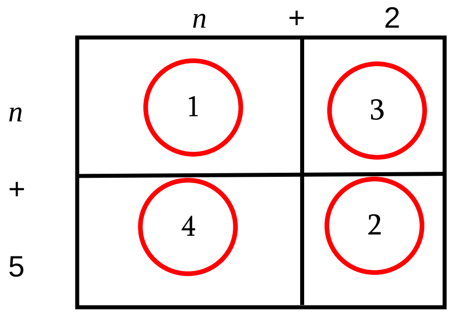


2. *Smiley face*

Smiley face can be illustrated as

$$(n + 2)(n + 5) = n^2 + 10 + 2n + n$$

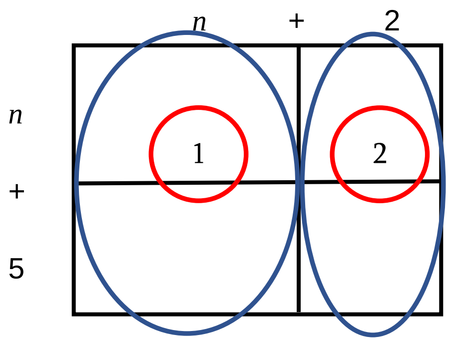
Note that this would just be the addition of the four areas in the order given…



3. *n(n + 5) + 2(n + 5)*

This is often explained as 'the first thing in the brackets multiplied by everything in the second set of brackets, plus the second thing multiplied by everything in the second set of brackets'.

In this way of thinking we just see it as two strips. The area of the first plus the area of the second. Then multiply out each of those …



4. *The grid method*

It looks like the area representation, but in the area representation we can see why you multiply the letters together, and more importantly, why we add them together.

So, all these different ways of doing it are just convenient (or shorthand) ways to remember a process of what to do. You can keep doing any of them – but they all benefit from understanding that they can be linked to finding areas.

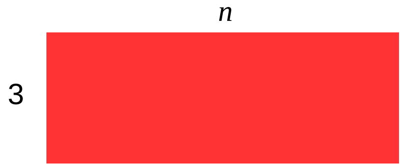
Ideas for Closure

1. For the mathematical goal ‘understand that *n* can represent a variable’;

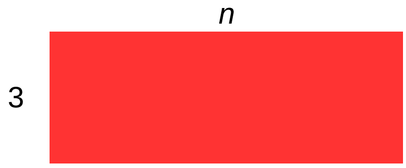
So let’s just take a moment to think about what we’ve done today.

We saw that we use letters in algebra to encourage a number of different ways of thinking. It often depends on the context. In this lesson we’ve worked with n representing a variable – something that can change. (Remember the rectangles that grew and shrunk).

So the area of the carpet at the start of the lesson was 3n, where n can take many different values.



But notice how the letter takes on a slightly different meaning if we did the following….

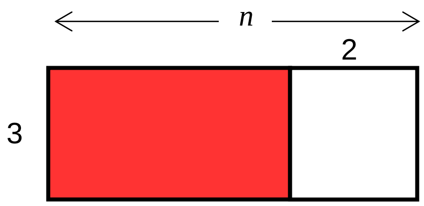


Find n if the area is 27.

In this case, n was a variable – but now there is only one value of that variable that ‘satisfies’ the problem. You could be asked to do either of these in an exam and you need to be aware of when examiners are thinking of algebra as a variable and when it is one unknown that you can solve.

2. For the mathematical goal ‘create algebraic expressions from area representations’.

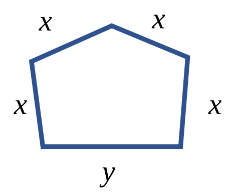
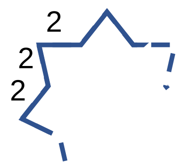
   Secondly, you may get questions in an exam that require you to create an algebraic expression from an area diagram. We did one, but how would you approach writing an algebraic expression for the red area in this one…



(Encourage students to work in pairs here, think how long the red rectangle is, encourage two different ways to consider the problem. Firstly, write an expression of two lengths that are multiplied together, secondly write as an area subtract another area).

Handouts for teachers session 3

Cut up the cards. Ask teachers to match them.

Letters as a specific unknown	$x + 5 = 13$
Letters as an object	If $x + y = 10$ what is $x + y + 2$ equal to?
Letters evaluated	 P = ________
Letters ignored	There are x sides, each of length 2.  P = ________
Letters as a variable	What can you say about x if $x + y = 10$ and $x < y$?
Letters as a generalised number	Which is larger, $2x$ or $x + 2$?

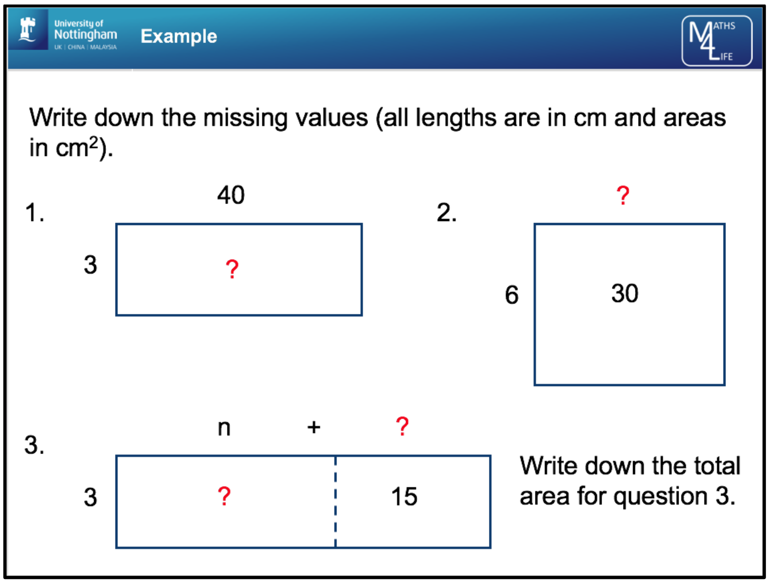
PD3.2 Opening Task

In Maths-for-Life the opening tasks have been designed with four key principles in mind. The task should:

1. Facilitate students to access the main lesson activity.
2. Create opportunities for dialogue.
3. Help students to see the relevance of mathematics by being set in a realisable[3] context.
4. Lead to a model of structure that can help make sense of the mathematics.

Version 1

The original design of the opening task is shown below along with a summary of how the opening task principles were addressed.



Principle	Intentions when the task was designed
1. Facilitate students to access the main lesson activity.	Students would attempt questions very similar to those shown in example number 3.
2. Create opportunities for dialogue.	The question marks indicate where students may need to discuss their

[3] The word "realisable" has been used as opposed to "real-world". This is used in the same sense that Hans Freudenthal (Realistic Mathematics Education) uses the term. That is a student can understand and relate to the situation, and the underlying structure, even though the task may be imaginary rather than real-world.

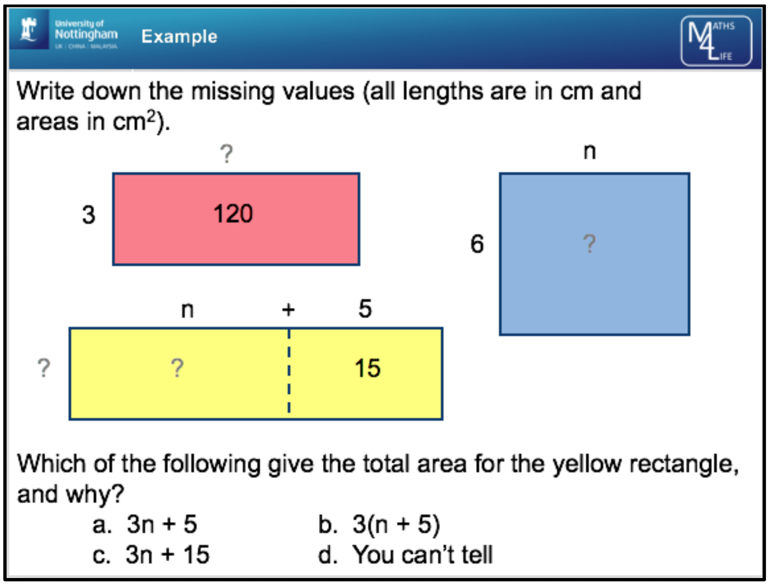
	understanding as well as indicating missing values to be found.
3. Help students to see the relevance of mathematics by being set in a realisable context.	The context chosen was to find expressions for lengths and areas using rectangles.
4. Lead to a model of structure that can help make sense of the mathematics.	The area of rectangles provides a model of structure by which multiplying out of brackets (single and double) can be understood.

Work through version 1 of the initial task.

- What are the strengths of the design?
- What are the weaknesses of the design?

Version 2

Version 2 of the opening task addressed a number of the practical and surface issues found with version 1.

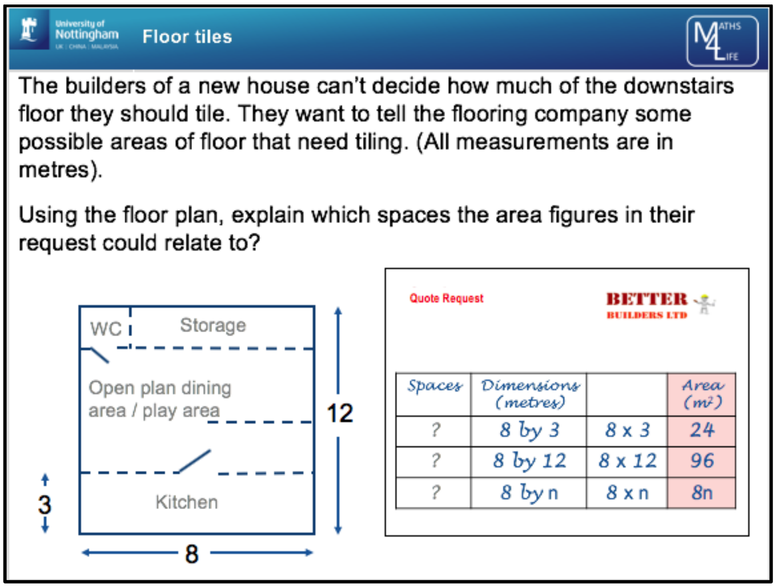


- Identify the changes that were made, and suggest reasons for those changes?
- What issues regarding the real-life context remain (principle 3)?
- What issues regarding how students understand the meaning of *n* remain (principle 4)?

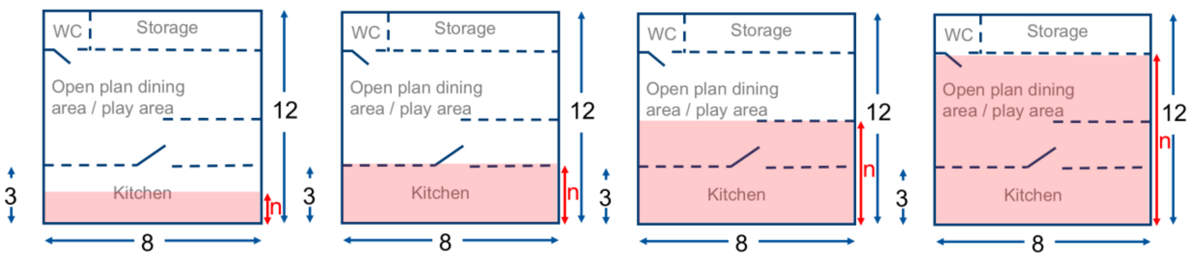
Version 3

Version 2 still did not provide a good real-life context to help students see the relevance of mathematics. Furthermore, it failed to address the issue that students were struggling to understand that *n* represents a variable and hence that an area could be written as 6*n*.

Version 3 attempted to address these two issues.

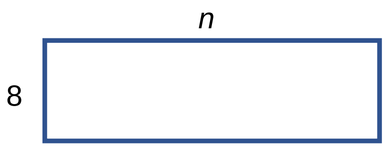


As part of the PowerPoint the following areas were then shown as possible spaces represented by 8*n*.



This version of the task was rejected before being trialled with a class.

- What issues remain if this task is meant to provide a real-world context?
- Does the way that *n* is handled in this task help students to see that *n* represents a variable in the following diagram?



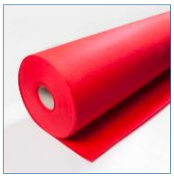
Now work through the opening task in lesson 3 and discuss how the four principles for opening tasks are addressed.

Floors-R-Us

A new trainee is about to start work at Floors-R-Us selling carpets.

A roll of red carpet is 3 metres wide, but the customer may want any length

Draw a diagram (with values marked on) to help the manager show the trainee how to find the area.



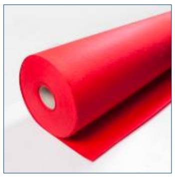
Floors-R-Us Extension

The manager usually recommends that an extra 2 metres (to allow for mistakes in carpet fitting) is bought **in addition** to whatever amount the customer needs.

The manager says that this is the same as doing the calculation $(n + 2) \times 3$

The assistant manager disagrees and says the calculation should be $6 + 3n$

Draw a diagram to help explain how each of them is thinking.



V3.1 Video Clip: Working Through the Task

Transcript of dialogue

Teacher 1: Lots of question marks. Oh I like that, the fact that they've added ..

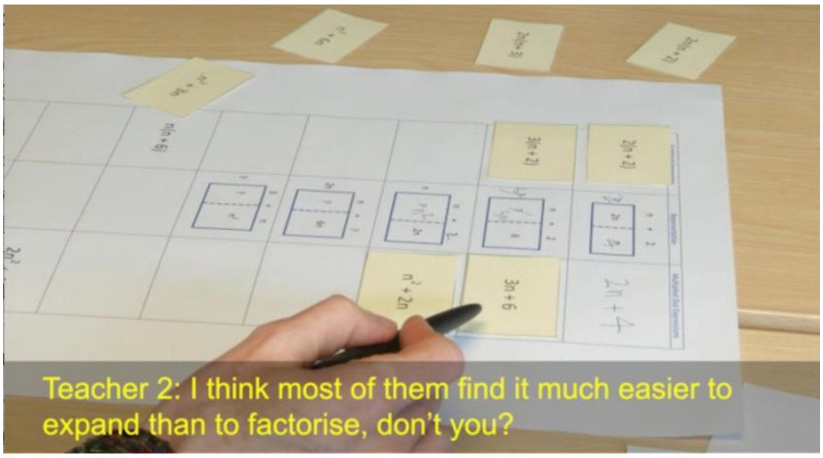
Teacher 2: Yes that's nice, a quadratic. Because we asked about that last time, I think.

Here we go. What do we know? …

Teacher 1: That was the hard bit, actually, going back that way for them.

So that way is an easier entry point into the activity.

Teacher 2: I think most of them find it much easier to expand than factorise, don't you?



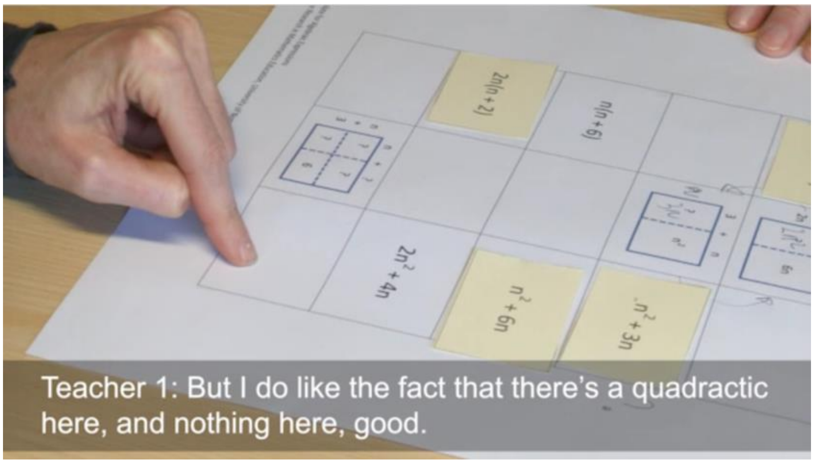
Teacher 1: So now, the next one here would be … so they've got to multiply these by two …

So now we're taught to use the *n* squared notation.

So two n squared, and here … err…

So in a way, same again. This one here is easier because what makes n square is n

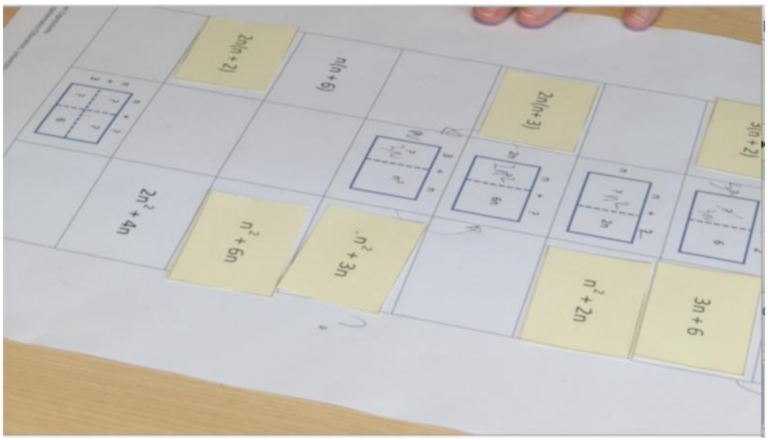
Teacher 1: But I do like the fact that there's a quadratic here, and nothing here, good.



I like the fact that here there's a … erm … factorisation to work out. And here the same

Teacher 2: Yes. So we've got two, two expands to identify, two factorised and two pictures and then …

I think the number of cards is actually okay.



Teacher 3: They've got something to refer back to.

It adds a little bit more confidence.

Teacher 2: They've got an anchor, haven't they?

Teacher 3: Because we're talking about some students who haven't a clue with this have they really …

Teacher 2: No.

Teacher 3: … initially?

PD3.4 Algebraic Expressions Closure

The three mathematical goals of the Algebraic Expression lesson are to help students:

- understand that *n* can represent a variable;
- understand multiplicative algebraic structure using an area representation;
- create algebraic expressions from area representations.

The following two teacher plans cover their ideas about how they would seek to bring closure around the second bullet point – "understand multiplicative algebraic structure using an area representation".

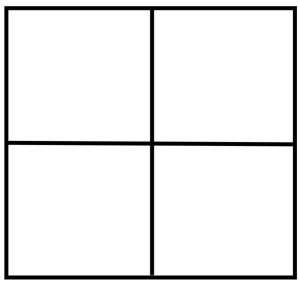
Teacher 1

One of the things we've seen today is that it helps us to understand multiplying algebraic terms together by thinking about an area representation.

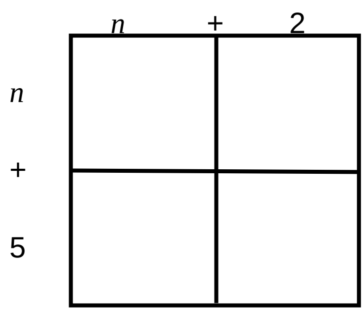
It's important that you remember that n times n is n^2 *for you to get this right.*

So, if we're given a question such as (n + 2)(n + 5) then we need to do the following steps:

1. *Draw a two by two grid.*



2. *Write the first n at the top, followed by the number, then the second n n followed by the number down the side.*



3. *Fill in the boxes working left to right and top to bottom … n times n is n^2, 2 times n is 2n, 5 times n is 5n and 5 times 2 is 10.*

	n	2
n	n^2	$2n$
5	$5n$	10

4. *Add up all of the boxes to get the final answer … $n^2 + 7n + 10$*

This should be clearer than methods you've been shown before.

Teacher 2

One of the things we've seen today is that it helps us to understand multiplying algebraic terms together by thinking about an area representation.

Remember, we saw that a width (of 3) x length (of n) gives us a variable area of 3n.

With that in mind let's look at how we've tried to make sense of multiplying out double brackets.

As a class, we've probably seen loads of different ways to expand an expression such as (n + 2)(n + 5).

Let's list some….

For each one, see how they relate to an area representation …

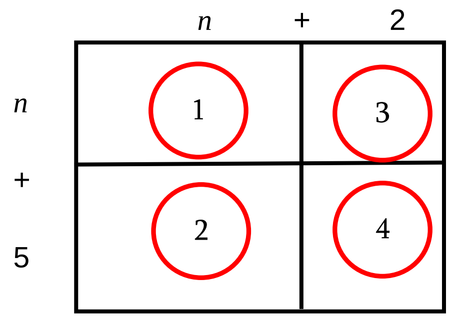
(As the area representation is drawn remind students that n indicates a variable length, the numbers a fixed length).

FOIL and the claw

First Outside Inside Last and the claw can both be illustrated as

$$(n + 2)(n + 5) = n^2 + 5n + 2n + 10$$

Note that this would just be the addition of the four areas in the order given…

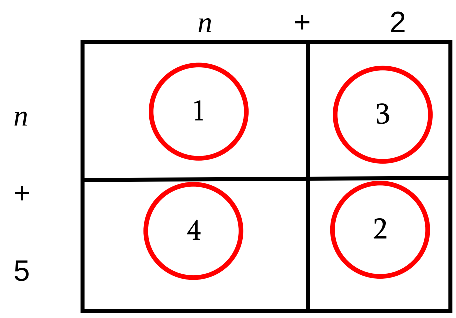


Smiley face

Smiley face can be illustrated as

$$(n + 2)(n + 5) = n^2 + 10 + 2n + n$$

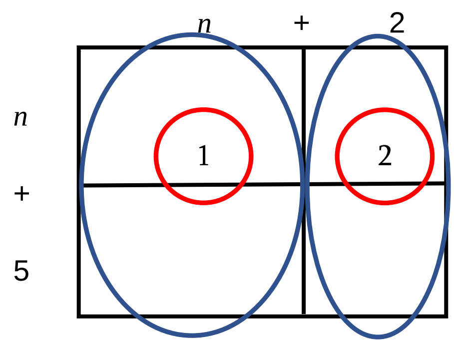
Note that this would just be the addition of the four areas in the order given…



n(n + 5) + 2(n + 5)

This is often explained as 'the first thing in the brackets multiplied by everything in the second set of brackets, plus the second thing multiplied by everything in the second set of brackets'.

In this way of thinking we just see it as two strips. The area of the first plus the area of the second. Then multiply out each of those …



The grid method
It looks like the area representation, but in the area representation we can see why you multiply the letters together, and more importantly, why we add them together.

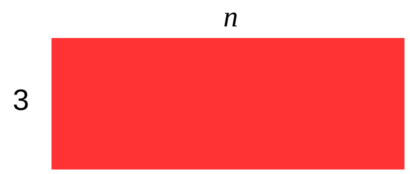
So, all these different ways of doing it are just convenient (or shorthand) ways to remember a process of what to do. You can keep doing any of them – but they all benefit from understanding that they can be linked to finding areas.

Ideas for Closure

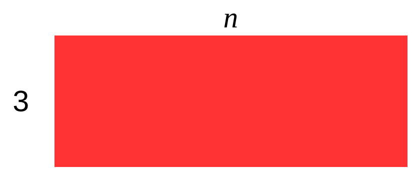
For the mathematical goal 'understand that *n* can represent a variable';

So let's just take a moment to think about what we've done today.

We saw that we use letters in algebra to encourage a number of different ways of thinking. It often depends on the context. In this lesson we've worked with n representing a variable – something that can change. (Remember the rectangles that grew and shrunk).
So the area of the carpet at the start of the lesson was 3n, where n can take many different values.



But notice how the letter takes on a slightly different meaning if we did the following….

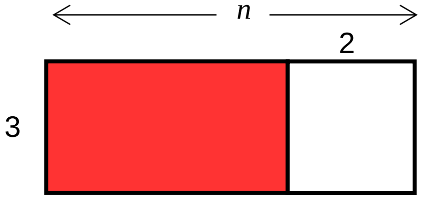


Find n if the area is 27.

In this case, n was a variable – but now there is only one value of that variable that 'satisfies' the problem. You could be asked to do either of these in an exam and you need to be aware of when examiners are thinking of algebra as a variable and when it is one unknown that you can solve.

For the mathematical goal 'create algebraic expressions from area representations'.

Secondly, you may get questions in an exam that require you to create an algebraic expression from an area diagram. We did one, but how would you approach writing an algebraic expression for the red area in this one…



(Encourage students to work in pairs here, think how long the red rectangle is, encourage two different ways to consider the problem. Firstly, write an expression of two lengths that are multiplied together, secondly write as an area subtract another area).

PD3.5 Closure

In Maths-for-Life we use the term 'closure' to indicate drawing a part of the lesson together to ensure a shared understanding of the mathematics that is taking place.

Closure could take place at any point in the lesson but is particularly important towards the end where we want students to leave the classroom feeling that they have progressed in their understanding. There is a risk with lessons based around dialogue and cognitive conflict that as teachers we ignore the students' need for clarity.

Reviewing of the task that took place should be seen as the pre-cursor to closure. This is most effective when the focus is on the methods used – not just on the answer. It is most powerful when different methods are shared and a common understanding of mathematical structure shared.

What makes a good closure phase?

1. **Understand the original problem**
   A return to the initial problem (or context of the original problem) allows students to feel that the lesson had purpose and highlights the importance of the work covered.

2. **Understand implications for exam questions**
   An exam style question helps students to see how the activities and reasoning that they have undertaken during the lesson can help them in an exam.

3. **Understand how the work has developed mathematical thinking**
   The mathematical purpose of the task should be shared as part of the reflection. This requires that the teacher has had clarity throughout the lesson on what the purpose has been. This can then be linked to the learning moments that took place.

Teacher Actions

The following are good principles for encouraging collaborative dialogue during closure and whole group discussion.

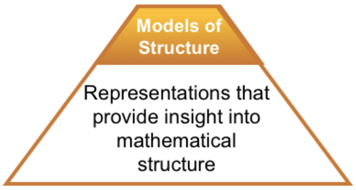
Action	**Reason**
Listen before interrupting	Listen to the what the students are actually saying. It is all too easy to interrupt with a predetermined agenda, diverting attention from their ideas.
Join in, don't judge	Try to listen as an equal member of the class rather than as an authority figure. When teachers adopt judgmental roles, students tend to try and 'guess what's in the teacher's head' rather than try to think for themselves.
Ask students to describe, explain and interpret	The purpose of any teacher intervention is to increase the depth of reflective thought. Challenge students to describe and explain by asking questions such as: • *can you say what that means?* • *can you show us why you're thinking that?*
Make students do the thinking	Many students are experts at making their teachers do the work. They know that if they 'play dumb' long enough, then the teacher will eventually take over. If a student says that he or she cannot explain something, ask another student in the group to explain, or ask the student to choose some part of the problem that they can explain. When a student asks the teacher a question, don't answer it (at least not straight away). Ask someone else in the group to do so.

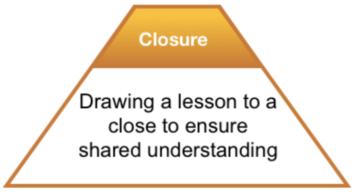
PD3.6 Lesson 3 Research Form

Observe a pair/group of students working during the lesson.

Note down the key moments and development in their mathematical thinking.

Note down examples of the Maths-for-Life pedagogies seen during this lesson.





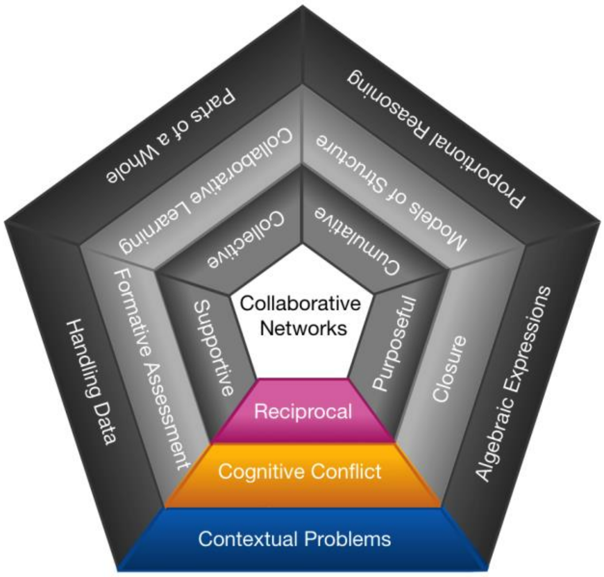
Research Question

How does purposeful dialogue contribute to student understanding during the closure phase of the lesson?

PD4 Session 4: Contextual Problems

Overview

The focus in the lesson, **Contextual Problems**, is on how **cognitive conflict** contributes towards **reciprocal** dialogue.



Cognitive conflict occurs when a student's thinking is challenged by new information that prompts thinking that contradicts/challenges their prior understanding.

Reciprocal dialogue occurs when participants listen, share ideas and consider alternative views with others.

Aims of Professional Development Session 4

1. Enable teachers to successfully teach the lesson, **Contextual Problems**.
2. Explore how the design of resources affects **cognitive conflict**.
3. Develop an understanding of what **reciprocal** dialogue looks like in the classroom.
4. Understand the stages of **cognitive conflict** that lead to effective learning.

Note: These problems are set in simplistic real-world contexts to encourage students to focus on the relationships between the knowns and unknowns.

Checklist for Session 4

☐	Prior to the meeting, ask teachers to look at the documents • **L4.1 Lesson Outline** (in the resource file) • **PD4 Cognitive Conflict** (see teacher handouts)
☐	At the *planning session*, work through **PD4 Professional Development**. This will help teachers understand the resources, pedagogy and aspect of dialogic learning focused on in the lesson, Contextual Problems.
☐	At the *planning session*, ensure that everyone is clear how a research lesson operates including use of the **Collaborative Lesson Research Form**.
☐	At the *planning session*, schedule the *research lesson*. Agreed date of research lesson: ☐
☐	Carry out the *research lesson*.
☐	At the *research lesson*, discuss the learning that has taken place in the lesson alongside a focus on the research question.
☐	At the research lesson, (if necessary) schedule the date for the next planning session (for lesson 5). Agreed date: ☐

1. **Enable teachers to successfully teach the lesson, Contextual Problems.**

Teachers should use the summary **L4.1 Lesson Outline** to understand this lesson.

Lead teachers are additionally supported by the summary in the centre of this page and **PD4.0 Lesson Pedagogy**.

Use **PD4.2 Guided Discussion of Videos** to guide post-video discussions. You may also decide to give teachers a copy of the transcripts to read through (**PD4.2 Guided Discussion of Videos _Teachers**).

"The challenges of seeing the mathematical structure of real-world problems"

After discussing the problem **PD4.1 An Opening Task,** which was used in the trials of the lesson, watch the video **V4.**1 to see how students attempt to make sense of the same problem.

2. **Explore how the design of resources affects cognitive conflict.**

"Misconceptions that can be tackled through cognitive conflict"

Cognitive conflict occurs when students are challenged by new information that contradicts their prior conceptions. In other words, students' prior ideas are incompatible with making sense of the new situations. Confronted in this way, students may be prompted to review and revise their thinking. Knowing the misconceptions students are likely to hold when tackling these types of problems will help teachers provide better support when students' thinking is conflicted. One such misconception can occur when establishing the mathematical structure of a problem. Students often mistakenly identify relationships between just the known values rather than between unknown ones. The video clip **V4.2** exemplifies this type of misconception.

Then discuss the *new* opening task (see **PD4.1 An Opening Task)**. Establish with the teachers, how the task has been changed and improved.

3. **Develop an understanding of what reciprocal dialogue looks like in the classroom.**

"Using resources to facilitate reciprocal dialogue"

Reciprocal dialogue can take place at any point in the lesson. However, it is more likely to occur when problems are challenging - students are then more inclined to use each other as resources to move their thinking on. Asking questions, listening, and responding to each other's explanations can help students better understand the problem and to overcome any hurdles. Such dialogue can be explicitly encouraged through the careful design of the tasks and the actions of the teacher.

Watch the video clip **V4.3**. This illustrates how pre-designed student responses can be used to facilitate productive discussion.

4. **Understand the stages of cognitive conflict that lead to effective learning.**

"Let students have time to explore"
Look at the whole group task *Selling Samosas* in **L4.1 Lesson Outline.** Consider mistakes that may be made and the mathematical focus that needs to be drawn out. Also discuss the benefits of using the student solutions to facilitate cognitive conflict.

Using file **PD4.3 Cognitive Conflict**, note the stages of cognitive conflict. Consider how each of these relate to the *Selling Samosas* task and/or the questions on 2b. Cards. Discuss ways that class teachers' actions can facilitate and inhibit cognitive conflict.

5. **Enable teachers to successfully teach the lesson, Contextual Problems.**

"Help students not to fear algebraic expressions"
Watch video clip **V4.4 Connecting Solutions** to see how teacher and students synthesise the different approaches taken by students.

Introduce the research question for this lesson:

How can cognitive conflict provide the opportunity to develop reciprocal dialogue?

Mathematical goals

To help students:

- determine the unknown in a problem;
- translate statements that relate to an unknown into a mathematical form;
- determine the value of an unknown in a problem.

Engaging with the lesson outline

Engage Question	Prompts
1. What are the advantages of using trial and improvement, and when might it be problematic?	Trial and improvement will give students the confidence to at attempt these kinds of questions. Students should notice that it is potentially inefficient, and difficult when values are 'difficult'. The important positive aspect to take from trial and improvement are that you are able to identify which object gets the initial guess, the relative values of the related objects and that parts add to make a whole.
2. What does a box in the blue method represent?	One box represents the unknown value of our base item. All other boxes relate to this. So, two boxes indicate two lots of the unknown value. Clicking on the 'Hint' button should help students to see that the trial and improvement method has the same mathematical structure as the diagrammatic representation.
3. How do you expect students to tackle this question?	Some students may still attempt trial and improvement. If so, then ask them to draw a box for their initial guess and to proceed from there.
4. How can you encourage students to check whether their answer is sensible?	Ask students to see if their final answers make sense in the question and check that their totals are correct. Sometimes, the total may be correct, but student answers are incorrect. When this is the case, ask students to check the relationships between the amounts.
5. What do you expect students to find difficult in this question?	Some students will find it difficult to spot that the relationship between the three objects has changed. In question 1 the structure is that B and C are both expressed in relation to A. In question 2 the structure is that B is expressed in relation to A, but C is expressed in relation to B.
6. What is the best way to help them?	Once again, it may be worth encouraging students to try trial and improvement so that they can see how the problem works.

	From here they may then be able to construct a diagrammatic representation.
7. What are the important differences in the two methods that should be highlighted to students?	The intention of the two solutions is to prompt students to address a commonly held misconception that variables have a fixed value. The boxes, although drawn with similar dimensions, have different values on the two days. The situation on Monday is different from the situation on Tuesday – the relationship between the two types of samosas is different. Some students may want to resolve this issue by drawing different size boxes.
8. What do you expect students to find difficult in this question?	Some students set this problem up in the form $3x+12=132$. (Three child tickets and £12 to give the total), rather than $3x + (x + 12) = 132$. If students have used a representation, ask them to point to each box and explain what it represents.
9. What do you expect students to find difficult in this question?	Some students may have difficulty in using the fact that the angles of a triangle add up to 180^o. Encourage students to write what they know on the diagram.
10. What are the key messages from the lesson that you expect to draw out?	It is vital that thinking is brought together at the end of the lesson. Some of what you share should be based on aspects of the problems that students found straightforward, and those they found difficult. These tasks have required students to identify a 'base' value that everything else relates to. Once that is identified a representation can be drawn that captures the structure of the relationships between values in a way that allows first one unknown, and then the others, to be evaluated. Additional points to highlight may include reminding students of the difference between "more" and "times as many". Also stress the power of the diagrammatic representation to model problems, and how it helps us to make sense of algebraic models. You may also want to mention that the value the box represents, will vary from one situation to the next.
11. What is different about this extension question?	Students are expected to remember that angles on a straight line add up to 180^o. Angle 3 is **smaller** than angle 2. This may cause students some problems, but once again, a trial and improvement strategy may help them to see the structure of the problem, before attempting to model with a diagram.

PD4.1 An Opening Task

The opening task has been designed with three key principles in mind. These are outlined below:

1. **Facilitate access to the main activities**.
   Students would attempt questions very similar to the *Buying Food* problem.
2. **Create opportunities for reciprocal dialogue**.
   Students may struggle to represent the situation mathematically. This in turn may encourage students to listen carefully to each other's ideas and respond by agreeing, challenging, or rejecting them.
3. **Generate cognitive conflict.**
   This may occur as students use their diverse prior understanding to decide how to translate the real-world setting into mathematics that will allow them to figure out the unknowns.

Opening task in the trials of the lessons

The original design of the opening task is shown below. This was used in the trials of the materials.

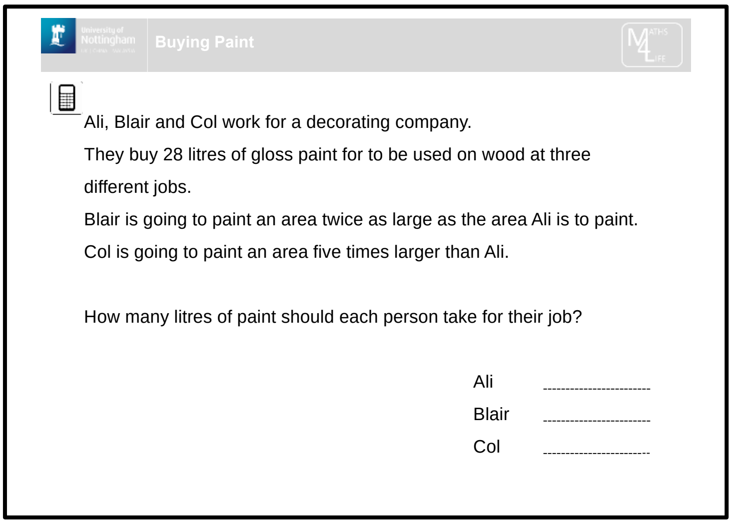
> University of Nottingham | **Buying Food**
>
> Joel, buys a meal consisting of a drink, chips and a burger from a restaurant.
>
> The chips cost 2 times more than the drink.
>
> The burger costs 3 times more than the drink.
>
> All together the meal costs £6.60.
>
> Find the individual cost of the drink, the chips and the burger.
>
> Drink -----------------------
>
> Chips -----------------------
>
> Burger -----------------------

Work through the task *Buying Food.* Consider with the teachers:

- How their students would tackle the task.
  Including the advantages and disadvantages of different approaches students may use.
- The mistakes students are likely to make.

New opening task

After watching and discussing video clips V4.1 and V4.2, consider with the teachers the new, replacement opening task:



Ask the teachers to:

- Identify the changes that were made, and suggest reasons for those changes.
- For students who make sense of the problem in a way similar to those seen in the Video (V4.2), consider how teachers could facilitate cognitive conflict in order to encourage students to review their thinking.

PD4.2(LT) Guided discussions: Video Clips

V4.1 Video Clip: Seeing mathematical structure

Correctly identifying the mathematical structure within these real-world problems requires establishing the relationships between known and unknown values. An algebraic, or diagrammatic approach can help, but a trial and improvement approach can obscure some relationships. This clip shows students encountering difficulties understanding the overall relationship between the values, when using a trial and improvement approach.

Introductory video captions

Two students have been working on the Buying Food problem.

(This problem was used in the trial version of the materials. It is not included in the final release.)

They have used trial and improvement to figure out the values for the drink, chips and burger.

As you watch the video, notice students' understanding of the relationship between the unknowns and knowns.

Think about how their trial and error approach facilitates these relationships – or not.

Problem tackled

Buying Food

Joel, buys a meal consisting of a drink, chips and a burger from a restaurant.

The chips cost 2 times more than the drink.

The burger costs 3 times more than the drink.

All together the meal costs £6.60.

Find the individual cost of the drink, the chips and the burger.

Drink ....

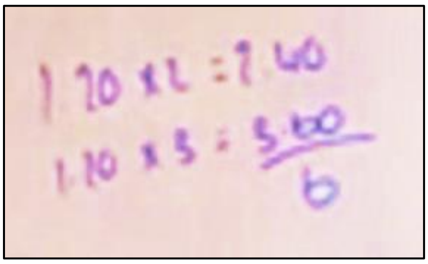
Chips ....

Burger ....

Transcript of dialogue

The transcript is not included here because the talk on the video is clear.

Student work on the white-board:



Post video guided discussion

Discuss with the teachers:

- The mistakes the students have made, and what these mistakes reveal about their understanding of the relationship between the unknowns.
  Teachers may suggest students understand the relationship between the price of the drink and the price of the chips and burger, but do not appear to understand how all three prices relate to the total price of £6.60.
- How the teacher attempted to challenge the pair's thinking.
  Teachers may consider how, through his questioning, the teacher attempts to separate out the different parts of the total cost. They may have also noticed the difficulty of ensuring students do not lose face in front of their class mates, but at the same time ensuring mathematical standards are maintained.
- Alternative ways the pair could be challenged to rethink their approach.
  Teachers may suggest other students could present their approaches, or ask students to write out the receipt Joel would receive.

V4.2 Video Clip: Tackling a Misconception

Problems set in a real-world context provide students with some values, and the students are tasked to figure out other unknown values. Students can find it challenging to establish relationships between unknown values, and then use this to establish a relationship with known values. They often assume that what is needed is the creation of a simpler relationship - between the known values and an unknown one. The video clip illustrates students working with such a misconception in their approach.

Introductory captions for first part of video clip

Students are working in small groups to solve the Buying Food problem.

(This problem was used in the trial version of the materials. It is not included in the final release.)

Notice how students try to make mathematical sense of the real-world problem.

Think about the mathematical relationships they established.

Problem tackled in video clip

University of Nottingham | **Buying Food** | M4L

Joel, buys a meal consisting of a drink, chips and a burger from a restaurant.

The chips cost 2 times more than the drink.

The burger costs 3 times more than the drink.

All together the meal costs £6.60.

Find the individual cost of the drink, the chips and the burger.

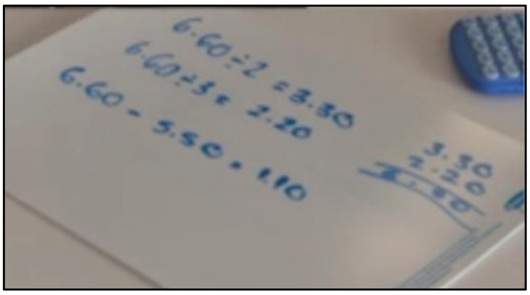
Drink ....

Chips ....

Burger ....

Transcript of dialogue of first part of video clip

Student 1	What yer doing?
Student 2	Oh, I’ve done it wrong.
Student 1	It’s what you have left for the six sixty.
Student 2	Oh … I get it.
Student 1	Whilst you pay for the chips.
Student 2	Six sixty … what do you have to divide it by; can’t remember. Is it two, three, five …. I’ve got this ….
Student 2	One pound ten
Student 3	So what’s is that for?
Student 2	The drink init? Yeah the drink. There the chips,
Student 3	Yeah three.
Student 2	Then that’s the burger.
Student 3	Burger, yeah.
Student 2	Then you add them up and that makes five pound fifty altogether; then you have to take that away from that to get the drink.
Student 3	Yeah I get you now.
Student 2	Do you get it?
Student 3	Errr ..
Student 2	I get it. Do you get it?
Student 3	Because you add that ..
Student 2	That add that is five pound fifty, then you six pound sixty take away five pound fifty is one pound ten which is for the drink.
Student 1	Even better add ‘em all up to see if it gets to six sixty.

Student 3	Yeah.
Student 2	Add what up? Oh yeah.
Student 1	Are we supposed to do these without a calculator?
Student 2	I don't care.
Student 2	Yeah I can. oh wait add
Student 3	What did you get?
Student 2	Six pound sixty.
Student 1	Yeah … I 've done it; so happy.
Student work:	

Discussion prompt for the first part of the video clip
Once the video has paused, discuss with the teachers what they have noticed about the mathematical relationships established by the students. Reflect on why the student found it difficult to work out the cost of the drink using division.
Teachers may suggest students appear to simply set up relationships between the known amounts (e.g. 6.60 divided by 2 – both figures appear in the question). To use a division to work out the value of the drink was difficult because the two known figures of two and three had already been 'used up'). Establishing relationships between two unknowns can be challenging for students.

Introductory captions for second part of video clip
The same student now explains their approach to solving the problem to the whole class. As you watch, think about ways to help the students review their thinking through the use of 'cognitive conflict'.

Transcript of second part of video clip
The transcript is not included here because the talk on the video is clear.

Post video guided discussion
Discuss with the teachers:

- How can the teacher create a 'cognitive conflict' situation to help students review and revise their thinking?
  The teachers may suggest asking other students with a different approach to explain it to the class, or giving the students another problem in which the rules they are applying would not work (e.g. the task shown below).

- What is it about the problem that confirms students' erroneous thinking?
  Teachers may consider that the numbers in the problem allow students to obtain the correct answers using an incorrect approach.

Buying Paint

Ali, Blair and Col work for a decorating company.
They buy 28 litres of gloss paint for to be used on wood at three different jobs.
Blair is going to paint an area twice as large as the area Ali is to paint.
Col is going to paint an area five times larger than Ali.

How many litres of paint should each person take for their job?

Ali ....
Blair ....
Col ....

V4.3 Encouraging reciprocal dialogue

Working with anonymous designed student responses to a problem can encourage students to focus on understanding rather than performance. The video clip exemplifies how students can work productively together employing reciprocal dialogue in order to arrive at a joint understanding of the responses.

Introductory video captions

Students have just tackled the Buying Food problem.

They are now attempting to understand some student responses to the same problem. These are responses that the teacher has shared with them.

Whilst watching the video, notice how the students respond to each other's comments.

In this clip the discussion refers to the second method (shown in the green box).

Problem and designed student responses discussed in video clip

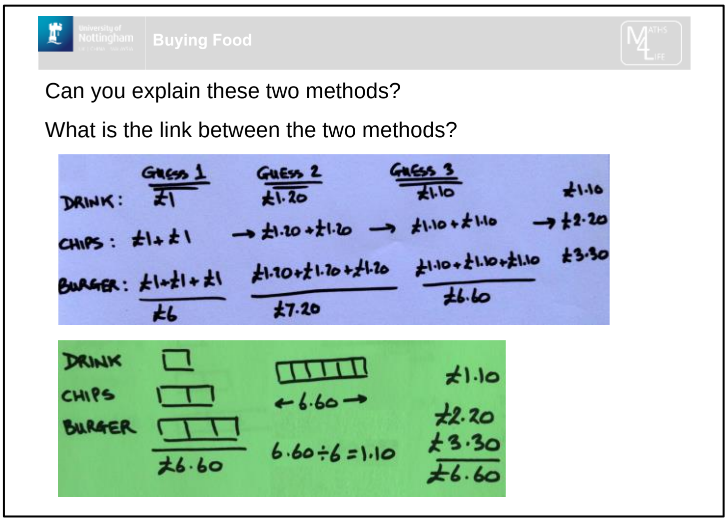
Buying Food

Joel, buys a meal consisting of a drink, chips and a burger from a restaurant.
The chips cost 2 times more than the drink.
The burger costs 3 times more than the drink.
All together the meal costs £6.60.

Find the individual cost of the drink, the chips and the burger.

Drink ....
Chips ....
Burger ....

Designed student responses:



Transcript of dialogue

Student 1	When it says six sixty divide like six.
Student 2	Yeah.
Student 1	It is one err one pound ten yeah.
Student 2	Yeah.
Student 1	Why do they do like divide by six?
Student 2	because you have got one drink haven't you
Student 1	Yeah.
Student 2	Then two chips and three burgers.
Student 1	Oh yeah.
Student 2	Where more .. six.
Student 1	So you've got one
Student 3	The second one looks confusing
Student 2	Do you think?
Student 3	Yeah, because of all those little boxes
Student 1	Yeah well, so if you add them up you've got six and so you have to divide this by six, so you've got one pound ten. So what is the price for? For the chips or?
Student 2	One pound ten will be for the drink.
Student 1	Yeah.
Student 1	One pound ten times it by two.
Student 2	Oh yeah.
Student 1	Which is two twenty.
Student 2	Yeah.

Student 1	Then times the one pound ten by three, which is three thirty. So add them all up gives you six pound sixty.
Student 4	I get it.
Student 1	So the drink is like one.
Student 1	You'd want errm .. the chips is like two times of the drink, and then the err.
Student 2	Burger.
Student 3	Burger.
Student 2	Is three times.
Student 1	You've got three boxes. So when you write it out, which it'd be six.
Student 2	Yeah.
Student 1	So six.
Student 2	One ten.
Student 1	sixty divided by six equals one ten, which that will be ten which that will be the drink. Then you do one pound ten, times that by the two, which is two twenty. Add the one pound ten again, times it by three which is three thirty.
Student 4	Which is the burger.
Student 2	The burger.
Student 1	For the burger. So add them all up, gives you the six pound sixty.

Post video guided discussion

Discuss with the teachers:

- How students responded to each other's comments.

Teachers may reflect on the way students confirmed each other's contributions, answered questions and were prepared to say when they did not understand. They may have also noticed that one student in particular did not extensively contribute to the conversation. This did not mean, however, that they were not learning.

- The ways the designed student responses facilitated the students' discussion.

Teachers may suggest that, unlike students' evolving and often incomplete understanding, designed student responses are coherent, correct and static. This lowers the demand for students. Also, students can criticise them without worrying about other students' feelings. Likewise, as the responses are anonymous, the perceived mathematical prowess of the author cannot influence students' evaluation of the solution. As such students may feel more able to negatively critique the responses.

Handouts for teachers session 4

PD4.1 An opening Task

The opening task has been designed with three key principles in mind. These are outlined below:

1. **Facilitate access to the main activities**.
   Students would attempt questions very similar to the *Buying Food* problem.
2. **Create opportunities for reciprocal dialogue**.
   Students may struggle to represent the situation mathematically. This in turn may encourage students to listen carefully to each other's ideas and respond by agreeing, challenging, or rejecting them.
3. **Generate cognitive conflict.**
   This may occur as students use their diverse prior understanding to decide how to translate the real-world setting into mathematics that will allow them to figure out the unknowns.

Opening task in the trials of the lessons

The original design of the opening task is shown below. This was used in the trials of the materials.

Buying Food

Joel, buys a meal consisting of a drink, chips and a burger from a restaurant.

The chips cost 2 times more than the drink.

The burger costs 3 times more than the drink.

All together the meal costs £6.60.

Find the individual cost of the drink, the chips and the burger.

Drink ....

Chips ....

Burger ....

New opening task

After watching and discussing video clips V4.1 and V4.2, consider with the teachers the new, replacement opening task:

Buying Paint

Ali, Blair and Col work for a decorating company.

They buy 28 litres of gloss paint for to be used on wood at three different jobs.

Blair is going to paint an area twice as large as the area Ali is to paint.

Col is going to paint an area five times larger than Ali.

How many litres of paint should each person take for their job?

Ali ------------------------

Blair ------------------------

Col ------------------------

PD4.2(T) Transcript of selected video clips

V4.2 Video Clip: Tackling a Misconception

Problem tackled in video clip

Buying Food

Joel, buys a meal consisting of a drink, chips and a burger from a restaurant.

The chips cost 2 times more than the drink.

The burger costs 3 times more than the drink.

All together the meal costs £6.60.

Find the individual cost of the drink, the chips and the burger.

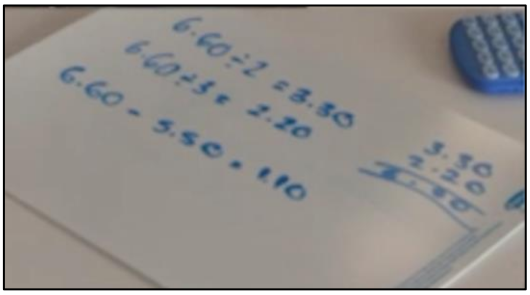
Drink ....

Chips ....

Burger ....

Transcript of dialogue of first part of video clip

Student 1	What yer doing?
Student 2	Oh, I've done it wrong.
Student 1	It's what you have left for the six sixty.
Student 2	Oh … I get it.
Student 1	Whilst you pay for the chips.
Student 2	Six sixty … what do you have to divide it by; can't remember. Is it two, three, five …. I've got this ….
Student 2	One pound ten
Student 3	So what's is that for?
Student 2	The drink init? Yeah the drink. There the chips,
Student 3	Yeah three.
Student 2	Then that's the burger.
Student 3	Burger, yeah.
Student 2	Then you add them up and that makes five pound fifty altogether; then you have to take that away from that to get the drink.

Student 3	Yeah I get you now.
Student 2	Do you get it?
Student 3	Errr ..
Student 2	I get it. Do you get it?
Student 3	Because you add that ..
Student 2	That add that is five pound fifty, then you six pound sixty take away five pound fifty is one pound ten which is for the drink.
Student 1	Even better add ‘em all up to see if it gets to six sixty.
Student 3	Yeah.
Student 2	Add what up? Oh yeah.
Student 1	Are we supposed to do these without a calculator?
Student 2	I don’t care.
Student 2	Yeah I can. oh wait add
Student 3	What did you get?
Student 2	Six pound sixty.
Student 1	Yeah … I‘ve done it; so happy.
Student work:	

V4.3 Video Clip: Encouraging reciprocal dialogue

Problem and designed student responses discussed in video clip

Buying Food

Joel, buys a meal consisting of a drink, chips and a burger from a restaurant.

The chips cost 2 times more than the drink.

The burger costs 3 times more than the drink.

All together the meal costs £6.60.

Find the individual cost of the drink, the chips and the burger.

Drink ------------------------

Chips ------------------------

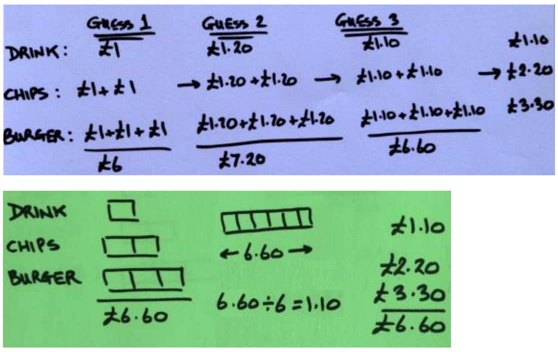
Burger ------------------------

Designed student responses:

Buying Food

Can you explain these two methods?

What is the link between the two methods?



Transcript of dialogue

Student 1	When it says six sixty divide like six.
Student 2	Yeah.
Student 1	It is one err one pound ten yeah.
Student 2	Yeah.
Student 1	Why do they do like divide by six?
Student 2	because you have got one drink haven't you
Student 1	Yeah.
Student 2	Then two chips and three burgers.

Student 1	Oh yeah.
Student 2	Where more .. six.
Student 1	So you've got one
Student 3	The second one looks confusing
Student 2	Do you think?
Student 3	Yeah, because of all those little boxes
Student 1	Yeah well, so if you add them up you've got six and so you have to divide this by six, so you've got one pound ten. So what is the price for? For the chips or?
Student 2	One pound ten will be for the drink.
Student 1	Yeah.
Student 1	One pound ten times it by two.
Student 2	Oh yeah.
Student 1	Which is two twenty.
Student 2	Yeah.
Student 1	Then times the one pound ten by three, which is three thirty. So add them all up gives you six pound sixty.
Student 4	I get it.
Student 1	So the drink is like one.
Student 1	You'd want errm .. the chips is like two times of the drink, and then the err.
Student 2	Burger.
Student 3	Burger.
Student 2	Is three times.
Student 1	You've got three boxes. So when you write it out, it'd be six.
Student 2	Yeah.
Student 1	So six.
Student 2	One ten.
Student 1	sixty divided by six equals one ten, which that will be ten which that will be the drink. Then you do one pound ten, times that by the two, which is two twenty. Add the one pound ten again, times it by three which is three thirty.
Student 4	Which is the burger.
Student 2	The burger.
Student 1	For the burger. So add them all up, gives you the six pound sixty.

PD4.3 Cognitive Conflict

Cognitive conflict occurs when thinking is challenged by new information that contradicts prior ideas.

A student may find that new information in a newly introduced situation challenges their existing thinking in ways that their understanding is insufficient to produce a solution and therefore they must act to resolve this conflict. The situation of being ‘not sure’ creates a space for new learning to occur. As the conflict has been discovered by the students they are motivated to resolve the inadequacy of their current understanding.

It is vital that the teacher has predicted issues that are likely to cause cognitive conflict. This is an important precursor to knowing how long students can be left wrestling with a task to gain maximum benefit.

The following table provides a helpful structure for understanding the stages of cognitive conflict in Maths-for-Life lessons.

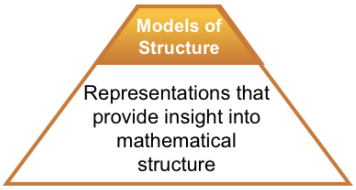
Establish	An initial period of students working within bounds of their current understanding helps to set up the moment of cognitive conflict. Students start by working with ideas that they are comfortable with. These may be with, or without, a reminder from the teacher of the thinking required to start the task.
Explore	Students must have time to explore and experiment as they attempt the task. Opportunities for cognitive conflict are often missed by teachers prematurely trying to resolve issues.
Identify	The ideal is that students notice for themselves that their prior ideas and/or methods are not adequate for the new problem situation. This heightens the impact of the cognitive conflict on learning. If necessary, the teacher may help students to identify the issue. For example, this could be by asking students to check whether their suggested solution method ‘works’ and if not, why not? The following kinds of phrases may help students: • ‘What is it that is different about this question…?’ • ‘Can you explain …?’ • ‘Why do you think that?’ • ‘How do you know that?’

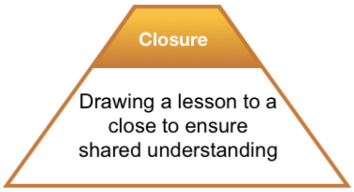
PD4.4 Lesson 4 Research Form

Observe a pair/group of students working during the lesson.

Note down the key moments and development in their mathematical thinking.

Note down examples of the Maths-for-Life pedagogies seen during this lesson.





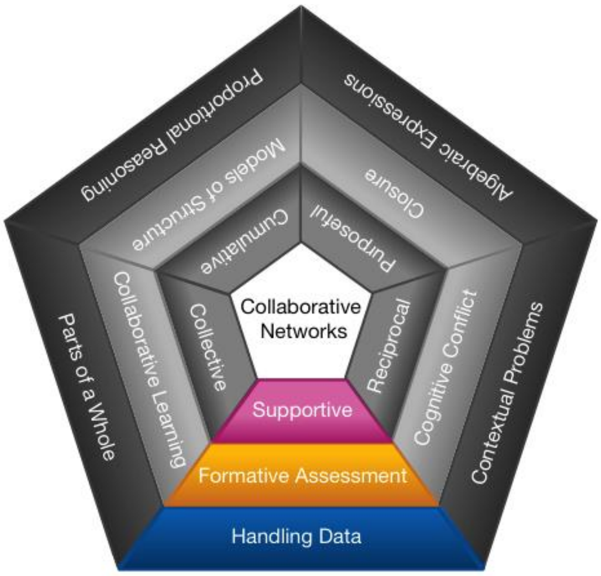
Research Question

How does the management of cognitive conflict help to develop a culture of reciprocal dialogic learning?

PD5 Session 5: Handling Data

Overview

The focus in the lesson, **Handling Data**, is on how **formative assessment** contributes towards **supportive** dialogue.



Formative assessment is assessment that provides information on what to do next.

Supportive dialogue is when ideas are expressed freely, without risk of embarrassment.

Aims of Professional Development Session 5

1. Enable teachers to successfully teach the lesson, **Handling Data**.
2. Understand the stages of **formative assessment** that lead to effective learning.
3. Develop an understanding of what a **supportive** dialogue looks like in the classroom.

Checklist for Session 5

☐	Prior to the meeting, ask teachers to look at the documents • **L5.1 Lesson Outline** (in the resource file) • **PD5 Formative Assessment** (see teacher handouts)
☐	At the *planning session*, work through **PD5 Professional Development**. This will help teachers understand the resources, pedagogy and aspect of dialogic learning focused on in the lesson, Handling Data.
☐	At the *planning session*, ensure that everyone is clear how a research lesson operates including use of the **Collaborative Lesson Research Form**.
☐	At the *planning session*, schedule the *research lesson*. Agreed date of research lesson: ☐
☐	Carry out the *research lesson*.
☐	At the *research lesson*, discuss the learning that has taken place in the lesson alongside a focus on the research question.

Professional Development Session 5

1. **Enable teachers to successfully teach the lesson, Handling Data.**

   Teachers should use the overview **L5.1 Lesson Outline**, to understand this lesson.

   Lead teachers are additionally supported by the summary in the centre of this page and **PD5.0 Lesson Pedagogy**.

   Use **PD5.2 Guided Discussions of Video Clips** to guide the post video discussions. There are no transcripts for the teachers as all the talk on the videos is clear.

2. **Effective formative assessment requires both an understanding of the challenges students are likely to face, and the use of teaching strategies to help students address them.**

   "We need to know the common issues students may encounter"

   To engage productively in formative assessment requires teachers to, in-the-moment of the lesson, note students' thinking, and then if necessary, adapt their instruction. To support this process, it is important to know, before the lesson, issues students are likely to encounter. Then teachers can consciously look out for them in the lesson.
   Ask teachers to tackle the pre-assessment problem. Then discuss the issues that are likely to arise**.**
   Hand out, **PD5.1 Student Work** and discuss the specific issues students have encountered.
   Also refer the teachers to PD5.2 Formative Assessment
   Show video **V5.1 Common issues students face**.

   "We need to make student thinking visible"

   Ask teachers to, in pairs, place the cards on the template (**L5.2 and L5.3**). The charts have been carefully designed to draw out common issues. For each chart, discuss with the teachers, the mistakes students are likely to make, and possible ways to help students address them. You may now also want to consider the table of student issues and suggested prompts in **L5.1 Lesson Outline.**

3. **Enable teachers to successfully teach the lesson, Handling Data.**

   "Which teaching strategies will help students address issues"
   Deciding how to best support students can be difficult. It can be tempting to tell students what to do next. This may improve performance in the short term, but is unlikely to address underlying issues. Show video **V5.2 Teaching strategies to support learning.**

4. **Explore how the structure of the task can help with formative assessment.**

   "We need to make sure that students understand what they need to learn"

   Recognising the gap between students' current thinking and the learning goals of the lesson requires a clear understanding of the goals. When students as well as teachers understand the goals, they too can judge the quality of their own work, and take steps to repair any deficiencies.
   First discuss with the teachers whether it is their usual practice to inform their students of the learning goals of the lesson, why they do/do not do this, and any difficulties they've encountered doing so.
   Then show video **V5.3 Ensuring a purposeful lesson.**

5. **Develop ways to encourage supportive learning in the classroom.**

   "Encourage students with being supportive in their learning"

   Whole class discussions are not only an opportunity to progress the collective learning of the class, but teachers can use them to model what supportive learning looks like. Discuss with teachers why whole class discussions can play a key role in promoting a supportive classroom.

   Show video **V5.4 Creating a supportive Classroom.**
   Introduce the research question for this lesson:

In the Handling Data lesson, how does the use of formative assessment help to develop an environment of supportive dialogic learning?

PD5.0 Lesson Pedagogy

Mathematical goals

To help students:

- understand the relationship between data and its representations;
- understand and be able to find measures of location and spread.

Engaging with the lesson outline

Engage Question	Prompts
1. How will the information you find out about the students' understanding affect the way you teach this lesson?	This may depend on what mistakes are seen, so it is worth thinking about the table of common issues and how you might adjust your lesson accordingly. You may be prompted to change the amount of time you spend on clarifying one of the measures of location. It may help you to see mistakes such as students wanting to leave out people that ate zero portions from the mean calculation.
2. Is this the 'best' definition of the median?	There are a number of definitions that could be used – the aim in this presentation was to give one that is simple and accessible to students. You may also want to ask students why the data must be in order.
3. How can you ensure that the data you collect from students and yourself is as helpful as possible?	Try to get an odd number of pieces of data (as this will make finding the median simpler). If there are an even number of students in the classroom then include yourself in the data collection. As an added challenge, you may like to try to get an integer mode, and even try to make the total divisible by the number of pieces of data!
4. Why do we use the word 'frequency'?	Frequency is a general term that can be used when working with data. It is usually an abbreviation of a longer, more specific phrase.
5. What different ways can be used to help students make sense of finding the median?	Many students will have simply been told to find the median by adding one to the number of pieces of data and halving to find where the median is positioned. However, it is difficult to make sense of why we do this. Two alternatives are: 1. Repeat removing one value from each end before being left with the middle value. 2. Understand that there must be the same number of pieces of data either side of the median value. This

	means that we subtract one from the number of pieces of data (take out the middle value) and halve.
6. Why is this an important stage in the lesson?	It is helpful to remind students that we find summary statistics to allow comparison of data. There is purpose to what we do.
7. How will you react if students are finding the task difficult?	You may like to bring the class back together and ask them to share strategies with one another. Basing matchings around mode and range, as far as possible, is a good strategy for many students.
8. Which bar charts in the matching can be referred to in these questions?	1. All cards 2. B3, B6, B7 3. B6, B7
9. How can you convince students of this?	The mode and median and range are relatively straightforward. For the mean, *n* people eat one more portion each, giving a total of *n* more portions. If this is divided by the number of people, then the mean increases by one. It may help students to demonstrate this with their own data.
10. What are the key messages from the lesson that you expect to draw out?	It is vital that thinking is brought together at the end of the lesson. Some of what you share should be based on good thinking, or difficulties, that you saw in the classroom. Points to highlight may include reminding students of the importance of understanding how data is represented. (It may be in a list, table, or bar chart). It may be worth looking back at the original definitions of the measures of location.
11. Can you identify the mistakes and misconceptions that have been made in the electronic presentation?	Note that there is a helpful animation designed to show that each block in the bar chart represents a person. All of the slides are not provided in the lesson outline as they take up too much space. It is well worth going through each of the slides to understand what has happened. 1. A common mistake is to subtract frequencies to find the range. 2. A second mistake in calculating range is to subtract the lowest value on the *x* axis from the highest. To help students make sense of this, ask how many more penalties must the person who scored the least score to match the person that scored the most? 3. The modal value is often taken from the frequency. 4. A common mistake in finding the mean is to add the frequencies. 5. Students should also watch out for calculator mistakes. Typing 0+4+9+28÷4 will not give the correct answer. 6. Students should discuss whether this answer would get them a mark in an exam.

Teachers will probably need to see each video clip at least twice, in order to fully understand what is happening in them.

V5.1 Video Clip: Common issues students face

The purpose of the clip is to highlight the common practice of students referring to averages simply as numbers disconnected from context and the subsequent misunderstandings this can engender. Separation of the problem from the context can encourage students to simply follow procedures, and may result in students confusing the different definitions of the averages. This in turn will limit how students are able to tackle unfamiliar, or challenging problems.

Introductory captions for first part of video clip

The video clip shows how students describe range, mean, and median when working on the various problems from the project.

As you watch the video, think about why these descriptions are problematic, and how you could support students to improve their descriptions.

Transcript of dialogue

The transcript is not included here because the video is clear.

Guided discussion prompt for the first part of video clip

The clip highlights the issue of students defining averages in a way that excludes the units of measure.

Once the video has paused, discuss:

- Whether teachers have encountered the ways students in the video describe measures, and the limitations of these descriptions. You may want to particularly refer to issues of finding averages from charts when the categories, as well as the frequencies are numbers.
  Teachers may suggest that the use of descriptions such as 'divide how many there are by the total' lacks specificity and can lead to errors in finding the measures. It can also lead to incorrect, or confused understanding of the meaning of the measures. This lack of understanding may prompt students to mix up the definitions of the various averages.

Transcript of second part of video clip

The transcript is not included here because the video is clear.

Post video guided discussion

The inclusion of the video clip after the pause, is to confirm the insufficiency of the description of the median in terms of the ‘Middle number’. Here it is important to emphasise how each of the mini-whiteboads represents a person and we want to find the number of pieces of fruit the middle person eats (when people are ranked in order of how many pieces of fruit they eat).

V5.2 Video Clip: Teacher strategies to support learning

The two-part video clip shows two instances of the teacher supporting student learning. The first relates primarily to how the teacher encourages students to relate the numbers they have calculated back to the context of the problem. In this way, the calculations and the measure they find, are more likely to be meaningful. The second instance relates to how the teacher seeks to draw attention to the students, what progress they have made. This strategy not only helps motivate students, but helps secure their new understandings.

Introductory captions for first part of video clip

Two students have just used a chart to explain how they worked out the mean portion of fruit consumed. This is part of a whole class discussion

The teacher now facilitates a review of this explanation.

As you watch, think about the purpose of the teacher’s questions and his contributions.

Notice also any difficulties students have with explaining their thinking.

Transcript of first part of video clip

Occasionally student’s talk is not clear. These subtitles are included in the video.

Guided discussion for the first part of the video clip

After pausing the video, discuss with teachers:

- The purpose of the teacher’s questions, and how he supported the students’ thinking. *Teachers may suggest the students were following the procedures but not relating them to the real-world context. Asking them to add the unit of measure to each of the numbers makes the calculations more meaningful to both the pair at the front of the class and the rest of the class. On occasions the teacher echoed the student’s explanations. This not only encourages student autonomy, but helps embed understanding.*

Introductory captions for second part of video clip

The students are working in pairs on the card problems. The teacher has notices that one pair have just revised how they figure out the median from the information on the chart.

Notice how the teacher supports the students in their newly established way of thinking.

Transcript of second part of video clip
Occasionally student's talk is not clear. These subtitles are included in the video.

Post video guided discussion
After watching the video, discuss with teachers:

- How the teacher supports the students in their newly established way of thinking. *Teachers may suggest the teacher in the clip, through his questioning, draws attention to the way the students have altered their understanding of median. Helping students to recognise the progress they have made can be motivational, and help to secure their new understanding. Teachers may also note that the teacher in the clip asks **both** students to express in their **own** words the meaning of a bar. Asking students to express out loud their understanding can prompt students to organise more coherently their thinking and in so doing encourages aspects of dialogic learning.*

V5.3 Video Clip: Ensuring a purposeful lesson
It can be useful to let students know the learning goals of the lesson. Such information gives both purpose to the activities, and supports students in monitoring the extent their current performance is helping them achieve the goals. It may be difficult, however, for teachers to express the goals in a way that is meaningful to students. Having a specific task to refer to helps make the goals explicit for both teacher and students. The clip shows the teacher making use of the knowledge gained from reviewing the students' work on the pre-lesson assessment task.

Introductory captions for first part of video clip

The teacher has reviewed students' work on the pre-lesson assessment task. At the start of the lesson he comments on this review to the whole class.

Whilst watching the video, how the teacher's approach to informing students on the learning goals of the lesson, differs from your own.

Transcript of dialogue
The transcript is not included here because the video is clear.

Post video guided discussion
Discuss with the teachers how the pre-lesson assessment facilitates:

- The teachers' own understanding of the learning goals.
  Teachers may reflect on how the specificity within solutions to a problem can help clarify the learning goals.

- Communication to students of the learning goals.
  Teachers may suggest having a problem to refer to may also help clarify the learning goals for students. The second part of the clip highlights how the teacher can help use the results from the pre-lesson assessment to highlight the learning goals of the lesson, and how students are progressing towards them.

V5.4 Video Clip: Creating a supportive classroom

Creating a supportive classroom culture entails both the teacher supporting students, and students supporting each other. The teacher can play a crucial role in fostering this culture. The video clip shows just two ways of doing this – there are many other ways. The first clip depicts an episode in which students are encouraged to support each other's learning. The second clip shows how the teacher supportively deals with students' incorrect contribution to the whole class discussion.

Note: Although the clip is split into two sections, there is no need to pause for a discussion after the first section. Allow the clip to play to the end before instigating a review of how teacher and students behaved.

Introductory captions for first part of video clip

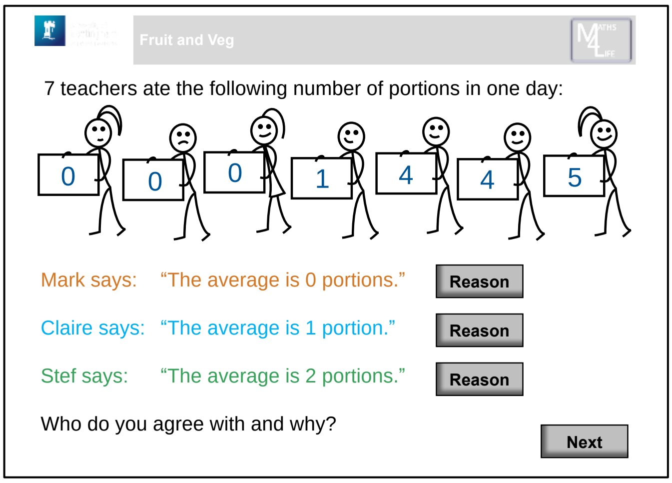
The clip shows two events. In each, students are discussing what is meant by average.

As you watch each, think of how teacher facilitates a supportive culture.

Event 1:
The teacher is holding a whole class discussion to explore students' understanding of the term average, and the meaning of mean, mode and median. He is using the Fruit and Veg problem.
Together they are evaluating three statements.
Students have already considered Mark and Claire's statement, and are now evaluating Stef's comments.
Two students have just declared that Stef is incorrect.

Event 2: The teacher is holding a whole class discussion.
Students have just worked out that the mean portion of fruit the class ate yesterday was" 2.5".
The clip opens with the teacher encourages the students to explain the meaning of 'the mean'.

The task the whole class is discussing during the first event



Transcript of dialogue
The transcript is not included here because the dialogue is clear.

Post-video guided discussion of both events

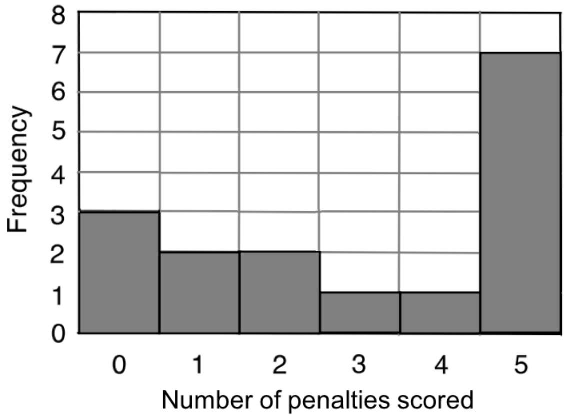
After watching the video, discuss

- Event 1: How the teacher handled the mistake, and how his actions could support students.
  Teachers may suggest that students getting things wrong in a public situation, such as a whole class discussion, has the potential to damage their social status. Such incidents need to be handled carefully (but not avoided). In this case, the teacher has been working with the class for some time and has already developed a supportive classroom where making mistakes is accepted as an integral part of learning. The teacher is prioritising understanding rather than performance: he does not make a big deal of the mistake. Instead, he uses it to provoke further discussion. The teachers may also have noticed how the pair who initially made the mistake are prompted to change their minds after following another student's explanation.
- Event 2: The advantages and disadvantages of asking students to work in small groups.
  Teachers may suggest that in smaller groups students are more accountable, and less likely to leave others to do work. Also, the social pressures of contributing are lessened compared to a whole class discussion. Using small group work in this way can encourage students to develop social ways of working that help encourage a supportive culture in the classroom. A disadvantage could be that there will be a limited range of perspectives on the problem. This in turn may restrict progress.

Handouts for teachers session 5

The examples of work shown below are taken from part b of the pre-lesson assessment task associated with Lesson 5.

The bar chart represents the outcome of a penalty shoot-out competition. Each person in the competition was allowed five shots at the goal. The graph shows, for example, that two people only scored one penalty with their five shots.



Student 1

b. Complete the table with values and explain how you calculate each answer.

Mode number of penalties scored	?	Add up divide by how many numbers there are. 3+2+2+1+1+7=16 16÷6=2.4?
Median number of penalties scored		
Mean number of penalties scored	2	~~[illegible]~~ mean is in the middle ~~1~~, ~~1~~, (2, 2), ~~3~~, ~~5~~
Range of the number of penalties scored		~~[illegible]~~ 7 − 1 = Can't remember if it's divide or subtract

Student 2

b. Complete the table with values and explain how you calculate each answer.

Mode number of penalties scored	/	Do not Know what mode is.
Median number of penalties scored	3.5	It is between 3 and 4 when displaying all numbe/s
Mean number of penalties scored	3.3 [redacted]	16 ÷ 5 = 3.3
Range of the number of penalties scored	6	It ranges from 0 - 5 in penalties scored.

Student 3

b. Complete the table with values and explain how you calculate each answer.

Mode number of penalties scored	7	Because it has the bigest frequenc. 5
Median number of penalties scored	2+1 = 2/2 = 1	3+2+2+1+1+1+7 = 16/6 = 2.6
Mean number of penalties scored	2.6	
Range of the number of penalties scored	8	7 - 3 = 4 7 - 1 = 6

1, 1, 2, 2, 3, 7

b. Complete the table with values and explain how you calculate each answer.

Mode number of penalties scored	1+2	This is the most common
Median number of penalties scored	2.5	This is the number in the middle
Mean number of penalties scored	2.6	Add up all the numbers and divide by how many there is.
Range of the number of penalties scored	7 - 1 = 6 6	This is the biggest number take away the smallest

PD5.2(T) Guided discussions: Video clips

V5.1 Video Clip: Common issues students face

The purpose of the clip is to highlight the common practice of students referring to averages simply as numbers disconnected from context and the subsequent misunderstandings this can engender. Separation of the problem from the context can encourage students to simply follow procedures, and may result in students confusing the different definitions of the averages. This in turn will limit how students are able to tackle unfamiliar, or challenging problems.

Introductory captions for first part of video clip

The video clip shows how students describe range, mean, and median when working on the various problems from the project.

As you watch the video, think about why these descriptions are problematic, and how you could support students to improve their descriptions.

Transcript of dialogue

The transcript is not included here because the video is clear.

Guided discussion prompt for the first part of video clip

The clip highlights the issue of students defining averages in a way that excludes the units of measure.

Once the video has paused, discuss:

- Whether teachers have encountered the ways students in the video describe measures, and the limitations of these descriptions. You may want to particularly refer to issues of finding averages from charts when the categories, as well as the frequencies are numbers.
  Teachers may suggest that the use of descriptions such as 'divide how many there are by the total' lacks specificity and can lead to errors in finding the measures. It can also lead to incorrect, or confused understanding of the meaning of the measures. This lack of understanding may prompt students to mix up the definitions of the various averages.

Transcript of second part of video clip

The transcript is not included here because the video is clear.

Post video guided discussion

The inclusion of the video clip after the pause, is to confirm the insufficiency of the description of the median in terms of the 'Middle number'. Here it is important to emphasise how each of the mini-whiteboads represents a person and we want to find the number of pieces of fruit the middle person eats (when people are ranked in order of how many pieces of fruit they eat).

V5.2 Video Clip: Teacher strategies to support learning

The two-part video clip shows two instances of the teacher supporting student learning. The first relates primarily to how the teacher encourages students to relate the numbers they have calculated back to the context of the problem. In this way, the calculations and the measure they find, are more likely to be meaningful. The second instance relates to how the teacher seeks to draw attention to the students, what progress they have made. This strategy not only helps motivate students, but helps secure their new understandings.

Introductory captions for first part of video clip

Two students have just used a chart to explain how they worked out the mean portion of fruit consumed. This is part of a whole class discussion

The teacher now facilitates a review of this explanation.

As you watch, think about the purpose of the teacher's questions and his contributions.

Notice also any difficulties students have with explaining their thinking.

Transcript of first part of video clip

Occasionally student's talk is not clear. These subtitles are included in the video.

Guided discussion for the first part of the video clip

After pausing the video, discuss with teachers:

- The purpose of the teacher's questions, and how he supported the students' thinking. *Teachers may suggest the students were following the procedures but not relating them to the real-world context. Asking them to add the unit of measure to each of the numbers makes the calculations more meaningful to both the pair at the front of the class and the rest of the class. On occasions the teacher echoed the student's explanations. This not only encourages student autonomy, but helps embed understanding.*

Introductory captions for second part of video clip

The students are working in pairs on the card problems. The teacher has notices that one pair have just revised how they figure out the median from the information on the chart.

Notice how the teacher supports the students in their newly established way of thinking.

Transcript of second part of video clip
Occasionally student's talk is not clear. These subtitles are included in the video.

Post video guided discussion
After watching the video, discuss with teachers:

- How the teacher supports the students in their newly established way of thinking. *Teachers may suggest the teacher in the clip, through his questioning, draws attention to the way the students have altered their understanding of median. Helping students to recognise the progress they have made can be motivational, and help to secure their new understanding. Teachers may also note that the teacher in the clip asks* ***both*** *students to express in their* ***own*** *words the meaning of a bar. Asking students to express out loud their understanding can prompt students to organise more coherently their thinking and in so doing encourages aspects of dialogic learning.*

V5.3 Video Clip: Ensuring a purposeful lesson
It can be useful to let students know the learning goals of the lesson. Such information gives both purpose to the activities, and supports students in monitoring the extent their current performance is helping them achieve the goals. It may be difficult, however, for teachers to express the goals in a way that is meaningful to students. Having a specific task to refer to helps make the goals explicit for both teacher and students. The clip shows the teacher making use of the knowledge gained from reviewing the students' work on the pre-lesson assessment task.

Introductory captions for first part of video clip

The teacher has reviewed students' work on the pre-lesson assessment task. At the start of the lesson he comments on this review to the whole class.

Whilst watching the video, how the teacher's approach to informing students on the learning goals of the lesson, differs from your own.

Transcript of dialogue
The transcript is not included here because the video is clear.

Post video guided discussion
Discuss with the teachers how the pre-lesson assessment facilitates:

- The teachers' own understanding of the learning goals.
  Teachers may reflect on how the specificity within solutions to a problem can help clarify the learning goals.
- Communication to students of the learning goals.
  Teachers may suggest having a problem to refer to may also help clarify the

learning goals for students. The second part of the clip highlights how the teacher can help use the results from the pre-lesson assessment to highlight the learning goals of the lesson, and how students are progressing towards them.

V5.4 Video Clip: Creating a supportive classroom

Creating a supportive classroom culture entails both the teacher supporting students, and students supporting each other. The teacher can play a crucial role in fostering this culture. The video clip shows just two ways of doing this – there are many other ways. The first clip depicts an episode in which students are encouraged to support each other's learning. The second clip shows how the teacher supportively deals with students' incorrect contribution to the whole class discussion.

Note: Although the clip is split into two sections, there is no need to pause for a discussion after the first section. Allow the clip to play to the end before instigating a review of how teacher and students behaved.

Introductory captions for first part of video clip

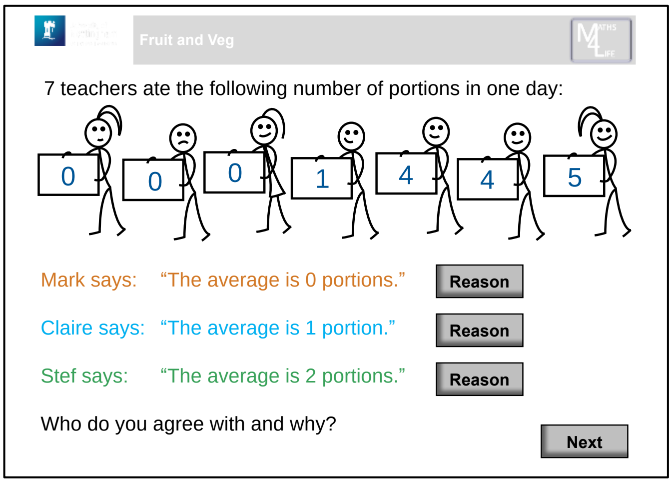
The clip shows two events. In each, students are discussing what is meant by average.

As you watch each, think of how teacher facilitates a supportive culture.

Event 1:
The teacher is holding a whole class discussion to explore students' understanding of the term average, and the meaning of mean, mode and median. He is using the Fruit and Veg problem.
Together they are evaluating three statements.
Students have already considered Mark and Claire's statement, and are now evaluating Stef's comments.
Two students have just declared that Stef is incorrect.

Event 2: The teacher is holding a whole class discussion.
Students have just worked out that the mean portion of fruit the class ate yesterday was" 2.5".
The clip opens with the teacher encourages the students to explain the meaning of 'the mean'.

The task the whole class is discussing during the first event



Transcript of dialogue
The transcript is not included here because the dialogue is clear.

Post-video guided discussion of both events

After watching the video, discuss

- Event 1: How the teacher handled the mistake, and how his actions could support students.
  Teachers may suggest that students getting things wrong in a public situation, such as a whole class discussion, has the potential to damage their social status. Such incidents need to be handled carefully (but not avoided). In this case, the teacher has been working with the class for some time and has already developed a supportive classroom where making mistakes is accepted as an integral part of learning. The teacher is prioritising understanding rather than performance: he does not make a big deal of the mistake. Instead, he uses it to provoke further discussion. The teachers may also have noticed how the pair who initially made the mistake are prompted to change their minds after following another student's explanation.
- Event 2: The advantages and disadvantages of asking students to work in small groups.
  Teachers may suggest that in smaller groups students are more accountable, and less likely to leave others to do work. Also, the social pressures of contributing are lessened compared to a whole class discussion. Using small group work in this way can encourage students to develop social ways of working that help encourage a supportive culture in the classroom. A disadvantage could be that there will be a limited range of perspectives on the problem. This in turn may restrict progress.

PD5.3 Formative Assessment

Formative assessment may be defined as:

> … all those activities undertaken by teachers, and by their students in assessing themselves, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching work to meet the needs.
>
> (Black & Wiliam, 1998)

The following points provide good guidelines for effective formative assessment in Maths-for-Life lessons.

1. Make the objectives of the lesson explicit	Making objectives explicit doesn't necessarily mean writing them on the board at the beginning of the lesson. However, it is vital that the teacher of the lesson has absolute clarity on the mathematical aims. Linking a task to one of the objectives is often a better way to engage students effectively. In Maths-for-Life many of the objectives are to develop particular mathematical concepts that are important in the GCSE exams. This is why the focus during review and closure stages should be on comparing approaches that work with mathematical understanding, rather than answers.
2. Assess groups as well as individual students	Group activities allow many opportunities to observe, listen, and question students. They help students to externalise reasoning and allow the teacher to see quickly where difficulties have arisen. It is, therefore, helpful before intervening in a group discussion, wait and listen and try to follow the line of reasoning that students are taking. When you do intervene, begin by asking students to explain their thinking. If they are unsuccessful then ask another student to help.

3. Use divergent assessment methods	*Divergent assessment* involves asking open questions that allow students opportunities to describe and explain their thinking and reasoning. These questions allow students to surprise us - the outcome is not predetermined. In contrast, convergent assessment strategies are characterised by tick lists and can-do statements. The teacher asks closed questions in order to ascertain whether or not the student knows, understands or can do a predetermined thing. This is the type of assessment most used in written tests.
4. Give constructive, useful feedback	Research shows that responding to students' work with marks or levels is ineffective and may even obstruct learning. Quantitative feedback of this type results in students comparing marks or levels and detracts from the mathematics itself. It is found that qualitative oral comments that help students recognise what they can do and how to develop their thinking are most helpful.
5. Change teaching to take account of assessment	As well as providing feedback to students, good assessment feeds forward into teaching. Be flexible and prepared to change your teaching plans in mid-course as a result of what has been discovered.

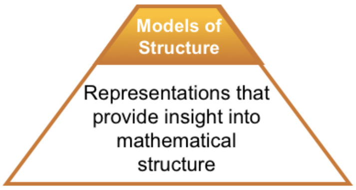
Adapted from: Improving Learning in Mathematics, Department for Education and Skills, 2005.

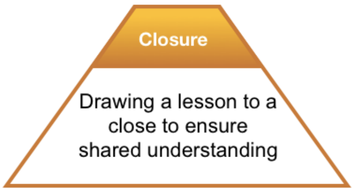
PD5.4 Lesson 5 Research Form

Observe a pair/group of students working during the lesson.

Note down the key moments and development in their mathematical thinking.

Note down examples of the Maths-for-Life pedagogies seen during this lesson.





Research Question

How does the use of formative assessment help to develop an environment of supportive dialogic learning?