JSPS London Symposium

Non-Equilibrium Dynamics, Thermodynamics and Fluctuations

From Fundamentals to the Next Generation of Microscopic Thermal Machines

University of Nottingham

12-15 December 2022

Organisers

Kay Brandner, *University of Nottingham, UK*

Keji Saito, *Keio University, Japan*

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Mission

With the next technological revolution underway in the quantum world, there is now an increasing need to develop thermal devices such as heat engines or refrigerators that operate on small length and energy scales. To uncover the fundamental principles that govern such microscopic machines and to realize them in experiments, a whole breadth of challenges must be overcome. Key topics in this context include the nature of non-equilibrium thermal and quantum fluctuations, non-Markovian time evolution laws, collective effects in many-body systems as well as the role of strong coupling and finite reservoirs.

This Symposium brings together researchers from Japan, Europe, and the University of Nottingham, who work in stochastic and quantum thermodynamics, cold-atom physics, and wider areas of quantum many-body physics. Its overarching aim is to find new avenues towards understanding the complex interplay between the dynamics, thermodynamics, and fluctuations of classical and quantum systems out of equilibrium and thus to help paving the way for the next generation of microscopic thermal machines. Besides the scheduled talks and poster session, the program leaves ample time for discussions and to build new connections between Japan, the UK and Europe.

Participants

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University of Tübingen, Germany

Programme

Monday 12 December		
Welcome		
9:00-9:15	Welcome - Organisers	K. Brandner K. Saito
9:15-9:30	Welcome - Head of School of Physics and Astronomy	M. Fromhold
9:30-10:00	Welcome - JSPS London	Prof. Kobayashi Mr. Takizawa
10:00-10:30	Coffee	
Session 1: Fluctuations in Mesoscopic Systems out of Equilibrium		
10:30-11:10	Thermodynamics of clocks	P. Pietzonka
11:10-11:50	Inverse thermodynamic uncertainty relations	J. P. Garrahan
11:50-12:30	Fluctuation and response of time-symmetric current around nonequilibrium stationary states	N. Shiraishi
12:30-13:30	Lunch	
Session 2: Quantum Many-Body Systems and Quantum Information Processing		
13:30-14:10	Nonequilibrium phase transitions in open quantum systems with absorbing states	F. Carollo
14:10-14:50	Time evolution of uniform sequential quantum circuits	A. Smith
14:50-15:30	Growth of Rényi entropies in interacting integrable models and the breakdown of the quasiparticle picture	B. Bertini
15:30-16:00	Coffee	
16:00-16:40	Entanglement negativity and mutual information after a quantum quench: Exact link from space-time duality	K. Klobas
16:40-17:00	Brief info-session on administrative matters	K. Brandner
19:00 - 21:00: Joint Dinner		

Tuesday 13 December		
	Session 3: Mesoscopic and Quantum Thermal Machines	
9:00-9:30	Geometric bounds on the power of adiabatic thermal machines	J. Eglinton
9:30-10:00	Exploiting coherence for quantum thermodynamic advantage	K. Hammam
10:00-10:30	Coffee	
10:30-11:10	Making statistics work: a quantum engine in the BEC-BCS crossover	T. Busch
11:10-11:50	A Bose-Einstein condensate thermal engine	G. De Chiara
11:50-12:30	Consistent thermodynamics of finite baths	P. Strasberg
12:30-13:30	Lunch	
	Session 4: Cold Atoms and Rydberg Ions	
13:30-14:10	Spin-phonon coupled dynamics with trapped Rydberg ions	W. Li
14:10-14:50	Exploring non-equilibrium thermodynamics in the quantum regime with ultracold atoms	G. Barontini
14:50-15:30	Non-equilibrium molecule association in Lithium-6 revealing coherence enhanced efficiency	L. Hackermüller
15:30-16:00	Coffee	
	16:00 - 18:00: Poster Session	
	Three-dimensional quantum spin dynamics and mean force Gibbs states	C. Hogg
	Concentration inequalities for output statistics of quantum Markov processes	F. Girotti
	Towards quantum engines in ultracold mixtures	T. Hewitt
	Generalised hydrodynamics of particle creation and decay	C. De Fazio
	Growth of entanglement of generic states under dual-unitary dynamics	A. Foligno
	Non-equilibrium quantum Rydberg ion engine	W. Martins
	Non-equilibrium time-crystal quantum engine	P. J. P. de Souza
	Reversible cellular automata: exactly solvable models of nonequilibrium statistical mechanics	J. Wilkinson

Wednesday 14 December		
Session 5: Thermodynamics Beyond Weak Coupling, Markovian Time Evolution and Infinite Reservoirs		
9:00-9:30	Dynamics and thermodynamics of open quantum systems interacting with finite baths	A. R.-Campeny
9:30-10:00	Nonequilibrium quantum thermodynamics in open systems: the influence of initial correlations	A. Colla
10:00-10:30	Coffee	
10:30-11:10	Taming environmental modifications to quantum states - some predictions and solutions	J. Anders
11:10-11:50	Mitigating work fluctuations in driven quantum systems	H. Miller
11:50-12:30	Exact numerical methods for non-Markovian open quantum systems	N. Lambert
12:30-13:30	Lunch	
Session 6: Information and Measurements		
13:30-14:10	Work extraction in the quantum regime: correlations versus ignorance	G. Adesso
14:10-14:50	Inferring non-equilibrium thermodynamics in continuously monitored systems: the role of information	M. Paternostro
14:50-15:30	Adaptive measurement filter: efficient strategy for optimal estimation of quantum Markov chains	M. Guta
15:30-16:00	Coffee	
Session 7: Speed Limits and Optimal Control in Open Systems		
16:00-16:40	Speed limits for ergodicity	A. Dechant
16:40-17:20	Thermodynamics of optimal transport and speed limits	T. Vu
17:20-18:00	General bound on the performance of shortcuts to adiabaticity under environmental effects	K. Funo

Thursday 15 December		
	Session 8: Thermometry and Stochastic Dynamics in the Quantum Regime	
9:00-9:30	A healthier semi-classical dynamics	I. Layton
9:30-10:00	Quantum thermometry with adaptive Bayesian strategies: a case study for release-recapture experiments	J. Rubio
10:00-10:30	Coffee	
10:30-11:10	Taking the temperature of a pure quantum state	M. Mitchison
11:10-11:50	Finite-time quantum Landauer principle and quantum coherence	K. Saito
11:50-12:30	Concluding remarks and perspectives	K. Brandner K. Saito
12:30-13:30	Lunch	
	End	

Abstracts

Thermodynamics of Clocks

Patrick Pietzonka

I will briefly review the basic ideas behind the thermodynamic uncertainty relation (TUR). It establishes a seemingly universal trade-off between cost and precision for classical non-equilibrium systems in a steady state. Applied to clocks subject to thermal noise, it states that the product of the energy used for the driving and the squared relative uncertainty of the displayed time is always greater than 2kT. The TUR has been proven for models based on Markov jump dynamics or overdamped Brownian motion. It had also been conjectured to hold for underdamped Brownian dynamics, i.e., systems where inertia plays a role. This conjecture can now be disproven. I will present a counterexample that is inspired by a pendulum clock, consisting of an underdamped oscillator and a discrete counter, with thermal noise accounted for in both degrees of freedom. As it turns out analytically, this classic design principle of a clock allows one to overcome the bounds on precision set by the TUR . Finally, I will also show numerically that the TUR can be broken in a fully continuous model with two underdamped degrees of freedom.

Inverse thermodynamic uncertainty relations

Juan Garrahan

Thermodynamic uncertainty relations (TURs) are general lower bounds on the size of fluctuations of dynamical observables. They have important consequences, one being that the precision of estimation of a current is limited by the amount of entropy production. I will describe the existence of corresponding general upper bounds on the size of fluctuations of any linear combination of fluxes (including all time-integrated currents or dynamical activities) for continuous-time Markov chains. These general relations are obtained by means of concentration bound techniques. These inverse TURs are valid for all times and not only in the long time limit. I will illustrate these analytical results with a simple model, and discuss wider implications of these new relations.

Fluctuation and response of time-symmetric current around nonequilibrium stationary states

Naoto Shiraishi

Fluctuation-response relation around equilibrium states is one of the most important relations in nonequilibrium statistical physics. This relation connects two different quantities, the fluctuation of currents around an equilibrium state and the response to nonequilibrium driving. Many studies tried to generalize the fluctuation-response relation applicable to nonequilibrium stationary states. Most of previous attempts employ time-antisymmetric quantities including currents in their relations. Instead, we investigate fluctuation-response relation of time-symmetric quantities. We introduce a novel quantity called time-symmetric current, which is time-symmetric but whose ensemble average is equal to the conventional current. We prove that the time-symmetric current satisfies the fluctuation-response relation in a conventional form around nonequilibrium stationary stalling states [1]. Here, the word “stalling” stands for the situation that the current in interest has zero average while a whole system can be highly nonequilibrium. The time-symmetric current together with the fluctuation-response relation serves as a probe of a bare transition rate in experiments. In addition, we can extend this fluctuation-response relation to general nonequilibrium stationary state beyond stalling states [2]. If time allows, we discuss another important relation between fluctuation and response around nonequilibrium stationary states, the thermodynamic uncertainty relation (TUR). We provide a unified view of TUR-type inequalities and derive the optimal one [3].

[1] N. Shiraishi, Phys. Rev. Lett. 129, 020602 (2022).
[2] N. Shiraishi, in preparation
[3] N. Shiraishi, J. Stat. Phys. 185, 19 (2021).

Nonequilibrium phase transitions in open quantum systems with absorbing states

Federico Carollo

A paradigmatic setting for the study of nonequilibrium phenomena is that of stochastic processes featuring absorbing states, i.e., configurations which, once reached during the dynamics, cannot be left. These systems typically follow elementary rules – for instance describing the dynamics of epidemic spreading – but display intriguingly complex nonequilibrium behavior. In classical settings, a famous conjecture asserts that generic models with a single absorbing state display emergent physics in the directed percolation universality class. In this talk, I will present different models of open quantum systems possessing absorbing states and discuss how quantum effects can alter the universal behavior of these systems, give rise to a new type of absorbing state phase transitions and even stabilize phases which are apparently not possible in related classical problems.

Time evolution of uniform sequential quantum circuits

Adam Smith

Simulating time evolution of generic quantum many-body systems using classical numerical approaches has an exponentially growing cost either with evolution time or with the system size. We present a polynomially scaling hybrid quantum-classical algorithm for time evolving a one-dimensional uniform system in the thermodynamic limit. This algorithm uses a layered uniform sequential quantum circuit as a variational ansatz to represent infinite translation-invariant quantum states. We show numerically that this ansatz requires a number of parameters polynomial in the simulation time for a given accuracy. Furthermore, this favourable scaling of the ansatz is maintained during our variational evolution algorithm. All steps of the hybrid optimisation are designed with near-term digital quantum computers in mind. After benchmarking the evolution algorithm on a classical computer, we demonstrate the measurement of observables of this uniform state using a finite number of qubits on a cloud-based quantum processing unit. With more efficient tensor contraction schemes, this algorithm may also offer improvements as a classical numerical algorithm.

Growth of Rényi Entropies in Interacting Integrable Models and the Breakdown of the Quasiparticle Picture

Bruno Bertini

Rényi entropies are conceptually valuable and experimentally relevant generalisations of the celebrated von Neumann entanglement entropy. After a quantum quench in a clean quantum many-body system they generically display a universal linear growth in time followed by saturation. While a finite subsystem is essentially at local equilibrium when the entanglement saturates, it is genuinely out-of-equilibrium in the growth phase. In particular, the slope of the growth carries vital information on the nature of the system's dynamics, and its characterisation is a key objective of current research. In the talk I will show that the slope of Rényi entropies can be determined by means of a spacetime duality transformation. I will argue that the slope coincides with the stationary density of entropy of the model obtained by exchanging the roles of space and time. Therefore, very surprisingly, the slope of the entanglement can be expressed as an equilibrium quantity. I will use this observation to find an explicit exact formula for the slope of Rényi entropies in all integrable models treatable by thermodynamic Bethe ansatz and evolving from integrable initial states. Interestingly, this formula can be understood in terms of a quasiparticle picture only in the von Neumann limit.

Entanglement negativity and mutual information after a quantum quench: Exact link from space-time duality

Katja Klobas

I will present recent results on the growth of entanglement between two adjacent regions in a tripartite, one-dimensional many-body system after a quantum quench. Combining a replica trick with a space-time duality transformation a universal relation between the entanglement negativity and Renyi-1/2 mutual information can be derived, which holds at times shorter than the sizes of all subsystems. The proof is directly applicable to any local quantum circuit, i.e., any lattice system in discrete time characterised by local interactions, irrespective of the nature of its dynamics. The derivation indicates that such a relation can be directly extended to any system where information spreads with a finite maximal velocity. The talk is based on Phys. Rev. Lett. 129, 140503 (2022).

Geometric bounds on the power of adiabatic thermal machines

Joshua Eglinton

The laws of thermodynamics put fundamental bounds on the efficiencies of thermal machines such as heat engines or refrigerators. These Carnot bounds can typically be attained only if the machine is operated quasi-statically, which leads to vanishing power output. How this trade-off between power and efficiency can be captured quantitatively for meso- and micro-scale thermal machines is a question that has attracted significant attention over the last years.

In this talk I will present a new family of power-efficiency trade-off relations that imply a quadratic rather than a linear decay of power at Carnot efficiency, for devices operating between two fixed temperatures. Notably, these relations depend only on geometric quantities such as the thermodynamic length of the driving cycle and hold for essentially any thermodynamically consistent micro-dynamics such as classical Markov-jump processes, adiabatic Lindblad dynamics or coherent transport. This analysis is based on a new general scaling argument, with which we show that the efficiency of such devices reaches the Carnot bound only if heat-leaks between the baths can be fully suppressed. Furthermore, we find that their power is in fact determined by second-order terms in the temperature difference between the two baths, which are neglected in standard linear-response theory.

https://doi.org/10.1103/PhysRevE.105.L052102

Exploiting coherence for quantum thermodynamic advantage

Kenza Hammam

The introduction of the quantum analogue of a Carnot engine based on a bath comprising of particles with a small amount of coherence initiated an active line of research on the harnessing of different quantum resources for the enhancement of thermal machines beyond the standard reversible limit, with an emphasis on non-thermal baths containing quantum coherence. In our work, we investigate the impact of coherence on the thermodynamic tasks of a collision model which is composed of a system interacting, in the continuous time limit, with a series of coherent ancillas of two baths at different temperatures. Our results show the advantages of utilising coherence as a resource in the operation of the machine, and allows it: (i) to exhibit unconventional behavior such as the appearance of a hybrid refrigerator, capable of simultaneous refrigeration and generation of work, and (ii) to function as an engine or a refrigerator with efficiencies larger than the Carnot bound. Moreover, we find an effective upper bound to the efficiency of the thermal machine operating as an engine in the presence of a coherent reservoir.

1- Exploiting coherence for quantum thermodynamic advantage. K. Hammam, H. Leitch, Y. Hassouni, G. De Chiara, arXiv preprint arXiv:2202.07515, (2022).
2- Optimizing autonomous thermal machines powered by energetic coherence. K. Hammam, Y. Hassouni, R. Fazio, G. Manzano, New Journal of Physics 23 (4), 043024 19, (2021).
3- Quantum Discord for Information Transmission Using Coherent States. A. El Allati, K. Hammam, H. Amellal, Y. Hassouni, Journal of Russian Laser Research 39 (5), 524-532, (2018).

Making statistics work: a quantum engine in the BEC-BCS crossover

Thomas Busch

Heat engines convert thermal energy into mechanical work and are well studied in the classical and in the quantum regimes. However, in the quantum realm genuine nonclassical forms of energy exist, different from heat, which can also be exploited in cyclic engines to produce useful work.

In this presentation I will introduce the concept of a Pauli engine, a novel quantum many-body engine fuelled by the energy difference between fermionic and bosonic ensembles of ultracold particles that follows from the Pauli exclusion principle. The difference in symmetry between these two systems leads to a change in the population distribution, which can be used to replace the traditional heat strokes in a quantum Otto engine.

I will also show that such a system has been realised by cycling the working medium of the engine between a Bose-Einstein condensate of bosonic molecules and a unitary Fermi gas (and back) through a magnetic field in a system of trapped superfluid 6-Li atoms. The experiments obtain a work output of several 10^6 vibrational quanta per cycle with an efficiency of up to 25%, therefore establishing quantum statistics as a useful thermodynamic resource for work production.

J. Koch, K. Menon, E. Cuestas, S. Barbosa, E. Lutz, T. Fogarty, Th. Busch, A. Widera, arXiv:2209.14202 (2022).

A Bose-Einstein condensate thermal engine

Gabriele De Chiara

The study of out-of-equilibrium thermodynamics of quantum systems has received increasing attention in recent years thanks to tremendous theoretical and experimental progress. While most of the studies in quantum thermodynamics bear a close resemblance to their classical counterparts, especially close to equilibrium, there are only a few examples of genuine quantum features, e.g. coherence and indistinguishability, that provide an advantage over classical thermodynamic devices. In this contribution, I will show the functioning and performance of an endoreversible Otto cycle operating with a harmonically confined Bose gas as the working medium. I analyze the engine operation in three regimes, with the working medium in the BEC phase, in the gas phase, and driven across the BEC transition during each cycle. The unique properties of the BEC phase allow for enhanced engine performance, including increased power output and higher efficiency at maximum power.

N. M. Myers, F. J. Peña, O. Negrete, P. Vargas, G. De Chiara, S. Deffner, New J. Phys. 24, 025001 (2022).

Consistent thermodynamics of finite baths

Philipp Strasberg

It was already known to Clausius that a bath can change its temperature when exchanging energy with another system. Remarkably, until recently this basic insight was not reflected in the quantum thermodynamics literature, despite proclaiming the universal validity of a "Clausius inequality" that Clausius himself would had refuted. Here, I show how to derive the true Clausius inequality based on a consistent microscopic definition of temperature valid out of equilibrium. I reveal if and when Clausius inequality is the second law, and I show that taking into account finite baths effects universally improves the efficiency of any thermodynamic process.

Exploring non-equilibrium thermodynamics in the quantum regime with ultracold atoms

Giovanni Barontini

In our experiment, we utilise mixtures of cold atoms to investigate non-equilibrium thermodynamics at the quantum level. With our architecture, it is possible to realize states of matter that are not accessible for equilibrium or closed systems. Specifically, by immersing an evaporating ultracold Bose gas of Rb in a cloud of K with substantially higher temperature, we realise a controlled source of dissipation that allows us to produce supercritical Bose-Einstein condensates, i.e., Bose-Einstein condensates above the critical temperature [1]. We are also working towards the implementation of single atom quantum engines, where a single atom of K is trapped in an optical tweezer and is immersed in a bath of Rb atoms [2]. In this framework, we have developed with our colleagues in Exeter a method based on adaptive Bayesian strategies that substantially improves our ability to perform single atom temperature estimation. We demonstrate that adaptively choosing the release–recapture times to maximise information gain substantially reduces the number of measurements needed and delivers much more reliable estimates [3]. I will finally present our latest results on Feshbach spectroscopy in our system, that provides us with the ability of controlling the interactions between the single atom and the bath.

[1] J. Mellado Muñoz, X. Wang, T. Hewitt, A. U. Kowalczyk, R. Sawant, and G. Barontini,

”Dissipative Distillation of Supercritical Quantum Gases”, Phys. Rev. Lett. 125, 020403 (2020)
[2] G. Barontini and M. Paternostro, ”Ultra-cold single-atom quantum heat engines”, New J. Phys. 21, 063019 (2019)
[3] J. Glatthard, J. Rubio, R. Sawant, T. Hewitt, G. Barontini, L. A. Correa, “Optimal cold atom thermometry using adaptive Bayesian strategies”, arXiv:2204.11816

Non-equilibrium molecule association in lithium-6 revealing coherence enhanced efficiency

Lucia Hackermüller

We study non-equilibrium association of Li_2 Feshbach molecules over a range of temperatures $T \gg T_F$ to $T \ll TF$. We observe an enhancement of the atom–molecule coupling efficiency as the fermionic atoms reach degeneracy demonstrating the importance of many-body coherence not captured by the conventional LZ model [1]. We develop and detail a theoretical model that can explain the temperature dependence of the atom–molecule coupling and use it to predict a shortcut to adiabaticity in molecular association efficiency.I will also report on our recent results of a microscopic atom-photon interface [2] as well as miniature 3D-printed vapour cells.

[1] V. Naniyil et al., ”Observation of collectivity enhanced magnetoassociation of 6Li in the quantum degenerate regime”, New J. Phys. 24 113005 (2022).
[2] E. da Ros et al., ”Cold atoms in micromachined waveguides: a new platform for atom-photon interaction ”, Phys. Rev. Res. 2, 033098 (2020).

Dynamics and thermodynamics of open quantum systems interacting with finite baths

Andreu Riera-Campeny

An open quantum system in contact with an ideal infinite bath approaches equilibrium, while the state of the bath remains unchanged. If the bath is finite, the open system still relaxes to equilibrium, but it induces a dynamical evolution of the bath state. In this talk, I will present our approach to describe the dynamics of an open quantum system in contact with a finite bath. I'll show how we obtain a hierarchy of master equations that improve their accuracy by including more dynamical information of the bath. I will also present the connection of the open system dynamics with the thermodynamics of those systems.

[1] Andreu Riera-Campeny, Anna Sanpera, and Philipp Strasberg, PRX Quantum 2, 010340,
[2] Philipp Strasberg, María García Díaz, and Andreu Riera-Campeny, Phys. Rev. E 104, L022103,
[3] Andreu Riera-Campeny, Anna Sanpera, and Philipp Strasberg, Phys. Rev. E 105, 054119

Nonequilibrium quantum thermodynamics in open systems: the influence of initial correlations

Alessandra Colla

Finding a consistent formulation of quantum thermodynamics for general nonequilibrium processes is a fundamental and relevant question of recent research that has prompted the development of several proposals for the definition of basic thermodynamics quantities such as work, heat and entropy production. We have recently put forth one such approach (Phys. Rev. A 105, 052216) which relies on techniques of open quantum systems to determine an effective Hamiltonian as the operator for internal energy. The original formulation of the approach assumes factorizing initial conditions between the system of interest and the bath. We show here that the theory may be extended to any initial system-bath correlations with the expected result that the effective Hamiltonian is unaffected by the presence of the initial correlations.

Entropy production and the role of correlations in quantum Brownian motion (Phys. Rev. A 105, 052216)
Open-system approach to nonequilibrium quantum thermodynamics at arbitrary coupling (Phys. Rev. A 105, 052216)

Taming environmental modifications to quantum states - some predictions and solutions

Janet Anders

The dynamical convergence to the Gibbs state is a standard assumption across much of classical and quantum thermodynamics. However, for nanoscale and quantum systems the interaction with their environment becomes non-negligible. Then so-called mean force (MF) states are the equilibrium states [2,3]. I will discuss general analytic expressions for such MF states in the limits of weak and ultrastrong system-environment coupling [1]. Deviations from the predictions of standard Gibbs physics are exemplified on the V-system.

In the second part of my talk I will discuss our recent results on a very different aspect: Experiments often encode qubit states in physical systems that have many more physical dimensions. Unfortunately, environmental noise can cause the logical qubit to leak into these dimensions, compromising the qubit nature of the state. This causes unwanted artefacts, such as increased entropies. I will describe a new method to recover a meaningful qubit state from a known noisy high-dimensional state [4].

[1] Cresser, Anders, PRL 127, 250601 (2021)
[2] Trushechkin, Merkli, Cresser, Anders, AVS Quantum Sci. 4, 012301 (2022)
[3] Cerisola, et al arXiv:2204.10874
[4] Anders, ..., Huard, in preparation.

Mitigating work fluctuations in driven quantum systems

Harry Miller

In quantum thermodynamics, optimal control methods are typically employed to find Hamiltonian protocols that reduce the average dissipation generated when a small system is driven out of equilibrium. However, dissipation is not the only figure of merit for optimising a thermodynamic process; one should also take care not to produce large fluctuations in work as this can lead to instability and unreliable performances. For driven quantum systems, one issue that arises is the fact that finite-time processes with minimal dissipation do not typically produce minimal fluctuations [1]. This means we are presented with a multi-objective optimisation problem, and a compromise between small fluctuations versus small dissipation must be made. In this talk I will outline a range of general optimisation principles needed for fine-tuning this balance in open systems that are driven either slowly or rapidly. In slow driving regimes, methods from quantum information-geometry can be used to identify optimal processes as smooth paths of minimal action with respect to a metric assigned to the space of control parameters [2]. I will contrast this with the regime of rapid driving, where I will show that optimal processes consist of discontinuous jumps instead of smooth changes in the Hamiltonian control parameters [3].

[1] H. Miller, H. Mohammady, M. Perarnau-Llobet, G. Guarnieri, 'Thermodynamic uncertainty relation in slowly driven quantum heat engines', Phys. Rev. Lett. 126, 210603 (2021)
[2] H. Miller, M. Mehboudi, 'Geometry of work fluctuations versus efficiency in microscopic thermal machines', Phys. Rev. Lett. 125, 260602 (2020)
[3] A. Rolandi, M. Perarnau-Llobet, H. Miller 'Optimal work statistics for rapidly driven quantum systems', unreleased (2022)

Spin-phonon coupled dynamics with trapped Rydberg ions

Weibin Li

In this talk, I will discuss how to engineer multi-body interaction and novel states with trapped Rydberg ions. I will show that the coupling between Rydberg pair interactions and collective motional modes of the ion crystal gives rise to effective long-range and multibody interactions consisting of two, three, and four-body terms. Their shape, strength, and range can be controlled via the ion trap parameters and strongly depends on both the equilibrium configuration and vibrational modes of the ion crystal. In particular, a three-body antiblockade effect can be induced and employed as a sensitive probe to detect structural phase transitions in Rydberg ion chains. By tuning the two-body dipolar interaction between a pair of ions, I will show that a conical intersection can be generated for studying non adiabatic processes. Our numerical simulations show that the conical intersection affects both the nuclear and electronic dynamics. These novel effects can be monitored in real time via direct spectroscopic measurement of the Rydberg population. Our study unveils the possibilities offered by trapped Rydberg ions for studying exotic phases of matter and quantum dynamics driven by engineered Rydberg interactions.

Exact numerical methods for non-Markovian open quantum systems

Neill Lambert

Understanding the influence of non-perturbative and non-Markovian environments on a quantum system is an important task for modelling certain physical systems correctly, like photosynthesis and strongly-correlated transport, and for understanding the fundamental limits of quantum thermodynamics. I will summarize two numerical methods designed to model such environments "exactly': the powerful hierarchical-equations-of-motion, and an approach based on constructing a finite set of discrete pseudomodes [1]. This pseudomode method differs from other discrete approaches to this problem via the unphysical nature of the modes. I will describe how to treat both bosonic and fermionic environments [2,3] with these methods, and give examples of modelling dressed system-bath states, heat transport between finite temperature environments, dynamical decoupling of noise [4], fitting arbitrary bath spectral densities, and Kondo resonance.

[1] N. Lambert, S. Ahmed, M. Cirio, F. Nori, Nature communications 10, 3721 (2019)

[2] M. Cirio, P. C. Kuo, Y. N. Chen, F. Nori, N. Lambert, Physical Review B 105 , 035121 (2022)
[3] M. Cirio, N. Lambert, P-C. Kuo, Y-N. Chen, P. Menczel, K. Funo, F. Nori arXiv:2207.05780
[4] N. Lambert, T. Raheja, S.Cross, P. Menczel, S. Ahmed, A. Pitchford, D. Burgarth, F. Nori arXiv:2010.10806

Work extraction in the quantum regime: correlations versus ignorance

Gerardo Adesso

We investigate constrained scenarios for work extraction in the resource theory of quantum thermodynamics. On one hand, we consider bipartite scenarios when one party wants to locally distil as much work as possible but is restricted to thermal operations whereas the other party can perform general quantum operations and they are allowed to communicate classically. We demonstrate that this question is intimately related to the distillation of classical/quantum correlations. On the other hand, we consider entropy production upon mixing two gases. Classically, an ignorant observer who cannot distinguish the gases has no way of extracting work by mixing them. Moving the thought experiment into the quantum realm, we find that the ignorant observer can extract as much work as if the gases were fully distinguishable, even without direct access to the gases' intrinsic degrees of freedom: a quantum Gibbs paradox. We discuss the implications of both scenarios for genuinely quantum modifications to thermodynamics.

Inferring non-equilibrium thermodynamics in continuously monitored systems: the role of information

Mauro Paternostro

I will put forth a unifying formalism for the description of the thermodynamics of continuously monitored systems, where measurements are only performed on the environment connected to a system. I will show, in particular, that the conditional and unconditional entropy production, which quantify the degree of irreversibility of the open system's dynamics, are related to each other by the Holevo quantity. This, in turn, can be further split into an information gain rate and loss rate, which provide conditions for the existence of informational steady-states, i.e. stationary states of a conditional dynamics that are maintained owing to the unbroken acquisition of information. I will illustrate the applicability of such framework through several examples, including the modelling of a recent experiment in the field of ultracold atoms and cavity optomechanics.

Adaptive measurement filter: efficient strategy for optimal estimation of quantum Markov chains

Madalin Guta

Continuous-time measurements are instrumental for a multitude of tasks in quantum engineering and quantum control, including the estimation of dynamical parameters of open quantum systems monitored through the environment. However, such measurements do not extract the maximum amount of information available in the output state, so finding alternative optimal measurement strategies is a major open problem.

In this presentation I will show how we can solve this problem in the setting of discrete-time input-output quantum Markov chains. We present an efficient algorithm for optimal estimation of one-dimensional dynamical parameters which consists of an iterative procedure for updating a 'measurement filter' operator and determining successive measurement bases for the output units. A key ingredient of the scheme is the use of a coherent quantum absorber as a way to post-process the output after the interaction with the system. This is designed adaptively such that the joint system and absorber stationary state is pure at a reference parameter value. The scheme offers an exciting prospect for optimal continuous-time adaptive measurements, but more work is needed to find realistic practical implementations.

Speed limits for ergodicity

Andreas Dechant

Most noisy physical systems are ergodic: For a sufficiently long time, the time-average of an observable over a single measurement will be close to the ensemble-average. For finite times, on the other hand, the time-average fluctuates due to the presence of noise, which can be characterized by the variance of the time-average. For a wide class of systems, the variance decays as the inverse of the measurement time, characterizing the approach of the system towards ergodic behavior. In this talk, I will discuss a new class of speed limits that bound the rate at which the system approaches ergodicity. For equilibrium systems, this is directly related to the first eigenvalue of the evolution equation, which determines the relaxation towards the equilibrium state. For non-equilibrium systems, on the other hand, we find a speed limit involving the rate of entropy production in the system. Interestingly, this can be inverted to give a lower bound on the entropy production rate from the measurement of the time-average of an observable.

Thermodynamics of optimal transport and speed limits

Tan Vu

Optimal transport is a mature field in mathematics and statistics, and its theory concerns the optimal planning and optimal cost of transporting a distribution. In recent years, a deep connection between optimal transport and stochastic thermodynamics has been elucidated in the context of overdamped Langevin dynamics, revealing that the problem of minimizing entropy production can be mapped to the optimal transport problem. This intimate connection has led to essential applications in stochastic thermodynamics. In this talk, I describe an analogous connection for discrete-variable cases by developing a thermodynamic framework for discrete optimal transport. Specifically, I present variational formulas that connect the discrete Wasserstein distances to stochastic and quantum thermodynamics of discrete Markovian dynamics described by master equations. These formulas not only unify the relationship between thermodynamics and the optimal transport theory for discrete and continuous cases but also generalize it to quantum cases. Notably, they lead to remarkable applications in stochastic and quantum thermodynamics, such as stringent thermodynamic speed limits and the finite-time Landauer principle. In addition, I demonstrate that the optimal transport approach yields a topological speed limit, which can reveal fundamental limitations that cannot be obtained with speed limits reported thus far.

General bound on the performance of shortcuts to adiabaticity under environmental effects

Ken Funo

Shortcuts to adiabaticity (STA) is a method to reproduce the target state obtained from quantum adiabatic dynamics without the requirement of slow driving, and hence has applications in state preparation, quantum computing, and quantum thermodynamics. Motivated by the theoretical progress of STA, experimental implementations of the STA protocol have been studied, aiming to control the system quickly enough such that the environmental effects are suppressed. However, STA was originally designed for isolated systems, and the performance of the STA is expected to degrade under environmental effects. In this talk, we study the performance of the STA via counter-diabatic driving when the system interacts with non-Markovian environments. In particular, we consider a spin-boson model (and its generalized version) and study the fidelity between the final state of the system and the target adiabatic state. We first show that when the system-bath coupling can be externally controlled, a unit fidelity protocol can be obtained, showing that realizing STA in open systems is possible. Next, we consider a more realistic situation such that the system-bath coupling is fixed and only the system degrees of freedom can be externally controlled. In this case, we derive an analytical lower bound on the fidelity, characterizing the maximum error of the STA induced by environmental effects. In addition, we utilize the obtained bound and show how the external control on the system can be optimized to increase the fidelity.

K. Funo, N. Lambert and F. Nori, PRL 127, 150401 (2021).

A healthier semi-classical dynamics

Isaac Layton

We study the back-reaction of quantum systems onto classical ones. Taking the starting point that the system should be described at all times by a point in classical phase space and a quantum state in Hilbert space, we consider an unravelling approach, describing the system in terms of a classical-quantum trajectory. We derive the general form of the dynamics under the assumptions that the classical trajectories are continuous and the evolution is autonomous, and the requirement that the dynamics is linear and completely positive in the combined classical-quantum state. This requirement is necessary in order to consistently describe probabilities, and forces the dynamics to be stochastic when the back-reaction is non-zero. The resulting equations of motion take the form of a pair of stochastic differential equations, with the quantum side resembling quantum state diffusion, and the classical side a stochastic generalisation of the standard approach where the back-reaction is determined by an expectation value of the quantum state. The dynamics are able to consistently generate correlations between the classical and quantum systems, resolving issues associated with other semi-classical approaches. In addition, despite a breakdown of predictability in the classical degrees of freedom, the quantum state evolves deterministically conditioned on the classical trajectory, provided a trade-off between decoherence and diffusion is saturated. As a result, the quantum state remains pure when conditioned on the classical trajectory.

The above is based on upcoming joint work with Jonathan Oppenheim (UCL) and Zachary Weller-Davies (Perimeter Institute)

Quantum thermometry with adaptive Bayesian strategies: a case study for release-recapture experiments

Jesús Rubio

Precise temperature measurements on systems of few ultracold atoms is of paramount importance in quantum technologies. These can however be very resource-intensive. This problem has led to an in-depth revision of the foundations of quantum thermometry, culminating in a Bayesian reformulation of fundamental precision limits in temperature estimation. In this talk, an adaptive Bayesian framework that substantially boosts the performance of cold atom temperature measurements is put forward. First, we shall see how the data from release–recapture thermometry experiments are to be optimally processed. Such experiments are performed on few potassium atoms which have been cooled down to the microkelvin range in an optical tweezer. Using simulations, it will then be demonstrated that adaptively choosing the release–recapture times as to maximise information gain substantially reduces the number of measurements needed for the estimate to converge to a final reading. Unlike conventional methods, this proposal systematically avoids capturing and processing uninformative measurements. Furthermore, the new approach is able to produce more reliable estimates, especially when the measured data are scarce and noisy. The new method could be adapted as to enhance the precision and resource-efficiency of many other techniques running on different experimental setups, thus opening new avenues in quantum thermometry.

J. Glatthard, J. Rubio, R. Sawant, T. Hewitt, G. Barontini, and L. A. Correa, Optimal cold atom thermometry using adaptive Bayesian strategies, arXiv:2204.11816 (2022)
J. Rubio, J. Anders and L. A. Correa, Global Quantum Thermometry, Phys. Rev. Lett., 127:190402 (2021)
J. Rubio, Quantum scale estimation, arXiv:2111.11921 (2021)

Taking the temperature of a pure quantum state

Mark Mitchison

Temperature is a deceptively simple concept that still raises deep questions at the forefront of quantum physics research. The observation of thermalisation in completely isolated quantum systems, such as cold-atom quantum simulators, implies that a temperature can be assigned even to individual, pure quantum states. Here, we propose a scheme to measure the temperature of such pure states through quantum interference. Our proposal involves interferometry of an auxiliary qubit probe, which is prepared in a superposition state and subsequently decoheres due to weak coupling with a closed, thermalised many-body system. Using only a few basic assumptions about chaotic quantum systems – namely, the eigenstate thermalisation hypothesis and the emergence of hydrodynamics at long times – we show that the qubit undergoes pure exponential decoherence at a rate that depends on the temperature of its surroundings. We verify our predictions by numerical experiments on a quantum spin chain that thermalises after absorbing energy from a periodic drive. Our work provides a general method to measure the temperature of isolated, strongly interacting systems under minimal assumptions.

M. T. Mitchison et al., Phys. Rev. A 105, L030201 (2022)

Finite-time quantum Landauer principle and quantum coherence

Keiji Saito

The Landauer principle states that any logically irreversible information processing must be accompanied by dissipation into the environment. In this study, we investigate the heat dissipation associated with finite-time information erasure and the effect of quantum coherence in such processes. By considering a scenario wherein information is encoded in an open quantum system whose dynamics are described by the Markovian Lindblad equation, we show that the dissipated heat is lower-bounded by the conventional Landauer cost, as well as a correction term inversely proportional to the operational time. To clarify the relation between quantum coherence and dissipation, we derive a lower bound for heat dissipation in terms of quantum coherence. This bound quantitatively implies that the creation of quantum coherence in the energy eigenbasis during the erasure process inevitably leads to additional heat costs. The obtained bounds hold for arbitrary operational time and control protocol. We also discuss finite-time Landauer principle in the low-temperature regime using the speed limit.

Three-Dimensional Quantum Spin Dynamics and Mean Force Gibbs States

Charlotte Hogg

Understanding the quantum dynamics of spin systems is increasingly important in light of the current efforts to scale down magnetic data storage devices [1]. Recent work [2] has gone beyond the phenomenological LLG equation used in most classical simulations and developed a fully-quantum, three-dimensional spin dynamics equation that can also include memory effects. In contrast to the semiclassical simulation in [2], here we simulate the system fully quantum-mechanically by employing an open systems 'reaction coordinate' approach [3]. In contrast to most models that use this technique, we extend it to three dimensions, motivated by the application to real systems considered by the magnetism community. We aim to examine the impact of coupling dimensionality on the spin's dynamical properties, as well as its steady state [4].

[1] I. Radu et al. Ultrafast and Distinct Spin Dynamics in Magnetic Alloys. SPIN,05(03):1550004, September 2015.
[2] J. Anders et al. Quantum Brownian motion for magnets. New Journal of Physics, 24(3):033020, March 2022.
[3] Jake Iles-Smith et al. Environmental dynamics, correlations, and the emergence of noncanonical equilibrium states in open quantum systems. Physical Review A, 90(3):032114, September 2014
[4] F. Cerisola et al. Quantum-classical correspondence in spin-boson equilibrium states at arbitrary coupling (arXiv preprint). 2022.

Concentration inequalities for output statistics of quantum Markov processes

Federico Girotti

We present new concentration bounds for time averages of measurement outcomes in quantum Markov processes. They generalize well-known bounds for classical Markov chains which provide constraints on finite time fluctuations of time-additive quantities around their averages. More precisely, we derived a Bernstein-type and a Hoeffding-type concentration bounds for time averages of the measurement outcomes of a quantum

Markov chain and we generalized the Bernstein-type bound to counting processes of continuous time quantum Markov processes. If specialized to the classical setting, the bounds provide new concentration inequalities for empirical fluxes of classical Markov chains. If time allows, we present extensions and suggest potential applications of our results. The talk is based on joint work with Madalin Guta and Juan P. Garrahan.

Generalised hydrodynamics of particle creation and decay

Cecilia De Fazio

In this poster we study the out-of-equilibrium dynamics of an integrable quantum field theory possessing an unstable excitation in its spectrum. In the standard scattering picture, unstable particles result from complex poles of the two-particle scattering matrix located in the unphysical sheet of rapidity space. Because of their finite life-time, they are not part of the asymptotic spectrum, and their presence is only felt through the effect they have on physical quantities associated to the stable constituent particles (i.e. energy/particle densities). Those quantities can be computed by employing the Generalised Hydrodynamic approach. We will see that, for an initial state given by a spacial Gaussian profile of temperatures peaked at the origin, time evolution gives rise to particle and spectral particle densities that exhibit hallmarks of the creation and decay of unstable particles. These signatures of instability can be used to provide a better understanding of the dynamics of the unstable excitations that goes beyond the pole structure of the scattering matrix.

[1] O. A. Castro-Alvaredo, C. De Fazio, B. Doyon, and A. A. Ziolkowska, Generalised Hydrodynamics of Particle Creation and Decay, JHEP 2022(35) (2022), arXiv:2112.05462.
[2] O. A. Castro-Alvaredo, C. De Fazio, B. Doyon, and A. A. Ziolkowska, Tails of Instability and Decay: a Hydrodynamic Perspective, SciPost Phys. 12 (115) (2022) arXiv:2103.03735

Towards quantum engines in ultracold mixtures

Thomas Hewitt

Our experiment is devoted to the study of quantum thermodynamics and aims to realise a single atom quantum heat engine. We take advantage of low-field interspecies Feshbach resonances to control the interactions between an ultracold atomic bath of rubidium-87 and a single potassium-41 atom which is trapped within a species-selective optical tweezer. Engine cycles, including the Carnot, Otto and Diesel engine can then be achieved by the implementation of basic quantum thermodynamic transformations using these tunable interactions.

New J. Phys. 21, 063019 (2019); Phys. Rev. Lett. 125, 020403 (2020); arXiv:2204.11816v2

Growth of entanglement of generic states under dual-unitary dynamics

Alessandro Foligno

Dual-unitary circuits are a class of locally-interacting quantum many-body systems displaying unitary dynamics also when the roles of space and time are exchanged. These systems have recently emerged as a remarkable framework where certain features of many-body quantum chaos can be studied exactly. In particular, they admit a class of ”solvable” initial states for which, in the thermodynamic limit, one can access the full non-equilibrium dynamics. This reveals a surprising property: when a dual-unitary circuit is prepared in a solvable state the quantum entanglement between two complementary spatial regions grows at the maximal speed allowed by the local structure of the evolution.

Here we investigate the fate of this property when the system is prepared in a generic pair-product state. We show that in this case the entanglement increment during a time step is sub-maximal for finite times, however, it approaches the maximal value in the infinite-time limit. This statement is proven rigorously for dual-unitary circuits generating high enough entanglement, while it is argued to hold for the entire class.

Nonequilibrium quantum Rydberg ion engine

Wilson Martins

Given the growing miniaturisation of technologies, microscopic devices are key tools for applications in science and beyond. Using trapped Rydberg ions, a quantum simulation platform in which internal and external degrees of freedom can be controlled precisely, we investigate engines operating in the quantum regime. In these devices, as in their classical counterparts, the main goal is to generate usable power from a protocol based on heat exchange. We study a quantum engine where work is stored in the relative motion of two trapped laser-driven Rydberg Ions. This becomes possible since the electronic and oscillatory motion are coupled. Rather than an equilibrium setting, we investigate a non-equilibrium situation, which is typically encountered experimentally when laser driving is competing with strong interactions and dissipation. We investigate whether one can realise a non-thermal energy source, which delivers mechanical work to an external load.

Martins, W.S.; Soares-Pinto, D.O. Suppressing information storage in a structured thermal bath: Objectivity and non-Markovianity. arXiv 2021, arXiv:2110.03490.

Nonequilibrium time-crystal quantum engine

Paulo José Paulino de Souza

A continuous time-crystal is a nonequilibrium phase of Markovian open quantum many-body systems, in which the time-translation symmetry of the (time-independent) dynamical generator is broken and the system state spontaneously approaches an asymptotic limit cycle. In a recent Letter [Carollo et al. Phys. Rev. Lett. 125, 240602 (2020)], it was theoretically demonstrated that this phase can be exploited to realize a new type of nonequilibrium many-body quantum engine, based on a driven-dissipative optomechanical cavity-atom setup, able to deliver power to a load (the mirror of the cavity) without the need of any periodic driving protocol. Here, we extend the analysis of such a nonequilibrium time-crystal engine and consider several dynamical regimes which were not previously addressed. We show that, in certain regimes, the nonequilibrium engine can generate power with a nonzero efficiency and we further discuss the effects of the motion of the mirror on the time-evolution of the engine working fluid.

[1] Hu, C. K., Qiu, J., Souza, P. J., Yuan, J., Zhou, Y., Zhang, L., ... Yu, D. (2022). Optimal charging of a superconducting quantum battery. Quantum Science and Technology.
[2] Souza, P. J., Mendonça, T. M., de Oliveira, E. V., Villas-Boas, C. J. (2021). Adiabatic Quantum Computation: From Adiabatic Theorem to D-Wave Computer. Revista Brasileira de Ensino de Física, 43.
[3] Souza, P. J., Comin, C. H., Costa, L. D. F. (2018). Topology and dynamics in complex networks: The role of edge reciprocity. EPL (Europhysics Letters), 122(2), 26001.
[4] Máximo, C. E., de Souza, P. P., Ianzano, C., Rempe, G., Bachelard, R., Villas-Boas, C. J. (2021). Bright and dark states of light: The quantum origin of classical interference. arXiv preprint arXiv:2112.05512.

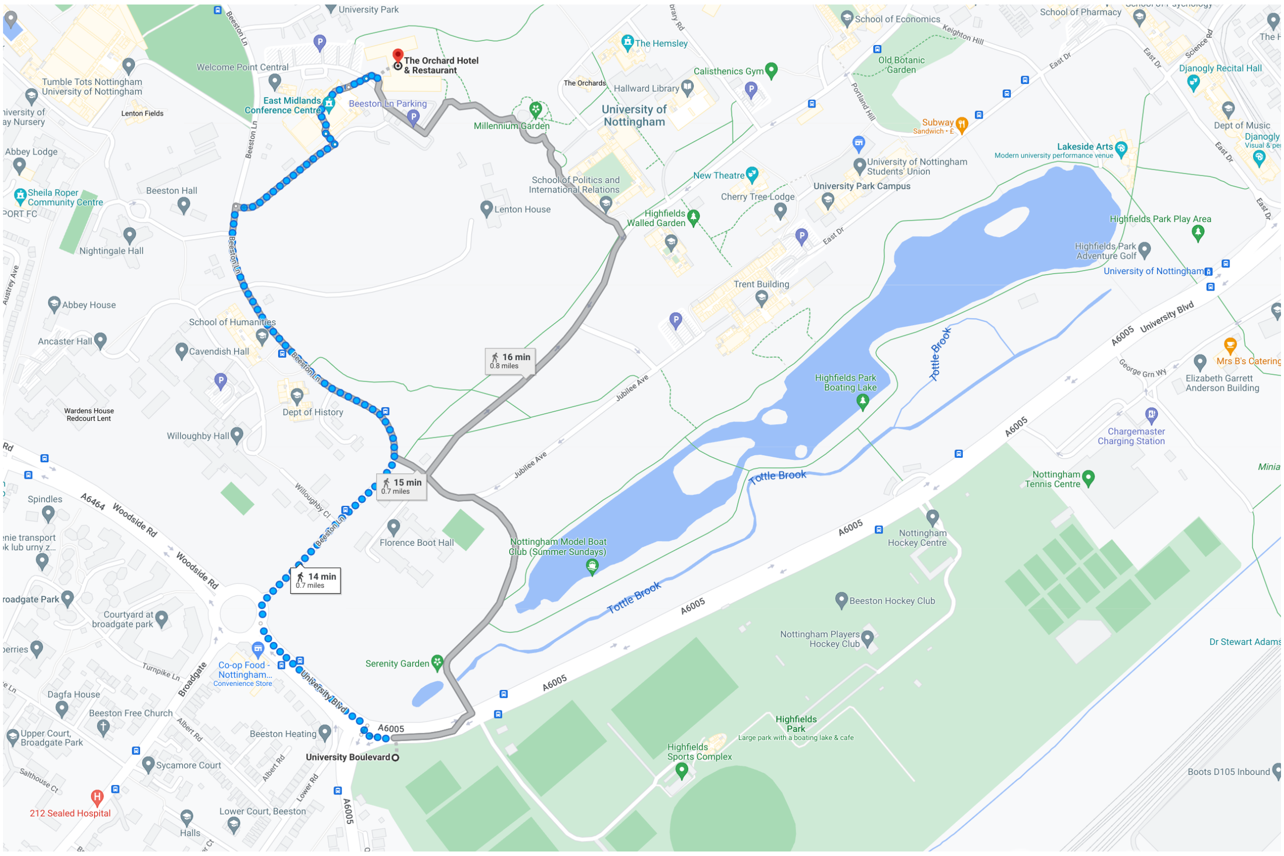
Reversible cellular automata: exactly solvable models of nonequilibrium statistical mechanics

Joseph Wilkinson

We study the dynamical, statistical, and spectral properties of reversible cellular automata. These are one-dimensional dynamical systems defined on discrete spacetime lattices over finite scalar fields. The microscopic time evolution governing the macroscopic properties of these models is defined locally in terms of a deterministic and reversible unitary operator which is applied periodically throughout space and time. The local dynamical rules act as kinetic constraints and thus the reversible cellular automata can be identified as kinetically constrained models, specifically, Fredrickson-Anderson spin chain models. We demonstrate that these models are integrable and exhibit physically intuitive interpretations in terms of locally interacting ballistically propagating quasiparticles. Using the mathematical methods and theoretical techniques developed for such interacting integrable systems, we solve these models and obtain many exact analytic results, including the nonequilibrium stationary states, large deviations statistics, thermodynamics and hydrodynamics, and full spectral statistics.

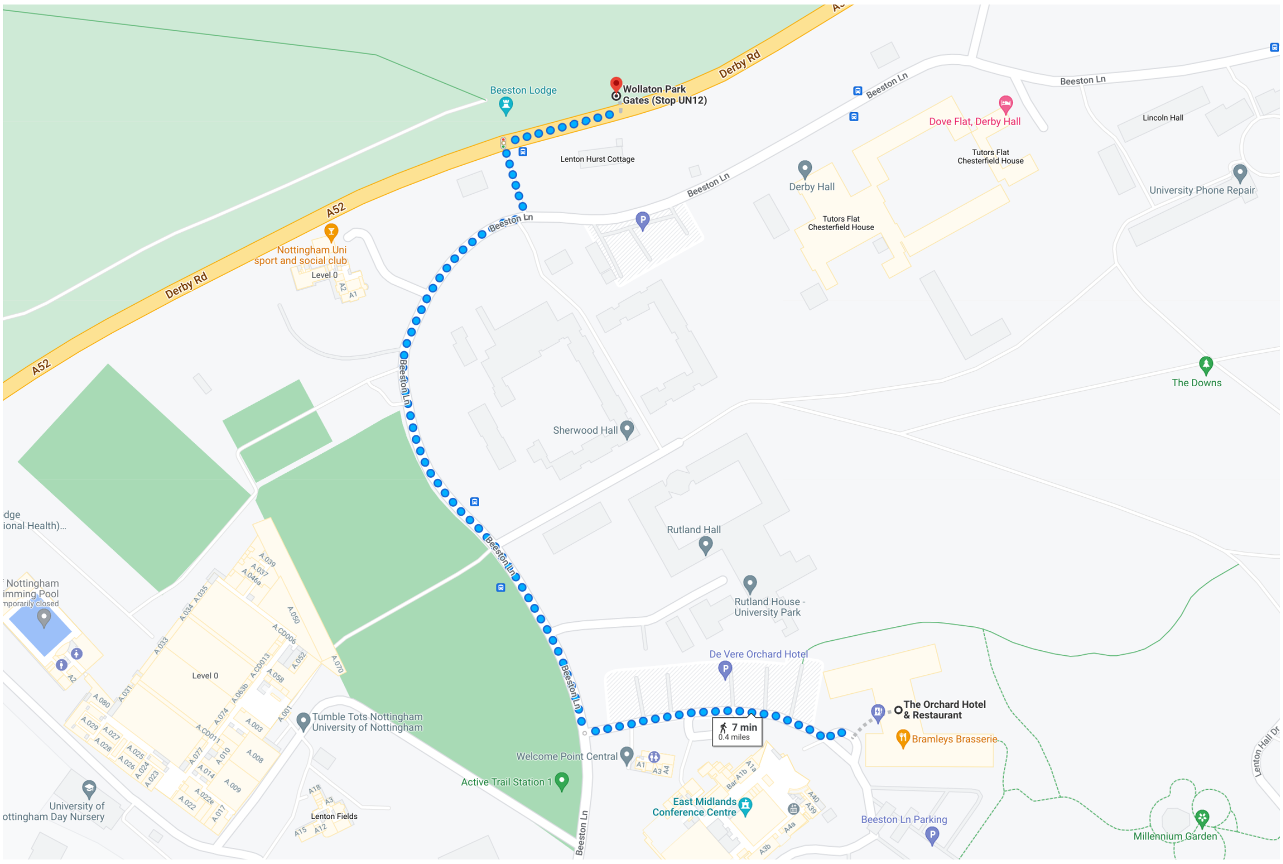
Travel Information

Tram



The easiest way to commute between the city center of Nottingham and *The Orchard Hotel*, where the Symposium will take place, is the tram. From *Nottingham Station*, take the tram for *Toton Lane* and get off at *University Boulevard*. From there, you can walk to the hotel in about 15 minutes, please see the map above. To go to the city center, take the tram for *Hucknall* from *University Boulevard* and get off at *Lace Market* or *Old Market Square*, which is the closest stop for the dinner on Monday. The trams run approx every 10 minutes. You may check the exact times on *Google Maps*. Tickets can be bought with the app *NETGO!* or from vending machines at the stops; credit cards, *Google Pay* and *Apple Pay* are all accepted. The single ticket is GBP 2.80, the day ticket is GBP 4.70.

Bus



You may also commute to the city center via bus. The closest stop to the hotel is *Wollaton Park Gates*, where you can walk in about 10 minutes, please see the map above. Buses run quite frequently, usually every 15 minutes, and there are many stops in the city center, for instance *Victoria Centre* or *Friar Lane*, which is the closest stop for the dinner. Please check the exact times on *Google Maps* or on the app *Nottingham City Transport (NCTX Buses)*. Tickets can be bought via the app or from the bus driver; credit cards, *Google Pay* and *Apple Pay* are all accepted. The single ticket is GBP 2.50 the day ticket is GBP 4.70.

Taxi and Uber

There are usually taxis waiting at *Nottingham Station*, which can take you directly to the hotel. Otherwise *Uber* is a very convenient way to travel. An *Uber* or taxi from the station to the hotel should cost roughly GBP 10-15.